Tests of the Relations Among Marketwide Factors, Firm-specific Variables, and Stock Returns Using a Conditional Asset Pricing Model

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ABSTRACT

In this paper we generalize Harvey's (1989) empirical specification of conditional asset pricing models to allow for both time-varying covariances between stock returns and marketwide factors and time-varying reward-to-covariabilities. The model is then applied to examine the effects of firm size and book-to-market equity ratios. We find that the traditional asset pricing model with commonly used factors can only explain a small portion of the stock returns predicted by firm size and book-to-market equity ratios. The results indicate that allowing time-varying covariances and time-varying reward-to-covariabilities does little to salvage the traditional asset pricing models.

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Asset pricing theory posits that the expected excess return on a financial asset is the (summed) product of the conditional covariance (or beta) of the asset return with marketwide factors and the reward-to-covariabilities (or factor premiums). Models based on this basic relationship are widely used in financial decisions. However, the unconditional versions of the models, especially the Sharpe (1964)-Lintner (1965) Capital Asset Pricing Model (CAPM), are frequently rejected by data and are known to leave some anomalies. Fama and French (1992) conclude that in explaining the cross-section of asset returns, betas are overwhelmed by two firm specific variables: the market value of a firm's equity (ME), and the book-to-market ratio of equity (BM). The Fama-French result has stimulated many subsequent studies to explain the ME and BM anomaly. In this paper, we investigate whether the cross-sectional explanatory power of ME and BM is consistent with a conditional multi-factor asset pricing model.

At the most general level, the asset pricing theory does not specify the functional form of the relation between conditional covariances and conditioning information, nor the functional form of the reward-to-covariability. This relation is empirically determined.[1] Since the ME and BM effects are cross-sectional relations, they are more relevant to the covariance terms in the asset pricing model. Many previous studies (including Fama and French (1992)) assume that covariances, or betas, are constant over a fixed period of time, while others adopt some functional forms. The failure of such constant specifications to explain the ME and BM effects may well be due to the misspecification of the functional form of the conditional covariances. To avoid the consequence of misspecifying conditional covariances, we adopt in this paper an empirical multi-factor model based on Harvey (1989). The main advantage of Harvey's specification is that it admits a general structure for conditional covariances between stock returns and marketwide factors. This is particularly suitable for the question we address in this paper. In addition to allowing time-varying covariances, we generalize Harvey's (1989) model to allow time-varying reward-to-covariabilities. The model is then used to examine the relations among the firm-specific variables, ME and BM, a set of commonly used marketwide factors, and expected stock returns. The purpose is to see how different are the predictive powers of ME and BM on the stock returns, with and without asset pricing restrictions, where the restrictions allow for both time-varying covariances and time-varying reward-to-covariabilities, and whether the predictive power of ME and BM is consistent with the implications of the asset pricing models.

We use returns on 25 portfolios sorted by size and book-to-market value as in Fama and French (1993). We examine three sets of marketwide factors. The first set consists of only the market return, so this model corresponds to the CAPM. Consistent with Fama and French (1992), we show that it captures little variation in expected returns even though time-varying covariances and time-varying reward-to-covariability are allowed. Almost none of the predictive power of ME and BM can be explained by the one-factor model. The second set of factors consists of the market return and two bond market factors, as used in Fama and French (1993). This three-factor model fares better than the one-factor model, but it still cannot capture much of the effect of ME and BM. In the third set of factors, we add the two characteristic portfolios used in Fama and French (1993) to the three-factor model. Even for this five-factor model, only about 30% of the cross-sectional predictive power of ME and BM is explained.

The rest of the paper is organized as follows. Section I presents the methodology, Section II describes the data and discusses the empirical results, and the final section provides our interpretation of the results.

I. Methodology

A. An Empirical Model

The discrete time equilibrium conditional asset pricing model we examine takes the form

$$\mu_t = \mathrm{Cov}_t(r_{t+1}, f_{t+1})\gamma_t, \tag{1}$$

where r_{t+1} is the return on n assets in excess of a risk-free rate, $\mu_t = E_t r_{t+1}$, the conditional expected excess return, and f_{t+1} is a k-vector of marketwide factors realized at time $t+1$. Here E_t and Cov_t are the expectation and covariance conditioned on all the information at time t. γ_t is a k-vector of the reward-to-covariabilities parameters. The components of γ_t measure expected excess return per unit of covariance of the return with the corresponding factors. (1) can be viewed as a discretized version of the models developed in continuous time by Merton (1973), Breeden (1979) and Cox, Ingersoll and Ross (1985). The essence of this equation is that expected excess returns on a large number of assets are determined by their covariances with a small number of factors. This relation has been the focus of empirical studies of asset pricing theory. In this paper we treat (1) as a starting point. To test the conditional multi-factor model above, ancillary assumptions are required to put the model in terms of observables and parameters.[2]

Harvey (1989) initiated a testing method which allows time-varying covariance between returns and factors. The conditional multi-factor model can be written as

$$\mu_t = E_t[(r_{t+1} - \mu_t)(f_{t+1} - \phi_t)']\gamma_t, \tag{2}$$

where $\phi_t = E_t f_{t+1}$ is the conditional mean of the factors. In the case of the CAPM, f_{t+1} is the excess return on the market portfolio, and γ_t can be taken as the ratio of the conditional expected excess return on the market portfolio to the conditional variance of the market portfolio. Harvey first assumes that γ_t is a constant. Let z_t be an l-vector of marketwide information at time t. The model is further parameterized by assuming

$\mu_t = Dz_t$, and $\phi_t = Cz_t$ where matrices D and C are constants. The model is tested by examining the moment conditions

$$E_t[r_{t+1} - (r_{t+1} - Dz_t)(f_{t+1} - Cz_t)'\gamma] = 0, \tag{3}$$

$$E_t[r_{t+1} - Dz_t] = 0, \tag{4}$$

$$E_t[f_{t+1} - Cz_t] = 0. \tag{5}$$

When applied to testing the CAPM, the above model is rejected by the data. Harvey presents evidence that the assumption of constant γ_t is one reason the model is rejected. To avoid rejecting the CAPM due to the assumption of constant γ_t, he considers a more general methodology which does not restrict γ_t to be constant.[3] This methodology, however, depends on the assumptions that there is only one factor, that the factor is the return on a portfolio, and that the functional form for the conditional expected returns on all individual assets must be pre-specified. Using this more general model, Harvey still rejects the CAPM even though γ_t is allowed to be time-varying. Two natural questions arise. The first is whether a multi-factor model with time-varying reward-to-covariabilities could explain the data better. The second question is whether the assumption that the conditional expected return is linear in the chosen instrumental variables is too restrictive.

For the multi-factor model using economic factors, we make two changes to generalize Harvey's model of (3)–(5). It should be emphasized that (4), (5) and the assumption of constant γ_t are ancillary assumptions in order to test the asset pricing relation. Therefore it is sensible to minimize their effect on the test. We first relax the constant γ_t assumption to the specification that $\gamma_t = Az_t$, where A is a constant matrix. Since z_t is assumed to contain a constant as one of its components, such a specification nests the assumption of constant γ_t. Second, we drop the Dz_t term in (3) without changing the content of (3), because $\mathrm{Cov}_t(r_{t+1}, f_{t+1}) = E_t[(r_{t+1} - \mu_t)(f_{t+1} - \phi_t)'] = E_t[r_{t+1}(f_{t+1} - \phi_t)']$. This makes it possible to drop (4). The generalized model is then written as

$$E_t[r_{t+1} - r_{t+1}(f_{t+1} - Cz_t)'Az_t] = 0, \tag{6}$$

$$E_t[f_{t+1} - Cz_t] = 0. \tag{7}$$

The hypothesis of constant reward-to-covariabilities can be formally tested. If the first component of z_t is the constant 1, then the hypothesis is represented as $A = [a_1, O]$, where a_1 is a k-vector and O is a conformable matrix of zeros. The advantage of generalizing the reward-to-covariabilities from a constant to functions of conditioning information is obvious if the hypothesis $A = [a_1, O]$ is rejected. The choice of linear functional form of the reward-to-covariability is the most natural and parsimonious one, given the absence of guidance from the theory.[4] This, however, can not be done without the second change we made. Harvey (1991), in estimating a world version of the CAPM, also used this trick of eliminating the assumption $\mu_t = Dz_t$. With the assumption that γ_t is a linear function of z_t, the assumption $\mu_t = Dz_t$ implies that the covariance is the ratio of two linear functions of z_t. The advantage of eliminating the assumption $\mu_t = Dz_t$ is that the covariances as functions of the information z_t are unrestricted.

One important feature of our model is that the parameters are all marketwide. Unlike betas, they are not associated with individual assets. The model avoids unnecessary assumption on the functional form of the covariances and the expected returns of individual assets and, consequently, its contribution to the rejection of the model. This feature is also consistent with the spirit of asset pricing models in that only the properties of a small number of factors are modeled while the properties of a large number of assets are left unrestricted.

The only ancillary assumptions in our model in addition to (1) are $\gamma_t = Az_t$ and $\phi_t = Cz_t$. The linearity of ϕ_t and γ_t in z_t can be derived under strong assumptions on preferences and the joint distribution of f_{t+1} and z_t (for example, the normal distribution). Since these assumptions can rarely be examined with accuracy, the assumption of linearity is best viewed as a first approximation of more general functional forms. The model can be further generalized at the cost of more parameters and more complicated estimation methods by including nonlinear terms. The linear functional form may not reflect all the restrictions from asset pricing theory. For example, the CAPM places a positivity constraint on γ_t which cannot be guaranteed from a linear function of the information variables. While this is a shortcoming of the specification, there is a positive aspect. If our purpose is not just to estimate a model known to be correct, but also to test the validity of the model, then the flexibility of the linear functional form can help detect deviations from positivity of γ_t.

Harvey's type of empirical model can be compared with the models which parameterize betas, such as the constant-beta model of Gibbons (1982), Gibbons, Ross and Shanken (1989) and the time-varying-beta model of Shanken (1991). Both approaches use one-step estimation, and are therefore free of the errors-in-variables problem associated with two-step estimation. But neither approach is more general than the other. For example, the constant-beta model of Gibbons, Ross and Shanken restricts betas to be constant, but does not make assumptions on risk premiums. In Harvey's type of model, the covariance can be entirely unrestricted but ancillary assumptions about ϕ_t and γ_t have to be made. One advantage that Harvey's methods have is that they can deal with models with economic factors while the early empirical methods which parameterize betas can only be applied to the models with mimicking portfolios as factors. This is so because when factors are economic variables, there is no constraint on the intercept in the regression of individual asset returns on the economic variables. The traditional approach to testing models with economic factors is the Fama and MacBeth (1973) two-step method used in Chen, Roll and Ross (1986). The test conducted in this paper is an attempt to test asset pricing models with economic variables as factors in a one-step approach.

A recent concurrent study by Jagannathan and Wang (1996) also deals with conditional asset pricing models. Their approach is to derive the unconditional moment of the conditional CAPM with specific assumptions on the conditional beta. Both their approach and our approach require ancillary assumptions, and it is not obvious which set of assumptions are more reasonable. The test we conduct is a conditional test which is more powerful than unconditional tests in principle because more restrictions of conditional moments are tested, although statistically, with a larger system of equations, a conditional test might not be more powerful than unconditional tests.

B. Testing the Multi-factor Model with Predetermined Variables

According to the specification in the last subsection,

$$E_t[r_{i,t+1} - r_{i,t+1}(f_{t+1} - Cz_t)'Az_t] = 0, \qquad i = 1, \ldots, n. \tag{8}$$

where n is the number of assets. Let $x_{it} = (1,\ \ln(\text{ME})_{it},\ \ln(\text{BM})_{it})'$ be the vector of a constant and the i-th firm-specific variables at time t and let z_t^* be the marketwide information variables excluding the constant. Since x_{it} and z_t^* are part of the time t information set, the law of iterated expectations implies that in the following system of regressions,

$$E[r_{i,t+1} - r_{i,t+1}(f_{t+1} - Cz_t)'Az_t | x_{it}, z_t^*] = \theta' x_{it} + c_i' z_t^*, \qquad i = 1, \ldots, n. \tag{9}$$

the coefficients θ and c_i should all be zero.

An alternative way is to investigate the following two sets of regressions.

$$\begin{aligned} E[r_{i,t+1} | x_{it}, z_t^*] &= d' x_{it} + h_i' z_t^*, \qquad i = 1, \ldots, n, & (10) \\ E[r_{i,t+1}(f_{t+1} - Cz_t)'Az_t | x_{it}, z_t^*] &= \delta' x_{it} + \eta_i' z_t^* \qquad i = 1, \ldots, n. & (11) \end{aligned}$$

The first set is the expected return conditional on x_{it} and z_t^*. This is a generalization of the Fama-French (1992) equation,

$$Er_{i,t+1} = d' x_{it}, \qquad i = 1, \ldots, n,\ \ t = 1, \ldots, T. \tag{12}$$

They estimated (12) and found the coefficient on $\ln(\text{ME})_{it}$ to be negative and the coefficient on $\ln(\text{BM})_{it}$ to be positive.[5] Since (10) is empirically driven, we refer to (10) as an empirical model of expected returns. The second set of equations can be viewed as the conditional expected returns under the restriction (8) of the specification of the asset pricing model. Therefore, if the asset pricing model is properly specified, the two sets of coefficients should be the same, i.e., $h_i = \eta_i$ and $d = \delta$. The comparison between d and δ reflects the degree to which the Fama-French (1992) result is consistent with the asset pricing model. The essence of the asset pricing theory is that the cross-sectional difference in expected asset returns is determined by their covariances with marketwide factors.[6] Predetermined firm-specific variables can explain the cross-section of expected return only if they proxy the covariance between returns and marketwide factors. Therefore, in order for the cross-sectional predictive power of the $d'x_{it}$ term in (10) to be consistent with the asset pricing model, the same $d'x_{it}$ term should be able to predict the realized covariation between returns and marketwide factors in (11).

While testing the equality $d = \delta$ is equivalent to testing $\theta = 0$ in (9), estimation of (10) and (11) does provide additional information. Asset pricing theory in its most general form does not specify the marketwide factors f_{t+1} and the information variables z_t. There always exists a possibility that the theory is correct, but the set of f_{t+1} and z_t specified in an asset pricing model is incomplete. From that perspective, it is more sensible to examine the *adequacy* of the specified asset pricing model. Therefore, in the case that the hypothesis $\theta = 0$ is rejected, a measure of goodness-of-fit is desired to gauge the inadequacy of the specified asset pricing model in explaining the predictive power of firm-specific variables. Ferson and Harvey (1991, 1993) define variance ratios to accomplish this task where the denominator of the ratio is the variance of predicted returns from an empirical equation, and the numerator is the variance of the expected return according to an asset pricing model. To emphasize the cross-sectional predictive power of firm-specific variables, we define the cross-sectional variance ratios as well as time series variance ratios to measure the adequacy of the asset pricing model in explaining the cross-sectional and time-series predictability of the empirical model, respectively.

More specifically, we define the following versions of variance ratios.

$$\mathrm{VR}^{RS}(x) = \frac{\sum_{t=1}^{T} \overline{\mathrm{Var}}(\delta' x_{it})}{\sum_{t=1}^{T} \overline{\mathrm{Var}}(d' x_{it})}, \tag{13}$$

$$\mathrm{VR}^{RS}(x, z^*) = \frac{\sum_{t=1}^{T} \overline{\mathrm{Var}}(\delta' x_{it} + \eta_i' z_t^*)}{\sum_{t=1}^{T} \overline{\mathrm{Var}}(d' x_{it} + h_i' z_t^*)}, \tag{14}$$

$$\mathrm{VR}^{MR}(x) = \frac{1}{T} \sum_{t=1}^{T} \frac{\overline{\mathrm{Var}}(\delta' x_{it})}{\overline{\mathrm{Var}}(d' x_{it})}, \tag{15}$$

$$\mathrm{VR}^{MR}(x, z^*) = \frac{1}{T} \sum_{t=1}^{T} \frac{\overline{\mathrm{Var}}(\delta' x_{it} + \eta_i' z_t^*)}{\overline{\mathrm{Var}}(d' x_{it} + h_i' z_t^*)}, \tag{16}$$

$$\mathrm{VR}_i^{TS}(x) = \frac{\widehat{\mathrm{Var}}(\delta' x_{it})}{\widehat{\mathrm{Var}}(d' x_{it})}, \qquad i = 1, \ldots, n, \tag{17}$$

$$\mathrm{VR}_i^{TS}(x, z^*) = \frac{\widehat{\mathrm{Var}}(\delta' x_{it} + \eta_i' z_t^*)}{\widehat{\mathrm{Var}}(d' x_{it} + h_i' z_t^*)}, \qquad i = 1, \ldots, n, \tag{18}$$

where an overbar represents the cross-sectional variance and a wide hat represents the time series variance. RS represents the ratio of sums, MR the mean of ratios, and TS time series.[7] Under the specification of the asset pricing models, these variance ratios are all equal to one. The deviation from one serves as a measure of inadequacy of the model being considered.

II. Empirical Results

A. The Data

Stock Returns and Firm-specific Variables

To investigate the cross-sectional relation between average returns and economic fundamentals, Fama and French (1993) employ monthly returns on twenty-five ME-BM formed stock portfolios in excess of a risk-free rate for the period July 1963 to December 1991. We adopt Fama and French's method of forming stock portfolios and use roughly the same test period (July 1964 to December 1992). Thus, we have ensured that any differences between our results and Fama and French's are not due to the different test assets employed. Monthly data on non-financial NYSE, AMEX, and (after 1972) NASDAQ stocks used to construct these portfolios are provided by the Center for Research in Security Prices at the University of Chicago, and accounting data are obtained from Compustat.

Factors and Characteristic Portfolios

The economic variables we select include two bond factors (DEF and TERM) and a stock market portfolio (MKT). DEF is the difference between the monthly returns on a market portfolio of long-term corporate bonds and long-term government bonds, and TERM is the difference between the monthly returns on long-term government bonds and one-month Treasury bill rates. We use excess returns on the value-weighted market portfolio of NYSE, AMEX, and (after 1972) NASDAQ stocks as a proxy for MKT. The market portfolio of long-term corporate bonds is the composite portfolio of corporate bonds available from the Ibbotson Corporate Bond Module, while the long-term government bond returns and Treasury bill rates are from Ibbotson's Stocks, Bonds, and Bills. Further, we use the two stock portfolios in Fama and French (1993), namely small-minus-large capitalization (SMB) and high-minus-low book-to-market (HML), to mimic the underlying risk factors in returns related to ME and BM. The construction of the two characteristic portfolios is identical to that of Fama and French (1993).

Information Variables

Motivated by earlier empirical evidence,[8] we choose five information variables: the Standard and Poor 500 composite stock index (S&P), the difference between three-month and one-month Treasury bill returns (HB3), the difference between the yields on a portfolio of Baa-rated bonds and a portfolio of Aaa-rated bonds (Baa−Aaa) constructed from the Ibbotson Corporate Bond Module, the dividend yield on the Standard and Poor's 500 composite stock index (DIV), and the one-month Treasury bill rate (TB1). All the information variables are standardized to have zero unconditional means and unit variances so that the estimated intercepts as well as slope coefficients are easier to interpret.

B. Estimates of the Conditional Model

The model consisting of (6) and (7) can be estimated by Hansen's (1982) generalized method of moments, using z_t as instrumental variables. We consider three sets of factors. The first set contains only the market portfolio (MKT), so it corresponds to the CAPM. The second set consists of default premium (DEF), term premium (TERM), and the market portfolio (MKT). The last set adds the two characteristic portfolios in Fama and French (1993), namely SMB and HML.

Table I presents the estimated A matrices for all three sets of factors. Both the unconstrained model and the constrained model with $A = [a_1, O]$ are presented.[9] Panel A of Table I reports the estimates for the one-factor model. For the constrained one-factor model, the estimated a_1 is insignificant: the asymptotic z-ratio is only 0.32. This finding is consistent with evidence provided by Fama and French (1992). It is in contrast with the result of Harvey (1989) where he uses ten size-sorted portfolio returns and reports a z-ratio of approximately 6.0. The difference is due to the choice of portfolio returns.[10] This suggests that the market risk is more relevant to portfolios sorted by size, even though the market beta does not subsume size in explaining returns. For the unconstrained one-factor model, the reward-to-covariability Az_t is a random variable, and the pricing of the covariance risk associated with MKT can be determined by testing two different hypotheses. The first hypothesis is that the unconditional mean of Az_t, a_1, is zero. (Recall that the information variables are standardized.) The second hypothesis is that Az_t degenerates to zero, as given by the hypothesis $A = 0$. It can be seen in Panel A of Table I that a_1 is positive but statistically insignificant, similar to the constrained case. The stronger hypothesis of $A = 0$ is rejected, however, because of its significant components in A associated with DIV and TB1. When DIV is above its mean, or when TB1 is below its mean, the reward-to-covariability for the market factor is higher than a_1, its unconditional mean. The time series plot of the estimated $\gamma_t = Az_t$ in Figure 1 reveals that γ_t visits the negative region frequently, and sometimes becomes very negative. That implies, in these periods, the portfolios which have higher covariation with the market portfolio would have lower (or even negative) expected returns. If we take our proxy for the market portfolio as the true market portfolio and the one-factor model as the CAPM, then this evidence goes against one of the implications of the CAPM, namely that the market premium should always be positive.[11]

Table I about here

Panel B of Table I reports the estimates for the three-factor model. As in the one-factor model, the unconditional mean of the reward-to-covariability with MKT is small and insignificant. In fact, inclusion of the two bond factors has little effect on the reward-to-covariability associated with MKT. The unconditional mean of each of the reward-to-covariabilities with the two bond factors is positive and significant. Although most coefficients corresponding to the time-varying part of z_t are not very significant individually, the hypothesis $A = [a_1, O]$ is still strongly rejected. The hypothesis $A = 0$ is also strongly rejected.

Figure 1 about here

Panel C of Table I reports the estimates for the five-factor model. In the unconstrained case, the unconditional mean of the reward-to-covariability with MKT remains small and insignificant. The magnitude of the time-varying part, however, is somewhat reduced. In Figure 1, we also present the time series plot of γ_t associated with MKT and it shows less variations than its counterpart in the one-factor and three-factor cases.[12]

In the constrained model, the reward-to-covariability for MKT becomes negative and significant. The unconditional mean of γ_t for TERM remains positive and significant, but the unconditional mean of γ_t for DEF becomes insignificant. Both the hypothesis $A = [a_1, O]$ and the hypothesis $A = 0$ are strongly rejected.

To see if the set of three factors (DEF, TERM, and MKT) and the two characteristic portfolios represent the same set of underlying factors, we perform in Panel C a test of whether the covariabilities with each set of factors are priced in the presence of the other. Under the five-factor model, none of these two sets of factors can dominate the other. It indicates that neither the traditional economic factors nor the characteristic portfolios are redundant.

Although the three unconstrained models generally fit the data better than the models with the constraint $A = [a_1, O]$, the unconstrained models are still rejected by the over-identification test. This indicates that our specification of the conditional asset pricing model, including in the choice of factors even the two characteristic portfolios, is still inadequate to fully describe the behavior of the stock returns.

C. The Effects of ME and BM

In this subsection we examine the degree of inadequacy of the specified asset pricing model along with the choice of marketwide factors and information variables in explaining the predictive power of ME and BM. Each equation of (9), (10) and (11) is estimated by GMM for 25 portfolios. For (10) and (11), the estimates of A and C from Table I are used.[13] Table II presents the estimates of d and δ, the coefficients of firm-specific variables, in the three equations for three cases corresponding to the three sets of factors. (The estimates of h_i and η_i, the coefficients of marketwide information, do not show any specific pattern and are too numerous to report here.)

The first equation is the same for all cases. Consistent with Fama and French (1992), the coefficient of ln(BM) is positive, the coefficient of ln(ME) is negative, and they are all statistically significant.

Table II about here

The following patterns are observed for the expected returns under asset pricing restrictions. Comparing the expected return under the asset pricing restrictions for asset pricing models with constraint $A = [a_1, O]$ and without the constraint, the magnitude and the significance of the coefficients of ln(BM) and ln(ME) are higher for those without constraint, except for the coefficient of ln(BM) for the five-factor model. The expected return under asset pricing restrictions can be contrasted with that in the empirical model by comparing the slope coefficients δ and d. The fact that the magnitude and significance of δ are all smaller than those of d implies that the asset pricing models explain much less cross-sectional variation of the returns than the empirical model. Comparing the expected return under the asset pricing restrictions for asset pricing models with different sets of factors, we find the magnitude and the significance increase with the number of factors included in the asset pricing model. For the one-factor model, the magnitude of the δ coefficients are so small and θ is almost identical to d. This means that for the one-factor model, $r_{it}(f_t - Cz_t)Az_t$ is almost unrelated to BM and ME while r_{it} is. Thus the one-factor model is incapable of explaining the BM and ME effect. The three-factor model does a little better. But the major improvement comes from the five-factor model, as one may expect. The hypothesis that the slope coefficients of δ are zero can not be rejected, except for the unconstrained five-factor model at the 5% level. Correspondingly, the hypothesis $\theta = 0$ is rejected at conventional 5% levels for all cases, except for the unconstrained five-factor model.

The more telling result is in Table III where the degree of inadequacy of the asset pricing model in explaining the predictive power of ME and BM is measured by the variance ratio. Only those for models without constraint $A = [a_1, O]$ are reported. The models with the constraint of constant reward-to-covariability have much lower variance ratios than their unconstrained counterparts.

A clear pattern from the table is that the variance ratios calculated with both the firm-specific variables and marketwide variables are higher than those calculated with firm-specific variables alone. This indicates that returns over time can be better predicted by the marketwide information than by firm-specific variables, and the expected returns under asset pricing restrictions capture more of the variations explained by the marketwide information than by the firm-specific information. This is identical to the conclusion reached by Ferson and Harvey (1993) with respect to world and local information in the context of international asset pricing.[14]

The cross-sectional variance ratios increase with the number of factors included in the model. If we look at the variances attributed to the firm-specific variables alone, the one-factor model basically does not explain much of the predictive power of ln(ME) and ln(BM). The three-factor model adds little. The five-factor model makes a substantial contribution. But even for the five-factor model whose added characteristic portfolios are designed to trace the common variation that causes firms to exhibit the size effect and the BM effect, only about 20–30% of the cross-sectional variation is explained.

Table III about here

III. Concluding Remarks

In this paper we tackle the question of whether the variations in expected returns captured by conditional asset pricing models is adequate to explain the predictive power of the firm-specific variables. We present an empirical model that generalizes the multi-factor model of Harvey (1989) to allow time-varying reward-to-covariabilities as well as time-varying covariances.

The hypothesis that expected returns on 25 portfolios conditioned on the firm-specific variables, ME and BM, are equal with and without the asset pricing model restrictions specified in our model is rejected, except for the five-factor model with time-varying reward-to-covariabilities. By estimating the regression slope coefficients of the fitted returns under conditional asset pricing models on BM and ME, we find that the firm-specific variables typically capture less variation in returns under the asset pricing restrictions. The one-factor model explains virtually nothing about the predictive power of ME and BM. The three-factor model with time-varying reward-to-covariability can do little better. These results indicate that allowing time-varying covariances and time-varying reward-to-covariabilities does little to salvage the asset pricing models.

Since the specification of our empirical asset pricing model involves less restrictive assumptions than unconditional models, rejection of the asset pricing models is less likely to be due to violations of the ancillary assumptions than to misidentification of marketwide factors. The linearity assumption on the expectations of the factors ($E_t f_{t+1}$) and reward-to-covariabilities (γ_t) can be viewed as approximations of arbitrary functional forms. Refining the functional forms of $E_t f_{t+1}$ and γ_t among well behaved functions is unlikely to change our results qualitatively, for the given choice of factors and information variables.

The difficulty in improving our understanding regarding the interaction among marketwide factors, information variables and firm-specific variables in expected returns, lies in identifying the correct set of factors. The fact that the two characteristic portfolios can substantially improve the performance of the asset pricing model opens wide possibilities of reconciling the predictive power of the firm-specific variables with asset pricing theory. The challenge is to identify the underlying factors which the characteristic portfolios proxy. Jagannathan and Wang (1996) identify human capital as one component of the true market portfolio, and find stronger support to a multi-factor version of the conditional CAPM and a much weakened role of ME in explaining the cross-section of stock returns.[15] The results in both Jagannathan and Wang (1996) and this study point to the need of searching for economic factors whose covariance with asset returns determine the expected rate of return on the financial assets.

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FOOTNOTES

1. There have been considerable efforts made in specifying conditional asset pricing models in empirical studies (see, for example, Ferson, Kandel, and Stambaugh (1987), Bollerslev, Engle, and Wooldridge (1988), Harvey (1989) and Shanken (1991)).

2. The model can be equivalently expressed as $\mu_t = B_t \lambda_t$ where $B_t = \text{Cov}_t(r_{t+1}, f_{t+1}) \, \text{Var}_t(f_{t+1})^{-1}$, is the conditional beta matrix, and $\lambda_t = \text{Var}_t(f_{t+1})\gamma_t$ is the factor premiums. In testing the CAPM, the most popular assumption is to set B_t to be constant for a fixed period. A rejection of the hypothesis, however, may be attributed to the misspecification of (1) or to the misspecification that B_t is constant.

3. In the case of the CAPM, $\gamma_t = E_t(f_{t+1})/\text{Var}_t(f_{t+1})$ where f_{t+1} is the excess return on the market portfolio. With (4) and (5), (2) can be rewritten as

$$E_t[Dz_t(f_{t+1} - Cz_t)^2 - (r_{t+1} - Dz_t)(f_{t+1} - Cz_t)Cz_t] = 0$$

The assumption on γ_t can be avoided.

4. One can add quadratic terms, cubic terms, etc. to enrich the model, but the number of parameters increases explosively and the model becomes intractable. For example, if the quadratic terms of z_t are included in γ_t, the number of parameters increases by kl^2.

5. Fama and French (1992) estimate (12) cross-sectionally and obtain an estimate d_t for each month t and then make inferences based on the time series average of d_t. The inclusion of the $h_i' z_t^*$ term in (10) is motivated by the literature (for example, Fama and French (1988) and Ferson and Harvey (1991)) that suggests some marketwide information variables have predictive power on the time-series of stock returns. By including the $h_i' z_t^*$ term in (10), we would be able to separate the time-series predictability of z_t^* from the cross-sectional predictability of x_{it}.

6. Recall in the decomposition of expected return in the asset pricing model, the covariance term is firm specific, while the reward-to-covariability is marketwide and can not explain any cross-sectional difference in expected returns.

7. RS can be viewed as a weighted time series average of the cross-sectional variance ratios, while MR is a simple time series average.

8. See, for example, Fama and Schwert (1977), Keim and Stambaugh (1986), Campbell (1987), Campbell and Shiller (1988), Fama and French (1988), and Cochrane (1992).

9. The C matrices are not reported because they are of less interest here. In all three cases, most components of C are significant.

10. In an early version of this paper, we estimate the unconstrained model using twenty size-sorted portfolio returns and find a significant a_1 as in Harvey (1989).

11. In an early version of this paper, we tried one variation of the one-factor model in which the reward-to-covariability for MKT is an exponential function of the information variables. The fit of that model is much poorer. Harvey and Siddique (1994) suggest that if investors are concerned with not just mean and variance but also higher moments of the distribution of their portfolio returns, then the market premium could be negative at times as long as some of its moments are time-varying.

12. To avoid clutter in Figure 1, we do not plot the reward-to-covariability associated with MKT for the three-factor case.

13. In principle, all the equations in (6)–(7) and (10)–(11) can be estimated altogether. However, the number of orthogonal conditions in such a system is too large relative to the number of time series observations.

14. We calculated the variance ratios for the 25 portfolios using z_t alone (with different intercept terms for different portfolios) as predetermined information variables, like those in Ferson and Harvey (1991). In general, these ratios are much higher than those in Table III, although they are slightly lower than those in Ferson and Harvey (1991). We also calculated variance ratios using firm-specific variables alone as predictors. The variance ratios for the five-factor model are around 50–60%.

15. Apart from the difference in the methodologies and the data used, Jagannathan and Wang (1996) have a more positive tone towards the CAPM than ours. The added

explanatory power of their version of the CAPM mainly comes from the human capital variable. The risk premium associated with the value-weighted index of market return remains negative in their estimated models.

Table I
GMM Estimation of the Conditional Asset Pricing Models

The table presents the estimates of A matrices in the following model over the period July 1964 to December 1992,

$$\begin{aligned} E[(f_{t+1} - Cz_t) \otimes z_t] &= 0, \\ E[(r_{t+1} - r_{t+1}(f_{t+1} - Cz_t)' A z_t) \otimes z_t] &= 0, \end{aligned}$$

using 25 portfolios sorted by size and book-to-market ratio. r_{t+1} is the returns, f_{t+1} is the factors, and z_t is the information variables. Panel A presents the one-factor model where the factor is the value-weighted market portfolio of the NYSE, AMEX and NASDAQ (MKT). Panel B presents the three-factor model where factors are default premium (DEF), term premium (TERM), and the market portfolio (MKT). Panel C presents the five-factor model where two additional factors are the return on small firm portfolio minus the return on large firm portfolio (SMB) and the return on the portfolio of high book-to-market firms minus the return on the portfolio of low book-to-market firms (HML), as in Fama and French (1993). $z_t = (1, \text{S\&P}_t, \text{HB3}_t, \text{Baa-Aaa}_t, \text{DIV}_t, \text{TB1}_t)'$ is a vector of six information variables observable at the end of month t, where S&P is the return on S&P 500 index, HB3 the difference between the returns on three-month and one-month Treasury bills, Baa-Aaa the difference between the yield on a portfolio of Baa-rated bonds and the yield on a portfolio of Aaa-rated bonds, DIV the dividend yield on the S&P 500 index, and TB1 the one-month Treasury bill rate. Numbers in parentheses below the estimates are asymptotic z-ratios. The J-test is the over-identifying test of Hansen (1982). LR is the likelihood ratio test of the constraint that the reward-to-covariability is constant: $A = [a_1, O]$ or the hypothesis that the covariance risk is not priced: $A = 0$.

Table I (continued)

Panel A: One-factor Model

Factor	Unconstrained A Information variables						Constrained $A = [a_1, O]$
	Constant	S&P	HB3	Baa−Aaa	DIV	TB1	Constant
MKT	1.5539	−0.3369	1.6547	−1.6238	7.8250	−7.8333	0.3991
	(0.99)	(−0.29)	(1.36)	(−1.23)	(5.71)	(−4.14)	(0.32)

Test of over-identifying restrictions:
$J = 186.28 \sim \chi^2_{144}$ p-value $= 0.0101$
Test of hypothesis that the reward-to-covariability is constant, $A = [a_1, O]$:
$LR = 55.234 \sim \chi^2_5$ p-value $= 0.0000$
Test of hypothesis that the covariability is not priced, $A = 0$:
$LR = 79.665 \sim \chi^2_6$ p-value $= 0.0000$

Panel B: Three-factor Model

Factors	Unconstrained A Information variables						Constrained $A = [a_1, O]$
	Constant	S&P	HB3	Baa-Aaa	DIV	TB1	Constant
DEF	12.9967	16.2282	−5.0218	−0.7113	10.4313	−8.7470	10.1511
	(2.80)	(3.55)	(−1.19)	(−0.26)	(1.80)	(−1.11)	(2.95)
TERM	12.2057	−1.6210	−2.7225	0.1203	0.4186	−4.4290	7.2327
	(3.97)	(−0.62)	(−1.46)	(0.04)	(0.14)	(−1.29)	(3.63)
MKT	1.0878	−0.9701	1.6869	−1.8027	7.9506	−8.8815	0.5072
	(0.68)	(−0.78)	(1.32)	(−1.38)	(4.76)	(−3.94)	(0.46)

Test of over-identifying restrictions:
$J = 184.83 \sim \chi^2_{132}$ p-value $= 0.0017$
Test of hypothesis that the reward-to-covariability is constant, $A = [a_1, O]$:
$LR = 172.37 \sim \chi^2_{15}$ p-value $= 0.0000$
Test of hypothesis that the covariability is not priced, $A = 0$:
$LR = 350.97 \sim \chi^2_{18}$ p-value $= 0.0000$

Table I (continued)

Panel C: Five-factor Model

Factors	Unconstrained A Information variables						Constrained $A = [a_1, O]$
	Constant	S&P	HB3	Baa-Aaa	DIV	TB1	Constant
DEF	0.6343	19.1705	−8.4651	−2.1750	10.8282	−2.7858	4.6300
	(0.12)	(3.91)	(−1.82)	(-0.51)	(1.49)	(−0.33)	(1.20)
TERM	7.7262	0.4611	−3.8095	−1.3795	3.7567	−3.5923	7.3330
	(2.33)	(0.18)	(−1.79)	(−0.42)	(1.05)	(−0.97)	(3.47)
MKT	0.0605	−2.8299	0.9904	0.5292	2.2094	−5.9122	−4.3609
	(0.03)	(−1.63)	(0.63)	(0.23)	(0.96)	(−1.80)	(−2.86)
SMB	7.4388	0.9161	3.1522	−1.9916	7.9306	−5.5940	5.5633
	(3.26)	(0.45)	(1.32)	(−0.77)	(2.51)	(−1.45)	(3.43)
HML	4.6113	0.1151	1.9725	2.0143	−5.1490	0.5922	3.3254
	(2.18)	(0.08)	(1.02)	(0.80)	(−1.88)	(0.17)	(2.16)

Test of over-identifying restrictions:
$J = 185.19 \sim \chi^2_{120}$ p-value $= 0.0001$

Test of hypothesis that the reward-to-covariability is constant, $A = [a_1, O]$:
$LR = 190.74 \sim \chi^2_{25}$ p-value $= 0.0000$

Test of hypothesis that the covariability is not priced, $A = 0$:
$LR = 880.44 \sim \chi^2_{30}$ p-value $= 0.0000$

Test of hypothesis that the covariability associated with (DEF, TERM, MKT) is not priced, $A = [O, A_2]'$:
$LR = 135.68 \sim \chi^2_{18}$ p-value $= 0.0000$

Test of hypothesis that the covariability associated with (SMB, HML) is not priced, $A = [A_1, O]'$:
$LR = 47.386 \sim \chi^2_{12}$ p-value $= 0.0000$

Table II
Consistency of the Asset Pricing Models with the Empirical Model

The table presents the estimates of d, δ and θ in the following models over the period July 1964 to December 1992,

$$\begin{aligned} E[(r_{i,t+1} - d'x_{it} - h_i'z_t^*) \otimes (x_{it}, z_t^*)] &= 0, \qquad i = 1, \ldots, n, \\ E[(r_{i,t+1}(f_{t+1} - Cz_t)'Az_t - \delta'x_{it} - \eta_i'z_t^*) \otimes (x_{it}, z_t^*)] &= 0, \qquad i = 1, \ldots, n, \\ E[(r_{i,t+1} - r_{i,t+1}(f_{t+1} - Cz_t)'Az_t - \theta'x_{it} - c_i'z_t^*) \otimes (x_{it}, z_t^*)] &= 0, \qquad i = 1, \ldots, n, \end{aligned}$$

where $r_{i,t+1}$ is the portfolio return, f_{t+1} the marketwide factors, $z_t^* =$ (S&P$_t$, HB3$_t$, Baa-Aaa$_t$, DIV$_t$, TB1$_t$)$'$ is a vector of five information variables observable at the end of month t, where S&P is the return on S&P 500 index, HB3 the difference between the returns on three-month and one-month Treasury bills, Baa-Aaa the difference between the yield on a portfolio of Baa-rated bonds and the yield on a portfolio of Aaa-rated bonds, DIV the dividend yield on the S&P 500 index, and TB1 the one-month Treasury bill rate. $x_{it} = (1, \ln(\mathrm{BM})_{it}, \ln(\mathrm{ME})_{it})'$ is the firm-specific variables where ln(BM) is the logarithm of book-to-market ratio and ln(ME) is the logarithm of market value of equity. A and C are taken as given from the estimation in Table I. . The returns are on 25 portfolios sorted by size and book-to-market ratio. Each of the three sets of equations is estimated separately. Equations for all 25 portfolios within each set are estimated jointly to obtain cross-sectional parameters d, δ or θ. Numbers in parentheses below the estimates are asymptotic z-ratios. LR is the likelihood ratio test of the hypothesis that the coefficients of ln(BM) and ln(ME) are jointly zero, and the number in parenthesis below is its asymptotic p-value.

Table II (continued)

		Constant	ln(BM)	ln(ME)	LR test $\sim \chi^2_2$
Empirical model:	d	2.244	0.162	−0.123	34.48
		(5.48)	(2.23)	(−4.46)	(0.000)
1-factor, unconstrained A:	δ	0.272	−0.015	−0.016	0.32
		(0.66)	(−0.21)	(−0.57)	(0.850)
3-factor, unconstrained A:	δ	0.604	0.049	−0.018	1.31
		(1.47)	(0.68)	(−0.66)	(0.520)
5-factor, unconstrained A:	δ	1.057	0.089	−0.055	7.77
		(2.58)	(1.23)	(−1.99)	(0.021)
1-factor, $A = [a_1, O]$:	δ	0.040	−0.005	−0.002	0.01
		(0.10)	(−0.08)	(−0.07)	(0.996)
3-factor, $A = [a_1, O]$:	δ	0.383	0.009	−0.012	0.25
		(0.94)	(0.12)	(−0.43)	(0.881)
5-factor, $A = [a_1, O]$:	δ	0.627	0.098	−0.033	4.68
		(1.53)	(1.35)	(−1.18)	(0.097)
1-factor, unconstrained A:	θ	2.146	0.164	−0.114	31.82
		(5.26)	(2.24)	(−4.17)	(0.000)
3-factor, unconstrained A:	θ	1.496	0.101	−0.097	22.53
		(3.93)	(1.44)	(−3.73)	(0.000)
5-factor, unconstrained A:	θ	0.740	0.052	−0.039	4.61
		(1.97)	(0.77)	(−1.55)	(0.100)
1-factor, $A = [a_1, O]$:	θ	2.188	0.172	−0.119	33.93
		(5.35)	(2.36)	(−4.33)	(0.000)
3-factor, $A = [a_1, O]$:	θ	1.772	0.162	−0.105	29.83
		(4.45)	(2.31)	(−3.92)	(0.000)
5-factor, $A = [a_1, O]$:	θ	1.444	0.015	−0.086	12.61
		(3.75)	(0.22)	(−3.29)	(0.002)

Table III
Degrees of Adequacy of Asset Pricing Models

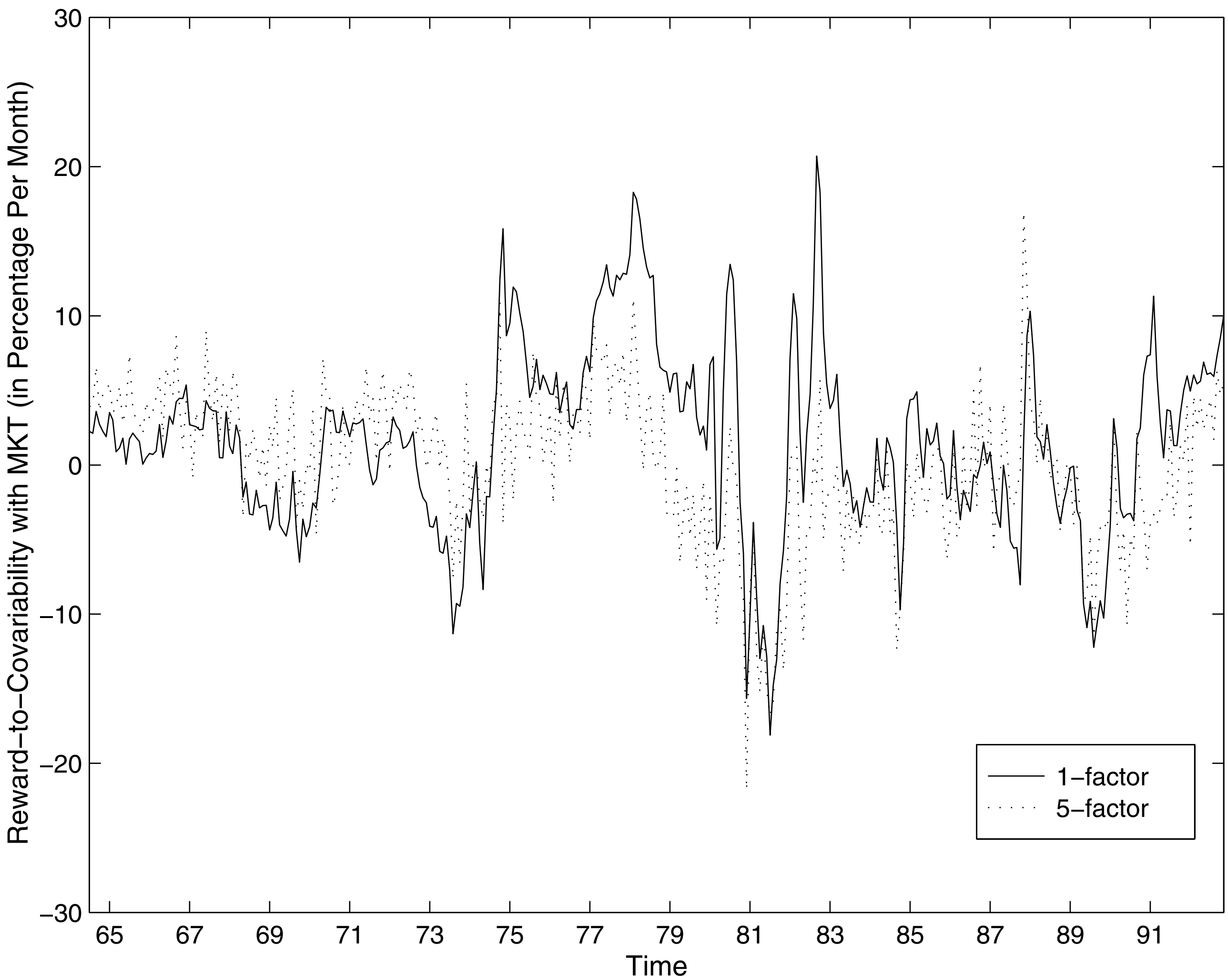
The table presents the cross-sectional variance ratios and selected time series variance ratios estimated over the period July 1964 to December 1992. The variance ratios are defined as

$$
\begin{aligned}
\mathrm{VR}^{RS}(x) &= \frac{\sum_{t=1}^{T} \overline{\mathrm{Var}}(\delta' x_{it})}{\sum_{t=1}^{T} \overline{\mathrm{Var}}(d' x_{it})}, \\
\mathrm{VR}^{RS}(x, z^*) &= \frac{\sum_{t=1}^{T} \overline{\mathrm{Var}}(\delta' x_{it} + \eta_i' z_t^*)}{\sum_{t=1}^{T} \overline{\mathrm{Var}}(d' x_{it} + h_i' z_t^*)}, \\
\mathrm{VR}^{MR}(x) &= \frac{1}{T} \sum_{t=1}^{T} \frac{\overline{\mathrm{Var}}(\delta' x_{it})}{\overline{\mathrm{Var}}(d' x_{it})}, \\
\mathrm{VR}^{MR}(x, z^*) &= \frac{1}{T} \sum_{t=1}^{T} \frac{\overline{\mathrm{Var}}(\delta' x_{it} + \eta_i' z_t^*)}{\overline{\mathrm{Var}}(d' x_{it} + h_i' z_t^*)}, \\
\mathrm{VR}_i^{TS}(x) &= \frac{\widehat{\mathrm{Var}}(\delta' x_{it})}{\widehat{\mathrm{Var}}(d' x_{it})}, \qquad i = 1, \ldots, n, \\
\mathrm{VR}_i^{TS}(x, z^*) &= \frac{\widehat{\mathrm{Var}}(\delta' x_{it} + \eta_i' z_t^*)}{\widehat{\mathrm{Var}}(d' x_{it} + h_i' z_t^*)}, \qquad i = 1, \ldots, n,
\end{aligned}
$$

where $z_t^* = (\text{S\&P}_t, \text{HB3}_t, \text{Baa-Aaa}_t, \text{DIV}_t, \text{TB1}_t)'$ is a vector of five information variables observable at the end of month t, where S&P is the return on S&P 500 index, HB3 the difference between the returns on three-month and one-month Treasury bills, Baa-Aaa the difference between the yield on a portfolio of Baa-rated bonds and the yield on a portfolio of Aaa-rated bonds, DIV the dividend yield on the S&P 500 index, and TB1 the one-month Treasury bill rate. $x_{it} = (1, \ln{(\text{BM})_{it}}, \ln{(\text{ME})_{it}})'$ is the firm-specific variables where ln(BM) is the logarithm of book-to-market ratio and ln(ME) is the logarithm of market value of equity. δ, d, η_i and h_i are Parameters defined in (10)–(11). An overbar represents the cross-sectional average and a wide hat represents the time series average. All the asset pricing models have an unconstrained A matrix.

Table III (continued)

		1-factor	3-factor	5-factor
Cross-sectional variance ratios				
$\mathrm{VR}^{RS}(x)$		1.3%	3.5%	21.9%
$\mathrm{VR}^{RS}(x, z^*)$		17.2%	24.3%	29.9%
$\mathrm{VR}^{MR}(x)$		1.3%	3.5%	21.9%
$\mathrm{VR}^{MR}(x, z^*)$		14.3%	29.4%	32.4%
Time series variance ratios				
minimal	$\mathrm{VR}_i^{TS}(x)$	0.2%	3.7%	22.5%
median	$\mathrm{VR}_i^{TS}(x)$	0.7%	4.1%	23.0%
maximal	$\mathrm{VR}_i^{TS}(x)$	0.9%	5.3%	25.0%
minimal	$\mathrm{VR}_i^{TS}(x, z^*)$	1.8%	23.0%	20.2%
median	$\mathrm{VR}_i^{TS}(x, z^*)$	5.8%	29.5%	30.5%
maximal	$\mathrm{VR}_i^{TS}(x, z^*)$	18.8%	77.6%	58.5%



30
20
10
0
−10
−20
−30
Reward−to−Covariability with MKT (in Percentage Per Month)
65
67
69
71
73
75
77
79
81
83
85
87
89
91
Time
1−factor
5−factor

Figure 1: Time series plot of reward-to-covariability with the market portfolio estimated under the one-factor and five-factor models (1964/7–1992/12)

The figure plots the time series of reward-to-covariability with the value-weighted market portfolio of the NYSE, AMEX and NASDAQ (MKT). The solid line represents the reward-to-covariability with the market portfolio estimated under the one-factor model. The dotted line represents the reward-to-covariability with the market portfolio estimated under the five-factor model.