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AN EFFICIENCY FRONTIER MODEL
AN ANALYSIS OF THE MACROECONOMIC IMPLICATIONS OF STRUCTURAL SHOCKS

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I. *Introduction*

Much of the current debate concerning the microelectronics "revolution", and related policy issues such as industrial strategy, takes place without the aid of explicit economic models. Nevertheless, it is generally recognized that the introduction of microelectronics-based technologies can be an important growth stimulus, as well as the source of potential adjustment problems. This paper proposes a tractable capital-theoretic framework for comparing alternative adjustment scenarios in response to such structural shocks.

The basic component of our framework is a dual efficiency frontier structure[1]. The steady-state efficiency curves are extended by distinguishing between *restricted* and *unrestricted* efficiency frontiers[2]. That structure is then "closed" by various intertemporal objective criteria which are based upon alternative expectations hypotheses and the consequent consumption/savings-investment behaviour.

The severity of the restrictions, for example, on capital-stock malleability, determines the generality (that is, steady-state or non-steady-state) of the solution. Since the efficiency frontier structure embodies the restrictions, any alleviation of the latter causes that structure to evolve. The combination of intertemporal optimization with the endogenously evolving efficiency frontier structure determines the dynamic path of the

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[1] The seminal paper was Bruno (1969). Also see Bruno (1967), Craven (1979), Morishima (1971), Orosel (1978), Schweinberger (1980) and Woodland (1977).

[2] The concept of a *restricted* efficiency frontier was used in an Austrian context by Hicks (1973). Craven (1979) analysed its effect on the existence of steady-state equilibria, and Orosel (1978) provided a comprehensive analysis of steady-state efficiency curves in the presence of restrictions due to vintages of capital. Our paper uses the restricted/unrestricted structure to model *non-steady-state* transition paths.

solution, and captures "development" aspects of the growth process as opposed to steady-state proportional dynamics or exogenous adjustment speeds toward a long-run equilibrium.

The convenience of the efficiency frontier model (EFM) is that it provides a particularly simple solution method with which to mimic sequences of temporary equilibria. Utilizing results in Becker (1981, 1982), this paper illustrates an equivalence between the control solution to the EFM and the perfect foresight competitive equilibrium solution to the corresponding decentralized market model. While correspondences of the perfect foresight type are relatively well-known, the EFM can mimic sequences of temporary equilibria under various alternative hypotheses about expectations[3], and the severity of restrictions on technology and the malleability of factors.

The restrictions imposed recognize the possibility of different degrees of flexibility for physical endowments and outputs versus that for prices. The source of restrictions on outputs is due to capital-stock non-malleability which can be summarized by the extent of the choice of techniques, the shiftability of machines, and the degree of embodiment of technical changes. These restrictions prevent a long-run solution from being attained immediately, and hence allow an explicit analysis of sectoral transition paths from short-run solutions, which have fixed stocks, to long-run growth paths[4]. On the other hand, we will assume that prices are always flexible, although their non-steady-state behaviour depends crucially on the degree of foresight inherent in the expectations assumption. A perfect foresight, dynamically efficient path provides a benchmark against which the effects of imperfect foresight can be measured. For example, we compare perfect foresight versus a rule-of-thumb savings adjustment in response to a labour-augmenting innovation. This comparison provides one explanation for

[3] Burmeister and Graham (1975) compare a planning model solution to that of a competitive model in which expectations are formed according to an "adaptive-type scheme" for which perfect foresight is a limiting case. Our paper uses a different solution method and focuses on restrictions on the flexibility of output as well as on restricted foresight. Cass and Shell (1976) provide a comprehensive analysis of the application of "Hamiltonian dynamics" to optimal growth and competitive growth models. The latter have instantaneously adjusted expectations about price changes and focus primarily on the two-class "classical" consumption/savings case. Other analyses of the relationship between optimal control solutions and perfect foresight competitive equilibria include Abel and Blanchard (1983) and Becker (1981, 1982).

[4] Dynamic adjustment paths ("traverses") were suggested, in a growth context, by Hicks (1965). The trade and factor-market distortions literature has analysed dynamic stability and modelled explicit adjustment mechanisms in the presence of short-run factor specificity and/or sluggish price adjustment. See, for example, Mussa (1982), Neary (1978a, 1978b, 1982) and Neary and Purvis (1981). Bruno and Sachs (1982) use a factor price frontier to analyse the dynamic adjustment to an exogenous input price shock assuming perfect foresight and sluggish price adjustment.

the often observed resistance to such changes (for example, by Fleet Street newspaper unions), and suggests ways of improving the economic adjustment to structural shocks.

Section II presents the dual efficiency frontier structure emphasizing the method of incorporating various realistic restrictions on flexibility. Section III "closes" that structure with various intertemporal objective criteria (based on alternative expectations hypotheses), and compares the solutions. In Section IV, the tractability of our solution method is illustrated by comparing alternative adjustment paths in response to a labour-saving innovation. This exercise also emphasizes the crucial importance of analysing explicit adjustment paths. Section V provides some brief concluding comments.

II. *An Efficiency Frontier Structure*

In general, the production possibilities are described by a set $T(M)$ of non-negative per capita outputs and inputs where M represents the available techniques of production. Assume that:

A1: the technology is "productive" although each activity requires positive input of at least one factor in order to produce positive output;

A2: $T(M)$ is a closed, convex cone;

A3: the attainable production set is bounded from above by the ultimate scarcity of primary factors and A1.

In addition, we assume that:

A4: there is free disposal of output;

A5: machines have infinite durability;

A6: production is instantaneous but this period's output cannot be utilized until next period[5];

A7: there are no externalities;

A8: there is perfect competition in all markets.

[5] The period may be arbitrarily short so that in the limit we can use continuous time. For notational convenience, time-scripts are not included unless necessary for clarity.

We begin with a prototype version which avoids commodity inputs (circulating capital) by vertically integrating, and aggregates all the firms in each of two sectors into one large firm. There are two inputs to production – capital (K) and labour (L) – and, unless otherwise stated, quantities are normalized by L[6]. Therefore, for the fixed-coefficients version[7], the production possibilities are summarized by $T(M^*)$ in which M^* represents a positive-definite input-per-unit-output coefficient matrix:

$$\text{A9:} \qquad M^* = \begin{matrix} \alpha & a \\ \lambda & l \end{matrix}\ , \quad \frac{\alpha}{\lambda} > k^* > \frac{a}{l}\ ;$$

where (α, λ) are the capital and labour input requirements for producing a unit of the consumption good, and (a, l) are those for producing a machine.

The basic component of the various versions of the non-steady-state dynamic growth model is a dual efficiency frontier[8] structure. The frontier of the static (that is, for a given initial endowment $k^* \equiv K^*/L^*$) production possibilities set is defined as the quantity efficiency curve:

$$(1) \qquad QEC \equiv \hat{c}(x; k^*, M^*) = \max\{c \mid c(x; k^*, M^*) \in T(M^*)\}\ ;$$

that is, the maximum per capita consumption $(\hat{c})$, for each feasible level of investment per capita (x), subject to the production constraints:

$$(2) \qquad k^* \geqslant \alpha c + ax\ ,$$

$$(3) \qquad 1 \geqslant \lambda c + lx\ ;$$

and non-negativity constraints on outputs (c, x). The QEC combines the technological possibilities (productivity) with the available factor endowment (k^*), and in doing so, illustrates the set of statically efficient outputs for the two sectors. Since $x \equiv gk$ where $g \equiv$ growth

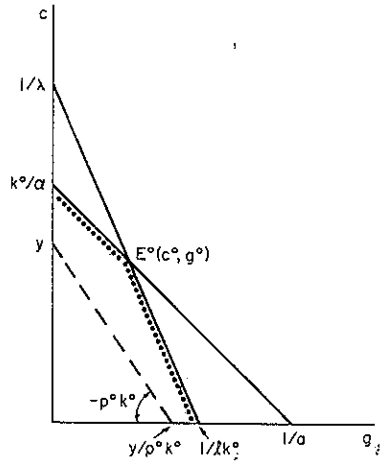
[6] For simplicity, there is no distinction between endowments and factor supplies.

[7] See the Appendix for a version which incorporates choice of technique, in which case M is the cost-minimizing technique.

[8] A constraint refers to a column or row of a technique matrix; an efficiency surface (or curve in the two-sector case) to a technique matrix; and an efficiency frontier to the case where there is a choice of techniques such that there are several efficiency surfaces (one for each technique matrix) with the efficiency frontier consisting of their envelope.

rate of the capital stock, the QEC can be plotted in (c, g) space (figure 1) scaling the horizontal axis by a factor $1/k^{\circ}$.

Fig. 1 - Quantity Efficiency Curve (QEC)



The underlying (that is, for individual sectors) duality is between production and cost functions – or, given A2 and A8, between isoquants and factor price constraints. Factor price possibilities for the economy are given by the non-negative set $P(M^{\circ})$. The frontier of the "momentary" (that is, for a given $p^{\circ} \equiv$ price of the capital good in terms of the consumption good) factor price possibilities set is defined as the price efficiency curve:

(4) $\quad \text{PEC} \equiv \hat{w}(q; p^{\circ}, M^{\circ}) = \min \{ w \mid w(q; p^{\circ}, M^{\circ}) \in P(M^{\circ}) \}$,

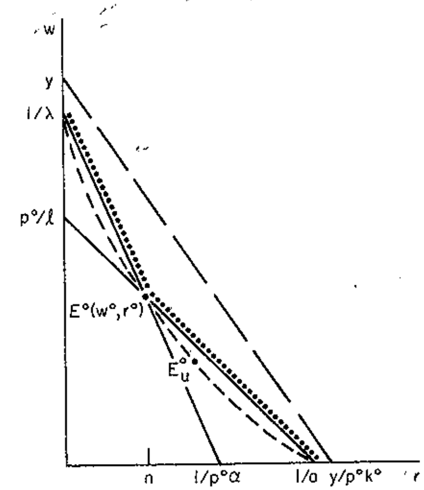
which is the minimum real wage rate ($\hat{w}$), for each feasible rental rate for capital (q), subject to the factor price constraints which ensure that excess profits are zero in equilibrium:

(5) $\quad 1 \leqslant q\alpha + w\lambda$,

(6) $\quad p^{\circ} \leqslant qa + wl$.

Since the rental rate of return per machine is $\frac{q}{p} \equiv r$, the PEC can be drawn in wage/"normal" rate of profit space (Figure 2).

Fig. 2.- Price Efficiency Curve (PEC) and Unrestricted Efficiency Curve (UEC)



Thus far, the production technology M° has been combined with the given endowment k° to derive the economy's QEC, and with the given p° to derive the economy's PEC. However, both k° and p° are fixed or exogenous in the above formulation of the efficiency relationships. In order to allow for endogenous (k, p), the QEC/PEC structure is now extended to a restricted versus unrestricted efficiency curve structure (REC/UEC).

If p is non-predetermined, and thus flexible, we can repeat (4) for all p to derive an envelope curve defined as an unrestricted efficiency curve:

(7) $\quad \text{UEC} \equiv \hat{w}(r; M^{\circ}) = \min \{ w \mid w(r; M^{\circ}) \in P(M^{\circ}) \}$

$$\text{or} \quad w = \frac{1 - ra}{\lambda + r(\alpha l - a\lambda)}$$

Figure 2 shows that varying p causes the price constraints to swivel on the endpoints $\frac{1}{\lambda}$ and $\frac{1}{a}$ tracing-out the UEC. Notice that each PEC (the w/r trade-off for a given p) will touch the UEC at a point such as E – that is, the zero excess profits position for both sectors.

For output, analogously to (7), it is possible to repeat (1) for all k to derive the envelope curve:

$$\text{(8)} \qquad UEC' \equiv \hat{c}(g; M^*) = \max\{c \mid c(g; M^*) \in T(M^*)\}$$

$$\text{or} \quad c = \frac{1 - ga}{\lambda + g(\alpha l - a\lambda)}$$

Note the well-known[9] analytical equivalence between (7) and (8). However, that equivalence is only realized in a steady-state framework. That is, (8) is the efficiency locus of *steady-state* combinations of (c, g).

In order to model non-steady-state dynamics, we introduce restrictions which reflect the different degree of flexibility for physical endowments and outputs versus that for prices. The output efficiency relationship (8) can be restricted by the predetermined endowments (k^*) whose dynamic flexibility may be limited by capital-stock non-malleability which is summarized by the extent of the choice of techniques, the shiftability of machines, and the degree of embodiment of technical changes. An instantaneous, costless, infinite choice of techniques across a wide range of capital intensities could allow immediate adjustment to a new steady state following a shock – without any change in k^*. But this "perfect substitution" or complete malleability is a very strong assumption. In this paper, the range of alternative efficient techniques is more limited. In particular, we model two versions – a fixed technique case which restricts the UEC' to (1) and a choice of techniques version (Appendix) which has an alternative activity in each sector making k more malleable and thus the output efficiency relationship slightly less restricted. The model can be extended to analyse the effects of two other sources of non-malleability – sector-specific factors and embodied technical change.

The implication of the above restrictions is that it takes resources (labour and savings flows) to transmute a fixed capital stock to the structure required by a new steady-state equilibrium. Thus a sequence

[9] See, for example, Bruno (1969) or Burmeister and Kuga (1970).

of non-steady-state solutions may be required. On the other hand, prices are completely flexible, although their non-steady-state behaviour will depend crucially on the degree of foresight inherent in the expectations assumption. Therefore, the basic optimizing structure consists of a restricted efficiency curve (such as (1)) for outputs, and the unrestricted efficiency curve (7) for prices.

III. *Closing the Efficiency Frontier Structure*

III. 1. *A control solution.* Thus far, the structure of our model consists of two efficiency relationships. Closing the model requires some criterion by which to rank or choose from among the set of efficient plans.

Consider a consumption-optimal growth objective[10];

$$\text{(9)} \qquad \text{Maximize} \int_0^T c(t)\, e^{-(\delta - n)t}\, dt,$$

subject to: the production (technological and factor endowment) constraints (2), (3); non-negativity constraints on the controls (c, x); the change in endowments:

$$\text{(10)} \qquad x - nk = \frac{dk}{dt};$$

and boundary feasibility constraints:

$$\text{(11)} \qquad k(0) \leqslant k^*,$$

$$\text{(12)} \qquad k(T) \geqslant k^T \leqslant k_m^T = k(0) + \int_0^T (g_m(t) - n)\, k(t)\, dt.$$

Notice that (10) – which links investment and the exogenous rate of growth of the labour force (n) to changes in endowments – defines dk/dt as net investment per capita since gross per capita output of the investment good ($x \equiv gk$) will be used to "widen" (nk) and, in the

[10] It is assumed that the planner wishes to maximize total utility. Therefore, the planner weights the typical agent's objective criterion $\int_0^T c(t)\, e^{-\delta t}\, dt$ by the population. Expressed in per capita terms the planner's objective criterion becomes (see Burmeister, 1980, p. 250):

$$\max \int_0^T c(t)\, e^{-(\delta - n)t}\, dt.$$

In the example discussed in the text, it is assumed that the typical agent's pure rate of time preference (δ) is equal to the exogenous rate of growth of the labour force or population (n) so that the planner's steady-state solution corresponds to a golden rule path. This assumption is not necessary, nor is it meant to imply what the planner's discount rate "ought" to be.

presence of alternative techniques, "deepen" the existing per capita capital stock. The boundary feasibility conditions require that the k utilized at $t = 0$ cannot be greater than the initial endowment (k°); and the actual k at time T must not be less than the target (k^T) which is a parameter open to choice[11] but subject to a feasibility constraint determined by accumulating at the maximum rate ($g_m(t)$) for all $0 \leqslant t \leqslant T$.

The above problem can be solved using optimal control theory. For example, applying Pontryagin's maximum principle, the optimal control variables (c°, x°), can be derived as functions of (p, k) by maximizing the Hamiltonian, defined as net national product per capita $[y - pnk \equiv c + p[x - nk]]$ subject to the quantity or restricted efficiency curve summarized by (1). Due to the Kuhn - Tucker saddle-point theorem, that problem is equivalent to:

$$\text{(13)} \qquad \max_{c,\,x;\; w,\,q \geqslant 0} \min L(c, x, w, q) \equiv c + p(x - nk) + w[1 - \lambda c - lx] + q[k - \alpha c - ax]$$

A necessary and sufficient condition for $(c^\circ, x^\circ, w^\circ, q^\circ)$ to be a saddle-point of L over $(c, x, w, q) \geqslant 0$, is that $(c^\circ, x^\circ, w^\circ, q^\circ)$ satisfy the Kuhn-Tucker conditions[12]:

$$\text{(14)} \qquad \begin{aligned} \frac{\partial L^\circ}{\partial c} &= 1 - w\lambda - q\alpha \leqslant 0 \leqslant c^\circ, \\ \frac{\partial L^\circ}{\partial x} &= p - wl - qa \leqslant 0 \leqslant x^\circ, \\ \frac{\partial L^\circ}{\partial w} &= 1 - \lambda c - lx \geqslant 0 \leqslant w^\circ, \\ \frac{\partial L^\circ}{\partial q} &= k - \alpha c - ax \geqslant 0 \leqslant q^\circ, \end{aligned}$$

[11] Notice that having specified k^T, the time horizon (T) required to reach that k^T along an intertemporally efficient output path is endogenous. When $T \to \infty$, $k^T \to k^*$ (the new steady-state per capita capital stock), and with $\delta = n$ the maximand can be changed to a Ramsey criterion

$$\int_0^\infty [c(t) - c^*]\, dt,$$

in which $c^* \equiv$ the new steady-state per capita consumption level, in order to ensure convergence. Alternatively, the von Weizsacker (1965) "overtaking criterion" can be used – see, for example, Burmeister (1980) or McKenzie (1979).

[12] As indicated in A2 above, the non-joint sectoral production activities plus input stock adding up constraints ensure the concavity of the quantity or restricted efficiency frontiers.

in which, for example, $u^\circ \equiv 1 - \lambda c - lx$ is a slack variable representing the rate of unemployment, and the notation $u^\circ \geqslant 0 \leqslant w^\circ$ implies that $u^\circ \geqslant 0$, $w^\circ \geqslant 0$, and $u^\circ w^\circ = 0$. That is, the Kuhn - Tucker conditions apply the rule of free goods (RFG) to factors, and the rule of profitability (RoP) to production.

Now using the Valentine - Berkovitz method[13], the dynamics of the intertemporally *efficient* paths are specified by[14]:

$$\text{(15)} \qquad \frac{\partial L^\circ}{\partial p} \equiv \frac{dk}{dt} = x^\circ - nk,$$

$$\text{(16)} \qquad -\frac{\partial L^\circ}{\partial k} \equiv \frac{dp}{dt} = -q^\circ + pn,$$

in which the superscript° refers to solutions to (13) for given p, k.

The given endowments k° and technology M° determine the position of the restricted efficiency curve, and the given p (for example, p° if we start at a steady state) ranks the alternative efficient production plans so that the one (c°, x°) which maximizes y is chosen. In addition, (w°, q°) is the minimum factor cost (or payments) of producing the optimum output subject to the constraint that the inputs absorb the entire value of the output (zero excess profits). Since the capital intensities span the endowments $(\alpha / \lambda > k^\circ > a / l)$, it is an interior solution, that is, the efficient output levels are able to fully employ the factors of production (k°) and thus factor prices are positive $((w, q) > 0)$. On Figure 1 the objective function, $y - p^\circ nk^\circ \equiv c + p^\circ(gk^\circ - nk^\circ)$, is pushed out until it supports the QEC at (c°, g°) and on Figure 2 the objective function supports the PEC at (w°, r°).

Thus (15) follows from (10) and from the static optimally requirements – in particular, the full employment/full utilization

[13] As in Appelbaum (1975). Notice that (13) can be re-arranged to give:

$$\min \max L \equiv w + k(q - pn) + c(1 - q\alpha - w\lambda) + x(p - qa - wl)$$

so that the dual problem gives the same solution as the primal described in the text.

[14] In the event of corner solutions, subdifferentials would replace derivatives – see Rockafellar (1970) or McKenzie (1979). To ensure that (p, k) are continuous along the maximal (and efficient) path(s), criteria analogous to the Weierstrass - Erdmann corner conditions for the classical calculus of variations solution method must be satisfied. This issue is particularly relevant for multisectoral models and/or where switches in technique are possible – see, for example, Burmeister and Graham (1975) or van den Heuvel (1983).

solution derived from (13) for each t. As can be seen from the diagrams, the dynamically efficient outputs are at the full employment/full utilization "kink" of the REC corresponding to each period or "point" in time. The dynamic efficiency condition (16) is the intertemporal zero excess profit condition consistent with perfect foresight and perfect competition (which invokes arbitrage). This price adjustment ensures that the period-by-period REC "kinks" are always tangent with the UEC.

It is useful to analyse the stability of the above perfect foresight solution to the efficiency frontier model in more detail. From the production constraints for outputs, (2) and (3), derive a relationship $k = k(g)$:

(17) $$k = \frac{\alpha}{\lambda + g(\alpha l - a\lambda)}$$

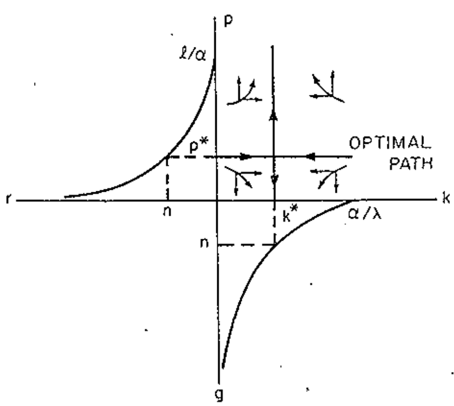
Similarly from (5) and (6)[15]:

(17′) $$p = \frac{l}{\lambda + r(l\alpha - \lambda a)}$$

Given A9, both (17) and (17′) are downward sloping and convex to the origin – in (k, g) and (p, r) space respectively.

On Figure 3, (17) is plotted in the south-east quadrant, and (17′) in the north-west quadrant. The phase diagram (north-east quadrant) is derived by plotting n in the south-east and north-west quadrants, and then recording the efficient directions of motion as dictated by (15) and (16).

Fig. 3- Phase Diagram of the Optimal Path for the State Variables



[15] Notice that both (17) and (17′) are alternative forms of the UEC and thus are appropriate for market-clearing non-steady-state dynamics. In other words, there is a REC "behind" each point of (17).

The optimal solution is a saddlepoint in (p, k) space. For the particular version of the model which is illustrated, $\frac{\alpha}{\lambda} > \frac{a}{l}$, the output path is stable. That is, as soon as an interior solution is possible, $\frac{\alpha}{\lambda} > k^{\circ} > \frac{a}{l}$, the path follows a Leontief trajectory (both stocks fully employed at each point or temporary equilibrium). When g becomes equal to n at $k = k^*$, $\frac{dk}{dt} = 0$ and the output side is in long-run (steady-state) dynamic equilibrium.

However, the price side is potentially unstable. Unless $p(0) = p^*$, prices will diverge (along an unrestricted efficiency curve (17′)) away from their long-run equilibrium solution. Given the inherited k° and choosing a k^T subject to the feasibility constraint (12), it is necessary to determine a unique $p^*(T)$ from which to compute the $p^*(0)$ – using (16) – which is consistent with k°. With perfect foresight, $\lim_{T\to\infty} k^T = k^*$ and the planner can calculate the constant prices necessary to support the maintenance of k^* that is, $q^*/p^* = n$. In other words:

(18) $$\lim_{t\to\infty} p(t)\,k(t) = p^*k^*.$$

This is illustrated by choosing the stable arm of the saddlepoint on the phase diagram. In the special case of fixed coefficients, $p(0) = p^*$ and the stable arm is a straight line in (p, k) space[16].

Interpreted graphically, a "momentary" solution is defined for given (k°, p°) using the QEC/PEC, and if a change in p° is required to reconcile cost and demand prices, the distribution point $E^{\circ}(w^{\circ}, r^{\circ})$ moves along UEC to $E^*(w^*, r^*)$ when p° jumps to $p^*(0)$. The production point $E^*(c^*, g^*)$ will be at the full employment/full utilization "kink" of the REC. Therefore, that point is on the UEC. The

[16] With no joint production the steady-state solution is unique, and given that the maximand (9) is linear, McKenzie (1979) shows that (theorem 7, pp. 66-67) "there is a maximal path from any expansible stock".

relative position of $E^*(c^* g^*)$ and $E^*(w^*, r^*)$, on the UEC, is determined by the relationship between r^* and g^* necessitated by the savings-investment required to satisfy (9) and to achieve k^* along an intertemporally efficient path.

III. 2. *Decentralized market solutions.* We now wish to illustrate the correspondence between the above planner's optimal control solution to the efficiency frontier model (EFM) and various competitive equilibrium solutions to the decentralized market version.

We know[17] that the competitive solution to a production model — formulated in terms of unit cost functions (such as, (5), (6)) and factor market equilibrium conditions ((2), (3)) — can be obtained as a solution $(c^\circ, x^\circ, w^\circ, q^\circ)$ to the linear programming problem (13). Therefore, the static or "momentary" (given p°, k°) solution to the EFM is equivalent to a market solution when p is a parameter[18].

The present formulation requires that the relative price of machines (in terms of consumption goods) be endogenous. Therefore, for the competitive equilibrium versions, one must model the market mechanisms which determine $\{p(t)\}_0^T$. The foresight assumption and the resulting consumption/savings behaviour will be crucial in determining the temporary equilibrium level of p, and thus whether or not the sequence of temporary equilibria will satisfy the *intertemporal* efficiency conditions derived above using the planner's control solution. Therefore, it is useful to compare the implications of static expectations versus perfect foresight about future spot prices[19].

For a perfect foresight decentralized market solution, the objective criterion (9) is replaced by:

$$(9') \qquad \max \int_0^T c(t)\, e^{-\delta t}\, dt ,$$

in which c and δ are the typical individual's level of consumption and discount rate[20] respectively. Given perfect foresight, the rate of

[17] Cf. Cass and Shell (1976) and Woodland (1977).

[18] As in a production model or a small open economy for which the relative commodity price is exogenous. Alternatively, that solution could refer to a case for which the non-substitution theorem holds so that any price changes are initiated by cost changes or, as in this paper, a "momentary" solution in a more general model. For the SOE interpretation, comparative-statics effects of changes in (p, k) can be interpreted in terms of Stolper - Samuelson and Rybczynski theorems (cf. Morishima, 1976; Woodland, 1977 and Dixit - Norman, 1980).

[19] A straightforward extension, which has complete spot markets and a futures market for bonds only, allows one to model the intermediate case of imperfect foresight.

[20] A discount rate $\delta = n$ for individual agents would be "consistent" with a zero discount rate $(\delta - n)$ for a planner who maximizes total utility and uses a per capita maximand as in (9).

return to holding a capital good includes a realized capital gain or loss component. Therefore, the portfolio equilibrium condition between saving and consuming is:

$$(19) \qquad \frac{q}{p} + \frac{dp/dt}{p} = \delta .$$

As shown in Becker (1981, 1982)[21], initial prices and thus the consumption/savings-investment allocation will be such that (19) holds at each t. Consequently, the $\{p(t)\}_0^T$ will be consistent with $\{k(t)\}_0^T$, intertemporal preferences, and market equilibrium conditions. When $\delta = n$, the latter coincide with the control solution efficiency conditions (14) and (16). However, in the competitive equilibrium version, those conditions result from utility maximization and cost minimization with competition ensuring that product prices equal cost of production ((5), (6)), that factor markets are in equilibrium ((2), (3)), and that intra- and intertemporal rates of return are equalized by arbitrage (19).

Therefore, the optimal control solution to the efficiency frontier framework is equivalent to the perfect foresight competitive equilibrium solution for the corresponding decentralized market model. While correspondence of this perfect foresight type are relatively well-known, the convenience of the EFM is that its REC/UEC structure, closed by the appropriate intertemporal objective criterion and asset markets, provides a particularly simple solution method with which to mimic sequences of temporary general equilibria under various alternative expectations hypotheses, and restrictions on technology and/or the malleability of factors.

As a comparison, consider the static expectations case for which agents are restricted to consumption/savings-investment behaviour prescribed by a rule-of-thumb — for example, the classical savings assumption which stipulates that workers do not save and owners of machines do not consume, or a simple uni-class assumption which

[21] Using the Benveniste and Scheinkman (1982) result that, under certain conditions — in particular, $\delta > 0$, infinite horizon concave dynamic optimization models exhibit capital value transversality as a necessary condition for optimality.

postulates a uniform and constant propensity to consume. The uni-class version would result in per capita demand functions:

$$d_1 = b(w + qk) \equiv by, \tag{20}$$

$$pd_2 = (1-b)(w + qk) \equiv (1-b)y \tag{21}$$

in which d_1 and d_2 are the typical individual's demand for consumption and savings/investment respectively, y is the full employment/full utilization level of per capita income and b is the propensity to consume[22]. One could undertake an "inverse optimal" exercise[23] to find a utility function for the representative individual which would be consistent with such rule-of-thumb myopic behaviour – for example, a time-invariant choice between units of current consumption and investment (future consumption) as in a Cobb-Douglas utility function,

$$U = d_1^b \, d_2^{1-b} \tag{9''}$$

Therefore, the demand/supply market equilibrium conditions for per capita consumption and capital goods, respectively are:

$$by \equiv d_1 = c, \tag{22}$$

$$(1-b)y \equiv pd_2 = px. \tag{23}$$

Walras' Law is:

$$(d_1 - c) + p(d_2 - x) + w(\lambda c + lx - 1) + q(\alpha c + ax - k) + \\ + (p - wl - qa)x + (1 - w\lambda - q\alpha)c \equiv 0 \tag{24}$$

which can be re-written in national-income-accounting form as:

$$d_1 + pd_2 = (w + qk) \equiv y. \tag{24'}$$

[22] In a two sector model, demands for the commodities are given by the consumption/savings-investment decisions. With no financial asset, savers acquire the durable physical capital goods directly and thus the per capita demand for investment goods d_2 is identically equal to that for savings. Including a financial asset such as bonds would introduce a portfolio choice by providing an alternative channel for the intertemporal transfer of purchasing power. Nevertheless, for the market-clearing equilibria analysed in this paper, aggregate investment demand equals aggregate savings so that the bond market can remain "behind the scenes".

[23] Cf. Kurz (1969).

The equilibrium conditions [(22), (23), (2), (3), (5), (6)], in combination with Walras' Law (24) show that the RFG applies to the factor markets[24], and that the RoP holds for production decisions. For example, if (3) happened to be a strict inequality in equilibrium, then w = 0; while if (5) is a strict inequality, then the technique (α, λ) would not be utilized.

With static expectations, q and p will be determined such that there is a portfolio equilibrium:

$$\frac{q}{p} = \delta. \tag{19'}$$

Therefore, the competitive equilibrium implies a rule of investment (RoI). That is, if the rate of return on machines is greater (less) than the trade-off between current and future consumption, then more (fewer) investment goods will be demanded. This ensures that, in equilibrium, the rate of return on machines $\left(r \equiv \frac{q}{p}\right)$ and the rate of time preference (or the cost and asset prices) are equal.

In summary, the static (fixed k^* plus static expectations) decentralized market model has six equilibrium conditions, [(2), (3), (5), (6), (22) and (23) plus the aggregate identity (24)], in five unknowns $(c, x (\equiv gk), w, q$ and $p)$. Given (24), only five of the equilibrium conditions are independent. One can combine (22) and (23) so that the equilibrium condition for the relative commodity price can be expressed as:

$$p = \frac{(1-b)c}{bx}, \tag{25}$$

or, using the classical savings assumption:

$$p = \frac{lk}{\alpha}. \tag{25'}$$

The competitive behaviour implied by this solution can be charac-

[24] For a non-trivial solution, at least one factor price is positive so that, given A9 and (5), (6), both commodity prices will be positive. In other words, commodities will never be free, allowing the consumption good to be used as numeraire and (22), (23) to hold as strict equalities.

terized by the rules of free goods, profitability, and investment (RFG, RoP, RoI). Notice that these are the same rules which were implied by the optimal control solution to the efficiency frontier structure – the RFG and RoP as complementary slackness conditions in (13), and the RoI in (16). However, the static decentralized market model was closed by the myopic consumption/savings behaviour (9″) as opposed to the explicit intertemporal optimizing implied by (9). Therefore, there is no reason to expect that the p (0) from (25) will be equal to the $p^*(0)$ which resulted from the optimal control solution and the perfect foresight competitive equilibrium solution.

With respect to alternative expectations hypotheses, we have shown that a static expectations competitive solution can be obtained by "closing" the REC/UEC structure with a myopic savings rule. For example, with a uni-class saving assumption or the corresponding objective criterion (9′), one can mimic the decentralized market solution using (13), (19′) and (25). If "b" is such that the implicit time preference is greater than n, the steady-state solution will correspond to $E^\circ(c^\circ, g^\circ = n)$ and $E^\circ_u\,(w^\circ_u, r^\circ_u > n, p^\circ_u < p^\circ)$ – (Figures 1 and 2). On the other hand, with a classical savings rule, (25) is replaced by (25′) and the long-run solution is the golden rule steady state $E^\circ(c^\circ, g^\circ = n)$, $E^\circ(w^\circ, r^\circ = n)$.

The EFM is easily extendable to consider foresight assumptions between the extreme cases of static expectations versus perfect foresight, and also to incorporate more severe restrictions on technology. The former can involve incomplete futures markets[25]. The latter results in movements along the REC towards the UEC.

IV. *An Example: The Explicit Adjustment Path to a Structural Shock*

Consider a disembodied, labour-saving innovation[26] in the consumption goods industry, which allows the activity $\binom{\alpha}{\lambda}$ to be

[25] As in Bliss (1976) and McCurdy (1982, chapter III.2). See also Arrow and Hahn (1971, pp. 136-151).

[26] For example, machines are re-organized on the consumption good assembly line such that fewer workers are needed per unit output. If that re-organization involves installing a microchip in existing machines, then – ignoring the initial cost of the chip – this shock proxies one aspect of the microelectronics revolution. The innovation is introduced instantaneously as soon as it is feasible, so that its impact effects are recognized in the static expectations case whereas its long-run implications are also foreseen in the perfect foresight case. The difference between these two cases, in the presence of non-malleable capital, will become clear below.

Fig. 4
Quantity or Restricted Efficiency Curves
RE'C and RE'C

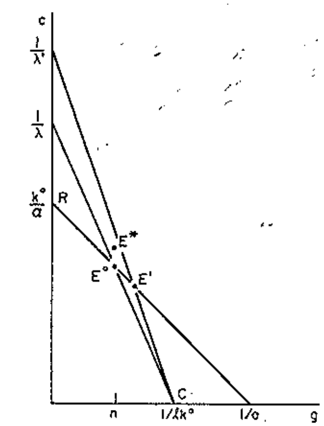


Fig. 5
Price Efficiency Curves PE'C and P'E'C

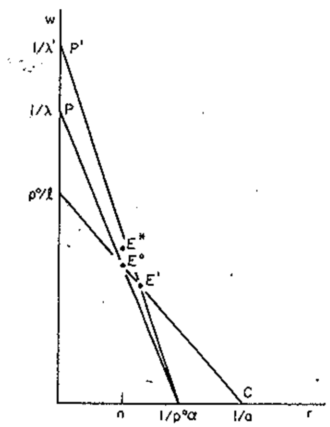


Fig. 6
Uni-class Price Efficiency Curves
PE'_uC and $P'E'_uC$

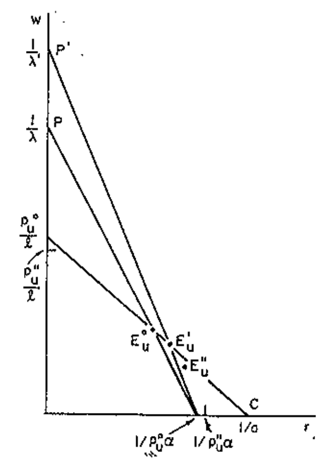
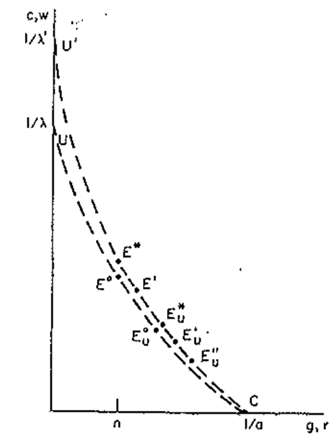


Figure 7
Unrestricted Efficiency Curves
$UE'E'_uC$ and $U'E''_uC$



replaced by $\begin{pmatrix} \alpha \\ \lambda' \end{pmatrix}$ with $\lambda' < \lambda$. Therefore, the technique matrix M° would be replaced by the cost minimizing $M' \left(\equiv \begin{matrix} \alpha & a \\ \lambda' & l \end{matrix}\right)$. This change in technology shifts the restricted efficiency curve from RE°C to RE′C (Figure 4), the price efficiency curve from PE°C to P′E′C (Figure 5) or P′E′$_u$C (Figure 6), and the unrestricted efficiency curve from UE°E$^\circ_u$C to U′E′E″$_u$C (Figure 7).

The impact effect of the innovation is as follows. On the output side, the efficient adjustment is to move along the capital constraint of the post-innovation RE′C to its full employment/full utilization point $E'(c', g')$ which is tangent to the post-shock U′E′C. This adjustment would remove the excess supply of labour by producing more of the relatively labour intensive machine and fewer consumption goods.

With the classical savings assumption, the efficient factor price adjustment would be to move to $E'(w' < w^\circ, r' > r^\circ)$ on P′E′C (Figure 5). That is, a decrease in w and an increase in r will remove the excess profits in the consumption good industry. Due to the classical savings assumption, changes in aggregate demand (or factor income) match the changes in output required to keep the factors fully employed. Therefore, as indicated by (25′), there is no change in p° required to ensure that supply prices are consistent with demand prices, and thus $E'(w', r' p^\circ)$ represents the temporary equilibrium price solution.

On the other hand, with the uni-class savings assumption, the temporary equilibrium is

$$E''_u\,(w''_u < w'_u < w^\circ_u, r''_u > r'_u > r^\circ_u, p''_u < p^\circ_u)$$

on Figure 6 or 7. As indicated by (25), there is an induced price effect in addition to the initial innovation effect. That is, with unchanged consumption/savings behaviour (constant b), p°_u must decrease in order to ensure that demand and supply prices are equal. The decrease in p°_u results in a movement along the post-shock unrestricted efficiency curve from E'_u to E''_u.

Given the impact effects, $g' > n$ so that k° will increase. This shifts the RE′C "period-by-period", moving $E'(c', g')$ up along the post-shock unrestricted efficiency curve towards the new steady-state position at $E^*(c^* > c^\circ, g^* = n = g^\circ, k^* > k^\circ)$. By (25′) or (25), p increases during the sequence of myopic temporary equilibria towards the new steady state $E^*(w^* > w^\circ, r^* = r^\circ = n, p^* > p^\circ)$ or

$E^*_u\,(w^*_u > w^\circ_u, r^*_u > r^\circ_u, p^*_u)$[27]. The technological assumption A9 and the savings assumptions ensure that the supply and demand effects reinforce each other. That is, as factor incomes change taking relative demands (and thus the demand p) in a particular direction, the capital intensity assumption ensures that the supply price moves in the same direction[28]. Therefore, by assuming static expectations and requiring p to adjust to clear the current consumption/savings-investment relations, there is sufficient "friction" to ensure joint stability.

However, as illustrated above, the perfect foresight solution is a saddlepoint and thus exhibits dual stability/instability. Nevertheless, perfect foresight and the flexibility of prices ensure that the long-run ($dp/dt = 0$) price solutions $E^*(w^*, r^*, p^*)$ are attained immediately by a discrete jump in p° to p^*. But on the output side we cannot adjust instantaneously from E° to $E^*(c^*, g^*, k^*)$[29]. Due to the non-malleability of k – and the consequent asymmetric flexibility of the price and output sides – non-steady-state temporary equilibria are required while k° is "transmuted" to be appropriate (k^*) to the new technology (M′) and the corresponding long-run price solution.

Each "momentary" equilibrium must be efficient (on the RE′C and the P′E′C) and maximal (at E′ – that is, at the points determined by (2), (3), (5), (6), RFG and RoP). In addition, temporary and long-run equilibria must be dynamically efficient (satisfying (15), (16)). Although the UE′C does not describe the production frontier during temporary equilibria (as it does the factor price frontier due to the flexibility of p), (15) indicates the dk/dt which shifts the RE′C each "period", and (16) ensures that the market-clearing points of the respective RE′Cs move along the UE′C. Figure 3 (p. 79) can also be used to illustrate the optimal sequence of market-clearing temporary equilibria.

By comparing the sequence of statically efficient temporary equilibria with the optimal adjustment, one can infer some implications of no foresight. The long-run equilibrium is determined by n, but during

[27] The relation between p°_u and p^*_u and thus the exact position of E^*_u relative to E°_u, depends on the relative size of the innovation induced price effect (E'_u to E''_u) versus the dynamic price effect (E''_u to E^*_u).

[28] This is consistent with well-known results (Burmeister and Dobell [1970, p. 122]). Alternatively, we could impose conditions on the aggregate demand functions. For example, in the uni-class case, stability results if (22), (23) satisfy the weak axiom of revealed preference which is the case if "b" remains constant in the presence of price and thus income distribution changes. Of course, in the case of identical individuals, this condition is satisfied. In fact, even the WARP condition is unnecessarily strong when n is constant (see Morishima [1969, pp. 52-57]). See Burmeister [1980, chapter 6] and references therein for discussion of the possibility of non-uniqueness and/or non-monotonic convergence. In our model the unique steady state is stable.

[29] On Figure 4, E^* is outside the feasible production set.

the adjustment $g > n$ in order to adjust k to the labour-saving shock. With perfect foresight, current consumption is sacrificed for higher future consumption and the rate of return ($r = \delta$) is kept equal to n throughout. In other words, the infinitely-lived agents maximize sustainable consumption by adjusting savings (initially increasing then decreasing towards $nk^* > nk^\circ$) to ensure full employment/full utilization during the non-steady-state adjustment to the shock.

However, with rule-of-thumb savings behaviour – for example, a constant savings rate due to static expectations – wages must fall to provide the necessary investment funds to ensure a sequence of statically efficient temporary equilibria towards the new steady state. This simple example illustrates the importance of explicitly analysing the dynamic adjustment paths. On the basis of comparative steady-state analysis, one would expect labour to support the implementation of labour-augmenting technical change since the model predicts an increase in the steady-state real wage ($w^* > w^\circ$). However, with static expectations and a non-malleable capital stock, the adjustment path illustrates that full employment can be maintained only if the wage falls[30] until the endowments adjust to the proportions required by the new technology. This result could explain the often observed myopic resistance (with the associated loss in productivity gains) to labour-augmenting technical change.

V. *Concluding Comments*

This paper uses a simple framework to compare alternative non-steady-state dynamic adjustments in response to a structural shock. The efficiency frontier model (EFM) provides a tractable solution method with which to mimic sequences of temporary general equilibria under various alternative hypotheses about expectations, the severity of restrictions on technology and the malleability of factors. The importance of analysing explicit transition paths, as well as the tractability of the solution method, is demonstrated using a labour-augmenting technical change shock. A comparison of the perfect foresight dynamically efficient adjustment versus the static expectations adjustment, in response to this labour-augmenting innovation, provides one explanation for the often observed myopic resistance to technical change. That

[30] See Ito (1980) for a similar result in a one-sector model with sluggish real wages. Also see Neary (1981) and Sinclair (1981).

is, with static expectations and a non-malleable capital stock, full employment can be maintained only if the wage falls until the endowments adjust to the proportions required by the new technology.

There are many other adjustment scenarios to such structural shocks which could be analysed using the EFM – for example, the potential technological unemployment if the decrease in current consumption is resisted with the consequence that the necessary savings-investment is not realized. In any event, it is useful to derive the "benchmark" optimal control path – which mimics a perfect foresight competitive equilibrium path – against which the implications of myopic behaviour and various potential adjustment problems can be measured. It is such comparisons which suggest ways to improve the efficiency of the economic adjustment to structural shocks.

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APPENDIX

The variable coefficient version has a choice of two alternative activities in each sector, and thus we have four possible techniques for the economy – M^{00}, M^{01}, M^{10}, M^{11} – plus the linear combinations of those pure techniques. (assuming the inputs and the outputs are divisible). For illustration we assume that $\alpha'/\lambda' > \alpha^\circ/\lambda^\circ > k^\circ > a^\circ/l^\circ > a'/l'$. The quantity and price efficiency frontiers are derived analogously to the fixed coefficient version.

Adding a choice of activity in each sector, that is, the possibility of substitution – in response to a change in w for example – to save on the relatively scarce factor, produces a kinked factor price constraint in *each* sector. Recall that for the fixed-coefficient version, the factor price constraint for each sector was a straight line while the PEC (which is the boundary of the intersection of the factor price possibilities sets for the two sectors) exhibited some "curvature" due to $\alpha/\lambda \neq a/l$. Since any (w, q) combination above the instantaneous (p° fixed) price efficiency curve is unprofitable and any combination below results in excess profits, the possibility of substitution (variable coefficients of production) permits either excess profits or, in equilibrium, higher factor payments relative to the fixed coefficients case. On the quantity side (with k° fixed), the variable coefficients permit potentially higher output.

Further illustration of the duality of the price and quantity sides of our production model can be seen from Figure 3A. The dual solution (w°, q°)

indicates that the cost minimizing choice of techniques is M^{00}. In fact, if we construct the normals to the factor price constraints for the consumption and the capital sectors at the dual factor price solution, they will have slopes $\frac{\lambda^*}{\alpha^*}$ and $\frac{l^*}{a^*}$ respectively. That is, we have super-imposed the cost-minimizing unit activity vectors onto Figure 3 at the dual solution. This result — which is known as Shephard's Lemma — says that if we differentiate a cost function with respect to the "optimal" factor prices we will get the cost-minimizing input/output coefficients. If the cost function is only subdifferentiable, Shephard's Lemma can be generalized using subgradients (see Woodland [1977, p. 54]). We illustrate the gradient of the cost function by drawing the normal to the appropriate cost contour (in this case the factor price constraint). Also, the direction perpendicular to the objective function y (minimum factor payments = $w^* + q^* k^*$) will be the endowment vector $(1, k^*)$ which is normal to the minimum iso-factor payment line which supports the instantaneous price efficiency curve for the economy. We can see, from Figure 3A, how the activity levels (c^*, x^*) can be derived such that the input "normals" fully employ the given initial endowment $(1, k^*)$. In the case of diversification, some linear combination (determined by the optimal output levels) of the "normals" span the endowment vector. If only one commodity is produced — for example, if k had been greater $(k' > k^*)$ such that the support point was at A — then the direction perpendicular or "normal" to the factor price constraint (for the consumption sector) is identical to that to the objective function, and the output level c' could be determined such that (2) holds, that is, $k' = \alpha c'$. Since A is a switching point, either $(\alpha' \lambda')'$ or $(\alpha^* \lambda^*)'$ could be used and c must vary accordingly to fully employ the given k'. Of course, solving for (c, x) using the above method should give the same results as using Figure 1, which is not surprising given the underlying duality and the consequent relationships between Figures 1 and 2.

Fig. 1A
QEC with Choice of Technique

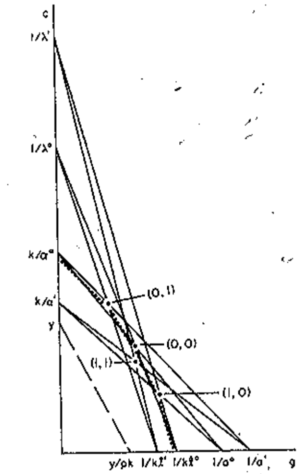


Fig. 2A
PEC with Choice of Technique

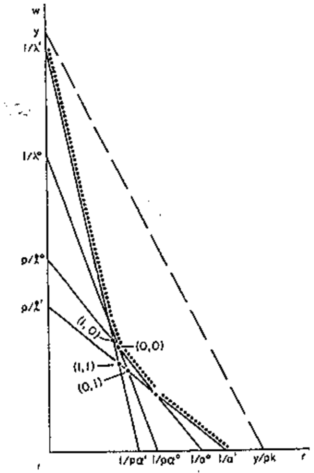
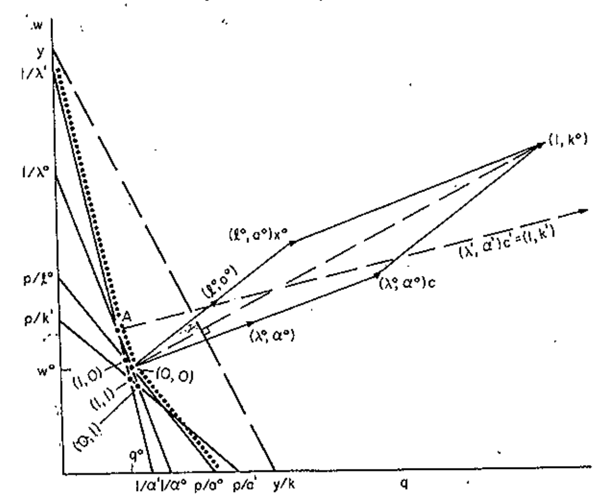


Fig. 3A
Duality of the Efficiency Frontier Structure



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