Rational expectations and efficiency in futures markets

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2 Foreign currency futures spreads and risk premiums

Thomas H. McCurdy and Ieuan G. Morgan

INTRODUCTION

The intertemporal asset pricing model was extended a few years ago to price nominal assets in an international setting (for example, Lucas 1982). Many applications of the model to financial forward markets followed, including those by Hansen and Hodrick (1983), Hodrick and Srivastava (1984) and Mark (1985). However, it is only recently that the work of Cox *et al.* (1981) and Richard and Sundaresan (1981) on futures has been adapted for financial futures. Applications to foreign currency futures include those by Hodrick and Srivastava (1987) and McCurdy and Morgan (1987, 1988a).

In these analyses, emphasis has been placed on testing for the potential time-varying risk premiums implied by the intertemporal asset pricing model. However, the applications to financial futures markets are very recent, and their results with respect to risk premiums are inconclusive. Hodrick (1987: 140–50) reviewed some of the evidence and concluded that more work was needed to resolve the issues raised by conflicting inferences drawn in different analyses. The aim of this chapter is to provide some new evidence concerning the temporal behaviour of risk premiums in foreign currency futures prices. This new evidence is derived from the study of spreads in the futures market in which a long position in one contract is offset by a short position in another contract for the same commodity but a different maturity.

Futures market analysis favours the use of high-frequency data. For example, tests of the time series behaviour of the basis, or difference between futures and spot prices, proposed by McCurdy and Morgan (1988b) require the use of daily data since cash flows are generated daily by the institutional practice of marking to market and overnight interest rates are stochastic. Direct testing of the martingale hypothesis for futures prices themselves is less restrictive in this respect, but it seems reasonable to assume that, if an interval of observation longer than one day is chosen for examining the rate of change of futures prices, the resulting loss of information may reduce the power of the tests to reject the hypothesis. It might be possible to reconcile some of the different inferences for weekly as opposed to daily data in this way, but a serious limitation of the published work to date has been the indirect nature of the testing for the risk premium hypothesis as the alternative to the martingale hypothesis. On the one hand, measurement of consumption and the general price level at a daily frequency is not feasible, and so testing for consumption and nominal (or purchasing power) risk in an intertemporal model must be indirect. On the other hand, the wealth-based single-period theory of the capital asset pricing model was adapted to futures pricing by Dusak (1973) and Black (1976), and it provides an observable measure for risk in the form of the daily excess returns from the market portfolio. Use of this measure in a multi-period consumption setting can be supported by a conditional capital asset pricing model or by the recent work of Epstein and Zin (1987) in which the standard additive expected utility specification is generalized.

Use of daily data also presents some potential problems. Test results rejecting a null hypothesis of no risk premium with daily data have been ambiguous because of the confounding influence of day of the week periodicity in futures prices. Even though the null hypothesis may have been rejected in these instances, the specific alternative of a risk premium was not being tested. In a futures spread formed from contracts of different maturities for the same commodity, the periodicity in the futures price of one contract may be offset by similar periodicity in the futures price of the other contract.

McCurdy and Morgan (1988c) found significant time-varying systematic or market risk in foreign currency futures prices. In this chapter we present evidence of small but significant systematic or market risk estimates in foreign currency futures spreads, and therefore of a systematic risk in futures prices related to the length of maturity of the contract. Our results indicate that the shorter maturity futures contract has the greater systematic risk.

Breeden (1980) estimated market risks as well as consumption betas for various real commodities and contract maturities with annual and quarterly futures data. His paper included the following remarks:

> Due to the very high correlations between spot and futures prices for most commodities, the determinants of futures' consumption betas are likely to be explained in large part by the same factors as the spots are. In applying the theory ... to the analysis of futures an important consideration is of the dependence of elasticities upon the time to maturity ... A possible result, which is seen in some commodities in the subsequent empirical estimates of consumption-betas, is that consumption-betas decline as the time to maturity increases, due to supply responses.

Breeden expressed his findings in this way:

> Note that, contrary to the generally negative quarterly market beta estimates ... verified here for many contract maturities, the consumption beta estimates show that several futures contracts have positive systematic risks as defined by the intertemporal CAPM [Capital asset pricing model]. For these contracts normal backwardation is predicted by the intertemporal CAPM, whereas the single period CAPM predicts contango.

Since we find positive estimates of systematic or market risk in foreign currency futures prices (McCurdy and Morgan 1988c) and negative estimates in spreads in which there is a long position in the longer maturity contract and a short position in the shorter maturity contract, our daily data analyses of market risks in foreign currency futures appear to be consistent with the pattern suggested by Breeden's work on consumption risks rather than market risks. However, our analyses of futures contracts deal with financial assets as opposed to real commodities, use daily as opposed to quarterly or annual data and explicitly model the time-varying variance of the futures prices as conditional heteroscedasticity.

In this chapter we use the intertemporal asset pricing paradigm to value a hedged position consisting of a futures spread. We present the univariate test equation for futures spreads risk premiums and the associated conditional variance function, and the bivariate test equation for time-varying spreads risk premiums. Issues related to the data are discussed, and univariate and bivariate results are summarized. Brief concluding comments are given in the final section.

THEORETICAL BACKGROUND

Let C_t be the number of units of the good consumed at t, p_t be the price per unit of the (consumption) good at t and $M_{t,\,t-1}$ be the intertemporal marginal rate of substitution of domestic currency between time t and time $t - 1$. For the special case of a constant time preference factor δ and an additively time-separable multi-period utility function,

$$M_{t,t-1} \equiv \delta \frac{u'(C_t)}{u'(C_{t-1})} \frac{p_{t-1}}{p_t}$$

$F_{T,t}$ is the price at t of a futures contract to deliver one unit of the foreign currency at T and R_{t-1} is 1 plus the US riskless rate of interest from $t - 1$ to t.

Given that a futures contract has zero present value because the investment outlay is zero, and that we have a long position in futures initiated at t and offset by a short position taken at $t + 1$, Theorem 1 of Richard and Sundaresan (1981), adapted for pricing nominal assets, implies that

$$0 = \mathrm{E}_t\left[\delta \frac{u'(C_{t+1})}{u'(C_t)} \frac{p_t}{p_{t+1}} (F_{T,t+1} - F_{T,t}) R_t\right]$$

Covariance decomposition gives

$$F_{T,t} = \mathrm{E}_t F_{T,t+1} + \mathrm{cov}_t\left[\delta \frac{u'(C_{t+1})}{u'(C_t)} \frac{p_t}{p_{t+1}} R_t, F_{T,t+1}\right] \qquad (2.1)$$

A direct implication of (2.1) for futures contracts with two different expiration dates, $T = 1$ and $T = 2$, is

$$\frac{F_{2,t-1} - F_{1,t-1}}{F_{1,t-1}} = \frac{\mathrm{E}_{t-1}(F_{2,t} - F_{1,t})}{F_{1,t-1}} + \mathrm{cov}_{t-1}\left(M_{t,t-1} R_{t-1}, \frac{F_{2,t} - F_{1,t}}{F_{1,t-1}}\right) \qquad (2.2)$$

which defines the value of a futures spread at $t - 1$ in terms of quantities expected for t. In (2.2) all quantities corresponding to those in (2.1) have been divided by $F_{1,t-1}$, the value of the futures price for the contract with the shorter time to maturity, to make them independent of scale. The price $F_{1,t-1}$ is, of course, known at $t - 1$.

The conditional covariance in (2.2) can be interpreted as a one-period risk premium for the spread. For example, for simplicity let $\mathrm{E}_{t-1}(F_{2,t} - F_{1,t})$ be zero. Then the left hand side of (2.2) has the same sign as that of $\mathrm{cov}_{t-1}(M_{t,t-1}, F_{2,t} - F_{1,t})$. If the marginal utility of consumption is declining in C_t, then $F_{2,t-1} < F_{1,t-1}$ if the shorter maturity futures contract is a better hedge against variation in consumption at t than the longer maturity contract or, equivalently, $\mathrm{cov}_{t-1}(M_{t,t-1}, F_{1,t}) > \mathrm{cov}_{t-1}(M_{t,t-1}, F_{2,t})$. A sufficient condition for the conditional covariance term in (2.2) to be zero is that the prices for the two futures contracts in the spread have the same covariation with the marginal utility from a dollar's worth of consumption.

The risk premium in (2.2) was derived from the intertemporal asset pricing model and, as indicated in the introduction, is not in a directly testable form in applications using daily data. Hansen *et al.* (1982) and Hansen and Hodrick (1983), building on an idea due to Breeden (1979), assumed the existence of a benchmark portfolio b with nominal return $R_{b,t}$ perfectly conditionally correlated with $M_{t,t-1}$, to derive a conditional capital asset pricing model. As in Campbell (1987), the perfect correlation assumption establishes that this portfolio must be efficient, i.e. on the minimum variance frontier; the existence of any asset assumed to have the same expected return but lower variance would require that the conditional correlation coefficient of the portfolio $\mathrm{corr}_{t-1}(M_{t,t-1}, R_{b,t})$ is greater than unity.

Let z be the minimum variance portfolio orthogonal to portfolio b. Since $\mathrm{E}_{t-1}(M_{t,t-1}, R_{z,t}) = 1$ and $\mathrm{cov}_{t-1}(M_{t,t-1}, R_{z,t}) = 0$, then $\mathrm{E}_{t-1} R_{z,t} = 1/\mathrm{E}_{t-1} M_{t,t-1}$. Given the assumption of a riskless nominal asset, for which $\mathrm{E}_{t-1}(M_{t,t-1} R_{t-1}) = 1$, $\mathrm{E}_{t-1} R_{z,t} = R_{t-1}$. In other words by Corollary 3A of Roll (1977), portfolio b with return perfectly conditionally correlated with $M_{t,t-1}$ must be the portfolio at the point of tangency between the efficient set and a straight line through the riskless rate in mean standard deviation space. In this model, the tangent portfolio is on the negatively sloped portion of the minimum variance frontier. Because portfolio b is efficient, corollary 3 of Roll (1977) establishes that returns on individual assets can be expressed as exact linear functions of its returns and those of its unique minimum variance orthogonal portfolio z. Then (2.2) can be rewritten in terms of the expected excess returns $\mathrm{E}_{t-1}(R_{b,t} - R_{t-1})$ in a conditional capital asset pricing model. That is, with the notation

$$R^*_{b,t} \equiv R_{b,t} - R_{t-1}$$

for the tangent portfolio return in excess of the riskless rate of interest and

$$b_{t-1} \equiv \frac{\mathrm{cov}_{t-1}[(F_{2,t} - F_{1,t})/F_{1,t-1}, R^*_{b,t}]}{\mathrm{var}_{t-1}(R^*_{b,t})},$$

for the conditional risk, (2.2) becomes

$$\frac{\mathrm{E}_{t-1}(F_{2,t} - F_{1,t})}{F_{1,t-1}} = \frac{F_{2,t-1} - F_{1,t-1}}{F_{1,t-1}} + b_{t-1}\mathrm{E}_{t-1}R^*_{b,t} \quad (2.3)$$

THE UNIVARIATE TEST EQUATION FOR FUTURES SPREADS RISK PREMIUMS

In the univariate analyses, tests for risk premiums associated with futures spreads must proceed under the assumption that the systematic risk is constant (i.e. the conditional and unconditional moments are equal). We obtain indirect tests for the existence of time-varying risk premiums by examining the behaviour of the test equation with respect to omitted variables that are potentially relevant *a priori*. Direct estimates of time-varying systematic risk premiums with a bivariate model will be described in the next section.

A link between theory and data for empirical work depends on $\mathrm{E}_{t-1}R_{z,t} = R_{t-1}$, with $R_{b,t}$ being the return from the tangent portfolio. To obtain a test equation from (2.3), we identify the tangent portfolio with the market portfolio, with excess return $R^*_{\mathrm{m},t}$, replace the rational expectation of the scaled spread by its realized value minus a forecast error with a conditional mean of zero, allow for a first-order moving-average error process in the time series behaviour of the test equation residuals and add indicator variables to capture any significant day of the week periodicity. Then, except for any indicator variables,

$$\frac{F_{2,t} - F_{1,t}}{F_{1,t-1}} - \frac{F_{2,t-1} - F_{1,t-1}}{F_{1,t-1}} = \gamma_0 + \gamma_1 R^*_{\mathrm{m},t} + \psi\varepsilon_{t-1} + \varepsilon_t \quad (2.4a)$$

We shall continue to refer to the quantity given by the dependent variable in (2.4a) as the spread.

For the conditional variance h_t of the error term

$$\varepsilon_t | I_{t-1} \sim \mathrm{N}(0, h_t)$$

we use an integrated generalized autoregressive conditional heteroscedasticity (IGARCH) model introduced by Engle and Bollerslev (1986). Let g_{it-1} represent an $n \times 1$ vector of variables in the information set at $t-1$. Then the IGARCH model for the conditional variance is

$$h_t = \alpha_0 + (1-\beta)\varepsilon^2_{t-1} + \beta h_{t-1} + \sum_{i=1}^{n}\phi_i g_{it-1} \quad (2.4b)$$

THE BIVARIATE TEST EQUATION FOR FUTURES SPREADS RISK PREMIUMS

Baillie and Bollerslev (1987) and Bollerslev (1987) have proposed a multivariate model for analysing vectors of rates of change of spot and forward prices of foreign currencies. Their model achieved a parsimonious specification by the assumption of constant conditional correlation between any pair of currencies. We use a bivariate version of this model, pairing each currency in turn with the market portfolio excess returns to analyse futures spreads while letting the systematic risk of the spread vary through time. One reason for doing this is to represent a possible effect of time to maturity of the contract on conditional systematic risk.

The bivariate test equation for the futures spread is similar to equation (2.3) but with the systematic risk specified to be the conditional covariance defined to be the product of the constant conditional correlation between the spread and the excess return from the market portfolio and the time-varying conditional standard deviations of these variables. The behaviour of the excess returns from the market portfolio itself is modelled as IGARCH with an MA(1) error term, as in Chou (1987).

Our application of the bivariate model suffers from one conceptual difficulty. The market portfolio represents an important part of world aggregate wealth but futures market contracts represent zero net wealth. Marcus (1984) has emphasized this point. Letting the futures data reflect the contemporaneous market portfolio behaviour makes sense, but any potential dependence in the other direction, stemming from the symmetric influence of the constant correlation coefficient on the conditional variances in the model, can make sense only if the futures market data are interpreted as representing components of wealth not included in the prices of New York Stock Exchange and American Exchange common stocks.

With the notation $R^*_{\mathrm{m},t} = R_{\mathrm{m},t} - R_{t-1}$ for the excess return from the market portfolio, $h_{\mathrm{m},t}$ for its conditional variance, $h_{\mathrm{f},t}$ for the conditional variance of the spread, ρ for the constant correlation between the two and $h_{f,m,t} = \rho h_{f,t}^{1/2} h_{m,t}^{1/2}$ for the corresponding conditional covariance, the bivariate model system of equations is

$$\frac{F_{2,t} - F_{1,t}}{F_{1,t-1}} - \frac{F_{2,t-1} - F_{1,t-1}}{F_{1,t-1}} =$$

$$\gamma_{0,f} + \frac{h_{f,m,t}}{h_{m,t}} (\gamma_{0,m} + \psi_m \varepsilon_{m,t-1}) + \psi_f \varepsilon_{f,t-1} + \varepsilon_{f,t} \quad (2.5a)$$

$$h_{f,t} = \alpha_{0,f} + (1 - \beta_f)\varepsilon^2_{f,t-1} + \beta_f h_{f,t-1} + \sum_{i=1}^{n} \phi_i g_{i,t-1} \quad (2.5b)$$

$$R^*_{m,t} = \gamma_{0,m} + \psi_m \varepsilon_{m,t-1} + \varepsilon_{m,t} \quad (2.6a)$$

$$h_{m,t} = \alpha_{0,m} + (1 - \beta_m)\varepsilon^2_{m,t-1} + \beta_m h_{m,t-1} \quad (2.6b)$$

This basic structure is augmented by the addition of an indicator variable with the value of unity on Monday or on a Tuesday following a holiday on Monday, and zero otherwise, to the regression equation for the market portfolio excess return. Similarly, for the British pound spread, an indicator variable with the value of unity on Friday and zero otherwise is added to the regression equation. These additions were indicated by our specification tests discussed below.

DATA

Futures prices for the British pound, the Canadian dollar, the Deutschmark, the Japanese yen and the Swiss franc (BP, CD, DM, JY and SF respectively) were taken from the 1985 version of the file provided by the Center for Research in Futures Markets (CRFM) of the University of Chicago. We used futures settlement prices for the contracts with the shortest and next to shortest maturities available at any time up to and including the Friday before the end of the life of the shortest contract. The data start on 1 October 1981 and end on 31 December 1985. Until 1 October 1981, US dollar cheques written to purchase spot foreign exchange for delivery on a given business day cleared for federal funds the following business day. The resulting artificially high demand for foreign currency before the weekend was described by Levi (1978); very closely related day of the week patterns in changes in futures prices for five currencies have been

described by McCurdy and Morgan (1987). By starting our sample period on 1 October 1981, we avoid the market imperfection that originated in the spot market and spilled over into the futures markets for foreign currencies. The series of 1,075 observations gives an effective sample size of 1,074 because the first observation is used in the start of the estimation.

In some of our tests for omitted variables we used short-term interest rates to compute a relative interest rate differential. The shortest debt maturity for which rates were quoted in readily available sources was two days for most actively traded foreign currencies, but overnight Eurocurrency interest rates were available for the United States and Japan. The Eurocurrency rate for Canada switched from being call to two-day notice part way through our sample period on 21 November 1983. On and after this date we used the 'most often quoted' call rate quoted by the Bank of Canada. When one or both interest rates were unavailable because of holidays we substituted both rates for the previous day.

For the market return in (2.4a), and in the bivariate analyses, we used the value-weighted market portfolio return in excess of the overnight rate of interest. The market portfolio return was taken from the daily data file of the Center for Research in Security Prices of the University of Chicago.

Futures prices are subject to rules specifying the maximum price change that can occur in one day. In the sample period, the limits were not tight for these five currencies and they were removed entirely on 22 February 1985. There were no occasions at all on which there were limit moves in BP and SF, six days in CD, two in DM and three in JY. When a limit move occurs for a given futures contract, the settlement price for that contract is clearly not an equilibrium price. When both contracts involved in a spread have limit moves on the same day, the resulting distortion in the dependent variable is probably not severe because the deviations from the equilibrium prices tend to cancel each other. A potentially serious problem occurs when one contract has a limit move but the other does not, as could happen towards the end of the life of the contract with shorter maturity. Under the rules in effect the limits did not apply in the 'spot' month beginning on the last day of trading of the month before the month in which the contact matured. In other words, except for most of the last year of the sample period, spreads were potentially awkward quantities to work with in the last three weeks or so of a contract's life because the near futures price was free but the far futures price was subject to the limit rules. For example, in

JY on 2 March 1984 there was a limit move on the June 1984 contract but not on the March 1984 contract, although the price change for this contract was greater. We treated this type of event by defining an indicator variable with the value of unity if on a given day the settlement-price was a limit move for one contract in the spread but free for the other.

UNIVARIATE RESULTS

We present the estimates of equation (2.4) for each currency in Table 2.1. Table 2.2 reports the results of some diagnostic test statistics on

Table 2.1 Univariate model estimates of futures spreads

		BP	*CD*	*DM*	*JY*	*SF*
Constant	$\hat{\gamma}_0$	−0.0198	−0.0015	−0.0018	−0.0235	−0.0052
		(0.0126)	(0.0059)	(0.0107)	(0.0096)	(0.0108)
$R_{m,t} - R_{t-1}$	$\hat{\gamma}_1$	−0.0075	−0.0011	−0.0040	−0.0097	−0.0026
		(0.0016)	(0.0010)	(0.0016)	(0.0016)	(0.0018)
F	$\hat{\gamma}_2$	−0.079				
		(0.033)				
ε_{t-1}	$\hat{\psi}$	−0.062	−0.191	−0.178	−0.244	−0.236
		(0.038)	(0.035)	(0.035)	(0.032)	(0.031)
Constant	$\hat{\alpha}_0$	0.0	0.0029	0.0061	0.0	−0.0080
			(0.0007)	(0.0016)		
IGARCH	$\hat{\beta}$	0.759	0.787	0.844	0.764	0.905
		(0.028)	(0.030)	(0.020)	(0.032)	(0.022)
F	$\hat{\phi}_1$	0.060				0.056
		(0.014)				(0.007)
N	$\hat{\phi}_2$				0.260	
					(0.090)	
$\lvert R_{t-1}/Z_{t-1} - 1\rvert$	$\hat{\phi}_3$				0.0091	
					(0.0024)	
Limit	$\hat{\phi}_4$				1.467	
					(0.590)	

Notes: Standard errors are shown in parenthesis.

F is an indicator variable for Friday, N is an indicator for the first day of trading of a spread with a new futures contract and 'Limit' is an indicator for the event that only one futures price of the two in the spread was restricted by the limit rules.

For the lower bound restrictions on α_0 the OPG LM test statistic p values were 0.22 for the BP, 0.99 for the JY and 0.46 for the SF.

Table 2.2 Diagnostic tests for the models in Table 2.1

	BP	*CD*	*DM*	*JY*	*SF*
R	−1.85	−4.83	−2.74	−0.79	−3.05
	(0.06)	(0.00)	(0.01)	(0.43)	(0.00)
$Q(10)$	6.37	14.43	4.11	10.30	5.17
	(0.78)	(0.15)	(0.94)	(0.41)	(0.88)
$Q^2(10)$	6.33	10.17	5.25	15.08	9.31
	(0.79)	(0.43)	(0.87)	(0.13)	(0.50)
S	4.44	1.30	0.09	4.21	0.70
	(0.04)	(0.25)	(0.76)	(0.04)	(0.40)
K	19.16	10.51	15.61	23.23	9.32
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

Notes: R is the test statistic for runs above the mean, $Q(10)$ is the Ljung–Box form of the portmanteau statistic for autocorrelation in the first ten lags of the standardized residuals, $Q^2(10)$ is the same statistic for the squared standardized residuals, and S and K are the conditional moment test statistics for skewness and kurtosis respectively. The p values, shown in parenthesis, are for the χ^2 distribution.

the standardized residuals $\varepsilon_t/h_t^{1/2}$ as well as conditional moment tests (Newey 1985; Tauchen 1985) for skewness and excess kurtosis. Outer product of the gradient Lagrange multiplier (OPG LM) test statistics for omitted variables (Godfrey and Wickens 1982; Davidson and MacKinnon 1988) are presented in Table 2.3. In all analyses, the spread, the relative interest rate differential and the excess market return values are scaled by multiplication by 1,000.

Table 2.1 shows very small but significantly negative values of the estimates of the coefficient γ_t for the constant systematic risk of the futures spread in three currencies: BP, DM and JY. The equation estimated is not the same for every currency. CD and DM have particularly simple equations. BP is slightly more complicated because the Friday indicator variable is included in both the regression and the conditional variance. The conditional variance function for JY is complicated: it includes an indicator variable with a value of unity on the first day of the changeover to a new pair of contracts in the spread and zero otherwise, the absolute value of the relative interest rate differential and an indicator variable with the value of unity if, at t, only one of the futures prices of the pair in the spread was constrained by the limit move rules of the exchange. The conditional variance function of SF was problematic until its constant term was allowed to assume a negative value. The constrained value

Table 2.3 Outer product of the gradient Lagrange multiplier tests for omitted variables

	BP		*CD*		*DM*		*JY*		*SF*	
	Mean	*Variance*	*Mean*	*Variance*	*Mean*	*Variance*	*Mean*	*Variance*	*Mean*	*Variance*
d.f.	1	2	1	1	1	1	1	2	1	2
$R_{t-1}/Z_{t-1} - 1$	1.22 (0.27)	1.55 (0.46)	2.53 (0.11)	0.71 (0.40)	1.44 (0.23)	0.07 (0.79)	2.02 (0.16)		0.35 (0.55)	3.49 (0.17)
$\lvert R_{t-1}/Z_{t-1} - 1\rvert^{1/2}$	2.87 (0.09)	1.96 (0.38)	1.75 (0.19)	0.36 (0.55)	1.90 (0.17)	1.14 (0.71)	0.03 (0.86)	4.39 (0.11)	0.22 (0.64)	3.24 (0.20)
M	2.07 (0.15)	1.50 (0.47)	0.01 (0.92)	0.30 (0.58)	2.25 (0.13)	1.08 (0.30)	0.41 (0.52)	0.00 (0.99)	0.14 (0.71)	0.57 (0.75)
T	0.03 (0.86)	1.49 (0.47)	0.46 (0.50)	1.35 (0.25)	1.75 (0.19)	0.06 (0.81)	0.38 (0.54)	4.75 (0.09)	2.26 (0.13)	0.60 (0.74)
W	0.05 (0.82)	4.84 (0.09)	0.25 (0.62)	2.80 (0.09)	0.00 (0.99)	3.51 (0.06)	0.04 (0.84)	0.13 (0.94)	0.02 (0.89)	0.57 (0.75)
Th	1.83 (0.18)	2.79 (0.25)	0.38 (0.54)	0.06 (0.81)	1.49 (0.22)	1.25 (0.26)	0.32 (0.57)	3.85 (0.15)	0.23 (0.63)	1.07 (0.59)
F			0.26 (0.61)	0.32 (0.57)	1.14 (0.29)	0.70 (0.40)	1.13 (0.29)	0.20 (0.90)	0.64 (0.42)	
N	0.99 (0.32)	2.78 (0.25)	0.11 (0.74)	0.00 (0.99)	0.46 (0.50)	3.28 (0.07)			0.10 (0.75)	0.84 (0.66)
$T - t$	1.63 (0.20)	6.01 (0.05)	1.53 (0.22)	0.50 (0.48)	2.85 (0.09)	0.56 (0.45)	1.15 (0.28)	0.26 (0.88)	0.35 (0.55)	9.43 (0.01)
$(T - t)^{1/2}$	1.17 (0.28)	5.13 (0.08)	1.21 (0.27)	0.93 (0.33)	2.60 (0.11)	0.90 (0.34)	1.49 (0.22)	0.51 (0.77)	0.29 (0.59)	6.94 (0.03)
$(T - t)^{-1/2}$	0.26 (0.61)	2.95 (0.23)	0.18 (0.67)	1.77 (0.18)	1.16 (0.28)	0.98 (0.32)	1.32 (0.25)	0.40 (0.82)	0.01 (0.92)	1.89 (0.39)

Notes: p values, for the χ^2 distribution with d.f. degrees of freedom are shown in parenthesis. For the time to contract expiration variables the square of the variable is tested in the variance, but for $R_{t-1}/Z_{t-1} - 1$ the absolute value is tested in the variance. For the other interest rate differential variable, the day of the week indicators and the indicator N for the first day of trading of a spread with a new futures contract, the same form is tested in the variance and the mean.

(a lower bound of −0.008) for the constant term, reported in Table 2.1, had a p value of 0.46, indicating that the constraint did not conflict with the data.

Significantly negative coefficient estimates of the MA(1) term were obtained in all currencies except BP. It is not clear what feature of the settlement prices of futures contracts for the same commodity induces the moving-average structure in the futures spread since the settlement prices are determined simultaneously. A known source of negative first-order autocorrelation in common stock returns, particularly where the minimum price fluctuation allowed by the exchange is large relative to the price, is the bid–ask spread (Roll 1984). Even if there were a counterpart in futures markets, it would be difficult to explain why the currency with the least noticeable moving-average structure in the spread is BP. The currency has a minimum price fluctuation of US$ 0.0005, five times that for the other major currencies (with the exception of JY), but its value in the sample period was only two or three times the value of the currencies with lower minimum price fluctuation.

A second possibility is that the longer-term contract of the spread, because it is typically less frequently traded than the nearer-term contract, has a settlement price that may have been determined by its last trade made earlier in the day than the last trade for the more actively traded contract. Even if the settlement prices are fixed simultaneously, the prices on which they were based would not have been for simultaneous trades. As shown by McCurdy and Morgan (1988b) for the basis, a first-order moving-average structure is then induced. The possibility that the moving-average term is induced by non-simultaneous trades, in turn related to less active trading of the longer-term contract, may also be consistent with the negative estimates obtained for the systematic risk of the spread. Scholes and Williams (1977), in a similar context, have shown how infrequent trading leads to a downward bias in the estimate of systematic risk of a security. A sufficiently strong bias of this kind could produce the observed negative systematic risk estimates for the spread even if the true risk for each of the futures is equal. Since we can offer no direct evidence to evaluate these possibilities, which depend on critical assumptions about the way that settlement prices are determined, we shall ignore them except to recognize their potential contribution to the moving-average structure and/or the systematic risk estimate for spreads.

Table 2.2 shows that there is some evidence of dependence in the standardized residuals, corresponding to positive autocorrelation,

since the number of runs above the mean is significantly smaller than expected in CD, DM, and SF. Neither the portmanteau test for the first ten lags of the autocorrelation function nor the estimated first-order autocorrelation coefficient itself (not shown) confirms this evidence. The portmanteau test for the first ten lags of the autocorrelation function for the squared standardized residuals does not detect any remaining heteroscedasticity. In other words, the IGARCH representation of the conditional variance seems to be satisfactory. Our choice of the more parsimonious model IGARCH, as opposed to generalized autoregressive conditional heteroscedasticity (GARCH), was based on log likelihood ratio tests of the restriction $\alpha_1 + \beta = 1$ (Bollerslev 1988). The choice was supported by the absence of a noticeable advantage, according to the residual diagnostic statistics, of GARCH over IGARCH in this application. Since the likelihood ratio statistics for the restriction had p values of 0.51, 0.35, 0.46, 0.00 and 0.05 for the five currencies, only JY indicated a possible advantage of GARCH. The final row of Table 2.2 reveals excess kurtosis in every currency.

Table 2.3 presents the OPG LM tests for missing variables. Evidence of a missing variable in the regression equation (or mean) would be important because it would indicate a possible inadequacy of the systematic risk term as a surrogate for the conditional covariance term in (2.2). No evidence of this kind is found in Table 2.3, although, of course, the BP spread regression equation was augmented by the inclusion of the Friday indicator variable when such tests detected the need for it.

The OPG LM tests for variables omitted from the conditional variance function are also generally satisfactory. Only in SF is there an indication of a potential problem; low p values are shown for the statistics for the time to maturity and one of its transformations – the square of the time to maturity. Attempts to include this variable or its square in the variance function would fail because its coefficient would be sufficiently negative to result in a negative conditional variance for some days.

We infer that Tables 2.2 and 2.3 have revealed only relatively minor faults with the univariate model. An unexpected feature of the analysis was that the day of the week patterns in price in the futures prices themselves, recorded by McCurdy and Morgan (1988c), were not entirely eliminated in the spreads. It also seems fair to conclude that representing the risk premium or conditional covariance term in (2.2) by a specification of constant systematic risk for the futures spread is reasonable. The bivariate estimates of the next section provide further evidence about the risk premium under a different specification.

BIVARIATE RESULTS

In the bivariate analyses we were generally able to fit the exact equivalents of the univariate models described in the previous section. Table 2.4 shows the estimates for the bivariate system of equations, and Table 2.5 shows the diagnostic checks and conditional moment tests on the standardized residuals. Where the bivariate and univariate models differ, as for BP and JY, the bivariate model is the simpler of the two. In BP, the MA(1) term for the futures spread was dropped because it added negligible explanatory power; the p value for the likelihood ratio test statistic of 1.40 was 0.24. In JY, the estimated coefficient for the absolute value of the relative interest rate differential in the conditional variance function fell to an insignificantly small positive value; the p value for the likelihood ratio test statistic of 1.16 was 0.28. At the same time, the estimated value of the IGARCH parameter β_f for JY increased from 0.764 in the univariate analysis to 0.919 in the bivariate analysis.

A major feature of Table 2.4 is the set of estimated constant conditional correlation coefficient values. The correlation coefficient estimate is significantly negative in the futures spreads for three of the five currencies (BP, DM and JY), and these are the same three currencies as those with significantly negative estimates of systematic risk for the spreads in the univariate analyses in Table 2.1. In other words, the inferences that can be drawn from the univariate and bivariate analyses with regard to systematic risk are quite similar. Again, the correlations are small: a coefficient of −0.1 can be interpreted roughly as implying that 1 per cent of the variation of the futures spread is accounted for by the market portfolio excess returns. The time series of the spread consists of observations from the two shortest term futures contracts for a given foreign currency. On the Friday before the expiration of one contract the spread relates to the difference between the expected spot prices at, typically, 96 days later and five days later. On the following Monday, data from a new contract are taken and the spread relates to expected spot prices 184 days and 93 days later. The difference in the maturities is constant at 91 days. If the relationship between the systematic risk of the futures and the time to maturity of the contract is not markedly non-linear over this range, the constant difference of 91 days may help to explain why the inferences from the univariate (constant systematic

Table 2.4 Bivariate model estimates of futures spreads

		BP	*CD*	*DM*	*JY*	*SF*
Constant	$\hat{\gamma}_{0,f}$	−0.016	−0.001	−0.001	−0.023	−0.005
		(0.014)	(0.006)	(0.011)	(0.011)	(0.011)
F	$\hat{\gamma}_{1,f}$	−0.071				
		(0.032)				
$\varepsilon_{f,t-1}$	$\hat{\psi}_{1,f}$		−0.186	−0.184	−0.216	−0.233
			(0.035)	(0.034)	(0.031)	(0.032)
Constant	$\hat{\alpha}_{0,f}$	0.0	0.0029	0.0037	−0.0006	−0.008
			(0.0007)	(0.0013)	(0.0010)	
IGARCH	$\hat{\beta}_{1,f}$	0.835	0.785	0.881	0.932	0.918
		(0.032)	(0.028)	(0.021)	(0.024)	(0.025)
F	$\hat{\phi}_{1,f}$	0.029				0.053
		(0.011)				(0.007)
N	$\hat{\phi}_{2,f}$				0.142	
					(0.038)	
Limit	$\hat{\phi}_{3,f}$				0.512	
					(0.245)	
$\frac{h_{f,t}^{1/2}}{h_{m,t}^{1/2}}(\gamma_{0,m}+\psi_m\varepsilon_{m,t-1})$	$\hat{\rho}$	−0.113	−0.018	−0.069	−0.140	−0.045
		(0.029)	(0.031)	(0.030)	(0.031)	(0.032)
Constant	$\hat{\gamma}_{0,m}$	0.076	0.073	0.078	0.080	0.077
		(0.026)	(0.028)	(0.029)	(0.026)	(0.024)
M	$\hat{\gamma}_{1,m}$	−0.203	−0.206	−0.209	−0.203	−0.202
		(0.048)	(0.052)	(0.052)	(0.052)	(0.046)
$\varepsilon_{m,t-1}$	$\hat{\psi}_{1,m}$	0.148	0.151	0.153	0.152	0.152
		(0.033)	(0.032)	(0.033)	(0.033)	(0.032)
Constant	$\hat{\alpha}_{0,m}$	0.0014	0.0014	0.0014	0.0014	0.0014
		(0.0011)	(0.0011)	(0.0011)	(0.0012)	(0.0011)
IGARCH	$\hat{\beta}_{1,m}$	0.960	0.960	0.960	0.960	0.960
		(0.010)	(0.010)	(0.010)	(0.010)	(0.010)

Notes: Standard errors are shown in parentheses

M is an indicator taking the value of unity on Monday or on a Tuesday following a holiday Monday, *F* is an indicator variable for Friday, *N* is an indicator for the first day of trading of a spread with a new futures contract and 'limit' is an indicator for the event that only one futures price of the two in the spread was restricted by the limit rules.

Table 2.5 Diagnostic tests for the models in Table 2.4

	BP	*CD*	*DM*	*JY*	*SF*
R_f	−0.95	−4.77	−4.70	−2.29	−4.15
	(0.34)	(0.00)	(0.00)	(0.02)	(0.00)
$Q_f(10)$	5.29	14.24	6.86	12.47	5.80
	(0.87)	(0.15)	(0.74)	(0.25)	(0.83)
$Q_f^2(10)$	8.09	9.35	4.08	17.40	10.12
	(0.62)	(0.50)	(0.94)	(0.07)	(0.43)
$Q_{f,m}(10)$	3.38	9.39	8.81	5.50	4.45
	(0.97)	(0.50)	(0.55)	(0.86)	(0.92)
R_m	1.24	1.38	1.49	1.84	1.36
	(0.22)	(0.17)	(0.14)	(0.07)	(0.17)
$Q_m(10)$	4.90	4.76	4.75	4.51	4.76
	(0.90)	(0.91)	(0.91)	(0.92)	(0.91)
$Q_m^2(10)$	2.94	2.87	3.03	3.29	2.97
	(0.98)	(0.98)	(0.98)	(0.97)	(0.98)

Notes: See notes to Table 2.2. Subscript f refers to futures spread, and subscript m to the market portfolio excess returns. $Q_{f,m}$ is the portmanteau statistic for the autocorrelation function for the cross-products of the standardized residuals of the two series.

risk) and bivariate (varying systematic risk) specifications are so similar.

The diagnostic test statistics for the standardized residuals from the bivariate analyses in Table 2.5 are also similar to those for the univariate analyses. There is an unexpectedly small number of runs of observations above the mean in the runs test for all spreads except BP. The portmanteau tests on the first ten lags do not detect this persistence, nor do the equivalent tests of the squared standardized residuals detect any remaining heteroscedasticity. Similarly, the autocorrelation function for the cross-products of the standardized residuals from the futures spreads and the market portfolio excess returns reveals no evidence of interaction that would suggest the need for a more complicated model of the joint evolution of the time series. The specification of constant correlation but time-varying covariance appears to be adequate.

An informal evaluation of the adequacy of the constant correlation model can be obtained by a comparison of the five sets of estimates for the market portfolio excess returns in Tables 2.4 and 2.5. Marked differences between the estimates for the market portfolio equations in the bivariate analyses of the various currencies would be cause for

concern. There do not seem to be important differences of this kind; the coefficient estimates for the market portfolio excess return equations in Table 2.4 do not change much when one currency is replaced by the next, nor are the p values for the diagnostic test statistics for the market portfolio returns sensitive to the choice of currency in Table 2.5. This robustness of the estimation of the market portfolio component is fortunate given the conceptual difficulty, discussed on p. 37, arising from the fact that the foreign currency spread position is not a component of net wealth. The results suggest that there is no serious difficulty with this particular empirical application of the bivariate model.

CONCLUSION

In this chapter we have provided new evidence about the temporal behaviour of risk premiums in foreign currency futures data. The evidence was found from the study of futures spreads in which a long position in one contract is offset by a short position in another contract for the same commodity. Very small but significant systematic or market risk estimates were found in three foreign currencies, indicating that the systematic risk in futures prices may be related to the length of maturity of the contract. Our results correspond to the shorter maturity futures contract's having the greater systematic risk. These conclusions hold for both the univariate and the bivariate sets of estimates and therefore are not sensitive to the specification of systematic risk as constant in the former or time varying in the latter. This suggests that the relationship between systematic risk and time to maturity of the futures contract is approximately linear over the range of times examined. The spread maintains a constant difference of 91 days between the times of maturity of the two contracts in the position: if the relationship is approximately linear it does not matter whether, for example, the shorter maturity contract has 91 days or only one day to expiration.

The conclusion that the shorter maturity contract has greater systematic risk should be qualified because it is not necessarily true that the analysis has controlled for all relevant factors. For example, it is apparent that the open interest and trading frequency of foreign currency futures contracts is much larger in the shorter-term than the longer-term contract. Until further more detailed evidence is obtained, our results suggest that the sensitivity of systematic risk to the time to maturity of the contract is slight enough for the relationship to have little quantitative economic importance.

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