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Behavior-Based Discrimination: Is it a winning play and if so when?

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Abstract

With advances in technology, the collection of information from consumers at the time of purchase is common in many categories. This information allows a firm to straightforwardly classify consumers as either "new" or "past" consumers. This opens the door for firms to implement marketing that a) discriminates between new and past consumers and b) entails making offers to them that are significantly different. Our objective is to examine the competitive effects of marketing that tailors offers to consumers based on their past buying behavior. In a two period model with two competing firms, we assume that each firm is able to commit about whether or not to implement behavior-based discrimination, i.e. to add benefits to its offer for past consumers in the second period. When the firms are identical in their ability to add value to the second period offer, behavior-based discrimination (BBD) generally leads to lower profits for both firms. Past customers are so valuable in the second period that BBD leads to cutthroat competition in the first period. As a result, the payoffs associated with the implementation of BBD form a Prisoners' Dilemma. Interestingly, when a firm has a significant advantage over its competitor (one firm has the capability to add more benefits for its past customers than the other), it can increase its profit versus the base case even when there is significant competition in the second period. Moreover, the firm at a disadvantage sometimes finds that the best response to BBD by a strong competitor is to respond with a uniform price and avoid the practice completely.

Keywords: dynamic games, price discrimination, customer data, product design.

1 Introduction

1.1 Background

Consider the following situations:

1. Air Canada announces at the end of February that any Aeroplan member who joined in February will earn double frequent flyer miles in March.

2. In the last week of December 2004, the SNVB Bank offers the Gold Visa card at 50% of the yearly fee to all clients who opened Codevi Savings accounts in the last quarter of 2004.[1]

3. All SAS travellers who sign up for Wireless Access at the Copenhagen airport find that they are offered the same access plus automatic flight information, the second time they login to purchase the same service.

All three situations entail firms taking actions that are directly related to a consumer's past behavior. The firms collect information at the time of the first purchase and know when interacting with the consumer a second time that the consumer is a "past consumer".[2] This information allows the firms to implement marketing that a) discriminates between new and past consumers and b) consists of offers to each segment that are significantly different.

The easiest action to discriminate between new and past customers is to offer them different prices (through couponing, introductory offers or a repeat-buyer discounts). Accordingly, there are a number of studies that analyze the impact of prices that depend on the past behavior of customers (Fudenberg and Tirole 2000 and Villas-Boas 1999). However, as the examples above demonstrate, a firm can go further than offering "tailored prices" when it bases marketing activity on past buying behavior. It can offer special services, options or accessories that are of added value to the customer.

Our objective is to examine the interaction between two competitors who decide whether to implement behavior-based discrimination in a two period model. The model focuses on markets where firms are able to commit not to implement behavior-based discrimination. In many markets, this may not be possible. In order for commitment to be possible, the implementation of behavior-based discrimination must be observable. For example, firms need to collect information and the

[1] SNVB is the acronym for the Sociètè Nancéienne Varin-Bernier, a major retail bank in France.

[2] Many firms collect information from consumers with whom they interact. Not only do consumers of durable goods complete warranty cards but many frequently purchased items are sold over internet sites or by retailers who have loyalty programmes.

identities of consumers in the first period (this information is used by firms to design benefits and to identify past consumers at the time of the second purchase). When the collection of identities and information is observable, firms can commit to not implementing behavior-based discrimination. We now review the key literature related to this topic.

1.2 Literature Review

In the last decade, a wide array of firms have initiated the use of customer-level information to guide their marketing. For example, firms now offer customer-specific prices through a combination of special offers, coupons, and bundling. In fact, there is an important body of research that examines the impact of prices that are tailored to consumers based on their characteristics (Thisse and Vives 1988 and Chen and Iyer 2002 and Chen, Iyer and Padmanabhan 2002).

Our interest however, is research that examines the impact of offers (modified products and corresponding prices) that are tailored to consumers based on past buying behavior.[3] In contrast to models where loyalty is exogeous or a function of network effects (Chen and Xie 2007), the incentive of past consumer to buy again from the same firm is driven by benefits that are offered to past consumers in the second period. The information to design these benefits is collected at the time of the first puchase through devices such as warranty cards and it allows a firm to make offers to consumers based on their past buying behavior.

The most important stream of research related to this topic examines the impact of offering tailored prices to consumers based on their past buying behavior (Fudenberg and Tirole 2000 and Villas-Boas 1999, 2004). As noted in Fudenberg and Tirole (2000) note that research on behavior-based price discrimination is recent because it does not correspond to the traditional categories of price discrimination such as first, second and third degree.

The models of Villas-Boas (1999 and 2004) consider markets with infinitely-lived firms and overlapping generations of consumers.[4] The Villas-Boas models are important because they identify the key forces that affect the prices and behavior of firms when firms charge a different price to the consumers whom they have served previously. However, these studies do not address situations where past consumers are offered an additional benefit along with a unique price.

Fudenberg and Tirole (2000) examine behavior-based price discrimination when there is not significant turnover. They show that when consumer preferences are fixed over time and the

[3] Syam, Ruan and Hess (2005) look at product customization in a context where firms position their offer along two dimensions. This is related to our work but does not address the impact of BBD due to its static nature.

[4] Villas-Boas (2004) considers the case of an infinitely-lived monopolist. Our study is focused on price discrinimation under competitive conditions so it relates primarily to Villas-Boas (1999).

market is characterized by short-term contracts, firms poach each others' previous customers and this leads to socially inefficient switching. In contrast, when consumer preferences over time are independent, they find short-term contracts to be efficient.

Our analysis builds on Fudenberg and Tirole's analysis in two important ways. First, we consider a situation where firms need to take observable action to engage in behavior-based discrimination.[5] Our objective is to examine the economic motivation for behavior-based discrimination given that there are costs associated with its implementation (the cost of collecting and processing information from past consumers and the cost of identifying consumers at the time of purchase).

Second, our interest is in situations where firms can do more than offer different prices to consumers based on their past purchase. To be specific, we wish to examine how offering an *additional benefit* and a unique price to past consumers affects firm performance under competition. Recent work demonstrates that adding benefits (such as embedded premiums) can be more effective than offering price discounts of similar value (Arora and Henderson 2007). In contrast to situations where consumers themselves design the benefits (Randall, Terwiesch and Ulrich 2007), we focus on situations where firms collect information from groups of consumers to design benefits themselves. Beyond the three examples provided in the introduction, the benefits we consider could be free business services to repeat business customers at a downtown hotel (internet access, printing and computer services), expedited check-out to past customers (at internet sites such as Amazon and Travelocity), and specialized promotions and services that are offered by casinos such as Harrah's to past customers (Loveman 2003).

To date there has been limited research on this topic. One paper that does examine the issue is Acquisti and Varian (2005). Most of the analysis in Acquisti and Varian is focussed on the impact of behaviour-based discrimination (BBD) in a monopoly context. The paper does however, include a section on the effect of BBD under competition. In this section, Acquisti and Varian assume that all competitors are identical and sell a homogenous good (after serving customers, firms are differentiated in the sense that previous customers obtain a benefit by buying again at the same firm). Because firms do not have market power, the solution is based on all firms earning zero profits. This analysis is incisive about what can happen in a market where BBD occurs but it does not provide guidance about a) whether a firm should implement BBD when it faces a *differentiated* competitor or b) the expected outcome in a market where each firm makes a choice about implementing BBD.[6]

[5] This also implies that firms can commit not to implement behavior-based discrimination.

[6] Acquisti and Varian also examine how the equilibrium changes when consumers have the ability hide their

A second paper by Zhang (2005) is related to this analysis but there are two key differences. First, Zhang's paper is a dynamic analysis of product positioning and product line management. The logic of Zhang's model is that firms make positioning decisions in each period and offer different products to consumers based on their past decisions. Second, past consumers are offered a repositioned product that provides a different combination of benefits; it does not necessarily provide the consumer with more benefits. In contrast, our objective is to examine the effect of adding a vertical benefit (and not of offering a repositioned product) to past consumers. In cases where BBD means redesigning the physical product attributes and not simply adding benefits for past-consumers, Zhang's model is more relevant.

It is important to note that our analysis has links to the literature on switching costs (Fudenberg and Villas-Boas 2005). The similarity is due to the fact that the second period decision for consumers is affected by a discrete change in the value of a previously consumed product. Chen (1997) finds that discriminatory pricing makes firms worse off in a two period market where consumers incur a cost to switch firms in the second period. Taylor (2003) extends Chen's two firm model to multiple firms and multiple periods. His analysis demonstrates how the addition of a third firm further exacerbates price competition driving economic profits to zero. These studies show that discriminatory pricing is not necessarily an advantage for firms in a competitive setting. In these models, switching costs occur endogenously as a function of the first purchase. In contrast, we analyze a context where the discrete change in the value of repurchasing from the same firm is a decision variable. In addition, the benefit a firm adds to its offer is not bounded in the way that switching costs are.

A first objective of our study is to examine the interaction between competing firms when they can add benefits for past consumers as well as charging them different prices. Second, we want to better understand the economic incentives to implement behavior-based discrimination if firms can commit not to do so. This will provide guidance about whether firms have an incentive to implement behavior-based discrimination (assuming the costs of implementation are sufficiently low). It will also allow identification of conditions (if they exist) where behavior-based discrimination is mutually beneficial for competing firms. In the following section, we preview our findings.

identity: for example in an internet purchasing situation, a consumer can hide her identity by deleting cookies. We do not consider this possibility and assume that past customers are identified correctly; however, our analysis shows that consumers do not have an incentive to hide their identity.

1.3 Preview of Findings

A key finding of our analysis is that when firms are *identical* in their ability to add value to the second period offer, behavior-based discrimination (BBD) often leads to a game that has the characteristics of a Prisoners' Dilemma: both firms earn less profits than they would in the absence of BBD. This is explained by considering the mechanism that underlies behavior-based discrimination. The benefits that are added to offers made to past consumers "tie" them to the firm they bought from previously. Consequently, firms charge these past consumers high prices. The problem for firms is that this dynamic also increases the value of a past consumer. Firms recognize the added value of past consumers and in the first period, this leads to cutthroat competition. In the first period, firms offer prices less than marginal cost to capture customers for the second period. (Interestingly, the findings are different when firms are restricted to charging prices greater than marginal cost in the first period; in this situation, BBD can be profitable even when both competitors implement it.)

When the benefits that firms can add for previous customers are below a threshold, the game has two equilibria, one where both firms implement behavior-based discrimination and one where neither of them do. Our analysis does not indicate which outcome is more likely even though the equilibrium where firms implement behavior-based discrimination leads to strictly lower profits for both firms. However, when the firms make sequential decisions to implement behavior-based discrimination, the unique outcome is that no behavior-based discrimination occurs. Because the capability to implement behavior-based discrimination is recent, it is possible that the absence of behavior-based discrimination in some markets is due to the fact that the competitors have acquired their capabilities sequentially as opposed to simultaneously.

Our analysis paints a stark picture of how BBD should affect industry profitability. However, the model is based on firms adding the benefits (for past consumers) at zero cost. This naturally leads to the question of whether firms might be better off were they to incur a cost to provide a benefit level to past consumers. In a modified version of the model, we show that the problem of the Prisoners' Dilemma is reduced but not eliminated by assuming that the level of benefit offered to past consumers is costly.

We also analyze the impact of BBD when firms have unequal capabilities to add benefits for past consumers, i.e. one firm has a stronger capability to add these benefits. When a firm has a sufficient advantage over its competitor, it can increase its profit versus the base case even when there is significant competition (poaching) in the second period. In addition, the firm at a disadvantage sometimes finds that the best response to BBD by a strong competitor is to respond with a uniform price and avoid the practice completely.

The model thus provides guidance to managers about the necessary conditions for behavior-based discrimination to be profitable. In addition. the model provides normative predictions about whether the lack of behavior-based discrimination in a market is due to the *choice* or *inability* of firms to implement the practice. In the following section, we present the modelling framework. In section 3, we present the analysis and we then conclude in the final section.

2 The Model

The model is comprised of two firms and two stages. In the first stage, each firm decides whether to implement behavior based discrimination. After the choices are made, the decisions become common knowledge. We also consider a modified game in which the firms make sequential decisions to implement behavior-based discrimination.

In the second stage, the firms engage in two periods of differentiated price competition. Thus, there are four potential second-stage games: both firms choose to implement behavior based discrimination, both choose not to implement BBD, and the two mixed cases where one firm chooses to implement BBD while the other does not. Our objective is to investigate a 2×2 meta-game where firms decide whether or not to implement BBD.

To model the second stage of the game, we use a standard unitary Hotelling market where each firm is located at either end of the market. We denote the firm at the left end of the market as Firm 1 and at the right end as Firm 2. Each firm produces a single product at a constant marginal cost of production, c. The products differ with respect to an attribute and each consumer is identified by an ideal point along the attribute that corresponds to her preferred brand. Consumers are assumed to be uniformly distributed along the market with a density of one and their location (their preferences) are fixed across both periods similar to Fudenberg and Tirole (2000).

We assume that value of surplus (realized by consumers) and profits (realized by firms) in both periods are of equivalent value.[7] Each consumer buys no more than one unit of product per period and places a value v on her ideal product in the first period.[8] Consumers however, cannot obtain their ideal product. In the first period, a consumer located a distance x from Firm i ($i = 1, 2$)

[7]Discounting the second period does not affect the findings. Reduced importance of the second period simply weakens the strategic importance of behavior-based discrimination (the directional impact on profits versus a base case of uniform pricing remains).

[8]We assume that v is sufficiently high such that the participation constraint does not bind for any consumer in the market. Given the unitary length of the market, this means that $v > \frac{3t}{2}$.

obtains a surplus $v - tx - p_i$ by consuming Firm i's product (t is the transportation cost and it reflects the degree of differentiation in the market).

If a firm implements behavior-based discrimination, there are two observable actions that the firm takes.[9] The first is that the identity of each consumer in the first period is recorded so that a consumer in the second period is correctly identified as either a "new consumer" or a "past consumer". This relates to situations where consumers need to provide personal information to make a purchase. For example to make a first purchase at the Price Club, a consumer needs to register and become a member. We assume that there are no legal restrictions against firms offering different "deals" to consumers based on their identity.

Second, in the first period, each firm collects information from its consumers related to needs that are not addressed by the first period offer. This information allows the firm to add a benefit to its second period offer that we denote by B_i $(i = 1, 2)$. In the examples listed in the introduction, the benefits would be double frequent flyer miles, the reduced rate for the Gold Card and the automatic flight information. The assumption is that the firms collect information at the time of the first purchase suggesting that these benefits would be of significant value. It is important to note that the benefits are specific to the group of consumers who purchase in the first period. We acknowledge that this representation of how information is collected and used to add benefits for past consumers is somewhat ad-hoc. Nevertheless, it reflects three key aspects of behavior-based discrimination: a) firms learn from consumers with whom they have had interactions (past consumers), b) these consumers are similar in terms of preferences and c) there is no reason why the benefits in question would be of value to "new consumers" who by definition have different preferences.

In the second period, a consumer in group j has purchased from Firm j in the first period $(j = 1, 2)$. When Firm i implements behavior-based discrimination then a consumer in group j located a distance x from Firm i where $j = i$ will obtain a surplus of $B_i + v - tx - p_i^j$ by consuming Firm i's product. Conversely, if $j \neq i$, then the same consumer obtains a surplus of $v - tx - p_i^j$ by consuming Firm i's product. Note that a consumer always pays a price that is specific to the group to which she belongs. However, only a consumer who is a "past consumer" obtains the benefit B_i associated with Firm i's product. If the firm does not implement behavior-based discrimination then all consumers in the second period obtain a surplus of $v - tx - p_i^b$ from Firm i's product. The superscript b indicates that all second period consumers are offered an identical price (the critical

[9] These actions are assumed to be observable by both consumers and the firms.

difference between the model with behavior-based discrimination and the standard model is that in the second period, consumers are offered different things based on their behavior in the first period). When the benefit B_i associated with behavior-based discrimination is 0, we obtain a strict model of price discrimination as in Fudenberg and Tirole (2000). We now specify demand in each period given these assumptions.

2.1 Consumer Demand

In the first period, we assume that consumers account for the expected surplus they will realize in the second period. This allows us to write the surplus for a consumer located at x_0 who buys from Firm 1 in the first period as:

$$CS_1^1(x) = v - tx_0 - p_1 + E(CS_2^1)$$

and from Firm 2 as

$$CS_1^2(x) = v - t(1 - x_0) - p_2 + E(CS_2^2) \tag{1}$$

Here CS_1^i represents the surplus from buying at Firm i in the first period and $E(CS_2^i)$ is the expected surplus in the second period given that the consumer purchased from Firm i in the first period. The consumer who is indifferent between buying at Firm 1 and 2 is found where these two expressions are equal. Let the indifferent consumer be located at a location we denote as q.

$$q = \frac{p_2 - p_1 + E(CS_2^1) - E(CS_2^2) + t}{2t} \tag{2}$$

In the second period, there are two groups of consumers: those who purchased from Firms 1 and 2 respectively in the first period. In period 2, let x_i be the location of the indifferent consumer in the group of consumer who purchased from Firm i in the first period.

We consider three cases. In the first case, we assume that neither firm implements behavior-based discrimination. This implies that the two firms charge a price of p_i^b in the second period ($i = 1, 2$) and no additional benefits are added to the products. In this situation, the surplus for a consumer at x who buys from Firm 1 will be $CS_2^1(x) = v - tx - p_1^b$ and from Firm 2 will be $CS_2^1(x) = v - t(1 - x) - p_2^b$. As a result, the indifferent consumer in the markets is found at $x^* = \frac{p_2^b - p_1^b + t}{2t}$. Here, the location of the indifferent consumer is independent of decisions made in the first period.

In the second case, we assume that both firms implement behavior-based discrimination. In the group who purchased from Firm 1 in the first period, the surplus for a consumer at x who buys from Firm 1 will be $CS_2^1(x) = B_1 + v - tx - p_1^1$ and from Firm 2 will be $CS_2^1(x) = v - (1-x) - p_2^1$. Therefore, in period 2, the indifferent consumer in the group that purchased from Firm 1 in period 1 is given by: $x_1 = \frac{B_1 + p_2^1 - p_1^1 + t}{2t}$. Similarly, $x_2 = \frac{p_2^2 - p_1^2 + t - B_2}{2t}$. Note that second period demands for Firms 1 and 2 respectively from the group that purchased from Firm 1 in period 1 are $d_1^1 = x_1$ and $d_2^1 = q - x_1$. Similarly second period demand for Firms 1 and 2 respectively from the group that purchased from Firm 2 are $d_1^2 = x_2 - q$ and $d_2^2 = 1 - x_2$.[10]

In the third case, we assume that only one firm (say Firm 1) implements behavior-based discrimination. Using similar reasoning to the first two cases, it is straightforward to show that the indifferent consumer in the group that purchased from Firm 1 in period 1 is given by: $x_1 = \frac{B_1 + p_2^b - p_1^1 + t}{2t}$. Similarly, $x_2 = \frac{p_2^b - p_1^2 + t}{2t}$. Note that second period demands for Firms 1 and 2 i.e., d_1^1, d_2^1, d_1^2 and d_2^2, are identical to the expressions presented for the second case.

In order to specify demand in the first period, we now show how consumers form expectations for consumer surplus in period 2. We consider each of the three cases in question. In the first case (when behavior-based discrimination is not implemented by either firm), consumers who bought from Firms 1 and 2 respectively in the first period know they are likely to buy from the same firm in the second period. These observations imply that:

$$E(CS_2^1) = v - x_0 t - p_1^b \tag{3}$$

$$E(CS_2^2) = v - (1 - x_0)t - p_2^b \tag{4}$$

In these expressions, $E(CS_2^1)$ and $E(CS_2^2)$ apply to the consumers to the left and right of the indifferent consumer in the first period respectively.

In the second case (when behavior-based discrimination is implemented by both firms), we assume that poaching occurs i.e., consumers close to q who bought from Firm 1 in the first period will switch to Firm 2 in the second period and vice versa.[11] These observations imply that:

$$E(CS_2^1) = v - (1 - x)t - p_2^1 \tag{5}$$

$$E(CS_2^2) = v - xt - p_1^2 \tag{6}$$

[10] These expressions assume poaching of the competitor's first period customers by both firms. Fudenberg and Tirole (2000) demonstrate that this is the equilibrium outcome in the symmetric case when $B_1 = B_2 = 0$. When poaching does not occur, $d_1^2 = d_2^1 = 0$.

[11] The expectations consumers form about second period surplus do not depend on the magnitude of B. In other words, the model does not rely on consumers having a precise estimate of the benefit they will be offered in the second period. The model simply requires that consumers have an idea about whether the benefit will be big enough to eliminate switching.

In the third case, we assume that poaching occurs (similar to the second case) and use the same expressions for $E(CS_2^1)$ and $E(CS_2^2)$.

Expected consumer surplus is a key input for the consumer's first-period decision. Because consumers are forward-looking, the first period decision is more complicated than the second period decision. The solution entails first solving for second period prices as a function of q (the split of the market in the first period). These expressions are then used to reduce the problem to a single period optimization for each of the three cases. We now move to the pricing decisions of the firms.

2.2 The Pricing Decision by Firms

When neither firms employs behavior-based discrimination, the pricing question in the first period is identical to the pricing question in the second period. To simplify the presentation of our analysis, we normalize t, the transportation cost to 1.[12] Straightforward calculations show that the equilibrium prices when neither firm implements behavior-based discrimination are $p_i = p_i^b = 1$ (for $i = 1, 2$) and both firms earn profits of $\frac{1}{2}$ per period.

We now consider the second and third cases discussed above. When one or more of the firms employs behavior-based discrimination, the second period decisions are a function of the fraction of the market that purchased from each firm in the first period. We write the second period objective functions, π_1 and π_2 (for Firms 1 and 2 respectively) assuming that each firm "poaches" some of the competitor's previous consumers:

$$\pi_1 = \left(p_1^1 - c\right) x_1 + \left(p_1^2 - c\right)(x_2 - q) \tag{7}$$

$$\pi_2 = \left(p_2^1 - c\right)(q - x_1) + \left(p_2^2 - c\right)(1 - x_2) \tag{8}$$

The values of x_1 and x_2 as presented in Section 2.1 are substituted into equations 7 and 8 to obtain second period demand as a function of second period prices. Each firm maximizes its profits with respect to it second period prices. This yields a series of first order conditions (provided in the appendix) which are used to calculate the equilibrium prices in the second period as a function of q, the location of the indifferent consumer in the first period (in the first period, Firms 1 and 2 obtain demand of q and $1 - q$ respectively). The optimal prices in the second period as a function

[12] Normalizing t to 1 implies that the critical levels of B will be identified in absolute terms. These levels can be multiplied by t to obtain the critical levels without the normalization.

of q are shown in equations 9 and 10.

$$p_1^1 = c + \frac{1}{3}B_1 + \frac{1}{3} + \frac{2}{3}q,\ p_1^2 = c + 1 - \frac{4}{3}q - \frac{1}{3}B_2$$
$$p_2^1 = c + \frac{4}{3}q - \frac{1}{3} - \frac{1}{3}B_1,\ p_2^2 = c + \frac{1}{3}B_2 + 1 - \frac{2}{3}q \tag{9}$$

$$p_1^1 = c + \frac{1}{6}q + \frac{5}{12}B_1 + \frac{2}{3},\ p_1^2 = c + \frac{2}{3} - \frac{1}{12}B_1 - \frac{5}{6}q,\ p_2^b = c + \frac{1}{3}q - \frac{1}{6}B_1 + \frac{1}{3} \tag{10}$$

When a consumer makes her first period purchase, naturally it depends on first period prices. But it also depends on the surplus that she expects to enjoy in the second period. As discussed earlier, consumers in the neighborhood of the indifferent consumer know that they will buy from different firms each period (they foresee poaching by both firms). We use the expressions for consumer surplus (equations 3 and 4) to write first period demand (in the symmetric and asymmetric cases) as a function of first period prices.

$$q_{both} = \frac{1}{8}B_1 - \frac{1}{8}B_2 - \frac{3}{8}p_1 + \frac{3}{8}p_2 + \frac{1}{2} \tag{11}$$
$$q_{Firm\ 1} = q = \frac{1}{14}B_1 - \frac{6}{7}p_1 + \frac{6}{7}p_2 + \frac{2}{7} \tag{12}$$

The next step in the solution is to determine the optimal prices in the first period. The first period objective functions for the symmetric and asymmetric cases can be written as functions of first period prices by substituting for x_1, x_2 and q into equations 7 and 8 (the expressions are provided in the appendix). For the two cases, these functions are optimized with regards to the first period price chosen by each firm.

Having solved for the optimal prices in the first period, equations 11 and 12 are used to identify first period demand and hence the optimal second period prices for each firm. In order to focus on the demand-side effects of behavior-based discrimination, we assume that the benefit and discriminatory price for past consumers are offered costlessly by a firm that implements behavior-based discrimination. We now consider the first stage of the game where firms make a decision about whether or not to implement behavior-based discrimination.

2.3 The Decision by Firms to Implement Behavior-Based Discrimination

The first stage of the game entails simultaneous decisions by the firms whether to implement behavior-based discrimination in the second period. The normal form of the game is represented by Figure 1. In Figure 1, $\Pi_{no\ bbd}$ and $\Pi_{no\ bbd}$ are the profits earned by each firms when both firms implement BBD and when neither firm implements BBD respectively. When only one firm

Firm 1 \ Firm 2	no Beh.-Based Discrimination	Beh.-Based Discrimination
no Beh.-Based Discrimination	$\Pi_{no\,bbd}, \Pi_{no\,bbd}$	Π_d , Π_a
Beh.-Based Discrimination	Π_a, Π_d	Π_{bbd}, Π_{bbd}

Figure 1: First Stage: Behavior-Based Discrimination or Not?

implements BBD, we define Π_a as the profit earned by the firm that implements BBD and Π_d as the profit earned by the firm employs standard pricing in both periods. A Nash equilibrium involves identifying an outcome where neither firm has an incentive to deviate. It is important to note that once a firm decides to implement BBD in the first stage, its implementation in the second stage is self-enforcing. That is, a firm that prices in the first period as if it were going to implement BBD is strictly worse off in the second period, if it does not implement BBD.[13]

In the following section, we present the analysis and findings.

3 Analysis

3.1 The Case of Firms with Identical Capability to Add Benefits

We first analyze the two period pricing subgame for firms with identical ability to add benefits ($B = B_1 = B_2$). Later in the paper, we consider a game between firms that are asymmetric, i.e. $B_1 > B_2$. The results from the two-period subgame are used to analyze the 2 by 2 meta-game of Figure 1.

3.1.1 The Two-Period Pricing Subgame

As discussed earlier, the symmetric game where neither firm engages in behavior-based discrimination is two one-period sub-games where each firm earns profits of $\frac{1}{2}$ per period. Accordingly, the role

[13] The analysis of Section 3.1.1 shows that BBD exacerbates price competition in Period 1 but allows high prices to be charged to past consumers in Period 2. This explains why the commitment to implement BBD is self-enforcing. A firm that abandons BBD in Period 2 walks away from the high prices that can be charged to past consumers.

of this section is to present the outcomes where either one or both firms engage in behavior-based discrimination. We start by examining the outcomes when poaching occurs in the (BBD, BBD) subgame.[14]

The Sub-Game Outcome When Poaching is Possible When poaching occurs in the (BBD, BBD) subgame, the objective functions are based on the case where "poaching" occurs in the second period (see Section 2.2). Proposition 1 summarizes the optimal prices in each period and the total profits when poaching occurs. All proofs are provided in the appendix.

Proposition 1 *When $B < 1$ and both firms engage in behavior-based discrimination, prices in the first period are $p_1 = p_2 = c + \frac{4}{3} - \frac{2}{3}B$ and the prices offered in the second period are $p_1^1 = p_2^2 = c + \frac{2}{3} + \frac{1}{3}B$ (to previous customers) and $p_1^2 = p_2^1 = c + \frac{1}{3} - \frac{1}{3}B$ (to previous customers of the competitor). The firms earn profits of $\frac{1}{9}B^2 - \frac{2}{9}B + \frac{17}{18}$.*

Proposition 1 shows that in the second period, each firm sets one price to retain its past consumers and another to capture demand from the competitor's past consumers. The targeted price for past consumers of the competitor is low because a) these consumers incur a high travel cost to change firms in the second period (consumers are located closer to the firm they buy from in the first period) and b) they lose the benefit B when they switch firms in the second period. This exacerbates price competition and prices are lower in the second period versus the base case (the optimal uniform prices are 1 in both periods).

Second, prices in the first period are higher than the base case when $B \in \left(0, \frac{1}{2}\right)$ and less than the base case when $B \in (\frac{1}{2}, 1)$. This happens because there are two opposing forces. The first is that firms have little incentive to expand their first period demand when they know that a significant fraction of their first period demand will be poached by the competitor. This increases the relative incentive to charge high prices to consumers who are nearby. The second force is the attractiveness of having a significant number of past consumers. This force is directly related to size of B because the second period price that can be charged to past customers is positively related to B. As B increases, the second force becomes stronger than the first and this explains why prices fall below the base case in the upper half of the $(0, 1)$ interval.

When only one firm (Firm 1) implements behavior-based discrimination, the sub-game equilibrium with poaching is based on the objective functions developed in Section 2.2. The conditions for poaching are satisfied when $B < \frac{23}{31}$. Proposition 2 summarizes the outcome in this range.

[14] The (BBD, BBD) subgame refers to the case where both firms implement behavior-based discrimination.

Proposition 2 *When Firm 1 employs behavior based discrimination and $B < \frac{23}{31}$, both firms poach the competitor's previous customers in the second period. Prices in the first period are $p_1 = c + \frac{2}{3} - \frac{11}{92}B$ and $p_2 = c + \frac{11}{12} - \frac{7}{46}B$. In the second period, the prices are $p_1^1 = c + \frac{39}{92}B + \frac{3}{4}, p_1^2 = -\frac{11}{92}B + \frac{1}{4}$ and $p_2^b = c - \frac{7}{46}B + \frac{1}{2}$. The profits of Firms 1 and 2 are $\Pi_1 = \frac{71}{276}B + \frac{777}{8464}B^2 + \frac{31}{48}$ and $\Pi_2 = \frac{63}{2116}B^2 - \frac{37}{138}B + \frac{17}{24}$.*

Proposition 2 shows that the main effects of one firm implementing behavior-based discrimination are to allow it to raise prices for its past consumers (due to the benefit that is added) and to increase price competition for the first-period consumers of Firm 2 in the second period. In particular, Firm 1 becomes aggressive in attacking its competitor when it implements BBD. Firm 2 prices aggressively precisely because it is vulnerable to the targeted marketing of Firm 1. Proposition 2 also underlines a second interesting characteristic of the outcome when only one firm implements BBD. When $B < \frac{23\sqrt{3553}-1334}{315} \approx 0.1173$, the firm implementing BBD earns lower profits than the competitor who neither adds benefits nor employs discriminatory pricing in the second period. The reverse is true when $B \gtrapprox 0.1173$.

When $B \in \left(\frac{23}{31}, 1\right)$, poaching occurs when both firms implement BBD but not when only one firm does. The firm that does not implement BBD (Firm 2) is too weak in the second period to attract previous customers of Firm 1 so the outcome does not entail poaching. Moreover, Firm 1 obtains more than 50% of the market due to the benefits it offers to its previous customers in the second period. The equilibrium is summarized in Proposition 3.

Proposition 3 *When Firm 1 employs behavior based discrimination and $B \in \left(\frac{23}{31}, 1\right)$, there are two possible equilibria:*

1. *Both firms serve exactly the same consumers in both periods. First period prices are $p_1 = c + \frac{9}{10} - \frac{2}{5}B$ and $p_2 = c + \frac{7}{10} - \frac{1}{5}B$. In the second period , the prices are $p_1^1 = c + \frac{4}{5}B + \frac{7}{10}, p_1^2 = c$ and $p_2^b = c + \frac{1}{2}$. The profits of Firms 1 and 2 are $\Pi_1 = \frac{8}{25}B + \frac{1}{25}B^2 + \frac{16}{25}$ and $\Pi_2 = \frac{1}{50}B^2 - \frac{6}{25}B + \frac{18}{25}$ respectively.*

2. *Firm 1 poaches Firm 2's past consumers. First period prices are $p_1 = c - \frac{4}{23}B - \frac{8}{69}$ and $p_2 = c - \frac{1}{23}B - \frac{2}{69}$. In the second period , the prices are $p_1^1 = \frac{11}{23}B + c + \frac{38}{23}$, $p_1^2 = c - \frac{6}{23}B + \frac{19}{23}$ and $p_2^b = c - \frac{3}{23}B + \frac{21}{23}$. The profits of Firms 1 and 2 are $\Pi_1 = \frac{66}{529}B + \frac{99}{1058}B^2 + \frac{573}{1058}$ and $\Pi_2 = \frac{9}{529}B^2 - \frac{80}{529}B + \frac{1243}{3174}$ respectively.*

Simple comparisons using Proposition 3 show that Firm 2 suffers by not implementing BBD. However, independent of the equilibrium that results in the sub-game, Firm 1's profits are lower than the base case. Note that the outcome with no poaching is strictly superior for both firms yet either outcome is possible. In sum, Propositions 2 and 3 show that both firms are strictly worse off than the base case of uniform pricing when $B < 1$. It also appears that when only Firm 1 implements BBD, the weak position of Firm 2 causes it to compete aggressively to the detriment of both firms. We now consider situations where B is sufficiently high to eliminate poaching in the second period.

The Sub-Game Outcome When Poaching cannot occur in the second period When the benefits that firms can offer in the second period are sufficiently high, firms do not poach in the second period and each firms serves exactly the same customers in both periods. As shown in the appendix, when $B > B^*$ where $B^* = \frac{197}{84} - \frac{69}{308}\sqrt{22} \approx 1.3$ poaching does not occur in the second period.[15] Proposition 4 describes the outcome in the sub-game where both firms implement BBD.

Proposition 4 *When $B > B^*$ and both firms implement behavior-based discrimination. First period prices are $p_1 = p_2 = 1 + c - B$, second period prices are $p_1^1 = p_2^2 = B + c$ and firms earn profits of $\Pi_1 = \Pi_2 = \frac{1}{2}$.*

Proposition 4 applies as long as marginal cost exceeds the threshold of $B - 1$ (otherwise first period prices will be negative). When marginal costs are positive and firms can sell for less than marginal cost (but at positive prices), the profits in the $(BBD,\ BBD)$ subgame are *independent* of the level of benefit and 50% less than the base case of $(no\ BBD, no\ BBD)$.

We now consider the case where only Firm 1 implements behavior-based discrimination and $B > B^*$. In the appendix, we show that Firm 2 is too weak to attract previous consumers of Firm 1 and an outcome where only Firm 1 poaches Firm 2's consumers is not stable. As a result, the equilibrium does not entail poaching and it summarized in Proposition 5.

Proposition 5 *When Firm 1 employs behavior based discrimination and*

1. *$B \in (B^*, 6)$, first period prices are $p_1 = 2c + \frac{9}{10} - \frac{2}{5}B$ and $p_2 = 2c + \frac{7}{10} - \frac{1}{5}B$ and second period prices are $p_1^1 = c + \frac{4}{5}B + \frac{7}{10}, p_1^2 = c$ and $p_2^b = c + \frac{1}{2}$. Firms 1 and 2 earn profits of $\Pi_1 = \frac{8}{25}B + \frac{1}{25}B^2 + \frac{16}{25}$ and $\Pi_2 = \frac{1}{50}B^2 - \frac{6}{25}B + \frac{18}{25}$ respectively.*

[15] When $B \in \left(1, \frac{669-39\sqrt{101}}{230} \approx 1.205\right)$, the equilibrium involves only one of the two firms poaching the competitor's past consumers (an asymmetric outcome even though the competitors are symmetric). Details of this outcome are provided in the appendix.

2. *$B > 6$, first period prices are $p_1 = c - \frac{27}{20}$ and $p_2 = c - \frac{3}{10}$ leaving Firm 1 with the entire market. Second period prices are $p_1^1 = B + c - 1$ and $p_2^b = c$. Firms 1 and 2 earn profits of $\Pi_1 = B - \frac{47}{20}$ and $\Pi_2 = 0$ respectively.*

Proposition 5 shows that when only Firm 1 implements BBD, Firm 1 earns higher profits than the base case of no BBD (because $\frac{8}{25}B + \frac{1}{25}B^2 + \frac{16}{25}$ and $B - \frac{47}{20}$ are greater than 1 in the relevant zone) at the expense of Firm 2. We now move the first stage of the game using the sub-game outcomes as a basis for analyzing each cell of Figure 1.

3.1.2 The First Stage of the Game

First, we consider the equilibrium outcome when poaching is possible i.e. when $B < 1$.

Proposition 6 *When $B \in (0, 1)$, there are two pure strategy Nash equilibria: both firms implement behavior-based discrimination or neither implements behavior-based discrimination.*

Proposition 6 shows that when the benefits that firms can add are less than the differentiation between products, the game has two pure strategy Nash equilibria.[16] This happens because the payoffs in the upper left and lower right cells of Figure 1 for both firms are strictly higher than the payoffs in the off-diagonal cells. Said differently, the best response to a competitor's strategy is to mimic it.

Proposition 6 leads to Corollary 1 which follow directly from the firm profit expressions. The corollaries highlight critical characteristics of the equilibrium as a function of the magnitude of benefits offered to past consumers.

Corollary 1 *When $B \in (0, 1)$, the equilibrium where neither firm implements behavior-based discrimination results in higher profits than the equilibrium where both firms implement behavior-based discrimination.*

Corollary 1 indicates that when the game has two pure strategy equilibria ($B < 1$), the equilibrium where firms implement behavior-based discrimination leads to strictly lower profits. When the firms make simultaneous decisions about the implementation of BBD, the model makes no prediction about which outcome is more likely. However, when the firms make *sequential* decisions

[16] As is always the case in a 2 player/2 action game with two Nash equilirbria, there is a mixed strategy equilibrium in which firms randomize between the two actions (implementing BBD and not implementing BBD). This equilibrium leads to strictly lower profits than either of the two pure strategy equilibrium so we focus the discussion on the pure strategy outcomes. A complete analysis of the mixed strategy equilibrium is available on request from the authors.

about the implementation of BBD, the unique outcome is that no BBD occurs. In other words, the plausibility of poaching by both firms (in equilibrium) depends on players making simultaneous and not sequential decisions to implement BBD. When the capability to implement behavior-based discrimination is acquired over time, the sequential game may be more relevant to understand whether BBD is likely to arise.

Note that the case of $B = 0$ is the situation where firms implement behavior-based *price* discrimination alone (no benefit is added to a past customer's offer). This is the case of short term contracts (where the consumer preferences are stable) examined by Fudenberg and Tirole (2000). Fudenberg and Tirole find that poaching and socially inefficient switching occur in equilibrium. When firms cannot commit to not implement BBD, this is exactly what happens. In our model however, firms are assumed to be able to make a credible commitment to not implement BBD. For that reason, the symmetric outcome of neither firm implementing behavior-based price discrimination is also possible.

We now consider the equilibrium when poaching cannot occur in the second period i.e. $B > B^*$.

Proposition 7 *When $B > B^*$, both firms implement behavior-based discrimination and earn profits of $\frac{1}{2}$.*

When $B > B^*$, the game has a unique Nash equilibrium where both firms implement behavior-based discrimination. Proposition 7 leads to Corollary 2.

Corollary 2 *When $B > B^*$, the firms are trapped in a Prisoners' Dilemma whereby both firms implement behavior based discrimination yet earn less profits than the base case.*

Corollary 2 shows that when the benefits that can be added are high, the unilateral incentive to implement behavior-based discrimination is positive but when the firms implement it, they are both worse off. This suggests that firms will implement behavior-based discrimination across many markets. But we should also expect significant press about how the effort has yielded little in terms of profitability. In addition, Corollary 2 highlights the fierceness of first-period competition. Despite the fact that consumers pay high prices for a benefit-enhanced offer in the second period, the impact of BBD on firm profits is negative due to unmitigated price competition in the first period.

3.2 Situations where Price Competition is Constrained in the First Period

The analysis of Section 3.1 is based on a market where the marginal cost, c, is sufficiently high to preclude negative prices being offered in the first period: when c is small the equilibrium first period prices of $1 + c - B$ can be negative. This may lead to a moral hazard problem in that non-customers will simply show up and ask for money. Moreover, in some situations, prices less than marginal cost may not be feasible.[17] In order to examine such situations, we consider another version of the model where the marginal cost of the product is normalized to zero and prices are restricted to being positive.

When $B < 1$ and behavior-based discrimination leads to poaching, the findings are analogous to the case when marginal cost is positive. The prices given in Propositions 1, 2 and 3 are the solutions with marginal cost set to zero (profits are unaffected).

As shown in the appendix, when both firms implement BBD and $B > B^{**} = \frac{669-39\sqrt{101}}{230} \approx 1.2$, the outcome where one or both firms poach the other's past customers is not possible. Lemma 1 summarizes the outcome when both firms implement BBD in this range.

Lemma 1 *When both firms engage in behavior-based discrimination and $B > B^{**}$, prices in the first period are $p_1 = p_2 = 0$ and the prices offered in the second period are $p_1^1 = p_2^2 = B$ (to previous customers) and $p_1^2 = p_2^1 = 0$ (to previous customers of the competitor). Both firms earn profits of $\frac{B}{2}$.*

When the firms are restricted to prices greater than or equal to zero (the marginal cost), firms give the product away in the first period and all profits obtain from second period sales. Lemma 1 also identifies a change in the relationship of profits to B as the level of B increases. In particular, the relationship between B and firm profits is negative when $B < 1$ but positive when $B > B^{**}$. In other words, when competition in the first period is attenuated by the inability of the firms to set prices below marginal cost: small benefits exacerbate competition but large benefits do not.

We now consider the case where only Firm 1 implements behavior-based discrimination and $B > B^{**}$. The outcome does not entail poaching and is similar to the case of positive marginal costs; however, when $B > \frac{9}{4}$, the outcome is different.

Lemma 2 *When Firm 1 employs behavior based discrimination and*

[17] If firms can buy product from each other, an equilibrium with prices less than marginal cost in the first period breaks down. Suppose Firms 1 and 2 sell identical products but are differentiated by location. If Firm 1 offers its product in the first period at a price less than marginal cost, Firm 2 can buy the product its sells from Firm 1 (versus paying the marginal cost c). The same is true for Firm 1.

1. $B \in \left(B^{**}, \frac{9}{4}\right)$, *first period prices are* $p_1 = \frac{9}{10} - \frac{2}{5}B$ *and* $p_2 = \frac{7}{10} - \frac{1}{5}B$. *In the second period , the prices are* $p_1^1 = \frac{4}{5}B + \frac{7}{10}, p_1^2 = 0$ *and* $p_2^b = \frac{1}{2}$. *Firms 1 and 2 earn profits of* $\Pi_1 = \frac{8}{25}B + \frac{1}{25}B^2 + \frac{16}{25}$ *and* $\Pi_2 = \frac{1}{50}B^2 - \frac{6}{25}B + \frac{18}{25}$ *respectively.*

2. $B > \frac{9}{4}$, *first period prices are* $p_1 = 0$ *and* $p_2 = \frac{1}{4}$. *In the second period , the prices are* $p_1^1 = B + \frac{1}{4}, p_1^2 = 0$ *and* $p_2^b = \frac{1}{2}$. *Firms 1 and 2 earn profits of* $\Pi_1 = \frac{5}{8}B + \frac{5}{32}$ *and* $\Pi_2 = \frac{9}{32}$ *respectively.*

Similar to the case of positive marginal costs, Lemma 2 demonstrates that when only Firm 1 implements BBD, Firm 1 earns higher profits than the base case and Firm 2 is strictly worse off.

The results for the first stage of the game are unaffected when poaching is possible: there are two possible symmetric equilibria where both or neither of the firms implement BBD. When poaching is not possible i.e. $B > B^{**}$, Proposition 8 describes the equilibrium in the first stage of the game.

Proposition 8 *When* $B > B^{**}$, *there is a unique equilibrium where both firms implement behavior-based discrimination and the firms earn profits of* $\frac{B}{2}$.

When $B > B^{**}$, the best response to a competitor that does not implement behavior-based discrimination is to implement behavior-based discrimination. As a result, the upper-left square in the Figure 1 is not stable. Corollary 3 follows directly from the profit expressions.

Corollary 3 *When* $B \in (B^{**}, 2)$, *the firms are trapped in a Prisoners' Dilemma whereby both firms implement behavior based discrimination yet generate strictly less profit than the base case. When* $B > 2$, *the firms increase profit versus the base case.*

Corollary 3 shows that firms may find themselves in a Prisoners' Dilemma when the benefits are sufficient to eliminate poaching in the second period. However, if B is sufficiently large (> 2), firms benefit from BBD even when both firms implement it. This does not happen when marginal costs are positive and firms set prices below marginal cost in the first period). In other words, when the ability of firms to compete on a basis of price is limited due to a) the inability to "pay consumers" to buy a product or b) the infeasibility of charging a price below marginal cost; behavior-based discrimination is attractive (on an industry basis) at benefit level that is much lower. This provides useful insight for managers. When competition for new customers is exogenously constrained, BBD can be profitable even when both firms implement the practice. In contrast, when competition for new customers is unfettered, the implementation of BBD invariably leads to lower profits for both firms.

3.3 Endogenizing the Level of Benefit offered to Past Consumers

Until now, we have assumed that the firms costlessly add benefits to the second period offer for past consumers. This raises a concern. The model predicts that the ability to implement BBD is often a curse because it drives the profits of firms down: the firms find themselves in a Prisoners' Dilemma. This may be an overstatement of the potential negative effects of BBD i.e., when the benefits offered to past consumers need to be financed, the likelihood of a Prisoners' Dilemma may be lower.

To address these concerns, we consider a version of the model where firms invest to add benefits to second period offers for past customers. In particular, we assume that firms invest αB^2 prior to the start of the game in order to have the ability to offer a benefit B to past consumers in the second period. These investments become public knowledge. As before, in the second stage of the game, the firms engage in 2 periods of differentiated price competition. We also assume that firms which invest to offer a benefit B do, in fact, implement BBD. In contrast, firms that have not invested do not implement the BBD and price uniformly to the market in the second period. These investments can be thought of as financing for systems that a) collect and process information from customers or b) use first period interactions with customers as a way of adding value in the second period.

The objective is to determine if firms will invest *a priori* to create benefits and if they do, what level of benefits will they offer. In other words, will the equilibrium involve investment by both firms (or only one) and what are the conditions on α that lead to different market outcomes? The following proposition describes the equilibrium outcome, benefit level and profits as a function of α.

Proposition 9

1. *When BBD leads to poaching* $\left(\alpha > \frac{13}{96}\right)$*, there are two Nash equilibria: both firms implement behavior-based discrimination or neither implements behavior-based discrimination. When both firms implement BBD, they choose a benefit level of* $B = \frac{23}{144\left(2\alpha - \frac{1}{9}\right)}$ *and earn profits of* $\Pi = \frac{529}{2304(18\alpha-1)^2} - \frac{23}{72(18\alpha-1)} + \frac{17}{18}$.

2. *When BBD does not lead to poaching* $\alpha < \frac{207\sqrt{22}+5863}{5175\sqrt{22}+54\,175}$*, there are two regions.*

(a) *When* $\alpha \in \left(\frac{1}{9}, \frac{207\sqrt{22}+5863}{5175\sqrt{22}+54\,175}\right)$*, firms do not implement BBD.*

(b) *When* $\alpha < \frac{1}{9}$*, both firms implement BBD and earn profits of* $\alpha\left(\frac{4531}{4312}\sqrt{22} - \frac{512\,597}{77\,616}\right) + \frac{1}{2}$.

When BBD leads to poaching $\left(\text{i.e. } \alpha > \frac{13}{96}\right)$, the asymmetric profits (where one firm implements BBD and the other does not) are strictly lower than the symmetric profits where firms poach. For this reason, there are two equilibria. As a result, if the firms make sequential decisions about the implementation of BBD when $\alpha > \frac{13}{96}$, the unique outcome is for no BBD to occur. In any event, the key observation is that when the costs of creating benefits are sufficiently high such that firms poach each other's past consumers; endogenzing the benefit level does not affect the results substantially. There are two possible equilibria and the creation of benefits simply becomes an added cost of doing business.

When BBD does not lead to poaching, point 2 in Proposition 9 shows that endogenizing the creation of the benefit reduces the Prisoners' Dilemma problem. For a significant fraction of the parameter zone, firms do not implement BBD. However, when the cost of creating benefits is sufficiently low $\left(\alpha < \frac{1}{9}\right)$, the Prisoners' Dilemma problem reappears. This is explained by recalling that the unilateral incentive to implement BBD remains high even when the level of B is endogenized.

3.4 The Case of Firms with Unequal Capability to Add Benefits

To analyze the case of asymmetric firms ($B_1 \neq B_2$), we assume without loss of generality that $B_1 > B_2$. As in the symmetric case, we derive the results for the two-period subgame where both firms implement BBD. Note that the asymmetric subgame where one firm implements BBD and the other does not is identical to the analysis of Section 3.1. We then use these results to solve the 2 by 2 meta-game of Figure 1.

The equilibrium outcomes in the sub-game are derived in the appendix. The analysis shows that when the benefits of both firms are low and they implement BBD, the outcome entails poaching by both firms. The details of the poaching outcome provided in the appendix show that Firm 1 (the firm that offers a larger benefit) prices more aggressively in the first period. As a result, Firm 1 captures more than 50% of the market in the first period (in fact, $q = \frac{1}{32}\Delta B + \frac{1}{2}$ where $\Delta B = B_1 - B_2$). Not surprisingly, in the second period, Firm 1 is able to charge higher prices to its former customers than does Firm 2. This explains why Firm 1 is more aggressive in the first period. In the second period, Firm 2 is more aggressive with Firm 1's former customers than is Firm 1 with Firm 2's former customers. Firm 2 has no choice but to be more aggressive because of the higher benefit offered by Firm 1.

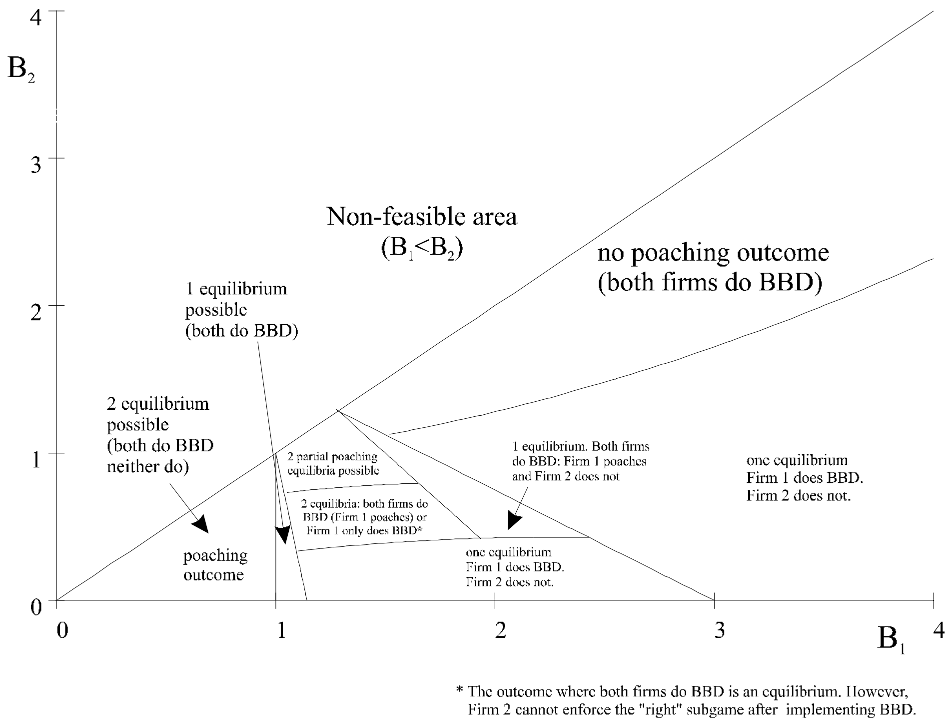

Figure 2: Summary of Meta-Game Equilibrium when $B_1 > B_2$

The analysis of the meta-game when firms have unequal capability to add benefits entails comparing the profits for each of the four cells in Figure 1. Note that the values of Π_a and Π_d are given straightforwardly by Propositions 1 to 5 but in contrast to the symmetric game, Π_a and Π_d are different depending on whether the strong firm (Firm 1) or the weak firm (Firm 2) implements BBD. A complete analysis of the meta-game is provided in the appendix. We provide a visual summary of the results in Figure 2. The figure is a sketch of how each of the conditions define different equilibrium zones as a function the benefit levels that each firm can add to its second period offer.

The figure shows that as we move from left to right, the equilibrium changes from poaching by both firms to partial poaching and ultimately to conditions where neither firm poaches. There are a number of insights generated the analysis that lies behind Figure 2. The first insight is that when the benefits levels are relatively low (less than the level of differentiation between firms), two equilibria are possible (one where both firms implement BBD and the other when they do not). Similar to the case of symmetric firms, the (BBD,BBD) outcome results in less profits for both firms than the (no BBD, no BBD) outcome.

Second, there is a *unique* equilibrium where both firms poach (the second zone in Figure 2 when moving from left to right). Interestingly, when $B_1 = B_2$, poaching by both firms is *never* unique. This suggests that poaching by both firms is more likely to be observed when firms have different capabilities to add benefits for past consumers.

Third and most surprisingly, for a significant fraction of the parameter space, the best response of the firm at a disadvantage (Firm 2) is to respond to BBD by pricing uniformly in the second period and offering no benefits (even when it has the ability to do so). When a firm makes a decision about implementing BBD, its decision is based on two effects. The first is the competition-increasing effect of BBD (because tailored prices can be set for sub-sections of the market, there are less infra-marginal consumers). The second effect is the ability to charge higher prices to past consumers as a result of the benefits that are added in the second period. Basically, when the benefits Firm 2 can offer are small, the first effect is larger than the second effect and Firm 2 benefits from lower competition associated with uniform pricing in the second period.

The profit expressions (provided in the appendix) lead to Proposition 10 which highlights a number of key differences between the cases of equal and unequal capability to add benefits.

Proposition 10

1. *When $B_1 < 1$ and firms implement BBD, Firm 1 increases profit versus the base case when* $\frac{23}{144}B_1 - \frac{55}{144}B_2 - \frac{7}{2304}B_1B_2 + \frac{263}{4608}B_1^2 + \frac{263}{4608}B_2^2 - \frac{1}{18} > 0.$

2. *When $B_1 \in \left(1, \frac{8}{7} - \frac{1}{7}B_2\right)$ and $\frac{23}{144}B_1 - \frac{55}{144}B_2 - \frac{7}{2304}B_1B_2 + \frac{263}{4608}B_1^2 + \frac{263}{4608}B_2^2 - \frac{1}{18} > 0$, Firm 1 increases profit versus the base case.*

3. *When both firms implement BBD and the equilibrium involves partial poaching by Firm 1, Firm 1 earns more than the base case when* $\frac{33}{169}B_1 - \frac{60}{169}B_2 - \frac{11}{676}B_1B_2 + \frac{99}{1352}B_1^2 + \frac{229}{4056}B_2^2 - \frac{125}{338} > 0.$

4. *When Firm 2 does not implement BBD, Firm 1 earns more than the base case.*

Proposition 10 is illustrated in Figure 3. The areas containing the letters a, b, c and d correspond to the regions specified by points 1-4 in the proposition. The most important insight highlighted by Proposition 10 is that BBD can be profit enhancing for a firm when it can offer benefits that provide higher value than those offered by the competitor. In particular, when Firm 1 has a sufficient advantage over Firm 2, BBD can lead to higher profits for Firm 1 (compared to no BBD)

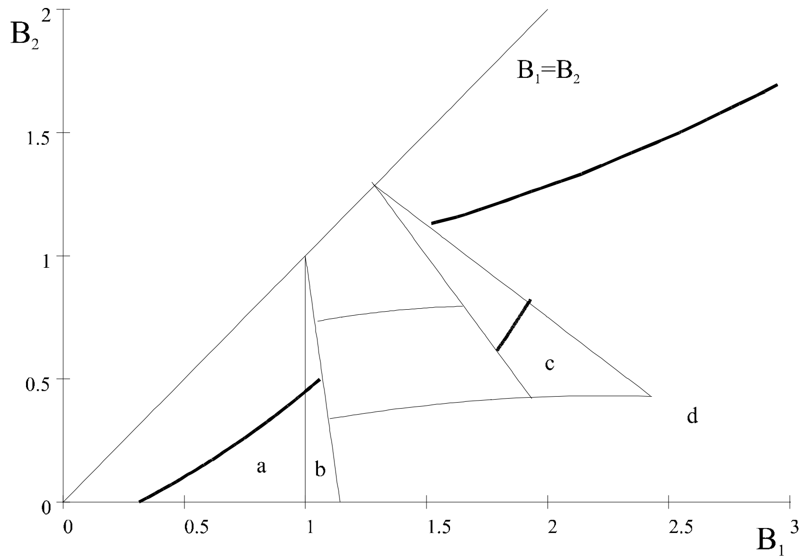

Figure 3: Graph showing zones where Firm 1 gains from BBD versus the base case

under conditions of poaching, partial poaching and no poaching. Moreover, in region b, poaching by both firms is the unique outcome and Firm 1 earns strictly higher profit than the base case. The analysis thus demonstrates that BBD can be an effective competitive tool for a firm when the benefits it adds are significantly higher than its competitor. The firm can increase profits versus the base case and its gains come largely at the expense of a competitor who is not able to respond effectively.

4 Conclusion

Our analysis shows that when firms have equal capability to add benefits for past consumers, behavior-based discrimination is generally a curse. For most parameter values, the ability to implement BBD creates a Prisoners' Dilemma. Only at low benefit levels (when there are two possible outcomes), can firms avoid the trap of reduced profits due to implementing BBD.

The deleterious effects of behavior-based discrimination occur because the benefit that a firm adds to its offer in the second period (for past consumers) indirectly leads to fiercer competition. Behavior-based discrimination makes the value of a first period sale high because firms obtain a significant benefit from first period customers in the second period. Firms account for this and price aggressively in the first period. We also consider a situation where firms are restricted to charging prices greater than marginal cost in the first period; in this situation, BBD can be profitable once the level of benefits is above a threshold.

In order to highlight the impact of adding benefits to past consumers' offers, we assume that benefits are added costlessly in the base model. This raises a question of whether the negative effects of BBD are due to this assumption. To investigate this possibility, we analyze a version of the model where the size of the benefits needs to financed. We show that the Prisoners' Dilemma problem is reduced by assuming that the level of benefit offered to past consumers is costly but it is not eliminated.

We also analyze the impact of BBD when firms have unequal capabilities to add benefits for past customers, i.e. one firm has a stronger capability to add benefits. Our analysis shows that when a firm has a sufficient advantage over its competitor, it can increases profit versus the base case. In addition, the firm at a disadvantage sometimes finds that the best response to BBD by a strong competitor is to respond with a uniform price and avoid implementing BBD.

The ultimate impact of behavior-based discrimination in a given market is a function of a) how important the benefits are to past consumers, b) how much the firm needs to invest in order to be able to offer these benefits and c) whether one firm has a significant advantage over its competitor in terms of its ability to add benefits.

It seems obvious that information collected from consumers is valuable to guide firms about how to treat consumers in the future. Moreover, many firms have implemented sophisticated forms of BBD, notably retailers and airline companies. Nevertheless, for many firms, these programs have increased the cost of doing business without delivering significant benefits in terms of higher profits. This is precisely the Prisoners' Dilemma outcome highlighted by the model. It happens because significant gains are possible when BBD is implemented unilaterally. As long as the competitors do not respond (or at least respond slowly), a significant advantage can be obtained. Perhaps that was the thinking behind American Airline's launch of the AAdvantage frequent flyer program in 1982. Unfortunately for American Airlines, competitive airlines launched their own programs within weeks and the advantage evaporated (Kearney 1990).

Another insight of the model relates to the impact of having firms make sequential versus simultaneous decisions. The model underlines how decisions become strategic when competitors make sequential versus simultaneous decisions. The sequential game allows the first mover to transmit information to the follower about which outcome it prefers. When the game is sequential, both firms benefit by avoiding the dominated equilibrium of implementing BBD (whenever B or $B_1 < 1$). This raises a question related to the previous American Airlines example. By 1982, a dynamic of leader-follower competition in the U.S. airline industry was well-established with American Airlines as the leader. The model suggests that American Airlines might have been able to avoid the profit-reducing outcome of (BBD, BBD) by choosing not to implement BBD. How do we explain this? First, it seems that American Airlines may have misjudged the ability of its competitors to respond quickly to the new program. Second, American Airlines may have misjudged the magnitude of the benefits provided by its frequent flyer program. Customers do appreciate the miles and upgrades. Nonetheless, the most notorious aspect of these programs is perhaps the huge balance of unclaimed awards (see "Fly me to the moon," *The Economist*, May 4, 2002, 68).

A final insight relates to the conditions where BBD is a winning play. When firms are equal in their capability to add benefits, BBD can be jointly beneficial when first period competition is limited by a) an inability to charge negative prices or b) an inability to sell products for less than their marginal cost.[18] Alternatively, a firm needs to have more ability to add benefits than its competitor for BBD to be a winning play. Regardless, the most likely situation for a firm to benefit from BBD is when the competitor is unable to respond *and* competition in the second period is eliminated. But the real winners with regards to the recent growth of BBD are consumers who often enjoy prices at or below cost for their first purchase.

[18] For example, with software, marginal costs are negligible so prices less than marginal cost may be infeasible.

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Appendix

The First order Conditions under Conditions of Poaching

Equations i apply to the case when both firms employ behavior based discrimination. The first order conditions in ii apply when only Firm 1 employs behavior based discrimination (p_2^b is the price chosen by Firm 2 in the second period when it charges the same price to all consumers).

$$\begin{aligned}\tfrac{\partial \pi_1}{\partial p_1^1} &= \tfrac{1}{2}\left(c + B_1 + 1 + p_2^1 - 2p_1^1\right) = 0\\ \tfrac{\partial \pi_1}{\partial p_1^2} &= \tfrac{1}{2}\left(c + 1 - 2q + p_2^2 - 2p_1^2 - B_2\right) = 0\\ \tfrac{\partial \pi_2}{\partial p_2^1} &= \tfrac{1}{2}\left(c + 2q - 1 - 2p_2^1 + p_1^1 - B_1\right) = 0\\ \tfrac{\partial \pi_2}{\partial p_2^2} &= \tfrac{1}{2}\left(c + B_2 + 1 - 2p_2^2 + p_1^2\right) = 0\end{aligned} \tag{i}$$

$$\begin{aligned}\tfrac{\partial \pi_1}{\partial p_1^1} &= \tfrac{1}{2}\left(c + B_1 + 1 + p_2^b - 2p_1^1\right) = 0\\ \tfrac{\partial \pi_1}{\partial p_1^2} &= \tfrac{1}{2}\left(c + 1 - 2q + p_2^2 - 2p_1^2\right) = 0\\ \tfrac{\partial \pi_2}{\partial p_2^b} &= \tfrac{1}{2}\left(2c + 2q - B_1 + p_1^1 - 4p_2^b + p_1^2\right) = 0\end{aligned} \tag{ii}$$

Proof of Proposition 1

As shown in the paper, the solution to the first order conditions when $B < 1$ in the second period implies:

$$p_1^1 = \frac{1}{3}B_1 + \frac{1}{3} + \frac{2}{3}q + c,\ p_1^2 = 1 - \frac{4}{3}q - \frac{1}{3}B_2 + c$$
$$p_2^1 = \frac{4}{3}q - \frac{1}{3} - \frac{1}{3}B_1 + c,\ p_2^2 = \frac{1}{3}B_2 + 1 - \frac{2}{3}q + c$$

We now write the following expresions for total profits over the two periods.

$$\Pi_1 = (p_1 - c)\, q + \left(p_1^1 - c\right) x_1 + \left(p_1^2 - c\right)(x_2 - q)$$

$$\Pi_2 = (p_2 - c)(1 - q)\left(p_2^1 - c\right)(q - x_1) + \left(p_2^2 - c\right)(1 - x_2)$$

Using the reasoning of Section 2.2, we substitute to obtain the following expression for q.

$$q = \frac{1}{8}B_1 - \frac{1}{8}B_2 - \frac{3}{8}p_1 + \frac{3}{8}p_2 + \frac{1}{2}$$

We then susbtitute first, for x_1 and x_2, second, for equilibrium second period prices (shown above) and finally, for q. This generates two objective functions in terms of first period prices (too long for presentation purposes). Taking the first order condition in terms of p_1 and p_2, we obtain the following system of two equations and two unknowns.

$$\frac{\partial \Pi_1}{\partial p_1} = \frac{1}{16}p_2 - \frac{3}{16}B_2 - \frac{7}{16}p_1 - \frac{1}{16}B_1 + \frac{1}{2} - \frac{1}{16}c = 0$$

$$\frac{\partial \Pi_2}{\partial p_2} = \frac{1}{16}p_1 - \frac{1}{16}B_2 - \frac{3}{16}B_1 - \frac{7}{16}p_2 + \frac{1}{2} - \frac{1}{16}c = 0$$

Solving these equations, we obtain equilibrium prices of

$$p_1 = \frac{4}{3} - \frac{11}{24}B_2 - \frac{5}{24}B_1 + c, p_2 = \frac{4}{3} - \frac{5}{24}B_2 - \frac{11}{24}B_1 + c$$

When $B = B_1 = B_2$, we obtain $p_1 = p_2 = \frac{4}{3} - \frac{2}{3}B + c$. Substitute this solution into equations 11 and 12 to derive the remainder of the expressions. In the second period, $x_2 > x_1$ for poaching to occur. This implies that $\frac{p_2^2 - p_1^2 + 1 - B_2}{2} > \frac{B_1 + p_2^1 - p_1^1 + 1}{2}$ Substitute the equilibrium values for each decision variable and once again assume $B = B_1 = B_2$. This inequality implies that $\frac{1}{3}B - \frac{1}{3} < 0$ which is only true if $B < 1$. The equilibrium prices imply that $\Pi = \frac{1}{9}B^2 - \frac{2}{9}B + \frac{17}{18}$ in this region. **Q.E.D.**

Proof of Proposition 2

When only Firm 1 implements behavior-based discrimination, the first order conditions are as follows when $B = B_1 = B_2$ (the firm objective functions) are:

$$\frac{\partial \Pi_1}{\partial p_1} = \frac{16}{49}p_2 - \frac{58}{49}p_1 - \frac{9}{98}B + \frac{24}{49} + \frac{6}{7}c$$

$$\frac{\partial \Pi_2}{\partial p_2} = \frac{34}{49}p_1 - \frac{15}{98}B - \frac{76}{49}p_2 + \frac{47}{49} + \frac{6}{7}c$$

Solving these equations, we obtain $p_1 = c + \frac{2}{3} - \frac{11}{92}B$ and $p_2 = c + \frac{11}{12} - \frac{7}{46}B$. Substitute this solution into equations 12 and then 10 to derive the remainder of the expressions. In the second period, $x_2 > q$ and $q > x_1$ for the poaching equilibrium to hold. Equation 12 implies that $q_{Firm\ 1} = \frac{1}{23}B + \frac{1}{2}$. Now $x_2 - q > 0$ implies that $\frac{1}{8} - \frac{11}{184}B > 0 \Rightarrow B < \frac{23}{11}$. Conversely, $q - x_1 > 0$ implies that $\frac{1}{8} - \frac{31}{184}B > 0 \Rightarrow B < \frac{23}{31}$. Because $\frac{23}{31} < \frac{23}{11}$, the limiting condition is $q > x_1$ and the solution applies when $B < \frac{23}{31}$. In this region, firms profits are $\Pi_1 = \frac{71}{276}B + \frac{777}{8464}B^2 + \frac{31}{48}$ and $\Pi_2 = \frac{63}{2116}B^2 - \frac{37}{138}B + \frac{17}{24}$. **Q.E.D.**

Proof of Proposition 3

When $B > \frac{23}{31}$, the outcome where both firms poach each other's previous customers is not feasible. Accordingly, we examine the three remaining outcomes that are possible: a)Firm 2 only poaches, b) Firm 1 only poaches and c) neither firm poaches. The analysis shows that a is infeasible and b is degenerate. Only c survives as a reasonable equilibrium.

Assume only Firm 2 poaches

The objective functions in the first period are $\pi_1 = \left(p_1^1 - c\right)x_L$ and $\pi_2 = \left(p_2^b - c\right)(1 - x_L)$ where $x_L = \frac{B + p_2^b - p_L + 1}{2}$. Substituting and differentiating, we obtain the first order conditions for the second period:

$$\frac{\partial \pi_1}{\partial p_1^1} = \frac{1}{2}\left(1 + B + p_2^b - 2p_1^1 + c\right) = 0$$

The solution is $p_2^b = c + 1 - \frac{1}{3}B$ and $p_1^1 = c + 1 + \frac{1}{3}B$ which implies that $x_L = \frac{1}{6}B_1 + \frac{1}{2}$. The indifferent consumer in the first period buys from Firm 2 in the second period independent of which firm she buys from in the first period. Therefore $CS_1 = 2v - p_1 - q - p_2^b - (1-q)$ equals $CS_2 = 2v - p_2 - 2(1-q) - p_2^b$. This implies that $q = \frac{1}{2}p_2 - \frac{1}{2}p_1 + \frac{1}{2}$. The first period objective functions are $\Pi_1 = (p_1 - c)\, q + \frac{1}{3}B_1 + \frac{1}{18}B_1^2 + \frac{1}{2}$ and $\Pi_2 = (p_2 - c)\,(1-q) + \frac{1}{18}B_1^2 - \frac{1}{3}B_1 + \frac{1}{2}$.

Substituting for q and differentiating, we obtain the first order conditions for Period 1.

$$\frac{\partial \Pi_1}{\partial p_1} = \frac{1}{2}c + \frac{1}{2}p_2 - p_1 + \frac{1}{2} = 0$$

$$\frac{\partial \Pi_2}{\partial p_2} = \frac{1}{2}c + \frac{1}{2}p_1 - p_2 + \frac{1}{2} = 0$$

The solution is $p_1 = 1 + c$ and $p_2 = 1 + c$ which implies that $q = \frac{1}{2}$ but $x_L = B_1 + \frac{1}{2} > \frac{1}{2}$ so an outcome where only Firm 2 poaches is inconsistent.

Assume only Firm 1 poaches

Then the marginal consumer who bought from Firm 1 in period 1 will be indifferent between Firm 1 and Firm 2. That is $B+v-q-p_1^1 = v-(1-q)-p_2^b$ which implies that $p_1^1 = B-2q+p_2^b+1$. Firm 1 and Firm 2 compete for the first period customers of Firm 2 which implies that $\pi_1^2 = \left(p_1^2 - c\right)(x_R - q)$ and $\pi_2^b = \left(p_2^b - c\right)(1 - x_R)$ where $x_R = \frac{p_2^b - p_1^2 + 1}{2}$. Substituting and differentiating, we obtain the following first order conditions:

$$\frac{\partial \pi_1^2}{\partial p_1^2} = \frac{1}{2}\left(c + 1 - 2q + p_2^b - 2p_1^2\right) = 0$$

$$\frac{\partial \pi_2^b}{\partial p_2^b} = \frac{1}{2}\left(c + 1 - 2p_2^b + p_1^2\right) = 0$$

The solution is $p_2^b = c + 1 - \frac{2}{3}q$ and $p_1^2 = c + 1 - \frac{4}{3}q$.

In the first period, we know that the marginal consumer between Firm 1 and Firm 2 must obtain equal surplus from both firms. That is, $CS_1 = v - p_1 - q + B + v - q - p_1^1$ (bought from Firm 1 in both periods) must equal $CS_2 = v - p_2 - (1-q) + v - q - p_1^2$ (bought from Firm 2 in period 1 but switches to Firm 1 in period 2). This implies that $q = \frac{1}{2}B - \frac{1}{2}p_1 + \frac{1}{2}p_2 - \frac{1}{2}p_1^1 + \frac{1}{2}p_1^2 + \frac{1}{2}$. Substituting, this implies that $q = \frac{3}{2}p_2 - \frac{3}{2}p_1$.The first period objective functions then are:

$$\pi_1 = p_2 - p_1 - \frac{3}{2}Bp_1 + \frac{3}{2}Bp_2 + \frac{19}{2}p_1p_2 - \frac{11}{2}(p_1)^2 - 4(p_2)^2 + \frac{1}{2} + \frac{3}{2}cp_1 - \frac{3}{2}cp_2$$

$$\pi_2 = p_1 + \frac{1}{2}p_1p_2 + \frac{1}{2}(p_1)^2 - (p_2)^2 + \frac{1}{2} - \frac{3}{2}cp_1 + \frac{3}{2}cp_2$$

The first order conditions imply a reaction function for Firm 1 of $p_1 = \frac{3}{22}c + \frac{19}{22}p_2 - \frac{3}{22}B - \frac{1}{11}$. For Firm 2, the reaction function is $p_2 = \frac{3}{4}c + \frac{1}{4}p_1$. These reaction functions intercept at $p_1 = c - \frac{4}{23}B - \frac{8}{69}$ and $p_2 = c - \frac{1}{23}B - \frac{2}{69}$ (prices that are less than marginal cost). In the relevant zone, the profits earned by the firms when only Firm 1 poaches ($\Pi_1 = \frac{66}{529}B + \frac{99}{1058}B^2 + \frac{573}{1058}$ and $\Pi_2 = \frac{9}{529}B^2 - \frac{80}{529}B + \frac{1243}{3174}$) are strictly less than the profits earned when neither firms poaches (the equilibrium presented below). Note that the *feasibility* conditions for this outcome ($x_R > q$) are satisfied as long as $B < \frac{19}{6}$.

Assume neither firm poaches

The equilibrium involves no poaching. The consumers who bought from each firm in the first period buy from them again. This implies two incentive compatibility conditions. In the first period: $B+2v-p_1-p_1^1-2q = 2v-p_2-p_2^b-2(1-q)$ and in the second period $B+v-q-p_1^1 = v-(1-q)-p_2^b$. The first period IC constraint implies that $q = \frac{1}{4}B + \frac{1}{4}p_2^b - \frac{1}{4}p_1 + \frac{1}{4}p_2 - \frac{1}{4}p_1^1 + \frac{1}{2}$. In the second period, Firm 2 must find p_2^b to be optimal and Firm 1 must have no demand (implying that $p_1^2 = c$. Therefore $\pi_2^b = \left(p_2^b - c\right)\left(1 - \frac{p_1^2-p_2^b+1}{2}\right) \Rightarrow \frac{\partial \pi_2^b}{\partial p_2^b} = p_2^b - \frac{1}{2}p_1^2 + \frac{1}{2} - \frac{c}{2} = 0$. When $p_1^2 = c$, $p_2^b = c + \frac{1}{2}$, the two IC constraints then imply that $p_1^1 = B + c + p_1 - p_2 + \frac{1}{2}$ and $q = \frac{1}{2}p_2 - \frac{1}{2}p_1 + \frac{1}{2}$.

The first period objective functions then become $\Pi_1 = \left(p_1 + p_1^1 - 2c\right)q$ and $\Pi_2 = (p_2 + p_2^b - 2c)(1-q)$. The first order conditions are:

$$\frac{\partial \Pi_1}{\partial p_1} = \frac{1}{2}c + \frac{3}{2}p_2 - 2p_1 - \frac{1}{2}B + \frac{3}{4} = 0$$
$$\frac{\partial \Pi_2}{\partial p_2} = \frac{1}{2}c + \frac{1}{2}p_1 - p_2 + \frac{1}{4} = 0$$

The solution is $p_1 = c + \frac{9}{10} - \frac{2}{5}B$ and $p_2 = c + \frac{7}{10} - \frac{1}{5}B$. Feasibility ($q < 1$) requires $B < 6$ which is trivially satisfed.[1] The profits that firms earn in this situation are $\Pi_1 = \frac{8}{25}B + \frac{1}{25}B^2 + \frac{16}{25}$ and $\Pi_2 = \frac{1}{50}B^2 - \frac{6}{25}B + \frac{18}{25}$. **Q.E.D.**

When is Poaching Impossible in the Second Period

As explained in Proposition 3 two possible outcomes exist in the subgame where only one firm implements BBD. However, in order for the outcome where Firm 1 (the firm that implements BBD) poaches past customers of Firm 2, its best response to Firm 2 must lead to an outcome that results in poaching. As shown in Proposition 3, the optimal price for Firm 1 if poaching occurs is $p_1 = \frac{3}{22}c + \frac{19}{22}p_2 - \frac{3}{22}B - \frac{1}{11}$ so the optimal price for Firm 1 given a price of $p_2 = c - \frac{1}{23}B - \frac{2}{69}$ by Firm 2 is $p_1 = c - \frac{4}{23}B - \frac{8}{69}$. However, if Firm 1 chooses the price such that poaching does not occur in the second period (a lower price consistent with the no poaching outcome of Proposition 3), i.e. $p_1 = \frac{3}{4}p_2 - \frac{1}{4}B + \frac{3}{8}$. Firm 1 will earn profits of $\Pi_1 = \frac{3751}{12\,696}B + \frac{121}{2116}B^2 + \frac{116\,281}{304\,704}$. When this is greater than $\Pi_1 = \frac{66}{529}B + \frac{99}{1058}B^2 + \frac{573}{1058}$ (the profits associated with poaching some of Firm 2's customers), the outcome where only Firm 1 poaches is not stable. This obtains when $B > \frac{197}{84} - \frac{69}{308}\sqrt{22}$ (the second root $\frac{69}{308}\sqrt{22} + \frac{197}{84}$ is outside the allowable zone for the poaching outcome). This implies that when only Firm 1 implements BBD and $B > \frac{197}{84} - \frac{69}{308}\sqrt{22}$, the subgame has a unique outcome where neither firms poaches.

[1] When $B > 6$, Firm 1 obtains the entire market.

Proof of Proposition 4

As shown in Proposition 1, when $B > 1$, an equilibrium where both firms implement BBD and poach is not feasible. When only one firms implements BBD, poaching does not occur when $B > \frac{197}{84} - \frac{69}{308}\sqrt{22}$. We focus our attention on the region where poaching is impossible independent of whether 1 or 2 firms implement it.

First, consider the case where both firms implement BBD. When neither firms poaches, the second period profits are $\pi_1 = \left(p_1^1 - c\right) q$ and $\pi_2 = \left(p_2^2 - c\right)(1-q)$. For no poaching to occur, we know that $p_1^2 = p_2^1 = c$ or either firm could reduce the price offered to new customers and poach the competitor's previous customers.

This implies that $p_1^1 = 1 - 2q + B$ and $p_2^2 = 2q - 1 + B$. Moving to the first period decision, the surplus from buying at Firm 1 is $CS_1 = B + 2v - 2tx - p_1 - p_1^1$ and from Firm 2 is $CS_2 = B + 2v - 2t(1-x) - p_2 - p_2^2$. Substituting the values for p_1^1 and p_2^2 at the indifferent consumer in the first period, we obtain $CS_1 = 2v - p_1 - 1$ and $CS_2 = 2v - p_2 - 1$. This implies that when competition is eliminated in the second period, the decision in the first period is based entirely on price (no matter which decision the consumer makes, he is fully compensated for his location by the price that is charged in the second period). Strictly speaking, any division of the market in the first period is consistent with these incentive compatbility constraints. To focus on a "reasonable division, one can assume that payoffs in the first period have a higher weight than the payoff in the second period ($\delta < 1$). As $\delta \to 1$, the outcomes are exactly as presented in the paper and $\delta < 1$ ensures that the split in the first period is symmetric.

The value of buying in period 1 for the indifferent consumer who buys from Firm 1 is $CS_1 = v - q - \delta - c\delta + v\delta + q\delta - p_1$ and from Firm 2 $CS_2 = v + q - c\delta + v\delta - q\delta - p_2 - 1$ and from Firm 2. Solving we obtain the following: $q = \frac{1}{2\delta - 2}(\delta + p_1 - p_2 - 1)$. The first period objective functions are $\Pi_1 = \left((p_1 - c) + \delta\left(p_1^1 - c\right)\right) q$ and $\Pi_2 = ((p_2 - c) + \delta\left(p_2^b - c\right))(1-q)$. Substituting and taking the first order conditions, we obtain

$$\frac{\partial \Pi_1}{\partial p_1} = \frac{1}{2\delta^2 - 4\delta + 2}\left(c - \delta - B\delta - c\delta - 2p_1 + p_2 + \delta p_2 + B\delta^2 + 1\right) = 0$$

$$\frac{\partial \Pi_2}{\partial p_2} = \frac{1}{2\delta^2 - 4\delta + 2}\left(c - \delta - B\delta - c\delta + p_1 - 2p_2 + \delta p_1 + B\delta^2 + 1\right) = 0$$

The solution to these equations is $p_1 = 1 + c - \delta B$ and $p_2 = 1 + c - \delta B$ in the first period. As $\delta \to 1$, the equilibrium prices in the first period approach $p_1 = 1 + c - B$ and $p_2 = 1 + c - B$. In this situation, $p_1^1 = p_2^2 = B + c$ and $p_1^2 = p_1^2 = c$ (in the second period) and firms earn profits of $\Pi_1 = \Pi_2 = \frac{1}{2}$. Note that the profits earned are independent of B. The feasibility of this outcome requires that $c > B - 1$.

The second possible outcome entails only one firm poaching. When only one firm poaches say Firm i:

1. Firm j must set $p_j^i = c$ in the second period or else it can reduce price and increase profits.

2. Firm i sets p_i^i such that the consumer at q (the indifferent consumer in the first period) is indifferent between Firm i and Firm j.$\Rightarrow p_i^i = B + 1 - 2q + c$

3. Optimization is needed to solve for the indifferent consumer in the segment of past customers of Firm j.

To simplify the exposition, assume that Firm 1 poaches (the analysis is silent on whether the poaching firm is 1 or 2). Then the second period objective functions are:

$$\pi_1 = \left(p_1^1 - c\right) q + \left(p_1^2 - c\right)(x_2 - q), \pi_2 = \left(p_2^2 - c\right)(1 - x_2)$$

where $p_1^1 = B + 1 - 2q + c$ and $x_2 = \frac{p_2^2 - p_1^2 + 1 - B}{2}$. Optimizing each firm's decision variable, we obtain $p_2^2 = \frac{1}{3}B - \frac{2}{3}q + 1 + c$ and $p_1^2 = 1 - \frac{1}{3}B - \frac{4}{3}q + c$. The consumer at q is indifferent from buying at Firm 1 twice versus buying from Firm 2 in period 1 and then Firm 1 in period 2. This implies that $q = \frac{1}{2}B - \frac{1}{2}p_1 + \frac{1}{2}p_2 - \frac{1}{2}p_1^1 + \frac{1}{2}p_1^2 + \frac{1}{2}$. Substituting the second period prices into this function, we express the first period indifferent consumer in terms of first period prices and B i.e., $q = \frac{3}{4}p_2 - \frac{3}{4}p_1 - \frac{1}{4}B + \frac{3}{4}$. The first period objective functions for each firm are:

$$\Pi_1 = \left(p_1 + p_1^1 - 2c\right) q + \left(p_1^2 - c\right)(x_2 - q), \Pi_2 = (p_2 - c)(1 - q) + \left(p_2^2 - c\right)(1 - x_2)$$

Substitute into the objective functions to obtain simplified profit functions for each firm in terms of p_1 and p_2.

$$\begin{aligned} \Pi_1 &= \frac{5}{4}B - \frac{3}{4}c + \frac{1}{4}Bc + \frac{9}{4}p_1 - \frac{3}{2}p_2 - \frac{7}{4}Bp_1 + \frac{3}{2}Bp_2 \\ &\quad + \frac{3}{4}cp_1 - \frac{3}{4}cp_2 + 2p_1p_2 - \frac{3}{8}B^2 - \frac{11}{8}(p_1)^2 - \frac{5}{8}(p_2)^2 - \frac{3}{8} \end{aligned}$$

$$\begin{aligned} \Pi_2 &= \frac{1}{4}B - \frac{1}{4}c - \frac{1}{4}Bc + \frac{1}{4}p_1 + \frac{1}{4}Bp_1 - \frac{3}{4}cp_1 \\ &\quad + \frac{3}{4}cp_2 + \frac{1}{2}p_1p_2 + \frac{1}{8}B^2 + \frac{1}{8}p_1^2 - \frac{5}{8}p_2^2 + \frac{1}{8} \end{aligned}$$

Optimizing these expressions with respect to p_1 and p_2, we obtain the solution for the first period:

$$p_1 = c + \frac{15}{13} - \frac{35}{39}B,\ p_2 = c + \frac{6}{13} - \frac{14}{39}B\ , q = \frac{2}{13}B + \frac{3}{13}$$

and the second period:

$$p_1^1 = c + \frac{9}{13}B + \frac{7}{13},\ p_2^2 = c + \frac{3}{13}B + \frac{11}{13},\ p_1^2 = c + \frac{9}{13} - \frac{7}{13}B,$$
$$p_2^1 = c \text{ and } x_2 = \frac{15}{26} - \frac{3}{26}B$$

The profits of Firms 1 and 2 under this regime are:

$$\Pi_1 = \frac{115}{1014}B^2 - \frac{27}{169}B + \frac{213}{338} \text{ and } \Pi_2 = \frac{83}{1014}B^2 - \frac{77}{507}B + \frac{241}{338}$$

Feasibility requires $x_2 > q$. Substituting, we obtain $\frac{15}{26} - \frac{3}{26}B > \frac{2}{13}B + \frac{3}{13} \Rightarrow B < \frac{9}{7}$ i.e. $B \in \left(1, \frac{9}{7}\right)$ for this regime to be feasible. Because our focus is on the region where neither firms poaches in the sub-game, $B > \frac{197}{84} - \frac{69}{308}\sqrt{22} > \frac{9}{7}$. This implies that the only feasible outcome in the sub-game where both firms implement BBD is for neither firm to poach. **Q.E.D**.

Proof of Proposition 5

In the proof of Proposition 3, we show that the solution is $p_1 = c + \frac{9}{10} - \frac{2}{5}B$ and $p_2 = c + \frac{7}{10} - \frac{1}{5}B$ when only Firm 1 implements BBD (this solution applies as long as $B > 6$). The profits that firms earn in this situation are $\Pi_1 = \frac{8}{25}B + \frac{1}{25}B^2 + \frac{16}{25}$ and $\Pi_2 = \frac{1}{50}B^2 - \frac{6}{25}B + \frac{18}{25}$. When $B > 6$ and only Firm 1 implements BBD, the optimal first period prices are $p_1 = c - \frac{27}{20}$ and $p_2 = c - \frac{3}{10}$ leaving Firm 1 with the entire market. Thus, Firm 2 no longer has an incentive to set $p_2^b = c + \frac{1}{2}$ (it has no past consumers) so $p_2^b = c$ (the lowest price that Firm 2 can set in order to capture past consumers of Firm 1). This implies that $p_1^1 = B + c - 1$. The resulting profits are $\Pi_1 = B - \frac{47}{20}$ and $\Pi_2 = 0$. Firm 1 captures the entire market in the first period. Because of the benefits Firm 1 offers in the second period, Firm 2 cannot obtain any demand in the second period. **Q.E.D.**

Proof of Proposition 6

When $B \in \left(0, \frac{23}{31}\right)$, the profit expressions are as follows.

$$\Pi_{bbd} = \frac{1}{9}B^2 - \frac{2}{9}B + \frac{17}{18}, \Pi_{no\ bbd} = 1$$
$$\Pi_a = \frac{71}{276}B + \frac{777}{8464}B^2 + \frac{31}{48}, \Pi_d = \frac{63}{2116}B^2 - \frac{37}{138}B + \frac{17}{24}$$

For the top left and lower right square of Figure 1 to be equilibria, we need $\Pi_{no\ bbd} > \Pi_a$ and $\Pi_{bbd} > \Pi_d$.

1. Show that $\Pi_{no\ bbd} > \Pi_a$. If the reverse is true then $\frac{71}{276}B + \frac{777}{8464}B^2 - \frac{17}{48} > 0 \Rightarrow B > \frac{23}{2331}\sqrt{59\,791} - \frac{3266}{2331} \approx 1.0116$ or $B < -\frac{23}{2331}\sqrt{59\,791} - \frac{3266}{2331} \approx -3.8138$ which implies a B outside the allowable range.

2. Show that $\Pi_{bbd} > \Pi_d$. If the reverse is true then $-\frac{19}{414}B - \frac{1549}{19\,044}B^2 - \frac{17}{72} > 0$ which is impossible ($-\frac{19}{414}B - \frac{1549}{19\,044}B^2 - \frac{17}{72}$ is a downward facing parabola with a maximum of $-\frac{8537}{37176} \approx -0.23$).

When $B \in \left(\frac{23}{31}, 1\right)$, the profit expressions are as follows.

$$\Pi_{bbd} = \frac{1}{9}B^2 - \frac{2}{9}B + \frac{17}{18}, \Pi_{no\ bbd} = 1$$
$$\Pi_a = \frac{8}{25}B + \frac{1}{25}B^2 + \frac{16}{25}, \Pi_d = \frac{1}{50}B^2 - \frac{6}{25}B + \frac{18}{25} \text{ or}$$
$$\Pi_a = \frac{66}{529}B + \frac{99}{1058}B^2 + \frac{573}{1058}, \Pi_d = \frac{9}{529}B^2 - \frac{80}{529}B + \frac{1243}{3174}$$

Recall, Proposition 3 shows there are two stable outcomes in the sub-game where only one firm implements BBD. For the top left and lower right square to be equilibria, we need $\Pi_{no\ bbd} > \Pi_a$ and $\Pi_{bbd} > \Pi_d$ (for both potential asymmetric outcomes).

1. Show that $\Pi_{no\ bbd} > \Pi_a$. If the reverse is true then $\frac{8}{25}B + \frac{1}{25}B^2 - \frac{9}{25} > 0 \Rightarrow B > 1$ or $B < -9$ which implies a B outside the allowable range. Similarly $\frac{66}{529}B + \frac{99}{1058}B^2 + \frac{573}{1058} > 0 \Rightarrow B > \frac{23}{33}\sqrt{11} - \frac{2}{3} \approx 1.6449$ or $B < -\frac{23}{33}\sqrt{11} - \frac{2}{3} \approx -2.9783$ which implies a B outside the allowable range.

2. Show that $\Pi_{bbd} > \Pi_d$. If the reverse is true then $-\frac{4}{225}B - \frac{41}{450}B^2 - \frac{101}{450} > 0$ which is impossible: $-\frac{4}{225}B - \frac{41}{450}B^2 - \frac{101}{450}$ is a downward facing parabola with a maximum of $-\frac{55}{246} \approx -0.22$. Similarly $\frac{338}{4761}B - \frac{448}{4761}B^2 - \frac{2632}{4761}$ is a downward facing parabola with a maximum of $-\frac{725}{1344} \approx -0.54$.

Q.E.D.

Proof of Corollary 1

When $B \in (0, 1)$, show that $\Pi_{no\ bbd} > \Pi_{bbd}$. If the reverse is true then $\frac{1}{9}B^2 - \frac{2}{9}B + \frac{17}{18} - 1 > 0 \Rightarrow B > \frac{1}{2}\sqrt{6} + 1 \approx 2.2247$ or $B < 1 - \frac{1}{2}\sqrt{6} \approx -0.22474$ which implies a B outside the allowable range. **Q.E.D.**

Proof of Proposition 7

When $B \in \left(\frac{197}{84} - \frac{69}{308}\sqrt{22} \approx 1.2945, 6\right)$, the profits of Firms 1 and 2 are:

$$\Pi_{bbd} = \frac{1}{2}, \Pi_{no\ bbd} = 1, \Pi_a = \frac{8}{25}B + \frac{1}{25}B^2 + \frac{16}{25}, \Pi_d = \frac{1}{50}B^2 - \frac{6}{25}B + \frac{18}{25}$$

For the lower right square of Figure 1 to be an equilibrium we need $\Pi_a > \Pi_{no\ bbd}$ and $\Pi_{bbd} > \Pi_d$.

1. Show that $\Pi_a > \Pi_{no\ bbd}$. If the reverse is true then $-\frac{8}{25}B - \frac{1}{25}B^2 - \frac{7}{50} > 0$. This implies that $B \in \left(-\frac{5}{2}\sqrt{2} - 4, \frac{5}{2}\sqrt{2} - 4\right) \approx (-7.54, -0.464)$ which implies a B outside the allowable range.

2. Show that $\Pi_{bbd} > \Pi_d$. If the reverse is true then $\frac{1}{50}B^2 - \frac{6}{25}B + \frac{18}{25} > \frac{1}{2}$. This implies that $B \in (1, 11)$.

Therefore when $B \in \left(\frac{197}{84} - \frac{69}{308}\sqrt{22}, 16\right)$, the unique equilibrium is (BBD, BBD). Conversely, when $B > 6$, the profits of Firms 1 and 2 are:

$$\Pi_{bbd} = \frac{1}{2}, \Pi_{no\ bbd} = 1, \Pi_a = B - \frac{19}{10}, \Pi_d = 0$$

As before, it is straightforward to show thet the lower right square of Figure 1 is the equilibrium because $\Pi_a > \Pi_{no\ bbd}$ and $\Pi_{bbd} > \Pi_d$. **Q.E.D.**

Proof of Corollary 2

When $B > \frac{197}{84} - \frac{69}{308}\sqrt{22}$, the equilibrium is the lower right square as shown in Proposition 7. This is a Prisoners' Dilemma because $\Pi_{no\ bbd} = 1 > \Pi_{bbd} = \frac{1}{2}$ throughout the range. **Q.E.D.**

Proof of Lemma 1

An equilibrium where both firms poach is not feasible when $B > 1$. This implies that there are two possible equilibria. The first is one is which neither firm poaches. The second is one in which only one of the two firms poaches.

When neither firms poaches, the second period profits are $\pi_1 = p_1^1 q$ and $\pi_2 = p_2^2(1-q)$. For no poaching to occur, p_1^2 and p_2^1 equal 0 or either firm could reduce the price offered to new customers and poach the competitor's previous customers.

Similar reasoning to the case of positive marginal cost implies that the indifferent consumer in the first period obtains $CS_1 = 2v - p_1 - 1$ by choosing Firm 1 and $CS_2 = 2v - p_2 - 1$ by choosing Firm 2. This implies that when competition is eliminated in the second period, the decision in the first period is based entirely on price (no matter which decision the consumer makes, he is compensated for his location by the price that is charged in the second period).[2] This implies that a) prices in the first period are negative or b) price in the first period are zero. Because we restrict our attention to positive prices, $p_1 = p_2 = 0$ in the first period and $q = \frac{1}{2}$. This outcome then results in profits of $\Pi = \frac{B}{2}$ for each firm.

The second possible equilibrium entails only one firm poaching. When only one firm poaches say Firm i:

1. Firm j must set $p_j^i = 0$ in the second period or else it can reduce price and increase profits.

2. Firm i sets p_i^i such that the consumer at q (the indifferent consumer in the first period) is indifferent between Firm i and Firm j.$\Rightarrow p_i^i = B + 1 - 2q$

3. Optimization is needed to solve for the indifferent consumer in the segment of past customers of Firm j.

To simplify the exposition, assume that Firm 1 poaches (the analysis is silent on whether the poaching firm is 1 or 2). Then the second period objective functions are:

$$\pi_1 = p_1^1 q + p_1^2\,(x_2 - q)\,, \pi_2 = p_2^2(1 - x_2)$$

where $p_1^1 = B + 1 - 2q$ and $x_2 = \frac{p_2^2 - p_1^2 + 1 - B}{2}$. Optimizing each firm's decision variable, we obtain $p_2^2 = \frac{1}{3}B - \frac{2}{3}q + 1$, and $p_1^2 = 1 - \frac{1}{3}B - \frac{4}{3}q$. The consumer at q is indifferent from buying at Firm 1 twice versus buying from Firm 2 in period 1 and then Firm 1 in period 2. This implies that $q = \frac{1}{2}B - \frac{1}{2}p_1 + \frac{1}{2}p_2 - \frac{1}{2}p_1^1 + \frac{1}{2}p_1^2 + \frac{1}{2}$. Substituting the second period prices into this function, we express the first period indifferent consumer in terms of first period prices and B i.e., $q = \frac{3}{4}p_2 - \frac{3}{4}p_1 - \frac{1}{4}B + \frac{3}{4}$. The first period objective functions for each firm are:

$$\Pi_1 = \left(p_1 + p_1^1\right) q + p_1^2(x_2 - q), \Pi_2 = p_2(1-q) + p_2^2(1 - x_2)$$

[2] Any division of the market in the first period is consistent with these incentive compatbility constraints. Similar to the case of positive marginal costs, we examine a reasonable division by assuming that payoffs in the first period have a higher weight than the payoff in the second period ($\delta < 1$). As $\delta \to 1$, the outcomes are as presented and $\delta < 1$ ensures that the split in the first period is symmetric.

Substitute into the objective functions to obtain simplified profit functions for each firm in terms of p_1 and p_2.

$$\Pi_1 = \frac{5}{4}B + \frac{9}{4}p_1 - \frac{3}{2}p_2 - \frac{7}{4}Bp_1 + \frac{3}{2}Bp_2 + 2p_1p_2 - \frac{3}{8}B^2 - \frac{11}{8}(p_1)^2 - \frac{5}{8}(p_2)^2 - \frac{3}{8}$$

$$\Pi_2 = \frac{1}{4}B + \frac{1}{4}p_1 + \frac{1}{4}Bp_1 + \frac{1}{2}p_1p_2 + \frac{1}{8}B^2 + \frac{1}{8}(p_1)^2 - \frac{5}{8}(p_2)^2 + \frac{1}{8}$$

Optimizing these expressions with respect to p_1 and p_2, we obtain the solution for the first period:

$$p_1 = \frac{15}{13} - \frac{35}{39}B,\ p_2 = \frac{6}{13} - \frac{14}{39}B\ , q = \frac{2}{13}B + \frac{3}{13}$$

and the second period:

$$p_1^1 = \frac{9}{13}B + \frac{7}{13},\ p_2^2 = \frac{3}{13}B + \frac{11}{13},\ p_1^2 = \frac{9}{13} - \frac{7}{13}B,\ p_2^1 = 0 \text{ and } x_2 = \frac{15}{26} - \frac{3}{26}B$$

The profits of Firms 1 and 2 under this regime are:

$$\Pi_1 = \frac{115}{1014}B^2 - \frac{27}{169}B + \frac{213}{338} \text{ and } \Pi_2 = \frac{83}{1014}B^2 - \frac{77}{507}B + \frac{241}{338}$$

Feasibility requires $x_2 > q$. Substituting, we obtain $\frac{15}{26} - \frac{3}{26}B > \frac{2}{13}B + \frac{3}{13} \Rightarrow B < \frac{9}{7}$ i.e. $B \in \left(1, \frac{9}{7}\right)$ for this regime to be feasible. The best response functions in the first period of the game for Firms 1 and 2 respectively are $p_1 = \frac{8}{11}p_2 - \frac{7}{11}B + \frac{9}{11}$ and $p_2 = \frac{2}{5}p_1$. Using the best response function for Firm 2, it is clear that Firm 1 can implement the equilibrium where neither firm poaches if its profits are less than $\frac{B}{2}$ (the amount it earns when neither firms poaches). Note that Firm 2's profits when Firm 1 poaches are higher than the no poaching profits, $\frac{B}{2}$, throughout the region $B \in \left(1, \frac{9}{7}\right)$ because $\frac{83}{1014}B^2 - \frac{77}{507}B + \frac{241}{338} > \frac{B}{2}$ is satisfied for all $B < \frac{661}{166} - \frac{13}{166}\sqrt{1165} \approx 1.3089 > \frac{9}{7}$.[3] We now compare the profits of Firm 1 when it poaches to a situation where neither firm poaches. For Firm 1 to prefer poaching, $\frac{115}{1014}B^2 - \frac{27}{169}B + \frac{213}{338} > \frac{B}{2} \Rightarrow B < \frac{669}{230} - \frac{39}{230}\sqrt{101} \approx 1.2046 < \frac{9}{7}$.[4] Therefore even though both outcomes are feasible when $B \in \left(1, \frac{669}{230} - \frac{39}{230}\sqrt{101}\right)$, the unique outcome is for one firm to poach and the other not. In contrast, when $B > \frac{669}{230} - \frac{39}{230}\sqrt{101}$, firms do not poach. As a result, there are two pairs of equilibrium profits when $B \in \left(1, \frac{669}{230} - \frac{39}{230}\sqrt{101}\right)$ and firms earn $\frac{B}{2}$ when $B > \frac{669}{230} - \frac{39}{230}\sqrt{101}$. **Q.E.D.**

Proof of Lemma 2

Similar reasoning to the case of positive marginal costs, the equilibrium is $p_1 = \frac{9}{10} - \frac{2}{5}B$ and $p_2 = \frac{7}{10} - \frac{1}{5}B$. However, this solution is valid if an only if $p_2 > 0 \Rightarrow B < \frac{9}{4}$. Therefore $p_1 = \frac{9}{10} - \frac{2}{5}B$ and $p_2 = \frac{7}{10} - \frac{1}{5}B$ when $B < \frac{9}{4}$ and $p_1 = 0$ and $p_2 = \frac{1}{4}$ when $B > \frac{9}{4}$. The equilibrium values imply that $\Pi_1 = \frac{8}{25}B + \frac{1}{25}B^2 + \frac{16}{25}$ and $\Pi_2 = \frac{1}{50}B^2 - \frac{6}{25}B + \frac{18}{25}$ when $B \in \left(\frac{669}{230} - \frac{39}{230}\sqrt{101}, \frac{9}{4}\right)$ and $\Pi_1 = \frac{5}{8}B + \frac{5}{32}$ and $\Pi_2 = \frac{9}{32}$ when $B > \frac{9}{4}$. **Q.E.D.**

[3] The second root is $B = \frac{13\sqrt{1165}}{166} + \frac{661}{166} \approx 6.6549$ but it is not relevent since it lies outside the feasible zone.

[4] The second root is $B = \frac{39\sqrt{101}+669}{230} \approx 4.6128$ but it is not relevent since it lies outside the feasible zone.

Proof of Proposition 8

When $B \in \left(\frac{669}{230} - \frac{39}{230}\sqrt{101}, \frac{9}{4}\right)$, the profit expressions are as follows.

$$\Pi_{bbd} = \frac{B}{2}, \Pi_{no\ bbd} = 1$$
$$\Pi_a = \frac{8}{25}B + \frac{1}{25}B^2 + \frac{16}{25}, \Pi_d = \frac{1}{50}B^2 - \frac{6}{25}B + \frac{18}{25}$$

Also note that in the zone $B \in \left(1, \frac{669}{230} - \frac{39}{230}\sqrt{101}\right)$, Π_{bbd1} and $\Pi_{bbd2} > \frac{B}{2}$. For the lower right square of Figure 1 to be an equilibrium we need $\Pi_a > \Pi_{no\ bbd}$ and $\Pi_{bbd} > \Pi_d$.

1. Show that $\Pi_a > \Pi_{no\ bbd}$. If the reverse is true then $\frac{9}{25} - \frac{1}{25}B^2 - \frac{8}{25}B > 0 \Rightarrow B \in (-9, 1)$ which implies a B outside the allowable range.

2. Show that $\Pi_{bbd} > \Pi_d$. First if $\Pi_d > \frac{B}{2}$ then $\frac{1}{50}B^2 - \frac{37}{50}B + \frac{18}{25} > 0 \Rightarrow B < 1$ or $B > 36$ which implies a B outside the allowable range. Because Π_{bbd1} and $\Pi_{bbd2} > \frac{B}{2}$, this implies that $\Pi_{bbd} > \Pi_d$ in the combined zone of $B \in \left(1, \frac{9}{4}\right)$.

When $B > \frac{9}{4}$, the profit expressions are as follows.

$$\Pi_{bbd} = \frac{B}{2}, \Pi_{no\ bbd} = 1$$
$$\Pi_a = \frac{5}{8}B + \frac{5}{32}, \Pi_d = \frac{9}{32}$$

For the lower right square of Figure 1 to be an equilibrium we need $\Pi_a > \Pi_{no\ bbd}$ and $\Pi_{bbd} > \Pi_d$.

1. Show that $\Pi_a > \Pi_{no\ bbd}$. If the reverse is true then $\frac{27}{32} - \frac{5}{8}B > 0 \Rightarrow B < \frac{27}{20}$ which implies a B outside the allowable range.

2. Show that $\Pi_{bbd} > \Pi_d$. If the reverse is true then $\frac{9}{32} - \frac{B}{2} > 0 \Rightarrow B < \frac{9}{16}$ which implies a B outside the allowable range.

Q.E.D.

Proof of Corollary 3

When $B \in (\frac{669}{230} - \frac{39}{230}\sqrt{101}, 2)$, the equilibrium is the lower right square of Figure 1. If $\Pi_{no\ bbd} > \Pi_{bbd}$ then firm profits are reduced by BBD. Using the profit expressions, if the reverse is true then $\frac{B}{2} - 1 > 0 \Rightarrow B > 2$ which implies a B outside the allowable range. When $B > 2$, the equilibrium is the lower right square as shown in Proposition 8. To show that profits increase because behavior-based discrimination, we need to show that $\Pi_{bbd} > \Pi_{no\ bbd}$. Assume the reverse, then $1 - \frac{B}{2} > 0 \Rightarrow B < 2$ which implies a B outside the allowable range. **Q.E.D.**

Proof of Proposition 9

Assume that both firms implement BBD and that the optimal level of benefit is less than 1 (poaching occurs). The profit function for each firm in terms of its benefit choice and the benefit choice of the competitor is given by substituting p_1 and p_2 as shown in the proof of Proposition 1 into the objective functions.

$$\Pi_1 = \frac{23}{144}B_1 - \frac{55}{144}B_2\text{-}\frac{7}{2304}B_1B_2 + \frac{263}{4608}B_1^2 + \frac{263}{4608}B_2^2 + \frac{17}{18}$$

$$\Pi_2 = \frac{23}{144}B_2 - \frac{55}{144}B_1 - \frac{7}{2304}B_1B_2 + \frac{263}{4608}B_1^2 + \frac{263}{4608}B_2^2 + \frac{17}{18}$$

Working backwards, firms will set B_1 and B_2 optimally and this implies that $\frac{\partial\Pi_i}{\partial B_i} = \frac{263}{2304}B_i - \frac{7}{2304}B_j - 2\alpha B_i + \frac{23}{144} = 0 \Rightarrow B_i = -\frac{23}{144\left(-2\alpha+\frac{1}{9}\right)}$. Substituting, this solution back into the objective functions, we obtain $\Pi_i = \frac{529}{2304(18\alpha-1)^2} - \frac{23}{72(18\alpha-1)} + \frac{17}{18}$. This solution applies when $\alpha > \frac{13}{96}$.

This is a Nash equilibrium as long as $\Pi_{bbd} > \Pi_d$. Following Propositions 2 and 3, $\Pi_d = \frac{63}{2116}B_1^2 - \frac{37}{138}B_1 + \frac{17}{24}$ when $B < \frac{23}{31}$ and $\Pi_d = \frac{1}{50}B_1^2 - \frac{6}{25}B_1 + \frac{18}{25}$when $B \in \left(\frac{23}{31}, 1\right)$.

1. When $B < \frac{23}{31}$, $\Pi_d = \frac{63}{331\,776\alpha^2-36\,864\alpha+1024} - \frac{37}{1728\alpha-96} + \frac{17}{24}$. For BBD to be an equilibrium, $\Pi_{bbd} - \Pi_d > 0$. Assume not. Then $\frac{235\,008\alpha^2-22\,464\alpha+1039}{3072(18\alpha-1)^2} < 0$. The denominator is positive. Therefore, $235008\left(\alpha - \frac{13}{272}\right)^2 + \frac{8537}{17} < 0$. But $235008\left(\alpha - \frac{13}{272}\right)^2 + \frac{8537}{17}$ is an upward facing parabola with its minimum at $\frac{8537}{17} \approx 502.18$. Hence $\Pi_{bbd} - \Pi_d > 0$.

2. When $B \in \left(\frac{23}{31}, 1\right)$, $\Pi_d = \frac{529}{4147\,200\alpha^2-460\,800\alpha+12\,800} - \frac{69}{3600\alpha-200} + \frac{18}{25}$. For BBD to be an equilibrium, $\Pi_{bbd} - \Pi_d > 0$. Assume not. Then $\frac{2792\,448\alpha^2-292\,608\alpha+14\,867}{38\,400(18\alpha-1)^2} < 0$. The denominator is positive. Therefore, $2792\,448(\alpha - \frac{127}{2424})^2 + \frac{727\,375}{101} < 0$. But $2792448(\alpha - \frac{127}{2424})^2 + \frac{727\,375}{101}$ is an upward facing parabola with its minimum at $\frac{727\,375}{101}$. Hence $\Pi_{bbd} - \Pi_d > 0$.

The proof to show that no BBD is an equilibrium is analogous to Proposition 6 (Π_a being strictly less compared to the case when benefits are costless).

To simplify our exposition, we ignore the region where partial poaching is possible and focus on the region where poaching does not occur $B > \frac{197}{84} - \frac{69}{308}\sqrt{22}$.

When only one firm implements BBD (say Firm 1), we use the expressions relative to one firm implementing BBD when $B > \frac{23}{31}$ from Proposition 3. The profits of Firm 1 (without accounting for the investment to create the benefit) and Firm 2 are $\frac{8}{25}B_1 + \frac{1}{25}B_1^2 + \frac{16}{25}$ and $\frac{1}{50}B_1^2 - \frac{6}{25}B_1 + \frac{18}{25}$ respectively. Note that $B < 6$ for these expressions to be feasible.

Rewriting and accounting for the cost of creating the benefit we have $\Pi_1 = \frac{8}{25}B_1 + \frac{1}{25}B_1^2 + \frac{16}{25} - \alpha B_1^2$. This implies that the optimal B_1 is $-\frac{8}{25\left(-2\alpha+\frac{2}{25}\right)}$. This is valid as long as $\alpha > \frac{1}{15}$ since $\alpha < \frac{1}{15}$ implies that $B > 6$. When $\alpha > \frac{1}{15}$, the relevant profit function for Firm 1 is then:

$$\Pi_1 = \frac{32}{625\alpha - 25} - 16\frac{\alpha}{625\alpha^2 - 50\alpha + 1} + \frac{16}{15\,625\alpha^2 - 1250\alpha + 25} + \frac{16}{25}$$

This function is declining in α and at $\alpha = \frac{1}{9}$, $\Pi_1 = 1$. This implies that in the second zone, a firm will unilaterally implement BBD whenever $\alpha \in \left(\frac{1}{15}, \frac{1}{9}\right)$.

When $\alpha < \frac{1}{15}$, the relevant profit functions are $\Pi_1 = B - \frac{19}{10}$ and $\Pi_2 = 0$ respectively. The relevant profit function for Firm 1 is then:

$$\Pi_1 = B - \frac{19}{10} - \alpha B^2$$

This implies an optimal $B = \frac{1}{2\alpha}$ and profits of $\frac{1}{4\alpha} - \frac{19}{10}$ for Firm 1 (Firm 2 in this situation earns 0).

When both firms implement BBD and benefits are high enough to eliminate poaching $B > \frac{197}{84} - \frac{69}{308}\sqrt{22}$, the profit function for each firm is $\Pi_i = \frac{1}{2} - \alpha B_i^2$. This function is declining in B_i so firms will choose the level B_i that ensures no poaching but maximises profit i.e. $B_i = \frac{197}{84} - \frac{69}{308}\sqrt{22}$ (if a firm chooses $B < \frac{197}{84} - \frac{69}{308}\sqrt{22}$, the competitor may poach the firm's past consumers leading to strictly lower profits).[5] Therefore the profits that firms earn when $\alpha < \frac{1}{9}$ are $\Pi_i = \alpha\left(\frac{4531}{4312}\sqrt{22} - \frac{512\,597}{77\,616}\right) + \frac{1}{2}$.

When $\alpha \in \left(\frac{1}{15}, \frac{1}{9}\right)$, we now determine the equilibrium. First, we assume that a pure strategy equilibrium exists. Second, if a pure strategy equilibrium exists, then there are only three possible benefit levels that a firm will consider. The first is to choose $B = 0$ (i.e. do not implement BBD). The second is to set the benefit level as if the firm faces a competitor that does not engage in BBD. The final level is to set the benefit level that would be chosen given that both firms decide to implement BBD. The meta-game can then be thought of as a 3 by 3, based on firms investing to achieve the three benefit levels described above.

		Firm 2		
		$B = 0$	$B = -\frac{8}{25\left(-2\alpha + \frac{2}{25}\right)}$	$B = \frac{197}{84} - \frac{69}{308}\sqrt{22}$
Firm 1	$B = 0$	$1, 1$	Π_0, Π_1	Π_3, Π_4
	$B = -\frac{8}{25\left(-2\alpha + \frac{2}{25}\right)}$	Π_1, Π_0	Π_2, Π_2	Π_6, Π_7
	$B = \frac{197}{84} - \frac{69}{308}\sqrt{22}$	Π_4, Π_3	Π_7, Π_6	Π_5, Π_5

Straightforward substitution allows us to write the profit expressions for each Π_i in the following table.

[5] This observation is based on the existence of a partial poaching equilibrium when $B < \frac{197}{84} - \frac{69}{308}\sqrt{22}$. The reduction in profits occurs in this equilibrium when a firm's past customers are poached.

The Variable	Value
Π_0	$\frac{8}{15\,625\alpha^2-1250\alpha+25} - \frac{24}{625\alpha-25} + \frac{18}{25}$
Π_1	$\frac{32}{625\alpha-25} - 16\frac{\alpha}{625\alpha^2-50\alpha+1} + \frac{16}{15\,625\alpha^2-1250\alpha+25} + \frac{16}{25}$
Π_2	$\frac{1}{2} - 16\frac{\alpha}{625\alpha^2-50\alpha+1}$
Π_3	$\frac{7061}{215\,600}\sqrt{22} + \frac{1122\,437}{3880\,800} \approx 0.44284$
Π_4	$\alpha\left(\frac{4531}{4312}\sqrt{22} - \frac{512\,597}{77\,616}\right) - \frac{12\,259}{107\,800}\sqrt{22} + \frac{3210\,677}{1940\,400}$
Π_5	$\alpha\left(\frac{4531}{4312}\sqrt{22} - \frac{512\,597}{77\,616}\right) + \frac{1}{2}$
Π_6	$\frac{1}{2} - 16\frac{\alpha}{625\alpha^2-50\alpha+1}$
Π_7	$\alpha\left(\frac{4531}{4312}\sqrt{22} - \frac{512\,597}{77\,616}\right) + \frac{1}{2}$

Point 1: when a firm invests in BBD, the best response is always to choose $B = \frac{197}{84} - \frac{69}{308}\sqrt{22}$ because this guarantees the highest profit in the sub-games where the competitor also chooses to implement BBD.

Point 2: when the competitor does not invest in BBD, the best response is always to choose, $B_1 = -\frac{8}{25\left(-2\alpha+\frac{2}{25}\right)}$. The question is: can "not investing" be the best response to a firm that does invest. (Is there a parameter range where $\Pi_0 > \Pi_5$). In other words, what is the parameter range where $\frac{8}{15\,625\alpha^2-1250\alpha+25} - \frac{24}{625\alpha-25} + \frac{18}{25} > \alpha\left(\frac{4531}{4312}\sqrt{22} - \frac{512\,597}{77\,616}\right) + \frac{1}{2}$. This reduces to a cubic equation in α that has three roots. The equation can be solved explicitly but the numerical solution is more informative. There are three roots $\alpha \approx 5.521\,8 \times 10^{-2}, \alpha \approx -0.22289$ and $\alpha \approx 0.116\,38$. None of roots lie in the allowable range $\alpha \in \left(\frac{1}{15}, \frac{1}{9}\right)$. This implies that the unique outcome is for both firms to implement BBD and choose $B = \frac{197}{84} - \frac{69}{308}\sqrt{22}$ when $\alpha \in \left(\frac{1}{15}, \frac{1}{9}\right)$.

Point 3: When $\alpha < \frac{1}{15}$, similar reasoning shows that the unqiue equilibrium is for both firms to implement BBD and $B = \frac{197}{84} - \frac{69}{308}\sqrt{22}$. Interestingly, firms earn strictly less than they would in the absence of BBD. Recall that firms earn $\alpha\left(\frac{4531}{4312}\sqrt{22} - \frac{512\,597}{77\,616}\right) + \frac{1}{2}$ when both firms implement BBD $\alpha\left(\frac{4531}{4312}\sqrt{22} - \frac{512\,597}{77\,616}\right) + \frac{1}{2}$ but this is strictly less than $\frac{1}{2}$ because $\frac{512\,597}{77\,616} > \frac{4531}{4312}\sqrt{22}$.

Q.E.D.

Proof of Proposition 10

Assume $B_1 > B_2$. From Proposition 1, the objective functions for the two firms when poaching occurs and the benefits are asymmetric are:

$$\Pi_1 = \frac{2}{9}B_1 - \frac{1}{9}B_2 + \frac{1}{2}p_1 - \frac{1}{144}B_1B_2 - \frac{1}{16}B_1p_1 + \frac{3}{16}B_1p_2 - \frac{3}{16}B_2p_1$$
$$+ \frac{1}{16}B_2p_2 + \frac{1}{16}p_1p_2 + \frac{29}{288}B_1^2 + \frac{5}{288}B_2^2 - \frac{7}{32}p_1^2 + \frac{5}{32}p_2^2$$
$$+ c\left(\frac{1}{8}B_2 - \frac{1}{8}B_1 + \frac{3}{8}p_1 - \frac{3}{8}p_2 - \frac{1}{2}\right) + \frac{5}{18}$$

$$\Pi_2 = \frac{2}{9}B_2 - \frac{1}{9}B_1 + \frac{1}{2}p_2 - \frac{1}{144}B_1B_2 + \frac{1}{16}B_1p_1 - \frac{3}{16}B_1p_2 + \frac{3}{16}B_2p_1$$
$$- \frac{1}{16}B_2p_2 + \frac{1}{16}p_1p_2 + \frac{5}{288}B_1^2 + \frac{29}{288}B_2^2 + \frac{5}{32}p_1^2 - \frac{7}{32}p_2^2$$
$$+ c\left(\frac{1}{8}B_1 - \frac{1}{8}B_2 - \frac{3}{8}p_1 + \frac{3}{8}p_2 - \frac{1}{2}\right) + \frac{5}{18}$$

Taking the first order conditions with respect to p_1 and p_2, we obtain:

$$\frac{\partial \Pi_1}{\partial p_1} = \frac{3}{8}c + \frac{1}{16}p_2 - \frac{3}{16}B_2 - \frac{7}{16}p_1 - \frac{1}{16}B_1 + \frac{1}{2} = 0$$

$$\frac{\partial \Pi_2}{\partial p_2} = \frac{3}{8}c + \frac{1}{16}p_1 - \frac{1}{16}B_2 - \frac{3}{16}B_1 - \frac{7}{16}p_2 + \frac{1}{2} = 0$$

To which the solution is first period prices of:

$$p_1 = c + \frac{4}{3} - \frac{11}{24}B_2 - \frac{5}{24}B_1, p_2 = c + \frac{4}{3} - \frac{5}{24}B_2 - \frac{11}{24}B_1$$

and second period prices of

$$p_1^1 = c + \frac{17}{48}B_1 - \frac{1}{48}B_2 + \frac{2}{3}, p_1^2 = c + \frac{1}{3} - \frac{7}{24}B_2 - \frac{1}{24}B_1$$
$$p_2^1 = c + \frac{1}{3} - \frac{1}{24}B_2 - \frac{7}{24}B_1, p_2^2 = c + \frac{17}{48}B_2 - \frac{1}{48}B_1 + \frac{2}{3}$$

This implies profits of:

$$\Pi_1 = \frac{23}{144}B_1 - \frac{55}{144}B_2 - \frac{7}{2304}B_1B_2 + \frac{263}{4608}B_1^2 + \frac{263}{4608}B_2^2 + \frac{17}{18}$$
$$\Pi_2 = \frac{23}{144}B_2 - \frac{55}{144}B_1 - \frac{7}{2304}B_1B_2 + \frac{263}{4608}B_1^2 + \frac{263}{4608}B_2^2 + \frac{17}{18}$$

This outcome is only possible as long as three conditions are satisfied. The first is that $q > x_1$ (the expression for q is equation 11 in the paper). The second is that $x_2 > q$. The third is that $x_2 < 1$. It is straightforward to show that the first condition to be violated is $q > x_1$. Because $q = \frac{1}{32}B_1 - \frac{1}{32}B_2 + \frac{1}{2}$ and $x_1 = \frac{17}{96}B_1 - \frac{1}{96}B_2 + \frac{1}{3}$, this obtains when $-\frac{7}{48}B_1 - \frac{1}{48}B_2 + \frac{1}{6} > 0$.

When $\frac{7}{48}B_1 - \frac{1}{48}B_2 + \frac{1}{6} < 0$, we investigate the possibility of partial poaching outcomes. First we consider a partial poaching outcome where only Firm 1 (the firm with stronger benefits) poaches Firm 2's past consumers. As noted earlier,

1. Firm 2 must set $p_2^1 = c$ in the second period or else it can reduce price and increase profits.
2. Firm 1 sets p_1^1 such that the consumer at q (the indifferent consumer in the first period) is indifferent between Firm 1 and Firm 2 $\Rightarrow p_1^1 = B_1 + 1 - 2q$
3. Optimization is needed to solve for the indifferent consumer in the segment of past consumers of Firm 2.

The objective functions are

$$\pi_1 = \left(p_1^1 - c\right) q + \left(p_1^2 - c\right)(x_2 - q), \pi_2 = \left(p_2^2 - c\right)(1 - x_2)$$

where $x_2 = \frac{p_2^2 - p_1^2 + 1 - B_2}{2}$. Optimizing each firm's decision variable, we obtain $p_2^2 = c + \frac{1}{3}B_2 - \frac{2}{3}q + 1$, and $p_1^2 = c + 1 - \frac{1}{3}B_2 - \frac{4}{3}q$. The consumer at q is indifferent from buying at Firm 1 twice versus buying from Firm 2 in period 1 and then Firm 1 in period 2. That is $CS_{11} = 2v + B_1 - p_1 - p_1^1 - 2q$ equals $CS_{21} = 2v - p_2 - p_1^2 - q - (1-q)$ at the indifferent consumer. Substituting, this implies that $q = \frac{3}{4}p_2 - \frac{3}{4}p_1 - \frac{1}{4}B_2 + \frac{3}{4}$. The first period objective functions for each firm are:

$$\Pi_1 = \left(p_1 + p_1^1 - 2c\right) q + \left(p_1^2 - c\right)(x_2 - q), \Pi_2 = (p_2 - c)(1 - q) + \left(p_2^2 - c\right)(1 - x_2)$$

We now substitute into the objective function to obtain profit functions for each firm in terms of p_1 and p_2.

$$\begin{aligned}\Pi_1 = &\frac{3}{4}B_1 + \frac{1}{2}B_2 + \frac{9}{4}p_1 - \frac{3}{2}p_2 - \frac{1}{4}B_1B_2 - \frac{3}{4}B_1p_1 + \frac{3}{4}B_1p_2 \\ &- B_2p_1 + \frac{3}{4}B_2p_2 + 2p_1p_2 - \frac{1}{8}B_2^2 - \frac{11}{8}p_1^2 - \frac{5}{8}p_2^2 \\ &+ c\left(\frac{1}{4}B_2 + \frac{3}{4}p_1 - \frac{3}{4}p_2 - \frac{3}{4}\right) - \frac{3}{8}\end{aligned}$$

$$\begin{aligned}\Pi_2 \;=\; &\frac{1}{4}B_2 + \frac{1}{4}p_1 + \frac{1}{4}B_2p_1 + \frac{1}{2}p_1p_2 + \frac{1}{8}B_2^2 + \frac{1}{8}p_1^2 - \frac{5}{8}p_2^2 \\ &+c\left(\frac{3}{4}p_2 - \frac{3}{4}p_1 - \frac{1}{4}B_2 - \frac{1}{4}\right) + \frac{1}{8}\end{aligned}$$

Optimizing these expressions with respect to p_1 and p_2, we obtain the solution for the first period:

$$p_1 = \frac{15}{13} - \frac{20}{39}B_2 - \frac{5}{13}B_1 + c, p_2 = \frac{6}{13} - \frac{8}{39}B_2 - \frac{2}{13}B_1 + c$$

and the second period:

$$\begin{aligned}p_1^1 \;=\; &\frac{17}{26}B_1 + \frac{1}{26}B_2 + \frac{7}{13} + c, p_1^2 = \frac{9}{13} - \frac{4}{13}B_2 - \frac{3}{13}B_1 + c \\ &p_2^1 = c, p_2^2 = \frac{9}{26}B_2 - \frac{3}{26}B_1 + \frac{11}{13} + c\end{aligned}$$

The profits of Firms 1 and 2 under this regime are:

$$\Pi_1 = \frac{33}{169}B_1 - \frac{60}{169}B_2 - \frac{11}{676}B_1B_2 + \frac{99}{1352}B_1^2 + \frac{229}{4056}B_2^2 + \frac{213}{338}$$
$$\Pi_2 = \frac{73}{507}B_2 - \frac{50}{169}B_1 - \frac{5}{676}B_1B_2 + \frac{45}{1352}B_1^2 + \frac{227}{4056}B_2^2 + \frac{241}{338}$$

The conditions that need to be checked to confirm the viability of this outcome are $x_2 < 1$ and $x_2 > q$. For the sake of brevity, we do not present the analysis of each condition but note that the limiting condition is $x_2 > q$ (when Firm 1 has sufficient advantage its aggressiveness eliminates the possibility of a partial poaching equilibrium). Since $q = \frac{9}{52}B_1 - \frac{1}{52}B_2 + \frac{3}{13}$and $x_2 = \frac{3}{52}B_1 - \frac{9}{52}B_2 + \frac{15}{26}$, the condition can be simplified to $\frac{9}{26} - \frac{2}{13}B_2 - \frac{3}{26}B_1 > 0$.

Second, consider a partial poaching outcome where only Firm 2 (the firm with a smaller benefit) poaches Firm 1's past consumers. The relevant derivation is identical to the derivation for the first partial poaching equilibrium replacing 1 by 2 in the relevant expressions. The equilibrium outcomes are:

$$p_1 = \frac{6}{13} - \frac{8}{39}B_1 - \frac{2}{13}B_2 + c, p_2 = \frac{15}{13} - \frac{20}{39}B_1 - \frac{5}{13}B_2 + c$$

and the second period:

$$p_1^1 = \frac{9}{26}B_1 - \frac{3}{26}B_2 + \frac{11}{13} + c, p_1^2 = c$$
$$p_2^1 = \frac{9}{13} - \frac{4}{13}B_1 - \frac{3}{13}B_2 + c,\ p_2^2 = \frac{17}{26}B_2 + \frac{1}{26}B_1 + \frac{7}{13} + c$$

The profits of Firms 1 and 2 under this regime are:

$$\Pi_1 = \frac{73}{507}B_1 - \frac{50}{169}B_2 - \frac{5}{676}B_1B_2 + \frac{45}{1352}B_2^2 + \frac{227}{4056}B_1^2 + \frac{241}{338}$$
$$\Pi_2 = \frac{33}{169}B_2 - \frac{60}{169}B_1 - \frac{11}{676}B_1B_2 + \frac{99}{1352}B_2^2 + \frac{229}{4056}B_1^2 + \frac{213}{338}$$

As before, there are conditions that need to be checked to confirm the viability of this outcome. They are $q < 1$ and $x_1 < q$. For the sake of brevity, we do not present the analysis of each condition but note that the limiting condition is $x_1 < q$ (when the benefits of Firm 1 are sufficiently high, Firm 2 cannot poach any of Firm 1's past consumers). Since $x_1 = \frac{9}{52}B_1 - \frac{3}{52}B_2 + \frac{11}{26}$ and $q = \frac{1}{52}B_1 - \frac{9}{52}B_2 + \frac{10}{13}$, the condition can be simplified to $\frac{2}{13}B_1 + \frac{3}{26}B_2 - \frac{9}{26} < 0$. This condition is more restrictive than the condition for the first partial poaching equilibrium.

This implies that when $-\frac{7}{48}B_1 - \frac{1}{48}B_2 + \frac{1}{6} < 0$ and $\frac{2}{13}B_1 + \frac{3}{26}B_2 - \frac{9}{26} < 0$, the outcome when both firms implement BBD could be either of two partial poaching outcomes. In contrast, when $\frac{2}{13}B_1 + \frac{3}{26}B_2 - \frac{9}{26} > 0$ and $\frac{9}{26} - \frac{2}{13}B_2 - \frac{3}{26}B_1 > 0$, the unique equilibrium is for Firm 1 (the strong firm) to poach Firm 2's past consumers in the second period.

When $\frac{9}{26} - \frac{2}{13}B_2 - \frac{3}{26}B_1 < 0$, the only viable outcome is for neither firm to poach the other's past consumers in the second period. In this situation:

1. Firm 2 sets $p_2^1 = c$ or else it could reduce price and increase profits.

2. Firm 1 sets $p_1^2 = c$ or else it could reduce price and increase profits

3. Firm 1 sets p_1^1 such that the consumer at q (the indifferent consumer in the first period) is indifferent between Firm 1 and Firm 2.$\Rightarrow p_1^1 = B_1 + 1 - 2q + c$.

4. Firm 2 sets p_2^2 such that the consumer at q (the indifferent consumer in the first period) is indifferent between Firm 1 and Firm 2$\Rightarrow p_2^2 = B_2 - 1 + 2q + c$.

5. Optimization is used to solve for the indifferent consumer in the first period.

Similar to the no-poaching equilibrium when firms are symmetric, the first period solution does not depend on the second period outcome i.e., the decision in the first period is based entirely on price. As before, we assume that payoffs in the first period have a higher weight than the payoff in the second period ($\delta < 1$). As $\delta \to 1$, the outcomes are exactly as presented in the paper and $\delta < 1$ ensures that the split in the first period is symmetric.

The value of buying in period 1 for the indifferent consumer who buys from Firm 1 is $CS_1 = v - q - \delta - c\delta + v\delta + q\delta - p_1$ and from Firm 2 $CS_2 = v + q - c\delta + v\delta - q\delta - p_2 - 1$ and from Firm 2. Solving we obtain the following: $q = \frac{1}{2\delta - 2}(\delta + p_1 - p_2 - 1)$. The first period objective functions are $\Pi_1 = \left((p_1 - c) + \delta\left(p_1^1 - c\right)\right) q$ and $\Pi_2 = ((p_2 - c) + \delta\left(p_2^b - c\right))(1 - q)$. Substituting and taking the first order conditions, we obtain:

$$\frac{\partial \Pi_1}{\partial p_1} = \frac{1}{2\delta^2 - 4\delta + 2}\left(c - \delta - B_1\delta - c\delta - 2p_1 + p_2 + \delta p_2 + B_1\delta^2 + 1\right) = 0$$

$$\frac{\partial \Pi_2}{\partial p_2} = \frac{1}{2\delta^2 - 4\delta + 2}\left(c - \delta - B_2\delta - c\delta + p_1 - 2p_2 + \delta p_1 + B_2\delta^2 + 1\right) = 0$$

The solution to these equations is:

$$p_1 = \frac{1}{\delta + 3}\left(3c + \delta + c\delta - 2\delta B_1 - \delta B_2 - \delta^2 B_2 + 3\right)$$
$$p_2 = \frac{1}{\delta + 3}\left(3c + \delta + c\delta - \delta B_1 - 2\delta B_2 - \delta^2 B_1 + 3\right)$$

As $\delta \to 1$, the equilibrium prices in the first period approach $p_1 = c - \frac{1}{2}B_1 - \frac{1}{2}B_2 + 1$ and $p_2 = c - \frac{1}{2}B_1 - \frac{1}{2}B_2 + 1$.In this situation, $p_1^1 = B_1 + c$ and $p_2^2 = B_2 + c$ and $p_1^2 = p_1^2 = c$ (in the second period). The firms earn profits of $\Pi_1 = \frac{1}{4}B_1 - \frac{1}{4}B_2 + \frac{1}{2}$ and $\Pi_2 = \frac{1}{4}B_2 - \frac{1}{4}B_1 + \frac{1}{2}$. Note that the profits earned are independent of B.

To summarize, the following are sub-game outcomes as a function of B_1 and B_2.

1. When $-\frac{7}{48}B_1 - \frac{1}{48}B_2 + \frac{1}{6} > 0$ and both firms implement BBD, the outcome involves poaching by both firms.

2. When $-\frac{7}{48}B_1 - \frac{1}{48}B_2 + \frac{1}{6} < 0$ and $\frac{2}{13}B_1 + \frac{3}{26}B_2 - \frac{9}{26} < 0$, there are two partial poaching outcomes. Either Firm 1 poaches Firm 2's past consumers or Firm 2 poaches Firm 1's past consumers.

3. When $\frac{2}{13}B_1 + \frac{3}{26}B_2 - \frac{9}{26} > 0$ and $\frac{9}{26} - \frac{2}{13}B_2 - \frac{3}{26}B_1 > 0$, the unique equilibrium is for Firm 1 (the strong firm) to poach Firm 2's past consumers in the second period. 1.

4. When $\frac{9}{26} - \frac{2}{13}B_2 - \frac{3}{26}B_1 < 0$ and both firms implement BBD, the unique equilibrium does not involve poaching by either firm.

Figure A1 summarizes the sub-game outcomes visually. Note that the area above the $B_1 = B_2$ line is not relevant because $B_1 > B_2$.

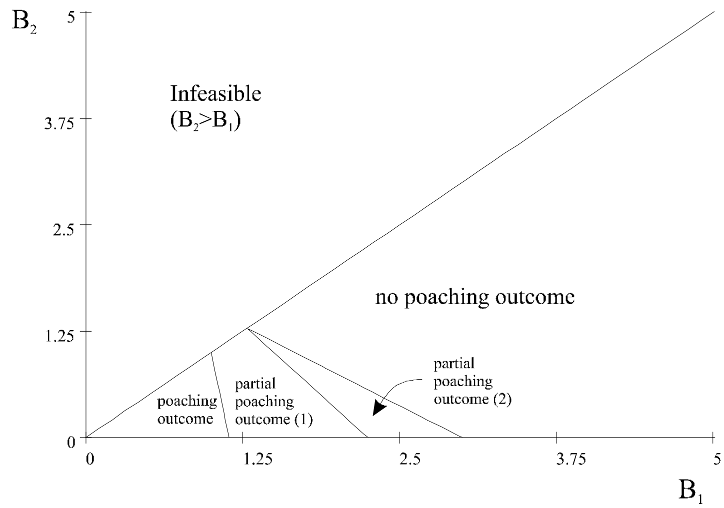

Figure A1: Outcomes when asymmetric firms implement BBD

These outcomes provide the basis for solving the meta-game.

Poaching Zone $\left(B_1 < \frac{23}{31}\right)$

1. We know from Proposition 6, that the no BBD outcomes are higher than asymmetric payoffs for both firms. Hence, (no BBD, no BBD) is a Nash equilibrium.

2. For BBD by Firm 1 to be the best response to BBD by Firm 2, we need $\Pi_{bbd1} > \Pi_{d1}$. For Firm 1 $\Pi_{d1} = \frac{63}{2116}B_2^2 - \frac{37}{138}B_2 + \frac{17}{24}$ (Firm 2 only implements BBD) and $\Pi_{bbd} = \frac{23}{144}B_1 - \frac{55}{144}B_2 - \frac{7}{2304}B_1B_2 + \frac{263}{4608}B_1^2 + \frac{263}{4608}B_2^2 + \frac{17}{18}$.
$$\Pi_{bbd} - \Pi_d = \frac{23}{144}B_1 - \frac{377}{3312}B_2 - \frac{7}{2304}B_1B_2 + \frac{263}{4608}B_1^2 + \frac{66\,551}{2437\,632}B_2^2 + \frac{17}{72}$$
This function has a global minimum given by the first order conditions:
$$\frac{\partial\left(\Pi_{bbd} - \Pi_d\right)}{\partial B_1} = \frac{263}{2304}B_1 - \frac{7}{2304}B_2 + \frac{23}{144} = 0$$
$$\frac{\partial\left(\Pi_{bbd} - \Pi_d\right)}{\partial B_2} = \frac{66\,551}{1218\,816}B_2 - \frac{7}{2304}B_1 - \frac{377}{3312} = 0$$
These conditions are satisfied at $B_1 = -\frac{61\,249}{45\,513}$ and $B_2 = \frac{91\,471}{45\,513}$. This is the global minimum because the second order conditions are satisfied everywhere:
$$\frac{\partial^2\left(\Pi_{bbd} - \Pi_d\right)}{\partial B_1^2} = \frac{263}{2304} > 0, \frac{\partial^2\left(\Pi_{bbd} - \Pi_d\right)}{\partial B_2^2} = \frac{66\,551}{1218\,816} > 0$$
At the global minimum, $\Pi_{bbd} - \Pi_d = \frac{15\,569}{1092\,312} > 0$. Hence the condition is always satisfied.

3. For BBD by Firm 2 to be the best response to BBD by Firm 1, we need $\Pi_{bbd2} > \Pi_{d2}$. For Firm 2 $\Pi_{d2} = \frac{63}{2116}B_1^2 - \frac{37}{138}B_1 + \frac{17}{24}$ (Firm 2 only implements BBD) and $\Pi_{bbd} = \frac{23}{144}B_2 - \frac{55}{144}B_1 - \frac{7}{2304}B_1B_2 + \frac{263}{4608}B_1^2 + \frac{263}{4608}B_2^2 + \frac{17}{18}$. An identical analysis shows that $\Pi_{bbd} - \Pi_d$ is strictly positive in this range.

Poaching Zone $B_1 \in \left(\frac{23}{31}, 1\right)$

1. For Firm 1. $\Pi_{no\ bbd} > \Pi_a$ since $\Pi_{no\ bbd} = 1$ and $\Pi_a = \frac{8}{25}B_1 + \frac{1}{25}B_1^2 + \frac{16}{25}$. For $\Pi_{no\ bbd} > \Pi_a$, $B_1 > 1$ which is outside the allowable zone. In addition, $\Pi_{bbd} > \Pi_d$ since $\Pi_{bbd} = \frac{23}{144}B_1 - \frac{55}{144}B_2 - \frac{7}{2304}B_1B_2 + \frac{263}{4608}B_1^2 + \frac{263}{4608}B_2^2 + \frac{17}{18}$ and $\Pi_d = \frac{1}{50}B_2^2 - \frac{6}{25}B_2 + \frac{18}{25}$. $\Pi_{bbd} - \Pi_d = \frac{23}{144}B_1 - \frac{511}{3600}B_2 - \frac{7}{2304}B_1B_2 + \frac{263}{4608}B_1^2 + \frac{4271}{115\,200}B_2^2 + \frac{101}{450}$. As shown in the Figure A2, the expression is positive throughout the feasible zone for B_1 and B_2.

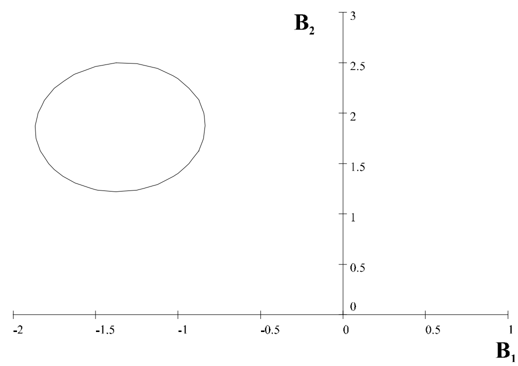

Figure A2: The Area where $\Pi_{bbd} - \Pi_d$ is negative

2. Similar reasoning applies to Firm 2.

In summary, the above reasoning implies that for all $B_1 < 1$, there are two Nash equilibria, one where both firms implement BBD and one where neither do.

Poaching Zone $B_1 \in \left(1, \frac{8}{7} - \frac{1}{7}B_2\right)$

The second limit is simply the limit for poaching $-\frac{7}{48}B_1 - \frac{1}{48}B_2 + \frac{1}{6} > 0$ rewritten for B_1 in terms of B_2. Note that $\Pi_a = \frac{8}{25}B_1 + \frac{1}{25}B_1^2 + \frac{16}{25}$ for Firm 1 is strictly greater than 1 so there is only one equilibrium (BBD,BBD) in this part of the parameter space.

Partial Poaching Zone (1) $B_1 \in \left(\frac{8}{7} - \frac{1}{7}B_2, \frac{9}{4} - \frac{3}{4}B_2\right)$

The second limit is simply the first limit for partial poaching $\frac{2}{13}B_1 + \frac{3}{26}B_2 - \frac{9}{26} = 0$ rewritten for B_1 in terms of B_2.

1. Firm 1 will always implement BBD because $\Pi_a > 1$. The only question is whether Firm 2 responds by implementing BBD as well.

2. When $\frac{33}{169}B_2 - \frac{486}{4225}B_1 - \frac{11}{676}B_1B_2 + \frac{3697}{101\,400}B_1^2 + \frac{99}{1352}B_2^2 - \frac{759}{8450} > 0$, there are two possible equilibria, one in which Firm 1 poaches (and Firm 2 does not) and vice versa. This is based on Π_2 (the profit of Firm 2 under partial poaching) being greater than Π_d for both cases. First we show that profits earned by Firm 2, when Firm 1 poaches, $\Pi_{2\ F1\ Poaches} = \frac{73}{507}B_2 - \frac{50}{169}B_1 - \frac{5}{676}B_1B_2 + \frac{45}{1352}B_1^2 + \frac{227}{4056}B_2^2 + \frac{241}{338}$, are greater than the profits earned by Firm 2 when Firm 2 poaches, $\Pi_{2\ F2\ Poaches} = \frac{33}{169}B_2 - \frac{60}{169}B_1 - \frac{11}{676}B_1B_2 + \frac{99}{1352}B_2^2 + \frac{229}{4056}B_1^2 + \frac{213}{338}$. $\Pi_{2\ F1\ Poaches} - \Pi_{2\ F2\ Poaches} > 0$ implies that $\frac{10}{169}B_1 - \frac{2}{39}B_2 + \frac{3}{338}B_1B_2 - \frac{47}{2028}B_1^2 - \frac{35}{2028}B_2^2 + \frac{14}{169} > 0$. Graphing this function in Figure A3, we see that this condition holds throughout the partial poaching zone $B_1 \in \left(\frac{8}{7} - \frac{1}{7}B_2, \frac{9}{4} - \frac{3}{4}B_2\right)$.

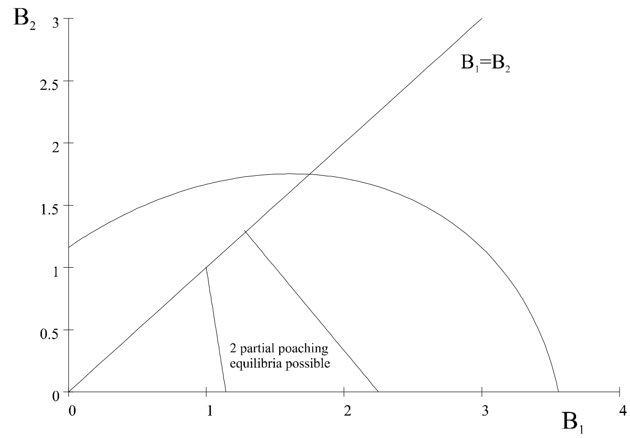

Figure A3: Graph showing that $\Pi_{2\ F1\ Poaches} - \Pi_{2\ F2\ Poaches} > 0$ in Partial Poaching (1)

This implies that if $\Pi_{2\ F2\ Poaches}$ exceeds the no BBD profits, then either partial poaching equilibrium is possible. $\Pi_{2\ F2\ Poaches} - \Pi_d > 0 \Rightarrow \frac{33}{169}B_2 - \frac{486}{4225}B_1 - \frac{11}{676}B_1B_2 + \frac{3697}{101\,400}B_1^2 + \frac{99}{1352}B_2^2 - \frac{759}{8450} > 0$. Graphing this limit in Figure A4 we see that it limits the area where (BBD,BBD) is an equilibrium to the part of the region near the $B_1 = B_2$ axis.

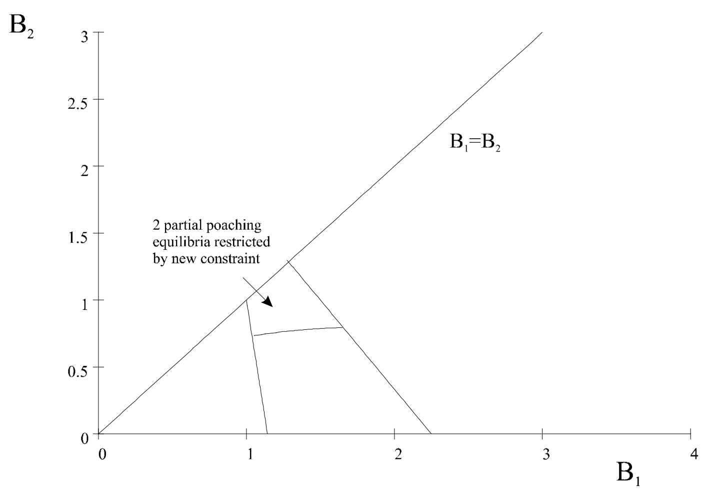

Figure A4: Partial Poaching (2 equilibria) Zone

3. When $\frac{33}{169}B_2 - \frac{486}{4225}B_1 - \frac{11}{676}B_1B_2 + \frac{3697}{101\,400}B_1^2 + \frac{99}{1352}B_2^2 - \frac{759}{8450} < 0$, the partial poaching equilibrium where Firm 2 poaches generates less profit for Firm 2 than not implementing BBD. The area in Partial Poaching (1) where $\Pi_{2\ F1\ Poaches} > \Pi_d > \Pi_{2\ F2\ Poaches}$ has an upper bound (B_1, B_2) as given above and a lower bound given by $\Pi_{2\ F1\ Poaches} - \Pi_d = 0$. This limit is $0 = \frac{73}{507}B_2 - \frac{236}{4225}B_1 - \frac{5}{676}B_1B_2 + \frac{449}{33\,800}B_1^2 + \frac{227}{4056}B_2^2 - \frac{59}{8450}$. The area is shown in Figure A5.

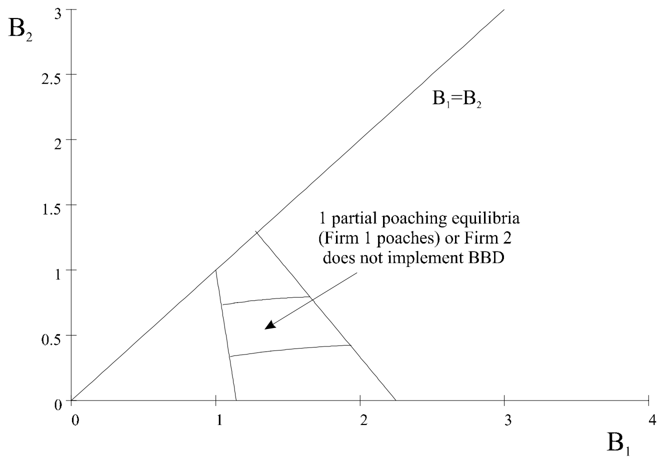

Figure A5: The Area where $\Pi_{2\ F1\ Poaches} > \Pi_d > \Pi_{2\ F2\ Poaches}$

In this area, there are two potential equilibrium. In the first, Firm 2 does not implement BBD. In the second, Firm 2 implements BBD (as does Firm 1) and Firm 1 poaches but Firm 2 does not. This equilibrium is valid analytically but can be questioned due to Firm 2's inability to implement the "right subgame" after implementing BBD.

4. When $0 > \frac{73}{507}B_2 - \frac{236}{4225}B_1 - \frac{5}{676}B_1B_2 + \frac{449}{33\,800}B_1^2 + \frac{227}{4056}B_2^2 - \frac{59}{8450}$, the equilibrium is for Firm 1 to implement BBD and Firm 2 to employ uniform pricing in period 2 (without adding benefits for past consumers). Note that in this equilibrium, Firm 1's profits are strictly higher than the base case (since $\Pi_a > \Pi_{no\ BBD}$).

Partial Poaching Zone (2) $B_1 \in \left(\frac{9}{4} - \frac{3}{4}B_2, 3 - \frac{4}{3}B_2\right)$

As noted earlier, Firm 1 will implement BBD. If Firm 2 also implements BBD, the equilibrium entails Firm 1 poaching Firm 2's customers (and not vice versa). Thus, the question is whether Firm 2's preferred response is to implement BBD or not. This obtains by comparing $\Pi_{2\ F1\ Poaches}$ to Π_d. As derived earlier, the boundary is given by $0 = \frac{73}{507}B_2 - \frac{236}{4225}B_1 - \frac{5}{676}B_1B_2 + \frac{449}{33\,800}B_1^2 + \frac{227}{4056}B_2^2 - \frac{59}{8450}$. Thus, when $\frac{73}{507}B_2 - \frac{236}{4225}B_1 - \frac{5}{676}B_1B_2 + \frac{449}{33\,800}B_1^2 + \frac{227}{4056}B_2^2 - \frac{59}{8450} > 0$, the equilibrium is for both firms to implement BBD. When the condition is not satisfied, only Firm 1 implements BBD.

No Poaching Zone

In this zone, Firm 1 will implement BBD. Firm 2 will implement BBD as long as its expected profits under BBD i.e. $\Pi_2 = \frac{1}{4}B_2 - \frac{1}{4}B_1 + \frac{1}{2}$ are greater the profits earned by not implementing BBD i.e. $\Pi_d = \frac{1}{50}B_1^2 - \frac{6}{25}B_1 + \frac{18}{25}$ as per Proposition 5. This implies that Firm 2 implements BBD when $B_1 \in \left(B_2, \frac{5}{4}\sqrt{8B_2 - 7} - \frac{1}{4}\right)$. When $B_1 > \frac{5}{4}\sqrt{8B_2 - 7} - \frac{1}{4}$, Firm 2 does not implement BBD.

1. When the equilibrium entails poaching by both firms, Firm 1's profit is $\Pi_1 = \frac{23}{144}B_1 - \frac{55}{144}B_2 - \frac{7}{2304}B_1B_2 + \frac{263}{4608}B_1^2 + \frac{263}{4608}B_2^2 + \frac{17}{18}$. Setting this equal to 1, generates the first boundary in the proposition.
2. When the equilibrium involves partial poaching by Firm 1, Firm 1's profit is given by $\Pi_1 = \frac{33}{169}B_1 - \frac{60}{169}B_2 - \frac{11}{676}B_1B_2 + \frac{99}{1352}B_1^2 + \frac{229}{4056}B_2^2 + \frac{213}{338}$. Setting this equal to 1, generates the second boundary in the proposition (under point 3).
3. When Firm 2 does not engage in BBD, Firm 1 earns Π_a. As noted earlier, $\Pi_a > 1$ for all $B_1 > 1$.

Q.E.D.