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Risk aversion, generalized correlation and investment in manufacturing capacities

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Abstract

This study analyzes the relationship between product demand correlation and risk on the investment in dedicated and flexible manufacturing capacities. It is proved that for a risk averse (or risk neutral) decision-maker, increased generalized correlation between output demands reduces the values of both dedicated and flexible manufacturing investments for all multivariate distributions. It is shown that no such monotonic relationship generally exists for the capacities themselves because of an economic tradeoff between risk and return. It is also proved that increased generalized correlation reduces the value of flexible capacity relative to dedicated capacity for the risk-neutral decision maker. However, contrary to the extant literature, flexible capacity is still shown to be potentially valuable even if product demands are perfectly positively correlated. Finally, it is shown that increases in the risk of product demand, holding correlation (of product demand) constant, reduces the value of the investment in manufacturing capacities when output demands are either jointly normal or independent.

1. Introduction

The increasing globalization of business and the attendant competitive pressures are expected to accelerate into the twenty-first century, see (Thurow, 1992). One of the major challenges facing corporate managers will be to lead the firm through the transformations necessary to prosper in a globally competitive and, hence, risky business environment. One of the crucial questions an operations manager faces in this regard is how does increased competition and risk affect manufacturing capacity decisions. Although the issue is important irrespective of the specific manufacturing technology, it is especially crucial for advanced manufacturing technologies such as flexible manufacturing systems. Not only do such systems require enormous initial capital outlays (per unit of capacity), they are also subject to more risks during downturns in the business cycle. Outputs from dedicated manufacturing facilities tend to be more disparate and unrelated whereas flexible manufacturing plants usually produce a variety of closely related substitutes. If the economy is strong, the demand for variety is likely to be strong as well, and the ability to produce closely related products at minimum cost (economies of scope) has potentially great strategic value, see (Goldhar and Jelinek, 1983; Banker et al., 1988; Gupta and Buzacott, 1989). In a weak economy, on the other hand, demand is likely to be slack for all related products simultaneously so

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that flexibility in production may be disadvantageous. [1] A good case in point is the recent recession in Japan. The experience from this downturn of the business cycle has caused a major shift in rethinking the strategy of excessive product variety induced by flexible manufacturing capabilities, especially in the automobile and electronics industries, see (Business Week, 1992; New York Times, 1993; Cusumano, 1994).

In addition to the correlation between product demands, risk of product demand per se is also important and, in fact, has been used to rationalize the investment in flexible manufacturing systems. In contradistinction to the problem of correlation, the riskier the product demand, the more beneficial flexibility appears to be relative to dedicated economies of scale. Black & Decker recently lost millions of dollars in sales because its plants were not sufficiently flexible to respond to rapid changes in market preferences for power tools. As a result, top management subsequently turned their attention to making their plants more fast and flexible, see (Fisher et al., 1994).

The purpose of this paper is to model the effects of product demand correlation and product demand risk on the decision to invest in manufacturing capacities in a competitive market environment. Both dedicated and flexible manufacturing technologies are considered. We analyze the optimal response to product demand risk and correlation for both risk-averse and risk-neutral decision makers. Given the practical difficulties in eliciting management utility functions and the well documented and ubiquitous nature of risk aversion among business managers, it is important to be able to generalize our results to risk-averse as well as risk-neutral decision makers.

Two distinct optimization paradigms for evaluating flexible and dedicated manufacturing capacities have evolved in the literature. The real options approach yields powerful multi-period valuation equations of manufacturing capacity, see (Kulatilaka, 1988; He and Pindyck, 1989; Triantis and Hodder, 1990). There are, however, two major problems with this approach. First, it assumes rather unrealistically that the firm's manufacturing process can be replicated by a portfolio of existing securities trading continuously in frictionless markets. Second, the analytical results are quite intractable so that the qualitative implications of the model can only be derived by simulating specific numerical examples.

The concave maximization approach analyzes the flexible-dedicated manufacturing capacity decision within the confines of a two period model [2]. In the first period, the firm decides on the optimal level of production capacity. In the second period, the firm satisfies the stochastic demand subject to the capacity constraints. Despite the two period simplification, the latter approach, as it is reflected in the current literature, is also problematic. First, the decision-maker is assumed to be risk neutral [3]. Second, all the results in this literature concerning the impact of product risk and correlation on the investment in manufacturing capacities have been derived by simulating specific numerical examples.

This paper extends the concave maximization approach to a risk-averse decision maker selling in competitive output markets. Of the extant literature, the studies by Fine and Freund (1990) and Gupta et al. (1989) are most closely related to this one. Fine and Freund study the optimal capacity decisions of a risk neutral decision maker operating in monopoly product markets. Both dedicated and flexible production technologies are available. They assume that the marginal distributions for all product demands are identical. Because of the complexity of their program, they analyze the effect of product demand risk and correlation for a specific numerical example that assumes symmetric products with identical capacity costs and linear demand functions. They find that, for their example, the expected value of profits is non-increasing in product demand correlation (hold-

[1] This statement presupposes that the capital costs of a flexible system are higher than those of a dedicated manufacturing system. In the absence of a cost differential, flexible systems can produce higher cash flows at every demand realization.

[2] For a development of this approach, see (Chakravarty, 1987; Gupta et al., 1989; Fine and Freund, 1990), and the citations provided in (Mensah and Miranti, 1989).

[3] One exception is (Lederer and Singhal, 1988) who use the Capital Asset Pricing Model to evaluate demand risk for new manufacturing technologies. Since their model suppresses the multiproduct nature of the firm, they cannot address the issues raised here.

ing product demand risk constant). They also find that there is no demand for flexibility if product demands are perfectly positively correlated. Moreover, holding product demand correlation constant, they show that there is no monotonic relationship between product demand risk and investment in flexible capacity (except for the case of perfect negative correlation). They also find (counter-intuitively) that expected profits are an increasing function of product demand risk, holding product demand correlation constant. Unfortunately, given the specific nature of their numerical example, it is impossible to determine whether these results are a function of risk neutrality, monopoly power in the output markets, the assumption that the marginal distributions over product demands are identical or some combination of these.

The assumptions underlying the Gupta, Gerchak and Buzacott study are closest to those of this study. They analyze the optimal manufacturing capacities decision of a risk neutral decision maker operating in competitive output markets. Again, both dedicated and flexible production technologies are available. The joint distribution over product demands is unconstrained. However, their inferences concerning the impact of product demand correlation and risk on investment in manufacturing capacities are also based entirely on a numerical example (involving a specific discrete joint distribution over product demands). They find that there is no incentive to purchase flexible capacity if demands are perfectly positively correlated. Although they find in their example that increased correlation reduces the value of flexible capacity relative to dedicated capacity, they also find that there is no monotonic relationship between correlation and the optimal size of the investment in flexible manufacturing capacity per se. Unfortunately, the economic factors underlying the latter result are not fully clarified.

We generalize the results of Fine and Freund and Gupta, Gerchak and Buzacott, and, with the exception of one counter-example, refrain from relying on specific examples. Some of our results hold generally for all risk-averse (or risk-neutral) decision makers *and* for all multivariate demand distributions. In establishing these results, we rely on a definition of "generalized" correlation that applies to arbitrary multivariate distributions. Generalizing Fine and Freund, we show that increased generalized correlation in product demands monotonically reduces the value of flexible and dedicated manufacturing capacities for all risk-averse (and risk-neutral) decision makers. We further study the relationship between correlation and manufacturing capacity for a special example and show the tradeoffs which cause manufacturing capacity *not* to be monotonically related to correlation. In another general result applicable to all risk-averse decision makers and multivariate distributions, we show that the more risk averse the decision maker, the smaller the investment in manufacturing capacities. We also generalize an important result from Gupta et al. (that they derive within a particular example) for a risk-neutral decision maker facing any multivariate demand distribution. Specifically, we prove that the smaller the generalized correlation, the greater the value of flexible capacity relative to dedicated capacity for any risk-neutral decision maker. In proving this result, we also demonstrate (contrary to the examples of Fine and Freund and Gupta et al.) that flexible capacity may be valuable even when product demands are perfectly positively correlated. Finally, we study the effect of increased risk on the value of manufacturing capacities for all risk-averse (or risk-neutral) decision makers under the assumption that the joint distribution of demand is either multivariate normal or independent. The multivariate normal case is precisely the one that underlies much of the finance literature approach to risk measurement as, for example, in the Capital Asset Pricing Model, see (Fama and Miller, 1972). When the joint distribution of product demands falls under either of these categories, an increase in risk decreases the value of investment in manufacturing capacities for all risk-averse and risk-neutral decision makers.

2. Preliminaries

We consider the optimal first period manufacturing capacity decision of a firm owned by a risk-averse (or risk-neutral) investor operating in competitive output markets. Demand for the firm's output is stochastic. Subject to capacity, the firm is assumed to sell two outputs X and Y (the generalization to an arbitrarily finite number of products is straightfor-

ward) demanded in the second period at the market prices p_x and p_y, respectively. Variable costs are denoted c_x and c_y and are assumed, without substantial loss of generality, to be identical across technologies. The contribution margin on X gross of fixed costs $(p_x - c_x)$ is denoted by α and the corresponding contribution margin on Y is denoted by β. Without loss of generality, α is assumed to be greater than β.

Two potential manufacturing technologies are available, dedicated and flexible. Using the dedicated technology, each product is produced separately on a specialized (dedicated) machine. Let K_x and K_y denote the manufacturing capacities devoted to products X and Y, respectively. The per unit capital costs of dedicated capacity are denoted by C_x and C_y. Using the flexible technology, both products are produced by one machine whose capacity is denoted by K and cost per unit of capacity by C. We assume, as does the rest of the literature, that $C \geqslant \max\{C_x, C_y\}$ so that the capital cost of flexible technology is at least as high as the cost of dedicated capacity. [4]

Some of the issues addressed in this study involve the relationship between investment in manufacturing capacities and the correlation between product demands. To properly address these issues necessitates a framework in which correlation aversion is related to (expected) utility. This is analogous to issues involving risk which can only be addressed in a framework relating risk aversion and (expected) utility. In what follows, we analyze the relationship between output demand correlation and investment in manufacturing capacities using the framework developed by Epstein and Tanny (1980) in which they relate correlation aversion and expected utility. Epstein and Tanny define a utility function to be correlation averse if, in the terminology of Milgrom and Shannon (1992), it is supermodular. More formally, they define correlation aversion as follows.

Definition 1 (correlation aversion). A function ψ: $\mathscr{U} \subseteq \mathbb{R}^2 \to \mathbb{R}$ is said to be *correlation averse (CAV)* if and only if for any points $(x_i, y_j) \in \mathscr{U}$ with $x_1 < x_2$ and $y_1 < y_2$, we have

$$\psi(x_1, y_1) + \psi(x_2, y_2) \leqslant \psi(x_2, y_1) + \psi(x_1, y_2).$$

The relationship between correlation aversion and risk aversion is complex. As Epstein and Tanny show, only if the two random variables are perfect substitutes in an investor's utility function are risk aversion and correlation aversion equivalent. However, it is always the case that a concave function of a CAV function is automatically CAV. [5] This feature is reflected in the next result which demonstrates why CAV functions play an important role in analyzing the effects of correlation.

Lemma 1 (normality and correlation aversion). [6] *Let $\tilde{X}$ and $\tilde{Y}$ be jointly normal with correlation τ_1. Let $\tilde{Z}$ and $\tilde{W}$ also be jointly normal with marginals identical to $\tilde{X}$, $\tilde{Y}$ but with correlation τ_2. Then for every concave (or linear) utility function $U(w)$ and every CAV function $\psi(x, y)$:*

$$\tau_1 > \tau_2 \quad \Leftrightarrow \quad \mathrm{E}\left[U\left(\psi(\tilde{X}, \tilde{Y})\right)\right] < \mathrm{E}\left[U\left(\psi(\tilde{Z}, \tilde{W})\right)\right].$$

Lemma 1 states that increased correlation reduces expected utility of all correlation averse functions provided that the underlying random variables are bivariate normal. For other joint distributions, however, this result no longer holds and so standard correlations are of limited theoretical validity. Lemma 1 is important, nevertheless, because it motivates a definition of generalized correlation which is not distribution dependent. Specifically, the following definition of generalized correlation is due to Epstein and Tanny.

[4] Fine and Freund and Gupta, Gerchak and Buzacott assume in addition that $C_x + C_y > C$. Such an assumption is unnecessary for our results.

[5] We do *not* assume in Sections 3 and 4 that the investor is correlation averse. Rather, we *prove* that the cash flow structure is CAV thus making all risk-averse and risk-neutral decision makers correlation averse with respect to the joint distribution of product demands.

[6] All proofs in this paper are supplied in Appendix A.

Definition 2 (generalized correlation). One bivariate distribution $F(x, y)$ exhibits greater generalized correlation than another, $G(x, y)$, if and only if (i) the marginals of F and G are identical and (ii) for every concave (or linear) utility function $U(w)$ and CAV function $\psi(x, y)$, we have

$$\int U(\psi(x, y))\ \mathrm{d}F(x, y) \leqslant \int U(\psi(x, y))\ \mathrm{d}G(x, y).$$

This definition of generalized correlation will prove useful in the analysis of manufacturing capacities.

In order to determine the impact of risk and correlation on manufacturing capacity, we must first establish the basic properties of the cash flow function for dedicated and flexible manufacturing. We begin by recording the form of the cash flow functions for some specified level of flexible capacity K, or alternatively, levels K_x, K_y for dedicated capacities. Then for realized demands x, y, the associated cash flow functions are given by

$$\begin{aligned}\mathrm{CF}(x, y \mid K) &= \alpha \min(x, K) + \beta \max\{\min(y, K-x), 0\} - CK \\ &= \beta \min(x+y, K) + (\alpha-\beta)\min(x, K) - CK,\end{aligned}$$

$$\begin{aligned}\mathrm{CD}(x, y \mid K_x, K_y) &= \alpha \min(x, K_x) + \beta \min(y, K_y) - C_x K_x - C_y K_y. \qquad (1)\end{aligned}$$

The form of the cash flow functions leads to the following basic result.

Proposition 1 (cash flows are concave and CAV).
1. *For fixed K (respectively, fixed K_x, K_y), the cash flow functions* $\mathrm{CF}(x, y \mid K)$ *(respectively,* $\mathrm{CD}(x, y \mid K_x, K_y)$*) are correlation averse and concave as functions of x, y.*
2. *For any fixed x, y, the cash flow functions* $\mathrm{CF}(x, y \mid K)$ *(respectively,* $\mathrm{CD}(x, y \mid K_x, K_y)$*) are concave as a function of K (respectively, K_x, K_y).*

The concavity of the cash flow functions implies a unique optimal manufacturing capacity for each of the dedicated and flexible manufacturing technologies as the next proposition shows.

Proposition 2 (optimal investment). *There are unique optimal investment levels K^* for flexible capacity and K_x^*, K_y^* for dedicated capacity.*

Two additional details that are of considerable use in developing our results need to be recorded. The first concerns the difference in payoffs when dedicated and flexible capacities are exactly matched. The second concerns the method of solving for optimal capacity decisions. We begin with the comparison of payoffs across technologies and summarize the conclusions in Lemma 2.

Lemma 2 (break-even cost differential). *Let $F(x, y)$ denote the joint distribution of $\tilde{X}$ and $\tilde{Y}$ and suppose that dedicated capacity costs are the same for $\tilde{X}$ and $\tilde{Y}$, that is, $C_x = C_y$. Then*
1. *If $C = C_x = C_y$, flexible capacity is preferred to dedicated capacity.*
2. *There is a maximal cost level, $C^*(F)$, with $C^*(F) \geqslant C_x = C_y$ such that the value of investing in flexible capacity becomes equal to the value of investing in dedicated capacity.*

Lemma 2 is to be expected in the sense that unless flexible capacity involves higher costs than dedicated capacity, the former is invariably preferred. Thus, some positive cost difference is needed in order for the values of the two types of investments to be equal. We would like to emphasize that this preference is typically a *strict* one even when product demand distributions are perfectly correlated (in contrast with Gupta, Gerchak and Buzacott and Fine and Freund). The point is that if the products have different contribution margins, flexibility always adds some value over dedicated capacity because of the ability to switch between products

whenever there is a large realized demand for the high margin product. [7]

The last step in this section of the paper is to characterize the level of optimal capacity implied by Proposition 2. We confine our analysis here to the discussion of flexible capacity. The analysis for dedicated capacity is similar. The optimal investment capacity, K^*, solves [8]

$$\begin{aligned}\frac{\partial}{\partial K}(\mathrm{E}U[\mathrm{CF}(x, y \mid K)]) &= -C\int_{\mathscr{A}(K)} U'[\mathrm{CF}(x, y \mid K)] f(x, y)\, \mathrm{d}x\, \mathrm{d}y \\ &\quad + (\beta - C)\int_{\mathscr{B}(K)} U'[\mathrm{CF}(x, y \mid K)] \times f(x, y)\, \mathrm{d}x\, \mathrm{d}y \\ &\quad + (\alpha - C)\int_{\mathscr{C}(K)} U'[\mathrm{CF}(x, y \mid K)] \times f(x, y)\, \mathrm{d}x\, \mathrm{d}y = 0, \qquad (2)\end{aligned}$$

where $\mathscr{A}(K) = \{(x, y) \mid x + y \leqslant K\}$, $\mathscr{B}(K) = \{(x, y) \mid x + y > K,\ x \leqslant K\}$, and $\mathscr{C}(K) = \{(x, y) \mid x + y > K,\ x > K\}$.

Intuitively, consider the addition of an extra unit of capacity starting at K. Then $\mathscr{A}(K)$ corresponds to the region where there is no additional cash flows from increasing capacity (even the current capacity is excessive). $\mathscr{B}(K)$ is the region where extra capacity leads to greater production of y, whereas $\mathscr{C}(K)$ is the region where adding capacity increases the production of x. The net benefit of adding a unit of additional capacity is the sum of these three effects.

The preliminary technical analysis of the problem is now complete and we apply these results in the subsequent sections of the paper to analyze various issues involving flexible and dedicated capacities.

3. Generalized correlation and manufacturing capacities

We begin our analysis by proving a general result linking the investment decision and correlation. Generalizing the example in Fine and Freund, this result shows unambiguously that, for any risk-averse investor and any multivariate distribution, an increase in the generalized correlation of output demands results in a reduction in the value of manufacturing capacity, whether flexible or dedicated.

Proposition 3 (generalized correlation and expected payoffs). *For any risk averse or risk neutral decision maker, an increase in the generalized correlation between $\tilde{X}$ and $\tilde{Y}$ results in a reduction of the expected utility from the investment.*

Proposition 3 states that the value of all forms of capacity decrease as the outputs become increasingly correlated. It fails to clarify, however, how the values of flexible and dedicated capacities change *relative* to each other. Consider a decision-maker who is trying to decide between dedicated capacity and flexible capacity. How would his choice be affected by changes in generalized correlation of x and y and the cost differential between flexible and dedicated capacities? In concert with the extant literature, we limit our solution to the case of a risk-neutral decision maker and assume that the cost of dedicated capacity is the same for both $\tilde{X}$ and $\tilde{Y}$, that is, $C_x = C_y$. We show that as generalized correlation decreases, a risk-neutral decision maker is prepared to tolerate a higher cost differential in switching over from dedicated to flexible capacity irrespective of the distribution of product demands. More precisely, we show that at least for a risk-neutral decision maker, the point of indifference (in terms of capacity-cost), $C^*(F)$, defined in Lemma 2 decreases in generalized correlation.

Proposition 4 (choosing between dedicated and flexible capacities). *Let F exhibit greater generalized correlation than G and assume in addition that*

[7] The inequality is strict as long as one of the regions $x > K_x$, $y < K_y$ or $y > K_y$, $x < K_x$ has positive probability. Alternatively, if x contributes strictly more than y, it suffices that the region $x > K_x$ have positive probability (see the proof of Lemma 2 in Appendix A). This last condition will be met whenever $\tilde{X}$ is not discrete.

[8] In view of the convexity of $\mathrm{E}[U[\mathrm{CF}(x, y \mid K)]]$, the obvious approach is to differentiate this function in K, set the derivative equal to zero and then solve for K^*. This procedure works in essence but there are some minor associated technical problems discussed in Appendix A.3.

the decision maker is risk-neutral. Then $C^*(F) < C^*(G)$, that is, the smaller the generalized correlation, the greater the value of flexible capacity relative to dedicated capacity.

Intuitively, for the risk-neutral decision maker, the value of dedicated capacity depends only on the marginal distributions of $\tilde{X}$ and $\tilde{Y}$ and hence is unaffected by changes in generalized correlation. Thus, the relative values of the investment are affected only through the impact of generalized correlation on flexible capacity. By Proposition 3, the smaller the level of generalized correlation, the greater the expected value of flexible capacity and Proposition 4 follows immediately.

The analysis so far has concentrated on the issue of investment value. We turn now to the issue of optimal investment levels. The analysis will be restricted to that of flexible capacity. The main reason is that the optimal level of flexible capacity is affected by the joint distribution even when the investor is risk-neutral. In other words, the optimal level of flexible capacity has a component deriving purely from the joint distribution irrespective of the investor's utility function.

An increase in generalized correlation systematically decreases the value of the investment. A reasonable conjecture is that this decrease in value is accompanied by a decrease in capacity. The example in Gupta et al. showed that this conjecture fails in general but the reasons for the failure in terms of economic tradeoffs are not apparent from their example. The next example yields the intuition relating changes in the level of correlation with changes in optimal investment levels.

Let $\tilde{X}$, $\tilde{Y}$ be uniformly distributed over $[0, L]$. Consider two cases: (i) perfect positive correlation and (ii) perfect negative correlation with relevant densities given by

$$\mathrm{PP}(x, y) = \begin{cases} 1/L & \text{if } x = y, \\ 0 & \text{if } x \neq y; \end{cases}$$

$$\mathrm{PN}(x, y) = \begin{cases} 1/L & \text{if } x = L - y, \\ 0 & \text{if } x \neq L - y. \end{cases}$$

$\mathrm{PP}(x, y)$ exhibits more ordinary correlation than $\mathrm{PN}(x, y)$. It also exhibits more generalized correlation. [9] We now solve for the optimal investment levels in each case starting with perfect negative correlation where the solution is intuitively transparent.

The optimal investment level (with perfect negative correlation) is the corner solution $K^*_{\mathrm{PN}} = L$ – investment levels $K > L$ are obviously inefficient, whereas for $K < L$ a slight increase ΔK increases expected profits for *every* possible joint realization of x and y. [10] The optimal investment level with perfect positive correlation, K^*_{PP}, may be either less than or greater than L depending on the values of α, β, C and the function U. A direct calculation based on the first-order condition (Eq. (2)) shows that when $\beta < 2C$ then $K^*_{\mathrm{PP}} < L = K^*_{\mathrm{PN}}$, whereas if $U(w) = 2\sqrt{w+2}$, $C = L = 1$, $\alpha = \beta = 8$, it follows that $K^*_{\mathrm{PP}} > L = K^*_{\mathrm{PN}}$.

This example illustrates that there is no one to one relationship between the correlation of product demands and the level of investment in manufacturing capacity. Intuitively, when generalized correlation of $\tilde{X}$ and $\tilde{Y}$ increases (while holding the marginals constant) there is a shift towards the tails of the distribution of $\tilde{X} + \tilde{Y}$. This increases both the mean cash flow and the risk resulting from higher levels of flexible investment. Therefore, optimal investment in manufacturing capacity may either increase or decrease in generalized correlation depending on the trade-off between increased mean and increased risk. As in all risky investment for which such tradeoffs exist, the optimal investment depends upon the specific details of the joint distribution and risk preferences.

It is worth emphasizing that this result has strong cautionary implications for policymakers concerned with industrial policy. It cannot be inferred from changes in manufacturing capacity if the decision-maker's investment decisions have been optimal or suboptimal. For instance, a decision to increase capacity may in fact be an optimal policy decision

[9] Clearly, the marginals of both $\mathrm{PP}(x, y)$ and $\mathrm{PN}(x, y)$ are uniform on $[0, L]$ and equal; the expectation inequality can be directly verified for every CAV function ϕ.

[10] This may be directly verified from Eq. (2) by noting that $\mathscr{A}(K)$ is empty for $K > L$, whereas $\mathscr{B}(K)$ and $\mathscr{C}(K)$ are empty for $K \leqslant L$. Hence, $K^*_{\mathrm{PN}} = L$.

despite the fact that one observes a simultaneous reduction in profits (induced by an increase in the generalized correlation of product demands).

4. Risk and manufacturing capacities

We now analyze how risk aversion and risk (rather than generalized correlation) affect the investment in manufacturing capacity. The basic feature driving the relation between risk-aversion and optimal flexible capacity levels is that an increase in capacity involves expending the capital cost C with certainty in exchange for uncertain future cash flows. It seems reasonable that as decision makers become progressively risk-averse, the value of the uncertain future cash flows decreases resulting in lower investment levels. This intuition is formalized in the next result.

Proposition 5 (optimal capacity and risk aversion). *Let U and V be the utility functions associated with two distinct decision makers such that decision maker U is more risk-averse than decision maker V in the sense that $-U''/U' > -V''/V'$. Then the optimal capacity chosen by the more risk-averse decision maker (U) is smaller than that chosen by the less risk-averse decision maker (V).*

Within the Epstein and Tanny framework, linking correlation aversion and expected utility requires that the marginal distributions be held constant. Intuitively, this is because correlation aversion is conceptually related to the joint characteristics of the bivariate distribution and conceptually independent of the marginals. In order to see how investment in manufacturing capacity varies with increases in product demand risk, we must allow the marginal distributions to vary. These considerations lead us to the next result.

Lemma 3 (investment increasing in joint distribution). *Let $\tilde{X}$, $\tilde{Y}$ be jointly distributed either according to F or G.*

1. *Then F is always preferred to G provided that $F(y \mid x)$ either first or second degree stochastically dominates $G(y \mid x)$ for every realization $\tilde{X} = x$.*
2. *The investment level with F is greater than the investment level for G provided that $F[y \mid x]$ first-degree stochastically dominates $G[y \mid x]$ for every realization $\tilde{X} = x$.*

One case where the first requirement of Lemma 3 holds, a case much emphasized in the finance literature, is when $\tilde{X}$ and $\tilde{Y}$ are jointly normal with covariance ρ.

Proposition 6 (normality and investment in manufacturing capacity). *Let $\tilde{X}$, $\tilde{Y}$ be jointly normal with covariance ρ. If the variance of $\tilde{Y}$ is increased while holding ρ constant, the value of the investment in manufacturing capacity decreases.* [11]

Another case where unambiguous conclusions may be drawn is when $\tilde{X}$ and $\tilde{Y}$ are independent.

Proposition 7 (independence and investment in manufacturing capacities). *Let $\tilde{X}$ and $\tilde{Y}$ be arbitrary independent distributions. Then holding $\tilde{X}$ fixed but changing the marginal distribution of $\tilde{Y}$ in the sense of first-order dominance leads to an increase in both the level of investment and (expected) payoff.*

This setting is, in some sense, complementary to our original analysis where marginals are held constant but generalized correlation is allowed to change. However, it should be emphasized that the assumption of independent demands is potentially appealing only when considering dedicated technologies since it precludes the cases when the two products are either substitutes or complements. It is precisely in the latter cases that flexible manufacturing is likely to be most valuable.

[11] The example provided earlier does not meet the conditions of Lemma 3. It is checked that $\mathrm{PP}[y \mid x] \geqslant \mathrm{PN}[y \mid x]$ for $x \leqslant L/2$ but $\mathrm{PP}[y \mid x] \leqslant \mathrm{PN}[y \mid x]$ whenever $x \geqslant L/2$. That is, there is no systematic first-degree dominance relationship between the posterior distributions induced by perfect negative correlation and perfect positive correlation (and, thus, no systematic ranking can be obtained with regard to investment levels).

5. Summary

In an increasingly globally competitive environment, operation managers must be ready to evaluate the impact of product demand risk and correlation on the investment in manufacturing capacities. This is especially true for advanced manufacturing technologies such as flexible manufacturing systems where the initial capital outlays are large and many of the costs and benefits are difficult to quantify.

The first fundamental issue we address in this paper is how does the correlation between product demands affect the value and the level of investment in manufacturing capacities whether of the dedicated or flexible variety. We show that the value of investment in manufacturing capacities of all types decreases with correlation in product demands irrespective of the decision-maker's risk preferences. This has important implications for the operations manager especially when evaluating flexible manufacturing technologies. One of the fundamental benefits of a flexible technology is the economies of scope inherent in producing similar varieties of products. While niche marketing can be an important strategic tool when market demand is strong, a downturn in the business cycle can mean a reduction in the demand for all varieties of a product. Therefore, one lesson to be derived from this study for the operations manager is to take the potential risk induced by product demand correlation into account when evaluating the investment in flexible (or dedicated) manufacturing capacities even if only on a qualitative level.

Unfortunately, there is no straightforward relationship between the correlation in product demand and the optimal level of manufacturing capacity. As we showed, an increase in product demand correlation is likely to simultaneously increase both the mean cash flow and the riskiness of the investment. Because increased mean cash flows stimulates investment whereas greater risk dampens investment levels, the net effect of greater correlation on investment levels may be ambiguous. The resolution of these two opposing effects ultimately depends on the specific joint demand distribution and the particular utility function of the decision maker. This is bad news for the operations manager and industrial policy makers. While increased product demand correlation reduces the value of the investment capacity, one cannot state a priori whether the optimal level of the investment should be scaled down, or vice-versa. It depends on the specific demand parameters and the decision-maker's risk-aversion.

The second fundamental issue addressed in this paper is how product demand correlation affects the decision to invest in flexible manufacturing versus dedicated manufacturing. We show unambiguously that for a risk-neutral decision maker, an increase in the correlation of product demands reduces the value of flexible manufacturing relative to dedicated manufacturing. This means that when evaluating the decision to invest in flexible capacity versus dedicated capacity, the operations manager needs to include product demand correlation as one of the negative factors in adopting a flexible manufacturing system. This is only one element in the evaluation process, however. Even if product demands are perfectly positively correlated, flexible manufacturing could still be valuable especially when one of the products has a higher contribution margin than the other.

Finally, we showed that increased risk in any one product market decreases the value of investing in all manufacturing capacities whatever the decision-maker's risk aversion. Although this result was shown only for the multivariate normal distribution (or when demands are independent of each other), it is worth remembering that many finance approaches to risk evaluation such as the Capital Asset Pricing Model are based on this very multivariate normal assumption. This result is important for the operations manager evaluating new investments in manufacturing capacities in risky competitive markets. It suggests that anything the operations manager can do to forecast demand accurately, including proactive forecasting as suggested by Fisher et al. (1994), may well increase the value of the firm's investment in manufacturing capacities.

Appendix A

The first section of the appendix collects technical results and proofs of the assertions made in Section 2. Proofs of the other results are presented subsequently in sequence. We begin by demonstrating the connection between ordinary correlation and generalized correlation.

A.1. Proof of Lemma 1

This section supplies the proof of Lemma 1. First, note that for any values μ, ν, the function $(x-\mu)(y-\nu)$ is CAV on the whole of $\mathbb{R}^2$. Therefore, the if part of the proposition is immediate. For the only if part, we begin with the case where the joint distribution of $\tilde{X}$ and $\tilde{Y}$ is given by $f(x, y) \equiv \exp\{-\frac{1}{2}[x^2 - 2\tau xy + y^2]\}$ with $\tau \geqslant 0$. Then we have to establish

$$\frac{\partial}{\partial\tau}\left(\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\psi(x, y)\exp\{-\tfrac{1}{2}[x^2 - 2\tau xy + y^2]\}\right)$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy\psi(x, y)f(x, y)\,\mathrm{d}x\,\mathrm{d}y < 0. \quad (3)$$

Splitting the integral into four ranges, it suffices to prove that

$$\int_0^{\infty}\int_0^{\infty} xy\psi(x, y)f(x, y)\,\mathrm{d}x\,\mathrm{d}y$$
$$+\int_0^{\infty}\int_{-\infty}^{0} xy\psi(x, y)f(x, y)\,\mathrm{d}x\,\mathrm{d}y$$
$$+\int_{-\infty}^{0}\int_0^{\infty} xy\psi(x, y)f(x, y)\,\mathrm{d}x\,\mathrm{d}y$$
$$+\int_{-\infty}^{0}\int_{-\infty}^{0} xy\psi(x, y)f(x, y)\,\mathrm{d}x\,\mathrm{d}y < 0. \quad (4)$$

Using the identities $xy = (-x)(-y)$, $(-x)(y) = (x)(-y) = -xy$ and $f(x, y) = f(-x, -y)$, $f(x, -y) = f(-x, y) = \exp\{4\tau xy\}f(x, y) \geqslant f(x, y)$ (recall that $\tau \geqslant 0$) we are reduced to showing

$$\int_0^{\infty}\int_0^{\infty} xy([\psi(x, y) + \psi(-x, -y)]$$
$$-[\psi(-x, y) + \psi(x, -y)])f(x, y)\,\mathrm{d}x\,\mathrm{d}y < 0. \quad (5)$$

The inequality in (5) follows directly from the definition of correlation aversion of ψ.

To prove the result when $\tau \leqslant 0$, we set $\tilde{X}^{(1)} \equiv -\tilde{X}$ and note that $\tilde{X}^{(1)}$ and $\tilde{Y}$ are positively correlated. Let $\hat{\psi}(s, t) \equiv -\psi(-s, t)$. Then $\hat{\psi}$ is CAV and $\mathrm{E}[\hat{\psi}(\tilde{X}^{(1)}, \tilde{Y})]$ is decreasing as $-\tau$ becomes more positive. It follows that $\mathrm{E}[\psi(\tilde{X}, \tilde{Y})] \equiv -\mathrm{E}[\hat{\psi}(\tilde{X}^{(1)}, \tilde{Y})]$ increases as τ becomes more negative.

For the general case, let $\tilde{X}$, $\tilde{Y}$ be distributed $\mathrm{N}(\mu_1, \sigma_1)$, $\mathrm{N}(\mu_2, \sigma_2)$, respectively. Then setting

$$\hat{\psi}(s, t) = \psi\left(\frac{s-\mu_1}{\sigma_1}, \frac{t-\mu_2}{\sigma_2}\right),$$

it is easily verified that $\hat{\psi}$ is CAV and that $\mathrm{E}_{X,Y}[\psi] = \mathrm{E}_{X^{(1)},Y(1)}[\hat{\psi}]$ where $\tilde{X}^{(1)}$ and $\tilde{Y}(1)$ are standard normals. Therefore, the only if part of Lemma 1 holds for all joint normal distributions. □

A.2. Proofs of Propositions 1 and 2 and Lemma 2

The following lemma is needed for developing the proofs.

Lemma A: concavity of the minimum. (*Let H_i: $\mathbb{R}^n \to \mathbb{R}$ be concave functions. Then $H = \min\{H_1, \ldots, H_n\}$ is a concave function.*

Proof of Proposition 1. Because of the facts that (i) functions involving only x or y but not both are trivially CAV; (ii) $\min(x + y \mid K)$ is CAV as a function of x, y for fixed K; and (iii) the sum of two CAV functions is CAV, it follows that $\mathrm{CF}(x, y \mid K)$ and $\mathrm{CD}(x, y \mid K_x, K_y)$ (see Eq. (1)) are CAV as functions of x, y. Noting that constant functions and linear functions are trivially concave, an application of Lemma A yields that $\mathrm{CF}(x, y \mid K)$ and $\mathrm{CD}(x, y \mid K_x, K_y)$ are concave as functions respectively of K or $\{K_x, K_y\}$ with x, y fixed and concave as functions of x, y with K (respectively, K_x, K_y) fixed. □

Proof of Proposition 2. From Proposition 1, for fixed x, y and any concave U, $U(\mathrm{CF}(x, y \mid K))$ is concave in K. Therefore, for any joint density $f(x, y) \geqslant 0$, $\lambda, \mu \geqslant 0$, $\lambda + \mu = 1$,

$$\int_X\int_Y U[CF(x, y \mid \lambda K + \mu L)]f(x, y)\,\mathrm{d}y\,\mathrm{d}x$$
$$\geqslant \int_X\int_Y \{\lambda U[CF(x, y \mid K)]$$
$$+\mu U[CF(x, y \mid K)]\}f(x, y)\,\mathrm{d}y\,\mathrm{d}x.$$

In other words, for any joint distribution of $\tilde{X}$ and $\tilde{Y}$, $\mathrm{E}[U(\mathrm{CF}(x, y \mid K))]$ is concave in K. It follows that there is a unique maximal level of investment $K^* \in [0, \infty]$.

Similarly, $\mathrm{CD}(x, y \mid K_x, K_y)$ is concave as a function of K_x, K_y and there are optimal investment levels K_x^* and K_y^*. □

Proof of Lemma 2. Suppose that $C = C_x = C_y$ and let K_x^* and K_y^* denote the optimal investments in dedicated capacity. Then investing $K_x^* + K_y^*$ in flexible capacity provides greater payoffs at the same cost:

$$\mathrm{CF}(x, y \mid K_x + K_y) - \mathrm{CD}(x, y \mid K_x, K_y)$$
$$\begin{cases} = 0, & y \leqslant K_y,\ x \leqslant K_x, \\ = (\alpha - \beta)(x - K_x), & x \geqslant K_x \text{ and } y \geqslant K_y, \\ > 0, & \text{otherwise}. \end{cases}$$

Thus, flexible capacity is preferred to dedicated capacity if there is no cost differential. Further, when $\alpha > \beta$, this preference is strict as long as there is some positive probability of landing in the region $\{(x, y) \mid x > K_x^* \text{ or } y > K_y^*\}$.

In contrast, if $\beta = C > C_x$, let K^* denote the optimal investment in flexible capacity. Then investing $K_x = K^*$ in dedicated ($\tilde{X}$) capacity generates the same cash flows as the optimal flexible investment but at a lower cost. So at $C = \beta$, dedicated capacity is always preferred.

Finally, noting that (optimal) payoffs decrease as capacity cost increases, we reach the conclusion that there is a unique cost level $C^*(f)$, $\beta > C^*(f) > C_x = C_y$ where the value of (optimal) flexible investment equals the value of (optimal) dedicated investment. □

A.3. The first-order condition in K

The basic technical problem is that

$$\frac{\partial}{\partial K}(U[CF(x, y \mid K)])$$
$$= \begin{cases} -CU'(\alpha x + \beta y - CK), \\ \quad x \leqslant K,\ x + y \leqslant K, \\ (\beta - C)U'(\alpha x + \beta(K - x) - CK), \\ \quad x \leqslant K,\ x + y > K, \\ \alpha U'(\alpha K), \quad x > K, \end{cases} \tag{6}$$

and the derivatives from the left and right are not equal along the lines $x + y = K$ and $x = K$. Consequently, the expression obtained in Eq. (2) may not be continuous. Nevertheless, $\mathrm{E}[U(\mathrm{CF}(x, y \mid K))]$ is a concave function of K with left and right derivatives at every point. so that we can then solve for the optimum K^* as the supremum of all values of K where the (left) derivative is positive:

$$K^* = \sup\left\{K \,\middle|\, \frac{\partial}{\partial K}(\mathrm{E}[U[\mathrm{CF}(x, y \mid K)]]) \geqslant 0\right\}.$$

Notice that even though the derivative of $\mathrm{CF}(x, y \mid K)$ may be undefined at certain points, $\mathrm{E}U[\mathrm{CF}(x, y \mid K)]$ could be continuously differentiable everywhere. In particular, as long as the joint distribution of $\tilde{X}$ and $\tilde{Y}$ is continuous, the set of points on the *lines* $x = K$ and $x + y = K$ will have measure zero and, consequently, the expected cash flow function $\mathrm{E}U[\mathrm{CF}(x, y \mid K)]$ will be differentiable in K and this derivative would be continuous.

One particular property of the derivative of the cash flow function, employed below, is sufficiently important to record as a separate result.

Lemma B: derivative of the cash flow function increasing. *For fixed x, $(\partial/\partial K)(U[\mathrm{CF}(x, y \mid K)])$ is nondecreasing in y.*

Proof. We use Eq. (6) to derive this result. If $x > K$, the derivative is independent of y. When $x < K$ and $y < K - x$, because U is increasing and concave and $\mathrm{CF}(x, y \mid K)$ increases in y, $-CU'[\mathrm{CF}(x, y \mid k)]$ is negative and increases in y. Because $(\beta - C)U'[\mathrm{CF}(x, y \mid K)] > 0$ for $y \geqslant K - x$, it follows that the derivative increases in y for $x < K$. Finally, at $x = K$, the left and right derivatives are different, but both increase in y as they are limits of the derivatives when $x > K$ and $x < K$, respectively. □

A.4. Proofs of Propositions 3 through 7 and Lemma 3

The proofs of all the remaining results are provided below.

Proof of Proposition 3. From Proposition 1, both $\mathrm{CF}(x, y \mid K)$ and $\mathrm{CD}(x, y \mid K_x, K_y)$ are CAV. Hence by Epstein and Tanny (1980) Proposition 2, $U(\mathrm{CF}(x, y \mid K))$ is CAV (as a function of x, y) for

every K and $U(\mathrm{CD}(x, y \mid K_x, K_y))$ is CAV for every K_x, K_y. Hence it follows from Definition 2 that the expected utility value of the investment decreases as generalized correlation increases. □

Proof of Proposition 4. For a risk-neutral investor, the value of the investment in dedicated capacity, denoted by V_0, is unchanged by the level of generalized correlation. Let $K^*(F)$ denote the optimal investment level with F. By definition, the value of flexible investment at level $K^*(F)$ and cost $C^*(F)$ is V_0. However, from Proposition 3, when the cost of flexible capacity is $C^*(F)$,

$$E_G[U(\mathrm{CF}(x, y \mid K^*(F)))] > E_F[U(\mathrm{CF}(x, y \mid K^*(F)))] = V_0.$$

Thus, the value of flexible investment when its cost is $C^*(F)$ and product demands follow the distribution G is greater than the value of dedicated capacity. Hence, the break-even cost, $C^*(G)$, satisfies $C^*(G) > C^*(F)$. □

Proof of Proposition 5. Let K^* denote the optimal investment level for U. It suffices to show that

$$\begin{aligned}\frac{\partial}{\partial K}(\mathrm{E}[V(\mathrm{CF})])|_{K^*} &= -C\int_{\mathscr{A}(K^*)} V'[\mathrm{CF}]f \\ &\quad + (\beta - C)\int_{\mathscr{B}(K^*)} V'[\mathrm{CF}]f \\ &\quad + (\alpha - C)\int_{\mathscr{C}(K^*)} V'[\mathrm{CF}]f > 0 \qquad (7)\end{aligned}$$

(where we have suppressed the arguments x, y for expositional convenience in the derivation above). Now define

$$w_1 = \sup\left\{\frac{V'(\mathrm{CF}(x, y \mid K))}{U'(\mathrm{CF}(x, y \mid K))}\,\middle|\,(x, y) \in \mathscr{A}(K^*)\right\}$$

and

$$w_2 = \inf\left\{\frac{V'(\mathrm{CF}(x, y \mid K))}{U'(\mathrm{CF}(x, y \mid K))}\,\middle|\,(x, y) \in \mathscr{B}(K^*) \cup \mathscr{C}(K^*)\right\}.$$

From the facts that $\mathrm{CF}(x, y \mid K)$ increases as we move from $\mathscr{A}(K)$ through $\mathscr{B}(K)$ to $\mathscr{C}(K)$ and that the hypothesis on U and V is equivalent to the fact that V'/U' is increasing, we obtain $w_1 < w_2$. Further, from the definitions of w_1 and w_2 and the fact that $w_1 < w_2$, we obtain

$$\begin{aligned}&\frac{\partial}{\partial K}(\mathrm{E}[V(\mathrm{CF})])\Big|_{K^*} \\ &\geqslant w_1\left[-C\int_{\mathscr{A}(K^*)} U'[\mathrm{CF}]f\right] \\ &\quad + w_2\left[(\beta - C)\int_{\mathscr{B}(K^*)} U'[\mathrm{CF}]f \right. \\ &\quad \left. + (\alpha - C)\int_{\mathscr{C}(K^*)} U'[\mathrm{CF}]f\right] \\ &\geqslant w_1\left[-C\int_{\mathscr{A}(K^*)} U'[\mathrm{CF}]f \right. \\ &\quad + (\beta - C)\int_{\mathscr{B}(K^*)} U'[\mathrm{CF}]f \\ &\quad \left. + (\alpha - C)\int_{\mathscr{C}(K^*)} U'[\mathrm{CF}]f\right] = 0, \qquad (8)\end{aligned}$$

where the equality in Eq. (8) obtains since K^* is optimal for U (the expression in parentheses in the second-last equation is the first-order condition for U as in Eq. (2)). We have established (7). □

Proof of Lemma 3. Because cash flows are concave and increasing in realized demand, it follows that F is preferred to G whenever $F(y \mid x)$ either first or second order stochastically dominates $G(y \mid x)$ for every x.

Denote the derivative of $\mathrm{E}[U(\mathrm{CF}(x, y \mid K))]$ with respect to K by $\mathrm{E}[U'(\mathrm{CF}(x, y \mid K))]$. Because $\mathrm{E}[U'(\mathrm{CF}(x, y \mid K))]$ increases in y for every x (Lemma B), it follows from Eq. (2) that $\mathrm{E}_F[U'(C(x, y \mid K))] \geqslant \mathrm{E}_G[U'(C(x, y \mid K))]$ and hence that

$$\begin{aligned}K_F^* &= \sup\{K \mid \mathrm{E}_F[U'(C(x, y \mid K))] \geqslant 0\} \\ &\geqslant \sup\{K \mid \mathrm{E}_G[U'(C(x, y \mid K))] \geqslant 0\} \\ &= K_G^*. \quad \square\end{aligned}$$

Proof of Proposition 6. Let $\tilde{X}$, $\tilde{Y}$ be jointly normal with variance-covariance matrix

$$\begin{bmatrix} \sigma_X^2 & \rho \\ \rho & \sigma_Y^2 \end{bmatrix}.$$

Then the posterior distribution of $\tilde{Y}$ conditional on $\tilde{X}$ is normal with mean $(\rho/\sigma_X^2)x$ and variance $\sigma_Y^2 - \rho^2/\sigma_X^2$. If the covariance, ρ, is held fixed and σ_Y^2 is increased, the posterior distribution decreases in the sense of second-order stochastic dominance. Therefore, by the first part of Proposition 3, the value of the investment in manufacturing capacity decreases as the variance of $\tilde{Y}$ increases.

Proof of Proposition 7. Follows immediately from Lemma 3(2).

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