Ride-hailing Platforms: Competition and Autonomous Vehicles

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Abstract. Problem definition: Ride-hailing platforms, which compete over drivers and riders, assert that autonomous vehicles (AVs) will transform their operations by reducing variable cost payments to drivers. This paper explores the implications of competition and access to AVs for the management of ride-hailing platforms. **Academic/Practical Relevance:** Ride-hailing, which has been transformed by platforms' use of independent driver-workers, has the potential to be transformed again by AVs. **Methodology:** We employ a game-theoretic model that captures platforms' AV fleet size, price and wage decisions. **Results:** A platform's access to supply-side (namely, AV) technology changes prescriptions for its demand-side (namely, pricing) decisions: The intuitive prescription from the setting without AVs, that price increases in the intensity of competition in the labor market, is reversed. The presence of demand-side competition changes prescriptions for a platform's supply-side (namely, AV fleet size) decisions: The intuitive prescription from the setting without demand-side competition, that the AV fleet size increases in the intensity of competition in the labor market, is reversed. We characterize the conditions under which these reversals occur and explain the driving forces behind the reversals. Finally, whether a platform benefits from its rival's access to AV technology depends on a simple comparison between the relative wage sensitivity of labor and the relative price sensitivity of demand. **Managerial Implications:** Competition and access to AVs each reverse intuitive prescriptions for the management of ride-hailing platforms.

1 Introduction

A central feature of ride-hailing platforms, such as Lyft and Uber, is that they simultaneously compete over a common pool of supply–namely, independent driver-workers–and a common pool of demand–namely, rider-consumers. Driver (rider) decisions are influenced by wage (price) as well as other factors. For example, some drivers (riders) might prefer to serve (seek service from) a platform with more-friendly or more-professional positioning (Dessaint 2015, Bhuiyan 2019b). Similar, some drivers and riders may prefer a platform because of the distinctive features of the smartphone application through which they access the platform (Dessaint 2015, Wisniewski 2017). The intensity of competition over drivers and riders depends on the degree of differentiation between the platforms.

Perhaps no development is perceived to have a larger potential for transforming the operations of ride-hailing platforms than the introduction of autonomous vehicles (AVs). Wage payments to drivers constitute the largest expense for ride-hailing platforms. With the specific purpose of eliminating the variable cost of payments to drivers, Lyft and Uber have aggressively pursued the development of AVs, with each investing billions in their efforts (Siddiqui and Bensinger 2019). Uber and Lyft comprise 98% of the U.S. ride-hailing market (Bosa 2018), and both platforms anticipate that AVs will be an important component of their operating model (Siddiqui and Bensinger 2019).

Uber has said that AVs are "existential" to its future. Uber initiated its AV efforts in 2015. In late 2017, Uber agreed to purchase 24,000 AVs from Volvo, stating that "everything we're doing right now is about building autonomous vehicles at scale" (Boston 2017, Isaac 2017). Uber invested $457 million in AV development in 2018 and announced a $1 billion investment in 2019 (Conger 2019). Lyft launched its in-house development of AVs in 2017, devoting one-tenth of its engineers to the effort. The goal of Lyft's AV efforts in-house and with industry partners (e.g., Waymo) is to bring "hundreds of thousands" of AVs to its platform (Bensinger 2017). Lyft intends to launch AVs by 2024, and intends AVs to provide the majority of its trips by 2029 (Lyft 2019). It is unclear which platform will be first to overcome the technological barriers to deploying AVs. In terms of in-house development, Uber has invested more and for a longer period of time. In terms of the broader industry, Lyft's partner Waymo is viewed as being farthest ahead in developing AVs (Welch 2018).

Each platform anticipates that after initially launching AVs, it will, for a period, serve customers with a mix of AVs and human-driven vehicles (Lyft 2019, Uber 2019). Uber intends to own and operate its own AVs (Isaac 2017, Uber 2019). Lyft has said it will "most likely" lease AVs if it does not own them outright (Murphy 2016). Lyft envisions that its AV offering will be "asset intensive," which is consistent with Lyft owning AVs (Lyft 2019). Although Lyft and Uber's in-house efforts are each aimed at developing technology for the platform's own AVs, each platform has expressed an openness to allowing AVs it does not wholly own on its network (Murphy 2016, Isaac 2017). One possibility is that a ride-hailing platform would partner with an outside entity that would put AVs on the platform's network. A second possibility is that a platform would allow AVs fully owned by external parties on its network. In the setup we explore, the platform determines its AV fleet size and incurs an associated cost. This is consistent with the platform owning or leasing the AVs on its network. It is also consistent with the partnership model, to the extent that the partners make decisions with the objective of maximizing their combined profit. To the extent that independent AV fleet owners with market power seek to put their AVs on a platform's network, a different setup would be required. Because examining this scenario would require a significant level of speculation about how the various entities (including the fleet owners, which do not as yet exist) would interact, we defer its discussion to §5.

What are the implications of competition and access to AV technology for the management of ride-hailing platforms? While competition over consumers is common in many settings, competition over independent-contractor workers that provide the service is a more novel feature of ride-hailing platforms. As such, we focus on the intensity of competition in the labor market, or more precisely, the cross-wage sensitivity of labor (i.e., the sensitivity of a platform's labor supply to changes in its rival's wage). We examine prescriptions for how the intensity of competition in the labor market influences the platforms' price and AV fleet size decisions.

The intensity of competition in the labor market can vary temporally and geographically. For example, initially Lyft and Uber's brand positioning was quite distinct, but over time the distinction may have lessened (Bhuiyan 2019a). Similarly, the degree of differentiation in the platforms' smartphone applications has evolved over time (Wisniewski 2017). In the same way, the degree to which drivers perceive the platforms to be distinct varies by geographic market. At times and in markets where drivers perceive the platforms as being quite distinct (similar), the cross-wage sensitivity of labor will tend to be low (high), consistent with low (high) intensity of competition in the labor market.

We focus on the setting where one platform obtains access to AV technology, which enables it to procure AVs. We consider how this access affects the platform's decisions and its rival's profit. Next, we state three research questions and summarize our findings.

First, how does a platform's access to supply-side (namely, AV) technology change prescriptions for its demand-side (namely, pricing) decisions? In the benchmark scenario where the platforms do not have access to AV technology, the platforms' equilibrium prices increase in the intensity of competition in the labor market. Intuitively, intensified competition in the labor market pushes up the marginal cost of supply, prompting the platforms to increase their prices. When a platform has access to AV technology, this intuitive benchmark result is reversed. That is, a platform's equilibrium price *decreases* in the intensity of competition in the labor market–provided that simple sufficient conditions hold. We identify the drivers behind and explain the intuition for how the presence of AVs reverses the intuitive result.

Second, how does the presence of demand-side competition change prescriptions for a platform's supply-side (namely, AV fleet size) decisions? In the benchmark scenario without demand-side competition, a platform's equilibrium AV fleet size increases in the intensity of competition in the labor market. Intuitively, intensified competition in the labor market pushes up the marginal cost of labor supply, prompting a platform to increase her AV fleet size, so as to reduce her reliance on the more costly labor. The presence of demand-side competition reverses this intuitive benchmark result. That is, a platform's equilibrium AV fleet size *decreases* in the intensity of competition in the labor market–provided that simple sufficient conditions hold. We identify the drivers behind and explain the intuition for how the presence of demand-side competition reverses the intuitive result.

Third, how is a platform affected by its rival's access to AVs? It may be natural to conjecture that a platform would be hurt by its rival's access to AV technology. We provide a simple, easy to interpret necessary and sufficient condition under which a platform benefits by its rival's access to such technology.

This paper is related to three streams of literature on competition: competition between ride-hailing platforms, supply chain competition with a common supplier, and capacity investment decisions of competing firms.

Ride-hailing platforms have been widely studied in the operations management literature. A large share of this work has focused on pricing, including dynamic pricing (Banerjee et al. 2015, Cachon et al. 2017, Bai et al. 2018, Hu et al. 2018), spatial pricing (Castro et al. 2018, Bimpikis et al. 2019), and the impact of uncertainty (Taylor 2018). Other dimensions of ride-hailing platforms that have received attention are labor and staffing considerations (Hu and Zhou 2017, Afeche et al. 2018, Allon et al. 2018, Benjaafar et al. 2018, Gurvich et al. 2019), matching mechanisms (Benjaafar et al. 2015, Ozkan and Ward 2017) and routing (Su 2018). Chen et al. (2018) surveys research opportunities exploring ride-hailing and other platforms.

Competition between ride-hailing platforms has received relatively little attention. Cohen and Zhang (2017) examines profit sharing contracts between duopolist ride-hailing platforms, and characterize conditions under which such agreements benefit both platforms. Bai and Tang (2018) identifies key factors that determine whether competing platforms earn strictly positive profit. Bernstein et al. (2018) considers how equilibrium outcomes in a duopoly depend on whether drivers work for one or both platforms, and show that all parties may be worse off when drivers work for both platforms. Nikzad (2018) examines the welfare effects of competition, finding that competition increases driver welfare, but may reduce customer welfare. Lin et al. (2018) shows that mergers between competing platforms can be beneficial for both customers and drivers. Liu et al. (2019) examines platforms that compete over independent-contractor workers by offering bonuses for multi-period participation. Our work differs substantively from the existing literature on ride-hailing platform competition in that we focus on the impact that access to AVs has on the ensuing equilibrium. Competition in two-sided markets has been extensively studied in the economics literature (e.g., Rochet and Tirole 2003, 2006); in contrast to our work, firms do not face capacity constraints in those papers.

This paper also builds on literature in which competing firms source from a common supplier. Salop and Scheffman (1987) considers a setting where the cost of an input increases in the total quantity purchased by competing firms, and show that it can be advantageous for a firm to "over-buy" so as raise the input's cost to the firm's competitors. Similarly, Schrader and Martin (1998), Arya et al. (2008), Chen and Guo (2014) and Wu and Zhang (2014) consider competitive settings where a firm's supply decisions affects its rival, and show that higher supply costs can lift profits, either due to the cost implications for the rival firm or the softening of competition. Qi et al. (2019) study competing buyers that reserve capacity at a common supplier. Our work is related to these papers in the sense that the labor market serves as a common supply source for both platforms. A key distinction of our work is that we focus on how the *intensity* of competition over the shared supply source affects firm decisions in equilibrium, and how these prescriptions depend on the availability of AVs.

Lastly, our paper is also related to research on capacity decisions made by firms in competition. Van Mieghem and Dada (1999) and Anupindi and Jiang (2008) consider settings where firms compete by making capacity, price and production decisions, and characterize the value of postponing production decisions after the realization of demand uncertainty. Additional aspects of this setting that have been addressed include forecast quality (Chod and Rudi 2006), multiple capacity types (Goyal and Netessine 2007) and investment timing (Swinney et al. 2011). A related set of literature focuses on competition with "reactive" capacity, in which firms have the ability to secure additional capacity following the realization of demand uncertainty (Caro and Martinez de Albeniz 2010, Afeche et al. 2014). In all the aforementioned papers, a firm's supply costs are invariant to its competitor's decisions. In contrast, we consider a setting in which firms compete over a shared supply source in addition to making independent capacity decisions, which introduces externalities that are absent in the existing literature on capacity investment and competition.

2 Model

A platform's decision of its AV fleet size is made over a longer-term horizon than its price and wage decisions. As such, we divide the time horizon into two periods. Long-term decisions are made in the first period: Platform 1 chooses the size of its AV fleet K_1, incurring cost $\theta c(K_1)$, where $\theta > 0$ and $c(K_1)$ is weakly convex, strictly increasing, twice differentiable and satisfies $c(0) = 0$; we refer to θ as platform 1's *AV cost*. AVs are prohibitively costly for platform 2, so its AV fleet $K_2 = 0$. Each platform $i \in \{1, 2\}$ observes the AV fleet of its rival platform $j \neq i$, K_j. Short-term decisions are made in the second period: Platform $i \in \{1, 2\}$ chooses its price p_i and wage w_i. Platform i's demand under prices $\mathbf{p} = <p_1, p_2>$ is $D_i(\mathbf{p}) = \alpha - \beta p_i + \gamma p_j$, where $\beta > \gamma \geq 0$; we refer to α as the *demand state*. Platform i's labor supply under wages $\mathbf{w} = <w_1, w_2>$ is $L_i(\mathbf{w}) = bw_i - gw_j$, where $b > g \geq 0$. Parameter g is the *cross-wage sensitivity of labor* supply, and as such is a measure of the *intensity of competition in the labor market*; we use the latter label because of its more managerial interpretation. Parameter g can be interpreted as reflecting the degree of differentiation of the platforms from the perspective of prospective workers, with differentiation decreasing in g, such that $g = 0$ corresponds to no competition in the labor market. Similarly, γ can be interpreted as reflecting the degree of differentiation of the platforms from the perspective of prospective customers, with differentiation decreasing in γ, such that $\gamma = 0$ corresponds to no competition in the consumer market. The assumption that labor supply is linear in wages has been used in the labor economics literature (e.g., Hamilton et al. 2000, Bhaskar et al. 2002) and parallels the commonly used assumption that demand is linear in prices. A unit of demand can be fulfilled by a unit of AV, which was obtained in the first period, or a unit of labor, which is obtained in the second. Accordingly, we restrict attention to the natural parameter range $(\mathbf{p}, \mathbf{w})$ wherein platform i sources labor to satisfy the demand unmet by its AVs: $L_i(\mathbf{w}) = \max\{D_i(\mathbf{p}) - K_i, 0\}$, where $D_i(\mathbf{p}) \geq 0$ for $i \in \{1, 2\}$. That is, if the platform's demand outstrips its AV supply $D_i(\mathbf{p}) > K_i$, the platform sources labor to make its total supply $K_i + L_i(\mathbf{w})$ meet the demand $D_i(\mathbf{p})$.

Platform i chooses its price and wage (p_i, w_i) to maximize its second-period contribution

$$v_i(\mathbf{p}, \mathbf{w}) = p_i D_i(\mathbf{p}) - w_i L_i(\mathbf{w}),$$

where, as noted above, $L_i(\mathbf{w}) = \max\{D_i(\mathbf{p}) - K_i, 0\}$. Let $\mathbf{p}^*(K_1) = <p_1^*(K_1), p_2^*(K_1)>$ and $\mathbf{w}^*(K_1) = <w_1^*(K_1), w_2^*(K_1)>$ denote equilibrium prices and wages under platform 1 AV fleet K_1. Platform i's second-period contribution under AV fleet K_1 and equilibrium prices and wages $(\mathbf{p}^*(K_1), \mathbf{w}^*(K_1))$ is

$$r_i(K_1) = v_i(\mathbf{p}^*(K_1), \mathbf{w}^*(K_1)). \tag{1}$$

Platform 1 chooses its AV fleet to maximize its (first-period) profit

$$\pi_1(K_1) = r_1(K_1) - \theta c(K_1).$$

Platform 2's (first-period) profit is $\pi_2(K_1) = r_2(K_1)$. Let K_1^* be the maximizer of platform 1's profit $\pi_1(K_1)$. Hence, $(K_1^*, \mathbf{p}^*, \mathbf{w}^*)$ denotes an equilibrium in platform 1's AV fleet and the platforms' prices and wages, where, for compactness, we drop the argument in $\mathbf{p}^*(K_1^*)$ and $\mathbf{w}^*(K_1^*)$.

In §4, we extend the model to allow the demand state α to be uncertain, AVs to have technological limitations such that they can only serve a portion of the market, and both platforms to acquire AVs. Because our focus is on high-level, strategic decisions (namely, AV fleet size), we abstract away from the more detail-level issue of rider waiting times, in the same spirit as Cachon et al. (2017), Su (2018), Bimpikis et al. (2019), Gurvich et al. (2019) and Liu et al. (2019). Proofs of all results are in the appendix, with the exception of the results in §4.2 and §4.3, which are in the electronic companion.

3 Results

§3.1 establishes "building block" equilibrium results which are used to address our research questions in §3.2, §3.3 and §3.4. The reader interested more in managerial issues than in technical details can skip ahead to §3.2.

3.1 Equilibrium Prices, Wages and Autonomous Vehicle Fleet

This section establishes the equilibrium in platform 1's AV fleet and the platforms' prices and wages $(K_1^*, \mathbf{p}^*, \mathbf{w}^*)$ is unique, and characterizes the equilibrium.

We begin by considering the platforms' price and wage decisions, for a given platform 1 AV fleet K_1. If platform 1's price is low $p_1 < (\alpha + \gamma p_2 - K_1)/\beta$, then its demand exceeds its AV fleet $D_1(\mathbf{p}) > K_1$, which implies the platform sources labor to satisfy the demand unmet by its AV fleet $L_1(\mathbf{w}) = D_1(\mathbf{p}) - K_1 > 0$, which implies the platform's wage $w_1 = [D_1(\mathbf{p}) - K_1 + gw_2]/b$. In this case, platform 1's second-period contribution is

$$v^l(\mathbf{p}, w_2) = p_1 D_1(\mathbf{p}) - [(D_1(\mathbf{p}) - K_1 + gw_2)/b][D_1(\mathbf{p}) - K_1].$$

If platform 1's price is high $p_1 \geq (\alpha + \gamma p_2 - K_1)/\beta$, then $D_1(\mathbf{p}) \leq K_1$ and the platform does not source labor $L_1(\mathbf{w}) = 0$. In this case, platform 1's second-period contribution is

$$v^s(\mathbf{p}) = p_1 D_1(\mathbf{p}).$$

Thus, platform 1's second-period contribution is

$$v_1(\mathbf{p}, w_2) = \begin{cases} v^l(\mathbf{p}, w_2) & \text{if } p_1 < (\alpha + \gamma p_2 - K_1)/\beta \\ v^s(\mathbf{p}) & \text{if } p_1 \geq (\alpha + \gamma p_2 - K_1)/\beta, \end{cases} \tag{2}$$

where the argument w_1 is eliminated. By parallel argument, platform 2's wage $w_2 = [D_2(\mathbf{p}) + gw_1]/b$ and second-period contribution is

$$v_2(\mathbf{p}, w_1) = p_2 D_2(\mathbf{p}) - [(D_2(\mathbf{p}) + gw_1)/b] D_2(\mathbf{p}). \tag{3}$$

Lemma 1 characterizes the platforms' equilibrium prices and wages under platform 1 AV fleet K_1. Let $\tilde{p}_1^l(K_1) = [(\alpha + \gamma p_2)(2\beta + b) - 2\beta K_1 + \beta g w_2]/[2\beta(\beta + b)]$, $\tilde{p}_1^e(K_1) = (\alpha + \gamma p_2 - K_1)/\beta$, $\tilde{p}_1^s(K_1) = (\alpha + \gamma p_2)/(2\beta)$, $\tilde{w}_1^l(K_1) = [(\alpha + \gamma p_2 - 2K_1)b + (\beta + 2b)gw_2]/[2(\beta + b)b]$, $\tilde{w}_1^e(K_1) = \tilde{w}_1^s(K_1) = gw_2/b$, $\tilde{p}_2(K_1) = [(\alpha + \gamma p_1)(2\beta + b) + \beta g w_1]/[2\beta(\beta + b)]$ and $\tilde{w}_2(K_1) = [(\alpha + \gamma p_1)b + (\beta + 2b)gw_1]/[2(\beta + b)b]$. Let $(p_1^u(K_1), w_1^u(K_1), p_2^u(K_1), w_2^u(K_1))$ denote the unique solution to $p_1^u(K_1) = \tilde{p}_1^u(K_1)$, $w_1^u(K_1) = \tilde{w}_1^u(K_1)$, $p_2^u(K_1) = \tilde{p}_2(K_1)$ and $w_2^u(K_1) = \tilde{w}_2(K_1)$, where $u \in \{e, l, s\}$. Let

$\mathbf{p}^u(K_1) = <p_1^u(K_1), p_2^u(K_1)>$ and $\mathbf{w}^u(K_1) = <w_1^u(K_1), w_2^u(K_1)>$, where $u \in \{e, l, s\}$. Further, let

$$\psi^l = \frac{(4\beta^2 - \gamma^2)b(b^2 - g^2) + \beta(2\beta^2 - \gamma^2)(2b^2 - g^2) - \beta^2\gamma bg}{\beta[\beta(\beta+\gamma)(2b+g) + (2\beta+\gamma)b(b+g)](b-g)} \tag{4}$$

$$\psi^s = \frac{(4\beta^2 - \gamma^2)b(b^2 - g^2) + \beta(2\beta^2 - \gamma^2)(2b^2 - g^2)}{\beta[\beta(\beta+\gamma)(2b^2-g^2) + (2\beta+\gamma)b(b^2-g^2)]}. \tag{5}$$

Note $0 < \psi^s \leq \psi^l$, where the weak inequality is strict if and only if $g > 0$.

Lemma 1 *Under platform 1 AV fleet K_1, the equilibrium prices and wages are unique and given by*

$$(\mathbf{p}^*(K_1), \mathbf{w}^*(K_1)) = \begin{cases} (\mathbf{p}^l(K_1), \mathbf{w}^l(K_1)) & \text{if } K_1 < \alpha/\psi^l, \\ (\mathbf{p}^e(K_1), \mathbf{w}^e(K_1)) & \text{if } K_1 \in [\alpha/\psi^l, \alpha/\psi^s], \\ (\mathbf{p}^s(K_1), \mathbf{w}^s(K_1)) & \text{if } K_1 > \alpha/\psi^s. \end{cases} \tag{6}$$

Platform 1's equilibrium AV fleet deployment and labor sourcing decisions depend on its fleet size in a natural way. We say that platform 1 *deploys its full AV fleet* if $K_1 \leq D_1(\mathbf{p}^*(K_1))$ and *sources labor* if $L_1(\mathbf{w}^*(K_1)) > 0$; recall $L_1(\mathbf{w}) = \min\{D_1(\mathbf{p}) - K_1, 0\}$. If its AV fleet is small, then, under the equilibrium prices and wages, platform 1 deploys its full AV fleet and sources labor, i.e., $K_1 < D_1(\mathbf{p}^*(K_1))$. If its fleet size is moderate, then platform 1 deploys its full AV fleet but does not source labor, i.e., $K_1 = D_1(\mathbf{p}^*(K_1))$. If its fleet size is large, then platform 1 deploys only a portion of its AV fleet and does not source labor, i.e., $K_1 > D_1(\mathbf{p}^*(K_1))$. The superscript l is mnemonic for sourcing labor, s for slack AV capacity, and e for equating AV fleet with demand.

Lemma 2 characterizes platform 1's equilibrium AV fleet. For the case in which there is competition in the consumer market $\gamma > 0$, ensuring that the platform 1's profit $\pi_1(K_1)$ is well behaved requires that the intensity of competition in the labor market not be too large. Let $\tilde{g} = [\sqrt{\beta^4 + 4\gamma^2(2\beta^2 + 3b\beta + b^2)} - \beta^2]b/[2\gamma(\beta+b)]$ if $\gamma > 0$, and $\tilde{g} = b$ otherwise. Note if $\gamma > 0$, then $\tilde{g} > [\sqrt{\beta^2 + 8\gamma^2} - \beta]b/(2\gamma)$. Let $K^u = \arg\max_{K_1 \geq 0}\{v_1(\mathbf{p}^u(K_1), \mathbf{w}^u(K_1)) - \theta c(K_1)\}$, where $u \in \{e, l\}$. Let $\theta^e = \lim_{K_1 \downarrow (\alpha/\psi^l)}\{(\partial/\partial K_1)r_1(K_1)/(\partial/\partial K_1)c(K_1)\}$ and $\theta^l = \lim_{K_1 \uparrow (\alpha/\psi^l)}\{(\partial/\partial K_1)r_1(K_1)/(\partial/\partial K_1)c(K_1)\}$. Let $\theta^0 = \lim_{K_1 \to 0}\{[(\partial/\partial K_1)r_1(K_1)]/[(\partial/\partial K_1)c(K_1)]\}$ if $\lim_{K_1 \to 0}(\partial/\partial K_1)c(K_1) > 0$, and $\theta^0 = \infty$ otherwise. Let $\hat{\theta} = \sup_{g \in [0,b)} \theta^0$. Note θ^l, θ^e and θ^0 depend on α, β, γ, b, and g.

Lemma 2 *If the intensity of competition in the labor market $g < \tilde{g}$, then platform 1's profit $\pi_1(K_1)$ is quasi-concave in K_1 and platform 1's equilibrium AV fleet is unique and given by*

$$K_1^* = \begin{cases} K^e & \text{if } \theta < \theta^e, \\ \alpha/\psi^l & \text{if } \theta \in [\theta^e, \theta^l], \\ K^l & \text{if } \theta \in (\theta^l, \theta^0), \end{cases} \tag{7}$$

where $\theta^e \leq \theta^l < \theta^0$, where the first inequality is strict if and only if $\gamma > 0$. There exists $\underline{\underline{g}}$ such that $\theta^l > 0$ if and only if $g > \underline{\underline{g}}$. Further, $\underline{\underline{g}} = 0$ if $\gamma = 0$, and $\underline{\underline{g}} \in (0, \tilde{g})$ if $\gamma > 0$. Further, $K_1^* = 0$ if and only if $\theta \geq \theta^0$; $K_1^* = 0$ for $g \in [0, b)$ if and only $\theta \geq \hat{\theta}$.

Together, Lemmas 1 and 2 characterize the unique equilibrium in platform 1's AV fleet and the platforms' prices and wages $(K_1^*, \mathbf{p}^*, \mathbf{w}^*)$. Platform 1's equilibrium AV fleet and labor sourcing decisions depend on the intensity of competition in the labor market and the AV cost in a natural way. If the intensity of competition in the labor market is low $g \leq \underline{\underline{g}}$ or the AV cost is high $\theta > \theta^l$, then platform 1 chooses a sufficiently small AV fleet K_1^* such that it subsequently sources labor $L_1^* > 0$. If instead, the intensity of competition in the labor market is high $g \in (\underline{\underline{g}}, \tilde{g})$ and the AV cost is low $\theta \leq \theta^l$, then platform 1 chooses a sufficiently large AV fleet K_1^* such that it subsequently does not source labor $L_1^* = 0$.

In the sequel, for analytical tractability, we assume that $g < \tilde{g}$. (Note this assumption is non-restrictive when there is no competition in the consumer market, $\gamma = 0$, because in that case $\tilde{g} = b$.) In a numerical study that relaxes the $g < \tilde{g}$ restriction, we observe that the results are consistent with our Propositions; see Appendix E in the electronic companion.

3.2 Impact of Intensity of Competition in Labor Market on Price

This section focuses on the impact of the intensity of competition in the labor market on platform 1's price.

How does a platform's access to supply-side (namely, AV) technology change prescriptions for its demand-side (namely, pricing) decisions? It is natural to conjecture that platform 1's price increases in the intensity of competition in the labor market. Increasing the intensity of competition in the labor market increases the marginal cost of labor, which makes it attractive for the platform to decrease its labor supply. Reducing its labor supply pushes the platform to serve a smaller market, which the platform accomplishes by setting a higher price. Proposition 1i confirms this intuition for the case in which the AV cost is prohibitive. Proposition 1ii reveals how this prescription is reversed when AV technology becomes accessible.

Proposition 1 *(i) Suppose the AV cost is high $\theta \geq \hat{\theta}$. Then platform 1's equilibrium AV fleet $K_1^* = 0$, and platform 1's equilibrium price p_1^* strictly increases in the intensity of competition in the labor market g on $g \in (0, b)$. (ii) Suppose the AV cost is low $\theta < \bar{\theta}$, where $\bar{\theta} \in (0, \hat{\theta}]$. Then there exist $\bar{\gamma} > 0$, $\underline{g} \geq 0$ and $\bar{g} > \underline{g}$ such that platform 1's equilibrium price p_1^* strictly decreases in the intensity of competition in the labor market g on $g \in (\underline{g}, \bar{g})$ for all $\gamma \in [0, \bar{\gamma}]$.*

Proposition 1ii provides sufficient conditions under which platform 1's equilibrium price *decreases* in the intensity of competition in the labor market g. That the conditions do not require the presence of competition in the consumer market $\gamma > 0$ reveals that the decreasing-price result is not driven by demand-side competition. To isolate the driving force behind the decreasing-price result, we begin by considering the setting with no competition in the consumer market $\gamma = 0$. (One can show $\gamma = 0$ implies $\underline{g} = 0$.)

To see the intuition, observe that increasing the intensity of competition in the labor market g has two effects on platform 1's equilibrium price: a price-increasing *labor cost effect* and a price-decreasing *fleet size effect*

$$\frac{dp_1^*}{dg} = \underbrace{\frac{\partial p_1^*}{\partial g}}_{\substack{>0 \\ \text{labor cost effect}}} + \underbrace{\frac{\partial p_1^*}{\partial K_1}\frac{dK_1^*}{dg}}_{\substack{<0 \\ \text{fleet size effect}}}.$$

As noted before Proposition 1, increasing the intensity of competition in the labor market increases the marginal cost of labor. This makes it attractive for platform 1 to decrease its labor supply and so serve a smaller consumer market, which is accomplished by setting a higher price (the labor cost effect).

The effect of the intensity of competition in the labor market on the price through the AV fleet size is more subtle. Increasing the intensity of competition in the labor market g makes the labor market comparatively less attractive as a source of supply, prompting platform 1 to increase its AV fleet in period one, $(d/dg)K_1^* > 0$. Platform 1's price decreases in its AV fleet, $(\partial/\partial K_1)p_1^* < 0$. To see why, let $S_1 = K_1 + L_1(\mathbf{w})$ denote platform 1's total supply in period two, and observe that platform 1's price decision (equivalently, its quantity decision of how much demand to serve) is driven by its marginal cost of total supply in period two. Increasing the AV fleet reduces the labor quantity required to achieve a given total supply $L_1(\mathbf{w}) = S_1 - K_1$, which drives down the cost of supply because the marginal cost of labor increases in the labor quantity. Hence, as platform 1's AV fleet increases, its marginal cost of total supply in period two decreases, prompting the platform to serve a larger market, which it does by setting a lower price. When there is competition in the consumer market, $\gamma > 0$, a second force emerges to reinforce the effect of platform 1's AV fleet expansion decreasing its marginal cost of total supply in period two. Platform 1 increasing its AV fleet in period one commits the platform to compete more aggressively in the consumer market, making that market less attractive to platform 2, which pushes platform 2 to source less labor and offer a lower wage. Platform 2 reducing its wage pushes down the platform 1's marginal cost of total supply in period two.

When the AV cost is prohibitively high $\theta \geq \hat{\theta}$ such that platform 1's equilibrium AV fleet $K_1^* = 0$, only the labor cost effect is at work, and, consequently, platform 1's equilibrium price p_1^* strictly increases in the intensity of competition in the labor market g (Proposition 1i). In contrast, when the AV cost is low $\theta < \bar{\theta}$, platform 1's equilibrium AV fleet is sensitive to the intensity of competition in the labor market, and the price-decreasing fleet size effect dominates the price-increasing labor cost effect, for a range of g (Proposition 1ii). (When the AV cost is moderate $\theta \in (\bar{\theta}, \hat{\theta})$, both effects are at work, and, depending on the parameters, either effect can dominate.) The restriction in Proposition 1ii that the cost AV be low $\theta < \bar{\theta}$ need not be onerous. For example, when the cost of an AV fleet is linear in the fleet size K_1 (i.e., $\theta c(K_1) = \theta K_1$), the AV cost threshold $\bar{\theta} = \theta^0$; thus, the condition $\theta < \bar{\theta}$ is equivalent to the very mild condition that the AV cost be sufficiently low that platform 1's equilibrium AV fleet is strictly positive $K_1^* > 0$ (see Lemma 2).

A purpose of this paper is to identify what factors drive reversals in intuitive prescriptions for platform decisions. As noted above, the presence of demand-side competition is not necessary for the reversal in price prescription. The next section identifies a prescription whose reversal *is* driven by the presence of demand-side competition.

3.3 Impact of Intensity of Competition in Labor Market on AV Fleet

This section focuses on the impact of the intensity of competition in the labor market on platform 1's AV fleet.

How does the presence of demand-side competition change prescriptions for a platform's supply-side (namely, AV fleet) decisions? Proposition 2 characterizes how platform 1's equilibrium AV fleet K_1^* changes with the intensity of competition in the labor market g. Because, when the AV cost is high $\theta \geq \hat{\theta}$, $K_1^* = 0$ and so is invariant to g (see Lemma 2), in Proposition 2, we restrict attention to the interesting case $\theta < \hat{\theta}$.

Increasing the intensity of competition in the labor market g increases the marginal cost of labor, which makes labor a less attractive supply source relative to AVs and pushes platform 1 to increase its AV fleet. Proposition 2i confirms this intuition for the case with no competition in the consumer market $\gamma = 0$. Proposition 2ii reveals how this prescription is reversed when there is competition in the consumer market $\gamma > 0$.

Proposition 2 *(i) Suppose there is no competition in the consumer market, $\gamma = 0$. There exist $\hat{g} > \check{g} \geq 0$ such that platform 1's equilibrium AV fleet K_1^* increases in the intensity of competition in the labor market g, strictly so if and only if $g \in (\check{g}, \hat{g})$. (ii) Suppose there is competition in the consumer market, $\gamma > 0$. There exists $\bar{\theta} > 0$, $\underline{g} > 0$, and $\bar{g} > \underline{g}$ such that if the AV cost is small $\theta < \bar{\theta}$, then platform 1's equilibrium AV fleet K_1^* strictly decreases in the intensity of competition in the labor market g on $g \in (\underline{g}, \bar{g})$.*

Platform 2ii provides sufficient conditions under which platform 1's equilibrium AV fleet K_1^* strictly *decreases* in the intensity of competition in the labor market g. To understand how this can occur, it is useful to understand the forces that drive platform 1's choice of AV fleet K_1^*, in equilibrium. Platform 1's expanding its AV fleet has two potentially-opposing effects on the platform's second-period contribution. In the absence of competition in the consumer market, $\gamma = 0$, a larger fleet always increases platform 1's contribution by reducing its need to source costly labor. We refer to this as the *supply substitution effect*. Increasing the intensity of competition in the labor market increases the marginal cost of labor, magnifying the supply substitution effect, making it attractive for platform 1 to reduce it total supply cost via a larger AV fleet.

In the presence of competition in the consumer market, $\gamma > 0$, selecting a large AV fleet in period one commits platform 1 to compete aggressively in period two, and excessive competition erodes the platform's revenue. We refer to this as the *price competition effect*. Increasing the intensity of competition in the labor market increases platform 2's marginal cost of labor, making platform 2 a less formidable rival. Facing a less formidable rival makes it attractive for platform 1 to soften competition by choosing a smaller AV fleet.

When the AV cost θ is small, platform 1 tends to source little labor relative to platform 2. Because platform 1 sources little labor, increasing the intensity of competition in the labor market has a small effect on platform 1: the fleet-increasing supply substitution effect is weak. Because platform 2 sources a larger labor quantity, increasing the intensity of competition in the labor market has a comparatively large effect in making platform 2 a less formidable rival: the fleet-reducing supply substitution effect is strong. The net result is that the fleet-reducing effect dominates, for a range of g.

The interesting result wherein platform 1's equilibrium AV fleet K_1^* strictly decreases in the intensity of competition in the labor market g occurs when platform 1's AV cost is "low." This occurs when a platform quite successful in developing (and driving down the cost of) AV technology competes in a market with a rival lacking access to AVs. The rival could lack such access because it lacked capital to invest in AV technology development, chose not to invest in such development, or invested but failed. To the extent that critical technological advances made by the successful platform are proprietary and granted strong intellectual property protection, the rival's access would be further thwarted. Nonetheless, the phenomenon that a platform's equilibrium AV fleet strictly decreases in the intensity of competition in the labor market does not require a sharp technological imbalance between platforms. §4.3 considers the setting in which both platforms have access to AVs and provides sufficient conditions under which the phenomenon occurs (see Proposition 2Cii).

3.4 Impact of AV Cost on Platform Profit

This section focuses on the impact of platform 1's AV cost on platform 2's profit.

How is a platform affected by its rival's access to AVs, or more precisely, a reduction of the rival's AV cost? Because a reduction in its AV cost makes platform 1 a more formidable rival, one might expect platform 2 would be hurt. Proposition 3 provides a simple necessary and sufficient condition for platform 2 to *benefit* from a reduction in platform 1's AV cost. Because g is the cross-wage and b is the own-wage sensitivity of labor, g/b is the *relative wage sensitivity of labor.* Because γ is the cross-price and β is the own-price sensitivity of demand, γ/β is the *relative price sensitivity of demand.*

Proposition 3 *Platform 2's equilibrium profit $\pi_2(K_1^*)$ strictly decreases in platform 1's AV cost θ on $\theta \in (\theta^l, \theta^0)$ if and only if the relative wage sensitivity of labor is greater than the relative price sensitivity of demand*

$$g/b > \gamma/\beta. \tag{8}$$

Further, $\pi_2(K_1^*)$ weakly increases in platform 1's AV cost θ on $\theta \in (0, \theta^l) \cup (\theta^0, \infty)$.

A reduction of platform 1's AV cost θ has two opposing effects on platform 2: a harmful *consumer market effect* and a beneficial *labor market effect.* Platform 1 responds to a reduction in its AV cost by expanding its AV fleet. This commits platform 1 to compete more aggressively on price in the consumer market (the consumer market effect), hurting platform 2. An increase in platform 1's AV fleet reduces platform 1's marginal value of labor, so platform 1 competes less aggressively on wage in the labor market (the labor market effect), benefiting platform 2.

Whether the consumer market effect or labor market effect dominates depends on platform 1's AV cost and the parameters that govern the demand and supply functions. In particular, the labor market effect is present if and only if platform 1 sources labor, which (by Lemma 2) occurs when the AV cost is moderate $\theta \in (\theta^l, \theta^0)$. In that case, which effect dominates depends on a simple comparison of the relative wage sensitivity of labor and the relative price sensitivity of demand (inequality (8)). When the relative wage sensitivity of labor is large, platform 2's marginal cost of labor is sensitive to platform 1's wage. Consequently, a small decrease in platform 1's wage translates to a relatively large reduction in platform 2's labor cost (the beneficial labor market effect is large). Conversely, when the relative price sensitivity of demand is small, a small decrease in platform 1's price translates to relatively small reduction in platform 2's revenue (the harmful consumer market effect is small). Thus, when the AV cost is moderate $\theta \in (\theta^l, \theta^0)$, platform 2 benefits from a reduction in platform 1's AV cost if and only if the relative wage sensitivity of labor is greater than the relative price sensitivity of demand. In contrast, if platform 1's AV cost is small $\theta < \theta^l$ such that the platform does not source labor, then the only effect of reducing platform 1's AV cost is to push the platform to compete more aggressively in the consumer market, hurting platform 2.

Currently, no platforms use AVs, which corresponds in our setting to platform 1's equilibrium AV fleet $K_1^* = 0$, or equivalently, platform 1's AV cost being prohibitive $\theta \geq \theta^0$ (see Lemma 2). When the first platform to obtain AVs initially does so, it is likely that AVs will be sufficiently costly that the platform will continue to employ labor alongside AVs (Lyft 2019, Uber 2019), which corresponds to the case where the AV cost $\theta \in (\theta^l, \theta^0)$. Thus, inequality (8) is the key condition which determines whether a platform will benefit by its rival's *initially* obtaining AVs.

The contribution of Proposition 3 is not in showing that is *possible* for a platform to benefit by its rival's access to AVs. Rather, the contribution is in providing a simple, readily interpretable necessary and sufficient condition under which a platform does benefit by its rival's access.

4 Extensions

In this section, we extend the model to allow the demand state α to be uncertain, AVs to have technological limitations such that they can only serve a portion of the market, and both platforms to acquire AVs. For each extension, we present analogues to Propositions 1, 2 and 3 that demonstrate the robustness of our central results.

4.1 Demand Uncertainty

This section considers an extension of our base model wherein the demand state α is uncertain at the time of platform 1's AV fleet size decision. Consistent with the practice of ride-hailing platforms of setting prices and wages in response to market conditions (Cachon et al. 2017), platform $i \in \{1, 2\}$ chooses its price p_i and wage w_i after observing the realized demand state α. At the time of making its long-term AV fleet decision in the first period, platform 1 anticipates repeatedly observing the demand state and making short-run state-dependent price and wage decisions. For simplicity, we collapse the platforms' short-run demand state observations and decisions into a single second period. Let $f(\alpha)$ denote the density function of the demand state; we assume f is continuously differentiable on the function's support $[\underline{\alpha}, \bar{\alpha}]$, where $\underline{\alpha} > 0$. Let $r_i(\alpha, K_1)$ denote platform i's second-period contribution under platform 1 AV fleet K_1, realized demand state α and equilibrium prices and wages $(\mathbf{p}^*(\alpha, K_1), \mathbf{w}^*(\alpha, K_1))$, as in (1). Under AV fleet K_1, platform i's expected second-period contribution is

$$R_i(K_1) = \int_{\underline{\alpha}}^{\bar{\alpha}} r_i(\alpha, K_1) f(\alpha) d\alpha,$$

platform 1's (first-period) expected profit is $\Pi_1(K_1) = R_1(K_1) - \theta c(K_1)$, and platform 2's (first-period) expected profit is $\Pi_2(K_1) = R_2(K_1)$. Define $\hat{\theta} = \sup_{g \in [0,b)} \theta^0(\bar{\alpha})$, where $\theta^0(\alpha)$ denotes the dependence on the demand state α in the special case where the demand state is deterministic. Let $p_1^*(\alpha)$ denote platform 1's equilibrium price under realized demand state α.

Proposition 1A *(i) Suppose the AV cost is high* $\theta > \hat{\theta}$*. Then platform 1's equilibrium AV fleet* $K_1^* = 0$*, and platform 1's equilibrium price* $p_1^*(\alpha)$ *strictly increases in the intensity of competition in the labor market* g *on* $g \in (0, b)$ *for all* $\alpha \in [\underline{\alpha}, \bar{\alpha}]$*. (ii) Suppose the AV cost is low* $\theta \leq \bar{\theta}$*, where* $\bar{\theta} \in (0, \hat{\theta}]$*. There exists* $\lambda < 1$ *such that if* $\underline{\alpha}/\bar{\alpha} > \lambda$*, then there exist* $\bar{\gamma} > 0$*,* $\underline{g} > 0$*,* $\bar{g} > \underline{g}$*,* $\check{\alpha} > 0$*, and* $\hat{\alpha} > \check{\alpha}$ *such that* $p_1^*(\alpha)$ *strictly decreases in the intensity of competition in the labor market* g *on* $g \in (\underline{g}, \bar{g})$ *for all* $\gamma \in [0, \bar{\gamma}]$ *and* $\alpha \in (\check{\alpha}, \hat{\alpha})$*.*

Parallel to Proposition 1ii, Proposition 1Aii provides sufficient conditions under which platform 1's equilibrium price strictly decreases in the intensity of competition in the labor market. Proposition 1Aii adds one condition: that the ratio of lower to upper limit of the support of the demand state $\underline{\alpha}/\bar{\alpha}$ not be too small. Similarly, in providing conditions under which platform 1's equilibrium AV fleet either increases or decreases in the intensity of competition in the labor market, Proposition 2A imposes this condition. The restriction on $\underline{\alpha}/\bar{\alpha}$ is a technical condition that ensures the quasi-concavity of platform 1's profit $\Pi_1(K_1)$. Appendix E in the electronic companion provides numerical evidence that the assumption is not particularly restrictive in that Propositions 1Aii and 2A hold in examples where $\underline{\alpha}/\bar{\alpha}$ is relatively small.

Proposition 2A *There exists* $\lambda < 1$ *such that if* $\underline{\alpha}/\bar{\alpha} > \lambda$*, then the following statements hold. (i) Suppose there is no competition in the consumer market,* $\gamma = 0$*. Then platform 1's equilibrium AV fleet* K_1^* *increases in the intensity of competition in the labor market* g *on* $g \in (0, b)$*. (ii) Suppose there is competition in the consumer market,* $\gamma > 0$*. There exists* $\bar{\theta} > 0$*,* $\underline{g} > 0$*, and* $\bar{g} > \underline{g}$ *such that if the AV cost is small* $\theta < \bar{\theta}$*, then platform 1's equilibrium AV fleet* K_1^* *strictly decreases in the intensity of competition in the labor market* g *on* $g \in (\underline{g}, \bar{g})$*.*

Proposition 3A reveals that the key necessary and sufficient condition under which platform 2 benefits by a reduction in platform 1's AV cost is unchanged by the extension to uncertain demand.

Proposition 3A *There exists* $\bar{\theta} > \underline{\theta} > 0$ *such that platform 2's expected equilibrium profit* $\Pi_2(K_1^*)$ *strictly decreases in platform 1's AV cost* θ *on* $\theta \in [\underline{\theta}, \bar{\theta}]$ *if and only if the relative wage sensitivity of labor is greater than the relative price sensitivity of demand (i.e., inequality* (8) *holds).*

4.2 Limited Sophistication of Autonomous Vehicles

This section considers an extension of our base model wherein AVs are limited in their sophistication such that they cannot serve all customers. Ride-hailing platforms (e.g., Lyft, Uber) anticipate that, at least initially, AVs will not be sufficiently sophisticated technologically to safely serve all customer origin-and-destination requests. AVs will not be able to serve origin-destination pairs that require routes that are particularly complex or challenging (e.g., involving bridges, tunnels, vehicle speed in excess of 35 miles per hour, heavy traffic, difficult pick-up or drop-off locations); these origin-destination pairs can only be served by human drivers. Origin-destination pairs which are less technologically demanding can be served by AVs as well as human drivers. Thus, AVs may be limited to geographic areas which involve low vehicle speeds and are well-mapped and well-understood by the platform in terms of routes and pick-up and drop-off locations (Lyft 2019, Uber 2019).

To capture that AVs are limited in their sophistication, we let $v \in \{a, o\}$ index two markets, where a represents the market that AVs can serve, and o represents the market that AVs cannot. Let p_i^v denote platform i's price in market v. Let $\mathbf{p}^v = <p_1^v, p_2^v>$ and $\mathbf{p} = <p_1^a, p_1^o, p_2^a, p_2^o>$. Platform i's market a demand $D_i^a(\mathbf{p}^a) = \tau(\alpha - \beta p_i^a + \gamma p_j^a)$ and market o demand $D_i^o(\mathbf{p}^o) = (1-\tau)(\alpha - \beta p_i^o + \gamma p_j^o)$, where $\tau \in (0, 1)$ reflects the relative size of the market that AVs can serve. Platform i sources labor to satisfy the demand unmet by its AVs $L_i(\mathbf{w}) = \max\{D_i^a(\mathbf{p}^a) - K_i, 0\} + D_i^o(\mathbf{p}^o)$, allocating labor $\max\{D_i^a(\mathbf{p}^a) - K_i, 0\}$ to market a and labor $D_i^o(\mathbf{p}^o)$ to market o. Platform i chooses its prices (p_i^a, p_i^0) and wage w_i to maximize its second-period contribution

$$v_i(\mathbf{p}, \mathbf{w}) = p_i^a D_i^a(\mathbf{p}^a) + p_i^o D_i^o(\mathbf{p}^o) - w_i L_i(\mathbf{w}), \tag{9}$$

where $L_i(\mathbf{w}) = \max\{D_i^a(\mathbf{p}^a) - K_i, 0\} + D_i^o(\mathbf{p}^o)$. Platform i's second-period contribution under platform 1 AV fleet K_1 and equilibrium prices $\mathbf{p}^*(K_1) = <p_1^{a*}(K_1), p_1^{o*}(K_1), p_2^{a*}(K_1), p_2^{o*}(K_1)>$ and wages $\mathbf{w}^*(K_1) = <w_1^*(K_1), w_2^*(K_1)>$ is $r_i(K_1) = v_i(\mathbf{p}^*(K_1), \mathbf{w}^*(K_1))$. Under platform 1 AV fleet K_1, platform 1's (first-period) profit is $\pi_1(K_1) = r_1(K_1) - \theta c(K_1)$ and platform 2's (first-period) profit is $\pi_2(K_1) = r_2(K_1)$.

Define $\theta^0 = \sup\{\theta > 0 | K_1^* > 0\}$. Note that θ^0 depends implicitly on g. Let $\hat{\theta} = \sup_{g \in [0,b)} \theta^0(g)$.

Proposition 1B *(i) Suppose the AV cost is high $\theta > \hat{\theta}$. Then platform 1's equilibrium AV fleet $K_1^* = 0$, and platform 1's equilibrium price p_1^{v*} in market $v \in \{a, o\}$ strictly increases in the intensity of competition in the labor market g on $g \in (0, b)$. (ii) Suppose the AV cost is low $\theta < \bar{\theta}$, where $\bar{\theta} \in (0, \hat{\theta}]$. Then there exists $\bar{\tau} \in (0, 1)$ such that for all $\tau > \bar{\tau}$, there exist $\bar{\gamma} > 0$, $\underline{g} \geq 0$ and $\bar{g} \in (\underline{g}, \tilde{g})$ such that platform 1's equilibrium price p_1^{v*} in market $v \in \{a, o\}$ strictly decreases in the intensity of competition in the labor market g on $g \in (\underline{g}, \bar{g})$ for all $\gamma \in [0, \bar{\gamma}]$.*

Parallel to Proposition 1ii, Proposition 1Bii provides sufficient conditions under which platform 1's equilibrium price p_1^{v*} in market $v \in \{a, o\}$ strictly decreases in the intensity of competition in the labor market. Proposition 1Bii adds one condition: that the size of the market AVs can serve τ is sufficiently large. This condition is *necessary* for platform 1's price to decrease in the intensity of competition in the labor market. To see why, recall that increasing the intensity of competition in the labor market g has a price-increasing *labor cost effect* and a price-decreasing *fleet size effect.*

If the size of the market that AVs can serve τ is very small, then the AV fleet size and the fleet size effect are small, and the price-increasing labor cost effect dominates. For the interesting price-decreasing result to occur, it must be that the size of the market that AV can serve is sufficiently large such that AVs play a non-trivial role in platform 1's decisions. Nonetheless, the restriction that the size of the market AVs can serve $\tau > \bar{\tau}$ need not be overly restrictive; Appendix E in the electronic companion provides a numerical example in which $\bar{\tau} = 0.1$.

Proposition 2B *(i) Suppose there is no competition in the consumer market, $\gamma = 0$. There exists $\hat{\theta} > \check{\theta} > 0$ such that if $\theta \in [\check{\theta}, \hat{\theta}]$, then platform 1's equilibrium AV fleet K_1^* strictly increases in the intensity of competition in the labor market g on $g \in (0, b)$. (ii) Suppose there is limited competition in the consumer market, $\gamma \in (0, \bar{\gamma}]$, where $\bar{\gamma} > 0$. There exists $\bar{\theta} > \underline{\theta} \geq 0$ and $\bar{g} > 0$ such that if the AV cost is moderate $\theta \in (\underline{\theta}, \bar{\theta}]$, then platform 1's equilibrium AV fleet K_1^* strictly decreases in the intensity of competition in the labor market g on $g \in (0, \bar{g})$.*

Parallel to Proposition 2ii, Proposition 2Bii provides sufficient conditions under which platform 1's equilibrium AV fleet strictly decreases in the intensity of competition in the labor market.

Proposition 3B reveals that the key necessary and sufficient condition under which platform 2 benefits by a reduction in platform 1's AV cost in unchanged by the extension in which AVs can only serve some customers.

Proposition 3B *Platform 2's equilibrium profit $\pi_2(K_1^*)$ strictly decreases in platform 1's AV cost θ on $\theta \in (\theta^l, \theta^0)$ if and only if the relative wage sensitivity of labor is greater than the relative price sensitivity of demand (i.e., inequality (8) holds).*

4.3 Both Platforms Can Acquire Autonomous Vehicles

This section considers an extension of our base model wherein both platforms can acquire AVs. More precisely, in the first period, platform $i \in \{1, 2\}$ chooses the size of its AV fleet K_i, incurring cost $\theta_i c(K_i)$. In the definitions of the second-period equilibrium prices $\mathbf{p}^*$ and wages $\mathbf{w}^*$ and platform i's second-period contribution under the equilibrium prices and wages r_i, we replace the argument K_1 with the vector of AV fleets $\mathbf{K} = <K_1, K_2>$. Platform i's first-period profit is $\pi_i(\mathbf{K}) = r_i(\mathbf{K}) - \theta_i c(K_i)$.

We begin by considering the case where the platforms are symmetric $\theta_1 = \theta_2 = \theta$. In the electronic companion we show that if the intensity of competition in the labor market is not too large $g \in [0, \tilde{g})$, where $\tilde{g} > 0$, then only one symmetric equilibrium, $K_1^* = K_2^* = K^*$, exists. Accordingly, we restrict attention to $g \in [0, \tilde{g})$ and symmetric equilibria. It is straightforward to verify that under the symmetric AV fleet equilibrium, there exists a unique equilibrium in prices and wages, and that equilibrium is symmetric: $p_1^* = p_2^* = p^*$ and $w_1^* = w_2^* = w^*$.

While Proposition 1 examines the effect of the intensity of competition in the labor market on platform 1's equilibrium price, the parallel result in Proposition 1C applies to both platforms' equilibrium prices. Define $\theta^0 = \sup\{\theta > 0 | K^* > 0\}$. Note that θ^0 depends implicitly on g. Let $\hat{\theta} = \sup_{g \in [0,b)} \theta^0(g)$.

Proposition 1C *(i) Suppose the AV cost is high $\theta > \hat{\theta}$. Then platform 1's equilibrium AV fleet $K_1^* = 0$, and the equilibrium price p^* strictly increases in the intensity of competition in the labor market g on $g \in (0, b)$. (ii) Suppose the AV cost is low $\theta < \bar{\theta}$, where $\bar{\theta} \in (0, \hat{\theta}]$. Then there exist $\bar{\gamma} > 0$, $\underline{g} > 0$ and $\bar{g} > \underline{g}$ such that the equilibrium price p^* strictly decreases in the intensity of competition in the labor market g on $g \in (\underline{g}, \bar{g})$ for all $\gamma \in [0, \bar{\gamma}]$.*

The conditions in Proposition 1C under which the equilibrium price strictly increases (or decreases) with the intensity of competition in the labor market are unchanged from those in the base model; see Proposition 1.

Proposition 2C *(i) Suppose there is no competition in the consumer market, $\gamma = 0$. The equilibrium AV fleet K^* strictly increases in the intensity of competition in the labor market g on $g \in (0, b)$. (ii) Suppose there is limited competition in the consumer market, $\gamma \in (0, \bar{\gamma}]$, where $\bar{\gamma} > 0$. There exists $\bar{\theta} > 0$ and $\bar{g} > 0$ such that if AV cost is small $\theta \in (0, \bar{\theta}]$, then the equilibrium AV fleet K^* strictly decreases in the intensity of competition in the labor market g on $g \in (0, \bar{g})$.*

Parallel to Proposition 2ii, Proposition 2Cii provides sufficient conditions under which platform 1's equilibrium AV fleet strictly decreases in the intensity of competition in the labor market. The next Proposition allows for the platform's AV costs to be asymmetric, $\theta_1 \neq \theta_2$.

Proposition 3C *Platform 2's equilibrium profit $\pi_2(K_1^*, K_2^*)$ strictly decreases in platform 1's AV cost θ_1 only if the relative wage sensitivity of labor is greater than the relative price sensitivity of demand (i.e., inequality (8) holds).*

Proposition 3C is weaker than Proposition 3 in that condition (8) is necessary but not sufficient for platform 2 to benefit by a reduction in platform 1's AV cost. More precisely, there exist parameters in which inequality (8) holds and yet platform 2's equilibrium profit $\pi_2(K_1^*, K_2^*)$ strictly increases in θ_1.

To see why inequality (8) is not sufficient to ensure that platform 2 benefits from a reduction in platform 1's AV cost θ_1, recall that such a reduction benefits platform 2 through the *labor market effect* (platform 1 competes less aggressively in the labor market) and hurts platform 2 through the *consumer market effect* (platform 1 competes more aggressively in the consumer market). When platform 2 lacks access to AVs (the setting of Proposition 3), the labor market effect dominates the consumer market effect if and only if the relative wage sensitivity of labor is greater than the relative price sensitivity of demand (inequality (8)). Platform 2's access to AVs reduces the platform's dependence on the labor market, dampening the beneficial labor market effect. Consequently, even if inequality (8) holds, the beneficial labor market effect can be outweighed by the harmful consumer market effect. This occurs, for example, when platform 2's AV cost θ_2 is small, such that the platform has a large AV fleet, finds labor relatively unattractive, and consequently benefits little from platform 1 competing less aggressively in the labor market.

5 Discussion

This paper provides three insights into the implications of competition and access to AV technology for the management of ride-hailing platforms. First, a platform's access to supply-side (namely, AV) technology changes prescriptions for its demand-side (namely, pricing) decisions: The intuitive prescription from the setting without AVs, that price increases in the intensity of competition in the labor market, is reversed. Second, the presence of demand-side competition changes prescriptions for a platform's supply-side (namely, AV fleet size) decisions: The intuitive prescription from the setting without demand-side competition, that the AV fleet size increases in the intensity of competition in the labor market, is reversed. We characterize the conditions under which these reversals occur and explain the driving forces behind the reversals. Third, whether a platform benefits from its rival's access to AV technology depends on a simple comparison between the relative wage sensitivity of labor and the relative price sensitivity of demand.

Our results are driven by the key feature that makes AVs attractive to ride-hailing platforms: the elimination of the variable cost payment to human drivers. The reduction in variable cost is most profound when a platform owns the AV fleet which provides rides on its network. However, as noted in §1, a ride-hailing platform may allow independently-owned AVs on its network. One possibility is that individual consumers own AVs, and a ride-hailing platform offers to pay a "wage" to AV owners for each ride the owner's AV provides (Higgins 2019). This scenario is similar to current practice, but with an additional class of "workers" alongside human drivers. A second possibility is that firms independent from the platform own AV fleets. Such firms might seek to compete with ride-hailing platforms by directly offering rides to consumers. Alternately, fleet-owning firms might negotiate with ride-hailing platforms to put their AVs on the platform's network. In contrast to the relatively simple arms-length financial transactions between a ride-hailing platform and a human driver (or an individual AV owner), the structure of the financial arrangement between a platform and a fleet-owning firm might be quite complex, specifying when and how many AVs the fleet owner would make available, how the platform would allocate consumer requests to the fleet owner's AVs versus other vehicles, fixed payments, revenue-dependent payments, etc. As AV technology develops and the manner in which AVs integrate with and/or compete against ride-hailing platforms comes into sharper focus, future research opportunities should abound.

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Appendix A: Base Model

The following lemma is useful in the proof of Lemma 1.

Lemma 3 *Under platform 1 AV fleet K_1, platform 2's best response price and wage to platform 1's price and wage (p_1, w_1) is $(\tilde{p}_2(K_1), \tilde{w}_2(K_1))$. Platform 1's best response price and wage to platform 2's price and wage (p_2, w_2) is*

$$(\tilde{p}_1(K_1), \tilde{w}_1(K_1)) = \begin{cases} (\tilde{p}_1^l(K_1), \tilde{w}_1^l(K_1)) & \text{if } K_1 < (\alpha + \gamma p_2 - g\beta w_2/b)/2, \\ (\tilde{p}_1^e(K_1), \tilde{w}_1^e(K_1)) & \text{if } K_1 \in [(\alpha + \gamma p_2 - g\beta w_2/b)/2, (\alpha + \gamma p_2)/2], \\ (\tilde{p}_1^s(K_1), \tilde{w}_1^s(K_1)) & \text{if } K_1 > (\alpha + \gamma p_2)/2. \end{cases}$$

Further, $K_1 < D_1(\tilde{p}_1(K_1), p_2)$ if and only if $K_1 < (\alpha + \gamma p_2 - g\beta w_2/b)/2$; $K_1 = D_1(\tilde{p}_1(K_1), p_2)$ if and only if $K_1 \in [(\alpha + \gamma p_2 - g\beta w_2/b)/2, (\alpha + \gamma p_2)/2]$; and $K_1 > D_1(\tilde{p}_1(K_1), p_2)$ if and only if $K_1 > (\alpha + \gamma p_2)/2$.

Proof of Lemma 3: It is straightforward to show that platform 2's second-period contribution $v_2(\mathbf{p}, w_1)$, as given in (3), is strictly concave in p_2. Platform 2's best response price is the unique solution to the first-order condition $(\partial/\partial p_2)v_2(\mathbf{p}, w_1) = 0$, namely, $p_2 = \tilde{p}_2(K_1)$. Further, platform 2's best response wage $w_2 = (D_2(p_1, \tilde{p}_2(K_1)) + gw_1)/b = \tilde{w}_2(K_1)$. It is straightforward to show that platform 1's second-period contribution $v_1(\mathbf{p}, w_2)$, as given in (2), is strictly concave in p_2. Further, $\lim_{p_1 \uparrow \tilde{p}_1^e(K_1)}(\partial/\partial p_1)v_1(\mathbf{p}, w_2) > \lim_{p_1 \downarrow \tilde{p}_1^e(K_1)}(\partial/\partial p_1)v_1(\mathbf{p}, w_2)$. If $K_1 < (\alpha + \gamma p_2 - g\beta w_2/b)/2$, then $\lim_{p_1 \uparrow \tilde{p}_1^e(K_1)}(\partial/\partial p_1)v_1(\mathbf{p}, w_2) < 0$ and platform 1's best response price is the unique solution to the first-order condition $(\partial/\partial p_1)v^l(\mathbf{p}, w_2) = 0$, namely, $p_1 = \tilde{p}_1^l(K_1)$; further, $K_1 < D_1(\tilde{p}_1^l(K_1), p_2)$. If $K_1 \in [(\alpha + \gamma p_2 - g\beta w_2/b)/2, (\alpha + \gamma p_2)/2]$, then $\lim_{p_1 \downarrow \tilde{p}_1^e(K_1)}(\partial/\partial p_1)v_1(\mathbf{p}, w_2) < 0 < \lim_{p_1 \uparrow \tilde{p}_1^e(K_1)}(\partial/\partial p_1)v_1(\mathbf{p}, w_2)$ and platform 1's best response price $p_1 = \tilde{p}_1^e(K_1)$; further, $K_1 = D_1(\tilde{p}_1^e(K_1), p_2)$. If $K_1 > (\alpha + \gamma p_2)/2$, then $\lim_{p_1 \downarrow \tilde{p}_1^e(K_1)}(\partial/\partial p_1)v_1(\mathbf{p}, w_2) > 0$ and platform 1's best response price is the unique solution to the first-order condition $(\partial/\partial p_1)v^s(\mathbf{p}, w_2) = 0$, namely,

$p_1 = \tilde{p}_1^s(K_1)$; further, $K_1 > D_1(\tilde{p}_1^s(K_1), p_2)$. If $K_1 < (\alpha + \gamma p_2 - g\beta w_2/b)/2$, then, platform 1's best response price is sufficiently small that the platform sources labor $L_1(\mathbf{w}) > 0$; thus, platform 1's best response wage is $w_1 = (D_1(\tilde{p}_1^l(K_1), p_2) - K_1 + g w_2)/b = \tilde{w}_1^l(K_1)$. If $K_1 \geq (\alpha + \gamma p_2 - g\beta w_2/b)/2$, then, platform 1's best response price is sufficiently large that the platform does not source labor $L_1(\mathbf{w}) = 0$; thus, platform 1's best response wage $w_1 = g w_2/b$. $\square$

Proof of Lemma 1: Lemma 3 implies that under platform 1 AV fleet K_1: prices and wages $(\mathbf{p}, \mathbf{w}) = (\mathbf{p}^l(K_1), \mathbf{w}^l(K_1))$ is an equilibrium if and only if $K_1 < D_1(\mathbf{p}^l(K_1))$; $(\mathbf{p}, \mathbf{w}) = (\mathbf{p}^e(K_1), \mathbf{w}^e(K_1))$ is an equilibrium if and only if $K_1 = D_1(\mathbf{p}^e(K_1))$; and $(\mathbf{p}, \mathbf{w}) = (\mathbf{p}^s(K_1), \mathbf{w}^s(K_1))$ is an equilibrium if and only if $K_1 > D_1(\mathbf{p}^s(K_1))$. Further, it is straightforward to verify: $K_1 < D_1(\mathbf{p}^l(K_1))$ if and only if $K_1 < \alpha/\psi^l$; $K_1 = D_1(\mathbf{p}^e(K_1))$ if and only if $K_1 \in [\alpha/\psi^l, \alpha/\psi^s]$; and $K_1 > D_1(\mathbf{p}^s(K_1))$ if and only if $K_1 > \alpha/\psi^s$.$\square$

Proof of Lemma 2: Using equation (2) and Lemma 1, we can write platform 1's second-period contribution under AV fleet K_1 and equilibrium prices and wages $(\mathbf{p}^*(K_1), \mathbf{w}^*(K_1))$ as $r_1(K_1) = v^l(\mathbf{p}^l(K_1), w_2^l(K_1))\mathbf{1}_{\{K_1<\alpha/\psi^l\}} + v^s(\mathbf{p}^e(K_1))\mathbf{1}_{\{K_1\in[\alpha/\psi^l,\alpha/\psi^s]\}} + v^s(\mathbf{p}^s(K_1))\mathbf{1}_{\{K_1>\alpha/\psi^s\}}$. It is straightforward to verify the following: $r_1(K_1)$ is continuous in K_1; $r_1(K_1)$ is strictly concave in K_1 for $K_1 < \alpha/\psi^l$ and $K_1 \in (\alpha/\psi^l, \alpha/\psi^s)$; $r_1(K_1)$ is invariant to K_1 on $K_1 > \alpha/\psi^s$; if $\gamma = 0$, then $\theta^e = \theta^l$; and $\theta^e < \theta^l$ if and only if $\gamma > 0$ and $g < \tilde{g}$. Thus, $\pi_1(K_1)$ is concave in K_1 for $K_1 < \alpha/\psi^l$ and $K_1 \in (\alpha/\psi^l, \alpha/\psi^s)$; $\pi_1(K_1)$ is concave and strictly decreasing on $K_1 > \alpha/\psi^s$. Further, if $\gamma = 0$ or $g < \tilde{g}$, then $\lim_{K_1\uparrow(\alpha/\psi^l)}(\partial/\partial K_1)r_1(K_1) \leq \lim_{K_1\downarrow(\alpha/\psi^l)}(\partial/\partial K_1)r_1(K_1)$, which implies that $\pi_1(K_1)$ is quasi-concave in K_1. If $\theta < \theta^e$, then $\lim_{K_1\downarrow(\alpha/\psi^l)}(\partial/\partial K_1)\pi_1(K_1) > 0$, which implies $K_1^* \in (\alpha/\psi^l, \alpha/\psi^s]$, which in turn implies $K_1^* = K^e$ (by Lemma 1). If $\theta \in [\theta^e, \theta^l]$, then $\lim_{K_1\downarrow(\alpha/\psi^l)}(\partial/\partial K_1)\pi_1(K_1) \leq 0 \leq \lim_{K_1\uparrow(\alpha/\psi^l)}(\partial/\partial K_1)\pi_1(K_1)$, which implies $K_1^* = \alpha/\psi^l$. If $\theta \in (\theta^l, \theta^0)$, then $\lim_{K_1\uparrow(\alpha/\psi^l)}(\partial/\partial K_1)\pi_1(K_1) < 0 < \lim_{K_1\to 0}(\partial/\partial K_1)\pi_1(K_1)$, which implies $K_1^* \in (0, \alpha/\psi^l)$, which in turn implies $K_1^* = K^l$ (by Lemma 1). Because $\theta \geq \theta^0$ if and only if $\lim_{K_1\to 0}(\partial/\partial K_1)\pi_1(K_1) \leq 0$,$K_1^* = 0$ if and only if $\theta \geq \theta^0$. Using Lemma 1, it is straightforward to verify that θ^l has the same sign as $\varphi^l(\beta, \gamma, b, g) = \beta b g(2\beta - \gamma)[\beta(2b - g) + 2b(b - g)] - \gamma^2(2b - g)[\beta(2b^2 - g^2) + b(b^2 - g^2)]$. Suppose $\gamma = 0$; then $\varphi^l(\beta, \gamma, b, g) \geq 0$, where the inequality is strict if and only if $g > 0$; this implies $\theta^l > 0$ if and only if $g > 0$. Suppose for the remainder of the proof that $\gamma > 0$. Note $(\partial^3/\partial g^3)\varphi^l(\beta, \gamma, b, g) < 0$, which implies that there exists $g^l \in [0, b]$ such that $\varphi^l(\beta, \gamma, b, g)$ is convex in g for $g \in [0, g^l]$ and concave in g for $g \in [g^l, b]$. It is straightforward to verify that $\lim_{g\to 0}(\partial/\partial g)\varphi^l(\beta, \gamma, b, g) > 0$ and $\lim_{g\to 0}\varphi^l(\beta, \gamma, b, g) < 0 < \lim_{g\to[\sqrt{\beta^2+8\gamma^2}-\beta]b/(2\gamma)}\varphi^l(\beta, \gamma, b, g)$. This implies existence of $\underline{\underline{g}} \in (0, [\sqrt{\beta^2 + 8\gamma^2} - \beta]b/(2\gamma))$ such that $\theta^l > 0$ if and only if $g > \underline{\underline{g}}$. Further, $\tilde{g} > [\sqrt{\beta^2 + 8\gamma^2} - \beta]b/(2\gamma)$ implies $\underline{\underline{g}} < \tilde{g}$. Because $K_1^* = 0$ if and only $\theta \geq \theta^0$, $K_1^* = 0$ for $g \in [0, b)$ if and only $\theta \geq \hat{\theta}$.$\square$

Proof of Proposition 1: (i) If $\theta \geq \hat{\theta}$, then for $g \in [0, b)$, $K_1^* = 0$ (by Lemma 2), which implies $(d/dg)K_1^* = 0$. Therefore, $(d/dg)p_1^* = [(\partial/\partial K_1)p_1^*][(d/dg)K_1^*] + (\partial/\partial g)p_1^* = (\partial/\partial g)p_1^*$. Because $K_1^* = 0$, $p_1^*(K_1^*) = p_1^l(K_1^*) = p_1^l(0)$, where the first equality follows by Lemma 1. It is straightforward to verify that $(\partial/\partial g)p_1^l(0) = (\alpha\beta^2 b^2)/[\beta(\beta-\gamma)(2b-g)+(2\beta-\gamma)b(b-g)]^2 > 0$. (ii) Let K_1^{0*} and p_1^{0*} denote platform 1's equilibrium AV fleet and price in the special case where $c(K_1) = K_1$. The proof proceed in three steps. First, we show that $p_1^{0*} = p_1^l(K_1^{0*})$ if $\gamma \in [0, \gamma^l(\theta)]$ and $g \in [0, g^l(\theta)]$ for some $\gamma^l(\theta) > 0$ and $g^l(\theta) > 0$. Second, we show that if $c(K_1) = K_1$, then there exists $\tilde{\theta} > 0$ such that for any $\theta \in (0, \tilde{\theta})$, there exist $\bar{\gamma} > 0$ and $\bar{g} > \underline{g} \geq 0$ such that $(d/dg)p_1^l(K_1^{0*}) < 0$ for all $\gamma \in [0, \bar{\gamma}]$ and $g \in (\underline{g}, \bar{g})$. Third, we show the result from the second step extends to when $c(K_1)$ is convex, increasing, when $\tilde{\theta}$ is replaced by $\bar{\theta}$. Step One: Let $\mu = \lim_{K_1 \to 0}(\partial/\partial K_1)c(K_1)$. Suppose $\mu > 0$. Then $\theta^0 = \lim_{K_1 \to 0}[(\partial/\partial K_1)r_1(K_1)]/\mu$. Let $\tilde{\theta} = \alpha/[(\beta+b)\mu]$. Because $\psi^l > 0$, Lemma 1 implies $\theta^0 = \lim_{K_1 \to 0}[(\partial/\partial K_1)v^l(\mathbf{p}^*(K_1), w_2^*(K_1))]/\mu$. Further, it is straightforward to show algebraically that $\lim_{\gamma \to 0}\lim_{g \to 0}\lim_{K_1 \to 0}[(\partial/\partial K_1)v^l(\mathbf{p}^*(K_1), w_2^*(K_1))] = \alpha/(\beta+b)$. Because $\lim_{\gamma \to 0}\lim_{g \to 0}\theta^0 = \tilde{\theta}$, $\lim_{\gamma \to 0}\lim_{g \to 0}\theta^l = 0$ (by Lemma 2), and θ^0 and θ^l are continuous in γ and g, for any $\theta \in (0, \tilde{\theta})$, there exist $\gamma^l(\theta) > 0$ and $g^l(\theta) > 0$ such that $\theta \in (\theta^l, \theta^0)$ for all $\gamma \in [0, \gamma^l(\theta)]$ and $g \in [0, g^l(\theta)]$. Let $\theta \in (0, \tilde{\theta})$. It follows that $p_1^{0*} = p_1^l(K_1^{0*})$ if $\gamma \in [0, \gamma^l(\theta)]$ and $g \in [0, g^l(\theta)]$. We have assumed $\mu > 0$; if instead $\mu = 0$, then the preceeding sentence holds by argument parallel to that above, where expressions with μ in the denominator are replaced by ∞. Step Two: It is straightforward to show algebraically that $\lim_{\gamma \to 0}\lim_{g \to 0}(d/dg)p_1^l(K_1^{0*}) = 0$ and $\lim_{\gamma \to 0}\lim_{g \to 0}(d^2/dg^2)p_1^l(K_1^{0*}) = -\theta(\beta+2b)/(4b(\beta+b)^2) < 0$. This implies that there exists $\bar{g} \in (0, g^l(\theta))$ such that $\lim_{\gamma \to 0}(d/dg)p_1^l(K_1^{0*}) < 0$ for all $g \in (0, \bar{g})$. Because $(d/dg)p_1^l(K_1^{0*})$ is continuous in γ and g, there exist $\bar{\gamma} \in (0, \gamma^l(\theta))$ and $\underline{g} < \bar{g}$ such that $(d/dg)p_1^l(K_1^{0*}) < 0$ for all $\gamma \in [0, \bar{\gamma}]$ and $g \in (\underline{g}, \bar{g})$. The result follows because $p_1^{0*} = p_1^l(K_1^{0*})$ for $\gamma \in [0, \gamma^l(\theta)]$ and $g \in [0, g^l(\theta)]$. Step Three: Let K_1^* and p_1^* denote platform 1's equilibrium AV fleet and price, when $c(K_1)$ is convex, increasing. We first show that for all $\epsilon > 0$, there exists $\bar{\theta}(\epsilon) > 0$ such that $|(d/dg)p_1^{0*} - (d/dg)p_1^*| < \epsilon$ if $\theta \in [0, \bar{\theta}(\epsilon)]$. To do so, we show that $\lim_{\theta \to 0}|(d/dg)p_1^{0*} - (d/dg)p_1^*| = 0$ and that $(d/dg)p_1^{0*}$ and $(d/dg)p_1^*$ are continuous in θ. Define $K^r = \arg\max_{K_1 \geq 0} r_1(K_1)$. Note that $|K_1^{0*} - K_1^*| \leq |K_1^{0*} - K^r| + |K_1^* - K^r|$ by the triangle inequality. Next, $0 \leq \pi_1(K_1^*) - \pi_1(K^r) = r_1(K_1^*) - r_1(K^r) + \theta[c(K^r) - c(K_1^*)] \leq \theta[c(K^r) - c(K_1^*)]$, where the first inequality follows because $\pi_1(K_1^*) \geq \pi_1(K^r)$ (by definition of K_1^*) and the second inequality follows because $r_1(K_1^*) \leq r_1(K^r)$ (by definition of K^r). Note that $0 \leq \pi_1(K_1^*) - \pi_1(K^r) \leq \theta[c(K^r) - c(K_1^*)]$ implies $\lim_{\theta \to 0}|\pi_1(K_1^*) - \pi_1(K^r)| = 0$. Because K_1^* is the unique maximizer of π_1, it must be that $\lim_{\theta \to 0}|K_1^* - K^r| = 0$. By an identical argument, $\lim_{\theta \to 0}|K_1^{0*} - K^r| = 0$. It follows that $\lim_{\theta \to 0}|K_1^{0*} - K_1^*| = 0$. Because $p_1^* = p_1^*(K_1^*)$, $p_1^{0*} = p_1^*(K_1^{0*})$, and $p_1^*(K_1)$ is continuous in K_1, it follows that $\lim_{\theta \to 0}|(d/dg)p_1^{0*} - (d/dg)p_1^*| = 0$. We now show continuity of $(d/dg)p_1^*$ and $(d/dg)p_1^{0*}$ in θ. Note $(d/dg)p_1^* = \{[(\partial/\partial K_1)p_1^*][(d/dg)K_1^*] + (\partial/\partial g)p_1^*]\}|_{K_1 = K_1^*}$. By the implicit function theorem, $(d/dg)K_1^* = -\{(\partial^2/\partial K_1 \partial g)r_1(K_1)/[(\partial^2/\partial K_1^2)r_1(K_1) - \theta(\partial^2/\partial K_1^2)c(K_1)]\}|_{K_1 = K_1^*}$. Therefore,

$$(d/dg)p_1^* = \{-[(\partial/\partial K_1)p_1^*][(\partial^2/\partial K_1 \partial g)r_1(K_1)]/[(\partial^2/\partial K_1^2)r_1(K_1) - \theta(\partial^2/\partial K_1^2)c(K_1)] + (\partial/\partial g)p_1^*]\}|_{K_1 = K_1^*}. \tag{10}$$

Using the equilibrium expressions in Lemma 1, it is straightforward to show that $(\partial^2/\partial K_1^2)r_1(K_1)$ and $(\partial^2/\partial K_1 \partial g)r_1(K_1)$ are continuous in K_1. Because $c(K_1)$ is twice differentiable, the right hand side of (10) is continuous in K_1. Because K_1^* is continuous in θ, it follows that $(d/dg)p_1^*$ is continuous in θ. Continuity of $(d/dg)p_1^{0*}$ in θ follows immediately. Because $\lim_{\theta\to 0}|(d/dg)p_1^{0*} - (d/dg)p_1^*| = 0$, and $(d/dg)p_1^{0*}$ and $(d/dg)p_1^*$ are continuous in θ, it follows that for any $\epsilon > 0$, there exists $\bar{\theta}(\epsilon) > 0$ such that $|(d/dg)p_1^{0*} - (d/dg)p_1^*| < \epsilon$ if $\theta \in [0, \bar{\theta}(\epsilon)]$. Lastly, from step one of the proof, there exists $\tilde{\theta} > 0$ such that for any $\theta \in (0, \tilde{\theta})$, there exist $\bar{\gamma} > 0$, $\underline{g} > 0$, and $\bar{g} > 0$ such that $(d/dg)p_1^{0*} < 0$ for all $\gamma \in [0, \bar{\gamma}]$ and $g \in (\underline{g}, \bar{g})$. The result follows from selecting ϵ to be sufficiently small and defining $\bar{\theta} = \min(\bar{\theta}(\epsilon), \tilde{\theta})$.$\square$

Proof of Proposition 2: (i) Because $\gamma = 0$, $\theta^e = \theta^l$ (by Lemma 2). It is straightforward to verify that: $(\partial/\partial g)\theta^e > 0$, $\lim_{g\to 0}\theta^e = 0$ and $\lim_{g\to b}\theta^e = \alpha/\beta$. Let $\mu = \lim_{K_1\to 0}(\partial/\partial K_1)c(K_1)$. Suppose $\mu > 0$. Then $\theta^0 = \lim_{K_1\to 0}[(\partial/\partial K_1)r_1(K_1)]/\mu$. It is straightforward to verify that: $(\partial/\partial g)\theta^0 > 0$, $\lim_{g\to 0}\theta^0 = \alpha/[(\beta + b)\mu]$ and $\hat{\theta} = \lim_{g\to b}\theta^0 = \alpha/(\beta\mu)$. Because $\theta^e < \theta^0$, the previous results imply that for any $\theta \in (0, \hat{\theta})$, there exist $0 \le \breve{g} < \hat{g}$ such that: $\theta > \theta^0$ if and only $g < \breve{g}$; $\theta \in (\theta^e, \theta^0)$ if and only if $g \in (\breve{g}, \hat{g})$; and $\theta < \theta^e$ if and only if $g > \hat{g}$. Therefore, $K_1^* = 0$ if $g < \breve{g}$, $K_1^* = K^l$ if $g \in (\breve{g}, \hat{g})$, and $K_1^* = K^e$ if $g > \hat{g}$ (by Lemma 2). We have assumed $\mu > 0$; if instead $\mu = 0$, then the previous sentence holds by parallel argument, where $\breve{g} = 0$ (because $\theta^0 = \infty$). It is straightforward to verify that $(\partial/\partial g)K^e = 0$. Therefore, $(\partial/\partial g)K_1^* = 0$ if $g < \breve{g}$ or $g > \hat{g}$. Because K_1^* is continuous in g, to establish the result, it is sufficient to show that $(\partial/\partial g)K^l > 0$ for $g \in (\breve{g}, \hat{g})$. By the implicit function theorem, $(\partial/\partial g)K^l = -[(\partial^2/\partial K_1\partial g)\pi_1(K_1)]/[(\partial^2/\partial K_1^2)\pi_1(K_1)]|_{K_1=K^l}$, where $\pi_1(K_1) = v_1(\mathbf{p}^l(K_1), \mathbf{w}^l(K_1)) - \theta c(K_1)$. Using Lemma 1, it is straightforward to verify $(\partial^2/\partial K_1^2)\pi_1(K_1)|_{K_1=K^l} < 0$. It remains to show that $(\partial^2/\partial K_1\partial g)\pi_1(K_1)|_{K_1=K^l} > 0$. It is straightforward to verify that $(\partial^3/\partial K_1^2\partial g)\pi_1(K_1) < 0$ and $\lim_{K_1\to\alpha/\psi^l}(\partial^2/\partial K_1\partial g)\pi_1(K_1) > 0$. Because $K^l \le \alpha/\psi^l$, this establishes $(\partial^2/\partial K_1\partial g)\pi_1(K_1)|_{K_1=K^l} > 0$, which in turn implies $(\partial/\partial g)K^l > 0$. (ii) First we show that there exists $\bar{g} \in (0, \tilde{g})$ such that $\theta^e < 0$ if and only if $g < \bar{g}$. Using Lemma 1, it is straightforward to verify that θ^e has the same sign as $\varphi^e(\beta, \gamma, b, g) = (2\beta+\gamma)\beta^3 bg[\beta(2b^2-g^2)+2b(b^2-g^2)] - \beta\gamma^2(b+g)[(\beta(2b-g)+2b(b-g)][\beta(2b^2-g^2)+b(b^2-g^2)] - \gamma^3[\beta(2b^2-g^2)+b(b^2-g^2)]^2$. Note $(\partial^3/\partial g^3)\varphi^e(\beta, \gamma, b, g) < 0$, which implies that there exists $g^e \in [0, b]$ such that $\varphi^e(\beta, \gamma, b, g)$ is convex in g for $g \in [0, g^e]$ and concave in g for $g \in [g^e, b]$. It is straightforward to verify that $\lim_{g\to 0}(\partial/\partial g)\varphi^e(\beta, \gamma, b, g) > 0$ and $\lim_{g\to 0}\varphi^e(\beta, \gamma, b, g) < 0 < \lim_{g\to[\sqrt{\beta^2+8\gamma^2}-\beta]b/(2\gamma)}\varphi^e(\beta, \gamma, b, g)$. This implies existence of $\bar{g} \in (0, [\sqrt{\beta^2+8\gamma^2} - \beta]b/(2\gamma))$ such that $\theta^e < 0$ if and only if $g < \bar{g}$. Further, $\tilde{g} > [\sqrt{\beta^2+8\gamma^2} - \beta]b/(2\gamma)$ implies $\bar{g} < \tilde{g}$. Next we show that $\bar{g} > \underline{\underline{g}}$. Because $\bar{g} < \tilde{g}$, at $g = \bar{g}$, $\theta^l > \theta^e = 0$, where the inequality follows from Lemma 2. Because $\theta^{\bar{l}} > 0$ if and only if $g > \underline{\underline{g}}$ (by Lemma 2), $\theta^l|_{g=\bar{g}} > 0$ implies $\bar{g} > \underline{\underline{g}}$. We conclude that for $g \in (\underline{\underline{g}}, \bar{g})$, $\theta^e < 0 < \theta^l$. Let $\underline{g} \in (\underline{\underline{g}}, \bar{g})$ and $\bar{\theta} = \inf_{g\in(\underline{g},\bar{g})}\theta^l$; note $\bar{\theta} > 0$. Because $g \in (\underline{g}, \bar{g})$ implies $\theta^e < 0 < \theta < \theta^l$, it follows from Lemma 2 that $K_1^* = \alpha/\psi^l$. It is straightforward to verify that $(\partial/\partial g)\psi^l > 0$. It immediately follows that $(\partial/\partial g)K_1^* < 0$ for $g \in (\underline{g}, \bar{g})$. $\square$

Proof of Proposition 3: Note $(d/d\theta)\pi_2(K_1^*) = (\partial/\partial K_1)\pi_2(K_1)|_{K_1=K_1^*}(d/d\theta)K_1^*$. Suppose $\theta \in (\theta^l, \theta^0)$. First we show $(d/d\theta)K_1^* < 0$. Because $\theta \in (\theta^l, \theta^0)$, $0 < K_1^* = K^l$ (by Lemma 2). By application of the implicit function theorem, $(d/d\theta)K^l < 0$. Therefore, $(d/d\theta)K_1^* < 0$. It remains to show that $(\partial/\partial K_1)\pi_2(K_1)|_{K_1=K_1^*} > 0$ if and only if $g/b > \gamma/\beta$. Because $\theta \in (\theta^l, \theta^0)$, $\pi_2(K_1^*) = v_2(\mathbf{p}^l(K_1^*), w_2^l(K_1^*))$ (by Lemma 2). By straightforward algebra, $(\partial/\partial K_1)\pi_2(K_1)|_{K_1=K_1^*} = (g/b - \gamma/\beta)\zeta(\alpha, \beta, \gamma, b, g, K_1^*)/\phi^l(\beta, \gamma, b, g)^2$ for some non-zero functions $\phi^l(\cdot)$ and $\zeta(\cdot)$. Note that if $\zeta(\cdot) > 0$, then $(\partial/\partial K_1)\pi_2(K_1)|_{K_1=K_1^*}$ has the same sign as $(g/b - \gamma/\beta)$. Therefore, it suffices to show $\zeta(\alpha, \beta, \gamma, b, g, K_1^*) > 0$. Writing $\zeta(\cdot)$ explicitly, $\zeta(\alpha, \beta, \gamma, b, g, K_1^*) = \alpha(b - g)[\beta(\beta + \gamma)(2b + g) + (2\beta + \gamma)b(b + g)] + (2\beta b + \gamma g)(\beta g - \gamma b)K_1^*$. If $\beta g \geq \gamma b$, then $\zeta(\cdot) > 0$. Suppose $\beta g < \gamma b$. Then $\zeta(\cdot) > 0$ if and only if $K_1^* \leq \alpha(b-g)[\beta(\beta+\gamma)(2b+g)+(2\beta+\gamma)b(b+g)]/[(2\beta b+\gamma g)(\gamma b-\beta g)]$. Because $K_1^* \leq \alpha/\psi^l$, it suffices to show $1/\psi^l \leq (b-g)[\beta(\beta+\gamma)(2b+g)+(2\beta+\gamma)b(b+g)]/((2b\beta+g\gamma)(\gamma b-\beta g))$. This inequality can be verified algebraically. If $\theta > \theta^0$, then $K_1^* = 0$ (by Lemma 2), which implies $(d/d\theta)K_1^* = 0$ and, hence, $(d/d\theta)\pi_2(K_1^*) = 0$. If $\theta \in [\theta^e, \theta^l]$, then $K_1^* = \alpha/\psi^l$, which implies $(d/d\theta)K_1^* = 0$ and hence $(d/d\theta)\pi_2(K_1^*) = 0$. If $\theta < \theta^e$, then $K_1^* = K^e > 0$ (by Lemma 2). By application of the implicit function theorem, $(d/d\theta)K^e < 0$. Therefore, $(d/d\theta)K_1^* < 0$. It remains to show that $(\partial/\partial K_1)\pi_2(K_1)|_{K_1=K_1^*} < 0$. It is straightforward to verify that $(\partial/\partial K_1)\pi_2(K_1)|_{K_1=K_1^*} = -[\alpha(\beta + \gamma) - \gamma K_1^*]/\phi^e(\beta, \gamma, b, g)^2$ for some non-zero function $\phi^e(\cdot)$; therefore, it remains to show $K_1^* < \alpha(\beta + \gamma)/\gamma$. By Lemma 1, $\pi_1(K_1)$ is strictly decreasing in K_1 on $K_1 > \alpha/\psi^s$, which implies $K_1^* \leq \alpha/\psi^s$. Thus, to establish $K_1^* < \alpha(\beta + \gamma)/\gamma$, it suffices to show $\alpha/\psi^s < \alpha(\beta + \gamma)/\gamma$, or equivalently, $(\beta + \gamma)\psi^s - \gamma > 0$. Note $(\beta + \gamma)\psi^s - \gamma = (2\beta + \gamma)[\beta(\beta^2 - \gamma^2)(2b^2 - g^2) + (2\beta^2 - \gamma^2)b(b^2 - g^2)]/\{\beta[\beta(\beta+\gamma)(2b^2 - g^2) + (2\beta+\gamma)b(b^2 - g^2)]\} > 0$, where the inequality follows because $\beta > \gamma$ and $b > g$.□

Appendix B: Demand Uncertainty

It is useful to introduce notation for the special case where the demand state is deterministic: let $\breve{K}_1(\alpha, \theta)$ denote platform 1's equilibrium fleet size under deterministic demand state α and AV cost θ; we write $\theta^0(\alpha)$ to make the dependence on the deterministic demand state explicit.

Lemma 4 *Suppose $g < \tilde{g}$. Then $\breve{K}_1(\alpha, \theta)$ increases in α. If $\breve{K}_1(\underline{\alpha}, \theta) > 0$, then $\breve{K}_1(\alpha, \theta)$ strictly increases in α on $\alpha \in [\underline{\alpha}, \bar{\alpha}]$.*

Proof of Lemma 4: We prove the two statements in order. Because $g < \tilde{g}$, $\pi_1(\alpha, K_1)$ is strictly quasi-concave in K_1 (by Lemma 2), and thus has a unique maximizer, $\breve{K}_1(\alpha, \theta)$. Therefore, by Berge's maximum theorem, $\breve{K}_1(\alpha, \theta)$ is continuous in α. By Lemma 2, there are four cases to consider: $\breve{K}_1(\alpha, \theta) = K^l$, $\breve{K}_1(\alpha, \theta) = K^e$, $\breve{K}_1(\alpha, \theta) = \alpha/\psi^l$, and $\breve{K}_1(\alpha, \theta) = 0$. For the first case, an application of the implicit function theorem yields $(d/d\alpha)\breve{K}_1(\alpha, \theta) = (d/d\alpha)K^l = -[(\partial^2/\partial K_1\partial\alpha)\pi_1(\alpha, K_1)]/[(\partial^2/\partial K_1^2)\pi_1(\alpha, K_1)]|_{K_1=K^l}$. Note $(\partial^2/\partial K_1^2)\pi_1(\alpha, K_1)|_{K_1=K^l} < 0$ because K^l is the maximizer of π_1; therefore, $(d/d\alpha)\breve{K}_1(\alpha, \theta)$ has the same sign as $(\partial^2/\partial K_1\partial\alpha)\pi_1(\alpha, K_1)$. The result follows because $(\partial^2/\partial K_1\partial\alpha)\pi_1(\alpha, K_1)|_{K_1=K^l} > 0$ for all $\alpha > 0$. The case where $\breve{K}_1(\alpha, \theta) = K^e$ follows by parallel argument to the $\breve{K}_1(\alpha, \theta) = K^l$ case. For the third case, $(d/d\alpha)\breve{K}_1(\alpha, \theta) = (d/d\alpha)\alpha/\psi^l = 1/\psi^l > 0$ because $\psi^l > 0$. For the fourth case, $\breve{K}_1(\alpha, \theta) = 0$ and is therefore invariant to α. It follows that $\breve{K}_1(\alpha, \theta)$ is nondecreasing in α. The second statement follows by a parallel argument to the first, with the exception that $\breve{K}_1(\underline{\alpha},\theta) > 0$ eliminates the case where $\breve{K}_1(\alpha,\theta) = 0$.□

Lemma 5 *If $g < \tilde{g}$, then $\Pi_1(K_1)$ is strictly concave on $K_1 \in (0, \underline{\alpha}/\psi^s)$ and any maximizer is attained on the interval $[\breve{K}_1(\underline{\alpha}), \breve{K}_1(\bar{\alpha})]$.*

Proof of Lemma 5: Suppose $g < \tilde{g}$. Let K_1^* be a maximizer of $\Pi_1(K_1)$. First, we show that $K_1^* \in [\breve{K}_1(\underline{\alpha}), \breve{K}_1(\bar{\alpha})]$. The argument is by contradiction. Suppose $K_1^* < \breve{K}_1(\underline{\alpha})$. Because $0 \leq K_1^* < \breve{K}_1(\underline{\alpha})$, $\breve{K}_1(\alpha)$ strictly increases in α on $\alpha \in [\underline{\alpha}, \bar{\alpha}]$ (by Lemma 4). Therefore, $K_1^* < \breve{K}_1(\alpha)$ for all $\alpha \in [\underline{\alpha}, \bar{\alpha}]$. Because $g < \tilde{g}$, following the proof of Lemma 2, $\pi_1(\alpha, K_1)$ can be shown to be strictly quasi-concave in K_1 on $K_1 \in (0, \alpha/\psi^s)$ for all $\alpha \in [\underline{\alpha}, \bar{\alpha}]$. Because $\breve{K}_1(\underline{\alpha}) \leq \breve{K}_1(\alpha) \leq \alpha/\psi^s$ for all $\alpha \in [\underline{\alpha}, \bar{\alpha}]$, this implies $\pi_1(\alpha, K_1)$ is strictly increasing in K_1 on $K_1 \in (0, \breve{K}_1(\underline{\alpha}))$ for all $\alpha \in [\underline{\alpha}, \bar{\alpha}]$. It follows that $\pi_1(\alpha, K_1^*) < \pi_1(\alpha, \breve{K}_1(\underline{\alpha}))$ for all $\alpha \in [\underline{\alpha}, \bar{\alpha}]$. Integrating over both sides of the preceding inequality yields $\int_{\underline{\alpha}}^{\bar{\alpha}} \pi_1(\alpha, K_1^*) f(\alpha) d\alpha < \int_{\underline{\alpha}}^{\bar{\alpha}} \pi_1(\alpha, \breve{K}_1(\underline{\alpha})) f(\alpha) d\alpha$, which contradicts the optimality of K_1^*. By a parallel argument, if $K_1^* > \breve{K}_1(\bar{\alpha})$, then $\pi_1(\alpha, K_1)$ is strictly decreasing in K_1 on $K_1 \in (\breve{K}_1(\bar{\alpha}), \infty)$ for all $\alpha \in [\underline{\alpha}, \bar{\alpha}]$, which leads to a similar contradiction. It follows that $K_1^* \in [\breve{K}_1(\underline{\alpha}), \breve{K}_1(\bar{\alpha})]$. Second, we show that $\Pi_1(K_1)$ is strictly concave on $K_1 \in (0, \underline{\alpha}/\psi^s)$. Because $c(K_1)$ is weakly convex for $K_1 \geq 0$, it suffices to show that $R_1(K_1)$ is strictly concave in K_1 for $K_1 \in (0, \bar{\alpha}/\psi^s)$. By Lemma 1, if $K_1 \leq \alpha/\psi^l$, then $r_1(\alpha, K_1) = v^l(\alpha, p^l, w_2^l)$. It is straightforward to show that $(\partial^2/\partial K_1^2) v^l(\alpha, p^l, w_2^l) < 0$ for $K_1 \in (0, \alpha/\psi^l)$. For $K_1 \in (\alpha/\psi^l, \alpha/\psi^s)$, $r_1(\alpha, K_1) = v^s(\alpha, p^e)$. By straightforward algebra, $(\partial^2/\partial K_1^2) v^s(\alpha, p^e) < 0$. Further, because $g < \tilde{g}$, $\lim_{K_1 \downarrow \alpha/\psi^l} (\partial/\partial K_1) v^s(\alpha, p^e) < \lim_{K_1 \uparrow \alpha/\psi^l} (\partial/\partial K_1) v^l(\alpha, p^l, w_2^l)$. Therefore, $r_1(\alpha, K_1)$ is strictly concave in K_1 on $K_1 \in (0, \alpha/\psi^s)$. This implies $r_1(\alpha, K_1)$ is strictly concave in K_1 on $K_1 \in (0, \alpha'/\psi^s)$ for any $\alpha' \leq \alpha$. It follows that for any $\alpha \in [\underline{\alpha}, \bar{\alpha}]$, $r_1(\alpha, K_1)$ is strictly concave in K_1 on $K_1 \in (0, \underline{\alpha}/\psi^s)$. It immediately follows that $R_1(K_1)$ is strictly concave in K_1 on $K_1 \in (0, \underline{\alpha}/\psi^s)$.□

Lemma 6 *For all $\theta > 0$, there exist $\lambda < 1$ and $g_0 > 0$ such that if $\underline{\alpha}/\bar{\alpha} > \lambda$ and $g < g_0$, then $K_1^*(\theta) = K^l(E[\alpha], \theta) = \breve{K}_1(E[\alpha], \theta) \leq \breve{K}_1(\bar{\alpha}, \theta)$, where $K_1^*(\theta)$ is the unique maximizer of $\Pi_1(K_1)$.*

Proof of Lemma 6: The proof proceeds in two steps. First, we show that for any $\theta > 0$, there exist $\lambda < 1$ and $g_0 > 0$ such that if $\underline{\alpha}/\bar{\alpha} > \lambda$ and $g < g_0$, then $\breve{K}_1(\bar{\alpha}, \theta) < \underline{\alpha}/\psi^l$. Second, we use the result from the first step to prove $K_1^*(\theta) = K^l(E[\alpha], \theta) = \breve{K}_1(E[\alpha], \theta) \leq \breve{K}_1(\bar{\alpha}, \theta)$. Step One: It can be verified algebraically that $\lim_{K_1 \to \alpha/\psi^l} \lim_{g \to 0} (\partial/\partial K_1) r_1(\alpha, K_1) < 0$, which by quasi-concavity of $r_1(\alpha, K_1)$ in K_1 (Lemma 2) implies $\lim_{g \to 0} \breve{K}_1(\alpha, \theta) < \alpha/\psi^l$ for any $\alpha > 0$. It follows that for any $\underline{\alpha} > 0$, there exists $g_0 > 0$ such that $\breve{K}_1(\underline{\alpha}, \theta) < \underline{\alpha}/\psi^l$ for all $g < g_0$. For each $g < g_0$, let $\breve{\lambda}(g)$ be a solution to $\breve{K}_1(\underline{\alpha}/\lambda(g), \theta) = \underline{\alpha}/\psi^l$ if a solution exists, and let $\breve{\lambda}(g) = 0$ otherwise. Because $\breve{K}_1(\alpha, \theta)$ is continuous and strictly increasing in α (by Lemma 4), $\breve{\lambda}(g)$ is unique and $\breve{\lambda}(g) < 1$, for all $g < g_0$. Let $\lambda = \sup_{g \in [0, g_0]} \breve{\lambda}(g)$. It follows that if $\underline{\alpha}/\bar{\alpha} > \lambda$, then $\breve{K}_1(\bar{\alpha}, \theta) < \underline{\alpha}/\psi^l$ for $g < g_0$, as desired. Step Two: The inequality follows from Lemma 4 and $E[\alpha] \leq \bar{\alpha}$. Let $K_1^*(\theta)$ be a maximizer of $\Pi_1(K_1)$. We now prove the equality $K_1^*(\theta) = \breve{K}_1(E[\alpha], \theta)$. We write $\pi_1(\alpha, K_1)$, $r_1(\alpha, K_1)$, $v^l(\alpha, p, w_2)$, $p^l(\alpha, K_1)$ and $w_2^l(\alpha, K_1)$ to denote these quantities under deterministic demand state α. First, note for all $\alpha \in [\underline{\alpha}, \bar{\alpha}]$ and $K_1 < \underline{\alpha}/\psi^l$, $r_1(\alpha, K_1) = v^l(\alpha, p^l(\alpha, K_1), w_2^l(\alpha, K_1))$ (by Lemma

1), which is strictly concave in K_1 on $K_1 \in (0, \underline{\alpha}/\psi^l)$. Because $c(K_1)$ is weakly convex, $\pi_1(\alpha, K_1)$ is strictly concave in K_1 on $K_1 \in (0, \underline{\alpha}/\psi^l)$ for $\alpha \in [\underline{\alpha}, \bar{\alpha}]$. Therefore, $\Pi_1(K_1)$ is strictly concave in K_1 on $K_1 \in (0, \underline{\alpha}/\psi^l)$. Because $\breve{K}_1(\bar{\alpha}, \theta) < \underline{\alpha}/\psi^l$, and $K_1^*(\theta) \leq \breve{K}_1(\bar{\alpha}, \theta)$ (by Lemma 5), it follows that $K_1^*(\theta) < \underline{\alpha}/\psi^l$. Because $\Pi_1(K_1)$ is strictly concave in K_1 on $K_1 \in (0, \underline{\alpha}/\psi^l)$, this implies $K_1^*(\theta)$ is given by the unique solution to the first order condition $(\partial/\partial K_1)\Pi_1(K_1) = 0$. Expanding $\Pi_1(K_1)$ and applying Leibniz's rule to interchange the derivative and integral operators, it follows that $K_1^*(\theta)$ is the solution to

$$\int_{\underline{\alpha}}^{\bar{\alpha}} (\partial/\partial K_1) r_1(\alpha, K_1) f(\alpha) d\alpha - \theta(\partial/\partial K_1)c(K_1) = 0. \tag{11}$$

It is straightforward to show that $(\partial/\partial K_1)r_1(\alpha, K_1)$ is linear in α for all $K_1 \in (0, \underline{\alpha}/\psi^l)$ and $\alpha \in [\underline{\alpha}, \bar{\alpha}]$. By linearity of expectation, the first term in (11) simplifies to $(\partial/\partial K_1)r_1(E[\alpha], K_1)$ for $K_1 < \underline{\alpha}/\psi^l$. Because $\breve{K}_1(E[\alpha], \theta) < \underline{\alpha}/\psi^l$ (as established above), $\breve{K}_1(E[\alpha], \theta)$ also solves (11). Because $K_1^*(\theta)$ and $\breve{K}_1(E[\alpha], \theta)$ are both solutions to (11), and because strict concavity of $\pi_1(E[\alpha], K_1)$ in K_1 on $K_1 \in (0, \underline{\alpha}/\psi^l)$ implies (11) has a unique solution on $K_1 \in (0, \underline{\alpha}/\psi^l)$, it must be that $K_1^*(\theta) = \breve{K}_1(E[\alpha], \theta)$. Lastly, because $\breve{K}_1(E[\alpha], \theta) < \underline{\alpha}/\psi^l \leq E[\alpha]/\psi^l$, it follows from Lemma 1 that $K^l(E[\alpha], \theta) = \breve{K}_1(E[\alpha], \theta)$.□

Lemma 7 $\Pi_1(K_1)$ *is twice-differentiable for all* $K_1 > 0$.

Proof of Lemma 7. It suffices to show that $(\partial^2/\partial K_1^2)R_1(K_1)$ is continuous in K_1. Suppose $\bar{\alpha}/\psi^l > \underline{\alpha}/\psi^s$; we consider the case where the inequality is reversed subsequently. Then there are five cases: $K_1 < \underline{\alpha}/\psi^l$, $\underline{\alpha}/\psi^l \leq K_1 < \underline{\alpha}/\psi^s$, $\underline{\alpha}/\psi^s \leq K_1 < \bar{\alpha}/\psi^l$, $\bar{\alpha}/\psi^l \leq K_1 < \bar{\alpha}/\psi^s$, and $\bar{\alpha}/\psi^l \leq K_1$. We consider the case where $\underline{\alpha}/\psi^s \leq K_1 < \bar{\alpha}/\psi^s$; the remaining cases follow by similar arguments and are more straightforward. We write $v^l(\alpha, p, w_2)$, $v^s(\alpha, p)$, $w_2^l(\alpha, K_1)$ and $p^u(\alpha, K_1)$ where $u \in \{e, l, s\}$ to denote these quantities under deterministic demand state α. Let $r^l(\alpha, K_1) = v^l(\alpha, p^l(\alpha, K_1), w_2^l(\alpha, K_1))$, $r^e(\alpha, K_1) = v^s(\alpha, p^e(\alpha, K_1))$ and $r^s(\alpha, K_1) = v^s(\alpha, p^s(\alpha, K_1))$. By Lemma 1,

$$(\partial^2/\partial K_1^2)R_1(K_1) = (\partial^2/\partial K_1^2)\int_{\underline{\alpha}}^{\psi^s K_1} r^s(\alpha, K_1) f(\alpha) d\alpha + (\partial^2/\partial K_1^2)\int_{\psi^s K_1}^{\psi^l K_1} r^e(\alpha, K_1) f(\alpha) d\alpha$$
$$+ (\partial^2/\partial K_1^2)\int_{\psi^l K_1}^{\bar{\alpha}} r^l(\alpha, K_1) f(\alpha) d\alpha. \tag{12}$$

It suffices to show that each of the three terms on the right hand side of (12) are continuous in K_1. Consider the first term. By Leibniz's rule,

$$(\partial/\partial K_1)\int_{\underline{\alpha}}^{\psi^s K_1} r^s(\alpha, K_1) f(\alpha) d\alpha = \psi^s r^s(\psi^s K_1, K_1) f(\psi^s K_1) + \int_{\underline{\alpha}}^{\psi^s K_1} (\partial/\partial K_1) r^s(\alpha, K_1) f(\alpha) d\alpha.$$

By a second application of Leibniz's rule,

$$(\partial^2/\partial K_1^2)\int_{\underline{\alpha}}^{\psi^s K_1} r^s(\alpha, K_1) f(\alpha) d\alpha = (\psi^s)^2 f'(\psi^s K_1) r^s(\psi^s K_1, K_1) + \psi^s f(\psi^s K_1)(\partial/\partial K_1) r^s(\psi^s K_1, K_1)$$
$$+\psi^s f(\psi^s K_1)(\partial/\partial K_1) r^s(\psi^s K_1, K_1) + \int_{\underline{\alpha}}^{\psi^s K_1} (\partial^2/\partial K_1^2) r^s(\alpha, K_1) f(\alpha) d\alpha.$$

Because $f'(\alpha)$ is continuous and r^s is continuous in both arguments, $(\partial^2/\partial K_1^2)\int_{\underline{\alpha}}^{\psi^s K_1} r^s(\alpha, K_1)f(\alpha)d\alpha$ is continuous in K_1. It can be shown by a parallel argument that the second and third terms on the right hand side of (12) are each continuous in K_1. Therefore, $(\partial^2/\partial K_1^2)\Pi_1(K_1)$ is continuous at K_1. The case where $\bar{\alpha}/\psi^l \leq \underline{\alpha}/\psi^s$ follows by a parallel argument. $\square$

Proof of Proposition 1A: (i) For brevity, in what follows, we drop the second argument in $\breve{K}_1(\alpha,\theta)$. Because $\theta \geq \hat{\theta}$, $\breve{K}_1(\bar{\alpha}) = 0$ for all $g \in [0, b)$ (by Lemma 2). Because $K_1^* \leq \breve{K}_1(\bar{\alpha})$ by Lemma 5, this implies $K_1^* = 0$ for all $g \in [0, b)$. It follows that $(d/dg)K_1^* = 0$ over $g \in [0, b)$. The remainder of the proof follows by a parallel argument to the proof of Proposition 1(i). (ii) It is useful to introduce notation for the special case where the demand state is deterministic: let $\breve{p}_1(\alpha)$ denote platform 1's equilibrium price under deterministic demand state α; we write $\breve{K}_1(\alpha)$, $p_1^l(\alpha)$ and $p_1^l(\alpha, K_1)$ to make the dependence on the deterministic demand state α and AV fleet K_1 explicit. We write $p_1^*(\alpha, K_1)$ to denote the equilibrium price under AV fleet K_1. By Lemma 6, for all $\theta > 0$ there exists $\lambda < 1$ and $g_0 > 0$ such that if $\underline{\alpha}/\bar{\alpha} > \lambda$ and $g < g_0$, then $K_1^*(\theta) = \breve{K}_1(E[\alpha], \theta)$. This implies that $p_1^*(E[\alpha], K_1^*) = p_1^*(E[\alpha], \breve{K}_1(E[\alpha])) = \breve{p}_1(E[\alpha])$, where the first equality follows because $K_1^* = \breve{K}_1(E[\alpha])$ and the second equality follows by definition of $\breve{p}_1(E[\alpha])$. Therefore, $(d/dg)p_1^*(E[\alpha], K_1^*) = (d/dg)\breve{p}_1(E[\alpha])$ if $\underline{\alpha}/\bar{\alpha} > \lambda$ and $g < g_0$. Next, by Proposition 1(ii), there exists $\bar{\theta} > 0$ such that for all $\theta \in (0, \bar{\theta}]$, there exist $\bar{\gamma} > 0$, $\underline{g} \geq 0$ and $\bar{g} > \underline{g}$ such that $(d/dg)\breve{p}_1(E[\alpha]) < 0$ for $g \in (\underline{g}, \bar{g})$ and $\gamma \in [0, \bar{\gamma}]$. Following the proof of Proposition 1(ii), it can be shown that $\bar{g} < g_0$. Combining the preceding three statements yields the following: There exists $\bar{\theta} > 0$ such that for all $\theta \in (0, \bar{\theta}]$, there exists $\lambda < 1$, $\bar{\gamma} > 0$, $\underline{g} > 0$ and $\bar{g} > \underline{g}$ such that if $\underline{\alpha}/\bar{\alpha} > \lambda$, then $(d/dg)p_1^*(E[\alpha], K_1^*) < 0$ for all $\gamma \in [0, \bar{\gamma}]$ and $g \in (\underline{g}, \bar{g})$. The result follows from continuity of $(d/dg)p_1^*(\alpha, K_1^*)$ in α. $\square$

Proof of Proposition 2A: (i) The proof proceeds in two steps. First, we show that $\Pi_1(K_1)$ is strictly concave in K_1 on $K_1 \in (0, \bar{\alpha}/\psi^s)$ and has a unique maximizer $K_1^* \in [0, \bar{\alpha}/\psi^s]$. Second, we show that $(d/dg)K_1^* \geq 0$ for all $g \in (0, b)$. Step One: Note that because $\gamma = 0$, $\tilde{g} = b$ by definition.Then by Lemma 5, $\Pi_1(K_1)$ is strictly concave in K_1 on $K_1 \in (0, \underline{\alpha}/\psi^s)$. It remains to show that $(\partial^2/\partial K_1^2)R_1(K_1) < 0$ for $K_1 \in (\underline{\alpha}/\psi^s, \bar{\alpha}/\psi^s)$. We write $v^l(\alpha, \mathbf{p}, w_2)$, $v^s(\alpha, \mathbf{p})$, $w_2^l(\alpha, K_1)$ and $\mathbf{p}^u(\alpha, K_1)$ where $u \in \{e, l, s\}$ to make explicit the dependence on the demand state α. Let $r^l(\alpha, K_1) = v^l(\alpha, \mathbf{p}^l(\alpha, K_1), w_2^l(\alpha, K_1))$, $r^e(\alpha, K_1) = v^s(\alpha, \mathbf{p}^e(\alpha, K_1))$ and $r^s(\alpha, K_1) = v^s(\alpha, \mathbf{p}^s(\alpha, K_1))$. By Lemma 1,

$$(\partial/\partial K_1)R_1(K_1) = (\partial/\partial K_1)\int_{\underline{\alpha}}^{\psi^s K_1} r^s(\alpha, K_1)f(\alpha)d\alpha + (\partial/\partial K_1)\int_{\psi^s K_1}^{\psi^l K_1} r^e(\alpha, K_1)f(\alpha)d\alpha$$
$$+ (\partial/\partial K_1)\int_{\psi^l K_1}^{\bar{\alpha}} r^l(\alpha, K_1)f(\alpha)d\alpha.$$

It is straightforward to show that $r^s(\psi^s K_1, K_1) = r^e(\psi^s K_1, K_1)$, $r^e(\psi^l K_1, K_1) = r^l(\psi^l K_1, K_1)$ and $(\partial/\partial K_1)r^s(\alpha, K_1) = 0$. Applying Leibniz's rule and these equalities yields

$$(\partial/\partial K_1)R_1(K_1) = \int_{\psi^s K_1}^{\psi^l K_1} (\partial/\partial K_1)r^e(\alpha, K_1)f(\alpha)d\alpha + \int_{\psi^l K_1}^{\bar{\alpha}} (\partial/\partial K_1)r^l(\alpha, K_1)f(\alpha)d\alpha.$$

Next, it is straightforward to show that $\gamma = 0$ implies $(\partial/\partial K_1)r^e(\psi^s K_1, K_1) = 0$ and $(\partial/\partial K_1)r^e(\psi^l K_1, K_1) = (\partial/\partial K_1)r^l(\psi^l K_1, K_1)$. Applying Leibniz's rule and these equalities yields

$$(\partial^2/\partial K_1^2)R_1(K_1) = \int_{\psi^s K_1}^{\psi^l K_1} (\partial^2/\partial K_1^2)r^e(\alpha, K_1)f(\alpha)d\alpha + \int_{\psi^l K_1}^{\bar{\alpha}} (\partial^2/\partial K_1^2)r^l(\alpha, K_1)f(\alpha)d\alpha.$$

It is straightforward to show that $(\partial^2/\partial K_1^2)r^e(\alpha, K_1) < 0$ and $(\partial^2/\partial K_1^2)r^l(\alpha, K_1) < 0$. Therefore, $(\partial^2/\partial K_1^2)R_1(K_1) < 0$ for all $K_1 \in (\underline{\alpha}/\psi^s, \bar{\alpha}/\psi^s)$. Because $(\partial^2/\partial K_1^2)R_1(K_1) < 0$ for $K_1 \in (0, \underline{\alpha}/\psi^s) \cup (\underline{\alpha}/\psi^s, \bar{\alpha}/\psi^s)$, and because $(\partial^2/\partial K_1^2)R_1(K_1)$ is continuous in K_1 (by Lemma 7), $(\partial^2/\partial K_1^2)R_1(K_1) < 0$ for $K_1 \in (0, \bar{\alpha}/\psi^s)$. We now show that $\Pi_1(K_1)$ attains its maximum on $[0, \bar{\alpha}/\psi^s]$. It is straightforward to show that $(\partial/\partial K_1)v^s(\alpha, \mathbf{p}^s) = 0$ for $K_1 > \alpha/\psi^s$. Because $\pi_1(\alpha, K_1) = v^s(\alpha, \mathbf{p}^s) - \theta c(K_1)$ for $K_1 > \alpha/\psi^s$, it follows that $(\partial/\partial K_1)\pi_1(\alpha, K_1) < 0$ for $K_1 > \alpha/\psi^s$. Therefore, $\breve{K}_1(\alpha) \leq \alpha/\psi^s$ for all $\alpha \in [\underline{\alpha}, \bar{\alpha}]$. By Lemma 5, because $g < \tilde{g} = b$, any maximizer of $\Pi_1(K_1)$ must lie in $[0, \bar{\alpha}/\psi^s]$. Because $\Pi_1(K_1)$ is strictly concave in K_1 for $K_1 \in (0, \bar{\alpha}/\psi^s)$, it follows that $\Pi_1(K_1)$ has a unique maximizer, K_1^*. Step Two: Note that if $K_1^* = 0$, then it immediately follows that $(d/dg)K_1^* \geq 0$. Suppose $K_1^* > 0$. Therefore, by the implicit function theorem, $(d/dg)K_1^* = -\left[(\partial^2/\partial K_1 \partial g) \int_{\underline{\alpha}}^{\bar{\alpha}} \pi_1(\alpha, K_1)f(\alpha)d\alpha \Big/ (\partial^2/\partial K_1^2)\Pi_1(K_1)\right]\Big|_{K_1=K_1^*}$. Because K_1^* is the unique maximizer of $\Pi_1(K_1)$, $(\partial^2/\partial K_1^2)\Pi_1(K_1)|_{K_1=K_1^*} < 0$. Further, it is straightforward to show algebraically that because $\gamma = 0$, $\pi_1(\alpha, K_1)$ is differentiable everywhere in K_1 and g. Therefore, by Leibniz's rule, $(\partial^2/\partial K_1 \partial g) \int_{\underline{\alpha}}^{\bar{\alpha}} \pi_1(\alpha, K_1)f(\alpha)d\alpha = \int_{\underline{\alpha}}^{\bar{\alpha}} (\partial^2/\partial K_1 \partial g)\pi_1(\alpha, K_1)f(\alpha)d\alpha$. It remains to show that $\int_{\underline{\alpha}}^{\bar{\alpha}} (\partial^2/\partial K_1 \partial g)\pi_1(\alpha, K_1)f(\alpha)d\alpha|_{K_1=K_1^*} \geq 0$. Note that $(\partial/\partial K_1)\pi_1(\alpha, K_1) = -\theta(\partial/\partial K_1)c(K_1)$ for $\alpha < \psi^s K_1^*$, and thus $(\partial^2/\partial K_1 \partial g)\pi_1(\alpha, K_1)|_{K_1=K_1^*} = 0$ for $\alpha < \psi^s K_1^*$. Further, it is straightforward to show that $\gamma = 0$ implies $(\partial^2/\partial K_1 \partial g)\pi_1(\alpha, K_1) = 0$ for $\alpha \in (\psi^s K_1^*, \psi^l K_1^*)$. Because, as noted above, $\pi_1(\alpha, K_1)$ is differentiable in K_1 and g, it follows that $(\partial^2/\partial K_1 \partial g)\pi_1(\alpha, K_1)$ is continuous at $\alpha = \psi^s K_1^*$, which implies $(\partial^2/\partial K_1 \partial g)\pi_1(\alpha, K_1) = 0$ for $\alpha \in (0, \psi^l K_1^*)$. If $\bar{\alpha} \leq \psi^l K_1^*$, then $(\partial^2/\partial K_1 \partial g)\pi_1(\alpha, K_1)|_{K_1=K_1^*} = 0$ for all $\alpha \in [\underline{\alpha}, \bar{\alpha}]$, and thus $(d/dg)K_1^* = 0$. Suppose $\bar{\alpha} > \psi^l K_1^*$. Then by Lemma 1,

$$\int_{\underline{\alpha}}^{\bar{\alpha}} (\partial^2/\partial K_1 \partial g)\pi_1(\alpha, K_1)f(\alpha)d\alpha\Big|_{K_1=K_1^*} = \int_{\psi^l K_1^*}^{\bar{\alpha}} (\partial^2/\partial K_1 \partial g)[r^l(\alpha, K_1) - \theta c(K_1)]f(\alpha)d\alpha\Big|_{K_1=K_1^*}.$$

It remains to show that there exists $\lambda_a < 1$ such that if $\underline{\alpha}/\bar{\alpha} > \lambda_a$, then $(\partial^2/\partial K_1 \partial g)r^l(\alpha, K_1)|_{K_1=K_1^*} \geq 0$ for all $\alpha \in (\psi^l K_1^*, \bar{\alpha}]$. Following the proof of Proposition 2(i), it is straightforward to show that for any $\alpha > 0$, $(\partial^2/\partial K_1 \partial g)r^l(\alpha, K_1) > 0$ for all $K_1 \in [0, \alpha/\psi^l]$. Therefore, $(\partial^2/\partial K_1 \partial g)r^l(\bar{\alpha}, K_1) > 0$ for all $K_1 \in [0, \bar{\alpha}/\psi^l]$. Because $(\partial^2/\partial K_1 \partial g)r^l(\alpha, K_1)$ is continuous in α, there exists $\lambda_a < 1$ such that if $\underline{\alpha}/\bar{\alpha} \geq \lambda_a$, then $(\partial^2/\partial K_1 \partial g)r^l(\alpha, K_1) > 0$ for all $\alpha \in [\underline{\alpha}, \bar{\alpha}]$ and $K_1 \in [0, \bar{\alpha}/\psi^l]$. The result follows because $K_1^* \in (0, \bar{\alpha}/\psi^l)$. (ii) First, we show that there exists $\lambda_b < 1$, $\bar{\theta} > 0$, $\underline{\underline{g}} > 0$ and $\bar{\bar{g}} > \underline{\underline{g}}$ such that if $\underline{\alpha}/\bar{\alpha} \geq \lambda_b$, $\theta < \bar{\theta}$ and $g \in (\underline{\underline{g}}, \bar{\bar{g}})$, then $\Pi_1(K_1)$ attains a unique maximizer K_1^*, where $K_1^* \in [\underline{\alpha}/\psi^l, \bar{\alpha}/\psi^l]$. Second, we show that there exists $\underline{g} > 0$, $\bar{g} > 0$ and $\lambda_c \in [\lambda_b, 1)$ such that if $\underline{\alpha}/\bar{\alpha} \geq \lambda_c$, then K_1^* is strictly decreasing in g on $g \in (\underline{g}, \bar{g})$, where $(\underline{g}, \bar{g}) \subseteq (\underline{\underline{g}}, \bar{\bar{g}})$. Step One: From the proof of Proposition 2(ii), there exists $\underline{\underline{g}} > 0$ and $\bar{\bar{g}} \leq \tilde{g}$ such that $\theta^e(\alpha) < 0 < \theta^l(\alpha)$ for $g \in (\underline{\underline{g}}, \bar{\bar{g}})$ and $\alpha > 0$. Let $\lambda_b = \sup_{g \in (\underline{\underline{g}}, \bar{\bar{g}})} \psi^s(g)/\psi^l(g)$, and note that $\lambda_b < 1$ because $\psi^s(g) < \psi^l(g)$ for all $g > 0$. Note that $\underline{\alpha}/\bar{\alpha} \geq \lambda_b$ implies $\underline{\alpha}/\bar{\alpha} > \psi^s(g)/\psi^l(g)$ for any $g \in (\underline{\underline{g}}, \bar{\bar{g}})$, and therefore $\bar{\alpha}/\psi^l < \underline{\alpha}/\psi^s$ for any $g \in (\underline{\underline{g}}, \bar{\bar{g}})$. By Lemma 5, $\Pi_1(K_1)$ is strictly concave in K_1 on $K_1 \in (0, \underline{\alpha}/\psi^s)$. Because $\bar{\alpha}/\psi^l < \underline{\alpha}/\psi^s$ for all $g \in (\underline{\underline{g}}, \bar{\bar{g}})$, $\pi_1(\alpha, K_1)$ is strictly concave in K_1 on $K_1 \in (0, \bar{\alpha}/\psi^l)$ for any $g \in (\underline{\underline{g}}, \bar{\bar{g}})$. Therefore, the maximizer of $\Pi_1(K_1)$ on $K_1 \in (0, \bar{\alpha}/\psi^l)$ is unique, for any $\theta > 0$. Next, let $\bar{\theta} = \inf_{\alpha \in [\underline{\alpha}, \bar{\alpha}]} \theta^l(\alpha)$. It remains to show that if $\theta < \bar{\theta}$, then any maximizer of $\Pi_1(K_1)$ must lie on $[\underline{\alpha}/\psi^l, \bar{\alpha}/\psi^l]$. By Lemma 5, any maximizer of $\Pi_1(K_1)$ must lie on $[\breve{K}_1(\underline{\alpha}), \breve{K}_1(\bar{\alpha})]$. Therefore, it suffices to show that $\breve{K}_1(\alpha) = \alpha/\psi^l$ for $\theta < \bar{\theta}$ and $\alpha \in [\underline{\alpha}, \bar{\alpha}]$. Note $\bar{\theta} \leq \theta^l(\alpha)$ for all $\alpha \in [\underline{\alpha}, \bar{\alpha}]$ by definition of $\bar{\theta}$. As noted above, $\theta^e(\alpha) < 0$ for all $g \in (\underline{\underline{g}}, \bar{\bar{g}})$ and $\alpha > 0$. It follows that if $\theta < \bar{\theta}$, then by Lemma 2, $\breve{K}_1(\alpha) = \alpha/\psi^l$ for all $\alpha \in [\underline{\alpha}, \bar{\alpha}]$. Step Two: It is straightforward to show that $(\partial/\partial g)\psi^l > 0$. Therefore, $\psi^l(\underline{\underline{g}})/\psi^l(\bar{\bar{g}}) < 1$. Let $\lambda_c = \max\{\lambda_b, \psi^l(\underline{\underline{g}})/\psi^l(\bar{\bar{g}})\}$, and note $\lambda_c < 1$. Suppose $\underline{\alpha}/\bar{\alpha} > \lambda_c$. Then, $K_1^*(\bar{\bar{g}}) \leq \bar{\alpha}/\psi^l(\bar{\bar{g}}) > \underline{\alpha}/\psi^l(\underline{\underline{g}}) \leq K_1^*(\underline{\underline{g}})$, where the first and third inequalities follow from Step One, and the second inequality follows because $\underline{\alpha}/\bar{\alpha} > \lambda_c \geq \psi^l(\underline{\underline{g}})/\psi^l(\bar{\bar{g}})$. Note that K_1^* is continuous in g by Berge's maximum theorem. Because $K_1^*(\bar{\bar{g}}) < K_1^*(\underline{\underline{g}})$, $\bar{\bar{g}} > \underline{\underline{g}}$ and K_1^* is continuous in g, there must exist $\underline{g} > 0$ and $\bar{g} > \underline{g}$ where $(\underline{g}, \bar{g}) \subseteq (\underline{\underline{g}}, \bar{\bar{g}})$ such that K_1^* strictly decreases in g on $g \in (\underline{g}, \bar{g})$. Lastly, to obtain the constant λ in the proposition statement, it suffices to set $\lambda = \max\{\lambda_a, \lambda_b, \lambda_c\}$. $\square$

Proof of Proposition 3A: Define $\bar{\theta} = \lim_{K_1 \to 0}\{(\partial/\partial K_1)R_1(K_1)/(\partial/\partial K_1)c(K_1)\}$ if $\lim_{K_1 \to 0}(\partial/\partial K_1)c(K_1) > 0$, and $\bar{\theta} = \infty$ otherwise. Define $\underline{\theta} = \lim_{K_1 \to \underline{\alpha}/\psi^l}\{(\partial/\partial K_1)R_1(K_1)/(\partial/\partial K_1)c(K_1)\}$. By Lemma 5, $\Pi_1(K_1)$ is strictly concave in K_1 on $K_1 \in (0, \underline{\alpha}/\psi^s)$. Because $\psi^l \geq \psi^s$, $\underline{\alpha}/\psi^l \leq \underline{\alpha}/\psi^s$. Therefore, $\Pi_1(K_1)$ is also strictly concave in K_1 on $K_1 \in (0, \underline{\alpha}/\psi^l)$. Because $\Pi_1(K_1)$ is strictly concave in K_1 on $K_1 \in (0, \underline{\alpha}/\psi^l)$, $\theta \in [\underline{\theta}, \bar{\theta}]$ implies $\lim_{K_1 \to \underline{\alpha}/\psi^l}(\partial/\partial K_1)\Pi_1(K_1) \leq 0 \leq \lim_{K_1 \to 0}(\partial/\partial K_1)\Pi_1(K_1)$. It follows that if $\theta \in [\underline{\theta}, \bar{\theta}]$, then $\Pi_1(K_1)$ has a unique maximizer $K_1^* \in [0, \underline{\alpha}/\psi^l]$. We wish to show that for $\theta \in [\underline{\theta}, \bar{\theta}]$, $(d/d\theta)\Pi_2(K_1^*) < 0$ if and only if $g/b > \gamma/\beta$. The total derivative of Π_2 with respect to θ is $(d/d\theta)\Pi_2(K_1^*) = [(\partial/\partial K_1)\Pi_2(K_1)(d/d\theta)K_1^*]|_{K_1 = K_1^*}$. First we show $(d/d\theta)K_1^* < 0$ for $\theta \in [\underline{\theta}, \bar{\theta}]$. Because $\Pi_1(K_1)$ has a unique maximizer K_1^* for $\theta \in [\underline{\theta}, \bar{\theta}]$, we may apply the implicit function theorem to obtain $(d/d\theta)K_1^* = -[(\partial^2/\partial K_1 \partial\theta)\Pi_1(K_1)]/[(\partial^2/\partial K_1^2)\Pi_1(K_1)]|_{K_1 = K_1^*}$. Note $(\partial^2/\partial K_1^2)\Pi_1(K_1)|_{K_1 = K_1^*} < 0$ because $\Pi_1(K_1)$ is strictly concave and $(\partial^2/\partial K_1 \partial\theta)\Pi_1(K_1) = -(\partial/\partial K_1)c(K_1) < 0$ because $c(K_1)$ is strictly increasing. It follows that $(d/d\theta)K_1^* < 0$ for $\theta \in [\underline{\theta}, \bar{\theta}]$. It remains to show that $(\partial/\partial K_1)\Pi_2(K_1)|_{K_1 = K_1^*} > 0$ if and only if $g/b > \gamma/\beta$. Note $K_1^* < \underline{\alpha}/\psi^l$ because $\theta \geq \underline{\theta}$. It is straightforward to show that $\pi_2(\alpha, K_1)$ is differentiable in K_1 on $K_1 \in [0, \underline{\alpha}/\psi^l]$ for all $\alpha \in [\underline{\alpha}, \bar{\alpha}]$. It follows from Leibniz's rule that $(\partial/\partial K_1)\Pi_2(K_1)|_{K_1 = K_1^*} = \int_{\underline{\alpha}}^{\bar{\alpha}}(\partial/\partial K_1)\pi_2(\alpha, K_1)f(\alpha)d\alpha|_{K_1 = K_1^*}$. It remains to show that $\int_{\underline{\alpha}}^{\bar{\alpha}}(\partial/\partial K_1)\pi_2(\alpha, K_1)f(\alpha)d\alpha|_{K_1 = K_1^*} > 0$ if and only if $g/b > \gamma/\beta$. The proof of Proposition 3 shows that if $K_1^* \leq \alpha/\psi^l$, then $(\partial/\partial K_1)\pi_2(\alpha, K_1) > 0$ if and only if $g/b > \gamma/\beta$. Because $K_1^* \leq \underline{\alpha}/\psi^l$, it follows that for all $\alpha \in [\underline{\alpha}, \bar{\alpha}]$, $(\partial/\partial K_1)\pi_2(\alpha, K_1) < 0$ if and only if $g/b > \gamma/\beta$. The result follows. □

Electronic Companion

Appendix C: Limited Sophisticated of Autonomous Vehicles

Because $L_i(\mathbf{w}) = \max\{D_i^a(\mathbf{p}^a) - K_i, 0\} + D_i^o(\mathbf{p}^o)$, platform 1's optimal wage $w_1 = [D_1^o(\mathbf{p}^o) + \min\{D_1^a(\mathbf{p}^a) - K_1, 0\} + gw_2]/b$. Platform 1 sources labor for market a, $D_1^a(\mathbf{p}^a) > K_1$, if and only if its market a price is sufficiently low $p_1^a < (\alpha + \gamma p_2^a - K_1/\tau)/\beta$. Therefore, platform 1's second-period contribution is given by (2), where: in the condition on the right hand side, p_i^a replaces p_i for $i \in \{1,2\}$ and K_1/τ replaces K_1; in $v^l(\mathbf{p},w_2)$, $p_1^a D_1^a(\mathbf{p}^a) + p_1^o D_1^o(\mathbf{p}^o)$ replaces $p_1 D_1(\mathbf{p})$ and $D_1^a(\mathbf{p}^a) + D_1^o(\mathbf{p}^o)$ replaces $D_1(\mathbf{p})$; and $v^s(\mathbf{p},w_2) = p_1^a D_1^a(\mathbf{p}^a) + p_1^o D_1^o(\mathbf{p}^o) - [(D_1^o(\mathbf{p}^o) + gw_2)/b]D_1^o(\mathbf{p}^o)$. By a parallel argument, platform 2's optimal wage $w_2 = (D_2^a(\mathbf{p}^a) + D_2^o(\mathbf{p}^o) + gw_1)/b$ and platform 2's second-period contribution is given by (3), where $p_2^a D_2^a(\mathbf{p}^a) + p_2^o D_2^o(\mathbf{p}^o)$ replaces $p_2 D_2(\mathbf{p})$ and $D_2^a(\mathbf{p}^a) + D_2^o(\mathbf{p}^o)$ replaces $D_2(\mathbf{p})$.

Let $\tilde{p}_1^{al}(K_1) = [b(\alpha+\gamma p_2^a) + \beta\{gw_2 + 2\alpha + \gamma[(1+\tau)p_2^a + (1-\tau)p_2^o] - 2K_1\}]/[2\beta(\beta+b)]$,$\tilde{p}_1^{ol}(K_1) = [b(\alpha+\gamma p_2^o) + \beta\{gw_2 + 2\alpha + \gamma[\tau p_2^a + (2-\tau)p_2^o] - 2K_1\}]/[2\beta(\beta+b)]$,$\tilde{p}_1^{ae}(K_1) = [(\alpha+\gamma p_2^a) - K_1/\tau]/\beta$, $\tilde{p}_1^{oe}(K_1) = \tilde{p}_1^{os}(K_1) = \{[2\beta(1-\tau)+b](\alpha+\gamma p_2^o) + \beta gw_2\}/\{2\beta[\beta(1-\tau)+b]\}$,$\tilde{p}_1^{as}(K_1) = (\alpha + \gamma p_2^a)/(2\beta)$, $\tilde{w}_1^l(K_1) = [(\beta+2b)gw_2 + b\{\alpha + \gamma[\tau p_2^a + (1-\tau)p_2^o] - 2K_1\}]/[2(\beta+b)b]$, $\tilde{w}_1^e(K_1) = \tilde{w}_1^s(K_1) = \{[\beta(1-\tau)+2b]gw_2 + (1-\tau)b(\alpha+\gamma p_2^o)\}/\{2[\beta(1-\tau)+b]b\}$,$\tilde{p}_2^a(K_1) = [b(\alpha+\gamma p_1^a) + \beta\{gw_1 + 2\alpha + \gamma[(1+\tau)p_1^a + (1-\tau)p_1^o]\}]/[2\beta(\beta+b)]$,$\tilde{p}_2^o(K_1) = [b(\alpha+\gamma p_1^o) + \beta\{gw_1 + 2\alpha + \gamma[\tau p_1^a + (2-\tau)p_1^o]\}]/[2\beta(\beta+b)]$ and $\tilde{w}_2(K_1) = [(\beta+2b)gw_1 + b\{\alpha + \gamma[\tau p_1^a + (1-\tau)p_1^o]\}]/[2(\beta+b)b]$. Let $(p_1^{au}(K_1), p_1^{ou}(K_1), w_1^u(K_1), p_2^{au}(K_1), p_2^{ou}(K_1), w_2^u(K_1))$ denote the unique solution to $p_1^{au}(K_1) = \tilde{p}_1^{au}(K_1)$,$p_1^{ou}(K_1) = \tilde{p}_1^{ou}(K_1)$,$w_1^u(K_1) = \tilde{w}_1^u(K_1)$,$p_2^{au}(K_1) = \tilde{p}_2^a(K_1)$,$p_2^{ou}(K_1) = \tilde{p}_2^o(K_1)$ and $w_2^u(K_1) = \tilde{w}_2(K_1)$, where $u \in \{e,l,s\}$. Let $\sigma^l = \tau\{b(\alpha+\gamma p_2^a) - \beta[gw_2 + \gamma(p_2^o - p_2^a)(1-\tau)]\}/\{2[\beta(1-\tau)+b]\}$ and $\sigma^s = \tau(\alpha+\gamma p_2^a)/2$.

Lemma 8 *Under platform 1 AV fleet K_1, platform 2's best response prices and wage to platform 1's prices and wage (p_1^a, p_1^o, w_1) is $(\tilde{p}_2^a(K_1), \tilde{p}_2^o(K_1), \tilde{w}_2(K_1))$. Platform 1's best response prices and wage to platform 2's price and wage (p_2^a, p_2^o, w_2) is*

$$(\tilde{p}_1^a(K_1), \tilde{p}_1^o(K_1), \tilde{w}_1(K_1)) = \begin{cases} (\tilde{p}_1^{al}(K_1), \tilde{p}_1^{ol}(K_1), \tilde{w}_1^l(K_1)) & \text{if } K_1 < \sigma^l, \\ (\tilde{p}_1^{ae}(K_1), \tilde{p}_1^{oe}(K_1), \tilde{w}_1^e(K_1)) & \text{if } K_1 \in [\sigma^l, \sigma^s], \\ (\tilde{p}_1^{as}(K_1), \tilde{p}_1^{os}(K_1), \tilde{w}_1^s(K_1)) & \text{if } K_1 > \sigma^s. \end{cases}$$

Further, $K_1 < D_1^a(\tilde{p}_1^a(K_1), p_2)$ if and only if $K_1 < \sigma^l$; $K_1 = D_1^a(\tilde{p}_1^a(K_1), p_2)$ if and only if $K_1 \in [\sigma^l, \sigma^s]$; and $K_1 > D_1^a(\tilde{p}_1^a(K_1), p_2)$ if and only if $K_1 > \sigma^s$.

Proof of Lemma 8: It is straightforward to show that platform 2's second-period contribution $v_2(\mathbf{p}, w_1)$ is strictly jointly concave in (p_2^a, p_2^o). Platform 2's best response prices are given by the unique solution to the first-order conditions $(\partial/\partial p_2^a)v_2(\mathbf{p}, w_1) = 0$ and $(\partial/\partial p_2^o)v_2(\mathbf{p}, w_1) = 0$, namely, $p_2^a = \tilde{p}_2^a(K_1)$ and $p_2^o = \tilde{p}_2^o(K_1)$. Further, platform 2's best response wage $w_2 = [D_2^a(p_1^a, \tilde{p}_2^a(K_1)) + D_2^o(p_1^o, \tilde{p}_2^o(K_1)) + gw_1]/b = \tilde{w}_2(K_1)$. Following (9), platform 1's second-period contribution can be written as $v_1(\mathbf{p}, w_1) = p_1^a D_1^a(\mathbf{p}^a) + p_1^o D_1^o(\mathbf{p}^o) - [(L_1)^2 + gw_2 L_1]/b$, where $L_1 = D_1^o(\mathbf{p}^o) + \max\{D_1^a(\mathbf{p}^a) - K_1, 0\}$.The function $p_1^a D_1^a(\mathbf{p}^a) + p_1^o D_1^o(\mathbf{p}^o)$ can be shown to be strictly concave in (p_1^a, p_1^o) because its Hessian is negative definite. Because $\max\{D_1^a(\mathbf{p}^a) - K_1, 0\}$ is the pointwise maximum of two functions that are convex in (p_1^a, p_1^o),L_1 is also convex in (p_1^a, p_1^o); further, because L_1^2 is the composite of the convex function L_1 and the convex, nondecreasing function $(\cdot)^2$, L_1^2 is also convex (Boyd and Vandenberghe 2004). It follows that $-[(L_1)^2 + gw_2 L_1]/b$ is weakly concave in (p_1^a, p_1^o). Because $v_1(\mathbf{p}, w_2)$ is the sum of strictly concave and weakly concave functions in (p_1^a, p_1^o),$v_1(\mathbf{p}, w_2)$ is strictly concave in (p_1^a, p_1^o). Let $\breve{p}_1^{ol}(p_1^a)$ denote the unique solution to the first-order condition $(\partial/\partial p_1^o)v^l(\mathbf{p}, w_2) = 0$, and define $\breve{v}^l(p_1^a, \mathbf{p}_2, w_2) = v^l(\mathbf{p}, w_2)|_{p_1^o=\breve{p}_1^{ol}(p_1^a)}$. Similarly, let $\breve{p}_1^{os}(p_1^a)$ denote the unique solution to $(\partial/\partial p_1^o)v^s(\mathbf{p}, w_2) = 0$, and define $\breve{v}^s(p_1^a, \mathbf{p}_2, w_2) = v^s(\mathbf{p}, w_2)|_{p_1^{os}=\breve{p}_1^{os}(p_1^a)}$. Note that in terms of platform 1's decision variables, platform 1's contribution functions $\breve{v}^l(p_1^a, \mathbf{p}_2, w_2)$ and $\breve{v}^s(p_1^a, \mathbf{p}_2, w_2)$ depend only on p_1^a. Parallel to (2), let $\breve{v}_1(p_1^a, \mathbf{p}_2, w_2) = \breve{v}^l(p_1^a, \mathbf{p}_2, w_2)$ if $p_1^a < \tilde{p}_1^{ae}(K_1)$ and $\breve{v}_1(p_1^a, \mathbf{p}_2, w_2) = \breve{v}^s(p_1^a, \mathbf{p}_2, w_2)$ otherwise. Note $\breve{v}_1(p_1^a, \mathbf{p}_2, w_2)$ can be shown to be strictly concave in p_1^a by parallel argument to that in Lemma 3. If $K_1 < \sigma^l$, then $\lim_{p_1^a \uparrow \tilde{p}_1^{ae}(K_1)}(\partial/\partial p_1)\breve{v}_1(p_1^a, \mathbf{p}_2, w_2) < 0$ and platform 1's best response price is the unique solution to the first-order conditions $(\partial/\partial p_1^a)v^l(\mathbf{p}, w_2) = 0$ and $(\partial/\partial p_1^o)v^l(\mathbf{p}, w_2) = 0$, namely, $p_1^a = \tilde{p}_1^{al}(K_1)$ and $p_1^o = \tilde{p}_1^{ol}(K_1)$. Further, $K_1 < D_1^a(\tilde{p}_1^{al}(K_1), p_2^a)$. Consequently, platform 1's best response wage $w_1 = [D_1^o(\tilde{p}_1^{ol}(K_1), p_2^o) + D_1^a(\tilde{p}_1^{al}(K_1), p_2^a) - K_1 + gw_2]/b = \tilde{w}_1^l(K_1)$. If $K_1 \in [\sigma^l, \sigma^s]$, then $\lim_{p_1^a \downarrow \tilde{p}_1^{ae}(K_1)}(\partial/\partial p_1)\breve{v}_1(p_1^a, \mathbf{p}_2, w_2) \le 0 \le \lim_{p_1^a \uparrow \tilde{p}_1^{ae}(K_1)}(\partial/\partial p_1)\breve{v}_1(p_1^a, p_2, w_2)$ and platform 1's best response prices are $p_1^a = \tilde{p}_1^{ae}(K_1)$ and $p_1^o = \tilde{p}_1^{oe}(K_1)$. Further, $K_1 = D_1^a(\tilde{p}_1^{ae}(K_1), p_2^a)$. Consequently, platform 1's best response wage $w_1 = [D_1^o(\tilde{p}_1^{oe}(K_1), p_2^o)+gw_2]/b = \tilde{w}_1^e(K_1)$. If $K_1 > \sigma^s$, then $\lim_{p_1 \downarrow \tilde{p}_1^{ae}(K_1)}(\partial/\partial p_1)\breve{v}_1(p_1^a, \mathbf{p}_2, w_2) > 0$ and platform 1's best response price is the unique solution to the first-order conditions $(\partial/\partial p_1^a)v^s(\mathbf{p}, w_2) = 0$ and $(\partial/\partial p_1^o)v^s(\mathbf{p}, w_2) = 0$, namely, $p_1^a = \tilde{p}_1^{as}(K_1)$ and $p_1^o = \tilde{p}_1^{os}(K_1)$.Further, $K_1 > D_1^a(\tilde{p}_1^{as}(K_1), p_2^a)$. Consequently, platform 1's best response wage $w_1 = [D_1^o(\tilde{p}_1^{os}(K_1), p_2^o) + gw_2]/b = \tilde{w}_1^s(K_1)$.□

Let $\mathbf{p}^{vu}(K_1) = <p_1^{vu}(K_1), p_2^{vu}(K_1)>$, $\mathbf{p}^u(K_1) = <p_1^{au}(K_1), p_1^{ou}(K_1), p_2^{au}(K_1), p_2^{ou}(K_1)>$ and $\mathbf{w}^u(K_1) = <w_1^u(K_1), w_2^u(K_1)>$, where $v \in \{a, o\}$ and $u \in \{e, l, s\}$. Define $\psi_n^l = (4\beta^2 - \gamma^2)b^2(b^2 - g^2) + (1 - \tau)\beta^2(\beta^2 - \gamma^2)(4b^2 - g^2) + (2 - \tau)\beta b[(2\beta^2 - \gamma^2)(2b^2 - g^2) - \beta\gamma bg]$, $\psi_d^l = \beta b\tau(b - g)[\beta(\beta + \gamma)(2b + g) + (2\beta + \gamma)b(b + g)]$, $\psi_n^s = b^4(4\beta^2 - \gamma^2)^2 + \beta b(2b^2 - g^2)(2\beta - \gamma)(2\beta + \gamma)(2\beta^2 - \gamma^2)(2 - \tau) - \beta^2(4b^2 - g^2)(((5 - \tau)\beta^2\gamma^2 - (4\beta^4 + \gamma^4))(1 - \tau)) - b^2(2g(1 - t)\beta^2\gamma(4\beta^2 - \gamma^2) + g^2(4\beta^2 - \gamma^2)^2)$, $\psi_d^s = \tau\beta(b^2(b^2 - g^2)(2\beta - \gamma)(2\beta + \gamma)^2 + b(2b^2 - g^2)\beta(2\beta + \gamma)(\beta\gamma + (2 - \tau)(2\beta^2 - \gamma^2)) + (1 - \tau)\beta^2(-b^2g\gamma(2\beta + \gamma) + (4b^2 - g^2)(\beta + \gamma)(2\beta^2 - \gamma^2)))$. Let $\psi^u = \psi_n^u/\psi_d^u$, where $u \in \{l, s\}$.

We write $\pi_i(K_1, \tau)$ and $r_i(K_i, \tau)$ to explicitly acknowledge dependence on τ; note that $\tau = 1$ corresponds to the model in §3. Recall that $p_i^l(K_1)$ for $i \in \{1, 2\}$ is defined in §3.1. Lemma 9 establishes that when platform 1 AV fleet K_1 is such that the platform sources labor, the equilibrium price pair is identical in the two markets, and the prices are invariant to τ.

Lemma 9 *(i) Under platform 1 AV fleet K_1, the equilibrium prices and wages are unique and given by (6). Further, there exists $g^+ > 0$ such that $\psi^l \ge \psi^s > 0$ if $g \in (0, g^+)$, where the first inequality is strict if $\tau < 1$, and $g^+ = b$ if $\gamma = 0$. (ii) $\mathbf{p}^{vl}(K_1) = <p_1^l(K_1), p_2^l(K_1)>$ for $v \in \{a, o\}$. (iii) If $\theta \in [\theta^l, \theta^0]$, then platform i's profit $\pi_i(K_1, \tau) = \pi_i(K_1, 1)$ for all $\tau \in (0, 1)$ and $i \in \{1, 2\}$.*

Proof of Lemma 9: (i) That the equilibrium is unique and given by (6) follows by the proof of Lemma 1, where $D_1^a(\mathbf{p}^{au}(K_1))$ replaces $D_1(\mathbf{p}^u(K_1))$ for $u \in \{e, l, s\}$. With some effort one can show that $\lim_{g\to 0} \psi^s(g) > 0$ and $\lim_{g\to 0} \psi^l(g) - \psi^s(g) \ge 0$, where the preceding inequality is strict if $\tau < 1$. It follows from the continuity of ψ^s and ψ^l in g that there exists $g^+ > 0$ such that $\psi^l \ge \psi^s > 0$ for $g \in (0, g^+)$. In the case where $\gamma = 0$, it can be verified that $\psi^l(g) \ge \psi^s(g)$ for all $g \in (0, b)$, which implies $g^+ = b$. (ii) The result follows from the definitions of $\mathbf{p}^{al}(\cdot), \mathbf{p}^{ol}(\cdot)$, $p_1^l(\cdot)$ and $p_2^l(\cdot)$. (iii) Suppose $\theta \in [\theta^l, \theta^0]$. Let $\mathbf{p}^{v*}(K_1) = <p_1^{v*}(K_1), p_1^{v*}(K_1)>$ for $v \in \{a, o\}$. Note

$\mathbf{p}^{v*}(K_1) = \mathbf{p}^{vl}(K_1) = <p_1^l(K_1), p_2^l(K_1)>$ for $v \in \{a, o\}$, where the first equality follows from part (i) and $\theta \in [\theta^l, \theta^0]$, and the second equality follows from part (ii). Consequently, platform i's second-period contribution under AV fleet K_1 and equilibrium prices and wages $(\mathbf{p}^*(K_1), \mathbf{w}^*(K_1))$, $r_i(K_i, \tau) = p_i^{a*}(K_1)D_i^a(\mathbf{p}^{a*}(K_1)) + p_i^{o*}(K_1)D_i^o(\mathbf{p}^{o*}(K_1)) - w_i^* L_i(\mathbf{w}^*(K_1)) = p_i^l(K_1)D_i(\mathbf{p}^l(K_1)) - w_i^l L_i(\mathbf{w}^l(K_1)) = r_i(K_i, 1)$ for $i \in \{1, 2\}$; the last equality follows because $\mathbf{p}^l(K_1)$ and $\mathbf{w}^l(K_1)$ are invariant to τ. The result is immediate.□

Lemma 10 *There exists $\tilde{g} > 0$ such that if the intensity of competition in the labor market $g < \tilde{g}$, then platform 1's profit $\pi_1(K_1)$ is quasi-concave in K_1 for all $\gamma \in [0, \beta)$, and platform 1's equilibrium AV fleet, K_1^*, is unique and given by (7), where $\theta^e < \theta^l < \theta^0$. If $\gamma = 0$, then $\tilde{g} = b$. Further, $K_1^* = 0$ if and only if $\theta \geq \theta^0$, and $K_1^* = 0$ for $g \in [0, b)$ if and only $\theta \geq \hat{\theta}$.*

Proof of Lemma 10: We proceed in three steps. First, to establish that $\tilde{g} = b$ if $\gamma = 0$, we show that if $\gamma = 0$, then $\pi_1(K_1)$ is strictly quasi-concave in K_1 for $g \in (0, b)$. Second, we address the case where $\gamma > 0$ and show that there exists $\tilde{g} > 0$ such that for all $\gamma \in (0, \beta)$, $\pi_1(K_1)$ is strictly quasi-concave in K_1 for $g \in (0, \tilde{g}]$. Third, we characterize K_1^*. Step One: Let $\gamma = 0$. Then by Lemma 9, $\psi^l(g) \geq \psi^s(g)$ for all $g \in (0, b)$. Next, it is straightforward to verify that $(\partial^2/\partial K_1^2)r(K_1) < 0$ for $K_1 \in (0, \alpha/\psi^l) \cup (\alpha/\psi^l, \alpha/\psi^s)$, $\lim_{K_1 \uparrow \alpha/\psi^l}(\partial^2/\partial K_1^2)r(K_1) > \lim_{K_1 \downarrow \alpha/\psi^l}(\partial^2/\partial K_1^2)r(K_1)$, and $(\partial^2/\partial K_1^2)r(K_1) < 0$ for $K_1 > \alpha/\psi^s$, which implies $r_1(K_1)$ is strictly quasi-concave for all $g \in (0, b)$. The result follows by weak concavity of $c(K_1)$. Step Two: First we show that for any $\gamma \in (0, \beta)$, there exists $\tilde{g}(\gamma) \in (0, b)$ such that $\pi_1(K_1)$ is strictly quasi-concave in K_1 for $g \in (0, \tilde{g}(\gamma)]$. Using Lemma 9, it is straightforward to verify that $r_1(K_1)$ is continuous in K_1 and $r_1(K_1)$ is invariant to K_1 on $K_1 > \alpha/\psi^s$. Therefore, it suffices to show that there exists $\tilde{g}(\gamma) \in (0, b)$ such that $\pi_1(K_1)$ is strictly concave in K_1 on $K_1 \in (0, \alpha/\psi^s)$ for $g \in (0, \tilde{g}(\gamma))$. It is straightforward to verify that $r_1(K_1)$ is strictly concave in K_1 for $K_1 < \alpha/\psi^l$ for all $g \in (0, b)$. Next, with some effort one can show that $\lim_{g \to 0}(\partial^2/\partial K_1^2)r_1(K_1) < 0$ for $K_1 \in (\alpha/\psi^l, \alpha/\psi^s)$, which implies that there exists $\breve{g}(\gamma) > 0$ such that $r_1(K_1)$ is strictly concave in K_1 on $K_1 \in (\alpha/\psi^l, \alpha/\psi^s)$ for $g \in (0, \breve{g}(\gamma))$.It remains to show that there exists $\tilde{g}(\gamma) \in (0, \breve{g}(\gamma))$ such that $\lim_{K_1 \uparrow (\alpha/\psi^l)}(\partial/\partial K_1)r_1(K_1) > \lim_{K_1 \downarrow (\alpha/\psi^l)}(\partial/\partial K_1)r_1(K_1)$ for $g \in (0, \tilde{g}(\gamma))$. Note $\lim_{g \to 0}\left[\lim_{K_1 \uparrow (\alpha/\psi^l)}(\partial/\partial K_1)r_1(K_1) - \lim_{K_1 \downarrow (\alpha/\psi^l)}(\partial/\partial K_1)r_1(K_1)\right] = [b^2\alpha(b + 2\beta)\gamma^2(4\beta^2 - \gamma^2)]/[\{2\beta(\beta - \gamma) + b(2\beta - \gamma)\}\{b^2(8\beta^4 - 6\beta^2\gamma^2 + \gamma^4) + 4(1 - \tau)\beta^2(2\beta^4 - 3\beta^2\gamma^2 + \gamma^4) + 2b\beta[(2 - \tau)(4\beta^4 + \gamma^4) - (8 - 3\tau)\beta^2\gamma^2]\}] > 0$, where the inequality follows because $\beta > \gamma$ and $\tau \in (0, 1)$. Therefore, there exists $\tilde{g}(\gamma) \in (0, b)$ such that $\pi_1(K_1)$ is strictly quasi-concave and $\theta^e < \theta^l$ for $g \in (0, \tilde{g}(\gamma))$. Next, define $\tilde{g} = \inf_{\gamma \in [0, b)} g(\gamma)$. It follows immediately that $\pi_1(K_1)$ is strictly quasi-concave and $\theta^e < \theta^l$ for $g \in (0, \tilde{g})$. Step Three: The characterization of K_1^* follows directly from the proof of Lemma 2. □

Proof of Proposition 1B: (i) By Lemma 10, $K_1^* = 0$ for all $g \in [0, b)$ if $\theta \geq \hat{\theta}$. By Lemma 9ii, this implies $p_1^{v*} = p_1^l(0)$ for $v \in \{a, o\}$. That $p_1^l(0)$ strictly increases in g follows directly from the proof of Proposition 1. (ii) Let $z(\tau) = \alpha(1 - \tau)/[\beta(1 - \tau) + b/2] - \theta(\partial/\partial K_1)c(K_1)|_{K_1 = \alpha/\psi^l(\tau)}$. The remainder of the proof proceeds in two steps. First, we show that there exists $\theta' > 0$ such that for $\theta \in (0, \theta']$, $z(\tau) = 0$ has a unique solution τ^* on $\tau \in (0, 1)$, and $z(\tau) < 0$ for $\tau \in (\tau^*, 1)$. Second, we use the result from the first step to prove the main result. Step One: First, we show that there exists $\theta' > 0$ such that for $\theta \in (0, \theta']$, $z(\tau)$ crosses 0 at least once on $\tau \in (0, 1)$. We do so by showing that for $\theta \in (0, \theta']$, $z(\tau)$ is continuous, $\lim_{\tau \to 0} z(\tau) > 0$ and $\lim_{\tau \to 1} z(\tau) < 0$. Continuity of $z(\tau)$ follows by definition of $z(\tau)$ and continuity of $\psi^l(\tau)$ in τ. Because $\lim_{\tau\to 0}\psi^l(\tau) = \infty$, $\lim_{\tau\to 0} z(\tau) = \alpha/(\beta+2b) - \theta(\partial/\partial K_1)c(K_1)|_{K_1=0}$. It follows that $\lim_{\theta\to 0}\lim_{\tau\to 0} z(\tau) = \alpha/(\beta+2b) > 0$, which implies that there exists $\theta' > 0$ such that $\lim_{\tau\to 0} z(\tau) > 0$ for $\theta \in (0,\theta']$. Note $\lim_{\tau\to 1} z(\tau) = -\theta(\partial/\partial K_1)c(K_1)|_{K_1=\alpha/\psi^l(1)} < 0$. Therefore, $z(\tau) = 0$ has a solution on the interval $\tau \in (0,1)$ for $\theta \in (0,\theta']$. Call this solution τ^*. We now show that τ^* is the unique solution to $z(\tau) = 0$ on $\tau \in (0,1)$, and that $z(\tau) < 0$ for $\tau \in (\tau^*,1)$. Note $(\partial/\partial\tau)\psi^l = -[(2b-g)\beta(\beta-\gamma) + b(b-g)(2\beta-\gamma)]/[b(b-g)\tau^2\beta] < 0$, which implies $\alpha/\psi^l(\tau)$ strictly increases in τ. Because $c(K_1)$ is weakly convex, it follows that $\theta(\partial/\partial K_1)c(K_1)|_{K_1=\alpha/\psi^l(\tau)}$ weakly increases in τ. Note also that $\alpha(1-\tau)/[\beta(1-\tau)+2b]$ strictly decreases in τ on $\tau \in (0,1)$. It follows that $z(\tau)$ strictly decreases in τ, and therefore $z(\tau)$ crosses 0 at most once on $\tau \in (0,1)$. Therefore, τ^* is the unique solution to $z(\tau) = 0$, and $z(\tau) < 0$ for $\tau \in (\tau^*,1)$. Step Two: For the remainder of the proof, fix $\theta \in (0,\theta']$. Note that τ^* depends implicitly on γ and g; we write $\tau^*(\gamma,g)$ to make this dependence explicit. Because ψ^l is continuous in γ, g and τ, $c(K_1)$ is continuous in K_1, $\tau^*(\gamma,g)$ is continuous in γ and g. Therefore, for any $\bar{\bar{\gamma}} > 0$ and $\bar{\bar{g}} \in (0,\tilde{g})$, $z(\tau) < 0$ for all $\tau > \sup_{\gamma\in[0,\bar{\bar{\gamma}}],\, g\in[0,\bar{\bar{g}}]} \tau^*(\gamma,g)$, $\gamma \in [0,\bar{\bar{\gamma}}]$ and $g \in [0,\bar{\bar{g}}]$. Next, note that for any $\tau \in (0,1)$, $\lim_{\gamma\to 0}\lim_{g\to 0}\lim_{K_1\uparrow\alpha/\psi^l(\tau)}(\partial/\partial K_1)\pi_1(K_1) = \alpha(1-\tau)/[\beta(1-\tau)+b] - \theta(\partial/\partial K_1)c(K_1)|_{K_1=\alpha/\psi^l(\tau)} < z(\tau)$, where the inequality follows by definition of $z(\tau)$. Define $\bar{\tau} = \sup_{\gamma\in[0,\bar{\bar{\gamma}}],g\in[0,\bar{\bar{g}}]} \tau^*(\gamma,g)$. It follows that there exist $\hat{\bar{\gamma}} \in [0,\bar{\bar{\gamma}}]$ and $\hat{\bar{g}} \in [0,\bar{\bar{g}}]$ such that $\lim_{K_1\uparrow\alpha/\psi^l(\tau)}(\partial/\partial K_1)\pi_1(K_1) < z(\tau) < 0$ for $\gamma \in [0,\hat{\bar{\gamma}}]$, $g \in [0,\hat{\bar{g}}]$ and $\tau > \bar{\tau}$. By Lemma 10, $\pi_1(K_1)$ is quasi-concave for $g \in (0,\tilde{g})$, which implies $K_1^* < \alpha/\psi^l(\tau)$ for $\gamma \in [0,\hat{\bar{\gamma}}]$, $g \in [0,\hat{\bar{g}}]$ and $\tau > \bar{\tau}$. Because $K_1^* \le \alpha/\psi^l(\tau)$, it follows from Lemma 10 that $K_1^* = K^l$ for $\gamma \in [0,\hat{\bar{\gamma}}]$, $g \in [0,\hat{\bar{g}}]$ and $\tau > \bar{\tau}$. Therefore, $p_1^{v*} = p_1^{vl}(K^l) = p_1^l(K^l)$ for $v \in \{a,o\}$, $\gamma \in [0,\hat{\bar{\gamma}}]$, $g \in [0,\hat{\bar{g}}]$ and $\tau > \bar{\tau}$, where the first equality follows by Lemma 9i because $K_1^* = K^l < \alpha/\psi^l$, and the second equality holds by Lemma 9ii. The existence of $\bar{\gamma} \in [0,\hat{\bar{\gamma}}]$, $\bar{g} < \hat{\bar{g}}$ and $\underline{g} \in [0,\bar{g})$ such that $p_1^l(K^l)$ strictly decreases in g on $(\underline{g},\bar{g})$ for all $\gamma \in [0,\bar{\gamma}]$ follows by a parallel argument to the proof of Proposition 1.□

Proof of Proposition 2B: (i) Let $\check{\theta} = \theta^l$ and $\hat{\theta} = \theta^0$, and note $\theta^l < \theta^0$ by Lemma 10. Consider $\theta \in [\check{\theta},\hat{\theta}]$. By Lemma 10, $K_1^* = K^l$. Because K_1^* is continuous in g, to establish the result, it is sufficient to show that $(\partial/\partial g)K^l > 0$ for $g \in (0,b)$. We write $\pi_1(K_1,\tau)$, $\psi^l(\tau)$ and $K^l(\tau)$ to explicitly acknowledge dependence on τ; note that $\tau = 1$ corresponds to the model in §3. Following the proof of Proposition 2(i), it suffices to show that $(\partial/\partial K_1\partial g)\pi_1(K_1,\tau)|_{K_1=K^l(\tau)} > 0$. By Lemma 9(iii), $\pi_1(K_1,\tau) = \pi_1(K_1,1)$ for $K_1 \in [0,\alpha/\psi^l(\tau)]$. It follows from the proof of Proposition 2(i) that $(\partial^2/\partial K_1\partial g)\pi_1(K_1,1) > 0$ for $K_1 \in [0,\alpha/\psi^l(1)]$. It remains to show $K^l(\tau) \le \alpha/\psi^l(1)$. It is straightforward to verify that $\psi^l(1) < \psi^l(\tau)$. The result follows because $K^l(\tau) \le \alpha/\psi^l(\tau)$ by Lemma 9. (ii) Let $\bar{g} = \tilde{g}$, $\bar{\theta} = \theta^l$, and $\underline{\theta} = \max\{\theta^e, 0\}$. By Lemma 10, for all $g \in (0,\tilde{g})$, $\theta^e < \theta^l$ and further if $\theta \in [\theta^e,\theta^l]$, then $K_1^* = \alpha/\psi^l$. It is straightforward to verify that $(\partial/\partial g)\psi^l(\tau) > 0$ for all $\tau \in (0,1)$. It remains to show that there exists $\bar{\gamma} > 0$ such that $\theta^l > 0$ for all $\gamma \in [0,\bar{\gamma}]$ and $g > 0$. This follows because θ^l is continuous in γ and $\lim_{\gamma\to 0}\theta^l = \alpha\{(1-\tau)[8b^5+16b^4\beta+2bg^2(g-\beta)\beta+8b^3\beta^2-4b^2g\beta^2+g^3\beta^2]-2(3-2\tau)b^2g^2\beta-4b^3g[g+(1-2\tau)b+(2-3\tau)\beta]\}/\{[2b(b-g)+(2b-g)\beta][4b^2(b^2-g^2)+2(2-\tau)b(2b^2-g^2)\beta+(1-\tau)(4b^2-g^2)\beta^2]\} > 0$, where the inequality can be verified algebraically. □

Proof of Proposition 3B: Note $(d/d\theta)\pi_2(K_1)|_{K_1=K_1^*} = [(d/d\theta)K_1^*][(\partial/\partial K_1)\pi_2(K_1)]$. By Lemma 9(iii), $\pi_i(K_1,\tau) = \pi_i(K_1,1)$ for $\theta \in [\theta^l,\theta^0]$. The remainder of the proof follows by a parallel argument to the proof of Proposition 3. □

Appendix D: Both Platforms Can Acquire Autonomous Vehicles

Let $(\tilde{p}_i^u(K_i), \tilde{w}_i^u(K_i), \tilde{p}_i(K_i))$ be given by $(\tilde{p}_1^u(K_1), \tilde{w}_1^u(K_1), \tilde{p}_1(K_1)$, wherein i replaces 1 and j replaces 2, for $u \in \{e, l, s\}$.

Lemma 11 *For fixed K_i, platform i's best response price and wage to platform j's price and wage (p_j, w_j) is*

$$(\tilde{p}_i(p_j, w_j), \tilde{w}_i(p_j, w_j)) = \begin{cases} (\tilde{p}_i^l(p_j, w_j), \tilde{w}_i^l(p_j, w_j)) & \text{if } K_i < (\alpha + \gamma p_j)/2 - g\beta w_j/2b, \\ (\tilde{p}_i^e(p_j, w_j), \tilde{w}_i^e(p_j, w_j)) & \text{if } K_i \in [(\alpha + \gamma p_j)/2 - g\beta w_j/2b, (\alpha + \gamma p_j)/2], \\ (\tilde{p}_i^s(p_j, w_j), \tilde{w}_i^s(p_j, w_j)) & \text{if } K_i > (\alpha + \gamma p_j)/2. \end{cases}$$

Under platform i AV fleet K_i, platform i's best response price and wage to platform j's price and wage (p_j, w_j) is $(\tilde{p}_i(K_i), \tilde{w}_i(K_i))$ for $i \in \{1, 2\}$ and $j \neq i$. Further, $K_i < D_1(\tilde{p}_i(K_i), p_j)$ if and only if $K_i < (\alpha + \gamma p_j - g\beta w_j/b)/2$; $K_i = D_i(\tilde{p}_i(K_1), p_j)$ if and only if $K_i \in [(\alpha + \gamma p_j - g\beta w_j/b)/2, (\alpha + \gamma p_j)/2]$; and $K_i > D_i(\tilde{p}_i(K_i), p_j)$ if and only if $K_i > (\alpha + \gamma p_j)/2$.

Proof of Lemma 11: Platform i's second-period contribution $v_i(\mathbf{p}, \mathbf{w})$ is unchanged from the base model, with the exception that the restriction that $K_2 \geq 0$ holds with equality is relaxed. Accordingly, for $i = 1$, the result follows immediately from Lemma 3. That the result holds for $i = 2$ holds because the platforms' second-period contribution functions are symmetric.□

Let $(p_1^{uv}(K_1), w_1^{uv}(K_1), p_2^{uv}(K_2), w_2^{uv}(K_2))$ denote the unique solution to $p_1^{uv}(K_1) = \tilde{p}_1^u(K_1)$, $\tilde{w}_1^{uv}(K_1) = \tilde{w}_1^u(K_1)$, $p_2^{uv}(K_2) = \tilde{p}_2^v(K_2)$ and $\tilde{w}_2^{uv}(K_2) = \tilde{w}_2^v(K_2)$, where $\{u, v\} \in \{e, l, s\}^2$. Further, let

$$\psi_L^l = \frac{(2\beta b + \gamma g)(\gamma b - \beta g)}{[\beta(\beta + \gamma)(2b + g) + b(2\beta + \gamma)(b + g)](b - g)}$$

$$\psi_H^l = \frac{(4\beta^2 - \gamma^2)b(b^2 - g^2) + \beta(2\beta^2 - \gamma^2)(2b^2 - g^2) - \beta^2\gamma bg}{\beta[\beta(\beta + \gamma)(2b + g) + b(2\beta + \gamma)(b + g)](b - g)}$$

$$\psi_L^s = \frac{\beta\gamma(2b^2 - g^2)}{\beta(\beta + \gamma)(2b^2 - g^2) + (2\beta + \gamma)b(b^2 - g^2)}$$

$$\psi_H^s = \frac{(4\beta^2 - \gamma^2)b(b^2 - g^2) + \beta(2\beta^2 - \gamma^2)(2b^2 - g^2)}{\beta[\beta(\beta + \gamma)(2b^2 - g^2) + b(2\beta + \gamma)(b^2 - g^2)]}$$

In Lemma 12 we assume, without loss of generality, that $K_1 \leq K_2$. This restriction implies that $(2 - \gamma/\beta)K_1 \leq \psi_L^s K_1 + \psi_H^s K_2 \leq \psi_L^l K_1 + \psi_H^l K_2$, where the inequalities are strict if and only if $K_1 < K_2$.

Lemma 12 *(a) Assume $K_1 \leq K_2$. Under AV fleets $\mathbf{K}$, the equilibrium prices and wages are unique and given by*

$$(\mathbf{p}^*(\mathbf{K}), \mathbf{w}^*(\mathbf{K})) = \begin{cases} (p_1^{ss}(K_1), p_2^{ss}(K_2), w_1^{ss}(K_1), w_2^{ss}(K_2)) & \text{if } \alpha < (2 - \gamma/\beta)K_1, \\ (p_1^{es}(K_1), p_2^{es}(K_2), w_1^{es}(K_1), w_2^{es}(K_2)) & \text{if } \alpha = (2 - \gamma/\beta)K_1 \text{ and } K_1 < K_2, \\ (p_1^{ee}(K_1), p_2^{ee}(K_2), w_1^{ee}(K_1), w_2^{ee}(K_2)) & \text{if } \alpha = (2 - \gamma/\beta)K_1 = (2 - \gamma/\beta)K_2, \\ (p_1^{ls}(K_1), p_2^{ls}(K_2), w_1^{ls}(K_1), w_2^{ls}(K_2)) & \text{if } \alpha \in ((2 - \gamma/\beta)K_1, \psi_L^s K_1 + \psi_H^s K_2), \\ (p_1^{le}(K_1), p_2^{le}(K_2), w_1^{le}(K_1), w_2^{le}(K_2)) & \text{if } \alpha \in [\psi_L^s K_1 + \psi_H^s K_2, \psi_L^l K_1 + \psi_H^l K_2], \\ (p_1^{ll}(K_1), p_2^{ll}(K_2), w_1^{ll}(K_1), w_2^{ll}(K_2)) & \text{if } \alpha > \psi_L^l K_1 + \psi_H^l K_2. \end{cases}$$

(b) If $K_2 < \alpha/(2 - \gamma/\beta)$, then

$$(\mathbf{p}^*(\mathbf{K}), \mathbf{w}^*(\mathbf{K})) = \begin{cases} (p_1^{sl}(K_1), p_2^{sl}(K_2), w_1^{sl}(K_1), w_2^{sl}(K_2)) & \textit{if } K_1 > (\alpha - \psi_L^s K_2)/\psi_H^s \\ (p_1^{el}(K_1), p_2^{el}(K_2), w_1^{el}(K_1), w_2^{el}(K_2)) & \textit{if } (\alpha - \psi_L^l K_2)/\psi_H^l \le K_1 \le (\alpha - \psi_L^s K_2)/\psi_H^s, \\ (p_1^{ll}(K_1), p_2^{ll}(K_2), w_1^{ll}(K_1), w_2^{ll}(K_2)) & \textit{if } \underline{K}_1 < K_1 < (\alpha - \psi_L^l K_2)/\psi_H^l, \\ (p_1^{le}(K_1), p_2^{le}(K_2), w_1^{le}(K_1), w_2^{le}(K_2)) & \textit{if } K_1 \le \underline{K}_1, \end{cases}$$

where $\underline{K}_1 = (\alpha - \psi_H^l K_2)/\psi_L^l$ *if* $g/b > \gamma/\beta$ *and* $\underline{K}_1 = -\infty$ *if* $g/b \le \gamma/\beta$. *If* $K_2 = \alpha/(2 - \gamma/\beta)$, *then*

$$(\mathbf{p}^*(\mathbf{K}), \mathbf{w}^*(\mathbf{K})) = \begin{cases} (p_1^{se}(K_1), p_2^{se}(K_2), w_1^{se}(K_1), w_2^{se}(K_2)) & \textit{if } K_1 > \alpha/(2 - \gamma/\beta), \\ (p_1^{ee}(K_1), p_2^{ee}(K_2), w_1^{ee}(K_1), w_2^{ee}(K_2)) & \textit{if } K_1 = \alpha/(2 - \gamma/\beta), \\ (p_1^{lv}(K_1), p_2^{lv}(K_2), w_1^{lv}(K_1), w_2^{lv}(K_2)) & \textit{if } K_1 < \alpha/(2 - \gamma/\beta), \end{cases}$$

where $v = e$ *if* $g/b \ge \gamma/\beta$ *and* $v = l$ *if* $g/b < \gamma/\beta$.

Proof of Lemma 12: (a) Lemma 11 implies that under AV fleets $\mathbf{K}$: prices and wages $(\mathbf{p}, \mathbf{w}) = (\mathbf{p}^{ss}(\mathbf{K}), \mathbf{w}^{ss}(\mathbf{K}))$ is an equilibrium if and only if $K_i > D_i(\mathbf{p}^{ss}(\mathbf{K}))$ for $i \in \{1, 2\}$; $(\mathbf{p}, \mathbf{w}) = (\mathbf{p}^{es}(\mathbf{K}), \mathbf{w}^{es}(\mathbf{K}))$ is an equilibrium if and only if $K_1 = D_1(\mathbf{p}^{es}(\mathbf{K}))$ and $K_2 > D_2(\mathbf{p}^{es}(\mathbf{K}))$; $(\mathbf{p}, \mathbf{w}) = (\mathbf{p}^{ee}(\mathbf{K}), \mathbf{w}^{ee}(\mathbf{K}))$ is an equilibrium if and only if $K_i = D_i(\mathbf{p}^{ee}(\mathbf{K}))$ for $i \in \{1, 2\}$; $(\mathbf{p}, \mathbf{w}) = (\mathbf{p}^{ls}(\mathbf{K}), \mathbf{w}^{ls}(\mathbf{K}))$ is an equilibrium if and only if $K_1 < D_1(\mathbf{p}^{ls}(\mathbf{K}))$ and $K_2 > D_2(\mathbf{p}^{ls}(\mathbf{K}))$; $(\mathbf{p}, \mathbf{w}) = (\mathbf{p}^{le}(\mathbf{K}), \mathbf{w}^{le}(\mathbf{K}))$ is an equilibrium if and only if $K_1 < D_1(\mathbf{p}^{le}(\mathbf{K}))$ and $K_2 = D_2(\mathbf{p}^{le}(\mathbf{K}))$, and $(\mathbf{p}, \mathbf{w}) = (\mathbf{p}^{ll}(\mathbf{K}), \mathbf{w}^{ll}(\mathbf{K}))$ is an equilibrium if and only if $K_i < D_i(\mathbf{p}^{ll}(\mathbf{K}))$ for $i \in \{1, 2\}$. Further, it is straightforward to verify: $K_i > D_i(\mathbf{p}^{ss}(\mathbf{K}))$, $i \in \{1, 2\}$ if and only if $\alpha < (2 - \gamma/\beta)K_1$; $K_1 = D_1(\mathbf{p}^{es}(\mathbf{K}))$ and $K_2 > D_2(\mathbf{p}^{es}(\mathbf{K}))$ if and only if $\alpha = (2 - \gamma/\beta)K_1$ and $K_1 < K_2$; $K_i = D_i(\mathbf{p}^{ee}(\mathbf{K}))$ for $i \in \{1, 2\}$ if and only if $\alpha = (2 - \gamma/\beta)K_1 = (2 - \gamma/\beta)K_2$; $K_1 < D_1(\mathbf{p}^{ls}(\mathbf{K}))$ and $K_2 > D_2(\mathbf{p}^{ls}(\mathbf{K}))$ if and only if $\alpha \in ((2 - \gamma/\beta)K_1, \psi_L^s K_1 + \psi_H^s K_2)$; $K_1 < D_1(\mathbf{p}^{le}(\mathbf{K}))$ and $K_2 = D_2(\mathbf{p}^{le}(\mathbf{K}))$ if and only if $\alpha \in [\psi_L^s K_1 + \psi_H^s K_2, \psi_L^l K_1 + \psi_H^l K_2]$; and $K_i < D_i(\mathbf{p}^{ll}(\mathbf{K}))$, $i \in \{1, 2\}$ if and only if $\alpha > \psi_L^l K_1 + \psi_H^l K_2$. (b) By interchanging indicies in part (a), it is straightforward to write $(\mathbf{p}^*(\mathbf{K}), \mathbf{w}^*(\mathbf{K}))$ in closed form for the case where $K_1 \ge K_2$; we refer to this, along with part (a), as the *extended part (a)*. If $K_2 = \alpha/(2 - \gamma/\beta)$, then $\alpha = (\psi_L^u + \psi_H^u)K_2$ for $u \in \{l, s\}$; the result follows from the extended part (a). For the remainder of the proof, suppose $K_2 < \alpha/(2 - \gamma/\beta)$. This implies $K_2 < (\alpha - \psi_u^s K_2)/\psi_u^s$ for $u \in \{L, H\}$. If $g/b > \gamma/\beta$, then $\psi_L^l < 0$ and $(\alpha - \psi_H^l K_2)/\psi_L^l < K_2 < (\alpha - \psi_L^l K_2)/\psi_H^l$; the result follows from the extended part (a). If $g/b \le \gamma/\beta$, then $\psi_L^l \ge 0$. Therefore, if $K_1 \le K_2$, then $\psi_L^l K_1 + \psi_H^l K_2 \le (\psi_L^l + \psi_H^l)K_2 < \alpha$, where the last inequality holds because $K_2 < \alpha/(2 - \gamma/\beta)$. The result follows from the extended part (a). □

Lemma 13 *If* $D_i(\mathbf{p}^*(\mathbf{K})) \ge K_i$ *and* $r_i(\mathbf{K})$ *is differentiable in* K_i *at* $\mathbf{K}$, *then* $(\partial^2/\partial K_i^2)r_i(\mathbf{K}) < (\partial^2/\partial K_i \partial K_j)r_i(\mathbf{K}) \le 0$. *If* $D_i(\mathbf{p}^*(\mathbf{K})) < K_i$ *then* $(\partial^2/\partial K_i^2)r_i(\mathbf{K}) = (\partial^2/\partial K_i \partial K_j)r_i(\mathbf{K}) = 0$.

Proof of Lemma 13: We prove the statements in order. First, if $D_i(\mathbf{p}^*(\mathbf{K})) \ge K_i$, then the price-and-wage equilibrium must be one of six types: *es*, *ee*, *el*, *ls*, *le*, or *ll*. Using the expressions for the equilibrium prices and wages $(\mathbf{p}^*(\mathbf{K}), \mathbf{w}^*(\mathbf{K}))$ in Lemma 12, $r_i(\mathbf{K})$ can be written in closed form for each of these equilibrium types. For equilibrium types *es*, *ee*, *el*, *ls*, and *le*, it can be verified algebraically that $(\partial^2/\partial K_i^2)r_i(\mathbf{K}) < (\partial^2/\partial K_i \partial K_j)r_i(\mathbf{K})$ and $(\partial^2/\partial K_i \partial K_j)r_i(\mathbf{K}) \le 0$. For the *ll* type equilibrium, it is straightforward to verify algebraically that $(\partial^2/\partial K_i^2)r_i(\mathbf{K}) < (\partial^2/\partial K_i \partial K_j)r_i(\mathbf{K})$, $\lim_{\gamma \to 0}(\partial^2/\partial K_i \partial K_j)r_i(\mathbf{K}) < 0$ and $(\partial^3/\partial K_i \partial K_j \partial \gamma)r_i(\mathbf{K}) < 0$ for all $\gamma \in [0, \beta)$; the latter two imply that $(\partial^2/\partial K_i \partial K_j)r_i(\mathbf{K}) \le 0$. Second, if $D_i(\mathbf{p}^*(\mathbf{K})) < K_i$, then the price-and-wage equilibrium must be one of three types: *ss*, *se* or *sl*. For each of these equilibrium types,

$(\partial/\partial K_i)r_i(\mathbf{K}) = 0$, which implies $(\partial^2/\partial K_i^2)r_i(\mathbf{K}) = (\partial^2/\partial K_i \partial K_j)r_i(\mathbf{K}) = 0.\square$

Lemma 14 *If $K_1^* = K_2^* = K^*$ is a symmetric equilibrium, then $K^* < \alpha/(2-\gamma/\beta)$ and $(\mathbf{p}^*, \mathbf{w}^*) = (\mathbf{p}^{ll}(K^*, K^*), \mathbf{w}^{ll}(K^*, K^*))$.*

Proof of Lemma 14: Let $\mathring{K}_i(K_j) = \max\{K : \tilde{K}_i(K_j) = K\}$, where $\tilde{K}_i(K_j)$ denotes platform i's best response AV fleet to platform j's AV fleet; in words, $\mathring{K}_i(K_j)$ denotes platform i's largest best response. To establish that a symmetric equilibrium cannot have $K^* \geq \alpha/(2-\gamma/\beta)$, it is sufficient to show that $\mathring{K}_i(K_j) < K_j$ when $K_j \geq \alpha/(2-\gamma/\beta)$. Suppose $K_j \geq \alpha/(2-\gamma/\beta)$. Using the expressions for $(\mathbf{p}^*, \mathbf{w}^*)$ given in Lemma 12a, it is straightforward to verify that $r_i(\mathbf{K})$ is invariant to K_i on $K_i \in [\alpha/(2-\gamma/\beta), \infty)$. Therefore, because $c(K_i)$ is strictly increasing, $\pi_i(\mathbf{K})$ is strictly decreasing in K_i on $K_i \in [\alpha/(2-\gamma/\beta), \infty)$. Therefore, $\mathring{K}_i(K_j) \leq \alpha/(2-\gamma/\beta)$. Hence, if $K_j > \alpha/(2-\gamma/\beta)$, $\mathring{K}_i(K_j) < K_j$. Suppose instead that $K_j = \alpha/(2-\gamma/\beta)$. Using the expressions for $(\mathbf{p}^*, \mathbf{w}^*)$ given in Lemma 12b, it is straightforward to verify that $\lim_{K_i \uparrow \alpha/(2-\gamma/\beta)}(\partial/\partial K_i)r_i(\mathbf{K}) < 0$, which implies $\lim_{K_i \uparrow \alpha/(2-\gamma/\beta)}(\partial/\partial K_i)\pi_i(\mathbf{K}) < 0$. Consequently, it cannot be that $\tilde{K}_i(K_j) = \alpha/(2-\gamma/\beta)$ is a best response for platform i. Hence, $\mathring{K}_i(K_j) < K_j$ when $K_j \geq \alpha/(2-\gamma/\beta)$. Because $\psi_L^l + \psi_H^l = \alpha/(2-\gamma/\beta)$, $K_i^* < \alpha/(2-\gamma/\beta)$ for $i \in \{1, 2\}$ implies $\alpha > \psi_L^l K_1^* + \psi_H^l K_2^*$, which by Lemma 12a implies $(\mathbf{p}^*, \mathbf{w}^*) = (\mathbf{p}^{ll}(K^*, K^*), \mathbf{w}^{ll}(K^*, K^*))$. $\square$

For use in Lemma 15, let $\bar{\bar{K}}_i = (\alpha - \psi_L^s K_j)/\psi_H^s$ if $K_j < \alpha/(2-\gamma/\beta)$ and $\bar{\bar{K}}_i = \alpha/(2-\gamma/\beta)$ if $K_j = \alpha/(2-\gamma/\beta)$. Note $\bar{\bar{K}}_i \in (0, \infty)$.

Lemma 15 *Suppose $K_j \in [0, \alpha/(2-\gamma/\beta)]$. Then there exists $\tilde{g} > 0$ such that the following statements hold for $g \in [0, \tilde{g})$: Platform i's profit $\pi_i(\mathbf{K})$ is continuous and strictly quasi-concave in K_i on $K_i \in [0, \infty)$; platform i's best response AV fleet to platform j's AV fleet, $\tilde{K}_i(K_j)$, is unique; $\tilde{K}_i(K_j) \in [0, \bar{\bar{K}}_i]$; and $\tilde{K}_i(K_j) \neq \underline{K}_i$. Further, if $\gamma = 0$, then $\tilde{g} = b$.*

Proof of Lemma 15: Because $c(K_i)$ is convex and strictly increasing, to establish that $\pi_i(\mathbf{K})$ is continuous and strictly quasi-concave in K_i it is sufficient to show that $r_i(\mathbf{K})$ is strictly quasi-concave in K_i on $K_i \in (0, \bar{\bar{K}}_i)$, invariant to K_i on $K_i \in [\bar{\bar{K}}_i, \infty)$ and continuous in K_i on $K_i \in [0, \infty)$. It is straightforward to verify the latter two properties algebraically using the expressions for $(\mathbf{p}^*, \mathbf{w}^*)$ given in Lemma 12a. It remains to show that there exists $\tilde{g} > 0$ such that $r_i(\mathbf{K})$ is strictly quasi-concave in K_i on $K_i \in [0, \bar{\bar{K}}_i)$. Let $i = 1$ without loss of generality. First, suppose $K_j = \alpha/(2-\gamma/\beta)$. Using the expressions for $(\mathbf{p}^*, \mathbf{w}^*)$ given in Lemma 12b, it is straightforward to show that $r_i(\mathbf{K})$ is differentiable in K_i for $K_i \in [0, \bar{\bar{K}}_i)$. Hence, $(\partial^2/\partial K_i^2)r_i(\mathbf{K}) < 0$ for $K_i \in [0, \bar{\bar{K}}_i)$ (by Lemma 13). Second, suppose $K_j < \alpha/(2-\gamma/\beta)$. By parallel argument, $(\partial^2/\partial K_i^2)r_i(\mathbf{K}) < 0$ for $K_i \in [0, \underline{K}_i) \cup (\underline{K}_i, (\alpha - \psi_L^l K_j)/\psi_H^l) \cup (\alpha - \psi_L^l K_j)/\psi_H^l, \bar{\bar{K}}_i)$. Using the expressions for $(\mathbf{p}^*, \mathbf{w}^*)$ given in Lemma 12b, it is straightforward to show the following: $\lim_{g \to 0} \lim_{K_i \uparrow (\alpha - \psi_L^l K_j)/\psi_H^l}(\partial/\partial K_i)r_i(\mathbf{K}) > \lim_{g \to 0} \lim_{K_i \downarrow (\alpha - \psi_L^l K_j)/\psi_H^l}(\partial/\partial K_i)r_i(\mathbf{K})$; and if $g/b > \gamma/\beta$, then $\lim_{g \to 0} \lim_{K_i \uparrow \underline{K}_i}(\partial/\partial K_i)r_i(\mathbf{K}) < 0$ and $\lim_{g \to 0} \lim_{K_i \downarrow \underline{K}_i}(\partial/\partial K_i)r_i(\mathbf{K}) < 0$. The former implies that there exists $\tilde{g}^x > 0$ such that $r_i(\mathbf{K})$ is strictly quasi-concave in K_i for $K_i \in (\max(0, \underline{K}_i), \bar{\bar{K}}_i)$ for $g \in [0, \tilde{g}^x)$. The latter implies if $g/b > \gamma/\beta$, then there exists $\tilde{g}^y > 0$ such that $\lim_{K_i \uparrow \underline{K}_i}(\partial/\partial K_i)r_i(\mathbf{K}) < 0$ and $\lim_{K_i \downarrow \underline{K}_i}(\partial/\partial K_i)r_i(\mathbf{K}) < 0$ for for $g \in [0, \tilde{g}^y)$. This implies that $r_i(\mathbf{K})$ is strictly quasi-concave in K_i for $K_i \in (0, \alpha - \psi_L^l K_2)/\psi_H^l)$ and $\tilde{K}_i(K_j) \neq \underline{K}_i$ for $g \in [0, \tilde{g}^y)$. Thus, $r_i(\mathbf{K})$ is strictly quasi-concave in K_i on $K_i \in (0, \bar{\bar{K}}_i)$ for $g \in [0, \tilde{g})$, where $\tilde{g} = \min\{\tilde{g}^x, \tilde{g}^y\}$. In the special case where $\gamma = 0$, it is straightforward to show by parallel argument to the above that $\tilde{g}^x = \tilde{g}^y = \tilde{g} = b$. Uniqueness of the best response $\tilde{K}_i(K_j)$

follows from strict quasi-concavity of $\pi_i(\mathbf{K})$ in K_i on $K_i \in [0, \infty)$. Because is $r_i(\mathbf{K})$ is invariant to K_i on $K_i \in [\bar{\bar{K}}_i, \infty)$, $\pi_i(\mathbf{K})$ is strictly decreasing in K_i on $K_i \in [\bar{\bar{K}}_i, \infty)$. This implies $\tilde{K}_i(K_j) \in [0, \bar{\bar{K}}_i]$. $\square$

Lemma 16 *If $\theta_1 = \theta_2$ and $g \in [0, \tilde{g})$, then only one symmetric equilibrium, $K_1^* = K_2^* = K^*$, exists.*
Proof of Lemma 16: We refer to $K_i \in [0, \alpha/(2-\gamma/\beta)]$ for $i \in \{1, 2\}$ as the *truncated strategy space* and $K_i \in [0, \infty)$ for $i \in \{1, 2\}$ as the *full strategy space.* The proof proceeds in three steps. First, we show that there exists only one equilibrium on the truncated strategy space, and that it is symmetric. We denote this equilibrium by $K_1^t = K_2^t = K^t$. Second, we show that K^t is also an equilibrium on the full strategy space. Third, we show that K^t is the only symmetric equilibrium on the full strategy space. Step One: Because the game is symmetric, the truncated strategy space $K_i \in [0, \alpha/(2-\gamma/\beta)]$ is compact and convex for $i \in \{1, 2\}$, and the profit functions $\pi_i(\mathbf{K})$ are continuous and quasi-concave in K_i on $K_i \in [0, \alpha/(2-\gamma/\beta)]$ for $i \in \{1, 2\}$ (from Lemma 15), there exists at least one symmetric equilibrium, K^t, on the truncated strategy space (Cachon and Netessine 2004). Next, we show that K^t is the only equilibrium on the truncated strategy space. By Lemma 15, $\tilde{K}_i(K_j)$ is unique for $K_j \in [0, \alpha/(2-\gamma/\beta)]$. It follows from Berge's maximum theorem that the best response $\tilde{K}_i(K_j)$ is continuous in K_j on $K_j \in [0, \alpha/(2-\gamma/\beta)]$ for $i \in \{1, 2\}$. Because an equilibrium exists on the truncated strategy space, to prove uniqueness, it suffices to show that the magnitude of the slopes of the best response functions are strictly less than one everywhere on the truncated strategy space (Cachon and Netessine 2004). Because the platforms are symmetric, and because $\tilde{K}_i(K_j)$ is continuous in K_j on $K_j \in [0, \alpha/(2-\gamma/\beta)]$, it is sufficient to show that $|(d/dK_j)\tilde{K}_i(K_j)| < 1$ for $K_j \in (0, \alpha/(2-\gamma/\beta))$. First, consider the case where $(\partial^2/\partial K_i \partial K_j)\pi_i(\mathbf{K})$ exists at $\mathbf{K} = (\tilde{K}_i(K_j), K_j)$. Because $\tilde{K}_i(K_j)$ is continuous and $\tilde{K}_i(K_j) \in (0, \alpha/(2-\gamma/\beta))$, by the implicit function theorem the slope of $\tilde{K}_i(K_j)$ is given by $|(d/dK_j)\tilde{K}_i(K_j)| = |[(\partial^2/\partial K_i \partial K_j) r_i(\mathbf{K})]/[(\partial^2/\partial K_i^2) r_i(\mathbf{K}) - \theta(\partial^2/\partial K_i^2)c(K_i)]_{K_i = \tilde{K}_i(K_j)}|$. We say that under AV fleets $\mathbf{K}$, a price-and-wage equilibrium is of type uv if $(\mathbf{p}^*(\mathbf{K}), \mathbf{w}^*(\mathbf{K})) = (p_1^{uv}(K_1), w_1^{uv}(K_1), p_2^{uv}(K_2), w_2^{uv}(K_2))$, where $\{u, v\} \in \{e, l, s\}^2$. Let $i = 1$ without loss of generality. Because $\tilde{K}_i(K_j) \in [0, \bar{\bar{K}}_i]$ (by Lemma 15), $\tilde{K}_i(K_j) \in (0, \alpha/(2-\gamma/\beta))$ and $K_j \in [0, \alpha/(2-\gamma/\beta)]$, under $\mathbf{K} = (\tilde{K}_i(K_j), K_j)$, the price-and-wage equilibrium must be of one of the following types el, ll or le (by Lemma 12b). Under these equilibrium types, $D_i(\mathbf{p}^*(\tilde{K}_i(K_j), K_j)) \geq \tilde{K}_i(K_j)$. Therefore, $|(d/dK_j)\tilde{K}_i(K_j)| \leq |[(\partial^2/\partial K_i \partial K_j) r_i(\mathbf{K})]/[(\partial^2/\partial K_i^2) r_i(\mathbf{K})]_{K_i = \tilde{K}_i(K_j)}| < 1$, where the first inequality follows because $c(\cdot)$ is weakly convex, and the second inequality follows from Lemma 13 because the existence of $(\partial^2/\partial K_i \partial K_j)\pi_i(\mathbf{K})$ at $\mathbf{K} = (\tilde{K}_i(K_j), K_j)$ implies $r_i(\mathbf{K})$ is differentiable in K_i at $\mathbf{K} = (\tilde{K}_i(K_j), K_j)$. Therefore, if $(\partial^2/\partial K_i \partial K_j)\pi_i(\mathbf{K})$ exists at $\mathbf{K} = (\tilde{K}_i(K_j), K_j)$, $|(d/dK_j)\tilde{K}_i(K_j)| < 1$. Second, consider the case where $(\partial^2/\partial K_i \partial K_j)\pi_i(\mathbf{K})$ does not exist at $\mathbf{K} = (\tilde{K}_i(K_j), K_j)$. By Lemma 12b, this can only occur if $\tilde{K}_i(K_j) = (\alpha - \psi_L^u K_j)/\psi_H^u$ for $u \in \{s, l\}$ or if $\tilde{K}_i(K_j) = \underline{K}_i$. By Lemma 15, $\tilde{K}_i(K_j) \neq \underline{K}_i$. If $\tilde{K}_i(K_j) = (\alpha - \psi_L^u K_j)/\psi_H^u$, then, $|(d/dK_j)\tilde{K}_i(K_j)| = \psi_L^u/\psi_H^u < 1$ for $u \in \{s, l\}$, where the inequality follows by straightforward algebra. It follows that $K_1^t = K_2^t = K^t$ is the unique equilibrium on the truncated strategy space. Step Two: By definition, $K^t = \arg\max_{K_i \in [0, \alpha/(2-\gamma/\beta))} \pi_i(K_i, K^t)$ for $i \in \{1, 2\}$. By Lemma 15, $\pi_i(K_i, K_j)$ is quasi-concave in K_i on $K_i \in [0, \infty)$ for $K_j \in [0, \alpha/(2-\gamma/\beta)]$ and $i \in \{1, 2\}$. Because $K^t \in [0, \alpha/(2-\gamma/\beta)]$, it follows that $K^t = \arg\max_{K_i \in [0, \infty)} \pi_i(K_i, K^t)$ for $i \in \{1, 2\}$, which implies

$\tilde{K}_i(K^t) = K^t$ for $i \in \{1,2\}$. Therefore, $K_1^t = K_2^t = K^t$ is also an equilibrium on the full strategy space. Step Three: Suppose that in addition to K^t, there exists a second symmetric equilibrium on the full strategy space, $K_1^a = K_2^a = K^a$. By Lemma 14, it must be that $K^a \in [0, \alpha/(2-\gamma/\beta))$. However, this contradicts the result in the first step of this proof that K^t is the unique equilibrium on the truncated strategy space $K_i \in [0, \alpha/(2-\gamma/\beta)]$ for $i \in \{1,2\}$. We conclude that $K^* = K^t$ is the only symmetric equilibrium on the full strategy space. □

Lemma 17 *If $K_i^* > 0$, then $(d/d\theta_i)K_i^* < 0$, for $i \in \{1,2\}$.*

Proof of Lemma 17: If $K_i^* > 0$, then $(d/d\theta_i)K_i^* < 0$, for $i \in \{1,2\}$. For convenience, define $U(K_i, K_j) = (\partial^2/\partial K_j \partial\theta_i)\pi_j(\partial^2/\partial K_i \partial K_j)\pi_i - (\partial^2/\partial K_i \partial\theta_i)\pi_i(\partial^2/\partial K_j^2)\pi_j$ and $V(K_i, K_j) = (\partial^2/\partial K_i^2)\pi_i(\partial^2/\partial K_j^2)\pi_j - (\partial^2/\partial K_i \partial K_j)\pi_j(\partial^2/\partial K_j \partial K_i)\pi_i$. It follows immediately from the analysis in Dixit (1986) that $(d/d\theta_i)K_i^* = U(K_i^*, K_j^*)/V(K_i^*, K_j^*)$. It suffices to show that $V(K_i^*, K_j^*) > 0$ and $U(K_i^*, K_j^*) < 0$. It follows from Lemma 13 that $V(K_i^*, K_j^*) > 0$. To see that $U(K_i^*, K_j^*) < 0$, note that $(\partial^2/\partial K_j \partial\theta_i)\pi_j = 0$, because θ_i does not appear in π_j, $(\partial^2/\partial K_i \partial\theta_i)\pi_i = -(\partial/\partial K_i)c(K_i) < 0$ because $c(K_i)$ is increasing, and $(\partial^2/\partial^2 K_j)\pi_j|_{(K_i,K_j)=(K_i^*,K_j^*)} < 0$ by Lemma 13. □

Proof of Proposition 1C: Because $p_1^*(K^*, K^*) = p_2^*(K^*, K^*)$ and $p_1^{ll}(K^*, K^*) = p_2^{ll}(K^*, K^*)$, we omit the subscript for conciseness. Note that if $\theta \geq \hat{\theta}$, then $K^* = 0$ for all $g \in [0, b)$, which implies $(d/dg)K^* = 0$ for all $g \in [0, b)$. Because $K^* = 0$, $p^*(K^*, K^*) = p^{ll}(K^*, K^*) = p^{ll}(0, 0)$, where the first equality follows from Lemma 12. It is straightforward to verify that $(\partial/\partial g)p^{ll}(0,0) = (\alpha\beta^2 b^2)/[\beta(\beta-\gamma)(2b-g) + (2\beta-\gamma)b(b-g)]^2 > 0$. (ii) By Lemma 14, $p^* = p^{ll}(K^*, K^*)$. The remainder of the proof follows by parallel argument to the second and third steps of the proof of Proposition 1(ii), with K^* in place of K_1^*, $p^{ll}(\mathbf{K})$ in place of $p_1^l(K_1)$, and where $g^l(\theta) = b$ and $\gamma(\theta) = \beta$. □

Proof of Proposition 2C: (i) Because $\gamma = 0$, $\tilde{g} = b$ by Lemma 15. Then for $g \in (0, b)$, by Lemma 16, only one symmetric equilibrium, K^*, exists. Note that (K^*, K^*) is the solution to $F_1(K^*, K^*) = 0$ and $F_2(K^*, K^*) = 0$, where $F_i(\mathbf{K}) = (\partial/\partial K_i)r_i(\mathbf{K}) - \theta(\partial/\partial K_i)c(K_i) = 0$ for $i \in \{1,2\}$. Taking the total derivative of these two equalities with respect to g yields $[(\partial/\partial K_1)F_i(d/dg)K^* + (\partial/\partial K_2)F_i(d/dg)K^* + (\partial/\partial g)F_i]_{K_1=K_2=K^*} = 0$, for $i \in \{1,2\}$. By symmetry, $(\partial/\partial K_1)F_1 = (\partial/\partial K_2)F_2$, $(\partial/\partial K_2)F_1 = (\partial/\partial K_1)F_2$, and $(\partial/\partial g)F_1 = (\partial/\partial g)F_2$ when evaluated at $K_1 = K_2 = K^*$. We can therefore rewrite both equations in terms of F_1. Rearranging for $(d/dg)K^*$ yields $(d/dg)K^* = -(\partial/\partial g)F_1/[(\partial/\partial K_1)F_1 + (\partial/\partial K_2)F_1]|_{K_1=K_2=K^*}$. Note that $(\partial/\partial K_1)F_1 + (\partial/\partial K_2)F_1 = [(\partial^2/\partial K_1^2)r_1 - \theta(\partial^2/\partial K_1^2)c(K_1) + (\partial^2/\partial K_1 \partial K_2)r_1]|_{K_1=K_2=K^*}$. Therefore, $(d/dg)K^* = -[(\partial/\partial K_1 \partial g)r_1]/[(\partial^2/\partial K_1^2)r_1 - \theta(\partial^2/\partial K_1^2)c(K_1) + (\partial^2/\partial K_1 \partial K_2)r_1]|_{K_1=K_2=K^*}$. Note that $D^*(\mathbf{p}^*) > K^*$ by Lemma 14. Because $(\partial^2/\partial K_1^2)c(K_1) \geq 0$ by weak convexity of $c(K_1)$, $(\partial^2/\partial K_1^2)r_1 - \theta(\partial^2/\partial K_1^2)c(K_1) + (\partial^2/\partial K_1 \partial K_2)r_1 < 0$ by Lemma 13. It follows that $(d/dg)K^*$ has the same sign as $(\partial/\partial K_1 \partial g)r_1|_{K_1=K_2=K^*}$. Let $\gamma = 0$. It can be shown that $(\partial/\partial K_1 \partial g)r_1|_{K_1=K_2=K^*} = (\alpha - 2K^*)\phi(\beta, b, g)/\zeta(\beta, b, g)^2$ for some functions ϕ and ζ, where $\phi(\beta, b, g) = 4b^2(b+\beta)[b^3\beta(4b + 3\beta) + (b^3 - g^3)(2b+\beta)^2 + 6bg(b-g)(b+\beta)(2b+\beta)]/[2b(b-g) + (2b-g)\beta]^3 > 0$ and the inequality follows because $g < b$. Because $\alpha - 2K^* > 0$ (by Lemma 14), $(\partial/\partial K_1 \partial g)r_1|_{K_1=K_2=K^*} > 0$. It follows that if $\theta > 0$, then $K^* < \alpha/2$. (ii) Set $\bar{g} = \tilde{g}$, where $\tilde{g}$ is defined in Lemma 15. Because $\pi_1(\mathbf{K})$ is strictly quasi-concave in K_1 for $g \in [0, \bar{g})$ (by Lemma 15), the expression for $(d/dg)K^*$ given in part (i) remains valid here. It can be shown $\lim_{\theta\to 0}(d/dg)K^* = -[(\partial/\partial K_1 \partial g)r_1]/[(\partial^2/\partial K_1^2)r_1 +$

$(\partial^2/\partial K_1\partial K_2)r_1]|_{K_1=K_2=K^*} = b^2\alpha\beta\gamma\phi(\beta,\gamma,b,g)/\zeta(\beta,\gamma,b,g)^2$ for some functions ϕ and ζ. Therefore, $\lim_{\theta\to 0}(d/dg)K^*$ has the same sign as ϕ. Next, $\lim_{\gamma\to 0}\phi(\beta,\gamma,b,g) = -2\beta^4\{16b^5(b-g)^2 + 8b^2(b-g)\beta[b(6b^2-4bg+g^2)+2\beta(3b^2-bg+g^2)]+\beta^3(2b-g)^2(4b^2+g^2)+4bg^3\beta(bg+b\beta+g\beta)\} < 0$, where the inequality can be verified algebraically using the fact that $g < b$. It follows that there exists $\bar{\gamma} > 0$ such that if $\gamma \in [0,\bar{\gamma}]$, then $\lim_{\theta\to 0}(d/dg)K^* < 0$. The result follows from continuity of K^* and $(d/dg)K^*$ in θ. □

Proof of Proposition 3C: The proof proceeds in two steps. First, we show that if $K_1^* > 0$, then $(d/d\theta_1)K_1^* < 0$, for $i \in \{1,2\}$. Second, we show the main result. Step One: For convenience, define $U(K_1,K_2) = (\partial^2/\partial K_2\partial\theta_1)\pi_2(\partial^2/\partial K_1\partial K_2)\pi_1 - (\partial^2/\partial K_1\partial\theta_1)\pi_1(\partial^2/\partial K_2^2)\pi_2$ and $V(K_1,K_2) = (\partial^2/\partial K_1^2)\pi_1(\partial^2/\partial K_2^2)\pi_2 - (\partial^2/\partial K_1\partial K_2)\pi_2(\partial^2/\partial K_2\partial K_1)\pi_1$. It follows immediately from the analysis in Dixit (1986) that $(d/d\theta_1)K_1^* = U(K_1^*,K_2^*)/V(K_1^*,K_2^*)$. It suffices to show that $V(K_1^*,K_2^*) > 0$ and $U(K_1^*,K_2^*) < 0$. It follows from Lemma 13 that $V(K_1^*,K_2^*) > 0$. To see that $U(K_1^*,K_2^*) < 0$, note that $(\partial^2/\partial K_2\partial\theta_1)\pi_2 = 0$, because θ_1 does not appear in π_2, $(\partial^2/\partial K_1\partial\theta_1)\pi_1 = -(\partial/\partial K_1)c(K_1) < 0$ because $c(K_1)$ is increasing, and $(\partial^2/\partial^2 K_2)\pi_2|_{(K_1,K_2)=(K_1^*,K_2^*)} < 0$ by Lemma 13. Step Two: It is sufficient to show that $g/b \le \gamma/\beta$ implies $(d/d\theta_1)\pi_2|_{(K_1,K_2)=(K_1^*,K_2^*)} \ge 0$. The total derivative of π_2 with respect to θ_1 is $(d/d\theta_1)\pi_2|_{(K_1,K_2)=(K_1^*,K_2^*)} = [(\partial/\partial K_1)\pi_2(d/d\theta_1)K_1^* + (\partial/\partial K_2)\pi_2(d/d\theta_1)K_2^* + (\partial/\partial\theta_1)\pi_2]|_{(K_1,K_2)=(K_1^*,K_2^*)}$. Note $(\partial/\partial\theta_1)\pi_2 = 0$ because θ_1 does not appear in π_2. Also, $(\partial/\partial K_2)\pi_2|_{K_2=K_2^*} = 0$ because K_2^* is a maximizer of π_2. It follows that $(d/d\theta_1)\pi_2|_{(K_1,K_2)=(K_1^*,K_2^*)} = [(\partial/\partial K_1)\pi_2(d/d\theta_1)K_1^*]|_{(K_1,K_2)=(K_1^*,K_2^*)}$. Because $(d/d\theta_1)K_1^* < 0$ by Step One, it follows that $(d/d\theta_1)\pi_2|_{(K_1,K_2)=(K_1^*,K_2^*)}$ and $(\partial/\partial K_1)\pi_2|_{(K_1,K_2)=(K_1^*,K_2^*)}$ have opposite signs. Thus, it is sufficient to show that $g/b \le \gamma/\beta$ implies $(\partial/\partial K_1)\pi_2|_{(K_1,K_2)=(K_1^*,K_2^*)} \le 0$. By Lemma 12, there are nine price-and-wage equilibria types to consider: *ee*, *es*, *se*, *ss*, *sl*, *le*, *ls*, *el* and *ll*. For the first six types, it is straightforward to verify algebraically that $(\partial/\partial K_1)\pi_2 \le 0$, where the inequality holds with equality for the first five types. For the *ls* equilibrium, is straightforward to verify algebraically that $(\partial^2/\partial K_1^2)\pi_2 > 0$ and $(\partial/\partial K_1)\pi_2|_{K_1=\alpha/(2-\gamma/\beta)} < 0$. Because for the *ls* equilibrium, $K_1^* < \alpha/(2-\gamma/\beta)$ (by Lemma 12), this implies $(\partial/\partial K_1)\pi_2|_{K_1=K_1^*} \le (\partial/\partial K_1)\pi_2|_{K_1=\alpha/(2-\gamma/\beta)} < 0$. For the *el* equilibrium, it is straightforward to verify algebraically that $(\partial^2/\partial K_1^2)\pi_2 > 0$. Because for the *el* equilibrium, $K_1^* \le (\alpha - \psi_L^s K_2)/\psi_H^s$ (by Lemma 12), this implies $(\partial/\partial K_1)\pi_2|_{K_1=K_1^*} \le (\partial/\partial K_1)\pi_2|_{K_1=(\alpha-\psi_L^s K_2)/\psi_H^s}$. Further, it is straightforward to verify algebraically that $(\partial/\partial K_1)\pi_2|_{K_1=(\alpha-\psi_L^s K_2)/\psi_H^s}$ is linear in K_2, $(\partial/\partial K_1)\pi_2|_{(K_1,K_2)=(\alpha/\psi_H^s,0)} < 0$ and $(\partial/\partial K_1)\pi_2|_{(K_1,K_2)=([\alpha-\psi_L^s\alpha/(2-\gamma/\beta)]/\psi_H^s,\alpha/(2-\gamma/\beta))} < 0$. Because for the *el* equilibrium, $K_2^* \in [0,\alpha/(2-\gamma/\beta)))$ (by Lemma 12), this implies $(\partial/\partial K_1)\pi_2|_{(K_1,K_2)=(K_1^*,K_2^*)} < 0$. For the *ll* equilibrium, we will show that $g/b \le \gamma/\beta$ implies $(\partial/\partial K_1)\pi_2|_{(K_1,K_2)=(K_1^*,K_2^*)} \le 0$. Suppose $g/b \le \gamma/\beta$. This implies $\psi_L^l \ge 0$. Note $\alpha > \psi_L^l \min(K_1^*,K_2^*) + \psi_H^l \max(K_1^*,K_2^*) \ge \psi_L^l(K_1^* + K_2^*) \ge \psi_L^l K_1^*$, where the first inequality holds (by Lemma 12) because the equilibrium is of the *ll* type, the second inequality holds because $\psi_H^l > \psi_L^l$, and the third inequality holds because $K_2^* \ge 0$ and $\psi_L^l \ge 0$. For the *ll* equilibrium, it is straightforward to verify algebraically that $(\partial/\partial K_1)\pi_2|_{(K_1,K_2)} = \alpha\eta_0(\beta,\gamma,b,g)+\eta_1(\beta,\gamma,b,g)K_1+\eta_2(\beta,\gamma,b,g)K_2$, where $\eta_2(\beta,\gamma,b,g) = (\partial^2/\partial K_1\partial K_2)\pi_2 \le 0$ (where the inequality holds by Lemma 13), $\eta_0(\beta,\gamma,b,g) = 2(g\beta-b\gamma)b\beta(b-g)(b+\beta)(2b\beta+g\gamma)[(2b+g)\beta(\beta+\gamma)+b(b+g)(2\beta+\gamma)]/\zeta(\beta,\gamma,b,g)^2$ and $\eta_1(\beta,\gamma,b,g) = 2(g\beta-b\gamma)^2b\beta(b+\beta)(2b\beta+g\gamma)^2/\zeta(\beta,\gamma,b,g)^2$ for some function ζ. If $g/b = \gamma/\beta$, then $\eta_0(\beta,\gamma,b,g) = \eta_1(\beta,\gamma,b,g) = 0$, which implies $(\partial/\partial K_1)\pi_2 =$

$\eta_2(\beta,\gamma,b,g)K_2 \leq 0$. For the remainder, suppose $g/b < \gamma/\beta$. Note $(\partial/\partial K_1)\pi_2|_{(K_1,K_2)=(K_1^*,K_2^*)} \leq \alpha\eta_0(\beta,\gamma,b,g) + \eta_1(\beta,\gamma,b,g)K_1^* < \alpha[\eta_0(\beta,\gamma,b,g) + \eta_1(\beta,\gamma,b,g)/\psi_L^l] = 0$, where the first inequality holds because $\eta_2(\beta,\gamma,b,g) \leq 0$, the second inequality holds because $K_1^* < \alpha/\psi_L^l$, and the equality holds by straightforward algebra. □

Appendix E: Numerical Study

We have restricted attention to $g < \tilde{g}$. (Note that for §3 and §4, $\tilde{g}$ is defined prior to Lemma 2; for §4.2, $\tilde{g}$ is defined in Lemma 10; and for §4.3, $\tilde{g}$ is defined in Lemma 15.) Here we provide evidence that when this restriction is relaxed, the results are consistent with our Propositions. The relevant results are Propositions 3, 3A, 3B and 3C. The remaining Propositions either do not make use of this restriction (Proposition 1(i), 1A(i), 1B(i), 1C(i), 2(i), 2A(i), 2B(i), 2C(i)) or provide sufficient conditions in the form of additional restrictions on the parameter g, which makes $g < \tilde{g}$ non-restrictive (Propositions 1(ii), 1A(ii), 1B(ii), 1C(ii), 2(ii), 2A(ii), 2B(ii), 2C(ii)).

Let Set I denote the 80,000 combinations of parameters $\beta \in \{0.1, 0.2, \ldots, 1\}$, $\gamma = \vartheta\beta$, where $\vartheta \in \{0, 0.1, \ldots, 0.9, 0.99\}$, $b \in \{0.1, 0.2, \ldots, 1\}$, $\theta \in \{0.05, 0.10, \ldots, 2\}$, and $z \in \{1, 2\}$, where $c(K_1) = K_1^z$. For each result, we considered $g = \varsigma b$, where $\varsigma \in \{0, 0.01, \ldots, 0.98, 0.99\}$. For Propositions 3, 3B and 3C, we considered Set I supplemented with $\alpha = 1$. For Proposition 3A, we considered Set I supplemented with $\alpha = 0.1 + \xi$, where $\xi \sim Beta(\sigma_1, \sigma_2)$, for the 16 parameter combinations of parameters $\sigma_1 \in \{2, 3, 4, 5\}$ and $\sigma_2 \in \{2, 3, 4, 5\}$. For each combination of parameters, we observed numerically that the equilibrium AV fleet K_1^* (or symmetric equilibrium AV fleet K^*) is unique. The numerical results are consistent with the analytical results in that no parameter combination contradicts either the sufficient condition in Proposition 3, 3A or 3B, or the necessary condition in Proposition 3C.

Propositions 1Aii and 2A require that $\underline{\alpha}/\bar{\alpha} > \lambda$ for some $\lambda < 1$. Here we provide evidence that this requirement is not particularly restrictive in that these results hold in examples where $\underline{\alpha}/\bar{\alpha}$ is relatively small. Suppose $\beta = b = 1$, $\theta = 0.1$ and $\alpha = 0.1 + \xi$ where $\xi \sim Beta(2, 2)$, so that $\underline{\alpha} = 0.1$ and $\bar{\alpha} = 1.1$. We observed numerically that Proposition 1Aii holds with $\gamma = 0.01$, Proposition 2Ai holds with $\gamma = 0$, and Proposition 2Aii holds with $\gamma = 0.8$. In these examples, $\underline{\alpha}/\bar{\alpha} = 0.091$.

Proposition 1Bii requires that $\tau > \bar{\tau}$ for some $\bar{\tau} \in (0, 1)$. Here we provide evidence that that $\bar{\tau}$ need not be particularly large for the result to hold. In particular, under parameters $\alpha = \beta = b = 1$, $\theta = 0.5$, $\gamma = 0.01$ and $\tau \in \{0.01, 0.02, \ldots, 0.99\}$, we observed numerically that the result holds for $\bar{\tau} = 0.1$.