Systematic Risk and the Price Structure of Individual Equity Options

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This study demonstrates the impact of systematic risk on the prices of individual equity options. The option prices are characterized by the level and slope of implied volatility curves, and the systematic risk is measured as the proportion of systematic variance in the total variance. Using daily option quotes on the S&P 100 index and its 30 largest component stocks, we show that after controlling for the underlying asset's total risk, a higher amount of systematic risk leads to a higher level of implied volatility and a steeper slope of the implied volatility curve. Thus, systematic risk proportion can help differentiate the price structure across individual equity options. (*JEL*: G10, G13)

Empirical work in the derivatives literature has uncovered some intriguing features of option prices: (i) the Black-Scholes implied volatility is higher than the historical or realized volatility and (ii) the risk-neutral negative skewness is more pronounced than that in the physical distribution, and the index options have a more pronounced volatility smile/smirk than individual equity options (e.g., Jackwerth, 2000; Dennis and Mayhew, 2002; and Bakshi, Kapadia, and Madan, 2003). Collectively, these features indicate structural differences between the risk-neutral and physical return distributions.

Our study is motivated by this observed structural difference. In particular, since the two distributions are linked through the risk premiums of the systematic risk factors, the structural difference must be related to the systematic risk of the underlying asset. Most of the existing studies focus only on index

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options, and therefore have little to offer in terms of the relative contribution of systematic risk in option pricing. Our study fills this gap by investigating how the price structure of individual equity options is affected by the amount of systematic risk in the underlying assets. Using the implied volatility to represent the option price structures, our empirical results demonstrate a clear link between option prices and the systematic risk. Specifically, after controlling for the overall level of total risk, a higher amount of systematic risk leads to a higher level of implied volatility and a steeper implied volatility curve.

The empirical study uses daily option quotes on the S&P 100 index and its 30 largest component stocks from 1 January 1991 to 31 December 1995, a data set identical to that in the study by Bakshi, Kapadia, and Madan (2003) (BKM hereafter). The systematic risk is measured by the systematic risk proportion, defined as the ratio of the systematic variance over the total variance. We test two null hypotheses: (i) the level of implied volatility is unrelated to the systematic risk proportion and (ii) the slope of the implied volatility curve is unrelated to the systematic risk proportion. Both hypotheses are strongly rejected, indicating that the systematic risk plays an important role in determining option prices. Our empirical findings are robust in subsamples and to different specifications and estimations. The empirical results can potentially be reconciled with several theories in which the premiums of the systematic risk factors drive a wedge between the risk-neutral and physical return distributions.

As mentioned, most of the existing studies focus on index options when attempting to explain the structural difference in distributions. For instance, Bates (2000); Buraschi and Jackwerth (2001); Bakshi and Kapadia (2003); and Jones (2006) documented the existence of additional risk factors, such as stochastic volatility or jump fears, in index option prices. Dennis and Mayhew (2002); and BKM (2003) are two exceptions.

Dennis and Mayhew (2002) investigated the relative importance of various firm characteristics (e.g., implied volatility, firm size, trading volume, leverage, and beta) in explaining the risk-neutral skewness implied from option prices. Among other things, they empirically established a link between the risk-neutral skewness and the systematic risk of the underlying stock. Specifically, the risk-neutral skewness tends to be more negative for stocks with larger betas, indicating the importance of market risk in option pricing. Their focus was on uncovering the driving factors for the risk-neutral skewness observed in option prices. BKM (2003) developed a theoretical relationship between the implied volatility and the risk-neutral skewness and kurtosis, and empirically demonstrated that the differential pricing of individual stock options and index options is indeed related to their differences in the risk-neutral skewness and kurtosis. Our study goes further by demonstrating that the price structure of individual equity options depends on the proportion of systematic risk in the total risk. We show that the systematic risk is the driver for the behavior of the implied volatility, the risk-neutral skewness and kurtosis.

Instead of relying on more general distributions or introducing more risk factors, Bollen and Whaley (2004) appealed to the demand-based arguments in resolving the different structures in option prices. Motivating their arguments based on limits to arbitrage (Shleifer and Vishny, 1997; and Liu and Longstaff, 2004), they showed that the extent of imbalance in demand and supply could determine the level and the slope of the implied volatility curve. Thus, the differential price structures among individual equity options is attributed to the different extents of imbalance. As we will argue later, their model does not answer why net buying or selling pressure exists. Therefore, the framework is more effective in explaining the steeper slope of the index options' implied volatility curve (due to net excess demand of out-of-the-money put options for insurance purposes), but less so in differentiating the price structures of individual equity options. In contrast, our systematic risk-based explanation is clear-cut and applies to all options.

The remainder of this paper is organized as follows. Section 1 presents the hypotheses and testing procedures, and reports the main results. Various robustness checks are documented in Section 2. Section 3 explores potential theoretical reconciliations of the empirical findings. Section 4 concludes the paper.

1. Empirical Relation Between Systematic Risk and the Structure of Option Prices

According to the Black-Scholes (1973) option pricing theory, option prices do not depend on how much systematic risk is contained in the underlying asset as long as its total risk is fixed. When the option prices are converted into implied volatilities, they should not be related to the proportion of systematic risk relative to the total risk. Therefore, we have the following two null hypotheses.

- **Hypothesis 1:** The implied volatility level of the options is unrelated to the systematic risk proportion of the underlying asset.
- **Hypothesis 2:** The slope of the implied volatility smile/smirk curve of the options is unrelated to the systematic risk proportion of the underlying asset.

As mentioned earlier, many empirical studies (e.g., Bates, 2000; Buraschi and Jackwerth, 2001; Bakshi and Kapadia, 2003; and Jones, 2006) indicate the existence of systematic risk factors (such as jumps and volatility risk) in option prices. These systematic risk factors become part of the pricing kernel, and how much they account for the total risk will obviously impact the characteristics of the risk-neutral distribution. Therefore, our alternative hypotheses are: (i) both the level and the slope of the implied volatility curve will depend on the systematic risk and (ii) the amount of systematic risk will differentiate the price structures of individual equity options.

The most natural definition for the systematic risk proportion, denoted as b_j, is the ratio of the systematic variance over the total variance. The logical variables representing the option price structures are the level and slope of the implied volatility. As in BKM (2003) and other studies, working with implied volatilities facilitates comparisons across option strikes and maturities for the same stock as well as comparisons across options on different stocks.

We will show as part of the preliminary tests why the systematic risk proportion is the appropriate metric to use. Here, we offer one theoretical motivation for the two hypotheses, with an emphasis on the importance of b_j. The key building blocks are from BKM (2003), and our specific arguments are provided in Appendix A. Assuming a one-factor model with a normally distributed idiosyncratic error term under the risk-neutral measure, the following relation for a given maturity is obtained:

$$\begin{aligned}\sigma_j^{\text{imp}}(K/S) \;\approx\; & q_1(K/S) + 3q_3(K/S) + q_2(K/S)\text{Skew}_m^{(rn)} b_j^{3/2} \\ & + q_3(K/S)(\text{Kurt}_m^{(rn)} - 3)b_j^2, \qquad (1)\end{aligned}$$

where σ_j^{imp} is the implied volatility, $\text{Skew}_m^{(rn)}$ and $\text{Kurt}_m^{(rn)}$ are the risk-neutral skewness and kurtosis for the market, and q_1, q_2, and q_3 are coefficients written explicitly as functions of the option's moneyness, K/S.

Clearly, the implied volatility is related to the systematic risk proportion of the underlying asset, and the degree to which it is related depends on the moneyness of the option. As long as the market return is skewed and/or leptokurtic, the level and slope of the implied volatility curve will be related to the systematic risk proportion. More significantly, the measure of the systematic risk for this purpose is not the absolute amount or beta; rather, it is the relative proportion.

Several empirical issues need to be sorted out before we proceed to the tests. To begin, how do we estimate the average volatility, or the overall level of total risk? Since we use the Black-Scholes implied volatility to characterize the option price structure, it is natural to use some version of the historical volatility to proxy the future average volatility. The key issue is how far back we should go in the estimation. Balancing between estimation efficiency from a larger sample and the relatively shorter options maturities, we opt for a one-year (250 days) rolling window. Specifically, we run daily, one-year rolling window OLS regressions in (A1) for stock j and estimate the systematic risk proportion as $\frac{\beta_j^2\sigma_m^2}{\sigma_j^2}$. If we need a measure of systematic risk proportion for a period of, say, four weeks, the daily estimates are averaged. In our study, we first average the daily variances over the period, and then calculate b_j.

Another issue is the empirical characterization of the implied volatility curve. BKM (2003) assumed a constant slope on the logarithmic scale for the curve. While this strategy simplifies the testing procedures, it tends to cloud the intricate features of the curve. To reveal potentially different features for different moneyness regions as apparent in Equation (1), we piecewise linearize the implied volatility curve into four distinct moneyness buckets, i.e., $K/S = [0.9, 0.95)$, $[0.95, 1.0)$, $[1.0, 1.05)$ and $[1.05, 1.10]$, and conduct tests within each bucket.

To test our hypotheses, we follow BKM (2003) and perform the Fama-MacBeth (1973) type two-pass regressions. We need to obtain time series of estimates for the level and slope of the implied volatility curve, which are used to run the cross-sectional regressions to determine whether they are related to the systematic risk proportion. The cross-sectional regression is repeated for nonoverlapping periods and the average regression coefficients are used to test the hypotheses.

While a weekly, nonoverlapping window in the first-pass regressions provides a sufficient number of observations in the study by BKM (2003), we must increase the window length because the option data have been further divided into four moneyness buckets. Thus, we adopt a window of one month (four weeks), and the second-pass regression is performed on a monthly basis.[1] The calculation of the risk-neutral skewness and kurtosis is based on the results in BKM (2003), and the procedure is briefly outlined in Appendix B.

With this in mind, we proceed with hypothesis testing. In the first-pass regression, for each stock and moneyness bucket, we lump all the observations in a four-week period and repeat the following regression for the jth stock:

$$\sigma_{jk}^{\text{imp}} - \sigma_j^{\text{his}} = a_{0j} + a_{1j}(y_{jk} - \bar{y}_j) + \varepsilon_{jk}, \qquad k = 1, 2, \ldots, I_j \tag{2}$$

for 65 times (260 weeks divided by 4). In the above expression, I_j is the number of options in a particular moneyness bucket for the jth stock, $y_{jk} = K_{jk}/S_{jk}$, and $\bar{y}_j$ is the sample average of y_{jk}. The intercept α_{0j} and regression coefficient a_{1j} are measures of the level and the slope of the implied volatility, after adjusting for the jth stock's total risk, σ_j^{his}.[2]

In the second pass, we perform three versions of cross-sectional regressions using the intercept from the first-pass regressions as the dependent variable: for $j = 1, 2, \ldots, 31$,

$$a_{0j} = \gamma_0 + \gamma_1 b_j + e_j, \tag{3}$$

$$a_{0j} = \gamma_0 + \gamma_2 \text{Skew}_j^{(rn)} + \gamma_3 \text{Kurt}_j^{(rn)} + e_j, \tag{4}$$

$$a_{0j} = \gamma_0 + \gamma_1 b_j + \gamma_2 \text{Skew}_j^{(rn)} + \gamma_3 \text{Kurt}_j^{(rn)} + e_j. \tag{5}$$

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1 Even with a window of four weeks, some stocks in certain buckets still have too few observations for the time-series regressions. To ensure that regressions are based on a reasonable sample size, we have set a minimum of 10 observations in both the time-series and the cross-section regressions. The screening criterion for cross-sectional regressions is not binding most of the time.

2 Historical volatility for the jth stock is actually day specific. The time subscript is omitted to simplify notation. We subtract the historical volatility from the implied volatility in order to control for the difference in total risk across stocks. Moreover, the moneyness variable y_{jk} is adjusted by its mean to ensure that the intercept α_{0j} is the average difference between the implied and the historical volatilities.

The time series of the regression coefficients, 65 in total, are then averaged and its corresponding t-statistic is calculated using the Newey-West standard error with three lags ($65^{1/4} \approx 3$). Regression (3) is an unconditional test of Hypothesis 1, which should not be rejected if $\gamma_1 = 0$. Regression (5) is a conditional test of Hypothesis 1, controlling for the effects of the risk-neutral skewness and kurtosis, and we should obtain $\gamma_1 = 0$ if the systematic risk proportion exerts no effect once the influence of risk-neutral skewness and kurtosis is considered. Regression (4) is performed purely for comparison purposes.

To test Hypothesis 2, we simply repeat the regressions in (3), (4), and (5) by using the slope a_{1j} from the first-pass regression as the dependent variable.

To ensure that our results are not due to the two-pass testing procedure, we will also perform a one-pass panel regression to test the two hypotheses. Moreover, we will control for some firm-specific characteristics.

1.1 Data summary and preliminary investigations

The option data used in this study are identical to those in BKM (2003), covering options written on the S&P 100 index and its 30 largest component stocks. Please see Appendix C for details.

Tables 1A and 1B report summary statistics. Table 1A reports the average implied volatility for each maturity-moneyness group. It also reports the average historical volatility and the average proportion of systematic risk for each stock. Several observations are in order. First, the volatility smile/smirk is clearly present for all stocks. The curve is mostly downward sloping. Second, within the same moneyness bucket, the implied volatility is generally lower for longer term options. Third, the average implied volatility is generally higher than the average historical volatility (for 19 out of 30 stocks), and the S&P 100 index has the highest volatility differential, which is 0.0327. Finally, excluding the S&P 100 index, the systematic risk proportions range from 0.089 for MCI Communications to 0.380 for General Electric. The average proportion across all stocks excluding the S&P 100 index is 0.235.

To see the general association between the stocks' key characteristics and the systematic risk proportion, we sort the stocks into quintiles by their systematic risk proportions and calculate the average value of the characteristic variables for each quintile. The variables we examine are the ones used for later tests, namely: (i) the average implied volatility minus the average historical volatility; (ii) the average slope of the implied volatility curve; (iii) the average risk-neutral skewness; and (iv) the average risk-neutral kurtosis. Since the last two quantities do not change across moneyness, we divide the sample only into maturity buckets. Given the magnitude of the S&P 100 index's systematic risk proportion, we put it in a separate group, the fifth quintile. The first quintile contains six stocks and the other three contain eight stocks each. The sorting is done monthly, and the average values are calculated for each quintile. We then average the monthly quantities for each quintile over 65 months. Table 1B

Table 1A
Summary statistics—implied volatility, historical volatility, and systematic risk proportion

			Short-term options: 20–70 days in maturity					Medium-term options: 71–120 days in maturity					Long-term Options: 121–180 days in Maturity							
			Moneyness, *K/S*					Moneyness, *K/S*					Moneyness, *K/S*							
			[0.90–0.95)	[0.95–1.00)	[1.00–1.05)	[1.05–1.10]	[0.90–1.10]	[0.90–0.95)	[0.95–1.00)	[1.00–1.05)	[1.05–1.10]	[0.90–1.10]	[0.90–0.95)	[0.95–1.00)	[1.00–1.05)	[1.05–1.10]	[0.90–1.10]	Average implied volatility	Average historical volatility	Systematic risk proportion
1.	AIG	American Int'l	0.2371	0.2281	0.2125	0.2146	0.2231	0.2282	0.2277	0.2126	0.2125	0.2207	0.2253	0.2268	0.2109	0.2099	0.2187	0.2214	0.2093	0.275
2.	AIT	Ameritech	0.2226	0.2056	0.1710	0.1806	0.1941	0.2189	0.2176	0.1684	0.1664	0.1928	0.2233	0.2273	0.1602	0.1583	0.1923	0.1933	0.1824	0.229
3.	AN	Amoco	0.2197	0.1927	0.1717	0.1910	0.1920	0.2003	0.1978	0.1676	0.1715	0.1842	0.2020	0.2028	0.1660	0.1662	0.1841	0.1879	0.1922	0.127
4.	AXP	American Express	0.3140	0.2935	0.2868	0.3009	0.2986	0.3060	0.2962	0.2979	0.3064	0.3010	0.3047	0.2948	0.2898	0.2959	0.2966	0.2986	0.2995	0.207
5.	BA	Boeing Company	0.2734	0.2539	0.2372	0.2481	0.2528	0.2563	0.2537	0.2316	0.2343	0.2434	0.2528	0.2498	0.2302	0.2292	0.2401	0.2473	0.2408	0.165
6.	BAC	Bank America Corp.	0.3078	0.2924	0.2664	0.2662	0.2838	0.2977	0.2989	0.2632	0.2588	0.2792	0.2929	0.2877	0.2564	0.2515	0.2723	0.2800	0.2700	0.257
7.	BEL	Bell Atlantic	0.2324	0.2084	0.1794	0.1978	0.2038	0.2219	0.2160	0.1816	0.1788	0.1995	0.2227	0.2227	0.1796	0.1723	0.1995	0.2017	0.2076	0.214
8.	BMY	Bristol-Myers	0.2304	0.2143	0.1884	0.2039	0.2088	0.2157	0.2110	0.1801	0.1849	0.1979	0.2147	0.2170	0.1783	0.1800	0.1970	0.2031	0.2003	0.290
9.	CCI	Citicorp	0.3403	0.3156	0.3058	0.3045	0.3168	0.3326	0.3241	0.3105	0.3033	0.3177	0.3279	0.3123	0.3006	0.2982	0.3101	0.3153	0.3357	0.208
10.	DD	DuPont	0.2512	0.2430	0.2188	0.2254	0.2347	0.2438	0.2451	0.2134	0.2151	0.2298	0.2433	0.2429	0.2117	0.2112	0.2273	0.2317	0.2211	0.261
11.	DIS	Walt Disney	0.2975	0.2820	0.2608	0.2603	0.2751	0.2921	0.2835	0.2629	0.2588	0.2743	0.2827	0.2807	0.2568	0.2547	0.2689	0.2733	0.2540	0.268
12.	F	Ford Motor	0.3200	0.3014	0.2807	0.2867	0.2974	0.3118	0.2974	0.2763	0.2752	0.2906	0.3089	0.3040	0.2723	0.2718	0.2895	0.2936	0.2928	0.237
13.	GE	General Electric	0.2402	0.2141	0.1849	0.1899	0.2073	0.2257	0.2187	0.1809	0.1788	0.2012	0.2253	0.2216	0.1789	0.1736	0.2002	0.2040	0.1862	0.380
14.	GM	General Motors	0.3125	0.2918	0.2880	0.2904	0.2960	0.3031	0.2846	0.2875	0.2852	0.2905	0.3008	0.2940	0.2869	0.2864	0.2921	0.2937	0.3010	0.234
15.	HWP	Hewlett-Packard	0.3323	0.3251	0.3095	0.3121	0.3199	0.3260	0.3232	0.3094	0.3094	0.3173	0.3127	0.3154	0.2935	0.2980	0.3051	0.3154	0.3230	0.212
16.	IBM	Int. Bus. Machines	0.2874	0.2675	0.2589	0.2616	0.2685	0.2787	0.2703	0.2527	0.2513	0.2630	0.2696	0.2647	0.2453	0.2452	0.2562	0.2642	0.2544	0.218
17.	JNJ	Johnson & Johnson	0.2531	0.2406	0.2243	0.2259	0.2363	0.2437	0.2425	0.2205	0.2153	0.2312	0.2416	0.2390	0.2135	0.2112	0.2274	0.2329	0.2336	0.303
18.	KO	Coca Cola Co.	0.2605	0.2382	0.2157	0.2142	0.2331	0.2403	0.2344	0.2096	0.1987	0.2216	0.2381	0.2334	0.2096	0.1951	0.2193	0.2267	0.2148	0.326
19.	MCD	McDonald's Corp.	0.2687	0.2416	0.2229	0.2287	0.2411	0.2504	0.2413	0.2236	0.2163	0.2328	0.2513	0.2448	0.2259	0.2219	0.2361	0.2378	0.2255	0.230
20.	MCQ	MCI Comm.	0.3574	0.3285	0.2983	0.3137	0.3255	0.3368	0.3253	0.2995	0.3051	0.3175	0.3311	0.3208	0.2980	0.3015	0.3134	0.3207	0.4037	0.089
21.	MMM	Minn Mining	0.2252	0.2044	0.1819	0.1883	0.1992	0.2147	0.2057	0.1761	0.1744	0.1928	0.2106	0.2057	0.1743	0.1721	0.1908	0.1956	0.1783	0.270
22.	MOB	Mobil Corp.	0.2079	0.1920	0.1675	0.1788	0.1856	0.1928	0.1933	0.1633	0.1625	0.1786	0.1969	0.1966	0.1572	0.1587	0.1773	0.1815	0.1777	0.122
23.	MRK	Merck & Co.	0.2710	0.2545	0.2392	0.2547	0.2549	0.2579	0.2552	0.2345	0.2413	0.2479	0.2530	0.2491	0.2309	0.2403	0.2439	0.2506	0.2332	0.356
24.	NT	Northern Telecom	0.3172	0.3013	0.2825	0.2902	0.2979	0.2955	0.2900	0.2766	0.2744	0.2843	0.3057	0.2937	0.2784	0.2809	0.2900	0.2930	0.2764	0.216
25.	PEP	PepsiCo Inc.	0.2732	0.2302	0.2258	0.2316	0.2404	0.2684	0.2375	0.2320	0.2194	0.2387	0.2533	0.2359	0.2118	0.2143	0.2297	0.2373	0.2438	0.272
26.	SLB	Schlumberger Ltd.	0.2567	0.2507	0.2344	0.2403	0.2451	0.2474	0.2484	0.2270	0.2241	0.2367	0.2459	0.2495	0.2227	0.2240	0.2356	0.2409	0.2506	0.118
27.	T	AT&T	0.2319	0.2035	0.1865	0.2019	0.2062	0.2185	0.2034	0.1839	0.1897	0.1990	0.2173	0.2020	0.1855	0.1836	0.1973	0.2024	0.1961	0.260
28.	WMT	Wal-Mart	0.3022	0.2819	0.2556	0.2698	0.2790	0.2818	0.2843	0.2524	0.2676	0.2716	0.2825	0.2778	0.2614	0.2556	0.2705	0.2749	0.2581	0.349
29.	XON	Exxon Corp.	0.1991	0.1710	0.1525	0.1649	0.1717	0.1831	0.1726	0.1425	0.1449	0.1613	0.1807	0.1770	0.1424	0.1377	0.1592	0.1661	0.1688	0.166
30.	XRX	Xerox Corp.	0.2715	0.2623	0.2361	0.2345	0.2512	0.2626	0.2630	0.2303	0.2263	0.2458	0.2612	0.2618	0.2203	0.2198	0.2412	0.2475	0.2333	0.180
31.	OEX	S&P 100 index	0.1846	0.1470	0.1162	0.1171	0.1444	0.1716	0.1503	0.1209	0.1136	0.1421	0.1667	0.1523	0.1256	0.1188	0.1422	0.1435	0.1108	0.952

This table reports the average implied volatilities within each moneyness bucket under a particular maturity range for options on the S&P100 index and its 30 largest component stocks. The third last column of the table contains the average implied volatility for the entire sample, while the second last column contains the average historical volatility. The last column contains the average proportion of systematic variance in the total variance.

Table 1B
Sorting of stocks' characteristics by systematic risk proportion

Quintile	Systematic risk proportion	Implied volatility minus historical volatility			
		Short term	Medium term	Long term	Overall
1	0.112	−0.009	−0.015	−0.019	−0.012
2	0.188	−0.00	−0.007	−0.008	−0.004
3	0.253	0.009	0.004	0.003	0.006
4	0.346	0.011	0.006	0.004	0.008
5	0.952	0.032	0.030	0.028	0.031

Quintile	Systematic risk Proportion	Slope of implied volatility curve			
		Short term	Medium term	Long term	Overall
1	0.112	−0.215	−0.220	−0.251	−0.224
2	0.188	−0.207	−0.221	−0.239	−0.216
3	0.253	−0.230	−0.213	−0.225	−0.225
4	0.346	−0.266	−0.250	−0.255	−0.258
5	0.952	−0.586	−0.497	−0.458	−0.550

Quintile	Systematic risk Proportion	Risk-neutral skewness			
		Short term	Medium term	Long term	Overall
1	0.112	−0.626	−0.778	−0.818	−0.806
2	0.188	−0.683	−0.800	−0.832	−0.812
3	0.253	−0.696	−0.764	−0.764	−0.757
4	0.346	−0.798	−0.857	−0.907	−0.853
5	0.952	−1.656	−1.517	−1.426	−1.588

Quintile	Systematic risk Proportion	Risk-neutral kurtosis			
		Short term	Medium term	Long term	Overall
1	0.112	4.033	3.622	3.003	3.564
2	0.188	4.181	3.757	3.140	3.750
3	0.253	4.145	3.559	2.925	3.554
4	0.346	4.428	3.651	3.095	3.749
5	0.952	6.452	4.152	3.385	5.305

This table summarizes the properties of five groups of individual stocks/index sorted by their systematic risk proportions. The properties are (i) the average implied volatility minus the average historical volatility, (ii) the average slope of the implied volatility curves, (iii) the average risk-neutral skewness, and (iv) the average risk-neutral kurtosis. The maturity ranges for short-term, medium-term, and long-term options are, respectively, 20–70 days, 71–120 days, and 121–180 days. The heading "Overall" is for all maturities combined. Given the large magnitude of the S&P 100 index's systematic risk proportion, we put in a separate group, the fifth quintile. The first quintile contains six stocks and the other three contain eight stocks each. To be consistent with the estimation procedures described in Section 2, we estimate the variables monthly. Thus, the sorting is also done monthly, and the average variables are calculated for each quintile. We then average the monthly quantities for each quintile.

contains the results. The most striking result is the positive association between the systematic risk proportion and the implied volatility differential. For the other three variables, although not entirely monotonic, we see a general positive association between the systematic risk proportion and the magnitude of the variables. Therefore, the sorting results already indicate a strong rejection of the two null hypotheses.

Before proceeding to the formal tests, we carry out two preliminary investigations. First, we perform a quick test of Hypothesis 1. Second, we demonstrate why the systematic risk proportion is a better measure than beta for our tests.

Table 2
Preliminary tests: Regressing the volatility differential on alternative measures of systematic risk

		Measure of systematic risk: systematic risk proportion		Measure of systematic risk: beta	
		With index	Without index	With index	Without index
Overall	$R^2(\%)$	25.89	32.77	1.94	1.95
	t	**3.183**	**3.694**	−0.757	−0.746
K/S: 0.90–0.95	$R^2(\%)$	35.02	33.28	3.86	4.20
	t	**3.954**	**3.746**	−1.079	−1.109
K/S: 0.95–1.00	$R^2(\%)$	20.67	26.39	6.01	6.16
	t	**2.748**	**3.169**	−1.361	−1.356
K/S: 1.00–1.05	$R^2(\%)$	19.85	30.48	0.01	0.06
	t	**2.680**	**3.504**	−0.149	−0.128
K/S: 1.05–1.10	$R^2(\%)$	15.32	27.65	0.48	0.45
	t	**2.290**	**3.271**	−0.376	−0.358
Short maturity	$R^2(\%)$	23.90	32.32	2.43	2.44
	t	**3.018**	**3.657**	−0.850	−0.838
Medium maturity	$R^2(\%)$	27.11	33.92	0.84	0.81
	t	**3.284**	**3.791**	−0.496	−0.479
Long maturity	$R^2(\%)$	27.10	31.23	2.20	2.24
	t	**3.283**	**3.566**	−0.808	−0.801

This table contains results for two univariate cross-sectional regressions under various sample constructions. In the first regression, the dependent variable is the volatility differential—the average difference between the implied volatility and the historical volatility—and the explanatory variable is the average systematic risk proportion, $\sigma_j^{\text{imp}} - \sigma_j^{\text{his}} = \gamma_0 + \gamma_1 b_j + e_j$. In the second regression, the explanatory variable is the average beta, i. e., $\sigma_j^{\text{imp}} - \sigma_j^{\text{his}} = \gamma_0 + \gamma_1 \beta_j + e_j$. The averages are taken or calculated from Table 1A. The regressions are run for the entire sample first, which corresponds to the "Overall" case. We then run the regressions for each of the four moneyness buckets. Finally, we run the regressions for each of the three maturity ranges. For each particular sample construction, we run regressions either with or without the S&P 100 index. For each pair of numbers, the first is the R^2 value, and the second is the t-value for the regression coefficient γ_1 (a negative t-value indicates that the regression coefficient is negative). The t-values in bold type are significant at the 10% level or higher for two-tailed tests. The maturity ranges for short term, medium term, and long term are, respectively, 20–70 days, 71–120 days, and 121–180 days.

To this end, we first regress the volatility differential on the average systematic risk proportion; we then do the same regression using average beta as the explanatory variable. The average volatilities and systematic risk proportions are from Table 1A. Average betas are calculated separately. OLS regressions are done for the entire sample and for various moneyness and maturity buckets. For each bucket, we run two versions of the regression: one with the S&P 100 index and one without it. The results are reported in Table 2. The R^2 and t-values show overwhelmingly that the volatility differential is positively related to the systematic risk proportion, while having no statistically significant relation to beta. This observation applies to all moneyness/maturity buckets, with or without the index. Thus, Hypothesis 1 is rejected with a high level of confidence.

It should not be surprising that beta is not a good measure of systematic risk for our purpose. Our hypotheses focus on the amount of systematic risk, *given* the amount of total risk. A higher beta does not always mean that the systematic risk accounts for most of the total risk. By the same token, equal betas do not mean equal systematic risk proportions. This point can be illustrated by a simple example. Suppose the market volatility is $\sigma_m = 0.2$ and there are two stocks, A and B, with $\sigma_A = 0.4$ and $\sigma_B = 0.5$. If the stocks' correlations with the market are $\rho_A = 0.75$ and $\rho_B = 0.60$ respectively, then the two stocks will have the same beta, 1.50, yet very different systematic risk proportions, 0.563 versus 0.360.

Finally, we perform a diagnostic check. Intuition would suggest that the systematic risk proportion may potentially be a proxy for the total volatility, especially for large stocks. To check if this is the case, we regress the total volatility (implied or historical) on the systematic risk proportion across the 30 stocks. The R^2 values are 0.49% and 6.74%, respectively, and the t-values for the regression coefficients are -0.372 and -1.422, respectively. Therefore, the total volatility and systematic risk proportion are unrelated.

1.2 Level-effect tests

We now proceed to the formal tests. Table 3 reports the test results for the level effect—i.e., tests pertaining to Hypothesis 1. To conserve space, we do not report the intercepts from the second-pass regressions. Panel A reveals a strong rejection of Hypothesis 1. The coefficient γ_1 is positive across all moneyness and maturity buckets, and all the corresponding t-values are significant. In fact, almost all of them are significant at the 1% level. The vast majority of the 65 γ_1 estimates are positive, as indicated by the percentages under $\gamma_1 > 0$. Moving to Panel B, where we control for the effects of the risk-neutral skewness and kurtosis, all the γ_1 estimates save two are still significant. Overall, the unconditional and conditional tests both show a strong level effect. The implied volatility levels, controlling for the stock-specific total volatilities, are significantly and positively related to the systematic risk proportion of the underlying stock. The empirical finding confirms the theoretical prediction derived from the relationship in Equation (1).

In terms of economic significance, the R^2 shows that the systematic risk proportion does a better job for the lower moneyness range in explaining the cross-sectional differences in the level of implied volatilities. For the univariate regressions covering all maturities, the systematic risk proportion alone explains 14.5%, 7.8%, 7.3%, and 5.4% of the cross-sectional variations in the implied volatility for the four moneyness buckets, respectively. When the risk-neutral skewness and kurtosis are added to the regressions, the corresponding numbers are 25.0%, 19.6%, 19.6%, and 17.8%.

The regression results also offer some other interesting insights. First of all, judging by the magnitude and t-value of the regression coefficient γ_1 as well as the percentage of positive entries, we see that the effect of systematic risk proportion itself also takes a smirk pattern across moneyness. The effect is much stronger for the lower moneyness buckets. As the exercise price becomes higher, the level effect becomes weaker. This is consistent with the pattern of the implied volatilities.

Second, in terms of maturities, it is clear that the effect is stronger for short-term options (20–70 days), and it becomes weaker as the maturity gets longer. This is true for both the unconditional and conditional tests.

Table 3
Regression tests for the level effect

Panel A: Separate regressions on systematic risk proportion, and skewness and kurtosis

		γ_1				γ_2		γ_3		
		Average	t	$\gamma_1 > 0$	$R^2(\%)$	Average	t	Average	t	$R^2(\%)$
Moneyness	All maturities	0.077	**19.239**	100.0%	14.5	−0.013	**−3.458**	0.000	0.336	12.7
K/S	Short term	0.074	**17.418**	100.0%	15.9	−0.015	**−3.561**	0.000	−0.021	16.1
0.90–0.95	Medium term	0.064	**12.813**	100.0%	23.1	−0.028	**−3.960**	−0.008	**−2.504**	22.2
	Long term	0.104	**4.573**	79.6%	13.1	−0.016	−1.603	−0.004	−1.050	23.2
Moneyness	All maturities	0.051	**14.240**	100.0%	7.8	−0.014	**−2.958**	−0.003	**−2.623**	11.3
K/S	Short term	0.044	**12.708**	98.5%	7.3	−0.013	**−3.685**	−0.002	**−2.128**	16.7
0.95–1.00	Medium term	0.036	**6.647**	93.6%	8.3	−0.009	−0.928	−0.003	−1.251	22.7
	Long term	0.032	**1.712**	63.9%	7.7	−0.012	**−2.986**	−0.006	**−4.757**	17.7
Moneyness	All maturities	0.047	**7.719**	98.5%	7.3	−0.002	−0.311	−0.001	−0.493	10.7
K/S	Short term	0.037	**6.257**	96.9%	5.6	0.002	0.404	−0.001	−1.340	13.1
1.00–1.05	Medium term	0.027	**5.664**	90.9%	9.0	−0.008	−1.121	−0.004	−1.635	21.3
	Long term	0.093	**4.139**	78.6%	14.9	0.007	1.046	−0.001	−0.235	22.4
Moneyness	All maturities	0.037	**6.660**	96.9%	5.4	0.002	0.552	0.001	0.708	10.9
K/S	Short term	0.024	**4.600**	78.5%	4.2	0.008	**2.181**	0.002	**1.943**	15.6
1.05–1.10	Medium term	0.023	**4.608**	84.0%	5.5	−0.004	−0.440	−0.004	−0.893	24.8
	Long term	0.051	**2.478**	71.4%	11.0	−0.001	−0.295	0.000	−0.120	17.5

Panel B: Combined regressions on systematic risk proportion, skewness and kurtosis

		γ_1			γ_2		γ_3		
		Average	t	$\gamma_1 > 0$	Average	t	Average	t	$R^2(\%)$
Moneyness	All maturities	0.074	**16.785**	100.0%	−0.009	**−2.218**	−0.001	−0.794	25.0
K/S	Short term	0.070	**12.319**	98.3%	−0.004	−1.011	−0.001	−0.655	29.6
0.90–0.95	Medium term	0.063	**6.729**	87.0%	−0.014	**−2.174**	−0.002	−0.667	38.9
	Long term	0.115	**2.920**	72.7%	−0.014	−1.327	−0.002	−0.516	34.0
Moneyness	All maturities	0.055	**8.997**	96.9%	−0.011	**−2.174**	−0.005	**−3.097**	19.6
K/S	Short term	0.041	**7.154**	89.7%	−0.008	**−2.136**	−0.003	**−2.359**	23.9
0.95–1.00	Medium term	0.026	**4.296**	75.0%	−0.005	−0.426	−0.003	−0.972	30.5
	Long term	0.029	0.696	50.0%	−0.012	**−4.033**	−0.007	**−5.371**	28.2
Moneyness	All maturities	0.052	**7.466**	93.8%	0.000	0.067	−0.002	−0.988	19.6
K/S	Short term	0.044	**6.594**	89.8%	0.006	1.332	−0.002	**−1.837**	22.6
1.00–1.05	Medium term	0.029	**4.994**	84.0%	−0.002	−0.333	−0.002	−0.694	28.5
	Long term	0.106	**2.163**	71.0%	0.008	0.922	0.000	−0.061	36.9
Moneyness	All maturities	0.044	**7.088**	95.4%	0.004	0.922	0.000	−0.231	17.8
K/S	Short term	0.025	**4.592**	75.9%	0.012	**2.814**	0.002	**2.004**	20.6
1.05–1.10	Medium term	0.033	**3.420**	84.2%	−0.001	−0.050	−0.004	−0.622	33.0
	Long term	0.052	1.679	61.1%	0.001	0.177	−0.001	−0.642	28.8

This table contains two-pass regression results for the level-effect tests. In the first pass, for each stock, we regress the difference between the implied volatility and the historical volatility on moneyness for nonoverlapping periods of one month (i.e., four weeks): $\sigma_i^{\mathrm{imp}} - \sigma_i^{\mathrm{his}} = a_0 + a_1(y_i - \overline{y}) + \varepsilon_i$. We thus obtain a monthly time series of the intercept a_0 and the slope coefficient a_1 for all stocks including the S&P100 index. The moneyness variable is adjusted by the sample mean within the month, so that the intercept a_0 is the average of the difference between the implied volatility and the historical volatility. In the second pass, we cross-sectionally regress the intercept on the systematic risk proportion, the risk-neutral skewness and kurtosis. This regression is done every month in three different forms: $a_{0j} = \gamma_0 + \gamma_1 b_j + e_j$, $a_{0j} = \gamma_0 + \gamma_2 \mathrm{Skew}_j^{(rn)} + \gamma_3 \mathrm{Kurt}_j^{(rn)} + e_j$ and $a_{0j} = \gamma_0 + \gamma_1 b_j + \gamma_2 \mathrm{Skew}_j^{(rn)} + \gamma_3 \mathrm{Kurt}_j^{(rn)} + e_j$. The monthly regression coefficients are then averaged, and the corresponding t-values calculated using the Newey-West standard error (with three lags). The results for the first two regressions are reported in Panel A, while those for the last are in Panel B. To conserve space, we omit the regression intercept and its t-value. The t-values in bold type are significant at the 10% level or higher for two-tailed tests. The entries under $\gamma_1 > 0$ are percentages of the monthly coefficient γ_1 that are positive. The reported R^2 is the average R^2 from monthly cross-sectional regressions. The risk-neutral skewness and kurtosis are estimated using the results in BKM (2003), and the calculation procedure is outlined in Appendix B. The maturity ranges for short term, medium term, and long term are, respectively, 20–70 days, 71–120 days, and 121–180 days. The regressions are performed separately for four moneyness buckets.

Finally, in both the unconditional and conditional tests, the coefficients for the risk-neutral skewness and kurtosis are mostly insignificant and the signs are mixed. Nevertheless, as shown in Panel B, the effect of the systematic risk proportion on the implied volatility level remains significant, even after controlling for the risk-neutral skewness and kurtosis.

1.3 Slope-effect tests

Table 4 reports the results for the slope-effect tests—i.e., tests pertaining to Hypothesis 2. The results are similar to those in Table 3, albeit slightly weaker, in terms of rejecting the hypothesis. For most parts, the slope of the implied volatility curve is related to the systematic risk proportion in a statistically significant fashion. The bigger the systematic risk proportion, the steeper the slope. The significance largely remains after controlling for the risk-neutral skewness and kurtosis. Therefore, Hypothesis 2 is also rejected.

Other observations regarding moneyness and maturity are also similar to those in Table 3. The weakening of the systematic risk effect on the slope is especially pronounced with the upper tail of the moneyness range, i.e., 1.05–1.10. This is due to the slight, curving back of the implied volatility curve in this region. As for maturity, we also observe a weaker effect with long-term options.

BKM (2003) predicted positive coefficients for the risk-neutral skewness and kurtosis in describing the slope of implied volatilities. We do observe positive (and sometimes significant) γ_2 and γ_3 for many cases, but the signs are by no means uniform across the moneyness buckets. When we combine the moneyness buckets and run a single regression as in BKM (2003), we do obtain the sign and significance as shown in BKM (2003). This implies that it is very crucial to separate moneyness buckets when examining the properties of the implied volatility.

In terms of economic significance, the R^2 is lower than its level-effect counterpart. For the univariate regressions covering all maturities, the systematic risk proportions explain 4.7%, 4.8%, 5.5%, and 1.6% of the cross-sectional variations in the slope for the four moneyness buckets, respectively. The numbers improve to 18.8%, 16.7%, 17.9%, and 12.8% when the risk-neutral skewness and kurtosis are added to the regressions.

1.4 A combined test of the level and slope effects using panel regressions

To make sure that our conclusions are not due to the two-pass testing procedure, we run a single-pass panel regression and test the two hypotheses therein. Specifically, we run the following panel regression for each moneyness/maturity bucket:

$$\begin{aligned}\sigma_{ij}^{\text{imp}} - \sigma_{ij}^{\text{his}} &= [\alpha_0 + \alpha_1(b_{ij} - \bar{b}_i)] + [\beta_0 + \beta_1(b_{ij} - \bar{b}_i)](y_{ij} - \bar{y}_j) + \varepsilon_{ij} \\ &= \alpha_0 + \alpha_1(b_{ij} - \bar{b}_i) + \beta_0(y_{ij} - \bar{y}_j) + \beta_1(b_{ij} - \bar{b}_i)(y_{ij} - \bar{y}_j) \\ &\quad + \varepsilon_{ij}, \end{aligned} \tag{6}$$

Table 4
Regression tests for the slope effect

Panel A: Separate regressions on systematic risk proportion, and skewness and kurtosis

		γ_1				γ_2		γ_3		
		Average	t	$\gamma_1 < 0$	R^2 (%)	Average	t	Average	t	R^2 (%)
Moneyness	All maturities	−0.431	**−5.213**	86.2%	4.7	0.121	**2.326**	−0.031	−1.287	14.7
K/S	Short term	−0.363	**−4.549**	78.5%	3.2	0.157	**1.933**	−0.033	−1.251	20.2
0.90–0.95	Medium term	−0.411	**−8.415**	93.6%	10.0	0.153	**3.990**	0.027	**2.579**	13.4
	Long term	−0.183	−1.180	54.6%	9.2	0.011	0.235	−0.032	−1.409	17.5
Moneyness	All maturities	−0.441	**−5.921**	92.3%	4.8	−0.006	−0.223	−0.046	**−3.002**	11.3
K/S	Short term	−0.583	**−10.520**	95.4%	6.1	−0.022	−0.397	−0.058	**−2.937**	17.4
0.95–1.00	Medium term	−0.534	**−14.074**	100.0%	15.8	0.037	0.656	−0.028	−1.275	17.4
	Long term	−0.212	**−2.004**	63.9%	5.6	0.032	0.763	0.007	0.346	13.7
Moneyness	All maturities	−0.557	**−7.193**	98.5%	5.5	0.172	**3.981**	0.076	**4.672**	10.7
K/S	Short term	−0.612	**−7.541**	93.9%	6.0	0.191	**2.647**	0.062	**3.286**	13.7
1.00–1.05	Medium term	−0.500	**−8.706**	97.0%	16.7	0.170	**1.831**	0.095	**3.933**	24.9
	Long term	−0.563	**−3.001**	73.8%	8.7	0.123	**3.627**	0.032	**2.274**	19.5
Moneyness	All maturities	0.003	0.047	49.2%	1.6	0.056	**1.996**	0.060	**3.387**	9.6
K/S	Short term	−0.053	−0.974	56.9%	2.1	−0.015	−0.287	0.024	1.555	16.0
1.05–1.10	Medium term	−0.158	**−2.339**	68.0%	6.0	0.167	**2.855**	0.047	**2.117**	17.1
	Long term	−0.311	−1.682	54.3%	9.0	−0.015	−0.284	−0.012	−0.439	21.0

Panel B: Combined regressions on systematic risk proportion, skewness and kurtosis

		γ_1			γ_2		γ_3		
		Average	t	$\gamma_1 < 0$	Average	t	Average	t	R^2 (%)
Moneyness	All maturities	−0.203	**−2.304**	66.2%	0.107	**2.141**	−0.026	−1.041	18.8
K/S	Short term	−0.070	−0.735	57.6%	0.126	1.474	−0.041	−1.397	24.3
0.90–0.95	Medium term	−0.372	**−5.621**	78.3%	0.041	0.881	−0.008	−0.429	20.9
	Long term	−0.043	−0.314	51.5%	0.006	0.151	−0.035	−1.295	28.0
Moneyness	All maturities	−0.476	**−7.207**	89.2%	−0.040	−1.551	−0.032	**−1.911**	16.7
K/S	Short term	−0.675	**−7.922**	91.4%	−0.131	**−2.446**	−0.059	**−2.930**	27.5
0.95–1.00	Medium term	−0.570	**−9.393**	95.0%	−0.093	−1.415	−0.055	**−2.358**	31.8
	Long term	−0.276	−1.346	55.0%	0.014	0.300	0.003	0.142	21.3
Moneyness	All maturities	−0.606	**−6.449**	93.8%	0.141	**3.149**	0.097	**5.271**	17.9
K/S	Short term	−0.552	**−4.264**	83.1%	0.102	1.370	0.059	**2.589**	21.3
1.00–1.05	Medium term	−0.480	**−6.328**	88.0%	0.087	0.821	0.076	**2.023**	37.0
	Long term	−0.460	−1.592	61.3%	0.147	**3.659**	0.047	**2.321**	32.3
Moneyness	All maturities	0.015	0.138	50.8%	0.086	**2.113**	0.077	**3.300**	12.8
K/S	Short term	0.000	0.001	48.3%	−0.010	−0.161	0.030	**1.967**	18.7
1.05–1.10	Medium term	−0.024	−0.387	42.1%	0.172	**2.746**	0.050	**1.832**	21.2
	Long term	−0.234	−1.053	55.6%	−0.021	−0.409	−0.006	−0.194	26.6

This table contains two-pass regression results for the slope-effect tests. In the first pass, for each stock, we regress the difference between the implied volatility and the historical volatility on moneyness for nonoverlapping periods of one month (i.e., four weeks): $\sigma_i^{\text{imp}} - \sigma_i^{\text{his}} = a_0 + a_1(y_i - \overline{y}) + \varepsilon_i$. We thus obtain a monthly time series of the intercept a_0 and the slope coefficient a_1 for all stocks, including the S&P100 index. The moneyness variable is adjusted by the sample mean within the month, so that the intercept a_0 is the average of the difference between the implied volatility and the historical volatility. In the second pass, we cross-sectionally regress the slope on the systematic risk proportion, the risk-neutral skewness and kurtosis. This regression is done every month in three different forms: $a_{1j} = \gamma_0 + \gamma_1 b_j + e_j$, $a_{1j} = \gamma_0 + \gamma_2 \text{Skew}_j^{(rn)} + \gamma_3 \text{Kurt}_j^{(rn)} + e_j$ and $a_{1j} = \gamma_0 + \gamma_1 b_j + \gamma_2 \text{Skew}_j^{(rn)} + \gamma_3 \text{Kurt}_j^{(rn)} + e_j$. The monthly regression coefficients are then averaged, and the corresponding t-values calculated using the Newey-West standard error (with three lags). The results for the first two regressions are reported in Panel A, while those for the last are in Panel B. To conserve space, we omit the regression intercept and its t-value. The t-values in bold type are significant at the 10% level or higher for two-tailed tests. The entries under $\gamma_1 < 0$ are percentages of the monthly coefficient γ_1 that are negative. The reported R^2 is the average R^2 from monthly cross-sectional regressions. The risk-neutral skewness and kurtosis are estimated using the results in BKM (2003), and the calculation procedure is outlined in Appendix B. The maturity ranges for short term, medium term, and long term are, respectively, 20–70 days, 71–120 days, and 121–180 days. The regressions are performed separately for four moneyness buckets.

where $\bar{b}_i$ is the observation-weighted, cross-sectional average of the systematic risk proportion for each day, $\bar{y}_j$ is the sample average of moneyness for stock j within the bucket. Broadly speaking, α_0 can be understood as the average differential between the implied volatility and the historical volatility over all names within the entire sample period. Similarly, β_0 can be understood as the average slope of the implied volatility curve. They are not exactly the said quantities due to the interaction term $b_{ij} * y_{ij}$. The coefficient α_1 indicates the level effect. If the systematic risk proportion does not affect the price level or the volatility differential, then α_1 should not be different from zero, statistically speaking. A positive α_1 would confirm the level effect. By the same token, the coefficient β_1 indicates the slope effect. A negative β_1 would imply that a stock with a higher than average systematic risk proportion will have a slope steeper than the average slope of all implied volatility curves.

Table 5 reports the results. Based on the t-values of the coefficient α_1, Hypothesis 1 is rejected at an extraordinary level of significance, reaffirming the level effect. As for the coefficient β_1, except for three cases, the t-values are significant and large. Therefore, Hypothesis 2 is also rejected, confirming the slope effect. If anything, the panel regressions lead to a stronger rejection of the two hypotheses than did our two-pass regression tests. We have also repeated the tests by calculating $\bar{b}_i$ as the simple average of the systematic risk proportions (i.e., not weighted by the number of observations). The results remain almost identical.[3]

1.5 Controlling for firm-specific characteristics

Dennis and Mayhew (2002) linked the risk-neutral skewness to the following firm-specific variables: (i) implied volatility as a measure of overall risk; (ii) trading volume of the underlying stock as a measure of liquidity; (iii) beta as a measure of systematic risk; (iv) leverage; (v) firm size; and (vi) trading pressure. The last variable, defined as the ratio of average daily put volume to average daily call volume within a week, is meant to measure the impact of imbalance in option demand. It is useful to know if the systematic risk can still explain the price structures in individual equity options after controlling for these firm-specific variables. Here, we include only the stock's trading volume, leverage, and firm size. We omit the implied volatility and beta, since they are already the subjects of our study. The trading pressure variable is not included because we do not have data on options' trading volumes, and this variable turned out to be insignificant in Dennis and Mayhew (2002). We rerun the panel regression in Equation (6) by adding the three control variables. We exclude the index in the tests because the control variables are available for the individual firms but not for the index. To ensure meaningful comparisons, we first rerun the panel regression in Equation (6) without the index.

[3] Incidentally, it is seen that the coefficient α_0 is negative for the moneyness measure K/S above 1.0. This should be intuitive given the downward-sloping feature of a typical implied volatility curve: implied volatilities in the moneyness range above 1.0 are lower than the average volatility at the midpoint or the at-the-money point.

Table 5
Level- and slope-effect tests based on panel regressions

		α_0	t	α_1	t	β_0	t	β_1	t	$R^2(\%)$
Moneyness	All maturities	0.032	**225.24**	0.068	**143.46**	−0.361	**−36.74**	−0.730	**−21.88**	23.7
K/S	Short term	0.038	**188.15**	0.066	**100.35**	−0.566	**−39.50**	−0.615	**−13.25**	23.8
0.90–0.95	Medium term	0.032	**129.59**	0.064	**85.19**	−0.275	**−16.16**	−0.636	**−12.06**	29.5
	Long term	0.016	**55.14**	0.088	**33.45**	−0.021	−1.07	−0.660	**−3.60**	6.6
Moneyness	All maturities	0.018	**141.22**	0.037	**85.62**	−0.229	**−24.92**	−0.616	**−20.58**	10.6
K/S	Short term	0.018	**95.65**	0.036	**60.20**	−0.318	**−24.25**	−0.654	**−15.68**	10.5
0.95–1.00	Medium term	0.022	**97.10**	0.038	**55.64**	−0.202	**−12.33**	−0.524	**−10.87**	15.5
	Long term	0.015	**51.96**	0.053	**21.06**	−0.026	−1.29	−0.511	**−2.79**	3.0
Moneyness	All maturities	−0.007	**−53.81**	0.028	**63.67**	−0.057	**−6.22**	−0.409	**−13.47**	5.3
K/S	Short term	−0.006	**−30.28**	0.022	**36.59**	−0.050	**−3.84**	−0.428	**−10.26**	3.5
1.00–1.05	Medium term	−0.004	**−18.34**	0.029	**43.31**	−0.090	**−5.56**	−0.427	**−8.92**	9.5
	Long term	−0.014	**−48.34**	0.075	**28.78**	−0.026	−1.26	−0.098	−0.53	5.0
Moneyness	All maturities	−0.008	**−53.31**	0.021	**31.73**	0.070	**6.40**	0.050	0.95	1.8
K/S	Short term	−0.003	**−13.91**	0.014	**15.16**	0.201	**12.50**	−0.117	−1.59	1.3
1.05–1.10	Medium term	−0.011	**−37.30**	0.021	**20.60**	−0.010	−0.50	0.240	**2.87**	2.9
	Long term	−0.017	**−56.35**	0.066	**22.52**	−0.055	**−2.66**	−0.415	**−1.96**	3.6

This table contains panel regression results for the level and slope effects. For each moneyness/maturity bucket, instead of running the Fama-MacBeth two-pass regressions, we lump the entire sample and run the following panel regression: $\sigma_{ij}^{\text{imp}} - \sigma_{ij}^{\text{his}} = [(\alpha_0 + \alpha_1(b_{ij} - \overline{b_i})] + [(\beta_0 + \beta_1(b_{ij} - \overline{b_i}))(y_{ij} - \overline{y_j})] + \varepsilon_{ij}$, where $\overline{b_i}$ is the cross-sectional average of the systematic risk proportion for each day, and $\overline{y_j}$ is the sample average of moneyness for stock j within the bucket. This panel regression tests the level and slope effects simultaneously. Specifically, if the systematic risk proportion does not affect the price level or the level of the implied volatility (after adjusting for the historical volatility), then the coefficient α_1 should not be significantly different from zero; likewise, if the systematic risk proportion does not affect the slope of the implied volatility curve, then the coefficient β_1 should not be significantly different from zero. The t-values in bold type are significant at the 10% level or higher for two-tailed tests.

Table 6 reports the results. All three control variables, especially the trading volume and firm size, exert strong influence. Nonetheless, the t-values for the level- and slope-effect tests, albeit smaller in most cases, retain their statistical significance.

2. Robustness Checks

2.1 Alternative ways of calculating and estimating the systematic risk proportion

Recall that the monthly systematic risk proportion, b_j, is calculated by using the average systematic and total risks within the four-week period. To see if our testing results are sensitive to how b_j is calculated, we repeat the tests by using the average $b_j's$ within the four-week period. In other words, we first calculate the daily proportions and then average them to obtain a single estimate for the four-week period. The results remain virtually the same, for both the two-pass regressions and the panel regressions. We have also repeated the tests by using $\sqrt{b_j} = |\frac{\beta_j \sigma_m}{\sigma_j}|$. The results are slightly weaker, but the statistical significance is retained in most cases. There is an intuitive justification for using the variance ratio rather than the standard deviation ratio. After all, variance is the natural measure of risk since it is additive for independent risks.

Another potential concern has to do with the estimation of the historical volatility and its composition, which employs a one-year rolling window with

Table 6
Panel regressions controlling for firm-specific characteristics

		Without firm-specific characteristics					With firm-specific characteristics: trading volume (γ_1), firm size (γ_2), and leverage (γ_3)										
		α_1	t	β_1	t	R^2 (%)	α_1	t	β_1	t	γ_1	t	γ_2	t	γ_3	t	R^2 (%)
Moneyness	All maturities	0.098	**55.332**	−0.756	**−6.081**	5.8	0.063	**32.318**	−0.694	**−5.674**	−0.007	**−29.265**	0.014	**41.449**	0.009	**9.399**	8.9
K/S	Short term	0.101	**38.277**	−0.755	**−4.058**	7.0	0.065	**22.272**	−0.681	**−3.708**	−0.005	**−13.246**	0.013	**26.374**	0.005	**3.884**	9.3
0.90–0.95	Medium term	0.095	**26.615**	−0.764	**−3.059**	5.4	0.065	**16.630**	−0.847	**−3.451**	−0.007	**−15.214**	0.013	**19.855**	0.010	**5.814**	8.6
	Long term	0.100	**31.830**	−0.578	**−2.607**	6.0	0.059	**17.441**	−0.388	**−1.816**	−0.011	**−28.663**	0.017	**30.523**	0.015	**9.583**	12.8
Moneyness	All maturities	0.073	**42.420**	−0.340	**−2.764**	3.4	0.050	**26.505**	−0.388	**−3.199**	−0.008	**−35.321**	0.011	**32.922**	0.003	**3.753**	6.3
K/S	Short term	0.074	**29.785**	−0.427	**−2.378**	3.5	0.056	**20.143**	−0.450	**−2.531**	−0.006	**−18.793**	0.008	**17.733**	0.000	0.148	5.2
0.95–1.00	Medium term	0.083	**23.009**	−0.165	−0.632	4.2	0.054	**13.845**	−0.291	−1.139	−0.007	**−17.029**	0.013	**19.309**	0.003	**1.960**	8.0
	Long term	0.063	**19.816**	−0.487	**−2.120**	2.7	0.036	**10.377**	−0.521	**−2.351**	−0.011	**−29.346**	0.014	**24.297**	0.011	**6.898**	9.3
Moneyness	All maturities	0.084	**50.026**	0.048	0.402	4.0	0.064	**34.261**	0.069	0.582	−0.002	**−10.616**	0.007	**22.886**	−0.001	−1.312	5.0
K/S	Short term	0.083	**34.317**	−0.212	−1.227	3.6	0.063	**23.669**	−0.194	−1.127	−0.002	**−7.627**	0.007	**15.620**	−0.002	**−1.901**	4.6
1.00–1.05	Medium term	0.080	**23.226**	0.553	**2.227**	4.0	0.066	**17.275**	0.577	**2.330**	−0.002	**−3.760**	0.005	**8.362**	0.003	**1.703**	4.6
	Long term	0.098	**30.359**	0.069	0.297	5.6	0.072	**20.062**	0.081	0.354	−0.003	**−6.355**	0.009	**15.707**	−0.001	−0.531	7.4
Moneyness	All maturities	0.071	**38.241**	−0.520	**−4.061**	2.9	0.043	**21.234**	−0.542	**−4.283**	−0.002	**−7.622**	0.010	**30.244**	−0.004	**−3.882**	5.3
K/S	Short term	0.065	**23.921**	−0.826	**−4.370**	2.7	0.030	**10.005**	−0.845	**−4.552**	−0.002	**−4.871**	0.012	**24.580**	−0.006	**−4.555**	6.1
1.05–1.10	Medium term	0.081	**21.149**	−0.287	−1.079	3.8	0.057	**13.836**	−0.267	−1.015	−0.002	**−3.674**	0.009	**13.628**	−0.001	−0.582	5.7
	Long term	0.080	**23.855**	−0.408	**−1.769**	4.1	0.057	**15.805**	−0.434	**−1.899**	−0.003	**−6.738**	0.009	**15.209**	0.001	0.698	5.9

This table reports panel regression results controlling for firm-specific characteristics used by Dennis and Mayhew (2002). The regression setup is the same as in Table 5, except that we exclude the index, since we are controlling for firm-specific characteristics. For meaningful comparisons, we rerun the panel regression in Table 5 without the index and report the results under the heading "Without firm-specific characteristics." To conserve space, we report only the R^2, the coefficients, and t-values for the level effect (α_1) and the slope effect (β_1). We then run the panel regression with the three control variables: log of the daily trading volume of the underlying stock, log of the firm size (which is the product of the number of shares outstanding and the share price), and the leverage (which is the sum of long-term debt and the par value of the preferred stock divided by the said sum plus the market value of equity). The results are reported under the heading "With firm-specific characteristics: trading volume (γ_1), firm size (γ_2), and leverage (γ_3)". The t-values in bold type are significant at the 10% level or higher for two-tailed tests.

daily frequency. The shorter window and higher data frequency raise the concern that the resulting risk estimates may be highly time-varying and do not necessarily reflect changes in the systematic risk proportion. To address this concern, we repeat the tests using a five-year rolling window at weekly frequency (by sampling data points on Wednesdays). For both the level-effect and the slope-effect tests, the statistical significance remains, albeit some t-values decrease slightly.

2.2 Systematic risk estimation using Fama-French factors

So far, all the tests use systematic risk estimates from a single factor model, the market model. We now reestimate the systematic risk by adding the two Fama-French factors (i.e., SMB and HML) to the market factor. By definition, the systematic risk proportion estimated with the two additional factors will be higher. The question is, will it increase proportionally across stocks so that our level and slope effects would hold up? To answer this question, we repeat the tests using the newly estimated systematic risk proportions. Since the panel regression results are stronger, to be conservative, we show the two-pass regression results. To conserve space, we report the test results for the level and slope effects in one table, Table 7. For brevity, we only report the regression coefficient and its t-value, together with R^2 for the univariate regression (with the systematic risk proportion being the only explanatory variable) and the multivariate regression (with the risk-neutral skewness and kurtosis as well as the systematic risk proportion as the explanatory variables).

Comparing Table 7 with Table 3 (level effect) and Table 4 (slope effect), we see that the results remain virtually the same. This is another indirect support for the choice of the systematic risk proportion over the beta for our study. Since we have controlled for the overall level of risk, what matters is the composition of the total risk, not the absolute magnitude of the components. As long as the same estimation procedure is applied to all stocks, the cross-sectional feature would manifest itself. Therefore, one may also infer that our results are likely robust to more sophisticated estimation methods, e.g., a Bayesian shrinkage estimator or a certain type of optimal estimator, for the systematic risk.

We have also conducted robustness checks on subsamples and the influence of the index. For the former, the results remain for the two equal-half samples; for the latter, after removing all index options, the significance remains for the level-effect tests, and weakens for the slope-effect tests. All said, our results are robust to various alternative specifications.

3. Reconciliation with the Existing Literature

As discussed in the introductory section, our study offers an alternative explanation to the observed difference in price structures across equity options. Existing explanations, some of which are reviewed earlier, include the volatility risk premium (e.g., Buraschi and Jackwerth, 2001; Bakshi and Kapadia, 2003;

Table 7
Level and slope effect tests using systematic risk estimates derived from Fama-French factors

Panel A: Level effects

		Univariate regressions			Multivariate regressions						
		γ_1			γ_1		γ_2		γ_3		
		Average	t	R^2 (%)	Average	t	Average	t	Average	t	R^2 (%)
Moneyness	All maturities	0.069	**15.681**	12.0	0.064	**13.180**	−0.014	**−3.379**	−0.004	**−3.640**	23.4
K/S	Short term	0.069	**17.573**	14.2	0.059	**12.827**	−0.013	**−3.295**	−0.004	**−4.536**	28.7
0.90–0.95	Medium term	0.064	**11.809**	21.9	0.051	**7.387**	−0.018	**−3.794**	−0.004	**−2.972**	36.2
	Long term	0.074	**5.789**	8.9	0.081	**2.872**	−0.019	**−1.847**	−0.004	−1.578	29.7
Moneyness	All maturities	0.045	**12.171**	6.3	0.038	**7.460**	−0.015	**−3.534**	−0.005	**−6.138**	19.7
K/S	Short term	0.041	**11.245**	6.6	0.029	**5.351**	−0.012	**−3.975**	−0.004	**−5.767**	23.0
0.95–1.00	Medium term	0.032	**5.769**	7.6	0.016	**2.320**	−0.009	−1.012	−0.002	−1.217	32.9
	Long term	0.010	0.664	6.5	0.041	1.268	−0.010	**−3.377**	−0.004	**−4.305**	30.3
Moneyness	All maturities	0.040	**5.931**	6.1	0.039	**5.453**	0.001	0.164	−0.001	−1.324	18.6
K/S	Short term	0.032	**4.847**	5.2	0.030	**3.903**	0.005	1.043	−0.001	−1.181	21.8
1.00–1.05	Medium term	0.025	**4.168**	8.9	0.022	**3.818**	−0.003	−0.576	−0.002	**−1.993**	28.4
	Long term	0.071	**3.240**	13.4	0.095	**2.290**	0.008	0.928	0.000	−0.027	38.6
Moneyness	All maturities	0.030	**4.998**	4.5	0.032	**5.407**	0.002	0.528	−0.001	−0.781	16.2
K/S	Short term	0.020	**3.790**	3.9	0.021	**3.875**	0.007	**2.123**	0.000	−0.110	18.7
1.05–1.10	Medium term	0.020	**3.390**	5.6	0.016	**2.083**	−0.002	−0.198	−0.002	−0.779	31.1
	Long term	0.024	1.461	10.6	0.045	**1.938**	−0.003	−0.476	−0.001	−0.933	28.7

Panel B: Slope effects

Moneyness	All maturities	−0.464	**−6.089**	5.1	−0.402	**−4.381**	0.080	1.327	0.000	−0.002	20.4
K/*S*	Short term	−0.372	**−4.870**	3.3	−0.263	**−3.440**	0.117	1.285	0.004	0.213	24.4
0.90–0.95	Medium term	−0.430	**−9.063**	10.1	−0.451	**−5.690**	0.038	0.595	0.005	0.277	24.1
	Long term	−0.231	−1.600	10.4	−0.016	−0.132	0.027	0.663	−0.015	−0.935	28.5
Moneyness	All maturities	−0.426	**−6.451**	4.8	−0.528	**−9.310**	−0.013	−0.458	−0.009	−1.001	15.5
K/*S*	Short term	−0.558	**−10.964**	5.5	−0.760	**−11.245**	−0.070	−1.414	−0.016	−1.362	24.7
0.95–1.00	Medium term	−0.522	**−16.746**	14.9	−0.666	**−10.417**	−0.107	**−2.263**	−0.060	**−4.557**	30.7
	Long term	−0.133	−1.315	5.2	0.136	0.821	0.011	0.224	−0.010	−1.067	19.6
Moneyness	All maturities	−0.507	**−6.644**	4.8	−0.420	**−5.572**	0.119	**3.328**	0.053	**6.068**	17.5
K/*S*	Short term	−0.564	**−6.812**	5.5	−0.373	**−3.131**	0.101	1.412	0.042	**3.050**	22.0
1.00–1.05	Medium term	−0.492	**−8.068**	15.6	−0.340	**−4.313**	0.059	0.743	0.037	**2.166**	35.1
	Long term	−0.481	**−2.777**	8.9	−0.438	**−1.883**	0.129	**2.920**	0.034	**2.973**	31.3
Moneyness	All maturities	−0.007	−0.129	1.7	0.071	0.863	0.070	**2.097**	0.031	**2.912**	11.9
K/*S*	Short term	−0.047	−0.786	2.3	0.146	**2.063**	0.068	1.316	0.035	**3.176**	18.8
1.05–1.10	Medium term	−0.158	**−2.481**	5.9	0.025	0.401	0.175	**3.033**	0.038	**2.155**	21.2
	Long term	−0.255	−1.447	8.9	−0.323	−1.526	0.030	0.756	0.019	1.405	25.6

This table contains two-pass regression results for the level and slope effects using systematic risk estimates derived from the Fama-French factors. The testing procedures are otherwise the same as those in Tables 3 and 4. Panel A corresponds to Table 3 and Panel B corresponds to Table 4. In Tables 3 and 4, the systematic risk is estimated by regressing the stock's returns on the market returns (S&P 500). Here, the systematic risk is estimated by regressing the stock's returns on the two Fama-French factors as well as on the market returns. To conserve space, we report only the regression coefficients, the t-values, and the average R^2. For brevity, we also omit the results for regressions whose explanatory variables are only the skewness and kurtosis. The t-values in bold type are significant at the 10% level or higher for two-tailed tests.

and Jones, 2006), the jump risk premium (e.g., Bates, 1988, 2000; and Buraschi and Jackwerth, 2001), the demand-based option pricing (e.g., Bollen and Whaley, 2004), and the GARCH option pricing model (e.g., Duan, 1995; and Duan and Wei, 2005). Except for Bollen and Whaley (2004), the studies lead to one common realization: the empirical regularities are ultimately due to a wedge driven between the physical and risk-neutral distributions of the underlying. This wedge is in turn caused by the entry of the risk premium into the second and/or higher moments of the risk-neutral distribution. In other words, as long as the additional risk factors are systematic or priced, they would influence option prices and determine the features of the risk-neutral distribution.

To check the above assertion, we regress the risk-neutral skewness and kurtosis on the systematic risk proportion. That is, we run the following cross-sectional regressions,

$$\text{Skew}_j^{(rn)} = \gamma_0 + \gamma_1 b_j + e_j \tag{7}$$

$$\text{Kurt}_j^{(rn)} = \gamma_0 + \gamma_2 b_j + e_j \tag{8}$$

on a monthly basis as in Section 1 and calculate the average regression coefficients and the associated t-statistics using Newey-West standard errors. For the risk-neutral skewness, we find $\hat{\gamma}_0 = -0.615$ with a t-value of -13.001 and $\hat{\gamma}_1 = -0.921$ with a t-value of -9.117. This result indicates that the risk-neutral return distributions are on average negatively skewed and the degree of the negative skewness is proportional to the systematic risk proportion. Our regression results for the risk-neutral kurtosis are $\hat{\gamma}_0 = 3.294$ with a t-value of 46.890 and $\hat{\gamma}_2 = 2.032$ with a t-value of 4.531. This finding suggests that the stocks in our sample have on average leptokurtic risk-neutral return distributions and the kurtosis is increasing in the systematic risk proportion.

In BKM (2003), the level and slope of the implied volatility curve have been found to be related to the risk-neutral skewness and kurtosis. Our results suggest that the level and slope of the implied volatility curve and the risk-neutral skewness and kurtosis are all influenced by the systematic risk proportion.

As for the channels through which the risk factors enter into the option price structure, different models postulate specific mechanisms. For instance, Bakshi and Kapadia (2003), through examining delta-hedge gains/losses, found conclusively that there is a negative risk premium for market volatility. A negative volatility risk premium would increase the drift of the risk-neutral volatility, which could explain the observed higher implied volatility relative to its physical counterpart. In contrast, jump risks, when priced by the market, enter the risk-neutral density via the third and fourth moments. As pointed out by Bakshi and Kapadia (2003), in a typical jump-diffusion setting, the jump size governs the risk-neutral skew while the jump intensity governs the risk-neutral kurtosis. A fear of market crashes would lead to a negative skew. Based on the results in BKM (2003), the more negatively skewed the risk-neutral distribution, the steeper the implied volatility curve.[4]

In a recent study, Bakshi and Madan (2006) theoretically showed how and when the risk-neutral and physical index volatilities can be different. Within their framework, the risk-neutral volatility is higher than its physical counterpart when investors are risk-averse and when the physical distribution is negatively skewed and leptokurtic. Therefore, combining this result with the finding in BKM (2003), it appears that a wider gap between the risk-neutral and physical volatilities would also be accompanied by a steeper slope of the implied volatility curve. A casual comparison of the S&P 100 index with stocks in Table 1A certainly confirms this (a rough measure of the slope can be obtained by finding the difference between the implied volatilities of the two extreme moneyness buckets). It is seen that, when the systematic risk proportion is approaching 1 (i.e., $b_j \to 1$), the spread between the implied and historical volatilities is the largest, and the implied volatility slope is the steepest, as manifested by the S&P 100 index options; the opposite is generally true when the total risk is mostly idiosyncratic (i.e., $b_j \to 0$), as apparent in the stocks with lower systematic risk proportions.

Which factor has a bigger systematic risk component, the stochastic volatility or jumps? Bakshi and Kapadia's (2003) is the only study that provides a direct answer: the stochastic volatility. Through examining the time series of hedge gains/losses, they found only minor impacts of jump fears. A more direct support for the bigger role of the stochastic volatility came from their finding that the implied volatility was already higher than the historical, even before the 1987 stock market crash. Some other studies also identified systematic risk factors in option prices, but did not identify the relative contributions of stochastic volatility and jumps. For instance, Buraschi and Jackwerth (2001) were able to rule out the potential role of a stochastic interest rate, but allowed both the stochastic volatility and jumps to be potential systematic risk factors. Jones (2006) uncovered two or three latent factors in the S&P 500 index option returns, and pointed to volatility risk and possibly jump risk.

In contrast to a continuous-time setup where the stochastic volatility or jumps exert their impact on option price structures through their risk premiums, the discrete-time GARCH setup requires only the risk premium of the underlying asset. As shown by Duan (1995), when the physical GARCH process is converted to its risk-neutral counterpart, the risk premium of the underlying asset does not vanish; instead, it enters into the volatility process. In this setting, the risk-neutral volatility, skewness, and kurtosis are all affected by the risk premium. Specifically, holding other parameters fixed, a bigger (positive) risk premium would lead to a higher volatility, a more negative skew and fatter tails

[4] As shown in Theorem 2 of BKM (2003), a negative skew under the physical measure would carry over to the risk-neutral measure. However, a negative risk-neutral skew can still exist even if the physical distribution is symmetrical, as long as there exist risk aversion and fat tails in the physical distribution. One way or the other, jump fears would lead to a negative risk-neutral skew.

under the risk-neutral measure. Therefore, under GARCH, when the risk premium is high, the higher implied volatility is a direct result, while the steeper slope of the implied volatility curve is through the more negative skew. Duan and Wei (2005), by assuming a one-factor stochastic discount factor with the GARCH feature, derived an explicit link between the risk premium and the systematic risk proportion. Specifically, a higher systematic risk proportion leads to a higher risk premium.

Finally, the arguments put forth by Bollen and Whaley (2004) represent a somewhat different line of reasoning. Instead of pursuing alternative distributions or risk factors, they appealed to demand-based pricing. Motivating their arguments based on limits to arbitrage (Shleifer and Vishny, 1997; and Liu and Longstaff, 2004), they put forth two hypotheses. First, the implied volatility would be higher than the actual realized volatility, since market makers demand a compensation for bearing hedging costs; second, the implied volatility curve for index options would be downward sloping if demand and supply are not balanced for different regions of the exercise price, particularly if the demand by institutional investors for out-of-the-money put index options is well over the supply. Their empirical results seem to support the hypotheses.

Though appealing on the intuitive level, this framework does not postulate *why* demand pressures are different across names and across moneyness for the same name. The framework can indeed explain the features of the implied volatility for the index in light of the well-known demand for out-of-the-money put options for insurance purposes, but it offers very little in explaining the difference in individual equity options. In contrast, our explanation is grounded on valuation theories (namely, risk premiums affect the moments of the risk-neutral distribution) and is capable of offering predictions for individual equity options as well as index options.

4. Summary and Conclusions

The derivatives literature has established, among other things, the following empirical findings: (i) the risk-neutral return distribution departs from its physical counterpart (e.g., Bakshi, Kapadia, and Madan, 2003); (ii) the index options have a more pronounced volatility smile/smirk than individual equity options (e.g., Jackwerth, 2000; and Bakshi, Kapadia, and Madan, 2003); and (iii) option prices seem to contain risk factors in addition to the Brownian innovation of the underlying asset (e.g., Buraschi and Jackwerth, 2001; Bakshi and Kapadia, 2003; and Jones, 2006). Collectively, these findings indicate that the additional risk factors may be the cause of the departure between the risk-neutral and physical distributions, and that the same factors may be priced differently across equity options.

Our study is motivated by the above realization. While most of the existing studies focus on index options in uncovering the price structure across moneyness, we instead examine the price structure across individual equity options. We demonstrate empirically how the systematic risk affects equity option prices. Insofar, as the wedge between the risk-neutral and physical distributions is driven by the premiums of the systematic risk factors, the systematic risk must be the factor that differentiates individual equity options in price structure.

We show conclusively that option prices are indeed related to the proportion of systematic risk in the total risk. Controlling for the overall risk level, a higher amount of systematic risk leads to a higher level of implied volatility and a steeper implied volatility curve. The effect remains robust to various alternative estimations of the variables and specifications of the tests.

Our empirical results could be reconciled with several theoretical paradigms. For instance, in a continuous-time setting, a negative risk premium for the volatility risk (as documented by Bakshi and Kapadia, 2003) would lead to option prices such that the higher the systematic risk, the higher the implied volatility relative to the historical volatility; alternatively, in a discrete-time GARCH setting, a higher systematic risk would predict a higher implied volatility and steeper slope of the implied volatility curve.

Appendix A: Relating Implied Volatility to the Systematic Risk Proportion

First, empirical evidence in the literature strongly indicates that the risk-neutral market return (manifested in, e.g., S&P 500 index options) is negatively skewed and leptokurtic. An individual stock's risk-neutral skewness and kurtosis are expected to be related to the market return's counterparts with the systematic risk proportion serving as the linkage. To this end, assume a standard one-factor market model for stock j,

$$R_{jt} = \alpha_j + \beta_j R_{mt} + \xi_{jt}. \tag{A1}$$

In addition to the usual assumptions for the factor model, we require ξ_{jt} to be a normal random variable. This assumption allows us to relate explicitly the implied volatility to the systematic risk proportion and the moments of the market return's distribution. Note also that the systematic risk proportion is essentially the R^2 of the regression model (A1).

Second, as in Theorem 3 in BKM (2003), we assume that the same factor model structure holds under the risk-neutral measure, except that α_j may undergo a mean shift. Preserving the same structure ensures that R^2 remains unchanged under the risk-neutral measure—i.e., $b_j^{(rn)} = b_j$ (hereinafter, the superscript rn stands for "risk-neutral"). By part (a) of Theorem 3 in BKM (2003) and our normality assumption, we can relate stock j's skewness to the market's as follows:

$$\text{Skew}_j^{(rn)} = b_j^{3/2}\text{Skew}_m^{(rn)}. \tag{A2}$$

One can also follow the same reasoning to relate stock j's kurtosis to its market counterpart,

$$\text{Kurt}_j^{(rn)} = b_j^2\big(\text{Kurt}_m^{(rn)} - 3\big) + 3. \tag{A3}$$

Note that if the market return is normally distributed under the risk-neutral measure, then so is the stock return irrespective of the value for b_j.

Third, Theorem 4 of BKM (2003) shows that, for a given maturity, the Black-Scholes implied volatility is related to the stock's risk-neutral skewness and kurtosis in the following fashion:

$$\sigma_j^{\text{imp}}(K/S) \approx q_1(K/S) + q_2(K/S)\text{Skew}_j^{(rn)} + q_3(K/S)\text{Kurt}_j^{(rn)}, \tag{A4}$$

where the coefficients q_1, q_2, and q_3 are written explicitly as functions of the option's moneyness, K/S.

Finally, substituting the skewness and kurtosis in Equation (A2) and Equation (A3) into Equation (A4), we have

$$\sigma_j^{\text{imp}}(K/S) \approx q_1(K/S) + 3q_3(K/S) + q_2(K/S)\text{Skew}_m^{(rn)} b_j^{3/2} + q_3(K/S)(\text{Kurt}_m^{(rn)} - 3)b_j^2.$$

Appendix B: Calculation of the Risk-Neutral Skewness and Kurtosis Using Option Prices

The calculation of the risk-neutral skewness and kurtosis is based on the theoretical results in BKM (2003). Since a continuum of out-of-the-money calls and puts can span any payoff (Bakshi and Madan, 2000), one must be able to span payoffs based on the second, third, and fourth moments of the stock returns. In other words, the risk-neutral moments can be backed out from out-of-the-money option prices.

Following BKM (2003), let $R(t, \tau) \equiv \ln \frac{S(t+\tau)}{S(t)}$ denote the τ-period stock return. Let $V(t, \tau)$, $W(t, \tau)$, and $X(t, \tau)$ represent the fair value of payoffs $R(t, \tau)^2$, $R(t, \tau)^3$, and $R(t, \tau)^4$, respectively. According to Theorem 1 of BKM (2003), the τ-period risk-neutral skewness and kurtosis can be expressed as

$$\begin{aligned}
\text{Skew}(t, \tau) &= \frac{e^{r\tau}W(t, \tau) - 3\mu(t, \tau)e^{r\tau}V(t, \tau) + 2\mu(t, \tau)^3}{[e^{r\tau}V(t, \tau) - \mu(t, \tau)^2]^{3/2}}, \\
\text{Kurt}(t, \tau) &= \frac{e^{r\tau}X(t, \tau) - 4\mu(t, \tau)e^{r\tau}W(t, \tau) + 6\mu(t, \tau)^2 e^{r\tau}V(t, \tau) - 3\mu(t, \tau)^4}{[e^{r\tau}V(t, \tau) - \mu(t, \tau)^2]^2},
\end{aligned}$$

where

$$\begin{aligned}
\mu(t, \tau) &\equiv e^{r\tau} - 1 - \frac{e^{r\tau}}{2}V(t, \tau) - \frac{e^{r\tau}}{6}W(t, \tau) - \frac{e^{r\tau}}{24}X(t, \tau), \\
V(t, \tau) &\equiv \int_{S(t)}^{\infty} \frac{2\left(1 - \ln \frac{K}{S(t)}\right)}{K^2} C(t, \tau, K)dK + \int_0^{S(t)} \frac{2\left(1 + \ln \frac{S(t)}{K}\right)}{K^2} P(t, \tau, K)dK, \\
W(t, \tau) &\equiv \int_{S(t)}^{\infty} \frac{6\ln \frac{K}{S(t)} - 3\left(\ln \frac{K}{S(t)}\right)^2}{K^2} C(t, \tau, K)dK \\
&\quad - \int_0^{S(t)} \frac{6\ln \frac{S(t)}{K} + 3\left(\ln \frac{S(t)}{K}\right)^2}{K^2} P(t, \tau, K)dK, \\
X(t, \tau) &\equiv \int_{S(t)}^{\infty} \frac{12\left(\ln \frac{K}{S(t)}\right)^2 - 4\left(\ln \frac{K}{S(t)}\right)^3}{K^2} C(t, \tau, K)dK \\
&\quad + \int_0^{S(t)} \frac{12\left(\ln \frac{S(t)}{K}\right)^2 + 4\left(\ln \frac{S(t)}{K}\right)^3}{K^2} P(t, \tau, K)dK.
\end{aligned}$$

In the above, $C(t, \tau, K)$ is the value of a call option with time-to-maturity τ and exercise price K, and $P(t, \tau, K)$ is the corresponding put option value. The integrals can be approximated in a straightforward fashion using options available on each day. Our implementation is similar to that of Dennis and Mayhew (2002). Specifically, we use the trapezoidal approximation and require that there are at least two calls and two puts for each maturity. To illustrate, suppose there are J call options available for maturity τ at time t, then the first integral in $W(t, \tau)$ can be approximated as

$$w(K_1)\,C(t, \tau, K_1)\Delta K_1 + \frac{1}{2}\sum_{j=2}^{J}[w\left(K_{j-1}\right)C(t, \tau, K_{j-1}) + w\left(K_j\right)C(t, \tau, K_j)]\Delta K_j,$$

where $K_J > K_{J-1} > \ldots > K_1 > S(t)$, $w(K_j) = \frac{6\ln\frac{K_j}{S(t)} - 3\left(\ln\frac{K_j}{S(t)}\right)^2}{K_j^2}$, $\Delta K_1 = K_1 - S(t)$, $\Delta K_2 = K_2 - K_1$ and so on. All the other integrals can be approximated in a similar fashion.

Appendix C: Data Description

The option data used in this study are identical to those in BKM (2003), covering the period of 1 January 1991 to 31 December 1995, for a total of 260 weeks. We refer readers to BKM (2003) for detailed descriptions. The data consist of triple-panel (stock, maturity, and exercise price) bid-ask quotes for options written on the 30 largest component stocks of the S&P 100 index and on the S&P 100 index itself. The options are American style and traded on the Chicago Board of Options Exchange. The data frequency is daily, and the bid-ask quotes are the last quotes prior to 3:00 p.m. (CST). Only out-of-the-money call and put options are retained in this data set. Since out-of-the-money puts (calls) correspond to in-the-money calls (puts), the data set effectively covers the whole moneyness spectrum.

As in BKM (2003), the data are screened on three fronts: (i) we retain only options that have both bid and ask quotes; (ii) we eliminate option prices that violate the arbitrage conditions (i.e., the option price must be smaller than the stock price, but larger than the stock price minus the present value of the exercise price and the dividends); and (iii) we eliminate the deep out-of-the-money puts (i.e., $K/S < 0.9$) and calls (i.e., $K/S > 1.1$) and retain the moneyness range from 0.9 to 1.1. BKM (2003) cleansed the very short and very long maturity options, and retained only those with more than nine days and less than 120 days to expiration. In our study, we extend the cutoff for the longer maturity to 180 days. (It turns out that the index option observations concentrate mostly in the maturity range shorter than 120 days. This is the main reason why BKM (2003) omitted maturities beyond 120 days. We decide to include the long-term range, since all individual stocks have enough observations in this range.) In addition, since we use a four-week window for time-series regressions, we set a lower cutoff of maturity to 20 days. Therefore, for our empirical study, we examine three maturity ranges: short term, 20–70 days; medium term, 71–120 days; and long term, 121–180 days.

For each particular option, the implied volatility based on the Black-Scholes formula is available. BKM (2003) showed that these implied volatilities are close to their counterparts backed out from the binomial tree. In other words, the difference between the precise American style implied volatilities and the European-style Black-Scholes volatilities is negligible. In our study, the implied volatility based on the Black-Scholes formula is used.

The daily stock prices, downloaded from www.finance.yahoo.com, are used to calculate historical volatilities and the proportion of systematic risk in the total risk. We use the S&P 500 index as a proxy for the market portfolio.

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