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Hedging foreign currency portfolios

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Abstract

This paper investigates dynamic and portfolio effects in a multi-currency hedging problem which incorporates both risk-reduction and speculative components for the futures demand. We model the joint evolution of daily spot portfolio returns and log-differences of the corresponding futures prices in a trivariate GARCH system allowing time-varying covariability between all the components of the system. Hedging performance is evaluated from both a risk-minimization and a utility standpoint. Our results show that accounting for portfolio effects in constructing a multi-currency hedge leads to efficiency and utility gains.

1. Introduction

The bulk of research on hedging is concerned with single commodity hedges. However, when hedging *portfolios* of currencies with multiple currency futures, the risk-minimizing and/or optimal position to take in each contract must not only reflect its own covariance with the cash position, but also its degree of covariability with the other currency futures and the other cash denominations. Estimation procedures which measure hedge ratios for each hedging instrument in isolation tend to over-estimate the number of futures contracts required to hedge the cash position and lead to a suboptimal mix in the composition of the hedge portfolio. We consider a situation where an *N*-asset portfolio is hedged with an *N*-futures contract portfolio in a dynamic setting.

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Our empirical application uses a multivariate version of the GARCH methodology developed by Engle (1982) and Bollerslev (1986) to evaluate the significance of both dynamic and portfolio effects in the optimal hedge for a two-currency portfolio. We consider speculative as well as risk reduction motives for hedging. Furthermore, hedging effectiveness is assessed from both a statistical and a utility standpoint.

Explicit modelling of the time-variation in currency hedge ratios yields improvements in hedging efficiency. Constructing the hedge in a portfolio context results in significantly smaller positions in each futures contract compared to the situation where each component of the portfolio is hedged separately. Accounting for portfolio effects in our dynamic hedging strategy results in significant variance reduction and utility gains over and above those achieved by the dynamic hedging strategy applied to each component in isolation.

2. Literature background

Previous studies based on the seminal work on efficient portfolio selection by Markowitz (1952) measure the risk-minimizing hedge ratio by dividing the unconditional covariance between cash and futures price changes by the unconditional variance of changes in futures prices. This approach was first introduced to the commodity hedging literature by Johnson (1960) and Stein (1961), and subsequently was applied to the hedging of a variety of sources of risk, such as short- and long-term interest rate risk in Ederington (1979), of foreign exchange risk in Hill and Schneeweiss (1981), and of market-wide risk using stock-index futures in Figlewski (1984).

This classical hedging approach used an OLS procedure which assumes that the joint distribution of cash and futures price changes remains constant over time. However, a substantial body of evidence indicates that the covariance matrix of cash and futures is time-varying. Recent empirical work has focused on utilizing that dynamic covariance structure to derive time-varying hedge ratios.

For example, Cecchetti et al. (1988) proposed a hedging model for financial futures based on the univariate autoregressive conditional heteroscedastic (ARCH) model introduced by Engle (1982). They assumed that the conditional correlation between cash and futures prices is constant, and report significant evidence of time-variation in the optimal hedge ratios. Myers (1991), Baillie and Myers (1991), and Sephton (1993) used the bivariate generalized ARCH (GARCH) model of Bollerslev (1986) to capture the time-variation of hedge ratios for agricultural commodities and report improved hedging performance using this approach. Kroner and Claessens (1991) illustrate the usefulness of the GARCH technique for hedging exchange-rate risk associated with external debt. Kroner and Sultan (1993) apply the bivariate GARCH framework to minimize exchange-rate risk using currency futures. These studies offer strong evidence of time variation in hedge ratios and show that, in at least some cases, substantial improvements in hedging performance can be realized by following a dynamic hedging strategy.

In situations where the spot position consists of two or more assets and in which futures contracts exist for each of these assets, any covariance between futures will give rise to portfolio effects. Therefore, information on the degree of covariability between each pair of futures, in addition to the information on the cash-futures covariance, should be taken into account in the construction of the hedge portfolio. Hedges which take portfolio effects into account should be more efficient than hedges which do not.

Eaker and Grant (1987) and Lypny (1988) considered portfolio effects assuming that the hedge ratio for each hedging instrument remains constant over time. These studies report little or no evidence of portfolio effects. However, failure to incorporate time-variation in the relevant covariances could lead to a mismeasurement of the true portfolio effects. In addition, those studies just compared coefficients of risk reduction. A utility-based criterion of hedging effectiveness could result in a different conclusion concerning the importance of portfolio effects.

A paper by Glen and Jorion (1993) also can be interpreted in the context of portfolio effects. They consider, among other things, whether the addition of foreign currency forward contracts to 'cross-hedge' a predetermined position in international stocks and bonds improves portfolio efficiency. They do not find evidence of improved performance for this case in an unconditional environment. However, a hedging strategy which conditions on the forward premium (interest rate differential) does improve the performance of international portfolios.

The issue of whether or not international portfolios are efficient, and consequently, whether or not hedging of foreign currency positions is effective, could be motivated in the context of the potential mispricing which results from market imperfections which lead to heterogeneity of investment opportunities across countries. We do not explicitly address this issue (see, for example, Smith and Stulz, 1985).

Section 3 reviews the derivation of risk-minimizing and optimal hedge ratios in a situation where an N-asset portfolio is hedged with N-futures contracts. Section 4 outlines the econometric implementation that explicitly models time variation in the conditional covariance structure of returns to spot and futures positions. Data and descriptive statistics are described in Section 5 and the maximum likelihood estimates for the dynamic model are presented in Section 6. Hedging performance, evaluated from both risk reduction and welfare viewpoints, is assessed in Section 7. Some concluding comments are offered in Section 8.

3. Hedging a multi-currency cash position

Consider a two-period world in which a fund manager chooses to invest his initial endowment, W_{t-1}, in a portfolio consisting of N currencies. Let $S_{t-1} =$

$(S_{1t-1} \dots S_{nt-1})'$ be the vector of initial currency spot rates expressed as the domestic currency price of a unit of the foreign currency. Let $\theta_{t-1} = (\theta_{1t-1} \dots \theta_{nt-1})'$ be the vector of spot currency holdings [1].

In addition to spot currencies, the fund manager has access to currency futures markets. We assume the existence of futures markets for each of the currencies held in the portfolio. Let $F_{t-1} = (F_{1t-1} \dots F_{nt-1})'$ represent the currency futures price vector and $\gamma_{t-1} = (\gamma_{1t-1} \dots \gamma_{nt-1})'$ the vector of holdings of currency futures. Since futures positions involve no initial investment, the initial portfolio satisfies the following condition

$$P_{t-1} = S'_{t-1}\theta_{t-1}. \tag{1}$$

where P is the value or price of the spot portfolio.

At time t, the currency holdings are liquidated and the futures positions are closed-out. Denote p_t as the natural logarithm of the spot portfolio price, and $s_t = (s_{1t} \dots s_{nt})'$ and $f_t = (f_{1t} \dots f_{nt})'$ as the vectors of natural logarithms of spot and futures prices, respectively. The return realized on the hedged portfolio between time $t-1$ and t is equal to

$$HR_t = (p_t - p_{t-1}) - (f_t - f_{t-1})'\gamma_{t-1}. \tag{2}$$

The covariance structure for the spot portfolio return, $(p_t - p_{t-1})$, and log-difference of futures prices, $(f_t - f_{t-1})$, conditional on the information set available at time $t-1$, Ω_{t-1}, is expressed as a $(1+n) \times (1+n)$ matrix Σ which may be partitioned as

$$\Sigma | \Omega_{t-1} = \begin{bmatrix} \sigma_p^2 & \Sigma_{pf} \\ \Sigma'_{pf} & \Sigma_{ff} \end{bmatrix} \tag{3}$$

where σ_p^2 is the conditional variance of spot portfolio returns, Σ_{ff} is an $n \times n$ conditional covariance matrix of changes in futures prices, and Σ_{pf} is an $1 \times n$ vector of conditional covariances between the spot portfolio returns and changes in futures prices. Time subscripts on these components of the conditional covariance matrix Eq. (3) are henceforth omitted for notational convenience.

[1] Since we are dealing with financial assets, we take the investment decision as a given and our focus is on the agent's hedging decision. For commodity producers and processors, interactions may exist between the production and the hedging decision. This problem was identified by Johnson (1960) and Stein (1961). Anderson and Danthine (1981) examine the conditions under which the separation of real and hedging decisions is permissible. Benninga et al. (1985) examine the interactions between hedging opportunities and production in the context of an exporting firm exposed to both commodity price and exchange-rate risk. Paroush and Wolf (1992) consider the impact of input price uncertainty in this framework.

Our agent is assumed to be a mean-variance utility maximizer [2]. The agent's problem is to select the futures position, γ_{t-1}, so as to maximize end-of-period utility:

$$\underset{\gamma}{\mathrm{Max}}\left[E\left(HR_t | \Omega_{t-1}\right) - \tfrac{1}{2}\phi \mathrm{Var}\left(HR_t | \Omega_{t-1}\right)\right] \tag{4}$$

where ϕ is a positive constant corresponding to the agent's risk tolerance parameter and HR_t is defined in Eq. (2). The first-order condition for a maximum is

$$\phi\left[\Sigma_{ff}\gamma_{t-1} - \Sigma_{pf}\right] - \left(E(f_t | \Omega_{t-1}) - f_{t-1}\right) = 0. \tag{5}$$

Rearranging, we obtain the vector of optimal demand for futures contracts, conditional on information available at $t-1$,

$$\gamma^{*}_{t-1} = \Sigma_{ff}^{-1}\Sigma_{pf} - \frac{1}{\phi}\Sigma_{ff}^{-1}\left(E(f_t | \Omega_{t-1}) - f_{t-1}\right). \tag{6}$$

This solution is unique if and only if Σ_{ff} is non-singular.

The optimal demand equation given by Eq. (6) consists of two terms. The first term is the pure-hedging term, more commonly referred to as the risk-minimizing hedge position. The second term may be seen as a speculative demand component which reflects the mean-variance trade-off inherent in uncovered futures positions in addition to the agent's attitude towards risk. Henceforth, on occasion we use the term optimal hedge to refer to the optimal demand for futures, given by Eq. (6), which includes both the risk-minimizing hedging component and the speculative demand component.

If futures prices follow a martingale, $E(f_t | \Omega_{t-1}) - f_{t-1} = 0$, then Eq. (6) reduces to

$$\gamma^{*}_{t-1} = \Sigma_{ff}^{-1}\Sigma_{pf}. \tag{7}$$

That is, in this framework, the martingale assumption for futures prices is a sufficient condition for the mean-variance optimal hedge and the risk-minimizing hedge to coincide. Under more general conditions, the risk-minimizing hedge is not consistent with utility maximization since an agent would optimally elect to

[2] This assumption is much less restrictive in our conditional environment than it would be in an unconditional version of the model. An explicitly multiperiod objective is provided in the consumption-based approach explored by Stulz (1984), Ho (1984), and Adler and Detemple (1988). With time-additive preferences, a multiperiod objective may be expressed as a sequence of one-period choices which are given by the rebalancing decision inherent in our conditional hedging strategy.

increase or reduce futures holdings, relative to the quantities dictated by Eq. (7), for mean-variance efficiency reasons.

The vector of conditional optimal futures demands Eq. (6), or conditional risk minimizing futures demands Eq. (7), changes over time as new information arrives to the market. Eqs. (6) and (7) generalize the typical hedging model to the case where a multi-currency portfolio is hedged with many different futures contracts. To the extent that the futures used in the hedge portfolio are correlated, Eqs. (6) and (7) will internalize portfolio effects and will yield a different hedging mix from the one obtained when each hedge ratio is measured in isolation. Recognizing portfolio effects in the construction of the optimal hedge Eq. (6), or the risk-minimizing hedge Eq. (7), should result in a more efficient hedge for the predetermined spot portfolio.

4. Econometric specification

In order to implement the hedging estimation procedure, we need to model the first two moments of the joint distribution of the spot portfolio value and the futures prices. We postulate a log linear process for both which consists of an expected component conditional on the information available at time $t-1$ and a random component which has an expected value of zero and is serially uncorrelated. That is, the linear rational expectations representation of the natural logarithms of the spot portfolio value and currency futures prices are

$$p_t = E(p_t | \Omega_{t-1}) + \epsilon_{pt} \tag{8}$$

and

$$f_t = E(f_t | \Omega_{t-1}) + \epsilon_{ft} \tag{9}$$

respectively. The prediction errors, ϵ_{it}, are assumed to be serially uncorrelated but their conditional covariances may change over time in response to shocks to the economy. The information set, Ω_{t-1}, available to all agents at the beginning of the period may contain a constant, lagged prices, and other weakly exogenous variables. Given the bulk of evidence for financial series, and for currency prices in particular, we adopt the multivariate GARCH (1,1) process to model time-dependence in the second moments.

Our econometric specification of the first two conditional moments is as follows

$$\Delta y_t = \mu + \delta(f_{t-1} - s_{t-1}) + \epsilon_t \tag{10}$$

$$\varepsilon_t | \Omega_{t-1} \sim N(0, H_t) \tag{11}$$

$$H_t = C'C + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + B'H_{t-1}B \tag{12}$$

where $y_t = (p_t, f_{1t}, \ldots, f_{nt})'$, μ is a vector of constants; $(f_{t-1} - s_{t-1})$ is the futures premium, H_t is the conditional covariance matrix; and $N(0, H_t)$ is the multivariate conditional normal density.

The conditional mean specification given in Eq. (10) includes a constant as well as the futures premium. The latter is $(f_{p,t-1} - p_{t-1})$, where $f_{p,t-1}$ is the natural logarithm of $F'_{t-1}\ \theta$, for the spot portfolio's conditional mean equation and $(f_{i,t-1} - s_{i,t-1})$ for the conditional mean equation associated with each futures position. Under an assumption of nonstochastic interest rates, a futures premium will equal the forward premium which, maintaining covered interest rate parity, will equal the differential between the domestic and the corresponding foreign interest rate over the remaining life of the contract. The potential importance of this time-varying component of the conditional mean can be motivated from a cost-of-carry relationship which links the foreign currency spot and futures prices [3].

Various parameterizations of the multivariate GARCH process have been proposed. Bollerslev (1986), Baillie and Bollerslev (1990), and Kroner and Sultan (1993) adopt the constant conditional correlation parameterization. We adopt the positive definite representation introduced by Engle and Kroner (1994), henceforth BEKK, to permit time-variation in the conditional correlations as well as the conditional variances. In addition, the BEKK functional form ensures that the conditional covariance matrix is positive definite so that conditional variances are always non-negative. We estimate the BEKK model with full, symmetric A and B matrices [4].

5. The data and descriptive statistics

We use daily spot and futures prices for the Deutsche Mark (DM), the Swiss Franc (SF), and the Japanese Yen (JY) from February 27, 1985 to February 28, 1990. Spot prices were obtained from Reuters. Futures prices were obtained from the 1985 version of the data file provided by the Center for Research in Futures Markets (CRFM) of the University of Chicago, and from Reuters in subsequent years. We chose this sample period in order to circumvent potential estimation problems caused by the existence of daily price limits imposed by the Chicago Mercantile Exchange's International Monetary Market on its foreign currency futures contracts. These limits were removed on February 22, 1985. Our empirical

[3] Kroner and Sultan (1993) use the negative of the futures premium in the conditional mean as an error correction term. This is motivated by the fact that, even though the futures price must equal the spot price on the maturity date of the contract, they were unable to reject that the spot and futures prices are cointegrated.

[4] See Bollerslev et al. (1988), and Engle and Kroner (1994) for alternative multivariate representations.

investigation is performed on the nearby contract for reasons of liquidity. Our futures positions are assumed to be rolled-over to the next contract maturity one day prior to the nearby contract's maturity date.

In order to evaluate the importance of the level of correlation between currencies in generating portfolio effects, we consider two alternative spot portfolios each of which represent a passive index of two currencies. That is, each portfolio consists of a fixed number of units of two of our sample currencies, with equal value weights in each currency set on the first day of our sample, February 27, 1985. Using a passive index eliminates quantity risk from the hedger's problem, since the composition of each portfolio is held fixed at the base-day weights, which are $\theta_1 = 1.66$ Deutsche Marks and $\theta_2 = 1.4176$ Swiss Francs for the DM–SF spot portfolio and $\theta_1 = 1.66$ Deutsche Marks and $\theta_2 = 1.2943$ Japanese Yen for the alternative DM–JY spot portfolio. Thus, the hedger will only rebalance her futures portfolio to hedge against changes in the underlying portfolio's value due to changes in price and will never have to adjust her futures position because the number of units of each currency held in the spot portfolio has changed.

In Table 1, we report descriptive statistics for the two spot currency portfolios as well as for futures contracts on the three underlying currencies. These statistics are computed on the spot portfolio returns and the log-differenced futures price series. The futures series exhibit significant departures from normality and appear to be leptokurtic. ARCH(24) test statistics indicate the presence of volatility persistence in the four series. This conditional heteroscedasticity is explicitly

Table 1
Descriptive statistics on the log-differenced prices multiplied by 100 for the two spot portfolios as well as Swiss Franc, Deutsche Mark, and Japanese Yen futures between March 1, 1985 and February 28, 1990. The DM–SF portfolio consists of 1.66 Deutsche Marks and 1.4176 Swiss Francs while the DM–JY portfolio consists of 1.66 Deutsche Marks and 1.2943 Japanese Yen

Statistic	DM–SF portfolio	DM futures	SF futures	DM–JY portfolio	JY futures
Mean	0.0419	0.0478	0.0480	0.0377	0.0375
Variance	0.4828	0.4875	0.6046	0.3563	0.3937
Kurtosis	2.6183	3.1650	3.5312	2.9734	4.6321
Skewness	0.0845	0.2704	0.3348	0.0169	0.0696
Range	6.2518	7.6288	9.1049	5.5525	7.2463
Minimum	−3.2141	−3.2638	−3.5688	−3.2471	−4.1325
Maximum	3.0377	4.3649	5.5361	2.3054	3.1138
Nobs	1264	1264	1264	1264	1264
Bera–Jarque	1.5045	15.3998	23.6149	0.0603	1.0197
(*p*-value)	0.4710	0.0000	0.0000	0.9700	0.6010
Q(24)	31.0255	35.5525	25.5095	32.8611	25.0735
(*p*-value)	0.1530	0.0606	0.3785	0.1070	0.4018
ARCH(24)	71.5346	53.5157	88.7565	53.6053	57.8509
(*p*-value)	0.0000	0.0000	0.0000	0.0001	0.0001

Table 2
Unconditional correlation matrix between the two spot currency portfolios and their respective components, i.e. the Deutsche Mark and the Swiss Franc futures and the Deutsche Mark and the Japanese Yen futures

	Portfolio	DM	SF
Portfolio	—	0.9363	0.9358
DM	0.9056	—	0.9516
JY	0.8856	0.7844	—

modelled in the next section. Table 2 contains the unconditional correlation matrix for the two series of spot portfolio returns and the log differences of their respective component currency futures price series.

6. Model estimates with time-varying covariance structures

6.1. Multivariate estimates

In Table 3, we report the parameter and standard error estimates for the full trivariate BEKK representation of the GARCH(1,1) model described in Eqs. (10)–(12) for the two alternative currency portfolios. Our standard errors are robust to departures from the maintained assumption of conditional normality (Bollerslev and Wooldridge, 1992). Note that the futures premium is statistically significant for every futures equation which implies that these futures prices do not follow a martingale [5]. An implication of this result for hedging is that hedge portfolios will include a speculative component in addition to the risk-minimizing position. Hence, the risk-minimizing hedge and the utility-maximizing (i.e. optimal) hedge imply two different futures positions.

The off-diagonal terms in the A and B matrices in our model of the conditional covariance matrix reveal significant cross-equation influences supporting our use of the full BEKK structure. In Table 4, we test parameter restrictions on the A and B matrices implied by the unconditional model where all elements of A and B are equal to zero, and to the diagonal BEKK model in which the off-diagonal elements of the A and B matrices are set to zero. Both sets of restrictions are rejected in favour of the full BEKK model which will be retained as the model underlying our multivariate conditional hedging strategy. These test results are only reported for

[5] For instance, McCurdy and Morgan (1992) report evidence of time-varying risk premiums in foreign exchange futures markets.

Table 3
Quasi-maximum likelihood estimates for the full trivariate BEKK representation of the GARCH(1,1) model. We model the conditional mean as $\Delta y = \mu + \delta(f_{t-1} - s_{t-1}) + \varepsilon_t$, where $\varepsilon_t | \Omega_t \sim N(0, H_t)$, and the conditional covariance as $H_t = C'C + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + B'H_{t-1}B$. Standard errors are robust to departures from the maintained assumption of conditional normality (Bollerslev and Wooldridge, 1992)

Parameter	DM–SF portfolio		Parameter	DM–JY portfolio	
	Estimate	Robust std. errors		Estimate	Robust std. errors
Mean			Mean		
μ_P	0.0438	0.0285	μ_P	0.0181	0.0266
δ_P	−0.0071	0.0437	δ_P	0.0273	0.0481
μ_{DM}	0.1417	0.0305	μ_{DM}	0.1286	0.0357
δ_{DM}	−0.2648	0.0460	δ_{DM}	−0.2658	0.0614
μ_{SF}	0.1505	0.0351	μ_{JY}	0.0968	0.0308
δ_{SF}	−0.2444	0.0468	δ_{JY}	−0.1736	0.0517
Covariance structure			Covariance structure		
$c_{P,P}$	0.1111	0.0413	$c_{P,P}$	0.0897	0.0679
$c_{P,DM}$	0.0811	0.0323	$c_{P,DM}$	0.0289	0.0185
$c_{P,SF}$	0.0736	0.0366	$c_{P,JY}$	0.0478	0.0259
$c_{DM,DM}$	0.2022	0.0621	$c_{DM,DM}$	0.1553	0.0249
$c_{DM,SF}$	0.1691	0.0329	$c_{DM,JY}$	0.1153	0.0182
$c_{SF,SF}$	0.2301	0.0366	$c_{JY,JY}$	0.1779	0.0249
$a_{P,P}$	0.4875	0.0946	$a_{P,P}$	0.5232	0.1189
$a_{P,DM}$	−0.1329	0.0480	$a_{P,DM}$	−0.1188	0.0632
$a_{P,SF}$	−0.1366	0.0737	$a_{P,JY}$	−0.1442	0.0622
$a_{DM,DM}$	0.3443	0.0511	$a_{DM,DM}$	0.3337	0.0456
$a_{DM,SF}$	0.0043	0.0286	$a_{DM,JY}$	0.0315	0.0336
$a_{SF,SF}$	0.3214	0.0528	$a_{JY,JY}$	0.3589	0.0528
$b_{P,P}$	0.5403	0.0903	$b_{P,P}$	0.3492	0.0518
$b_{P,DM}$	0.2007	0.0643	$b_{P,DM}$	0.2911	0.0313
$b_{P,SF}$	0.2022	0.0621	$b_{P,JY}$	0.2935	0.0202
$b_{DM,DM}$	0.8001	0.0635	$b_{DM,DM}$	0.7965	0.0295
$b_{DM,SF}$	−0.0934	0.2999	$b_{DM,JY}$	−0.1467	0.2056
$b_{SF,SF}$	0.8079	0.0474	$b_{JY,JY}$	0.7786	0.0264
Log-likelihood		2521.87			2137.79

the DM–SF portfolio in the interest of space since the DM–JY test results are qualitatively similar.

Table 5 reports Ljung–Box–Pierce portmanteau statistics on the squared and cross standardized residuals for the models reported in Table 3. These residual-based diagnostic tests suggest that, with the possible exception of the DM conditional variance in the DM–SF case, there is no neglected structure in the conditional variance–covariance parameterization. In other words, these residual-based diagnostic tests lend support to the maintained specifications reported in Table 3.

Table 4
Log-likelihood estimates and tests of parameter restrictions for the DM–SF portfolio case. Restrictions are imposed on parameters of matrices A and B from the conditional covariance structure. The critical values for the $\chi^2(6)$ and the $\chi^2(12)$ statistics at the 5% and the 1% confidence level are 12.6, 16.8, 21.0, and 26.2, respectively. In all cases, the models implied by the parameter restrictions are rejected in favour of the unrestricted model

	Log-likelihood	Number of parameters
Trivariate model		
No GARCH	2401.96	12
Diag. A and B	2489.62	18
Full A and B	2521.87	24
Bivariate DM		
No GARCH	1271.2	7
GARCH(1,1)	1371.55	13
Bivariate SF		
No GARCH	911.94	7
GARCH(1,1)	980.93	13

6.2. *Bivariate estimates*

In order to evaluate the magnitude of portfolio effects, we compare the performance of portfolio-effects versus no-portfolio-effects strategies. The first consists of hedging the spot portfolios using the hedge ratios derived from the estimates of the multivariate system described in Section 4. The second uses hedge ratios derived from the estimates of two separate bivariate systems, each of which involves one of the currencies and its corresponding futures contract. The second approach in effect ignores the covariance between the components of the spot position as well as the covariance between the futures contracts used to implement the hedge, and hence does not take portfolio effects into account.

The implementation of this no-portfolio-effects hedging strategy requires the estimation of two separate bivariate systems similar to the multivariate structure

Table 5
Residual-based diagnostic tests for neglected structure in the conditional variance–covariance parameterization. These Ljung–Box–Pierce portmanteau statistics are asymptotically distributed as $\chi^2(5)$ with critical values of 11.07 and 15.09 at the 5% and the 1% levels of significance, respectively

	DM–SF portfolio				DM–JY portfolio		
	portfolio	DM	SF		portfolio	DM	JY
Portfolio	3.56			portfolio	4.36		
DM	0.78	13.94		DM	6.65	2.60	
SF	3.55	6.32	5.13	JY	4.91	0.90	10.38

described in Section 4. Again, in the interest of space, coefficient and standard error estimates for these bivariate models are not reported but we can see, from Table 4, that the GARCH effects for these bivariate systems are highly significant.

7. Futures demands and hedging performance

We evaluate the performance of four separate hedging strategies for both risk-minimizing and utility-maximizing objectives. In assessing the risk reduction potential of the four competing hedging models considered in this paper, we adopt the traditional approach which compares the percentage reduction in the unconditional variance of the spot position achieved with each hedge [6].

The first strategy is referred to as the dynamic portfolio strategy because it rests on time-varying hedge ratios calculated from the multivariate system estimates of Eqs. (10)–(12) reported in Table 3. In particular, optimal and risk-minimizing futures demands per unit of the underlying spot position are calculated for the portfolio models as in Eqs. (6) and (7), respectively.

Secondly, we consider a dynamic no-portfolio-effects strategy which relies on time-varying hedge ratios estimated in the context of bivariate systems, thereby ignoring portfolio effects. The risk-minimizing hedge ratios for this strategy are obtained by dividing the conditional covariance between a spot currency price and its futures price by the conditional variance of its futures price, that is, h_{sft}/h_{ft}. Analogously to the comparison of Eqs. (6) and (7) for the portfolio-effects case, the optimal futures demand for this dynamic no-portfolio-effects strategy is calculated by subtracting from h_{sft}/h_{ft} a term equal to the expected log difference of the futures price divided by the product of its conditional variance and the agent's risk aversion coefficient ϕ. As discussed below, these bivariate time-varying hedge ratios are used, in conjunction with the value weights of each currency in the spot portfolio, to construct the hedged return series for the dynamic no-portfolio-effects strategy.

The third approach is called the static portfolio-effects strategy because it assumes that currency return distributions are time invariant. Hedge ratios for this model are obtained by multiple ordinary least squares regression (OLS).

The fourth approach assumes that distributions are constant over time and also ignores portfolio effects. Hedge ratios for this model are obtained by regressing the individual components of the spot portfolio against their corresponding futures contract. This is the static no-portfolio-effects strategy.

[6] Out-of-sample comparisons could also be calculated. However, it is unlikely that they would change the qualitative rankings of the alternative hedging strategies. Since we compare four alternative strategies for both risk-minimizing and utility-maximizing objectives, we restrict our analyses to the traditional metric of performance.

Table 6 presents descriptive statistics for the various futures demands (hedge ratios) associated with the above four hedging strategies for both the risk-minimizing and the utility-maximizing objectives. Note that the sign taken on by the futures demand or hedge ratios (futures demands per unit of the underlying spot position) estimates indicates whether the hedger should go short or long in order to implement her hedge. For instance, when the estimate for the hedge ratio is positive, the hedger will short futures contracts, and when the hedge ratio is negative, the hedger will initiate a long position in futures. By virtue of the fact that futures contracts are found in zero net-supply in the economy, there is no need to impose a restriction on the sign of the futures position. The only potentially binding constraint facing hedgers is the position limit imposed by the IMM. Since these limits are typically not very restrictive in the present case where the trader has a position in the underlying asset, we ignore them in these analyses.

In Table 7 we present descriptive statistics on the time series of hedged portfolio returns for each hedging strategy, along with risk reduction measures.

Table 6
Descriptive statistics for hedge ratios from risk-minimizing and utility-maximizing objectives. For each objective, we consider four alternative hedging strategies: static versus dynamic hedges with portfolio effects; and static versus dynamic hedges with no-portfolio effects. ϕ represents the hedger's risk tolerance parameter

	DM–SF portfolio				DM–JY portfolio		
		Mean	Var			Mean	Var
Risk-min hedge ratios							
Portfolio effects							
DM	dynamic	0.4584	0.0382	DM	dynamic	0.4789	0.0061
	static	0.4834			static	0.4685	
SF	dynamic	0.4559	0.0302	JY	dynamic	0.4246	0.0094
	static	0.4232			static	0.4338	
No-portfolio effects							
DM	dynamic	0.9062	0.0043	DM	dynamic	0.9062	0.0043
	static	0.9034			static	0.9034	
SF	dynamic	0.9093	0.0036	JY	dynamic	0.9002	0.0046
	static	0.8971			static	0.8975	
Utility-max hedge ratios ($\phi = 2$)							
Portfolio effects							
DM	dynamic	0.3546	0.1916	DM	dynamic	0.4538	0.0392
	static	0.3796			static	0.4434	
SF	dynamic	0.5013	0.1555	JY	dynamic	0.3937	0.0336
	static	0.4686			static	0.4334	
No-portfolio effects							
DM	dynamic	0.8548	0.0103	DM	dynamic	0.8548	0.0103
	static	0.8519			static	0.8519	
SF	dynamic	0.8538	0.0112	JY	dynamic	0.8448	0.0075
	static	0.8416			static	0.8420	

Table 7
Descriptive statistics for the portfolio returns realized by risk-minimizing and utility-maximizing objectives. For each objective we report returns from four alternative hedging strategies: static versus dynamic hedges with portfolio effects; and static versus dynamic hedges with no-portfolio effects. The incremental variance reduction (I.V.R.) associated with the static hedging strategy is measured against the variance of the unhedged portfolio while that for the dynamic hedging strategy is measured relative to the variance of the static hedge. ϕ represents the hedger's risk tolerance parameter

	DM–SF portfolio			DM–JY portfolio		
	Mean	Var	I.V.R.	Mean	Var	I.V.R.
Risk-min returns						
Portfolio effects						
Unhedged	0.0419	0.4828	0.00%	0.0378	0.3561	0.00%
Static	−0.0015	0.0492	89.81%	−0.0009	0.0357	89.98%
Dynamic	−0.0022	0.0471	4.27%	−0.0019	0.0354	0.84%
No-portfolio effects						
Unhedged	0.0419	0.4828	0.00%	0.0378	0.3561	0.00%
Static	−0.0012	0.0493	89.79%	−0.0008	0.0355	90.03%
Dynamic	−0.0012	0.0480	2.64%	−0.0001	0.0343	3.38%
Utility-max returns ($\phi = 2$)						
Portfolio effects						
Unhedged	0.0419	0.4828	0.00%	0.0378	0.3561	0.00%
Static	0.0012	0.0509	89.46%	0.0015	0.0369	89.64%
Dynamic	0.0202	0.0521	−2.36%	0.0107	0.0394	−6.78%
No-portfolio effects						
Unhedged	0.0419	0.4828	0.00%	0.0378	0.3561	0.00%
Static	0.0013	0.0510	89.44%	0.0015	0.0367	89.69%
Dynamic	0.0060	0.0485	4.90%	0.0048	0.0333	9.26%

Statistics for the risk-minimizing objective are provided in the first panel of Table 7 and those for the utility-maximizing objective are presented in the second panel of Table 7.

7.1. Hedging performance: risk-minimizing objective

In this subsection, we investigate the risk-reduction potential of the four hedging strategies described above and measure portfolio effects inherent in our multivariate hedge.

7.1.1. Evaluation of dynamic versus static hedging efficiency

Note from the first panel of Table 7 that up to 89.81% of the DM–SF spot portfolio's variance can be eliminated by following the static risk-minimizing strategy, and that 4.27% of the remaining variance can be eliminated by using the dynamic strategy. For the DM–JY portfolio, the static risk-minimizing strategy posts a slightly higher variance reduction performance, 89.97%, while the incremental variance reduction associated with the dynamic approach is slightly less than 85 basis points. Our efficiency comparisons between the dynamic and the static models confirm the evidence reported in earlier studies concerning the benefits of the dynamic approach for risk-reduction objectives.

7.1.2. Evaluation of portfolio effects

One of the main issues addressed in this paper is whether portfolio effects matter in the construction of a hedge. In order to assess the contribution of the portfolio approach to hedging, we consider a strategy in which portfolio effects are not taken into account. In this strategy, both components of the portfolio are hedged in isolation by taking the futures positions dictated by the bivariate hedging (dynamic or static) model. This procedure yields one hedged return time series per currency. These two series are then aggregated using the value weights $(S_{i,t-1}\theta_i/P_{t-1})$ of each currency in the spot portfolio. By construction, this strategy ignores the covariance between the components of the spot portfolio as well as the covariance between the two futures instruments. We measure the significance of portfolio effects in two ways. First, we compare the size of the futures position which would be required under the portfolio-effects and the no-portfolio-effects cases. Second, we compare the risk-reduction potential of both strategies.

Recall that descriptive statistics for the hedge ratios of both the static and dynamic versions of the risk-minimizing objective were reported in the first panel of Table 6. Note that the no-portfolio-effects strategies entail futures positions which are about twice as high as the positions that are necessary under the portfolio model. Consider for instance the dynamic minimum-risk no-portfolio-effects case when the underlying portfolio consists of Deutsche Marks and Swiss Francs. In this case, one would short 0.9062 DM futures per unit of spot DM and 0.9093 SF futures per unit of spot SF, on average. Contrastingly, by hedging the cash portfolio as a whole, the average position for the DM and the SF futures would fall to 0.4584 and 0.4559, respectively. Similar results are obtained for the DM–JY portfolio. We also observe that while hedge ratios for the portfolio model are about half the size of their no-portfolio counterparts, they also exhibit significantly more variance than when portfolio effects are ignored. This increase in variance is illustrated in Figs. 1 and 2 which show the time-series of DM and JY hedge ratios associated with the portfolio-effects and the no-portfolio-effects cases.

We now proceed to efficiency comparisons between the dynamic portfolio-effects and no-portfolio-effects models. From Table 7, we learn that models which account for portfolio effects are not necessarily more efficient at minimizing risk than those which ignore these effects. In the case of the DM–SF spot position, the risk-minimizing allocation obtained by accounting for portfolio effects has a variance of 0.0471 compared to 0.0480 for the situation where portfolio effects are ignored. This represents an incremental risk reduction equal to 1.88% per day. For

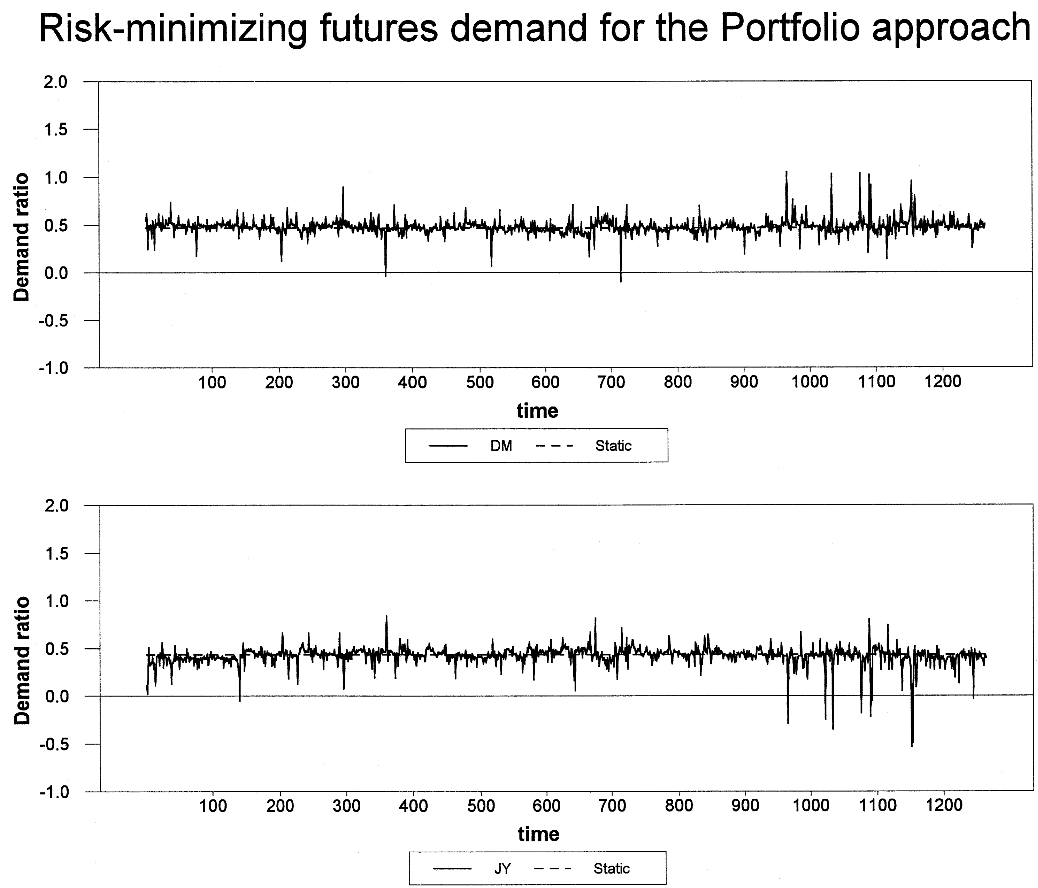

Fig. 1. Risk-minimizing futures demand for the portfolio approach.

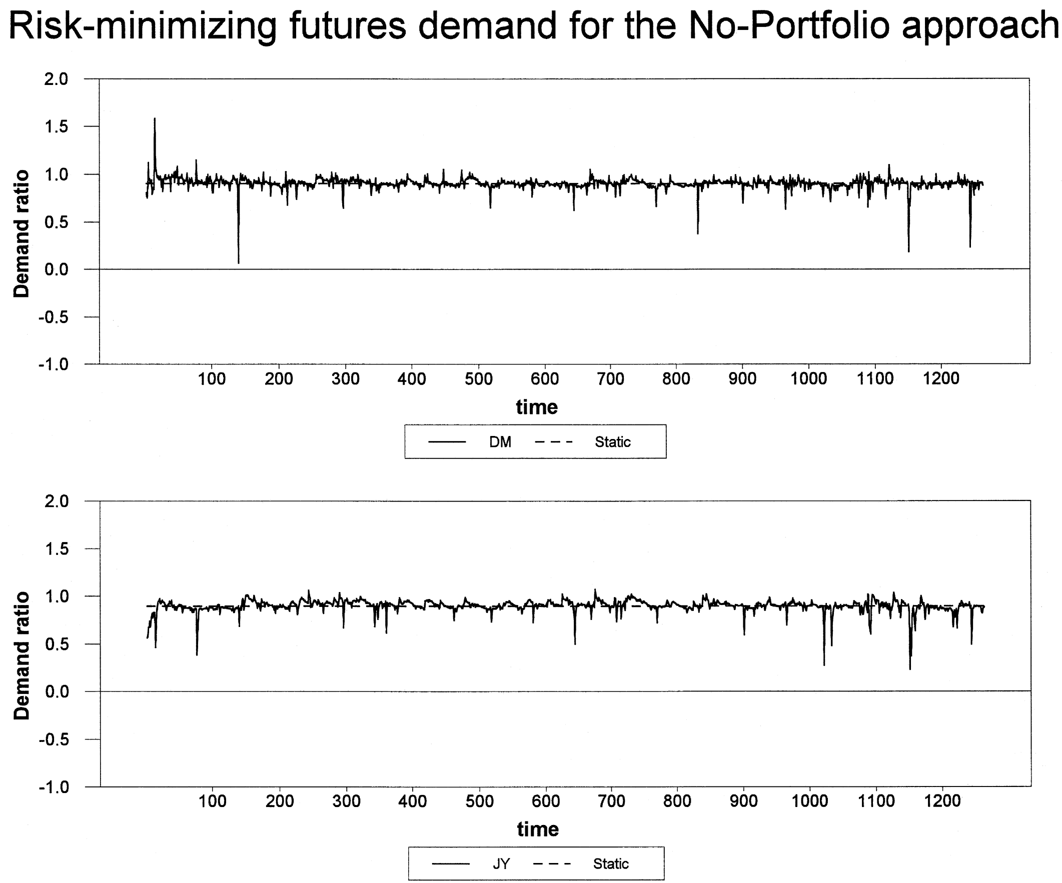

Fig. 2. Risk-minimizing futures demand for the no-portfolio approach.

the DM–JY portfolio, hedging each component of the portfolio in isolation does not result in an increase in risk but to a slight reduction from 0.0354 for the portfolio approach to 0.0343 for the isolation case. We now turn to utility-based comparisons of the four hedging strategies considered above.

7.2. Hedging performance: utility-maximizing objective

Based on the risk-reduction criterion, our evidence indicates that our dynamic portfolio hedging strategy is superior to its static counterpart and that hedging the currency positions as a portfolio may yield performance improvements over the isolation strategy. Although the risk-reduction criterion may constitute a useful benchmark for comparisons between alternative static hedging models, it has two principal drawbacks when used in comparisons between static and dynamic strategies. First, dynamic models involve significantly more transactions costs than their static counterparts because they entail potential daily rebalancings of the futures positions, whereas the static models require at most quarterly rollovers coinciding with the nearby contract maturity. Second, since the futures returns are not martingales, the risk-minimization criterion fails to capture the risk-return trade-off inherent in any hedge, whether static or dynamic.

In order to address the risk-return trade-off, we examine hedging performance from a utility standpoint as in Cecchetti et al. (1988), Kroner and Sultan (1993), and Sephton (1993). We represent the agent's preferences using the quadratic utility function presented in Eq. (4). Under the dynamic models, we compute the agent's utility for the entire sample period by averaging the daily utility levels which are based on the hedged portfolio's time-varying mean and variance of return. Average utility levels for the static models are computed from the portfolio's unconditional mean and variance of return.

7.2.1. Evaluation of dynamic versus static hedging efficiency

In the second panel of Table 7, we show that from the standpoint of an agent with a risk aversion coefficient $\phi = 2$, the dynamic optimal hedge is slightly more risky than its static counterpart. The static hedge eliminates 89.46% of the DM–SF spot portfolio's variance while the dynamic hedge eliminates 89.21%. However, the incremental risk associated with the dynamic hedge is compensated by a twentyfold increase in mean returns. We make the same observation for the DM–JY portfolio but the eightfold increase in mean returns is slightly more modest.

In Table 8, we perform utility comparisons associated with some hedging strategies for the DM–SF spot portfolio for a range of different risk aversion coefficients ϕ from 0.5 to 4. The utility levels associated with the unhedged position are presented in the second column of the table in order to provide a benchmark for comparisons. Note that the utility-maximizing dynamic strategy is always preferred to its static counterpart. For example, when $\phi = 2$, the dynamic

Table 8
Utility comparisons between dynamic hedging strategies, with and without portfolio effects, plus a static hedging strategy which includes portfolio effects for the DM–SF portfolio. Preferences are represented by the quadratic utility function Eq. (4) in which ϕ represents the hedger's risk tolerance parameter

(ϕ)	Unhedged	Dynamic		Static
		portfolio effects	no-portfolio effects	portfolio effects
0.5	−0.08246	0.03275	−0.00138	−0.00920
1.0	−0.20678	−0.00051	−0.01765	−0.02305
1.5	−0.33109	−0.01861	−0.03030	−0.03542
2.0	−0.45541	−0.03291	−0.04205	−0.04742
2.5	−0.57973	−0.04570	−0.05344	−0.05927
3.0	−0.70404	−0.05733	−0.06464	−0.07105
3.5	−0.82836	−0.06933	−0.07574	−0.08279
4.0	−0.95268	−0.08066	−0.08678	−0.09456

strategy yields a utility gain of 0.01451 relative to the static strategy. Although not reported in Table 8 for reasons of space, the corresponding utility gain for the DM–JY spot portfolio is 0.00685.

7.2.2. Evaluation of portfolio effects

Descriptive statistics for the utility-maximizing hedge ratios associated with the portfolio-effects and the no-portfolio-effects strategies are presented in the second panel of Table 6. We assume a coefficient of risk aversion $\phi = 2$. Considering the case of the DM–SF spot portfolio, an agent pursuing the portfolio strategy would short 0.3546 DM futures per unit of spot DM and 0.5013 SF futures per unit of spot SF, on average. The corresponding futures positions for the agent hedging each currency in isolation would be 0.8548 DM and 0.8538 SF. As we observed earlier for the minimum-risk strategies, the utility-maximizing futures positions are almost half as large when the spot position is hedged as a portfolio than when each of its components are hedged separately. We also note that optimal futures demand ratios (utility-maximizing hedge ratios) exhibit much more variability in the portfolio context than in the isolation approach as was the case for their risk-minimizing counterparts. This phenomenon is illustrated in Figs. 3 and 4 which also reveal a very distinct contract maturity pattern in the futures demand ratios which does not show up in Figs. 1 and 2 which pertain to risk-minimizing allocations as opposed to utility-maximizing ones.

The second panel of Table 7 illustrates the risk-return trade-off associated with the utility-maximizing objective for a risk aversion coefficient $\phi = 2$. For example, for the DM–SF spot portfolio, when we compare the risk reduction potential of the dynamic no-portfolio-effects case to that of the dynamic portfolio-effects case, we note that the latter strategy entails more risk (variance of 0.0521) than the first one (variance of 0.0485). However, the amount of risk is only one of the two

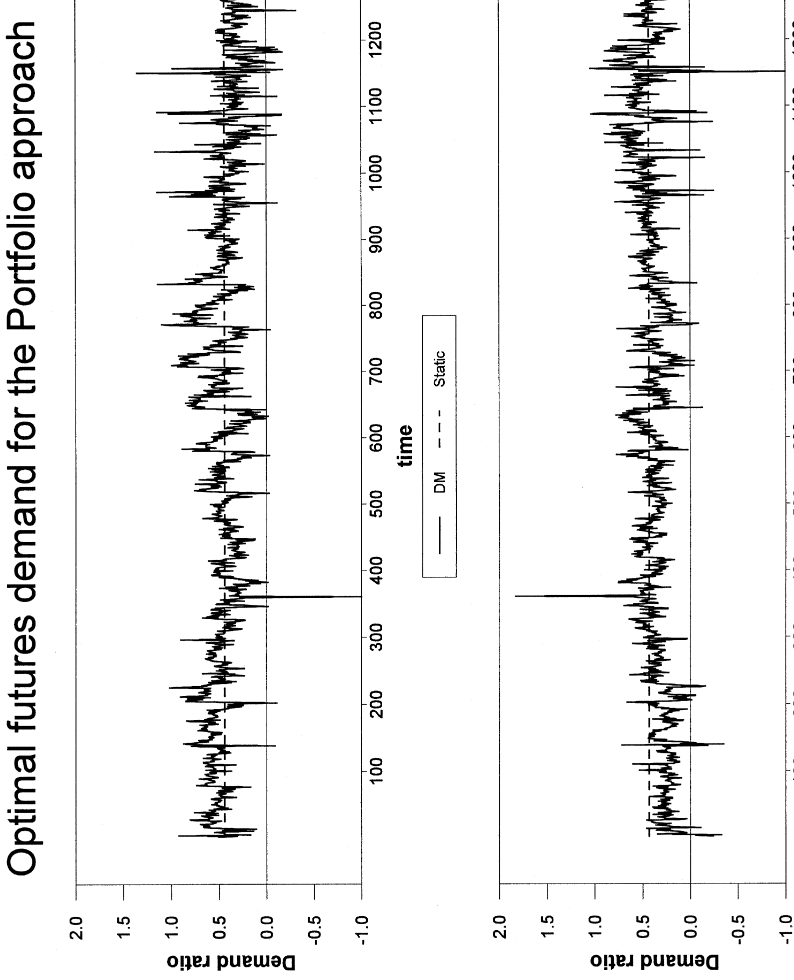

Fig. 3. Optimal futures demand for the portfolio approach.

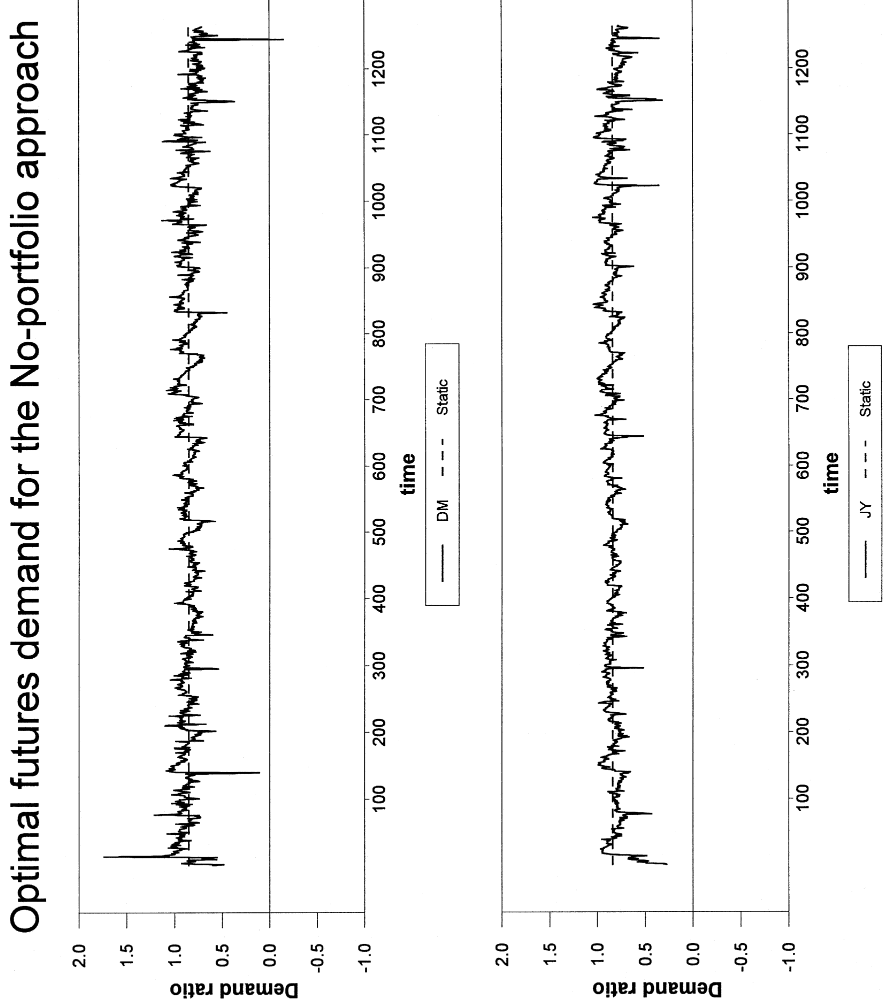

Fig. 4. Optimal futures demand for the no-portfolio approach.

dimensions entering the utility-maximizer's decision. The incremental risk borne by an agent pursuing the dynamic portfolio strategy would have been rewarded by almost three and a half times the return realized by someone pursuing the no-portfolio strategy. For the DM–JY portfolio, the incremental reward is about two and a half times. The relatively smaller pay-back (in terms of extra return per unit risk) to taking account of portfolio-effects for the DM–JY versus the DM–SF case reflects the lower correlation between the DM and the JY than between the DM and the SF. Nevertheless, there are significant portfolio effects associated with hedging either of our alternative spot portfolios. That is, although the portfolio approach was not always superior to the no-portfolio-effects strategy when our goal is to minimize variance (the first panel of Table 7), the portfolio approach is always the dominant strategy when we seek to maximize utility.

In Table 8, we present utility comparisons, for a range of ϕ, between our dynamic portfolio model and its no-portfolio counterpart for the DM–SF spot portfolio. Comparing columns 3 and 4, we see that from the perspective of the utility maximizer, the portfolio approach is always preferred to the no-portfolio model. These findings indicate that portfolio effects have a significant impact on agent's utility levels and that hedging the components of a portfolio in isolation is a suboptimal practice.

7.3. Transactions costs

Dynamic models are potentially more costly to implement than static models. One way to assess the size of the utility differences across hedging strategies is to ask whether they could be explained by average transactions costs. Since Eq. (4) expresses average daily utils in terms of certainty equivalent rates of return, we can subtract from the utils reported in Table 8 an approximate daily transactions cost measured as a percentage of the face value of the futures positions. For instance, in Table 8, if we assume that the round trip cost of trading one currency futures contract is $5 for an institutional investor, which represents a 0.005% cost for a contract with face value of $100,000, the average daily utility net of transactions costs achieved by a utility-maximizing agent (with $\phi = 1$) using the dynamic portfolio model would be -0.00551 ($-0.00051 - 0.005$). This is still much better than the utility level achieved by following the static model which yields a utility of -0.02305. The cost of trading futures would have to be equal to 0.02254%, or $22.54 per contract, in order for this agent to be indifferent between the dynamic and the static model. For an agent with $\phi = 4$, this indifference level is equal to 0.0139%, or $13.90 per contract. We conclude that the additional rewards associated with the dynamic portfolio strategy over the static one are more than sufficient to compensate for the potential additional transactions costs which the dynamic model might entail.

The utility gains reported here actually underestimate the true potential of our dynamic strategies because we assumed that portfolios were rebalanced every day.

In reality, hedgers have discretion as to whether or not to rebalance their portfolios at a given point in time. Typically, a hedger will elect to rebalance her position if the benefits of doing so outweigh the costs associated with the change.

8. Conclusion

This paper investigates using futures contracts to hedge two alternative spot currency portfolios, the first one consisting of Deutsche Marks and Swiss Francs and the second one consisting of Deutsche Marks and Japanese Yen. We model the joint evolution of the spot portfolio returns and the log-differences of futures prices in a trivariate GARCH(1,1) system using the full BEKK parameterization. We measure portfolio effects by comparing the performance of a strategy in which the spot portfolio is hedged jointly with the two futures instruments to an alternative approach which involves hedging the two components of the portfolio individually with their respective futures and then aggregating the two hedged positions on the basis of the value weights implicit in the spot currency portfolio. Hedging performance is examined from a risk-minimization as well as from a utility-maximizing standpoint. We compare the performance of dynamic and static risk-minimizing hedging strategies and show that the dynamic approach results in substantial risk reduction. Finally, accounting for portfolio effects in constructing a multi-currency hedge leads to considerable utility gains.

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References

Adler, M., Detemple, J., 1988. Hedging with futures in an intertemporal portfolio context. Journal of Futures Markets 8, 249–269.

Anderson, R.W., Danthine, J.P., 1981. Cross hedging. Journal of Political Economy 89, 1182–1196.

Baillie, R.T., Bollerslev, T., 1990. A multivariate generalized ARCH approach to modelling risk premia in forward foreign exchange markets. Journal of International Money and Finance 9, 309–324.

Baillie, R.T., Myers, R.J., 1991. Bivariate GARCH estimation of the optimal commodity futures hedge. Journal of Applied Econometrics 6, 109–124.

Benninga, S., Eldor, R., Zilcha, I., 1985. Optimal international hedging in commodity and currency forward markets. Journal of International Money and Finance 4, 537–552.
Bollerslev, T., 1986. Generalized autoregressive conditional heteroscedasticity. Journal of Econometrics 31, 307–327.
Bollerslev, T., Wooldridge, J.F., 1992. Quasi-maximum likelihood estimation and inference in dynamic models with time-varying covariances. Econometric Reviews 11, 143–172.
Bollerslev, T., Engle, R.F., Wooldridge, J.F., 1988. A capital asset pricing model with time varying covariances. Journal of Political Economy 96, 116–131.
Cecchetti, S.G., Cumby, R.E., Figlewski, S., 1988. Estimation of optimal futures hedge. Review of Economics and Statistics 70, 623–630.
Eaker, M., Grant, D., 1987. Cross-hedging foreign currency risk. Journal of International Money and Finance 6, 85–105.
Ederington, L.H., 1979. The hedging performance of the new futures markets. Journal of Finance 34, 157–170.
Engle, R., 1982. Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. Econometrica 50, 987–1007.
Engle, R.F., Kroner, K.F., 1994. Multivariate simultaneous generalized ARCH. Econometric Theory, forthcoming.
Figlewski, S., 1984. Hedging performance and basis risk in stock index futures. Journal of Finance 39, 657–669.
Glen, J., Jorion, P., 1993. Currency hedging for international portfolios. Journal of Finance 48, 1865–1886.
Hill, J., Schneeweiss, T., 1981. A note on the hedging effectiveness of foreign currency futures. Journal of Futures Markets 1, 659–664.
Ho, T.S.Y., 1984. Intertemporal commodity futures hedging and the production decision. Journal of Finance 39, 351–376.
Johnson, L.L., 1960. The theory of hedging and speculation in commodity futures. Review of Economics Studies 27, 139–151.
Kroner, K.F., Claessens, S., 1991. Optimal dynamic hedging portfolios and the currency composition of external debt. Journal of International Money and Finance 10, 131–148.
Kroner, K.F., Sultan, J., 1993. Time varying distributions and dynamic hedging with foreign currency futures. Journal and Financial and Quantitative Analysis 28, 535–551.
Lypny, G.J., 1988. Hedging foreign exchange risk with currency futures: portfolio effects. Journal of Futures Markets 8, 703–715.
Markowitz, H.M., 1952. Portfolio selection. Journal of Finance 7, 77–91.
McCurdy, T.H., Morgan, I.G., 1992. Evidence of risk premiums in foreign currency futures markets. Review of Financial Studies 5, 65–83.
Myers, R.J., 1991. Estimating time varying optimal hedge ratios on futures markets. Journal of Futures Markets 11, 39–53.
Paroush, J., Wolf, A., 1992. The derived demand with hedging cost uncertainty in the futures markets. Economic Journal 102, 831–844.
Sephton, P.S., 1993. Optimal hedge ratios at the Winnipeg commodity exchange. Canadian Journal of Economics 26, 175–193.
Smith, C.F., Stulz, R.M., 1985. The determinants of Firms' hedging policies. Journal and Financial and Quantitative Analysis 20, 391–406.
Stein, J.L., 1961. The simultaneous determination of spot and futures prices. American Economic Review 51, 1012–1025.
Stulz, R.M., 1984. Optimal hedging policies. Journal of Financial and Quantitative Analysis 19, 127–140.