Chi-squared tests for evaluation and comparison of asset pricing models

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Not-for-publication appendix

In this appendix, we provide additional tests and simulation results that are not included in the paper. We use the same notation as in the paper.

1 Tests for model specification and pairwise comparison

1.1 HJ-distance test of model specification

The following lemma presents the asymptotic distributions of the sample squared HJ-distance under correctly specified and misspecified models.

Lemma B.1. *Under Assumptions A, B and C in the paper,*

(a) *if* $\delta = 0$,

$$T\hat{\delta}^2 \overset{A}{\sim} F_{n-k}(\xi), \tag{1}$$

where the ξ_i*'s are the eigenvalues of*

$$A = P'U^{-\frac{1}{2}}SU^{-\frac{1}{2}}P, \tag{2}$$

with P *being an* $n \times (n-k)$ *orthonormal matrix whose columns are orthogonal to* $U^{-\frac{1}{2}}D$.

(b) *if* $\delta > 0$,

$$\sqrt{T}(\hat{\delta}^2 - \delta^2) \overset{A}{\sim} N(0, \sigma_b^2), \tag{3}$$

where $\sigma_b^2 = \sum_{j=-\infty}^{\infty} E[b_t b_{t+j}]$ *and* $b_t = \phi_t(\theta^*) - \delta^2$.

Proof. See Appendix.

The asymptotic distribution and matrix A in part (a) of Lemma B.1 coincide with the ones derived by Jagannathan and Wang (1996) and Parker and Julliard (2005) for the case of linear and nonlinear models, respectively. The asymptotic normality in part (b) of Lemma B.1 has been established by Hansen, Heaton and Luttmer (1995). To conduct inference, the covariance matrices in Lemma B.1 should be replaced with consistent estimators. In particular, in part (a), we can replace A with its sample analog

$$\hat{A} = \hat{P}'\hat{U}^{-\frac{1}{2}}\hat{S}\hat{U}^{-\frac{1}{2}}\hat{P}, \tag{4}$$

where $\hat{S}$ is obtained using a nonparametric heteroskedasticity and autocorrelation consistent (HAC) estimator (see, for example, Newey and West, 1987 and Andrews, 1991), $\hat{P}$ is an orthonormal matrix whose columns are orthogonal to $\hat{U}^{-\frac{1}{2}}\hat{D}$ with $\hat{D} = \frac{1}{T}\sum_{t=1}^{T}\left[x_t \frac{\partial y_t(\hat{\gamma})}{\partial \gamma'}\right]$. Similarly, in part (b) we can use a HAC estimator to estimate the variance σ_b^2.

1.2 Joint tests of correct specification of two non-nested models

Let $\mathcal{F} = \{y_t^{\mathcal{F}}(\gamma_{\mathcal{F}})\ ; \gamma_{\mathcal{F}} \in \Gamma_{\mathcal{F}}\}$ and $\mathcal{G} = \{y_t^{\mathcal{G}}(\gamma_{\mathcal{G}})\ ; \gamma_{\mathcal{G}} \in \Gamma_{\mathcal{G}}\}$ be two competing models with k_1 and k_2 parameter vectors and population squared HJ-distances $\delta_{\mathcal{F}}^2 = \min_{\gamma_{\mathcal{F}}} \max_{\lambda_{\mathcal{F}}} E[\phi_t^{\mathcal{F}}(\theta_{\mathcal{F}})]$ and $\delta_{\mathcal{G}}^2 = \min_{\gamma_{\mathcal{G}}} \max_{\lambda_{\mathcal{G}}} E[\phi_t^{\mathcal{G}}(\theta_{\mathcal{G}})]$, respectively. To test $H_0 : \delta_{\mathcal{F}}^2 = \delta_{\mathcal{G}}^2 = 0$ for non-nested models, we can use the test statistic $T(\hat{\delta}_{\mathcal{F}}^2 - \hat{\delta}_{\mathcal{G}}^2)$ based on the difference of the sample HJ-distances of models $\mathcal{F}$ and $\mathcal{G}$. Alternatively, we can employ an LM test that measures the distance of the Lagrange multipliers of the two models from zero. This will provide a joint test of correct model specification for models $\mathcal{F}$ and $\mathcal{G}$.

To set up the notation, define $e_t^{\mathcal{F}}(\gamma_{\mathcal{F}}^*) = x_t y_t^{\mathcal{F}}(\gamma_{\mathcal{F}}^*) - q_{t-1}$, $e_t^{\mathcal{G}}(\gamma_{\mathcal{G}}^*) = x_t y_t^{\mathcal{G}}(\gamma_{\mathcal{G}}^*) - q_{t-1}$, and

$$\mathcal{S} \equiv \begin{bmatrix} S_{\mathcal{F}} & S_{\mathcal{FG}} \\ S_{\mathcal{GF}} & S_{\mathcal{G}} \end{bmatrix} = \sum_{j=-\infty}^{\infty} E\left[\tilde{e}_t \tilde{e}_{t+j}'\right], \tag{5}$$

where $\tilde{e}_t = [e_t^{\mathcal{F}}(\gamma_{\mathcal{F}}^*)',\ e_t^{\mathcal{G}}(\gamma_{\mathcal{G}}^*)']'$. Let $P_{\mathcal{F}}$ and $P_{\mathcal{G}}$ denote orthonormal matrices with dimensions $n \times (n - k_1)$ and $n \times (n - k_2)$ whose columns are orthogonal to $U^{-\frac{1}{2}}D_{\mathcal{F}}$ and $U^{-\frac{1}{2}}D_{\mathcal{G}}$, respectively, where $D_{\mathcal{F}}$ ($D_{\mathcal{G}}$) is the D matrix for model $\mathcal{F}$ ($\mathcal{G}$) defined in the paper. Also, denote by $\hat{P}_{\mathcal{F}}$, $\hat{P}_{\mathcal{G}}$, $\hat{S}_{\mathcal{F}}$, $\hat{S}_{\mathcal{G}}$, $\hat{S}_{\mathcal{FG}}$, $\hat{S}_{\mathcal{GF}}$, $\hat{\lambda}_{\mathcal{F}}$, and $\hat{\lambda}_{\mathcal{G}}$ the sample counterparts of $P_{\mathcal{F}}$, $P_{\mathcal{G}}$, $S_{\mathcal{F}}$, $S_{\mathcal{G}}$, $S_{\mathcal{FG}}$, $S_{\mathcal{GF}}$, $\lambda_{\mathcal{F}}$ and $\lambda_{\mathcal{G}}$, respectively. Finally, let

$$\tilde{\lambda}_{\mathcal{FG}} = \begin{bmatrix} \hat{P}_{\mathcal{F}}'\hat{U}^{\frac{1}{2}}\hat{\lambda}_{\mathcal{F}} \\ \hat{P}_{\mathcal{G}}'\hat{U}^{\frac{1}{2}}\hat{\lambda}_{\mathcal{G}} \end{bmatrix}. \tag{6}$$

The following lemma provides the appropriate asymptotic distribution of the difference in the sample squared HJ-distances when both models are correctly specified and an LM test of $H_0 : \lambda_{\mathcal{F}} = \lambda_{\mathcal{G}} = 0_n$ (which is equivalent to testing $H_0 : \delta_{\mathcal{F}}^2 = \delta_{\mathcal{G}}^2 = 0$).

Lemma B.2. *Suppose that Assumptions A, B and C in the paper hold for each model and* $y_t^{\mathcal{F}}(\gamma_{\mathcal{F}}^*) \neq y_t^{\mathcal{G}}(\gamma_{\mathcal{G}}^*)$. *Then, under* $H_0 : \delta_{\mathcal{F}}^2 = \delta_{\mathcal{G}}^2 = 0$,

(a)

$$T(\hat{\delta}^2_{\mathcal{F}} - \hat{\delta}^2_{\mathcal{G}}) \overset{A}{\sim} F_{2n-k_1-k_2}(\xi), \tag{7}$$

where the ξ_i*'s are the eigenvalues of the matrix*

$$\left[\begin{array}{cc} P'_{\mathcal{F}}U^{-\frac{1}{2}}S_{\mathcal{F}}U^{-\frac{1}{2}}P_{\mathcal{F}} & -P'_{\mathcal{F}}U^{-\frac{1}{2}}S_{\mathcal{F}\mathcal{G}}U^{-\frac{1}{2}}P_{\mathcal{G}} \\ P'_{\mathcal{G}}U^{-\frac{1}{2}}S_{\mathcal{G}\mathcal{F}}U^{-\frac{1}{2}}P_{\mathcal{F}} & -P'_{\mathcal{G}}U^{-\frac{1}{2}}S_{\mathcal{G}}U^{-\frac{1}{2}}P_{\mathcal{G}} \end{array}\right], \tag{8}$$

(b)

$$LM_{\tilde{\lambda}_{\mathcal{F}\mathcal{G}}} = T\tilde{\lambda}'_{\mathcal{F}\mathcal{G}}\hat{\Sigma}^{-1}_{\tilde{\lambda}_{\mathcal{F}\mathcal{G}}}\tilde{\lambda}_{\mathcal{F}\mathcal{G}} \overset{A}{\sim} \chi^2_{2n-k_1-k_2}, \tag{9}$$

where

$$\hat{\Sigma}_{\tilde{\lambda}_{\mathcal{F}\mathcal{G}}} = \left[\begin{array}{cc} \hat{P}'_{\mathcal{F}}\hat{U}^{-\frac{1}{2}}\hat{S}_{\mathcal{F}}\hat{U}^{-\frac{1}{2}}\hat{P}_{\mathcal{F}} & \hat{P}'_{\mathcal{F}}\hat{U}^{-\frac{1}{2}}\hat{S}_{\mathcal{F}\mathcal{G}}\hat{U}^{-\frac{1}{2}}\hat{P}_{\mathcal{G}} \\ \hat{P}'_{\mathcal{G}}\hat{U}^{-\frac{1}{2}}\hat{S}_{\mathcal{G}\mathcal{F}}\hat{U}^{-\frac{1}{2}}\hat{P}_{\mathcal{F}} & \hat{P}'_{\mathcal{G}}\hat{U}^{-\frac{1}{2}}\hat{S}_{\mathcal{G}}\hat{U}^{-\frac{1}{2}}\hat{P}_{\mathcal{G}} \end{array}\right]. \tag{10}$$

Proof. See Appendix.

Since the eigenvalues ξ_i's in part (a) of Lemma B.2 can take on both positive and negative values, the test of the hypothesis $H_0 : \delta^2_{\mathcal{F}} = \delta^2_{\mathcal{G}} = 0$ should be two-sided. The LM test in part (b) of Lemma B.2 offers an alternative way of testing $H_0 : \delta^2_{\mathcal{F}} = \delta^2_{\mathcal{G}} = 0$ (using the equivalence between $H_0 : \delta^2_{\mathcal{F}} = \delta^2_{\mathcal{G}} = 0$ and $H_0 : \lambda_{\mathcal{F}} = \lambda_{\mathcal{G}} = 0_n$) but it is easier to implement and is expected to deliver power gains compared to the test in part (a). The reason is that the test in part (a) may have low power in finite samples when $\hat{\delta}^2_{\mathcal{F}} \approx \hat{\delta}^2_{\mathcal{G}} \neq 0$ although it is still consistent since under the alternative $\hat{\delta}^2_{\mathcal{F}} - \hat{\delta}^2_{\mathcal{G}} = O_p(T^{-1/2})$ and $|T(\hat{\delta}^2_{\mathcal{F}} - \hat{\delta}^2_{\mathcal{G}})| \to \infty$.

1.3 Tests for pairwise comparison of nested and non-nested models

In this section, we present the tests of $H_0 : y^{\mathcal{F}}_t(\gamma^*_{\mathcal{F}}) = y^{\mathcal{G}}_t(\gamma^*_{\mathcal{G}})$ and $H_0 : \sigma^2_d = 0$ that can be used for both nested and non-nested models. Let $H_{\mathcal{F}} = \frac{\partial^2 E[\phi^{\mathcal{F}}_t(\theta^*_{\mathcal{F}})]}{\partial\theta_{\mathcal{F}}\partial\theta'_{\mathcal{F}}}$ and $M_{\mathcal{F}} = \lim_{T\to\infty} \text{Var}\left[\frac{1}{\sqrt{T}}\sum_{t=1}^T \frac{\partial\phi_t(\theta^*_{\mathcal{F}})}{\partial\theta}\right]$ with $H_{\mathcal{G}}$ and $M_{\mathcal{G}}$ defined similarly. Marcellino and Rossi (2008) among others show that under $H_0 : \phi^{\mathcal{F}}_t(\theta^*_{\mathcal{F}}) = \phi^{\mathcal{G}}_t(\theta^*_{\mathcal{G}})$,

$$T(\hat{\delta}^2_{\mathcal{F}} - \hat{\delta}^2_{\mathcal{G}}) \overset{A}{\sim} F_{2n+k_1+k_2}(\xi), \tag{11}$$

where the ξ_i's are the eigenvalues of the matrix

$$\frac{1}{2}\left[\begin{array}{cc} -H^{-1}_{\mathcal{F}}M_{\mathcal{F}} & -H^{-1}_{\mathcal{F}}M_{\mathcal{F}\mathcal{G}} \\ H^{-1}_{\mathcal{G}}M_{\mathcal{G}\mathcal{F}} & H^{-1}_{\mathcal{G}}M_{\mathcal{G}} \end{array}\right]. \tag{12}$$

Several remarks regarding this inference procedure are in order. First, estimating the ξ_i's from the sample counterpart of the matrix in (12) produces more nonzero estimated ξ_i's than the theory suggests. In addition, the estimated ξ_i's do not have the same sign. This is problematic because for nested models, the larger model has a smaller sample HJ-distance by construction. By not imposing the constraints that the ξ_i's should have the same sign, the nonnegative test statistic $T(\hat{\delta}_{\mathcal{F}}^2 - \hat{\delta}_{\mathcal{G}}^2)$ is compared with a distribution that can take on both positive and negative values. Theorems 2 and 3 in the paper propose restricted versions of the test in (11) in addition to easier-to-implement chi-squared tests.

Alternatively, we can directly test $H_0 : \sigma_d^2 = 0$. In this case,

$$T\hat{\sigma}_d^2 \stackrel{A}{\sim} F_{2n+k_1+k_2}(\xi), \tag{13}$$

where $\hat{\sigma}_d^2$ is a consistent estimator of σ_d^2 and the ξ_i's are four times the squared eigenvalues of the matrix in (12) (see Golden, 2003).

It can be shown that for nested models (for $\mathcal{F} \subset \mathcal{G}$), the test in (13) simplifies to

$$T\hat{\sigma}_d^2 \stackrel{A}{\sim} F_{k_2-k_1}(\xi), \tag{14}$$

where the ξ_i's are four times the squared eigenvalues of the matrix

$$(\Psi_*^{\mathcal{G}} \tilde{H}_{\mathcal{G}} \Psi_*^{\mathcal{G}\prime})^{-1} \Psi_*^{\mathcal{G}} \Sigma_{\hat{\gamma}_{\mathcal{G}}} \Psi_*^{\mathcal{G}\prime}, \tag{15}$$

where $\Psi_*^{\mathcal{G}}$, $\tilde{H}_{\mathcal{G}}$ and $\Sigma_{\hat{\gamma}_{\mathcal{G}}}$ are defined in the paper.

Similarly, for overlapping models ($\mathcal{F} \not\subset \mathcal{G}$, $\mathcal{G} \not\subset \mathcal{F}$ and $\mathcal{H} = \mathcal{F} \cap \mathcal{G}$ with a k_3 parameter vector), the restricted version of the test of $H_0 : \sigma_d^2 = 0$ is given by

$$T\hat{\sigma}_d^2 \stackrel{A}{\sim} F_{k_1+k_2-2k_3}(\xi), \tag{16}$$

where the ξ_i's are four times the squared eigenvalues of the matrix

$$\begin{bmatrix} -(\Psi_*^{\mathcal{F}} \tilde{H}_{\mathcal{F}} \Psi_*^{\mathcal{F}\prime})^{-1} & 0_{(k_1-k_3)\times(k_2-k_3)} \\ 0_{(k_2-k_3)\times(k_1-k_3)} & (\Psi_*^{\mathcal{G}} \tilde{H}_{\mathcal{G}} \Psi_*^{\mathcal{G}\prime})^{-1} \end{bmatrix} \Psi_*^{\mathcal{FG}} \Sigma_{\hat{\gamma}_{\mathcal{FG}}} \Psi_*^{\mathcal{FG}\prime}. \tag{17}$$

Finally, for strictly non-nested models ($\mathcal{F} \cap \mathcal{G} = \emptyset$), the restricted version of the test of $H_0 : \sigma_d^2 = 0$ is

$$T\hat{\sigma}_d^2 \stackrel{A}{\sim} F_{2n-k_1-k_2}(\xi), \tag{18}$$

where the ξ_i's are four times the squared eigenvalues of the matrix

$$\begin{bmatrix} P'_{\mathcal{F}}U^{-\frac{1}{2}}S_{\mathcal{F}}U^{-\frac{1}{2}}P_{\mathcal{F}} & -P'_{\mathcal{F}}U^{-\frac{1}{2}}S_{\mathcal{F}\mathcal{G}}U^{-\frac{1}{2}}P_{\mathcal{G}} \\ P'_{\mathcal{G}}U^{-\frac{1}{2}}S_{\mathcal{G}\mathcal{F}}U^{-\frac{1}{2}}P_{\mathcal{F}} & -P'_{\mathcal{G}}U^{-\frac{1}{2}}S_{\mathcal{G}}U^{-\frac{1}{2}}P_{\mathcal{G}} \end{bmatrix}, \tag{19}$$

defined in the previous section.

2 Simulation inputs

Appendix D in the paper provides the details of the simulation designs. In this section, we include the necessary descriptive statistics about the data used to calibrate the parameters in the simulation study. Since the factor means are set equal to zero (see Appendix D in the paper), we only need to provide the population covariance matrix (V_{22}), the population cross-covariance matrix between the returns and the factors (V_{21}) and the population covariance matrix of the factors (V_{11}). In all simulations, the number of test assets (n) is 26 (gross risk-free rate plus gross returns on the 25 Fama-French size and book-to-market ranked portfolios).

In the linear SDF case, the first 13 columns of V_{22} (the first row and the first column of the matrix are multiplied by 1000) are given by

$$\begin{bmatrix}
0.0476 & -0.0875 & -0.0690 & -0.0478 & -0.0569 & -0.0739 & -0.0584 & -0.0452 & -0.0493 & -0.0240 & -0.0513 & -0.0733 & -0.0446 \\
-0.0875 & 0.0236 & 0.0189 & 0.0165 & 0.0154 & 0.0161 & 0.0194 & 0.0159 & 0.0134 & 0.0124 & 0.0132 & 0.0166 & 0.0132 \\
-0.0690 & 0.0189 & 0.0174 & 0.0148 & 0.0141 & 0.0148 & 0.0165 & 0.0143 & 0.0124 & 0.0117 & 0.0125 & 0.0143 & 0.0120 \\
-0.0478 & 0.0165 & 0.0148 & 0.0137 & 0.0128 & 0.0136 & 0.0143 & 0.0127 & 0.0112 & 0.0109 & 0.0115 & 0.0123 & 0.0108 \\
-0.0569 & 0.0154 & 0.0141 & 0.0128 & 0.0125 & 0.0132 & 0.0134 & 0.0121 & 0.0108 & 0.0106 & 0.0113 & 0.0116 & 0.0103 \\
-0.0739 & 0.0161 & 0.0148 & 0.0136 & 0.0132 & 0.0147 & 0.0138 & 0.0126 & 0.0114 & 0.0113 & 0.0124 & 0.0118 & 0.0107 \\
-0.0584 & 0.0194 & 0.0165 & 0.0143 & 0.0134 & 0.0138 & 0.0184 & 0.0146 & 0.0124 & 0.0115 & 0.0119 & 0.0158 & 0.0125 \\
-0.0452 & 0.0159 & 0.0143 & 0.0127 & 0.0121 & 0.0126 & 0.0146 & 0.0131 & 0.0111 & 0.0106 & 0.0111 & 0.0128 & 0.0109 \\
-0.0493 & 0.0134 & 0.0124 & 0.0112 & 0.0108 & 0.0114 & 0.0124 & 0.0111 & 0.0104 & 0.0097 & 0.0101 & 0.0109 & 0.0097 \\
-0.0240 & 0.0124 & 0.0117 & 0.0109 & 0.0106 & 0.0113 & 0.0115 & 0.0106 & 0.0097 & 0.0101 & 0.0102 & 0.0099 & 0.0093 \\
-0.0513 & 0.0132 & 0.0125 & 0.0115 & 0.0113 & 0.0124 & 0.0119 & 0.0111 & 0.0101 & 0.0102 & 0.0116 & 0.0103 & 0.0096 \\
-0.0733 & 0.0166 & 0.0143 & 0.0123 & 0.0116 & 0.0118 & 0.0158 & 0.0128 & 0.0109 & 0.0099 & 0.0103 & 0.0147 & 0.0111 \\
-0.0446 & 0.0132 & 0.0120 & 0.0108 & 0.0103 & 0.0107 & 0.0125 & 0.0109 & 0.0097 & 0.0093 & 0.0096 & 0.0111 & 0.0101 \\
-0.0477 & 0.0113 & 0.0107 & 0.0098 & 0.0095 & 0.0101 & 0.0108 & 0.0099 & 0.0090 & 0.0089 & 0.0092 & 0.0095 & 0.0088 \\
-0.0489 & 0.0106 & 0.0103 & 0.0094 & 0.0093 & 0.0100 & 0.0101 & 0.0094 & 0.0088 & 0.0088 & 0.0092 & 0.0088 & 0.0085 \\
-0.0223 & 0.0116 & 0.0112 & 0.0102 & 0.0101 & 0.0111 & 0.0106 & 0.0099 & 0.0092 & 0.0093 & 0.0102 & 0.0092 & 0.0089 \\
-0.0520 & 0.0145 & 0.0123 & 0.0105 & 0.0099 & 0.0099 & 0.0139 & 0.0111 & 0.0094 & 0.0085 & 0.0088 & 0.0127 & 0.0099 \\
-0.0606 & 0.0114 & 0.0105 & 0.0095 & 0.0091 & 0.0094 & 0.0113 & 0.0099 & 0.0087 & 0.0084 & 0.0087 & 0.0100 & 0.0090 \\
-0.0565 & 0.0102 & 0.0095 & 0.0087 & 0.0085 & 0.0090 & 0.0099 & 0.0089 & 0.0082 & 0.0080 & 0.0084 & 0.0087 & 0.0082 \\
-0.0513 & 0.0099 & 0.0094 & 0.0086 & 0.0085 & 0.0090 & 0.0094 & 0.0086 & 0.0080 & 0.0081 & 0.0085 & 0.0082 & 0.0079 \\
-0.0333 & 0.0113 & 0.0109 & 0.0099 & 0.0096 & 0.0105 & 0.0106 & 0.0099 & 0.0091 & 0.0091 & 0.0098 & 0.0092 & 0.0087 \\
-0.0621 & 0.0095 & 0.0083 & 0.0071 & 0.0067 & 0.0067 & 0.0095 & 0.0077 & 0.0067 & 0.0062 & 0.0063 & 0.0091 & 0.0070 \\
-0.0377 & 0.0084 & 0.0075 & 0.0067 & 0.0064 & 0.0066 & 0.0085 & 0.0073 & 0.0065 & 0.0062 & 0.0064 & 0.0078 & 0.0068 \\
-0.0616 & 0.0068 & 0.0063 & 0.0056 & 0.0055 & 0.0057 & 0.0067 & 0.0060 & 0.0054 & 0.0054 & 0.0054 & 0.0062 & 0.0057 \\
-0.0348 & 0.0070 & 0.0069 & 0.0063 & 0.0063 & 0.0068 & 0.0068 & 0.0065 & 0.0061 & 0.0063 & 0.0065 & 0.0063 & 0.0060 \\
-0.0506 & 0.0079 & 0.0077 & 0.0069 & 0.0070 & 0.0076 & 0.0075 & 0.0070 & 0.0066 & 0.0067 & 0.0074 & 0.0067 & 0.0065
\end{bmatrix}$$

and the last 13 columns are given by

−0.0477	−0.0489	−0.0223	−0.0520	−0.0606	−0.0565	−0.0513	−0.0333	−0.0621	−0.0377	−0.0616	−0.0348	−0.0506
0.0113	0.0106	0.0116	0.0145	0.0114	0.0102	0.0099	0.0113	0.0095	0.0084	0.0068	0.0070	0.0079
0.0107	0.0103	0.0112	0.0123	0.0105	0.0095	0.0094	0.0109	0.0083	0.0075	0.0063	0.0069	0.0077
0.0098	0.0094	0.0102	0.0105	0.0095	0.0087	0.0086	0.0099	0.0071	0.0067	0.0056	0.0063	0.0069
0.0095	0.0093	0.0101	0.0099	0.0091	0.0085	0.0085	0.0096	0.0067	0.0064	0.0055	0.0063	0.0070
0.0101	0.0100	0.0111	0.0099	0.0094	0.0090	0.0090	0.0105	0.0067	0.0066	0.0057	0.0068	0.0076
0.0108	0.0101	0.0106	0.0139	0.0113	0.0099	0.0094	0.0106	0.0095	0.0085	0.0067	0.0068	0.0075
0.0099	0.0094	0.0099	0.0111	0.0099	0.0089	0.0086	0.0099	0.0077	0.0073	0.0060	0.0065	0.0070
0.0090	0.0088	0.0092	0.0094	0.0087	0.0082	0.0080	0.0091	0.0067	0.0065	0.0054	0.0061	0.0066
0.0089	0.0088	0.0093	0.0085	0.0084	0.0080	0.0081	0.0091	0.0062	0.0062	0.0054	0.0063	0.0067
0.0092	0.0092	0.0102	0.0088	0.0087	0.0084	0.0085	0.0098	0.0063	0.0064	0.0054	0.0065	0.0074
0.0095	0.0088	0.0092	0.0127	0.0100	0.0087	0.0082	0.0092	0.0091	0.0078	0.0062	0.0063	0.0067
0.0088	0.0085	0.0089	0.0099	0.0090	0.0082	0.0079	0.0087	0.0070	0.0068	0.0057	0.0060	0.0065
0.0087	0.0081	0.0085	0.0084	0.0082	0.0077	0.0075	0.0084	0.0061	0.0061	0.0053	0.0059	0.0063
0.0081	0.0086	0.0087	0.0078	0.0077	0.0077	0.0077	0.0086	0.0057	0.0059	0.0052	0.0062	0.0065
0.0085	0.0087	0.0102	0.0079	0.0080	0.0078	0.0080	0.0094	0.0057	0.0059	0.0052	0.0063	0.0071
0.0084	0.0078	0.0079	0.0121	0.0092	0.0080	0.0075	0.0080	0.0086	0.0074	0.0059	0.0057	0.0061
0.0082	0.0077	0.0080	0.0092	0.0089	0.0078	0.0073	0.0080	0.0067	0.0066	0.0056	0.0059	0.0062
0.0077	0.0077	0.0078	0.0080	0.0078	0.0077	0.0071	0.0078	0.0060	0.0061	0.0052	0.0059	0.0061
0.0075	0.0077	0.0080	0.0075	0.0073	0.0071	0.0076	0.0080	0.0057	0.0057	0.0050	0.0059	0.0062
0.0084	0.0086	0.0094	0.0080	0.0080	0.0078	0.0080	0.0102	0.0060	0.0060	0.0053	0.0065	0.0073
0.0061	0.0057	0.0057	0.0086	0.0067	0.0060	0.0057	0.0060	0.0078	0.0062	0.0050	0.0050	0.0052
0.0061	0.0059	0.0059	0.0074	0.0066	0.0061	0.0057	0.0060	0.0062	0.0063	0.0050	0.0051	0.0053
0.0053	0.0052	0.0052	0.0059	0.0056	0.0052	0.0050	0.0053	0.0050	0.0050	0.0051	0.0046	0.0046
0.0059	0.0062	0.0063	0.0057	0.0059	0.0059	0.0059	0.0065	0.0050	0.0051	0.0046	0.0058	0.0056
0.0063	0.0065	0.0071	0.0061	0.0062	0.0061	0.0062	0.0073	0.0052	0.0053	0.0046	0.0056	0.0073

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In addition, V_{21} (the matrix is multiplied by 1000) is given by

−0.1039	−0.0028	0.0190	−0.0015	−0.0074	−0.0060	−0.0000
10.0010	6.6710	−3.1832	−0.0541	0.1961	0.2304	0.0009
8.8866	5.8508	−1.8130	−0.0695	0.1848	0.2532	0.0011
7.7876	5.2760	−1.0787	−0.0602	0.1630	0.2339	0.0012
7.4489	4.9435	−0.6097	−0.0492	0.1536	0.2160	0.0011
7.7372	5.3229	0.0261	−0.0513	0.1797	0.2385	0.0015
9.6234	5.4680	−3.3460	−0.0872	0.1711	0.3355	0.0013
8.1399	4.7476	−1.6474	−0.0724	0.1458	0.2524	0.0013
7.2145	4.0510	−0.8552	−0.0437	0.1481	0.2498	0.0013
6.8046	3.8339	−0.0197	−0.0602	0.1475	0.2785	0.0015
7.1070	4.1678	0.3161	−0.0670	0.1657	0.2150	0.0016
8.9141	4.4117	−3.3189	−0.0692	0.1556	0.3312	0.0012
7.3653	3.6984	−1.3671	−0.0678	0.1452	0.2759	0.0013
6.6115	3.2744	−0.4112	−0.0494	0.1384	0.2407	0.0013
6.3383	3.0177	0.2536	−0.0511	0.1369	0.2543	0.0015
6.4669	3.5102	0.6817	−0.0504	0.1603	0.2153	0.0016
8.2503	3.3307	−3.1970	−0.0576	0.1341	0.3014	0.0011
7.0069	2.8412	−1.2485	−0.0650	0.1203	0.2861	0.0015
6.3969	2.5004	−0.4126	−0.0611	0.1259	0.2336	0.0017
6.1369	2.5123	0.0642	−0.0352	0.1423	0.2100	0.0011
6.6610	3.0569	0.4514	−0.0436	0.1400	0.2542	0.0018
6.7358	1.3186	−2.4705	−0.0389	0.1312	0.2781	0.0010
6.0179	1.2755	−1.3505	−0.0519	0.0962	0.2654	0.0014
5.1085	0.8960	−0.6450	−0.0443	0.0771	0.2449	0.0013
5.2417	1.1248	0.2833	−0.0295	0.1110	0.2390	0.0015
5.5297	1.4114	0.5855	−0.0448	0.1109	0.2588	0.0017

,

where the seven columns correspond, in the order, to r_{mkt}, r_{smb}, r_{hml}, Δc_{dur}, Δc_{ndur}, cay, and $\Delta c_{ndur} * cay$. For example, for the linear YOGO model, the V_{21} matrix (multiplied by 1000) would be (26×3) (formed by taking columns one, four and five of the matrix above).

Finally, the V_{11} matrix (multiplied by 1000) is given by

6.5794	1.8969	−1.5929	−0.0443	0.1271	0.2729	0.0012
1.8969	3.0605	−0.4002	−0.0147	0.0448	0.0059	−0.0000
−1.5929	−0.4002	2.9897	0.0102	0.0017	−0.0509	0.0004
−0.0443	−0.0147	0.0102	0.0289	0.0065	−0.0322	−0.0001
0.1271	0.0448	0.0017	0.0065	0.0276	−0.0008	−0.0000
0.2729	0.0059	−0.0509	−0.0322	−0.0008	0.1824	0.0007
0.0012	−0.0000	0.0004	−0.0001	−0.0000	0.0007	0.0000

,

with the factors in the same order as above. For example, for the linear YOGO model, the V_{11} matrix (multiplied by 1000) would be (3×3), i.e.,

$$
\begin{bmatrix}
6.5794 & -0.0443 & 0.1271 \\
-0.0443 & 0.0289 & 0.0065 \\
0.1271 & 0.0065 & 0.0276
\end{bmatrix}.
$$

In the log-linear SDF case, the first 13 columns of V_{22} (the first row and the first column of the matrix are multiplied by 1000) are given by

$$
\begin{bmatrix}
0.0461 & -0.1027 & -0.0753 & -0.0553 & -0.0613 & -0.0769 & -0.0733 & -0.0544 & -0.0561 & -0.0281 & -0.0533 & -0.0861 & -0.0525 \\
-0.1027 & 0.0235 & 0.0185 & 0.0160 & 0.0149 & 0.0155 & 0.0194 & 0.0156 & 0.0133 & 0.0121 & 0.0128 & 0.0168 & 0.0131 \\
-0.0753 & 0.0185 & 0.0167 & 0.0140 & 0.0134 & 0.0139 & 0.0163 & 0.0138 & 0.0121 & 0.0112 & 0.0119 & 0.0142 & 0.0117 \\
-0.0553 & 0.0160 & 0.0140 & 0.0129 & 0.0120 & 0.0126 & 0.0140 & 0.0122 & 0.0108 & 0.0103 & 0.0108 & 0.0121 & 0.0104 \\
-0.0613 & 0.0149 & 0.0134 & 0.0120 & 0.0117 & 0.0122 & 0.0131 & 0.0116 & 0.0103 & 0.0100 & 0.0106 & 0.0114 & 0.0099 \\
-0.0769 & 0.0155 & 0.0139 & 0.0126 & 0.0122 & 0.0135 & 0.0134 & 0.0120 & 0.0108 & 0.0106 & 0.0115 & 0.0115 & 0.0102 \\
-0.0733 & 0.0194 & 0.0163 & 0.0140 & 0.0131 & 0.0134 & 0.0185 & 0.0144 & 0.0124 & 0.0112 & 0.0117 & 0.0159 & 0.0125 \\
-0.0544 & 0.0156 & 0.0138 & 0.0122 & 0.0116 & 0.0120 & 0.0144 & 0.0127 & 0.0109 & 0.0102 & 0.0106 & 0.0128 & 0.0106 \\
-0.0561 & 0.0133 & 0.0121 & 0.0108 & 0.0103 & 0.0108 & 0.0124 & 0.0109 & 0.0102 & 0.0093 & 0.0097 & 0.0109 & 0.0096 \\
-0.0281 & 0.0121 & 0.0112 & 0.0103 & 0.0100 & 0.0106 & 0.0112 & 0.0102 & 0.0093 & 0.0095 & 0.0097 & 0.0098 & 0.0090 \\
-0.0533 & 0.0128 & 0.0119 & 0.0108 & 0.0106 & 0.0115 & 0.0117 & 0.0106 & 0.0097 & 0.0097 & 0.0108 & 0.0102 & 0.0093 \\
-0.0861 & 0.0168 & 0.0142 & 0.0121 & 0.0114 & 0.0115 & 0.0159 & 0.0128 & 0.0109 & 0.0098 & 0.0102 & 0.0149 & 0.0111 \\
-0.0525 & 0.0131 & 0.0117 & 0.0104 & 0.0099 & 0.0102 & 0.0125 & 0.0106 & 0.0096 & 0.0090 & 0.0093 & 0.0111 & 0.0099 \\
-0.0527 & 0.0114 & 0.0105 & 0.0096 & 0.0093 & 0.0098 & 0.0110 & 0.0098 & 0.0090 & 0.0087 & 0.0090 & 0.0096 & 0.0088 \\
-0.0502 & 0.0104 & 0.0099 & 0.0090 & 0.0088 & 0.0094 & 0.0099 & 0.0091 & 0.0085 & 0.0084 & 0.0088 & 0.0087 & 0.0082 \\
-0.0246 & 0.0112 & 0.0107 & 0.0097 & 0.0095 & 0.0103 & 0.0104 & 0.0095 & 0.0089 & 0.0088 & 0.0096 & 0.0090 & 0.0085 \\
-0.0608 & 0.0144 & 0.0121 & 0.0103 & 0.0097 & 0.0097 & 0.0139 & 0.0110 & 0.0094 & 0.0084 & 0.0087 & 0.0127 & 0.0098 \\
-0.0680 & 0.0116 & 0.0104 & 0.0093 & 0.0089 & 0.0092 & 0.0114 & 0.0099 & 0.0088 & 0.0083 & 0.0085 & 0.0102 & 0.0089 \\
-0.0598 & 0.0100 & 0.0093 & 0.0084 & 0.0082 & 0.0086 & 0.0098 & 0.0087 & 0.0081 & 0.0077 & 0.0081 & 0.0086 & 0.0080 \\
-0.0531 & 0.0097 & 0.0091 & 0.0082 & 0.0081 & 0.0085 & 0.0093 & 0.0083 & 0.0078 & 0.0077 & 0.0081 & 0.0082 & 0.0077 \\
-0.0332 & 0.0109 & 0.0103 & 0.0093 & 0.0090 & 0.0098 & 0.0103 & 0.0094 & 0.0087 & 0.0086 & 0.0092 & 0.0090 & 0.0083 \\
-0.0655 & 0.0097 & 0.0083 & 0.0071 & 0.0067 & 0.0067 & 0.0097 & 0.0077 & 0.0068 & 0.0061 & 0.0063 & 0.0092 & 0.0070 \\
-0.0408 & 0.0085 & 0.0075 & 0.0066 & 0.0063 & 0.0065 & 0.0086 & 0.0073 & 0.0065 & 0.0061 & 0.0063 & 0.0078 & 0.0067 \\
-0.0627 & 0.0070 & 0.0063 & 0.0056 & 0.0054 & 0.0057 & 0.0068 & 0.0060 & 0.0054 & 0.0053 & 0.0054 & 0.0063 & 0.0056 \\
-0.0336 & 0.0069 & 0.0067 & 0.0061 & 0.0060 & 0.0064 & 0.0068 & 0.0064 & 0.0060 & 0.0060 & 0.0063 & 0.0062 & 0.0059 \\
-0.0458 & 0.0080 & 0.0076 & 0.0067 & 0.0068 & 0.0073 & 0.0076 & 0.0069 & 0.0065 & 0.0065 & 0.0071 & 0.0068 & 0.0064
\end{bmatrix}
$$

and the last 13 columns are given by

$$
\begin{bmatrix}
-0.0527 & -0.0502 & -0.0246 & -0.0608 & -0.0680 & -0.0598 & -0.0531 & -0.0332 & -0.0655 & -0.0408 & -0.0627 & -0.0336 & -0.0458 \\
0.0114 & 0.0104 & 0.0112 & 0.0144 & 0.0116 & 0.0100 & 0.0097 & 0.0109 & 0.0097 & 0.0085 & 0.0070 & 0.0069 & 0.0080 \\
0.0105 & 0.0099 & 0.0107 & 0.0121 & 0.0104 & 0.0093 & 0.0091 & 0.0103 & 0.0083 & 0.0075 & 0.0063 & 0.0067 & 0.0076 \\
0.0096 & 0.0090 & 0.0097 & 0.0103 & 0.0093 & 0.0084 & 0.0082 & 0.0093 & 0.0071 & 0.0066 & 0.0056 & 0.0061 & 0.0067 \\
0.0093 & 0.0088 & 0.0095 & 0.0097 & 0.0089 & 0.0082 & 0.0081 & 0.0090 & 0.0067 & 0.0063 & 0.0054 & 0.0060 & 0.0068 \\
0.0098 & 0.0094 & 0.0103 & 0.0097 & 0.0092 & 0.0086 & 0.0085 & 0.0098 & 0.0067 & 0.0065 & 0.0057 & 0.0064 & 0.0073 \\
0.0110 & 0.0099 & 0.0104 & 0.0139 & 0.0114 & 0.0098 & 0.0093 & 0.0103 & 0.0097 & 0.0086 & 0.0068 & 0.0068 & 0.0076 \\
0.0098 & 0.0091 & 0.0095 & 0.0110 & 0.0099 & 0.0087 & 0.0083 & 0.0094 & 0.0077 & 0.0073 & 0.0060 & 0.0064 & 0.0069 \\
0.0090 & 0.0085 & 0.0089 & 0.0094 & 0.0088 & 0.0081 & 0.0078 & 0.0087 & 0.0068 & 0.0065 & 0.0054 & 0.0060 & 0.0065 \\
0.0087 & 0.0084 & 0.0088 & 0.0084 & 0.0083 & 0.0077 & 0.0077 & 0.0086 & 0.0061 & 0.0061 & 0.0053 & 0.0060 & 0.0065 \\
0.0090 & 0.0088 & 0.0096 & 0.0087 & 0.0085 & 0.0081 & 0.0081 & 0.0092 & 0.0063 & 0.0063 & 0.0054 & 0.0063 & 0.0071 \\
0.0096 & 0.0087 & 0.0090 & 0.0127 & 0.0102 & 0.0086 & 0.0082 & 0.0090 & 0.0092 & 0.0078 & 0.0063 & 0.0062 & 0.0068 \\
0.0088 & 0.0082 & 0.0085 & 0.0098 & 0.0089 & 0.0080 & 0.0077 & 0.0083 & 0.0070 & 0.0067 & 0.0056 & 0.0059 & 0.0064 \\
0.0086 & 0.0079 & 0.0083 & 0.0085 & 0.0082 & 0.0076 & 0.0074 & 0.0081 & 0.0062 & 0.0061 & 0.0053 & 0.0058 & 0.0063 \\
0.0079 & 0.0082 & 0.0082 & 0.0077 & 0.0076 & 0.0074 & 0.0074 & 0.0081 & 0.0057 & 0.0058 & 0.0051 & 0.0059 & 0.0063 \\
0.0083 & 0.0082 & 0.0095 & 0.0078 & 0.0079 & 0.0075 & 0.0077 & 0.0088 & 0.0057 & 0.0058 & 0.0051 & 0.0060 & 0.0068 \\
0.0085 & 0.0077 & 0.0078 & 0.0119 & 0.0093 & 0.0079 & 0.0074 & 0.0078 & 0.0086 & 0.0074 & 0.0059 & 0.0057 & 0.0062 \\
0.0082 & 0.0076 & 0.0079 & 0.0093 & 0.0089 & 0.0077 & 0.0072 & 0.0077 & 0.0069 & 0.0067 & 0.0057 & 0.0058 & 0.0062 \\
0.0076 & 0.0074 & 0.0075 & 0.0079 & 0.0077 & 0.0075 & 0.0069 & 0.0075 & 0.0060 & 0.0060 & 0.0051 & 0.0057 & 0.0060 \\
0.0074 & 0.0074 & 0.0077 & 0.0074 & 0.0072 & 0.0069 & 0.0073 & 0.0076 & 0.0056 & 0.0056 & 0.0049 & 0.0056 & 0.0060 \\
0.0081 & 0.0081 & 0.0088 & 0.0078 & 0.0077 & 0.0075 & 0.0076 & 0.0095 & 0.0060 & 0.0059 & 0.0052 & 0.0061 & 0.0070 \\
0.0062 & 0.0057 & 0.0057 & 0.0086 & 0.0069 & 0.0060 & 0.0056 & 0.0060 & 0.0079 & 0.0062 & 0.0050 & 0.0049 & 0.0052 \\
0.0061 & 0.0058 & 0.0058 & 0.0074 & 0.0067 & 0.0060 & 0.0056 & 0.0059 & 0.0062 & 0.0062 & 0.0049 & 0.0050 & 0.0053 \\
0.0053 & 0.0051 & 0.0051 & 0.0059 & 0.0057 & 0.0051 & 0.0049 & 0.0052 & 0.0050 & 0.0049 & 0.0050 & 0.0045 & 0.0045 \\
0.0058 & 0.0059 & 0.0060 & 0.0057 & 0.0058 & 0.0057 & 0.0056 & 0.0061 & 0.0049 & 0.0050 & 0.0045 & 0.0056 & 0.0054 \\
0.0063 & 0.0063 & 0.0068 & 0.0062 & 0.0062 & 0.0060 & 0.0060 & 0.0070 & 0.0052 & 0.0053 & 0.0045 & 0.0054 & 0.0071
\end{bmatrix}.
$$

In addition, V_{21} (the matrix is multiplied by 1000) is given by

$$\begin{bmatrix} -0.0591 & -0.1067 & -0.0040 & 0.0158 & -0.0073 & -0.0063 & -0.0014 \\ 10.0119 & 10.2757 & 6.4540 & -3.2497 & 0.2008 & 0.1252 & -0.0560 \\ 8.7143 & 8.9235 & 5.5649 & -1.8180 & 0.1843 & 0.1501 & -0.0687 \\ 7.5990 & 7.7728 & 4.9715 & -1.1230 & 0.1605 & 0.1111 & -0.0559 \\ 7.2600 & 7.4329 & 4.6344 & -0.6840 & 0.1498 & 0.1170 & -0.0471 \\ 7.4879 & 7.6788 & 4.9401 & -0.1254 & 0.1747 & 0.1304 & -0.0461 \\ 9.6422 & 9.8683 & 5.3268 & -3.3531 & 0.1762 & 0.0914 & -0.0843 \\ 8.0169 & 8.1964 & 4.5564 & -1.6700 & 0.1465 & 0.0932 & -0.0672 \\ 7.1312 & 7.2987 & 3.8893 & -0.9080 & 0.1477 & 0.0978 & -0.0416 \\ 6.6456 & 6.7748 & 3.6301 & -0.0887 & 0.1455 & 0.0952 & -0.0547 \\ 6.9387 & 7.0958 & 3.9055 & 0.1875 & 0.1626 & 0.1157 & -0.0604 \\ 8.9031 & 9.1308 & 4.3296 & -3.3137 & 0.1629 & 0.0866 & -0.0670 \\ 7.2469 & 7.4129 & 3.5752 & -1.3770 & 0.1449 & 0.0706 & -0.0652 \\ 6.5775 & 6.7319 & 3.1840 & -0.4726 & 0.1382 & 0.0822 & -0.0456 \\ 6.1914 & 6.3347 & 2.8736 & 0.1554 & 0.1346 & 0.0768 & -0.0467 \\ 6.3149 & 6.4329 & 3.3084 & 0.5238 & 0.1556 & 0.0958 & -0.0466 \\ 8.1799 & 8.3684 & 3.2484 & -3.1243 & 0.1383 & 0.0603 & -0.0568 \\ 6.9831 & 7.1618 & 2.7972 & -1.2805 & 0.1217 & 0.0692 & -0.0625 \\ 6.2619 & 6.4176 & 2.4058 & -0.4697 & 0.1241 & 0.0588 & -0.0566 \\ 5.9680 & 6.1114 & 2.4021 & -0.0097 & 0.1407 & 0.0679 & -0.0317 \\ 6.4578 & 6.5863 & 2.8807 & 0.3242 & 0.1369 & 0.1103 & -0.0391 \\ 6.7234 & 6.8930 & 1.3037 & -2.4337 & 0.1327 & 0.0598 & -0.0344 \\ 5.9461 & 6.0785 & 1.2648 & -1.3354 & 0.0965 & 0.0442 & -0.0501 \\ 5.0138 & 5.1537 & 0.9070 & -0.6570 & 0.0750 & 0.0510 & -0.0430 \\ 5.1097 & 5.2180 & 1.0743 & 0.2292 & 0.1088 & 0.0617 & -0.0275 \\ 5.4627 & 5.5857 & 1.3680 & 0.4989 & 0.1107 & 0.1024 & -0.0425 \end{bmatrix},$$

where the seven columns correspond, in the order, to $\ln(R_{mkt})$, $\ln(1+r_{mkt})$, $\ln(1+r_{smb})$, $\ln(1+r_{hml})$, Δc_{ndur} at time t and $t-1$, respectively, and Δc_{dur}.

Finally, the V_{11} matrix (multiplied by 1000) is given by

$$\begin{bmatrix} 6.4069 & 6.5649 & 1.8502 & -1.5634 & 0.1199 & 0.0616 & -0.0428 \\ 6.5649 & 6.7740 & 1.8851 & -1.6090 & 0.1293 & 0.0690 & -0.0414 \\ 1.8502 & 1.8851 & 2.9867 & -0.4156 & 0.0447 & 0.0397 & -0.0158 \\ -1.5634 & -1.6090 & -0.4156 & 2.9453 & 0.0026 & 0.0240 & 0.0094 \\ 0.1199 & 0.1293 & 0.0447 & 0.0026 & 0.0276 & 0.0095 & 0.0065 \\ 0.0616 & 0.0690 & 0.0397 & 0.0240 & 0.0095 & 0.0283 & 0.0096 \\ -0.0428 & -0.0414 & -0.0158 & 0.0094 & 0.0065 & 0.0096 & 0.0289 \end{bmatrix},$$

with the factors in the same order as above.

3 Additional simulation results

3.1 Multivariate normally distributed factors and returns

3.1.1 Nested models

In Table 5, we evaluate the size and power properties of the tests of $H_0 : y_t^{\mathcal{F}}(\gamma_{\mathcal{F}}^*) = y_t^{\mathcal{G}}(\gamma_{\mathcal{G}}^*)$ and $H_0 : \sigma_d^2 = 0$ in pairwise nested model comparison tests. The results in this section complement the pairwise nested model comparison results reported in Panels A and C of Table 3 in the paper. Specifically, we consider three additional tests: 1) the unrestricted weighted chi-squared test (UFT) in (11); 2) the unrestricted variance test (UVT) in (13); and 3) the restricted variance test (RVT) in (14). The data and simulation designs are the same as the ones for Panels A and C of Table 3 in the paper.

Table 5 about here

Several interesting insights emerge from Table 5:

- UFT performs well in terms of size across linear and nonlinear asset pricing specifications. However, it is dominated in terms of power by the chi-squared test in part (b) of Theorem 2 in the paper;
- UVT is very conservative under the null and has substantially lower power than all the other tests;
- RVT exhibits better size and power properties than UVT but is still dominated in terms of size and power by UFT and especially by the chi-squared test in part (b) of Theorem 2 in the paper.

3.1.2 Non-nested models

In Table 6, we evaluate the size and power properties of the tests of $H_0 : y_t^{\mathcal{F}}(\gamma_{\mathcal{F}}^*) = y_t^{\mathcal{G}}(\gamma_{\mathcal{G}}^*)$ and $H_0 : \sigma_d^2 = 0$ in pairwise non-nested model comparison tests. The results in this section complement the pairwise non-nested model comparison results reported in Panels A and C of Table 4 in the paper. Specifically, we consider three additional tests: 1) the unrestricted weighted chi-squared test (UFT) in (11); 2) the unrestricted variance test (UVT) in (13); and 3) the restricted variance test (RVT) in (16). The data and simulation designs are the same as the ones for Panels A and C of Table 4 in the paper.

Table 6 about here

Several interesting insights emerge from Table 6:

- UFT performs well in terms of size (for $T > 120$) across linear and nonlinear asset pricing specifications. However, it is strongly dominated in terms of power by the chi-squared test in part (b) of Theorem 3 in the paper. The power of UFT is much lower for nonlinear models than for linear models;

- UVT is very conservative under the null and has substantially lower power than all the other tests;

- RVT exhibits better size and power properties than UVT but is still dominated in terms of size and power by the chi-squared test in part (b) of Theorem 3 in the paper.

In light of the discussion in Section 3 in the paper, if we reject $H_0 : y_t^{\mathcal{F}}(\gamma_{\mathcal{F}}^*) = y_t^{\mathcal{G}}(\gamma_{\mathcal{G}}^*)$, we should then test whether models $\mathcal{F}$ and $\mathcal{G}$ are correctly specified before applying the normal test. In Table 7, we evaluate the size and power properties of the tests of $H_0 : \delta_{\mathcal{F}}^2 = \delta_{\mathcal{G}}^2 = 0$ and $H_0 : \sigma_d^2 = 0$ for two overlapping distinct SDFs. We consider five tests: 1) the restricted weighted chi-squared test (RFT) in part (a) of Lemma B.2; 2) the LM test (LM) in part (b) of Lemma B.2; 3) the unrestricted weighted chi-squared test (UFT) in (11); 4) the unrestricted variance test (UVT) in (13); and 5) the restricted variance test (RVT) in (16).

To examine size in the linear SDF case, we consider two correctly specified one-factor models. The factor in each model is created by adding a normally distributed error to r_{mkt}. The error term in each model has a mean of zero and a variance of 20% of the variance of r_{mkt}. The two error terms are independent of each other as well as of the returns and the factor. The implied population HJ-distances of the two models are both equal to zero. To analyze power, we simply set the return means equal to the means estimated from the sample and consider the two models above. This implies that the population HJ-distances of these two models are both equal to 0.6524.

To examine size in the log-linear SDF case, we also consider two correctly specified one-factor models. The factor in each model is created by adding a normally distributed error to $\ln(R_{mkt})$. The error term in each model has a mean of zero and a variance of 20% of the variance of $\ln(R_{mkt})$. The two error terms are independent of each other as well as of the returns and the factor. The implied population HJ-distances of the two models are both equal to zero. To analyze power, we simply set the return means equal to the means estimated from the sample and consider the two models above. This implies that the population HJ-distances of these two models are both equal to 0.6377. Panels A and B of Table 7 are for linear and nonlinear SDFs, respectively.

Table 7 about here

Several interesting insights emerge from Table 7:

- RFT in part (a) of Lemma (B.2) enjoys the best size properties overall, followed by UFT;
- LM in part (b) of Lemma (B.2) is severely undersized but has the best power properties among the five tests. Given our simulation design, the underrejection problem under the null for the LM test occurs because the eigenvalues of $\Sigma_{\tilde{\lambda}_{\mathcal{FG}}}$ are of very different magnitudes and, in particular, half of them are very close to zero. Sampling error in $\hat{\Sigma}_{\tilde{\lambda}_{\mathcal{FG}}}$ then leads to many tiny eigenvalues, and $\hat{\Sigma}_{\tilde{\lambda}_{\mathcal{FG}}}^{-1}$ has many abnormally large eigenvalues. Since these large eigenvalues are associated with linear combinations of $\tilde{\lambda}_{\mathcal{FG}}$ with abnormally low variance, this will lead to a severe underrejection problem for LM in our simulation experiment.
- UVT and RVT are conservative under the null but perform well in terms of power for $T > 120$.

3.2 Multivariate t(8) distributed factors and returns

In Tables 8 and 9, we use the same data and simulation designs used in Tables 1 through 4 in the paper. The only difference is that we assume that the factors and the returns are multivariate t-distributed. The number of degrees of freedom of the t-distribution is set equal to 8 in all simulations.

Tables 8 and 9 about here

The t-distribution results are very similar to the ones obtained under normality. Tables 8 and 9 show that the only noteworthy difference is a slight increase in the empirical size of the tests in the t-distribution case.

Appendix: Proofs

Proof of Lemma B.1. (a) From the definition of H in the paper, we can use the partitioned matrix inverse formula to obtain

$$H^{-1} = \begin{bmatrix} 2C & 2D' \\ 2D & -2U \end{bmatrix}^{-1} = \frac{1}{2}\begin{bmatrix} \tilde{H} & \tilde{H}D'U^{-1} \\ U^{-1}D\tilde{H} & -U^{-1}+U^{-1}D\tilde{H}D'U^{-1} \end{bmatrix}, \tag{A.1}$$

where $\tilde{H} = (C + D'U^{-1}D)^{-1}$. Under the null hypothesis $H_0 : \delta = 0$, using Lemma A.1 in the paper we have

$$\hat{\delta}^2 = -\frac{1}{2}\bar{v}_T(\theta^*)'H^{-1}\bar{v}_T(\theta^*) + o_p\left(\frac{1}{T}\right) \tag{A.2}$$

since $\lambda^* = 0_n$ and $\phi_t(\gamma^*, 0_n) = E[\phi_t(\gamma^*, 0_n)] = 0$. Let $\bar{v}_T(\theta^*) = [\bar{v}_{1,T}(\theta^*)', \bar{v}_{2,T}(\theta^*)']'$, where $\bar{v}_{1,T}(\theta^*)$ denotes the first k elements of $\bar{v}_T(\theta^*)$. Under the null, $\bar{v}_{1,T}(\theta^*) = 0_k$ and $C = 0_{k\times k}$.

Then, it follows that

$$\begin{aligned} T\hat{\delta}^2 &= -\frac{1}{2}\sqrt{T}\bar{v}_T(\theta^*)'H^{-1}\sqrt{T}\bar{v}_T(\theta^*) + o_p(1) \\ &= \frac{1}{4}\sqrt{T}\bar{v}_{2T}(\theta^*)'[U^{-1} - U^{-1}D(D'U^{-1}D)^{-1}D'U^{-1}]\sqrt{T}\bar{v}_{2,T}(\theta^*) + o_p(1) \\ &= \frac{1}{4}\sqrt{T}\bar{v}_{2T}(\theta^*)'U^{-\frac{1}{2}}PP'U^{-\frac{1}{2}}\sqrt{T}\bar{v}_{2,T}(\theta^*) + o_p(1) \end{aligned} \tag{A.3}$$

by using the fact that $I_n - U^{-\frac{1}{2}}D(D'U^{-1}D)^{-1}D'U^{-\frac{1}{2}} = PP'$. Also, Assumptions A, B and C in the paper ensure that the empirical process $\sqrt{T}\bar{v}_{2,T}(\theta^*)$ obeys the central limit theorem and

$$\sqrt{T}\bar{v}_{2,T}(\theta^*) \overset{A}{\sim} N(0_n, M_{\lambda\lambda}). \tag{A.4}$$

Thus, using the fact that $M_{\lambda\lambda} = 4S$ under the null, we obtain

$$T\hat{\delta}^2 \overset{A}{\sim} z'S^{\frac{1}{2}}U^{-\frac{1}{2}}PP'U^{-\frac{1}{2}}S^{\frac{1}{2}}z, \tag{A.5}$$

where $z \sim N(0_n, I_n)$. Since $S^{\frac{1}{2}}U^{-\frac{1}{2}}PP'U^{-\frac{1}{2}}S^{\frac{1}{2}}$ has the same nonzero eigenvalues as $P'U^{-\frac{1}{2}}SU^{-\frac{1}{2}}P$, we have

$$T\hat{\delta}^2 \overset{A}{\sim} F_{n-k}(\xi), \tag{A.6}$$

where the ξ_i's are the eigenvalues of $P'U^{-\frac{1}{2}}SU^{-\frac{1}{2}}P$. This completes the proof of part (a).

(b) Now consider the case $\delta > 0$. In this situation, the asymptotic behavior of $\sqrt{T}(\hat{\delta}^2 - \delta^2)$ is determined by $\frac{1}{\sqrt{T}}\sum_{t=1}^{T}(\phi_t(\theta^*) - E[\phi_t(\theta^*)])$, which converges weakly to a Gaussian process. Under

Assumptions A, B and C in the paper, and since $E[\phi_t(\theta^*)] = \delta^2$, we have

$$\sqrt{T}(\hat{\delta}^2 - \delta^2) = \frac{1}{\sqrt{T}} \sum_{t=1}^{T} (\phi_t(\theta^*) - E[\phi_t(\theta^*)]) + o_p(1) \overset{A}{\sim} N(0, \sigma_b^2). \tag{A.7}$$

This completes the proof of part (b). ■

Proof of Lemma B.2. (a) From Lemma A.1 in the paper and under the null $H_0 : \delta^2_{\mathcal{F}} = \delta^2_{\mathcal{G}} = 0$, we obtain

$$\begin{aligned} &T(\hat{\delta}^2_{\mathcal{F}} - \hat{\delta}^2_{\mathcal{G}}) \\ &= \frac{1}{4} \begin{bmatrix} \sqrt{T}\bar{v}^{\mathcal{F}}_{2,T}(\theta^*_{\mathcal{F}}) \\ \sqrt{T}\bar{v}^{\mathcal{G}}_{2,T}(\theta^*_{\mathcal{G}}) \end{bmatrix}' \begin{bmatrix} U^{-\frac{1}{2}} P_{\mathcal{F}} P'_{\mathcal{F}} U^{-\frac{1}{2}} & 0_{n\times n} \\ 0_{n\times n} & -U^{-\frac{1}{2}} P_{\mathcal{G}} P'_{\mathcal{G}} U^{-\frac{1}{2}} \end{bmatrix} \begin{bmatrix} \sqrt{T}\bar{v}^{\mathcal{F}}_{2,T}(\theta^*_{\mathcal{F}}) \\ \sqrt{T}\bar{v}^{\mathcal{G}}_{2,T}(\theta^*_{\mathcal{G}}) \end{bmatrix} + o_p(1). \end{aligned} \tag{A.8}$$

From Assumptions A, B and C in the paper, we have

$$\begin{bmatrix} \sqrt{T}\bar{v}^{\mathcal{F}}_{2,T}(\theta^*_{\mathcal{F}}) \\ \sqrt{T}\bar{v}^{\mathcal{G}}_{2,T}(\theta^*_{\mathcal{G}}) \end{bmatrix} \overset{A}{\sim} N\left(0_{2n}, 4\mathcal{S}\right). \tag{A.9}$$

Hence,

$$T(\hat{\delta}^2_{\mathcal{F}} - \hat{\delta}^2_{\mathcal{G}}) \overset{A}{\sim} z'\mathcal{S}^{\frac{1}{2}} \begin{bmatrix} U^{-\frac{1}{2}} P_{\mathcal{F}} P'_{\mathcal{F}} U^{-\frac{1}{2}} & 0_{n\times n} \\ 0_{n\times n} & -U^{-\frac{1}{2}} P_{\mathcal{G}} P'_{\mathcal{G}} U^{-\frac{1}{2}} \end{bmatrix} \mathcal{S}^{\frac{1}{2}} z, \tag{A.10}$$

where $z \sim N(0_{2n}, I_{2n})$. Furthermore, the nonzero eigenvalues of

$$\mathcal{S}^{\frac{1}{2}} \begin{bmatrix} U^{-\frac{1}{2}} P_{\mathcal{F}} P'_{\mathcal{F}} U^{-\frac{1}{2}} & 0_{n\times n} \\ 0_{n\times n} & -U^{-\frac{1}{2}} P_{\mathcal{G}} P'_{\mathcal{G}} U^{-\frac{1}{2}} \end{bmatrix} \mathcal{S}^{\frac{1}{2}} \tag{A.11}$$

are the same as the eigenvalues of the matrix

$$\begin{bmatrix} P'_{\mathcal{F}} U^{-\frac{1}{2}} & 0_{(n-k_1)\times n} \\ 0_{(n-k_2)\times n} & P'_{\mathcal{G}} U^{-\frac{1}{2}} \end{bmatrix} \mathcal{S} \begin{bmatrix} U^{-\frac{1}{2}} P_{\mathcal{F}} & 0_{n\times(n-k_2)} \\ 0_{n\times(n-k_1)} & -U^{-\frac{1}{2}} P_{\mathcal{G}} \end{bmatrix}. \tag{A.12}$$

Then, it follows that

$$T(\hat{\delta}^2_{\mathcal{F}} - \hat{\delta}^2_{\mathcal{G}}) \overset{A}{\sim} F_{2n-k_1-k_2}(\xi), \tag{A.13}$$

where the ξ_i's are the eigenvalues of the matrix

$$\begin{bmatrix} P'_{\mathcal{F}} U^{-\frac{1}{2}} S_{\mathcal{F}} U^{-\frac{1}{2}} P_{\mathcal{F}} & -P'_{\mathcal{F}} U^{-\frac{1}{2}} S_{\mathcal{F}\mathcal{G}} U^{-\frac{1}{2}} P_{\mathcal{G}} \\ P'_{\mathcal{G}} U^{-\frac{1}{2}} S_{\mathcal{G}\mathcal{F}} U^{-\frac{1}{2}} P_{\mathcal{F}} & -P'_{\mathcal{G}} U^{-\frac{1}{2}} S_{\mathcal{G}} U^{-\frac{1}{2}} P_{\mathcal{G}} \end{bmatrix}. \tag{A.14}$$

This completes the proof of part (a).

(b) Using the result in part (b) of Lemma 1 in the paper, it can be shown that when $\lambda_{\mathcal{F}} = \lambda_{\mathcal{G}} = 0_n$,

$$\sqrt{T}\tilde{\lambda}_{\mathcal{F}\mathcal{G}} \overset{A}{\sim} N(0_{2n-k_1-k_2}, \Sigma_{\tilde{\lambda}_{\mathcal{F}\mathcal{G}}}), \tag{A.15}$$

where

$$\Sigma_{\tilde{\lambda}_{\mathcal{F}\mathcal{G}}} = \begin{bmatrix} P'_{\mathcal{F}}U^{-\frac{1}{2}}S_{\mathcal{F}}U^{-\frac{1}{2}}P_{\mathcal{F}} & P'_{\mathcal{F}}U^{-\frac{1}{2}}S_{\mathcal{F}\mathcal{G}}U^{-\frac{1}{2}}P_{\mathcal{G}} \\ P'_{\mathcal{G}}U^{-\frac{1}{2}}S_{\mathcal{G}\mathcal{F}}U^{-\frac{1}{2}}P_{\mathcal{F}} & P'_{\mathcal{G}}U^{-\frac{1}{2}}S_{\mathcal{G}}U^{-\frac{1}{2}}P_{\mathcal{G}} \end{bmatrix}. \tag{A.16}$$

Using the fact that $\hat{\Sigma}_{\tilde{\lambda}_{\mathcal{F}\mathcal{G}}}$ is a consistent estimator of $\Sigma_{\tilde{\lambda}_{\mathcal{F}\mathcal{G}}}$, we have

$$LM_{\tilde{\lambda}_{\mathcal{F}\mathcal{G}}} = T\tilde{\lambda}'_{\mathcal{F}\mathcal{G}}\hat{\Sigma}^{-1}_{\tilde{\lambda}_{\mathcal{F}\mathcal{G}}}\tilde{\lambda}_{\mathcal{F}\mathcal{G}} \overset{A}{\sim} \chi^2_{2n-k_1-k_2}. \tag{A.17}$$

This completes the proof of part (b). ■

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Table 5. Model selection tests for nested models

Panel A: Pairwise model comparison tests: Linear models

	UFT						UVT					
	Size			Power			Size			Power		
T	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
120	0.114	0.056	0.010	0.310	0.188	0.049	0.000	0.000	0.000	0.006	0.001	0.000
240	0.122	0.064	0.014	0.452	0.314	0.116	0.005	0.001	0.000	0.075	0.020	0.001
360	0.122	0.066	0.014	0.565	0.423	0.182	0.023	0.006	0.000	0.219	0.089	0.008
480	0.120	0.065	0.014	0.660	0.521	0.255	0.040	0.013	0.001	0.353	0.179	0.028
600	0.117	0.061	0.014	0.738	0.612	0.333	0.050	0.019	0.002	0.472	0.272	0.057

RVT

	Size			Power		
T	10%	5%	1%	10%	5%	1%
120	0.061	0.020	0.001	0.173	0.075	0.008
240	0.068	0.025	0.002	0.288	0.147	0.024
360	0.075	0.030	0.003	0.397	0.225	0.047
480	0.079	0.034	0.004	0.499	0.308	0.078
600	0.081	0.036	0.004	0.592	0.391	0.114

Panel B: Pairwise model comparison tests: Nonlinear models

	UFT						UVT					
	Size			Power			Size			Power		
T	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
120	0.082	0.033	0.003	0.254	0.132	0.021	0.000	0.000	0.000	0.002	0.000	0.000
240	0.118	0.059	0.012	0.426	0.285	0.095	0.001	0.000	0.000	0.030	0.008	0.000
360	0.121	0.063	0.015	0.527	0.381	0.154	0.002	0.000	0.000	0.096	0.033	0.002
480	0.117	0.061	0.015	0.612	0.467	0.210	0.006	0.001	0.000	0.182	0.077	0.009
600	0.111	0.057	0.014	0.686	0.547	0.274	0.010	0.002	0.000	0.275	0.136	0.019

RVT

	Size			Power		
T	10%	5%	1%	10%	5%	1%
120	0.025	0.006	0.000	0.113	0.041	0.004
240	0.041	0.012	0.001	0.242	0.120	0.022
360	0.047	0.015	0.001	0.325	0.185	0.043
480	0.048	0.016	0.001	0.395	0.241	0.066
600	0.047	0.016	0.001	0.463	0.299	0.093

The table presents the empirical size and power of pairwise model comparison tests for nested linear (Panel A) and nonlinear (Panel B) models. The tests considered are the unrestricted weighted chi-squared test (UFT) in (11), the unrestricted variance test (UVT) in (13), and the restricted variance test (RVT) in (14). We report results for different levels of significance (10%, 5% and 1% levels) and for different values of the number of time series observations (T) using 100,000 simulations, assuming that the factors and the gross returns (continuously compounded gross returns in the nonlinear case) are generated from a multivariate normal distribution.

Table 6. Model selection tests for non-nested models

Panel A: Pairwise model comparison tests: Linear models

	UFT						UVT					
	Size			Power			Size			Power		
T	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
120	0.076	0.036	0.006	0.416	0.326	0.171	0.000	0.000	0.000	0.008	0.002	0.000
240	0.089	0.045	0.009	0.721	0.661	0.527	0.001	0.000	0.000	0.076	0.019	0.001
360	0.094	0.048	0.010	0.870	0.838	0.760	0.011	0.002	0.000	0.226	0.082	0.005
480	0.097	0.049	0.010	0.940	0.923	0.882	0.026	0.007	0.000	0.383	0.169	0.018
600	0.097	0.049	0.010	0.973	0.964	0.943	0.038	0.013	0.001	0.531	0.274	0.040

	RVT					
	Size			Power		
T	10%	5%	1%	10%	5%	1%
120	0.038	0.010	0.000	0.116	0.039	0.002
240	0.049	0.015	0.001	0.209	0.081	0.006
360	0.058	0.021	0.001	0.324	0.145	0.016
480	0.066	0.025	0.002	0.451	0.224	0.031
600	0.070	0.027	0.003	0.577	0.319	0.053

Panel B: Pairwise model comparison tests: Nonlinear models

	UFT						UVT					
	Size			Power			Size			Power		
T	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
120	0.044	0.015	0.001	0.107	0.048	0.005	0.000	0.000	0.000	0.000	0.000	0.000
240	0.084	0.041	0.008	0.226	0.135	0.036	0.000	0.000	0.000	0.001	0.000	0.000
360	0.096	0.050	0.012	0.294	0.189	0.061	0.001	0.000	0.000	0.030	0.004	0.000
480	0.100	0.053	0.013	0.336	0.228	0.082	0.005	0.001	0.000	0.144	0.038	0.001
600	0.100	0.053	0.013	0.378	0.265	0.104	0.013	0.002	0.000	0.319	0.129	0.008

	RVT					
	Size			Power		
T	10%	5%	1%	10%	5%	1%
120	0.014	0.003	0.000	0.068	0.021	0.001
240	0.026	0.007	0.000	0.203	0.091	0.010
360	0.033	0.009	0.000	0.326	0.176	0.033
480	0.037	0.011	0.001	0.436	0.266	0.065
600	0.038	0.012	0.001	0.535	0.358	0.111

The table presents the empirical size and power of pairwise model comparison tests for overlapping linear (Panel A) and nonlinear (Panel B) models. The tests considered are the unrestricted weighted chi-squared test (UFT) in (11), the unrestricted variance test (UVT) in (13), and the restricted variance test (RVT) in (16). We report results for different levels of significance (10%, 5% and 1% levels) and for different values of the number of time series observations (T) using 100,000 simulations, assuming that the factors and the gross returns (continuously compounded gross returns in the nonlinear case) are generated from a multivariate normal distribution.

Table 7. Joint tests of correct specification

Panel A: Pairwise model comparison tests: Linear models

	RFT						LM					
	Size			Power			Size			Power		
T	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
120	0.148	0.080	0.018	0.387	0.289	0.138	0.002	0.000	0.000	0.660	0.456	0.116
240	0.127	0.067	0.014	0.493	0.407	0.254	0.002	0.001	0.000	1.000	0.999	0.987
360	0.120	0.062	0.013	0.555	0.475	0.331	0.003	0.001	0.000	1.000	1.000	1.000
480	0.116	0.058	0.013	0.599	0.525	0.389	0.004	0.001	0.000	1.000	1.000	1.000
600	0.113	0.058	0.013	0.632	0.563	0.436	0.005	0.001	0.000	1.000	1.000	1.000

	UFT					
	Size			Power		
T	10%	5%	1%	10%	5%	1%
120	0.068	0.026	0.002	0.179	0.097	0.019
240	0.079	0.034	0.004	0.313	0.214	0.076
360	0.085	0.037	0.005	0.396	0.301	0.146
480	0.090	0.040	0.007	0.461	0.366	0.206
600	0.091	0.043	0.007	0.507	0.419	0.260

	UVT						RVT					
	Size			Power			Size			Power		
T	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
120	0.001	0.000	0.000	0.016	0.004	0.000	0.003	0.001	0.000	0.505	0.319	0.087
240	0.002	0.000	0.000	0.524	0.245	0.016	0.014	0.004	0.000	0.997	0.992	0.950
360	0.006	0.001	0.000	0.966	0.898	0.499	0.025	0.008	0.001	1.000	1.000	1.000
480	0.013	0.003	0.000	0.998	0.991	0.920	0.036	0.013	0.001	1.000	1.000	1.000
600	0.020	0.005	0.000	1.000	0.999	0.990	0.044	0.017	0.002	1.000	1.000	1.000

Table 7 (continued). Joint tests of correct specification

Panel B: Pairwise model comparison tests: Nonlinear models

	RFT						LM					
	Size			Power			Size			Power		
T	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
120	0.092	0.042	0.006	0.309	0.214	0.083	0.002	0.000	0.000	0.694	0.499	0.147
240	0.097	0.046	0.008	0.458	0.366	0.211	0.002	0.000	0.000	1.000	0.999	0.992
360	0.100	0.048	0.009	0.535	0.453	0.305	0.002	0.001	0.000	1.000	1.000	1.000
480	0.098	0.049	0.009	0.587	0.511	0.372	0.004	0.001	0.000	1.000	1.000	1.000
600	0.100	0.050	0.009	0.625	0.552	0.420	0.005	0.002	0.000	1.000	1.000	1.000

	UFT					
	Size			Power		
T	10%	5%	1%	10%	5%	1%
120	0.046	0.014	0.000	0.129	0.054	0.004
240	0.069	0.028	0.002	0.284	0.183	0.053
360	0.081	0.035	0.004	0.381	0.279	0.124
480	0.083	0.039	0.006	0.447	0.350	0.184
600	0.088	0.041	0.006	0.494	0.401	0.238

	UVT						RVT					
	Size			Power			Size			Power		
T	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
120	0.000	0.000	0.000	0.003	0.001	0.000	0.002	0.000	0.000	0.413	0.235	0.049
240	0.001	0.000	0.000	0.244	0.049	0.001	0.011	0.003	0.000	0.995	0.984	0.904
360	0.003	0.000	0.000	0.951	0.814	0.157	0.022	0.006	0.000	1.000	1.000	0.999
480	0.008	0.001	0.000	0.997	0.988	0.851	0.031	0.010	0.001	1.000	1.000	1.000
600	0.015	0.004	0.000	1.000	0.999	0.985	0.040	0.014	0.001	1.000	1.000	1.000

The table presents the empirical size and power of pairwise model comparison tests for overlapping linear (Panel A) and nonlinear (Panel B) correctly specified distinct SDFs. The tests considered are the restricted weighted chi-squared test (RFT) in part (a) of Lemma B.2, the chi-squared test (LM) in part (b) of Lemma B.2, the unrestricted weighted chi-squared test (UFT) in (11), the unrestricted variance test (UVT) in (13), and the restricted variance test (RVT) in (16). We report results for different levels of significance (10%, 5% and 1% levels) and for different values of the number of time series observations (T) using 100,000 simulations, assuming that the factors and the gross returns (continuously compounded gross returns in the nonlinear case) are generated from a multivariate normal distribution.

Table 8. t-tests under model misspecification and specification tests

Panel A: t-tests under potentially misspecified models

	Size (r_{mkt})			Size ($\triangle c_{ndur}$)			Size ($\triangle c_{dur}$)		
T	10%	5%	1%	10%	5%	1%	10%	5%	1%
120	0.143	0.079	0.018	0.093	0.043	0.006	0.092	0.043	0.007
240	0.139	0.077	0.019	0.092	0.043	0.006	0.094	0.044	0.007
360	0.133	0.074	0.018	0.094	0.045	0.008	0.095	0.046	0.008
480	0.129	0.070	0.018	0.095	0.045	0.008	0.097	0.047	0.009
600	0.126	0.069	0.017	0.097	0.049	0.009	0.099	0.049	0.009

Panel B: t-tests under correctly specified models

	Size (r_{mkt})			Size ($\triangle c_{ndur}$)			Size ($\triangle c_{dur}$)		
T	10%	5%	1%	10%	5%	1%	10%	5%	1%
120	0.232	0.150	0.050	0.287	0.193	0.072	0.292	0.198	0.074
240	0.266	0.181	0.074	0.330	0.236	0.102	0.343	0.247	0.110
360	0.282	0.198	0.088	0.358	0.265	0.128	0.375	0.282	0.140
480	0.291	0.208	0.098	0.375	0.287	0.147	0.399	0.308	0.162
600	0.298	0.216	0.103	0.390	0.301	0.162	0.412	0.321	0.181

Panel C: Specification tests

	HJ-distance test						LM test					
	Size			Power			Size			Power		
T	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
120	0.280	0.180	0.063	0.991	0.983	0.948	0.136	0.065	0.010	0.989	0.974	0.897
240	0.165	0.093	0.024	1.000	0.999	0.997	0.114	0.056	0.011	1.000	0.999	0.998
360	0.139	0.074	0.017	1.000	1.000	1.000	0.110	0.054	0.010	1.000	1.000	1.000
480	0.126	0.067	0.014	1.000	1.000	1.000	0.107	0.053	0.010	1.000	1.000	1.000
600	0.119	0.061	0.014	1.000	1.000	1.000	0.104	0.051	0.010	1.000	1.000	1.000

The table presents the empirical size of t-tests of $H_0 : \gamma_i = 0$ in Lemma 1 in the paper and the empirical size and power of the conventional HJ-distance test in part (a) of Lemma B.1 and of the LM test in Theorem 1 in the paper. The various t-ratios are compared to the critical values from a standard normal distribution. We report results for different levels of significance (10%, 5% and 1% levels) and for different values of the number of time series observations (T) using 100,000 simulations, assuming that the factors and the gross returns are generated from a multivariate t-distribution with eight degrees of freedom.

Table 9. Model selection tests for nested and non-nested SDFs

Panel A: Pairwise nested model comparison tests

	Weighted χ^2 test						Wald test					
	Size			Power			Size			Power		
T	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
120	0.106	0.048	0.007	0.362	0.218	0.053	0.132	0.069	0.015	0.488	0.351	0.146
240	0.105	0.049	0.008	0.516	0.355	0.116	0.131	0.070	0.015	0.661	0.538	0.297
360	0.105	0.050	0.008	0.624	0.464	0.185	0.131	0.069	0.015	0.755	0.652	0.420
480	0.105	0.051	0.009	0.706	0.554	0.256	0.128	0.069	0.016	0.820	0.733	0.519
600	0.105	0.051	0.009	0.770	0.633	0.327	0.126	0.068	0.016	0.866	0.794	0.601

Panel B: Multiple nested model comparison test

	Size			Power		
T	10%	5%	1%	10%	5%	1%
120	0.110	0.053	0.010	0.398	0.269	0.099
240	0.112	0.055	0.011	0.584	0.454	0.227
360	0.116	0.060	0.012	0.704	0.589	0.354
480	0.115	0.059	0.012	0.789	0.691	0.469
600	0.114	0.058	0.013	0.846	0.766	0.564

Panel C: Pairwise non-nested model comparison tests

	Weighted χ^2 test						Wald test					
	Size			Power			Size			Power		
T	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
120	0.087	0.040	0.007	0.629	0.536	0.331	0.102	0.050	0.010	0.752	0.628	0.366
240	0.089	0.042	0.007	0.862	0.825	0.712	0.098	0.047	0.009	0.969	0.935	0.800
360	0.092	0.044	0.008	0.942	0.927	0.880	0.100	0.049	0.009	0.997	0.992	0.963
480	0.094	0.045	0.008	0.973	0.966	0.946	0.101	0.050	0.010	1.000	0.999	0.994
600	0.094	0.046	0.008	0.987	0.984	0.975	0.100	0.050	0.010	1.000	1.000	0.999

Panel D: Pairwise ($p = 1$) and multiple ($p = 2$) model comparison tests

	$p = 1$						$p = 2$					
	Size			Power			Size			Power		
T	10%	5%	1%	10%	5%	1%	10%	5%	1%	10%	5%	1%
120	0.135	0.068	0.011	0.434	0.314	0.132	0.134	0.065	0.010	0.364	0.250	0.093
240	0.111	0.054	0.007	0.597	0.467	0.237	0.107	0.049	0.006	0.507	0.378	0.171
360	0.105	0.051	0.007	0.711	0.590	0.351	0.101	0.046	0.006	0.625	0.501	0.270
480	0.103	0.050	0.007	0.792	0.686	0.452	0.100	0.047	0.007	0.718	0.601	0.367
600	0.103	0.049	0.008	0.851	0.761	0.543	0.100	0.048	0.007	0.789	0.686	0.458

The table presents the empirical size and power of pairwise and multiple model comparison tests for nested (Panels A and B) and non-nested (Panels C and D) models. In Panel A, we report results for the weighted chi-squared test and the Wald test in parts (a) and (b) of Theorem 2 in the paper, respectively. Panel B is for the Wald test for multiple nested model comparison analyzed in Section 3.2 of the paper. In Panel C, we report results for the weighted chi-squared test and the Wald test in parts (a) and (b) of Theorem 3 in the paper, respectively. Panel D is for the pairwise ($p = 1$) and multiple ($p = 2$) model comparison tests for non-nested misspecified (distinct) SDFs described in Sections 3.1 and 3.2 of the paper, respectively. We report results for different levels of significance (10%, 5% and 1% levels) and for different values of the number of time series observations (T) using 100,000 simulations, assuming that the factors and the gross returns are generated from a multivariate t-distribution with eight degrees of freedom.