Credit Default Swaptions

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Among the credit derivatives traded since 1991, credit default swaps (CDS) account for the vast majority of trading. Like interest rate swaptions in the interest rate marketplace, credit default swaptions represent a potentially important derivative product for credit markets.

A CDS that is cancelable includes an embedded credit default swaption. A cancelable long CDS position (where long means that the CDS trader is paying a fixed swap rate and is thus the buyer of credit protection) is a package of a straight (non-cancelable) long CDS plus a put-style CDS swaption—an option to enter a CDS short and thus close the outstanding long position. A cancelable short CDS represents a combination of a short position in a straight CDS plus a call-style CDS swaption.[1]

To the extent that a CDS is cancelable—and most are in practice—ignoring the value of the embedded CDS swaption can lead to pricing errors and thus arbitrage opportunities. We believe that methods used to establish initial swap rates on cancelable CDS, as well as methods used to value seasoned CDS that are cancelable, typically ignore the embedded swaption to terminate the position, so these CDS may be mispriced.[2]

Our purposes are threefold: to describe CDS swaptions; to illustrate some of their applications; and, most important, to present accessible valuation models.

I. PRODUCT DESCRIPTION

We describe CDS swaptions using an example. Assume that all counterparties (dealers and buy side) are AA-rated, which could be through credit enhancements such as collateral, midmarket, or netting agreements.

Assume the CDS that underlies the swaption has a three-year maturity, semiannual payment dates, and a swap rate (the strike rate on the swaption) of 150 basis points (bp). The strike rate assumes semiannual compounding—the same periodicity (or tenor) of the CDS. The credit default swap underlying the reference credit asset is a BB-rated ten-year 8% coupon bond with $100 million par. The CDS swaption is a call, European-style, with a maturity of six months. Thus the CDS swaption owner has the right, in six months, to enter the underlying CDS long, that is, paying 150 bp.

Suppose that in six months, when the swaption matures, the bid-offer swap rates on newly minted three-year credit default swaps (with semiannual tenors)—on the same reference credit asset (or pari passu asset)—are 200 bp by 220 bp.[3] The underlying bond has thus exhibited credit deterioration, perhaps having been downgraded to a weak single B. The call swaption is exercised, meaning that its owner can now long the same swap paying just 150 bp.

By engaging in a reversing trade (entering a short CDS), the swaption owner locks in an annuity of 50 bp (the bid of 200 bp less the strike rate of 150 bp) on $50 million for the next six semiannual periods. This annuity is present-valued (monetized) at the *interest rate swap* midrate on a new three-year semi-annual (s.a.) dollar-LIBOR swap since both counterparties are AA-rated.

If the swaption is a put and at expiration new CDS rates are 100 bp by 110 bp (perhaps because the bond is now a weak single A), the payoff to the CDS swaption would be the present value (again, discounted at the three-year interest rate swap midrate) of six annuity payments of $50 million times 40 bp (the strike rate of 150 bp less the offer of 110 bp).[4]

CDS swaptions that are traded outright are likely to be European, but a cancelable CDS will entail either an American or Bermudan swaption. For example, consider a long CDS giving the buyer of credit protection the option to terminate the swap every six months. Assume the underlying reference credit asset is unique and illiquid, and has no pari passu substitutes. Then this CDS represents a package of a straight CDS plus a potentially valuable Bermudan put swaption—the ability to short the CDS, at six-month intervals, thus closing the original long position.

If the reference credit asset (our 8% coupon bond) has a default-triggering event (such as a missed coupon date) before the six-month maturity of the swaption, the CDS would be terminated by physical or cash settlement, which means we have a CDS swaption with no underlying. If the CDS swaption is a put, the issue is moot; the put swaption owner would not want to exercise, because the credit spread on the defaulted bond would presumably explode to something well above the original (150 bp) strike rate.

If it is a call swaption, the owner would want to exercise. The call owner therefore needs a mechanism to capture value. European-style CDS call swaptions permit early exercise in the event of a default-triggering event for the reference credit asset before maturity of the CDS swaption.[5]

If the CDS call swaption is exercised early because of default of the reference credit asset, the swaption owner ought to be required to make a payment to the writer. Such a payment represents a type of premium accrual on a default insurance policy written on the reference credit asset. In our illustration, if at inception of the swaption there is a six-month bond insurance policy costing 10 bp of face value that pays the difference between the face value and the recovery value of the bond, then, assuming the reference credit asset defaults midway through the life of the swaption, the CDS call swaption owner (or its bond insurance company) would be required to pay 5 bp of $100 million.

Note that a pari passu provision on the underlying CDS does not eliminate the need for the call swaption owner to exercise in the event of default of the reference credit asset. Finally, these circumstances do not affect cancelable CDS. A default-triggering event terminates the CDS and therefore the embedded option to cancel the CDS.

Besides plain vanilla CDS swaptions—whether American, Bermudan, European, calls, puts, outright, or embedded in cancelable CDS—there are a variety of more exotic CDS swaptions, such as swaptions written on binary and basket CDS, or barrier CDS swaptions. It will be interesting to watch changes in the market for CDS swaptions as the market for credit derivatives in general continues to grow in size and innovation.

II. PRODUCT APPLICATION

To illustrate the use of CDS swaptions, we consider three product applications: to reduce a bank's regulatory capital; to create a synthetic credit-linked note; and to create a synthetic collateralized debt obligation.

Reducing Bank Regulatory Capital

Suppose a bank is carrying so many commercial loans as to compromise its regulatory capital. The bank cannot sell all the loans because most are not assignable; it needs to reduce its regulatory capital requirements.

The bank can sell one loan and use the proceeds to purchase a call credit default swaption whose underlying reference credit asset is a portfolio of the remaining loans (or a highly correlated basket of them). By purchasing this basket CDS call swaption, the bank should obtain regulatory capital relief much like being long a basket CDS.

The principal advantage of buying the CDS call swaption (versus entering a long position in a basket CDS) is the returns earned on the loans should their credit quality improve. The principal disadvantage of the swaption is its cost.

Creating a Synthetic Credit-Linked Note

Suppose a hedge fund buys a four-year floating-rate note issued by a highly rated bank sponsor. The note pays s.a. dollar-LIBOR plus 5 bp. The fund manager can enhance the coupon to s.a. $LIBOR plus 45 bp if she agrees to bear the default risk associated with an altogether different bond (in addition to the credit risk of the note she is buying). This is a common credit-linked note (representing a way dealers lay off their credit risk from engaging in short CDS positions).

Instead, the manager can effectively enhance the coupon on the note by writing a put CDS swaption on the same/second bond. By purchasing the floating-rate note and writing the put CDS swaption, the hedge fund manager is long a synthetic credit-linked note.

Creating a Synthetic Collateralized Debt Obligation

Suppose an asset manager wants to create a synthetic collateralized debt obligation (CDO), so he issues or sponsors a $200 million CDO (through a special-purpose vehicle) with four debt tranches and one equity tranche. $175 million represents the debt tranches and $25 million represents the equity tranche, which the sponsor keeps. The $200 million is then invested in high-quality agency securities.

The manager then shorts a CDS (as a credit protection seller) on 20 different high-yield bonds with an average notional principal of $10 million each. For writing these credit default swaps, the CDO will receive an average of 520 bp per year. The average yield on the agency bonds held is 4.41%. Thus, with the pick up of 5.20 percentage points, the synthetic high-yield assets are yielding 9.61%.

Suppose further that the funding costs (the debt tranches of the CDO) have an average yield of 5.63%. The manager wins if the losses from default are less than 398 bp per year. The losses will be determined by the number and amount of the high-yield bonds that default and the recovery rates on those defaulted bonds.

This CDO is said to be synthetic because the yield enhancement (on the agency bonds) is occasioned by shorting CDS, rather than holding junk bonds. But instead of shorting CDS, the sponsor could write a call CDS swaption whose underlying CDS references the same basket of high-yield bonds. Buying agency bonds and writing CDS call swaptions is an alternative way to create a synthetic CDO.

III. PRICING CDS SWAPTIONS

We first address the pricing of European CDS swaptions, and discuss how to obtain the two critical model inputs, the forward CDS swap rate and the forward volatility. We then illustrate the valuation of Bermudan CDS swaptions.

Schönbucher [2000], Jamshidian [2002], and Schmidt [2004] develop other valuation models for CDS swaptions that we find less accessible.

European CDS Swaption Pricing

If the forward credit default swap midrate is lognormal, European CDS swaptions can be priced using a straightforward modification of Black's [1976] model.[6] That is, the CDS swaption can be priced using a model that prices interest rate swaptions.

The notation is as follows:

R_0 = relevant forward CDS swap rate, expressed with compounding of m periods per year, at time 0;
R_K = strike rate on the CDS swaption, also expressed with compounding of m periods per year;
T = maturity of the CDS swaption;
σ = standard deviation of the change in the natural logarithm of R_0, i.e., the *forward vol*;
n = maturity of the underlying CDS;
m = periodicity (or tenor) of the underlying CDS;
$P(0, T_i)$ = price at time 0 of a $1 face value, pure discount bond maturing at times T_i, $i = 1$ to mn; and
L = notional principal of the underlying CDS, commonly the face value of the reference credit asset.

The model to value a European CDS call swaption, C^E, is given by:

$$C^E = LA[R_0N(d_1) - R_KN(d_2)] \tag{1}$$

where $A = (1/m)\Sigma\, P(0, T_i)$ (for the summation 1 through mn);

$$N(x) = \int_{-\infty}^{x}(1/\sqrt{2\pi})e^{-(x^2/2)}dx;$$

$$d_1 = [\ln(R_0/R_K) + \sigma^2T/2]/\sigma\sqrt{T}; \text{ and}$$
$$d_2 = d_1 - \sigma\sqrt{T}.$$

The corresponding model to value a European CDS put swaption, P^E, is:[7]

$$P^E = LA[R_KN(-d2) - R_0N(-d_1)] \tag{2}$$

Suppose the forward CDS swap rate is 150 bp (with semiannual compounding), so the call swaption is struck at the money. Assume the forward vol is 12%. Finally, assume that the interest rate swap curve is flat at 3% per year with continuous compounding.

With L = \$100 million, m = 2, n = 3, R_0 = 0.015 s.a., R_K = 0.015 s.a., σ = 0.12, and T = 0.50;

$$A = (1/2)[e^{(-0.03 \times 0.5)} + e^{(-0.03 \times 1.0)} + e^{(-0.03 \times 1.5)} + e^{(-0.03 \times 2.0)} + e^{(-0.03 \times 2.5)} + e^{(-0.03 \times 3.0)}] = 2.785295$$

$$d_1 = [\ln(0.015/0.015) + (0.12)^2(0.50)/2]/(0.12)\sqrt{0.50}] = 0.04246$$

$$d_2 = 0.04246 - (0.12)\sqrt{0.50} = -0.04239$$

$$N(0.04246) = 0.51696 \text{ and } N(-0.04239) = 0.48307$$

$$C^E = \$141,590.$$

The value of the put is the same, that is, P^E = \$141,590, since both are struck at the money.

Note that a long (short) position in a CDS call swaption combined with a short (long) position in a corresponding CDS put swaption creates a synthetic long (short) forward-starting CDS (starting at time T and with swap rate R_K). This in turn implies that we can use combinations of CDS swaptions to infer default rates and recovery rates for the underlying reference credit assets (see Hull [2003, p. 641]).

Obtaining R_0 and σ

The critical inputs in CDS swaption valuation are the relevant forward CDS swap rate R_0 and the forward vol σ. One can readily compute the forward CDS swap rate if there is a CDS swap curve for the reference credit (or pari passu) asset. The methodology is analogous to obtaining a forward interest rate swap rate from an interest rate swap curve.

In practice, there is typically a term structure of CDS swap rates. For example, in January 2001, Enron's rating was Baa1 (Moody's), and the bid-offer midrates on Enron three-, five-, seven-, and ten-year credit default swaps were 115 bp, 125 bp, 137 bp, and 207 bp, respectively.

If a CDS swap rate term structure is not available on the reference credit (or pari passu) asset, one must generate a credit spread term structure using a model such as Jarrow, Lando, and Turnbull [1997].

Obtaining the forward vol σ in practice is much more difficult. For interest rate swaptions, forward vols can be gleaned from the prices of actively traded over-the-counter dollar-LIBOR options, but there are now no other actively traded options on CDS from which to imply vols, vol term structures, and forward vols. Hence, one must generate a volatility term structure using historic data (on credit spreads) and an econometric time series model.

An example using historic data and a GARCH (1, 1) model to forecast forward vols is in Hull [2003, Chapter 17].[8]

IV. AMERICAN AND BERMUDAN CDS SWAPTION PRICING

The pricing of American and Bermudan CDS swaptions (interest rate swaptions) depends on the evolution of the entire relevant credit default swap rate term structure (interest rate swap curve), rather than a single credit default swap rate (interest rate swap rate) expected to prevail at option maturity. A no-arbitrage term structure model of credit default swap rates makes the pricing of American and Bermudan swaptions extremely complicated.

For the popularly traded Bermudan interest rate swaption, which permits the swaption owner to exercise on the net payment dates, most professional traders use a one-factor no-arbitrage interest rate term structure model. While some experts have argued that such an approach is prudent (Andersen and Andreasan [2001]), others contend it leads to substantial pricing error (Longstaff, Santa-Clara, and Schwartz [2001]). The pricing of American and European CDS swaptions is no less complicated and controversial.

Fortunately, end user demand for CDS swaptions is heavily concentrated in the European swaptions. And for cancelable CDS, the value of the embedded American or Bermudan option to terminate is largely minimized, or completely eliminated, if the underlying reference credit asset, or pari passu asset, is liquid (permitting a trader in a CDS to close the position by executing a reversing or opposite trade in a new CDS written on the same or pari passu asset).

We first discuss pricing using a one-factor credit spread term structure model. A second alternative is to price American and Bermudan CDS swaptions using a richer credit spread term structure model in conjunction with Monte Carlo simulation.

One-Factor Model Approach

In a simple one-factor model of credit spreads, in continuous time, the one factor would be the instantaneous credit spread. The model does not permit mean-reversion in the credit spread. It assumes a flat credit spread volatility term structure; that is, all credit spreads, whether short-dated or long-dated, have the same volatility.[9]

And as a one-factor model, it does not permit the possibility of short-term and long-term credit spreads moving in opposite directions contemporaneously (a credit spread twist). The model permits the credit spread term structure to shift in non-parallel ways, and it accommodates a level effect in that credit spreads become more (less) volatile with a rise (fall) in credit spreads.

We examine a discrete-time version of the model with a time increment equal to 0.5 years, so the one factor is the six-month credit spread. The reference credit assets are a series of risky high-yield bonds such as emerging market bonds. Given the credit quality of the bonds' issuer, suppose current (time 0) credit spreads (out to two years) are 554 basis points for a 0.5-year maturity, 545 bp for a 1.0-year maturity, 547 bp for a 1.5-year maturity, and 550 bp for a 2.0-year maturity.

These four credit spreads represent the relevant credit spread term structure. The rates are expressed with semiannual compounding and have been purged of any contaminating factors such as embedded options in the reference credit assets.

The change in the short-term six-month one-factor credit spread is given by the multiplicative term:

$$e^{mh \pm \sigma\sqrt{h}} \tag{3}$$

where m represents a drift term (or mean), h represents a time increment, and σ represents the volatility of the credit spread (the standard deviation of the percentage change in the natural logarithm of the credit spread).

Equation (3) implies a recombining binomial framework in that the one factor—the short-term credit spread—and therefore the entire credit spread term structure can move up or down after a discrete increment of time (h), which here = 0.5. We will assume that $\sigma = 0.17$; the volatility of the credit spread (of any maturity) is 17% per year, a high volatility accompanying a substantial average credit spread (due to the level effect).

The drift terms m_1, m_2, m_3, and m_4 are non-stochastic but can change each period. These terms are parameterized by forcing the model to fit the current credit spread term structure (a no-arbitrage approach). We also force the risk-neutral probabilities of the up jumps (and down jumps) in the single factor (and therefore entire credit spread term structure) to be 50%.

Given the initial credit spread term structure, we have the values: $m_1 = -0.0797$, $m_2 = 0.0422$, $m_3 = 0.0169$, and $m_4 = 0.0015$. These values are obtained via no-arbitrage arguments. For example, m_1 is calculated from:

$$0.5[e^{0.5m_1+0.17\sqrt{0.5}} + e^{0.5m_1-0.17\sqrt{0.5}}]0.0554 = 2\left[\frac{(1 + 0.0545/2)^2}{(1 + 0.0554/2)} - 1\right]$$

Exhibit 1 reports the resulting tree of credit spread term structures for the high-yield emerging market bonds. The top number in each node represents the prevailing (at time 0) or subsequently prevailing (depending on the jump in the term structure) 0.5-year credit spread. The second number (if there is one) represents the prevailing or subsequently prevailing 1.0-year credit spread. The third number represents the prevailing or subsequently prevailing 1.5-year credit spread; and the fourth number at time 0 is the current 2.0-year credit spread. The probabilities of upward and downward movements in the term structure are 50% each.

We can use this tree of credit spread term structures to value a Bermudan CDS swaption that permits exercise every six months.[10]

For illustration, consider a Bermudan CDS put swaption with two-year maturity and strike rate 550 bp (with semiannual compounding), whose underlying is an originally two-year CDS entailing $100 million face value of the reference credit asset having two-year maturity. In other words, the CDS swaption permits its owner—every six months for two years—to opt to sell credit protection and to receive 5.50% s.a., until year 2. The swaption grants the right to short the CDS for zero; that is, enter a no-cost short position in the CDS (receiving 5.5% s.a. and paying an amount contingent upon default of the two-year emerging market bond).

For simplicity, assume the relevant interest rate curve (used for discounting all cash flows) is flat at 3% s.a., so every forward rate is also 3% s.a., and each expected six-month discounting factor ($d_{0.5}$) is $1/[1 + (0.03/2)] = 0.9852$. Assume also for simplicity that the volatility of each forward rate is zero (of course unrealistic). We relax this assumption later.

Given the binomial term structure, Exhibit 2 shows the tree of values (in $millions) for a short position in the

EXHIBIT 1

Tree of Credit Spread Term Structures

Time 0	Time 0.5	Time 1.0	Time 1.5
			7.864%
		6.915%	
	6.004%	6.968%	
	6.089%		6.184%
	6.147%		
5.54%		5.437%	
5.45%		5.479%	
5.47%			
5.50%	4.721%		4.862%
	4.788%		
	4.834%	4.275%	
		4.308%	
			3.823%

underlying CDS. It assumes that at any node, the short CDS party can reverse-trade by entering a long CDS position, and therefore lock in an annuity of future inflows (or outflows) to be discounted at (here) 3% s.a. (the zero-volatility forward swap rate). The annuity itself is given by the difference between the original CDS rate (5.50% s.a.) and the new rate, times one-half of $100 million. The new rate is the appropriate swap rate on the new/long CDS. The length of the annuity is obvious—the remaining maturity of the original CDS or, equivalently, the maturity of the new/long CDS.

Determining the new swap rate is rather straightforward in this simplified environment. For example, at time 1.5 in the up-up-up-state (uuu), the new credit spread is 7.864%. As at this time there is just one more period remaining in the original swap, the reversing trade would entail assuming a long position in a 0.5-year CDS whose correct swap midrate must be 7.864%. Thus the short CDS is valued at (5.50% − 7.864%)($50MM)(0.9852) = −$1.1645MM. The same procedure produces the values presented in nodes uud, udd, and ddd of Exhibit 2.

Now consider an interior node like ud in Exhibit 2. Here we have a credit spread term structure (from Exhibit 1) of 5.437% (0.5-year) and 5.479% (1.0-year). These rates imply discount factors of $d_{0.5} = 0.9735$ and $d_{1.0} = 0.9474$. The par rate occasioned by these rates (the yield to maturity implied, which is also the correct CDS swap rate) is $2(1 - 0.9474)/(0.9735 + 0.9474) = 0.05478$. Thus the value of the short CDS in node ud of Exhibit 2 is given by two payments of (5.50% – 5.478%)($50MM), each discounted at 3% s.a. for a total of $0.021MM.

The other values (at nodes uu, dd, u, d, and at time 0) in Exhibit 2 are calculated in an analogous fashion.[11]

We can now compute in Exhibit 3 the four possible put swaption values at time 1.5. This is the first time point necessary to value the put swaption in the illustration (because the underlying CDS expires at the same time as the swaption, so the last time that any exercise would occur is at the 1.5-year mark).

We then compute in Exhibit 4 the three possible put swaption values at the 1.0-year mark, while checking for the prospect of early exercise. This entails comparing the "wait value" (if any) to the early exercise value and entering in each node the higher of the two values.

This process is repeated at times 0.5 and 0 in order to obtain the value of the Bermudan put swaption. Exhibits 5 (time 0.5) and 6 (time 0) give the value of the swaption at $498,200.

Exhibits 4 through 6 present intuitive results. The value of the swaption will change (directly) with σ. From these results we can compute the swaption's relevant risk metrics (delta or DV01, gamma, and vega).

Exhibit 2

Tree of Values for a Short CDS ($millions)

Time 0	Time 0.5	Time 1.0	Time 1.5
			uuu -1.1645
		uu -1.3867	
	u -0.9377		uud -0.3369
		ud +0.0210	
+0.0019			
	d +0.9727		udd +0.1595
		dd +1.4552	
			ddd +0.4193

Exhibit 3

Put Swaption at Time 1.5 ($millions)

Short CDS Time 1.5	Put Swaption Time 1.5
-1.1645	Max[− 1.1645,0] = 0
-0.3369	Max[− 0.3369,0] = 0
+0.1595	Max[+0.1595,0] = 0.1595
+0.4193	Max[+0.4193,0] = 0.4193

Jarrow-Lando-Turnbull Model

To overcome the concern that the one-factor model assumes a flat credit spread volatility term structure, we suggest a multifactor version of the Markov model as developed by Jarrow, Lando, and Turnbull [1997] (a similar credit spread term structure model is presented in Kijima [1998]). It permits a richer credit spread environment, and can be used to capture the potential evolution of the credit spread.

To permit the possibility of early exercise, we suggest using the Monte Carlo method of either Longstaff and Schwartz [2001] or Andersen [2000]. These methods are suitable because they accommodate American and Bermudan options that depend on two or more stochastic variables. Either method requires a procedure to correct for a suboptimal exercise boundary suggested by Andersen and Broadie [2001].

Exhibit 4

Put Swaption at Time 1.0 ($millions)

Short CDS Time 1.0	Put Swaption Time 1.0	Time 1.5
		0
-1.3867	Exercise value < 0 Wait value = 0 Swaption value = 0	
		0
+0.0210	Exercise value = 0.0210 Wait value = [0.5(0 + 0.1595)](0.9852) Swaption value = 0.0786	
		0.1595
+1.4552	Exercise value = 1.4552 Wait value = [0.5(0.1595 + 0.4193)](0.9852) Swaption value = 1.4552	
		0.4193

Exhibit 5

Put Swaption at Time 0.5 ($millions)

Short CDS Time 0.5	Put Swaption Time 0.5	Time 1.0
		0
-0.9377	Exercise value < 0 Wait value = [0.5(0 + 0.0786)](0.9852) Swaption value = 0.0387	
		0.0786
+0.9727	Exercise value = 0.9727 Wait value = [0.5(0.0786 + 1.4552)](0.9852) Swaption value = 0.9727	
		1.4552

Exhibit 6

Put Swaption at Time 0 ($millions)

Short CDS Time 0.5	Put Swaption Time 0	Time 0.5
		0.0387
+0.0019	Exercise value = 0.0019 Wait value = [0.5(0.0387 + 0.9727)](0.9852) **Swaption value = 0.4982**	
		0.9727

V. CONCLUSION

There is growing interest in credit default swaptions. We have offered some illustrations of their application to achieve a variety of financial goals and discussed valuation models. Two avenues for future research might include implying the default probabilities and recovery rates of the underlying reference credit assets from the market prices of CDS swaptions, and valuing more complex CDS swaptions such as swaptions written on or embedded in binary and basket credit default swaps.

ENDNOTES

[1]See Hull [2003, Chapter 27] for a discussion of straight credit default swaps.

[2]CDS are commonly cancelable because they are written on a particular reference credit asset such as a junk bond. To reverse-trade a CDS without an embedded option to cancel, the trader would have to find another counterparty willing to execute a CDS on the particular reference credit asset. This may not be realistic for liquidity reasons.

[3]A CDS that is physically settled commonly requires the long trader to deliver to the short the reference credit asset, or an equivalent asset, known as a pari passu asset. The short trader then pays the long the face value of the reference credit asset (the notional on the CDS). A cash-settled CDS requires the short trader to pay the long the difference between the face value and the post-default value of the reference credit asset, as determined by a calculation agent. The agent typically ascribes a value by taking the mean of the bid and offer prices quoted by dealers of the reference credit asset. CDS dealers tend to prefer physical settlement because they feel they can obtain better value than that indicated by the calculation agent.

The ability to trade pari passu assets tends to mitigate the value of the embedded swaption to terminate a CDS. It gives the CDS greater secondary market liquidity.

[4]Our illustrations ignore the day-count convention, and assume markets operate continually and time can be divided into perfect one-half year intervals. The usual day-count convention for a CDS or CDS swaption is actual/360.

[5]This is, of course, different from implying that the CDS call swaption is American-style. An American or Bermudan call swaption could be exercised prematurely for reasons other than the termination of the underlying CDS occasioned by the default of the reference credit asset.

[6]Another possibility is to assume the credit spread follows a process analogous to the interest rate process in the LIBOR market model of Brace, Gatarek, and Musiela [1997], Jamshidian [1997], and Miltersen, Sandmann, and Sondermann [1997]. There may be an analytic approximation for the pricing of European credit default swaptions. Hull and White [2000] derive an analytic approximation for the pricing of European interest rate swaptions whose swap reference interest rate is described by the LIBOR market model.

[7]Proofs of all equations are available on request. Note that the dynamic and size of the recovery rate are already reflected in the forward CDS swap rate. The formulas also abstract from the cancelable feature.

[8]See Engle [1982], Bollerslev [1986], Nelson [1990], and Cumby, Figlewski, and Hasbrook [1993].

[9]If each yield constituting the credit spread is itself mean-reverting, then, by definition, the credit spread itself will be mean-reverting, but probably at a much slower rate, so the degree of mean reversion may be nominal. It is probable also

that shorter-term credit spreads are more volatile than longer-term credit spreads.

[10]To value a quarterly Bermudan CDS swaption, one would need to change the value of h (to 0.25). To value an American CDS swaption, h would be much smaller, e.g., a single trading day, so one could frequently test for early exercise and thus obtain an accurate price.

[11]Note that the time 0 value of the short CDS in our illustration is not quite zero but $1,900. The at-market swap rate is slightly lower than 5.50% s.a. at 5.499% s.a.

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