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Components of Market Risk and Return

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ABSTRACT
This article proposes a flexible but parsimonious specification of the joint dynamics of market risk and return to produce forecasts of a time-varying market equity premium. Our parsimonious volatility model allows components to decay at different rates, generates mean-reverting forecasts, and allows variance targeting. These features contribute to realistic equity premium forecasts for the U.S. market over the 1840–2006 period. For example, the premium forecast was low in the mid-1990s but has recently increased. Although the market's total conditional variance has a positive effect on returns, the smooth long-run component of volatility is more important for capturing the dynamics of the premium. This result is robust to univariate specifications that condition on either levels or logs of past realized volatility (RV), as well as to a new bivariate model of returns and RV.

KEYWORDS: volatility components, long-run market risk premium, realized volatility

The expected return on the market portfolio is an important input for many decisions in finance. For example, accurate measures or forecasts of the equity

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premium are important for computing risk-adjusted discount rates, capital budgeting decisions involving the cost-of-equity capital, as well as optimal investment allocations.

The simplest approach to measuring the market premium is to use the historical average market excess return. Unfortunately, this assumes that the premium is constant over time. If the premium is time varying, as asset pricing theory suggests, then a historical average will be sensitive to the time period used. For example, if the level of market risk were higher in some subperiods than others, then the average excess return will be sensitive to the subsample chosen.

A better approach to estimating the premium is to directly incorporate the information governing changes in risk. For example, the Merton (1980) model implies that the market equity premium is a positive function of market risk, where risk is measured by the variance of the premium. Under certain conditions discussed in the next section, intertemporal asset pricing models (IAPM) reduce to a conditional version of Merton (1980). That is, if the conditional variance of the market portfolio return is larger, investors will demand a higher premium to compensate for the increase in risk.[1]

This positive risk–return relationship for the market portfolio has generated a large literature which investigates the empirical evidence. Historically, authors have found mixed evidence concerning the relationship between the expected return on the market and its conditional variance. In some cases a significant positive relationship is found, in others it is insignificant, and still others report it as being significantly negative.[2]

Recent empirical work investigating the relationship between market risk and return offers some resolution to the conflicting results in the early literature. Scruggs (1998) includes an additional risk factor implied by the model of Merton (1973), arguing that ignoring it in an empirical test of the risk–return relationship results in a misspecified model. Including a second factor, measured by long-term government bond returns, restores a positive relationship between expected return and risk. Campbell and Hentschel (1992), Guo and Whitelaw (2006) and Kim, et al. (2004) report a positive relationship between market risk and return when volatility feedbacks are incorporated. Using a latent VAR process to model the conditional mean and volatility of stock returns, Brandt and Kang (2004) find a negative conditional correlation between innovations to those conditional moments but a positive unconditional correlation due to significant lead–lag correlations. Lundblad (2007) reports a positive trade-off over a long time period, Engle and Lee (1999) find a positive relationship between return and the permanent volatility

[1] There are many other models of asset premiums, many of which can be thought of as versions of a multi-factor approach, such as the three-factor model of Fama and French (1992), or the arbitrage pricing theory of Ross (1976). For example, Claus and Thomas (2001), Fama and French (2002), and Donaldson, et al. (2004) use earnings or dividend growth to estimate the market premium. Pastor, et al. (2007) use implied cost of capital as a measure of expected return.

[2] Early examples include Campbell (1987), Engle, et al. (1987), French, et al. (1987), Chou (1988), Harvey (1989), Turner et al. (1989), Baillie and DeGennaro (1990), Glosten, et al. (1993) and Whitelaw (1994). Table 1 of Scruggs (1998) summarizes that empirical evidence.

component, and Ghysels, et al. (2005) find a positive trade-off using a mixed data sampling (MIDAS) approach to estimate variance. Pastor and Stambaugh (2001) find evidence of structural breaks in the risk−return relationship.[3]

Our article investigates a conditional version[4] of the risk−return specification. We exploit improved measures of *ex post* variance and incorporate them into a new component forecasting model in order to implement a time-varying risk model of the equity premium.

How do we achieve better forecasts of a time-varying market equity premium? First, we use a nonparametric measure of *ex post* variance, referred to as realized volatility (RV). Andersen and Bollerslev (1998) show that RV is considerably more accurate than traditional measures of *ex post* latent variance. Due to the data constraints of our long historical sample, in this article we construct annual RV using daily squared excess returns.[5]

Second, as in Andersen, et al. (2003), French, et al. (1987) and Maheu and McCurdy (2002), our volatility forecasts condition on past RV. Since RV is less noisy than traditional proxies for latent volatility, it is also a better information variable with which to forecast future volatility.

Third, we propose a new volatility forecasting function which is based on exponential smoothing. Our model inherits the good performance of the popular exponential smoothing filter but allows for mean reversion of volatility forecasts and targeting of a well-defined long-run (unconditional) variance. This feature adds to the parsimony of our forecasting function, which is important in our case given the relatively low frequency data necessary to allow estimation over a long time period. It also allows for multiperiod forecasts.

Fourth, motivated by the component-GARCH approach of Engle and Lee (1999) applied to squared returns, we extend our conditional variance specification, which conditions on past RV, to a component-forecasting model. This flexible conditioning function allows different decay rates for different volatility components. We also investigate whether or not total market risk or just some component of it is priced, that is, we allow our risk−return model to determine which components of the volatility best explain the dynamics of the equity risk premium.

Finally, in one of our parameterizations, we generalize the univariate risk−return model for the market equity premium by estimating a bivariate stochastic specification of annual excess returns and the logarithm of RV. In this case, the conditional variance of excess returns is obtained as the conditional

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[3] There is also a literature which investigates a nonlinear relationship between market risk and return, for example, Pagan and Hong (1990), Backus and Gregory (1993), Whitelaw (2000) and Linton and Perron (2003).

[4] As discussed below, our conditional parameterization is motivated by the IAPM of Campbell (1993) and Merton (1973), as well as the component model of Engle and Lee (1999).

[5] Early empirical applications of this measure at low frequencies, for example, using daily squared returns to compute monthly volatility, included Poterba and Summers (1986), French, et al. (1987), Schwert (1989), Schwert and Seguin (1990) and Hsieh (1991).

expectation of the RV process. Again, multiperiod forecasts are available from the assumed dynamics of the bivariate process.

We focus on the dynamics of the premium over the 1840–2006 period. Our volatility specification, which only requires one parameter per volatility component, produces precise estimates of the risk–return relationship. The forecasts of a time-varying premium match important features of the data. For example, our Figure 9 shows how well our forecasts captured the declining equity premium in the mid-1990s.

In summary, we use improved measures of volatility in a parsimonious forecasting model which allows components of volatility with different decay rates to be priced in a conditional risk–return model. This involves several new contributions. We introduce a new weighting function on past RV, and show how mean reversion can be imposed in the model to target the unconditional mean of RV. Building on Engle and Lee (1999), we focus on a multiple component formulation of our new-volatility forecasting function in order to allow components of volatility to decay at different rates and to investigate which component is priced. Exploiting our mean-reverting multiperiod variance forecasts, our models can generate multiperiod premium forecasts. We analyze a long, low-frequency dataset and show that our models produce realistic time-varying premium forecasts over the entire 1840–2006 time period.

Our empirical results show that for 167 years of the U.S. equity market, there is a significant positive relationship between market risk and the market-wide equity premium. The equity premium varies considerably over time and confirms that the average excess return associated with subperiods can be misleading as a forecast. Nevertheless, long samples of historical information are useful as conditioning information and contribute to improved estimates of the time-varying market premium.

In our two-component specifications of the conditional variance, one component tracks long-run moves in volatility while another captures the short-run dynamics. The two-component conditional variance specification provides a superior variance forecast. Furthermore, it is the long-run component in the variance that provides a stronger risk–return relationship.

The article is organized as follows. Section 1 introduces the models that motivate our empirical study, and discusses the importance of the measurement and modeling of the variance of market returns. Section 2 details our results on the significance of the risk–return relationship for several model specifications. We discuss the importance of volatility components, and the range of implied premiums that the models produce. Finally, Section 3 summarizes the results and future work.

1 THE RISK–RETURN MODEL

1.1 Background

Both static and intertemporal models of asset pricing imply a risk–return relationship. Examples of intertemporal models which do not require consumption data are the IAPM proposed by Merton (1973) and Campbell (1993), and also the conditional capital asset pricing model(CAPM).

The IAPM of Merton (1973) relates the expected market return and variance through a representative agent's coefficient of relative risk aversion and also allows sensitivity of the market premium to a vector of state variables (or hedge portfolios) which capture changing investment opportunities. Under some assumptions, the intertemporal model implies a market risk–return relationship with no additional factors, that is, market risk is captured by the variance of the market portfolio. Merton (1980) argues that this case will be a close approximation to the intertemporal asset pricing model in Merton (1973) if either the variance of the change in wealth is much larger than the variance of the change in the other factor(s), or if the change in consumption in response to a change in wealth is much larger than that associated with a change in other state variable(s). Sufficient conditions are if the investment opportunity set is essentially constant, or if the representative investor has logarithmic utility.

Campbell (1993) provides a discrete-time intertemporal model which substitutes out consumption. In this case, the expected market premium is a function of its variance as well as its covariance with news (revisions in expectations) about future returns on the market. As in Merton (1973), if the coefficient of relative risk aversion is equal to 1 or if the investment opportunity set is constant or uncorrelated with news about future market returns, the expected market premium will only be a function of the market return variance. However, the Campbell (1993) derivation provides an alternative, empirically plausible, condition under which that market risk–return relationship obtains. If the covariance of the market return with news about future investment opportunities is proportional to the variance of the market return, then the latter will be a sufficient statistic for market risk.[6] Section III of Campbell (1993) provides conditions that produce this conditional market risk–return relationship.

This motivates a risk–return model

$$E_{t-1}(r_{M,t}) = \gamma_1 \sigma^2_{M,t} \tag{1}$$

where $E_{t-1}(r_{M,t})$ is the conditional expectation of the excess return on the market, and $\sigma^2_{M,t}$ is the conditional variance of the market excess return (that is, the forecast of the variance conditional on time $t-1$ information). This model implies a proportional relationship between the market equity premium and its conditional variance. As discussed in introduction above, Merton (1980) argues that one should impose the prior that the *expected* equity premium is nonnegative. If Equation (1) were the risk–return model, this would be satisfied as long as $\gamma_1 > 0$.

[6] Campbell (1996) reports empirical evidence in support of an analogous condition in a cross-sectional application for which the restriction is that covariances of all asset returns with news about future returns on invested wealth are proportional to their covariances with the current return on wealth. In this case, most of the explained cross-sectional variation in returns is explained by cross-sectional variation in the assets' covariances with the market return.

1.2 Measuring and Forecasting Volatility

In this section, we discuss how we measure and then forecast the volatility which drives the time-varying risk premiums. Note that, throughout the article, we use the term volatility to refer generically to either the variance or standard deviation. Where necessary for clarity, we refer specifically to whether it is an *ex post* (realized) measure or a conditional estimate (forecast); and whether we are referring to a variance or a standard deviation. For ease of notation, we also drop the subscript M on the market excess return and its conditional variance so that henceforth $r_t \equiv r_{M,t}$ and $\sigma_t^2 \equiv \sigma_{M,t}^2$.

1.2.1 Measuring volatility. In this article, we employ a nonparametric measure of volatility. A traditional proxy for *ex post* latent volatility has been squared returns or squared residuals from a regression model. As shown by Andersen and Bollerslev (1998), this measure of volatility is very noisy and of limited use in assessing features of volatility such as its time-series properties.

Better measures of *ex post* latent volatility are available. In this article, we use a measure of *ex post* variance, termed (RV), developed in a series of papers by Andersen, Bollerslev, Diebold and co-authors, and Barndorff-Nielsen and Shephard. The increment of quadratic variation is a natural measure of *ex post* variance over a time interval. RV is computed as the sum of squared returns over this time interval. As shown by Andersen, et al. (2001b), as the sampling frequency is increased, the sum of squared returns converges to the quadratic variation over a fixed time interval for a broad class of models. Thus RV is a consistent estimate of *ex post* variance for that period. The asymptotic distribution of RV has been studied by Barndorff-Nielsen and Shephard (2002b) who provide conditions under which RV is also an unbiased estimate. Recent reviews of this growing literature are by Andersen, et al. (2004) and Barndorff-Nielsen, et al. (2004).

Defining RV of the market return for year t as RV_t, we construct annual RV as

$$RV_t = \sum_{j=1}^{D_t} r_{t,j}^2 \tag{2}$$

where $r_{t,j}$ is the continuously compounded return for the jth day in year t, and D_t is the number of days in year t for which market equity returns were available. Prior to 1886, RV_t is computed from squared monthly continuously compounded returns since we only have monthly data for that early part of our sample. This allows us to use a longer time period. We analyze whether computing annual RV prior to 1886 from lower frequency intra-period observations has any effect on our results.

1.2.2 Forecasting volatility. Our time-varying risk model of the equity premium is forward looking. That is, the expected market equity premium is a function of market equity risk. According to our test equations, the latter is measured by the conditional variance of market excess returns. Therefore, we need a forecast of the time t volatility, conditional on information at time $t-1$.

Our volatility forecasts condition on past RV. Given that RV has a superior signal-to-noise ratio for measuring latent volatility, it should be a superior conditioning variable for forecasting future volatility.

What functional form should be used to summarize information in past RV? Given our objective to model risk and return at annual frequencies, parsimony is an obvious objective. One candidate is the infinite exponential smoothing function. In this case, estimates can be derived from the recursion

$$\sigma_t^2 = (1-\alpha)RV_{t-1} + \alpha\sigma_{t-1}^2, \tag{3}$$

in which $0 \le \alpha < 1$, α is the smoothing parameter, and σ_t^2 is the conditional variance. A small value of α puts more weight on the most recent observable value of RV, that is RV_{t-1}, and less weight on the past forecast σ_{t-1}^2. Conversely, an α close to 1 puts less weight on recent observations and more weight on past forecasts which smooths the data. This model is analogous to the popular *RiskMetrics* filter, except for the fact that the standard practice is to smooth on lagged squared returns rather than on lagged RV. The recursion in Equation (3) implies the following weighting function on past RV,

$$\sigma_t^2 = (1-\alpha)\sum_{j=0}^{\infty} \alpha^j RV_{t-j-1}, \tag{4}$$

in which the weights sum to one.

Although infinite exponential smoothing provides parsimonious estimates, it possesses several drawbacks. Given our long time period and low sampling frequency, we prefer to estimate the smoothing parameter α rather than taking it as given as is common practice with filtering approaches such as *RiskMetrics*. The smoothing parameter α can be estimated using an objective criterion, such as maximum likelihood. However, as Nelson (1990) has shown in the case of squared returns or squared innovations to returns, filters with no intercept and whose weights sum to one are degenerate in the asymptotic limit in the sense that the distribution of the variance process collapses to zero variance asymptotically. This is problematic for maximum-likelihood estimation of the smoothing parameter. To circumvent these problems, but still retain the parsimony and accuracy of exponential smoothing, we propose the following volatility specification,

$$\sigma_t^2 = \omega + (1-\alpha)\sum_{j=0}^{\tau-1} \alpha^j RV_{t-j-1}, \tag{5}$$

where we truncate the expansion at the finite number τ.[7] In this specification the weights sum to less than one allowing mean reversion in volatility forecasts as shown below.

[7] In the empirical applications below, we fixed $\tau = 40$ due to presample data requirements. We provide some robustness checks for alternative choices of τ.

Based on Corollary 1 of Andersen, et al. (2003), we assume that the conditional expectation of annual quadratic variation (QV_t) is equal to the conditional variance of annual returns,[8] that is, $E_{t-1}(QV_t) = \text{Var}_{t-1}(r_t) \equiv \sigma_t^2$.[9] Assuming that RV is a unbiased estimator of quadratic variation it follows that $E_{t-1}(RV_t) = \sigma_t^2$, and we can derive the unconditional variance associated with specification (5) as[10]

$$E(\sigma_t^2) = \frac{\omega}{1 - (1-\alpha)\sum_{j=0}^{\tau-1} \alpha^j}. \tag{6}$$

This unconditional variance leads to our strategy of variance targeting by setting σ^2 from the data and using

$$\omega = \sigma^2 \left[1 - (1-\alpha) \sum_{j=0}^{\tau-1} \alpha^j \right]. \tag{7}$$

In summary, this new specification is similar in spirit to exponential smoothing but allows for mean reversion in volatility forecasts. In addition, the finite unconditional variance allows for variance targeting which means that only one parameter needs to be estimated. Our specification is also more parsimonious than the covariance-stationary GARCH(1,1) model.[11] As discussed later in the text, at least for our sample of annual data the more parsimonious specification is critical for precision of the estimates of the risk–return relationship and also for generating reasonable premium estimates.

As is evident from Equation (5), past data receive an exponentially declining weight. As we will see below, this is not flexible enough to capture the time-series dynamics of RV. A simple approach to providing a more flexible model is to allow different components of volatility to decay at different rates.[12] This can be achieved with a component volatility function which estimates the conditional variance as

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8 Barndorff-Nielsen and Shephard (2002a) also discuss the theoretical relationship between integrated volatility and RV.

9 We assume that any stochastic component in the intraperiod conditional mean is negligible compared to the total conditional variance.

10 To make the mean reversion in variance forecasts clear, note that with the assumption $E_{t-1}(RV_t) = \sigma_t^2$, the dynamics of RV implied by Equation (5) follow the ARCH-like AR representation, $RV_t = \omega + \sum_{j=1}^{\tau} \beta_j RV_{t-j} + v_t$, where $v_t = RV_t - \sigma_t^2$, with $E_{t-1}v_t = 0$, and $\beta_j = (1-\alpha)\alpha^{j-1}$. Conditional variance forecasts can be derived from $E_t\sigma_{t+i}^2 = E_t RV_{t+i}$; and, as $i \to \infty$, we have Equation (6).

11 The covariance-stationary GARCH (1,1) model, $\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2$, with $\epsilon_{t-1} \equiv r_{t-1} - E_{t-2}r_{t-1}$, can be written as $\sigma_t^2 = \omega/(1-\beta) + \alpha \sum_{i=0}^{\infty} \beta^i \epsilon_{t-1-i}^2$ which requires an extra parameter compared to (5).

12 Several papers, for example, long memory, mixtures of regimes, mixtures of jumps and stochastic volatility, have highlighted that a single exponential decay rate is inadequate to capture volatility dynamics over time. Examples include Engle and Lee (1999), Maheu and McCurdy (2000), Alizadeh, et al. (2002), Bollerslev and Zhou (2002), Chernov, et al. (2003), Chacko and Viceira (2003) and Ghysels, et al. (2006a). Engle and Rangel (2004) develop a two-component volatility specification with a GARCH(1,1) model for short-run volatility dynamics which mean revert to a nonparametrically estimated time-varying unconditional volatility.

the average of two or more components. Formally, define the k-component volatility model as

$$\sigma^2_{t,(k)} = \omega + \frac{1}{k}\sum_{i=1}^{k}\sigma^2_{t,i}, \tag{8}$$

where

$$\sigma^2_{t,i} = (1-\alpha_i)\sum_{j=0}^{\tau-1}\alpha_i^j RV_{t-j-1}, \quad i = 1, ..., k. \tag{9}$$

Note that the conditional variance components are projections on past RV. We do not specify the high-frequency dynamics of spot volatility. Indeed, one of the attractions of using RV, besides being an efficient estimate of *ex post* volatility, is that it will be a consistent estimate of volatility for a very large class of empirically realistic models. Therefore, our modeling assumptions are on the annual conditional variance process given observations on RV.

Related work on volatility modeling includes the component model of Engle and Lee (1999) and Ghysels, et al. (2005). Relative to component-GARCH models, our parameterization only requires 1 parameter per component rather than two. Another difference is that we smooth on past annual RV. Ghysels, et al. (2005) use a MIDAS approach to estimate volatility. In that paper, using data from 1928 to 2000, the monthly conditional variance of returns is modeled using a flexible functional form to estimate the weight given to each lagged daily squared return. They find that a two- parameter filter works well.[13] Our decay rates are additive. That is, in our case with two components, the coefficient on RV_{t-j-1} is $(1-\alpha_1)\alpha_1^j/2 + (1-\alpha_2)\alpha_2^j/2$, allowing us to separate out and price short-lived versus slower-decaying components. In contrast, the smoothing coefficients in Ghysels, et al. (2005) interact (multiplicatively) in a way that makes it difficult to isolate their separate effects on pricing.[14]

Our conditional variance specification maintains the parsimony of smoothing models but allows mean reversion. This allows us to use variance targeting which may be important to gain precision in our application. In the next section, we extend the existing literature to investigate a bivariate risk–return specification. This joint stochastic specification of returns and RV allows for multiperiod forecasts of the premium.

Our objective is to have a parsimonious and flexible function that summarizes information in past RV that might be useful for forecasting changes in the market equity risk premium. We allow for alternative components of volatility with different decay rates. Not only is this a more flexible way to capture the time-series

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[13] Their monthly variance is $V_t = 22\sum_{j=0}^{251} w_j r^2_{t-j}$, $w_j(\kappa_1, \kappa_2) = \frac{\exp(\kappa_1 j + \kappa_2 j^2)}{\sum_{i=0}^{251}\exp(\kappa_1 i + \kappa_2 i^2)}$.

[14] Note that their weight on past squared returns is proportional to $\exp(\kappa_1 j + \kappa_2 j^2) = \tilde{\kappa_1}^j \tilde{\kappa_2}^{j^2}$ with $\tilde{\kappa_1} = \exp(\kappa_1)$ and $\tilde{\kappa_2} = \exp(\kappa_2)$.

dynamics of volatility, but it also allows us to investigate whether a particular component, rather than the full conditional variance, is more important in driving the market premium.

1.3 The Empirical Risk–Return Models

As discussed in Section 1.1, our empirical models based on Equation (1) are motivated as special cases of an IAPM. Each of the empirical models implies a time-varying equity premium which is a function of its own conditional second moment, that is, a forecast of the equity premium's time t variance conditional on time $t-1$ information.

The conventional univariate test equation for (1) is a linear model,

$$r_t = \gamma_0 + \gamma_1 \sigma_t^2 + \epsilon_t \, . \tag{10}$$

For example, Scruggs (1998) motivates the addition of an intercept to account for market imperfections, such as differential tax treatment of equity versus Treasury-bill returns, which might account for a constant equity premium unrelated to risk.

In this subsection we introduce two alternative empirical specifications of the risk–return relationship. Each of our models jointly estimate the conditional mean and conditional variance parameters using maximum likelihood. We label the first specification univariate since it fits the stochastic excess return process by conditioning on variance forecasts which are estimated using a projection on past RV as in Equation (8).[15] The second specification is bivariate since we estimate a bivariate stochastic specification of annual excess returns and log(RV). In that case, the conditional variance of excess returns is obtained as the conditional expectation of the RV process.

1.3.1 Univariate risk–return specifications. The conditional mean is,

$$r_t = \gamma_0 + \gamma_1 \sigma_{t,(q)}^2 + \epsilon_t, \;\; \epsilon_t = \sigma_{t,(k)} z_t, \;\; z_t \sim N(0,1); \tag{11}$$

and the conditional variance is either applied to levels of RV, as in Equations (8) and (9), or applied to log(RV) as in

$$\log \sigma_{t,(k)}^2 = \omega + \frac{1}{k} \sum_{i=1}^{k} \log \sigma_{t,i}^2, \tag{12}$$

$$\log \sigma_{t,i}^2 = (1 - \alpha_i) \sum_{j=0}^{\tau-1} \alpha_i^j \log RV_{t-j-1}, \;\; i = 1, ..., k, \tag{13}$$

where k indexes the total number of variance components and q indexes the number of variance components that affect the conditional mean. That is, we specify $\sigma_{t,(q)}^2$ in

[15] The risk–return regressions in Ghysels, et al. (2006b) also estimate market risk based on past RV.

the conditional mean, in which q denotes the number of volatility components that affect the market premium when the conditional variance follows a k-component model. Note that $q \leq k$. For example, where we want the total variance to enter the mean, we set $q = k$. On the other hand, for models in which $q = 1, k = 2$, we let the maximum likelihood estimator determine which component of the variance is optimal in the mean specification.

The tth contribution to the loglikelihood is

$$l_t = -\frac{1}{2}\log(2\pi) - \frac{1}{2}\log(\sigma^2_{t,(k)}) - \frac{(r_t - \gamma_0 - \gamma_1\sigma^2_{t,(q)})^2}{2\sigma^2_{t,(k)}}, \tag{14}$$

where $\sigma^2_{t,(\cdot)}$ depends on the levels or log specification. The loglikelihood $\sum_{t=1}^{T} l_t$ is maximized with respect to the parameters $\gamma_0, \gamma_1, \alpha_1, ..., \alpha_k$.

1.3.2 Bivariate risk–return specification. In this case, we estimate a bivariate stochastic specification of annual excess returns and log(RV). The parameterization has conditional mean

$$r_t = \gamma_0 + \gamma_1\sigma^2_{t,(q)} + \epsilon_t, \quad \epsilon_t = \sigma_{t,(k)} z_t, \; z_t \sim N(0,1) \tag{15}$$

and the following conditional variance specification

$$\sigma^2_{t,(k)} \equiv E_{t-1}(RV_t|k) = \exp(E_{t-1}(\log RV_t|k) + .5\text{Var}_{t-1}(\log RV_t|k)) \tag{16}$$

$$\log RV_t = \omega + \frac{1}{k}\sum_{i=1}^{k}\log\sigma^2_{t,i} + \eta_t, \quad \eta_t \sim N(0,\phi^2) \tag{17}$$

$$\log\sigma^2_{t,i} = (1-\alpha_i)\sum_{j=0}^{\tau-1}\alpha_i^j \log RV_{t-j-1}, \quad i = 1, ..., k. \tag{18}$$

Again, $\sigma^2_{t,(q)} \equiv E_{t-1}(RV_t|q),\; q \leq k$, represents the conditional variance component(s) that affect the conditional mean. l_t consists of contributions from the return and volatility equation,

$$\begin{aligned} l_t = &-\log(2\pi) - \frac{1}{2}\log(\sigma^2_{t,(k)}) - \frac{1}{2}\log(\phi^2) \\ &- \frac{(r_t - \gamma_0 - \gamma_1\sigma^2_{t,(q)})^2}{2\sigma^2_{t,(k)}} - \frac{\left(\log RV_t - \omega - \frac{1}{k}\sum_{i=1}^{k}\log\sigma^2_{t,i}\right)^2}{2\phi^2}. \end{aligned} \tag{19}$$

In this case, by parameterizing the joint density of annual excess returns and log(RV), we explicitly model the annual log(RV) process as stochastic given the most recent information. However, making a conditional log-normal assumption for RV allows for a convenient calculation of the conditional variance as in Equation (16).

2 RESULTS

2.1 Data and Descriptive Statistics

We are evaluating the risk–return relationship associated with equity for the market as a whole. Therefore, we require data on a broad equity market index and on a riskfree security so that we can construct returns on the equity index in excess of the riskfree rate.

There are two considerations. First, as shown by Merton (1980) for i.i.d. returns, we can only increase the precision of expected return estimates by increasing the length of the time period (calendar span) covered by our historical sample. In other words, sampling more frequently (for example, collecting daily instead of monthly data) will not improve the precision of the market premium estimates. For this reason we want our historical sample to be as long as possible. Second, since we use a nonparametric measure of RV which is constructed as the sum of squared *intraperiod* returns, we will need data at a higher frequency than our estimation frequency in order to compute RV as in Equation (2). As noted in Section 1.2.1 above, we use monthly returns from 1802–1885 and daily returns from 1886–2006 to compute annual RV.

The U.S. market equity returns from 1802–1925 are from Schwert (1990). The Schwert equity index was constructed from various sources: railway and financial companies up to 1862 from Smith and Cole (1935); the Macaulay (1938) railway index (1863–1870); the value-weighted market index published by the Cowles Commission for the time period 1871–1885; and the Dow Jones index of industrial and railway stocks for 1885–1925.[16] For the 1926–2006 period, we use returns (including distributions) for the value-weighted portfolio of NYSE, NASDAQ and AMEX stocks compiled by the Center for Research in Security Prices (CRSP). The equity return data are converted to continuously compounded returns by taking the natural logarithm of one plus the monthly or daily equity return.

Annual bill yield data (riskfree rate) for the 1802–1925 period are from Jeremy Siegel. Siegel (1992) describes how these annual yields were constructed to remove the very variable risk premiums on commercial paper rates in the 19th century.[17] For the period 1926–2006, we use monthly bid yields associated with U.S. 3-month Treasury Bills from the Fama monthly riskfree file provided by CRSP. The annual riskfree yield was obtained by summing the 12 monthly yields.

Annual continuously compounded equity returns are computed as the sum of the monthly continuously compounded returns. Annual excess returns are then

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[16] Schwert adjusts for the time averaging in the original series for 1863–1885 and adds an estimate of dividends for the period 1802–1870.

[17] The market excess return is often measured with respect to a long-term (for example, 30 year) Treasury yield rather than the Treasury Bill yield used in this article. Although this choice depends on investment horizon and the purpose of the calculation, for example, capital budgeting versus asset pricing, our focus is on forecasting the market equity premium over a (approximately) riskfree rate rather than over an alternative risky asset. Booth (1999) compares several approaches and shows that for certain periods the risk associated with long-term Treasuries was almost as large as that associated with market equity.

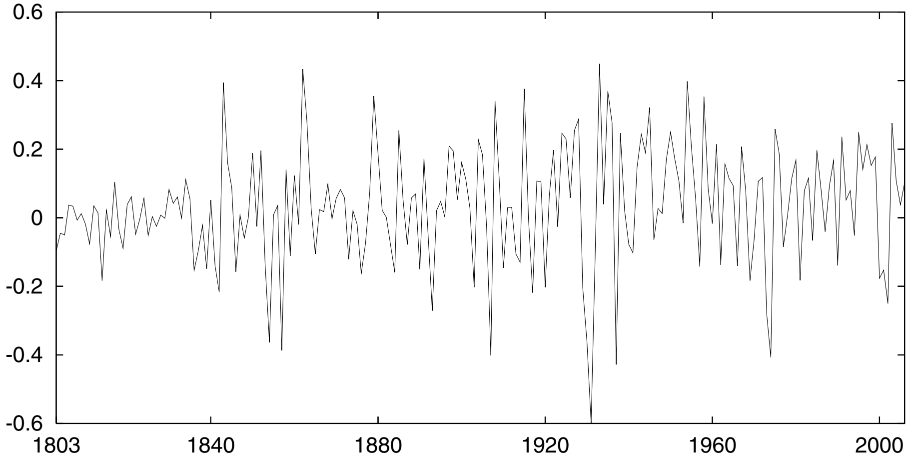

Figure 1 Annual realized excess returns: 1803–2006.

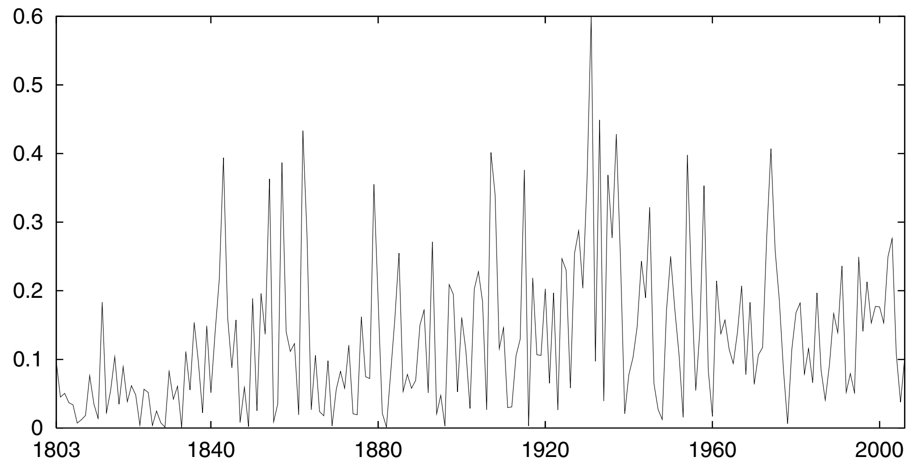

Figure 2 Annual absolute value measure of *ex post* volatility.

computed by subtracting the annual riskfree rate. Henceforth, unless otherwise indicated, returns and excess returns refer to continuously compounded rates.[18]

Figure 1 plots annual market excess returns for the entire sample from 1803–2006. Figures 2 and 3 plot two alternative measures of *ex post* volatility, the absolute value of the excess returns and the square root of RV, respectively. Notice how much smoother the square root of RV is than the absolute value of annual excess returns. It is also clear from these plots that the data for the

[18] We use continuously compounded returns to conform with our estimate of RV.

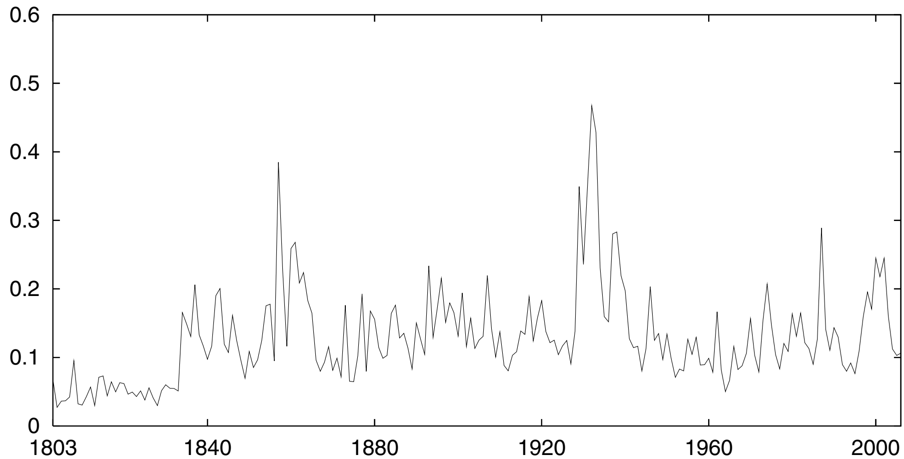

Figure 3 Realized volatility measure: $\sqrt{RV}$.

Table 1 Summary statistics.

	Excess return				Realized volatility	
Sample	Mean	StdDev	Min	Max	Mean	StdDev
1840−2006	0.0427	0.1824	−0.6017	0.4490	0.0250	0.0293

period 1803−1834 have a very different structure than that for the remainder of the sample. As noted in Schwert (1990), data from only a small number of companies was available for that subperiod. Both Schwert (1989) and Pastor and Stambaugh (2001) drop this period. For the same reason, our analyses focus on the time period 1840−2006. Starting in 1840 provides presample values to condition our time-varying volatility model.

Table 1 reports summary statistics for the time period, 1840−2006. The average excess return is 4.27%.

2.2 Univariate Risk−Return Results

As discussed in Section 1.3.1, the univariate specifications fit the stochastic excess return process by conditioning on variance forecasts which are estimated using a projection on past RV. We report results using both the levels of RV, as in Equations (8) and (11), and the natural logarithm of RV, as in Equations (12) and (11).[19] Due to the importance of the length of the time period necessary for efficient conditional mean forecasts, we use all of our data and thus forecasts are in-sample.

[19] Andersen, et al. (2001a) show that for equity returns the natural logarithm of RV is more normally distributed than the level of RV.

Table 2 Parameter estimates: univariate risk–return using RV levels

$$r_t = \gamma_0 + \gamma_1 \sigma^2_{t,(q)} + \epsilon_t, \quad \epsilon_t = \sigma_{t,(k)} z_t, \quad z_t \sim N(0,1)$$

$$\sigma^2_{t,(k)} = \omega + \frac{1}{k}\sum_{i=1}^{k} \sigma^2_{t,i}; \quad \sigma^2_{t,i} = (1-\alpha_i)\sum_{j=0}^{\tau-1} \alpha_i^j RV_{t-j-1}, \quad i = 1, \ldots, k$$

Model Labels: U(k) ≡ univariate risk–return model with k-component model of RV; U(2_s) ≡ univariate risk–return model with two-component volatility but the conditional mean prices the smooth volatility component.

	γ_0	γ_1	ω	α_1	α_2	L	$LRT_{\gamma_0=0}$	$LRT_{\gamma_1=0}$
U(1)	−0.032	3.122	0.0003	0.910		38.14	1.08	5.24
($q = k = 1$)	[−1.027]	[2.197]		(0.024)			(0.30)	(0.02)
Proportional Model		1.768	0.0002	0.903		37.60		12.58
		[3.543]		(0.027)				(0.00)
U(2)	−0.012	2.314	0.0009	0.950	0.396	39.71	0.24	3.68
($q = k = 2$)	[−0.479]	[1.930]		(0.020)	(0.294)		(0.62)	(0.06)
Proportional Model		1.794	0.0009	0.950	0.406	39.59		12.76
		[3.557]		(0.021)	(0.282)			(0.00)
U(2_s)	−0.052	4.338	0.0009	0.951	0.408	41.64	2.50	7.54
($q = 1, k = 2$)	[−1.573]	[2.624]		(0.019)	(0.256)		(0.11)	(0.01)
Proportional Model		1.986	0.0009	0.950	0.438	40.39		14.36
		[3.580]		(0.021)	(0.242)			(0.00)

Notes: Conditional mean coefficient estimates of γ_i with t-statistics in brackets; ω is the volatility function intercept which is consistent with targeting the median of RV; volatility function coefficient estimates α_i have standard errors in parenthesis; L is the log-likelihood function; and LRT are likelihood ratio test statistics with p-values in parenthesis.

See Inoue and Kilian (2004) for evidence in favor of using in-sample forecasts for model evaluation.

2.2.1 Using RV levels. Table 2 reports parameter estimates and likelihood ratio test (LRT) results associated with the risk–return relationship estimated using the alternative component volatility models applied to levels of RV. Note that ω is computed, as in Equation (7). To avoid influential observations in our small sample, we set the long-run target volatility, σ^2, to the sample median of annual RV over the period 1802–2006. For each model, U(1), U(2) and U(2_s), we report both the linear parameterization of the risk–return relationship, Equation (11), and the proportional parameterization with $\gamma_0 = 0$, as motivated by Equation (1).[20]

[20] See Lanne and Saikkonen (2006) and Bandi and Perron (2006) for statistical arguments in favor of estimating the proportional model when the estimated intercept is insignificant. For example, based on results showing low power of a Wald test when an unnecessary intercept is included, Lanne and Saikkonen (2006) conclude with a recommendation to always restrict the intercept to zero if it is implied

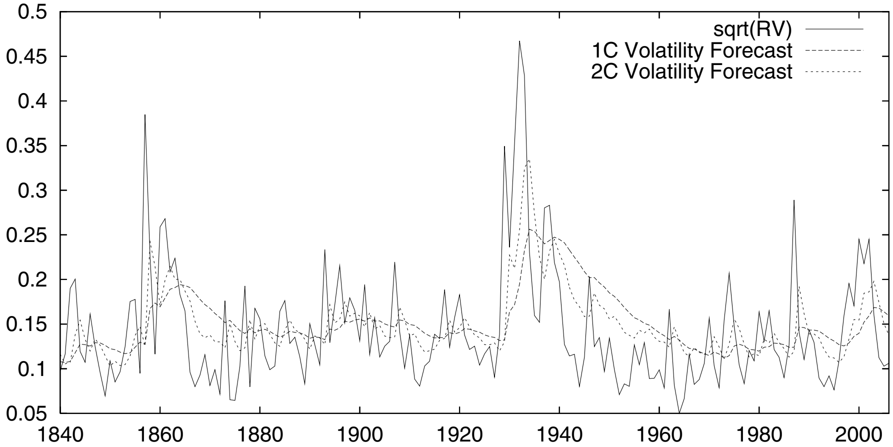

Figure 4 Volatility forecast vs realized: U(1) and U(2) models.

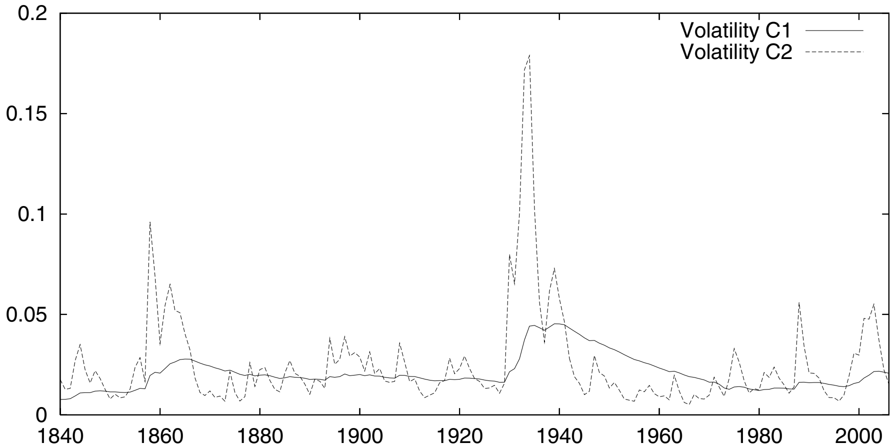

Figure 5 Volatility component forecasts ($\sigma^2_{t,i}$): U(2) model.

Models U(2) and U(2_s) have the same volatility specification but U(2_s) allows the risk–return model to determine which volatility component has the most explanatory power for the dynamics of the equity premium. We find that, when given the choice, the maximum likelihood estimator always chooses to price the long-run or smooth component.

by the theory being tested. Note that our models with an intercept all have a positive slope parameter so that restricting the statistically insignificant intercept to zero does not force the slope parameter to be positive but rather allows us to estimate it with more precision.

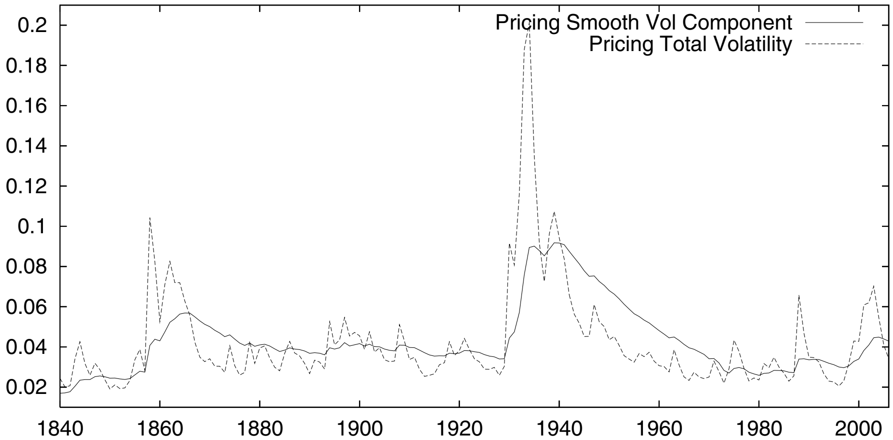

Figure 6 Proportional premium forecasts: U(2_s) model.

It is clear from Figure 4 that the two-component volatility specification tracks RV better than the one-component version.[21] Figure 5 plots the individual components of volatility from the U(2) specification, that is, Equation (9) for $k = 1, 2$. This shows that the smooth component is persistent, as expected from the smoothing coefficient estimate $\hat{\alpha}_1 = .950$ reported in Table 2. Note in particular how long it takes the smooth component to decay from its high level in the 1930s. On the other hand, the second component has much lower persistence, $\hat{\alpha}_2 = .396$, which implies that it is more influenced by the most recent RV.

Table 2 reveals that the risk–return relationship is positive for all specifications. For linear parameterizations of the premium, the LRT results reveal a statistically significant relationship between excess market returns and their conditional variance, although it is marginal (p-value = 0.06) for the U(2) case. For these linear parameterizations, the t-statistic on the risk–return slope coefficient, γ_1, is highest (2.624) for the U(2_s) model. That version also has the highest log-likelihood, 41.64, and a LRT statistic of 7.54 which rejects the hypothesis that $\gamma_1 = 0$.

However, for all volatility specifications, the more parsimonious proportional premium model cannot be rejected by the data. That is, the intercept in the conditional mean is statistically insignificant, both from the perspective of t-tests on γ_0 and from the LRT reported in the second-last column of Table 2. As noted above, by restricting the intercept to be zero, the precision of the estimate $\hat{\gamma}_1$ improves. For instance, the t-statistics associated with the hypothesis $\gamma_1 = 0$ increase, as do the LRT statistics which are more than twice as large for the proportional as opposed to the linear model. The one exception is the U(2_s) model which reveals a strongly positive market risk–return relationship for both the linear and the proportional parameterization of the premium.

[21] Note that these volatility plots are square roots of the volatility forecasts versus $\sqrt{RV_t}$.

Table 3 Model comparisons and diagnostics: univariate risk–return using RV levels

$$r_t = \gamma_0 + \gamma_1 \sigma^2_{t,(q)} + \epsilon_t, \quad \epsilon_t = \sigma_{t,(k)} z_t, \quad z_t \sim N(0,1)$$

$$\sigma^2_{t,(k)} = \omega + \frac{1}{k}\sum_{i=1}^{k} \sigma^2_{t,i}; \quad \sigma^2_{t,i} = (1-\alpha_i)\sum_{j=0}^{\tau-1} \alpha_i^j RV_{t-j-1}, \quad i = 1,\ldots,k$$

Model Labels: U(k) ≡ univariate risk–return model with k-component model of RV; U(2_s) ≡ univariate risk–return model with two-component volatility but the conditional mean prices the smooth volatility component.

		Diagnostics			Volatility Fit			Premium Fit		
Model	L	$LB^2(10)$	$LB(10)$	KS	MAE	RMSE	R^2	MAE	RMSE	R^2
U(1) ($q = k = 1$)	38.14	(0.54)	(0.19)	(0.03)	0.0165	0.0284	0.0727	0.1410	0.1800	0.0224
U(2) ($q = k = 2$)	39.71	(0.57)	(0.23)	(0.09)	0.0147	0.0250	0.2656	0.1410	0.1807	0.0162
U(2_s) ($q = 1, k = 2$)	41.63	(0.87)	(0.15)	(0.16)	0.0147	0.0251	0.2639	0.1405	0.1795	0.0272

L is the log-likelihood function. $LB^2(10)$ is the Ljung and Box (1978) heteroskedastic-robust portmanteau test statistic for serial correlation in the squared standardized residuals up to 10 lags; $LB(10)$ is the same for conditional mean residuals; and KS is the Kiefer and Salmon (1983) test for departures from the normal distribution for standardized residuals. We report p-values for these diagnostic tests. MAE ≡ mean absolute error; RMSE ≡ square root of the mean-squared error; R^2 from Mincer-Zarnowitz regressions applied to the volatility and premium forecasts.

The LRT statistics in the final column of the table all reject the null hypothesis that $\gamma_1 = 0$. This result is particularly strong for the proportional parameterizaton of the risk premium in which case the test statistics are all greater than 12 so that the p-value associated with the null hypothesis is very small. The t-statistic for γ_1 also supports this conclusion in that it is greater than 3.5 for the proportional model for all of the alternative volatility specifications.

Figure 6, plots the market equity premium forecast for the proportional risk premium parameterization. When total volatility from the two-component volatility specification is priced (the U(2) model), there is a high peak in the forecasted market equity premium which lasts for 2 years, 1933 and 1934. In contrast, pricing the smooth component of volatility (the U(2_s) model) results in a lower peak which decays more slowly. In this case, the premium forecast ranges from 1.7% to 9.2% with the average forecasted premium over the 1840–2006 sample equal to 4.29 percent.

Table 3 reports model diagnostics and statistics for volatility and premium fit. These include mean absolute error (MAE) and root mean squared error (RMSE), as well as the R^2 from Mincer and Zarnowitz (1969) forecast regressions for

Table 4 Parameter estimates: univariate risk–return using log(RV)

$$r_t = \gamma_0 + \gamma_1 \sigma^2_{t,(q)} + \epsilon_t, \quad \epsilon_t = \sigma_{t,(k)} z_t, \quad z_t \sim N(0,1)$$

$$\log \sigma^2_{t,(k)} = \omega + \frac{1}{k}\sum_{i=1}^{k} \log \sigma^2_{t,i}; \quad \log \sigma^2_{t,i} = (1-\alpha_i)\sum_{j=0}^{\tau-1} \alpha_i^j \log RV_{t-j-1}, \quad i = 1, \ldots, k$$

Model Labels: $U_{log}(k) \equiv$ univariate risk–return model with k-component model of the natural logarithm of RV; $U_{log}(2_s) \equiv$ univariate risk–return model with two-component volatility but the conditional mean prices the smooth volatility component.

	γ_0	γ_1	ω	α_1	α_2	L	$LRT_{\gamma_0=0}$	$LRT_{\gamma_1=0}$
$U_{log}(1)$	−0.007	2.617	−0.0025	0.830		20.81	0.60	3.01
$(q = k = 1)$	[−0.231]	[1.576]		(0.056)			(0.81)	(0.08)
Proportional Model		2.259	−0.0018	0.823		20.78		16.00
		[3.966]		(0.051)				(0.00)
$U_{log}(2)$	−0.008	2.734	−0.0931	0.924	0.309	23.04	0.12	3.98
$(q = k = 2)$	[−0.348]	[1.931]		(0.032)	(0.235)		(0.73)	(0.05)
Proportional Model		2.287	−0.0812	0.921	0.311	22.98		16.20
		[3.987]		(0.032)	(0.238)			(0.00)
$U_{log}(2_s)$	−0.067	5.631	−0.1688	0.938	0.236	26.07	3.96	10.04
$(q = 1, k = 2)$	[−1.940]	[3.294]		(0.021)	(0.207)		(0.05)	(0.00)
Proportional Model		2.355	−0.0796	0.921	0.308	24.09		18.42
		[4.149]		(0.029)	(0.230)			(0.00)

Notes: Conditional mean coefficient estimates of γ_i with t-statistics in brackets; ω is the volatility function intercept which is consistent with targeting the mean of log(RV); volatility function coefficient estimates α_i have standard errors in parenthesis; L is the log-likelihood function; and LRT are likelihood ratio test statistics with p-values in parenthesis.

a particular model's premium and volatility forecasts.[22] The results in Table 3 support the two-component models U(2) and U(2_s) over the more restrictive one-component version U(1). For example, as suggested by Figure 4, the R^2 increases from about 7% to 26% by adding a second volatility component. Note that the U(2_s) model, which had the highest log-likelihood, also has the best premium fit whether measured from the perspective of MAE, RMSE, or R^2.

2.2.2 Using Log(RV).

We also fit the univariate risk–return model (11) using variance forecasts which are estimated from a projection on the natural logarithm (rather than levels) of past RV, that is, Equations (12) and (13). Tables 4 and 5 report these results. Again, ω is computed to target long-run volatility, for which $\log \sigma^2$ is set to the 1802–2006 average of log(RV).[23]

[22] For example for the volatility forecasts, the regression is $RV_t^{.5} = a + b\hat{\sigma}_t + u_t$ in which $\hat{\sigma}_t$ is the square root of the particular model's volatility forecast for time t given information up to time $t-1$.

[23] The log transformation reduced the outliers present in the levels of RV.

Table 5 Model comparisons and diagnostics: univariate risk–return using log(RV)

$$r_t = \gamma_0 + \gamma_1 \sigma^2_{t,(q)} + \epsilon_t, \quad \epsilon_t = \sigma_{t,(k)} z_t, \quad z_t \sim N(0,1)$$

$$\log \sigma^2_{t,(k)} = \omega + \frac{1}{k}\sum_{i=1}^{k} \log \sigma^2_{t,i}; \quad \log \sigma^2_{t,i} = (1-\alpha_i)\sum_{j=0}^{\tau-1} \alpha_i^j \log RV_{t-j-1}, \quad i = 1, \ldots, k$$

Model Labels: $U_{log}(k) \equiv$ univariate risk–return model with k-component model of the natural logarithm of RV; $U_{log}(2_s) \equiv$ univariate risk–return model with two-component volatility but the conditional mean prices the smooth volatility component.

		Diagnostics			Volatility Fit			Premium Fit		
Model	L	$LB^2(10)$	$LB(10)$	KS	MAE	RMSE	R^2	MAE	RMSE	R^2
$U_{log}(1)$ $(q=k=1)$	20.81	(0.23)	(0.12)	(0.10)	0.0145	0.0281	0.1179	0.1414	0.1807	0.0133
$U_{log}(2)$ $(q=k=2)$	23.05	(0.36)	(0.19)	(0.19)	0.0136	0.0263	0.2608	0.1415	0.1809	0.0119
$U_{log}(2_s)$ $(q=1, k=2)$	26.07	(0.69)	(0.15)	(0.26)	0.0135	0.0263	0.2737	0.1406	0.1803	0.0199

L is the log-likelihood function. $LB^2(10)$ is the Ljung and Box (1978) heteroskedastic-robust portmanteau test for serial correlation in the squared standardized residuals up to 10 lags; $LB(10)$ is the same for the conditional mean residuals; and KS is the Kiefer and Salmon (1983) test for departures from the normal distribution for standardized residuals. We report p-values for these diagnostic tests. MAE $\equiv$ mean absolute error; RMSE $\equiv$ square root of the mean-squared error; R^2 from Mincer-Zarnowitz regressions applied to the volatility and premium forecasts.

The parameter estimates using log(RV), reported in Table 4, reveal that the risk–return relationship is again positive for all specifications and, with two exceptions, is very significant—even more so than when projecting on the levels of past RV. The two exceptions, with p-values of 0.08 and 0.05 respectively for the LR test of the restriction $\gamma_1 = 0$, are the linear parameterizations of the premium with one-component and two-component volatility specifications. However, the proportional parameterizations of the premium do result in very statistically significant positive risk–return relationships for all cases. The LRT statistics in the final column range from 16.00 to 18.42, even larger than in Table 2. The t-statistics associated with the risk–return slope parameter estimates, $\hat{\gamma}_1$, range from 3.97 to 4.15 in this case.

Table 5 shows that the two-component parameterizations of volatility dominate the one-component version from the perspective of all three criteria, MAE, RMSE, or R^2, for volatility fit. As in the levels case, the $U_{log}(2_s)$ model, which prices the smooth component of volatility, has the best overall fit.

Figure 8 shows that the smooth proportional premium forecasts using the univariate model that conditions on log(RV), that is, the $U_{log}(2_s)$ model, range from 1.6% to 9.5% with an average of 4.33% over the period 1840 to 2006. This

Table 6 Parameter estimates: bivariate return and log(RV) model

$$r_t = \gamma_0 + \gamma_1 \sigma^2_{t,(q)} + \epsilon_t, \quad \epsilon_t = \sigma_{t,(k)} z_t, \quad z_t \sim N(0,1)$$

$$\sigma^2_{t,(k)} \equiv E_{t-1}(RV_t|k) = \exp(E_{t-1}(\log RV_t|k) + .5\mathrm{Var}_{t-1}(\log RV_t|k));$$

$$\sigma^2_{t,(q)} \equiv E_{t-1}(RV_t|q), \; q \leq k$$

$$\log RV_t = \omega + \frac{1}{k}\sum_{i=1}^{k} \log \sigma^2_{t,i} + \eta_t, \quad \eta_t \sim N(0, \phi^2)$$

$$\log \sigma^2_{t,i} = (1-\alpha_i)\sum_{j=0}^{\tau-1} \alpha_i^j \log RV_{t-j-1}, \quad i = 1, \ldots, k$$

Model Labels: B(k) ≡ bivariate model of returns and the natural logarithm of RV with k-component volatility. B(2_s) ≡ bivariate model with two-component volatility but the conditional mean prices the smooth volatility component.

	γ_0	γ_1	ω	α_1	α_2	ϕ	L	$LRT_{\gamma_0=0}$	$LRT_{\gamma_1=0}$
B(1)	0.019	0.861	−0.2e-7	0.621		0.763	−143.72	0.74	0.94
($q = k = 1$)	[0.874]	[0.963]		(0.079)		(0.046)		(0.39)	(0.33)
Proportional Model		1.533	−0.7e-7	0.638		0.765	−144.09		10.88
		[3.272]		(0.078)		(0.046)			(0.00)
B(2)	−0.006	1.954	−0.0653	0.916	0.176	0.745	−136.60	0.04	4.48
($q = k = 2$)	[−0.239]	[1.693]		(0.034)	(0.140)	(0.045)		(0.84)	(0.03)
Proportional Model		1.706	−0.0596	0.914	0.175	0.745	−136.62		12.08
		[3.440]		(0.034)	(0.141)	(0.045)			(0.00)
B(2_s)	−0.066	4.289	−0.1429	0.934	0.159	0.745	−134.30	2.94	9.08
($q=1, k=2$)	[−1.648]	[2.859]		(0.027)	(0.140)	(0.045)		(0.09)	(0.00)
Proportional Model		1.791	−0.0612	0.915	0.171	0.744	−135.77		13.78
		[3.645]		(0.031)	(0.145)	(0.045)			(0.00)

Notes: Conditional mean coefficient estimates of γ_i with t-statistics in brackets; ω is the volatility function intercept which is consistent with targeting the mean of log(RV); volatility function coefficient estimates α_i have standard errors in parenthesis; L is the log-likelihood function; and LRT are likelihood ratio test statistics with p-values in parenthesis.

figure also shows that these premium forecasts are similar to the univariate case in which volatility forecasts were estimated from past levels of RV rather than log(RV), except that in the log(RV) case the peaks are slightly higher with faster decay.[24]

[24] The plots of the premium forecasts for all specifications show that the premiums are increasing from a low level over the first few years of the estimation period. This is partly due to the low volatility of the

Table 7 Model comparisons and diagnostics: bivariate return and log(RV) model

$$r_t = \gamma_0 + \gamma_1 \sigma^2_{t,(q)} + \epsilon_t, \quad \epsilon_t = \sigma_{t,(k)} z_t, \quad z_t \sim N(0,1)$$

$$\sigma^2_{t,(k)} \equiv E_{t-1}(RV_t|k) = \exp(E_{t-1}(\log RV_t|k) + .5\mathrm{Var}_{t-1}(\log RV_t|k))$$

$$\sigma^2_{t,(q)} \equiv E_{t-1}(RV_t|q), \ q \leq k$$

$$\log RV_t = \omega + \frac{1}{k}\sum_{i=1}^{k} \log \sigma^2_{t,i} + \eta_t, \quad \eta_t \sim N(0, \phi^2)$$

$$\log \sigma^2_{t,i} = (1-\alpha_i)\sum_{j=0}^{\tau-1} \alpha_i^j \log RV_{t-j-1}, \quad i = 1, \ldots, k$$

Model Labels: B(k) ≡ bivariate model of returns and the natural logarithm of RV with k-component volatility. B(2_s) ≡ bivariate model with two-component volatility but the conditional mean prices the smooth volatility component.

		Diagnostics			Volatility Fit			Premium Fit		
Model	L	$LB^2(10)$	$LB(10)$	KS	MAE	RMSE	R^2	MAE	RMSE	R^2
B(1) ($q=k=1$)	−143.72	(0.12)	(0.21)	(0.33)	0.0165	0.0268	0.2330	0.1413	0.1806	0.0133
B(2) ($q=k=2$)	−136.60	(0.34)	(0.17)	(0.20)	0.0149	0.0248	0.2771	0.1415	0.1810	0.0115
B(2_s) ($q=1, k=2$)	−134.30	(0.67)	(0.15)	(0.26)	0.0148	0.0248	0.2809	0.1407	0.1804	0.0196

L is the log-likelihood function. $LB^2(10)$ is the Ljung and Box (1978) heteroskedastic-robust portmanteau test for serial correlation in the squared standardized residuals up to 10 lags; $LB(10)$ is the same for the conditional mean residuals; and KS is the Kiefer and Salmon (1983) test for departures from the normal distribution for standardized residuals. We report p-values for these diagnostic tests. MAE ≡ mean absolute error; RMSE ≡ square root of the mean-squared error; R^2 from Mincer-Zarnowitz regressions applied to the volatility and premium forecasts.

2.3 Bivariate Risk–Return Results

In this section, we generalize our risk–return model for the market equity premium by estimating a joint stochastic specification of the conditional mean and annual log(RV) of market excess returns. The logarithmic specification of RV ensures that volatility is nonnegative during estimation of the joint stochastic process. The system of test equations is (15) and (16) to (18); and the results are summarized in Tables 6 and 7, as well as Figures 7 to 8.

presample data from 1802 to 1834 which the initial part of our estimation period require as conditioning information when $\tau = 40$.

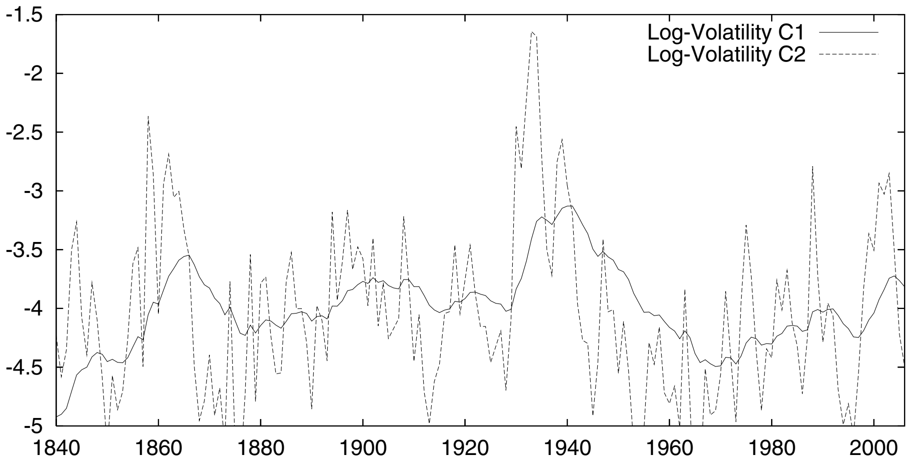

Figure 7 Log-Volatility component forecasts ($\log \sigma^2_{t,i}$): B(2) model.

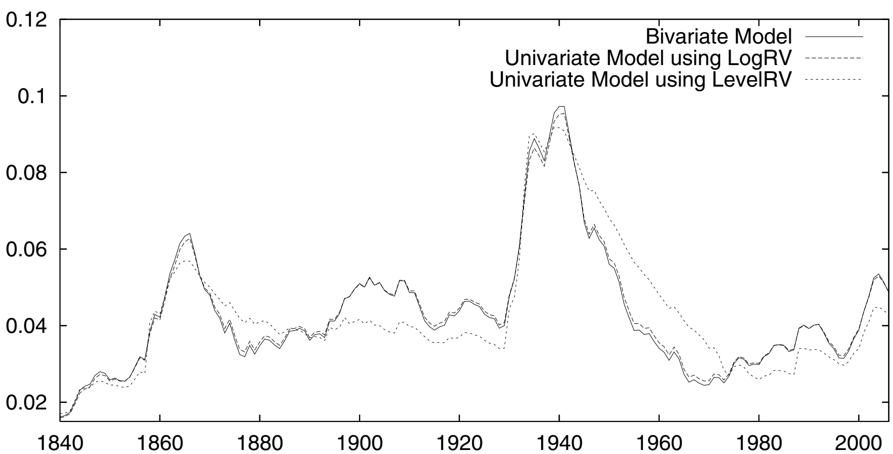

Figure 8 Proportional premium comparisons.

The results are very comparable to the univariate results reported above. Figure 7 plots the individual components of log volatility from the B(2) specification, that is, Equation (18) for $k = 1, 2$. Table 7 shows that the two-component volatility specification fits log(RV) better. For example, the R^2 for volatility fit increases from about 23% to 28% by adding the second volatility component.

Table 6 reports that the risk−return relationship is positive for all models and statistically significant for all of the models except two. As in the log(RV) univariate case, the t-statistic associated with γ_1 is smaller for the linear risk premium parameterizations B(1) and B(2). However, since we are unable to

Table 8 Robustness checks: bivariate return and log(RV) model

$$\log RV_t = \omega + \frac{1}{2}\sum_{i=1}^{2} \log \sigma_{t,i}^2 + \eta_t, \quad \eta_t \sim N(0, \phi^2);$$

$$\log \sigma_{t,i}^2 = (1-\alpha_i)\sum_{j=0}^{\tau-1} \alpha_i^j \log RV_{t-j-1}, \quad i = 1, 2$$

$$\text{Model B}(2_s): r_t = \gamma_0 + \gamma_1 \exp\left[\omega + \log \sigma_{t,1}^2 + .5\text{Var}_{t-1}(\log RV_t)\right] + \epsilon_t,$$

$$\epsilon_t = \sigma_{t,(2)} z_t, \quad z_t \sim N(0,1)$$

$$\text{Model B}(2_a): r_t = \gamma_0 + \gamma_1 \exp\left[\omega + \gamma_2 \log \sigma_{t,1}^2 + (1-\gamma_2)\log \sigma_{t,2}^2 + .5\text{Var}_{t-1}(\log RV_t)\right] + \epsilon_t, \quad \epsilon_t = \sigma_{t,(2)} z_t, \quad z_t \sim N(0,1)$$

	γ_0	γ_1	γ_2	ω	α_1	α_2	ϕ	L
Model B(2_a)	−0.064 [−1.569]	4.188 [2.603]	1.035 [5.149]	−0.1325	0.932 (0.028)	0.161 (0.142)	0.744 (0.045)	−134.29
Model B(2_s)	−0.066 [−1.648]	4.289 [2.859]		−0.1429	0.934 (0.027)	0.159 (0.140)	0.745 (0.045)	−134.30
$\tau = 5$	−0.024 [−0.365]	0.826 [1.042]		−1.437	0.920 (0.043)	0.229 (0.148)	0.741 (0.046)	−135.94
$\tau = 15$	−0.080 [−0.994]	1.924 [2.007]		−1.184	0.960 (0.025)	0.185 (0.140)	0.741 (0.045)	−134.56
1886−2006	−0.088 [−1.257]	4.636 [2.314]		−0.3689	0.957 (0.018)	0.1e−7 (0.167)	0.677 (0.047)	−87.87
1928−2006	−0.073 [−1.000]	4.224 [2.127]		−0.3882	0.958 (0.020)	0.1e−8 (0.225)	0.745 (0.067)	−65.67
1945−2006	−0.149 [−1.736]	7.483 [2.154]		−0.4059	0.959 (0.028)	0.2e−8 (0.365)	0.658 (0.064)	−34.56
RV from monthly (1840−2006)	−0.071 [−1.758]	3.804 [2.868]		−0.1281	0.932 (0.033)	0.269 (0.141)	0.771 (0.043)	−139.97

Notes: Conditional mean coefficient estimates of γ_i with t-statistics in brackets; ω is the volatility function intercept which is consistent with targeting the mean of log(RV); volatility function coefficient estimates α_i have standard errors in parenthesis; L is the log-likelihood function.

reject that $\gamma_0 = 0$, either by t-tests or by the LR tests reported in the second-last column of the table, the data support a proportional parameterization of the risk premium. In that case, the risk−return relationship is significantly positive for all volatility specifications. Again, as in Tables 2 and 4, the LRT results in the final column indicate that the relationship between excess market returns and their conditional variance is very strong for proportional parameterizations of the risk premium.

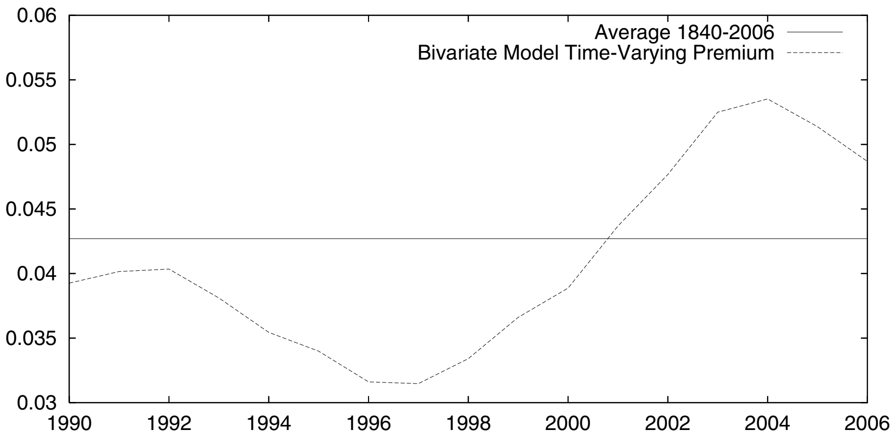

Figure 9 Proportional premium forecasts: 1990–2006.

Table 7 reports that the preferred model is, once again, the 2_s specification, that is, two volatility components with the smooth component being priced in the conditional mean.[25] As in the univariate cases, all of the premium fit statistics, MAE, RMSE, or R^2, support this conclusion; as does the log likelihood which is -134.30 for the B(2_s) model, as opposed to -143.72 for the B(1) model which has a one-component volatility specification.

As displayed in Figure 8, the proportional premium for the preferred B(2_s) specification is similar to that for the univariate case which conditions on past log(RV). The premium forecasts from this bivariate stochastic model range from 1.6% to 9.7% with an average of 4.31%. Note that all of our specifications deliver reasonable estimates of the market premium.[26]

Figure 9 illustrates the premium for the 1990-2006 period generated by our bivariate specification B(2_s). The time-varying premium estimates for the period 1990 to 2000 were below the long-run average of 4.27%, reaching a low of 3.15% in 1997. In addition, they are considerably lower than the 1980-2006 average continuously compounded excess return of 6.2%. This graphically illustrates the point, discussed in the introduction, that the average realized excess return is not a very reliable forecast of the market equity premium since it will be sensitive to the subsample chosen. Finally, the forecasted continuously compounded premium increased from the low of 3.15% in 1997 to 5.4% in 2004 then back down to 4.9% in 2006.

[25] Given our relatively small number of annual observations, we investigated potential small-sample bias for the maximum-likelihood estimator by simulating 1000 bootstrap samples using the parameter estimates for the $B(2_s)$ model in Table 6 as the DGP. The bias for all of the parameters was small. This suggests that the maximum-likelihood estimator is reliable for our sample which addresses one of the issues raised by Stambaugh (1999) in our context.

[26] We do not have enough annual data to get reliable estimates from a conventional GARCH-in-Mean parameterization using squared returns.

2.4 Robustness Analyses

We now check the robustness of the above results to alternative parameterizations and various subsamples. These results are summarized in Table 8.

The bivariate model with the most empirical support is labeled B(2_s). This specification specializes the bivariate system in Section 1.3.2 to the case with two volatility components ($k = 2$); allowing one of those components to affect the conditional mean ($q = 1$). That is,

$$r_t = \gamma_0 + \gamma_1 \exp\left[\omega + \log\sigma^2_{t,1} + .5\mathrm{Var}_{t-1}(\log RV_t)\right] + \epsilon_t, \tag{20}$$

$$\epsilon_t = \sigma_{t,(2)} z_t\ ,\ \ z_t \sim N(0,1) \tag{21}$$

$$\log RV_t = \omega + \frac{1}{2}\sum_{i=1}^{2}\log\sigma^2_{t,i} + \eta_t,\ \ \eta_t \sim N(0,\phi^2) \tag{22}$$

$$\log\sigma^2_{t,i} = (1-\alpha_i)\sum_{j=0}^{\tau-1}\alpha_i^j \log RV_{t-j-1}\ \ i = 1,2 \tag{23}$$

The results for this model for the sample 1840–2006 are repeated in the second panel of Table 8, labeled Model B(2_s).

To check that our parsimonious B(2_s) specification is able to adequately capture the potential differential effect of the smooth long-run volatility component on the dynamics of the premium, we introduce a new bivariate specification that replaces Equation (20) with

$$r_t = \gamma_0 + \gamma_1 \exp\left[\omega + \gamma_2\log\sigma^2_{t,1} + (1-\gamma_2)\log\sigma^2_{t,2} + .5\mathrm{Var}_{t-1}(\log RV_t)\right] + \epsilon_t \tag{24}$$

Imposing $\gamma_2 = 1/2$ corresponds to the total variance being priced, while restricting $\gamma_2 = 1$ is one component being priced. These cases were discussed in the previous subsection and Table 6 as models B(2) and B(2_s) respectively.

The results for the less parsimonious specification in Equation (24) are reported in the first panel of Table 8, labeled Model B(2_a). The estimate for parameter γ_2 is 1.035 with a standard error of 0.201. Therefore, we can reject that it is zero (the t-statistic is 5.149) which confirms the results from all of our earlier 2_s specifications that the persistent volatility component has a positive and very significant effect on the dynamics of excess returns. On the other hand, the coefficient on the second, less persistent volatility component is not significantly different from zero. While it is useful to see how the two volatility components are separately priced, notice that our more parsimonious model B(2_s) has an almost identical fit which confirms the robustness of the 2_s parameterizations reported earlier.

The third and fourth panels of Table 8 report results for the B(2_s) model with alternative values of τ, the number of lags used in the exponential smoother components. As compared to the default value, $\tau = 40$, the $\tau = 5$ case results in an insignificant risk–return relationship. As τ is increased, the t-statistic on the slope parameter γ_1 increases. We have found that the optimal τ is somewhat model and criterion specific. For example, a $\tau = 25$ produces a significantly better loglikelihood (−131.57) and higher t-statistic (3.153) associated with γ_1 than our base B(2_s) which has $\tau = 40$. However, the univariate specifications favored higher values of τ. Therefore, we chose a common value that was high enough to capture the smooth volatility component with our limited number of data.

The next three panels of Table 8 report subsample results for model B(2_s). This specification which prices the smooth volatility component of a two-component volatility model was able to deliver a positive and statistically significant risk–return relationship for subsamples such as 1886–2006, 1928–2006 and 1945–2006. Note that as the sample gets shorter, the persistence, α_2, of the second volatility component goes to zero. Nevertheless, the B(2_s) parameterization still always dominates a one-component volatility specification.

Finally, the last panel of Table 8 reports results for model B(2_s) for which annual RV is computed using monthly squared returns for the entire 1840–2006 sample, rather than daily squared returns when they became available in 1886. Our default model which uses the latter has a higher loglikelihood but otherwise the results are similar. We also tried using an indicator variable in the volatility function for the 1840–1885 part of the sample for which daily data were not available. Again, although that indicator variable was significantly different from zero indicating that annual RV was slightly lower over that first part of the sample, the risk–return results were similar. Also, note that the results for the 1886–2006 subsample refer to the period over which annual RV is always computed from daily squared returns. Again, the positive and significant risk–return relationship still obtains.

3 SUMMARY AND FUTURE DIRECTIONS

This article evaluates the market risk–return relationship for U.S. equity over the period 1840-2006 using a time-varying market premium for equity risk. We begin with a univariate specification of the risk–return relationship. This application models the stochastic market excess returns by conditioning on variance forecasts which are estimated by projecting onto past RV. We assess the robustness of those results by also estimating a univariate version which projects onto past log(RV).

We propose a parsimonious and flexible function that summarizes information in past RV that might be useful for forecasting changes in the market equity risk premium. We allow for alternative components of volatility with different decay rates. Not only is this a flexible way to capture the time-series dynamics of volatility, but it also allows us to investigate whether a particular component, rather than the full conditional variance, is more important in driving the market premium.

Our conditional variance specification maintains the parsimony of exponential smoothing functions but allows mean reversion in forecasts, and targets the implied long-run variance. In addition, we allow for increased flexibility by explicitly modeling more than one volatility component.

Finally, we generalize the risk–return model for the market equity premium by estimating a bivariate stochastic specification of annual excess returns and log(RV). In this case, the conditional variance of excess returns is obtained as the conditional expectation of the RV process.

In summary, in addition to our volatility specification, we consider two additional contributions to the literature. First, unlike the univariate specifications that dominate the literature, we extend the analysis to a bivariate risk–return model of returns and RV. Second, we investigate whether or not one volatility component is more important than total volatility in driving the dynamics of the equity premium.

All of the empirical specifications support a conclusion that the relationship between risk and return for the market equity premium is positive. The higher the expected market risk, the higher the market equity premium. This relationship is strongest for a two-component specification of volatility with the long-run smooth component being priced in the conditional mean.[27] In fact, the preferred model from the perspective of overall as well as equity premium fit, for both the univariate and bivariate specifications, is the 2_s model which uses a two-component volatility specification and prices the smooth component of volatility in the conditional mean. In future work, as in Engle and Rangel (2004) and Engle, et al. (2006), we plan to explore potential sources of this smooth component of volatility.

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