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On Testing Theories of Financial Intermediary Portfolio Selection

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1. INTRODUCTION

In papers published at roughly the same time, J. M. Parkin (1970) and J. M. Parkin, M. R. Gray and R. J. Barrett (1970) specified and estimated a model of financial intermediary portfolio behaviour based on expected utility maximization.[1] A principal objective of these studies was to

> . . . specify a model . . . so that the equations being estimated are directly deducible from plausible, but simple, basic assumptions and so that the resulting empirical relationships have a ready interpretation.
>
> (Parkin, Gray and Barrett (1970), p. 233)

The resulting portfolio demand equations possessed properties similar to those of consumer demand theory: symmetry, homogeneity, and " adding up " restrictions. When the system of so-restricted demand equations was estimated (using an inefficient trace minimization procedure), seemingly reasonable results were obtained. In particular, in both Parkin (1970) and Parkin, Gray and Barrett (1970) a great deal of emphasis was placed on the signs and significance tests of the individual parameters. Parkin *et al.* recognized that their estimation procedure was deficient, but claimed that the use of a more appropriate procedure " does not lead to any major changes in conclusions " (p. 241).

More recently, A. S. Courakis (1975) re-estimated the system of portfolio demand equations using a maximum likelihood procedure and the discount house data; he also examined the empirical validity of the various restrictions imposed by Parkin. Courakis found that these restrictions were inconsistent with the data, and concluded that " Parkin has presented what is certainly a theoretically more rigorous, but empirically unconvincing alternative " (p. 647).

The purpose of this paper is twofold. First, based on the data from the Parkin *et al.* study we estimate by maximum likelihood the portfolio demand equations, and examine whether Courakis' negative conclusions carry over to this other body of data. Our test results are very similar to those of Courakis. Second, we generalize the stochastic specification to account for first order vector autocorrelation in the system of portfolio demand equations and test for the validity of the " no autocorrelation " restrictions implicitly imposed by both Parkin *et al.* and Courakis. We conclude that the null hypothesis of no autocorrelation must be rejected. We then again examine whether the Parkin *et al.* restrictions are consistent with the data when vector autocorrelation in the residuals is recognized and modelled. We find that the negative results of our first tests are weakened considerably, and indeed in some cases we are unable to reject the symmetry and homogeneity restrictions. Hence the results of hypothesis tests depend critically on the generality of the stochastic specification.

We present these empirical findings (1) to make specific remarks within the context of the Parkin *et al.* portfolio demand model, and (2) to illustrate the more general principle that care should be taken in interpreting empirical findings, for the maintained hypothesis may not be sufficiently general. Our generalization of the Parkin *et al.* and Courakis maintained hypothesis is limited to a stochastic specification adjustment in which we allow first order vector autocorrelation within their static economic model. We could have generalized the analysis further by permitting higher order autoregressive processes and/or by introducing lagged variables into the estimating equations. Since the computational costs of this relatively simple first order vector autoregressive specification are already substantial, we judged further generalization of the stochastic structure to be " uneconomic " at this time. We also rejected the possibility of introducing *ad hoc* dynamic specifications involving lagged variables, since such formulations are not consistent with the static economic theory underlying the Parkin *et al.* model. The estimation and testing of a portfolio model in which the dynamic specification has been provided on the basis of economic theory is of course desirable, but beyond the scope of this paper.

The paper is organized as follows. In Section 2 we outline, with some qualifying observations, the basic structural model of Parkin (1970) and Parkin *et al.* (1970). In Section 3 we present maximum likelihood estimates and test results without allowing for autocorrelation in the residuals. In Section 4 we test for autocorrelation and then present empirical results based on the generalized stochastic specification. In Section 5 we offer concluding remarks.

2. THE STRUCTURAL MODEL: THEORETICAL AND ECONOMETRIC ASPECTS

The theoretical basis of the empirical results derives from a mean-variance model of portfolio choice. First, it is assumed that the optimal portfolio is chosen through maximizing the expected utility of " real profits ", where the utility function is given by

$$U(\pi) = a-c\exp(-b\pi) \quad b, c>0. \qquad ...(1)$$

Real profits, π, are defined by

$$\pi = m'v \qquad ...(2)$$

where m is a vector of yields and borrowing rates and v is a vector of assets and liabilities (the latter are treated as negative assets).[2] The real profit relationship (2) is decomposed as follows:

$$\begin{aligned}\pi &= (\hat{m}+u_m)'(\hat{v}+u_v)\\ &= \hat{m}'\hat{v}+\hat{m}'u_v+u_m'\hat{v}+u_m'u_v, \qquad ...(3)\end{aligned}$$

where $\hat{m}$ and $\hat{v}$ are vectors of forecasts and the u's are vectors of forecast errors. Parkin *et al.* assume that u_m and u_v are independently distributed with mean vectors equal to zero and constant covariance matrices; this enables them to express expected profits, μ_π, and variance of profits, σ_π^2, as

$$\mu_\pi = \hat{m}'\hat{v} \qquad ...(4)$$

$$\sigma_\pi^2 = \hat{m}'C_{vv}\hat{m}+\hat{v}'C_{mm}\hat{v}+\gamma \qquad ...(5)$$

where the C's are covariance matrices and γ is a constant not affected by the choice variables.

Parkin *et al.* assume further that the distributions of u_m and u_v are such that π is normally distributed.[3] Under this assumption, it is of course true that maximizing expected utility is equivalent to maximizing

$$W = \mu_\pi - \frac{b}{2}\sigma_\pi^2. \qquad \text{...(6)}$$

Maximization of (6) is carried out subject to a portfolio constraint

$$i_1'\hat{v}_1 + i_2'\hat{v}_2 = 0, \qquad \text{...(7)}$$

where $\hat{v}_1$ is a vector containing the subset of assets assumed to be in the choice set, and $\hat{v}_2$ is a vector containing the remaining assets and liabilities (defined as negatives), and " i " represents a vector of ones.

The resulting system of portfolio demand equations for a single firm is

$$\hat{v}_1 = (1/b)G\hat{m}_1 + H^*\hat{v}_2^*, \qquad \text{...(8)}$$

where G and H^* are coefficient matrices which depend on, and only on, the parameters in $C_{m_1m_1}$ and $C_{m_1m_2}$.[4] Vector $\hat{v}_2^*$ differs from $\hat{v}_2$ in that elements of $\hat{v}_2$ which have the same coefficients are consolidated into a single component.

In order to obtain estimable market portfolio demand equations, the vectors $\hat{v}_1$, $\hat{v}_2^*$, and $\hat{m}_1$ must be related to observable quantities, and the portfolio demand equations of individual firms must be aggregated into market relationships. Parkin *et al.* assume that $\hat{m}_1$ is given *without error* by $\overline{m}_1$, where the latter is defined as a simple average of thirteen weekly rates centred on quarter end. Furthermore, Parkin *et al.* assume that forecast errors on v_2^* are zero in aggregate across firms. These assumptions yield non-stochastic market demand equations. Errors are introduced on these market demand equations (presumably derived from the optimization process), and these errors are assumed to be independent of the other stochastic elements in the model. It is useful to explore these last steps in greater detail.

First, from (8) and (3) we can write

$$v_1 = \frac{1}{b}Gm_1 + H^*v_2^* + \left(u_{v_1} - H^*u_{v_2^*} - \frac{1}{b}Gu_{m_1}\right). \qquad \text{...(9)}$$

Equation (9) provides an error vector in the individual firm equations which is *not* independent of m_1 and v_2^*, unless one assumes in addition that actual values and forecast errors are independent. Alternatively, if one provides an explicit expectations hypothesis, such as the $\hat{m} = \overline{m}$ assumption of Parkin *et al.*, but allows for the possibility of an error in the expectations formulation so that $\overline{m} = \hat{m} + e$, then

$$v_1 = \frac{1}{b}G\overline{m}_1 + H^*v_2^* + \left(u_{v_1} - H^*u_{v_2^*} - \frac{1}{b}Ge\right), \qquad \text{...(10)}$$

and again, the natural presumption would be that $\overline{m}_1$ and e are correlated. The alternative, that the " measured value " $\overline{m}_1$ and the " measurement error " e are independent, is difficult to maintain.

Parkin *et al.* assume that all firms have identical point estimates of m and covariance matrices C_{mm}. With n firms, we sum over (10) and obtain,

$$\textstyle\sum_{i=1}^n v_{1i} = \left(\sum_{i=1}^n \frac{1}{b_i}\right)G\overline{m}_1 + H^*\sum_{i=1}^n v_{2i}^* + \sum_{i=1}^n\left(u_{v_{1i}} - H^*u_{v_{2i}^*} - \frac{1}{b_i}Ge\right) \qquad \text{...(11)}$$

which we write compactly as

$$V_1 = G^*\overline{m}_1 + H^*V_2^* + \textstyle\sum_{i=1}^n\left(u_{v_{1i}} - H^*u_{v_{2i}^*} - \frac{1}{b_i}Ge\right). \qquad \text{...(12)}$$

This suggests two problems with (12). The first is one of logical consistency. The composite disturbances in (12) and π in (3) are affected by the same random variables, and thus the question arises as to whether both π and the composite disturbances in (12) can be normally distributed. Secondly, consistent and efficient estimation of the parameters in (12) requires recognition of the correlations between these composite disturbances and the independent variables.[5] As noted above, Parkin *et al.* assume $e = 0$ and $\sum_{i=1}^{n} u_{v_{2t}^*} = 0$. Hence (12) becomes:

$$V_1 = G^*\overline{m}_1 + H^*V_2^* + \varepsilon^* \qquad \ldots(13)$$

where[6] $\varepsilon^* = \sum_{i=1}^{n} u_{v_{1i}}$.

Although we will not comment any further on this specification, we note that the disturbances in (13) can be interpreted as representing random errors resulting from the optimization process. Hence, we can assume that these disturbances in the aggregate demand equations are independent of the other stochastic components in the model and therefore of the independent variables in the regression equations.

Parkin *et al.* add seasonal dummy variables and a further data dummy.[7] No discussion is provided as to how seasonality fits into the theoretical framework.[8] One possible approach would have been to specify that the error vector ε^* is a composite of a seasonal and a remaining random stochastic component. An extensive literature has been developed which compares the properties of the two principal alternative methods of estimation for such models—covariance estimators and error components estimators (see Maddala (1971), Nerlove (1971), Swamy and Arora (1972) and Wallace and Hussain (1969)). Covariance estimation is computationally equivalent to the use of seasonal intercepts. Error component estimation is more cumbersome computationally. Wallace and Hussain (1969) and Nerlove (1971) have shown that under rather general conditions, the covariance and error component estimation procedures yield asymptotically equivalent parameter estimates and covariance matrices. Furthermore, Swamy and Arora (1972) have shown that in our context the covariance estimator is likely to have favourable finite sample properties.[9]

In any case, the set of estimable portfolio demand equations specified by Parkin (1970) and Parkin *et al.* (1970) is

$$V_1 = G^*\overline{m}_1 + H^*V_2^* + JD + J_5D_5 + \varepsilon^* \qquad \ldots(14)$$

where D is the matrix of seasonal dummies, J the matrix of seasonal coefficients, and D_5 and J_5 are respectively the data classification dummy and its coefficients.

The underlying economic theory provides strong predictions concerning the coefficents of (14). In particular, the matrix G^* is predicted to be symmetric (the portfolio equivalent of the symmetry of substitution effects in consumer theory) and to have zero row sums (homogeneity of degree zero with respect to $\overline{m}_1$). In addition, the portfolio balance restriction implies zero column sums in G^*, J, and J_5, and column sums of -1 in H^*. Since the underlying theory does not specify an intercept, J is restricted to have zero row sums. Of course, not all of these restrictions are independent.

Parkin *et al.* estimated the portfolio demand system (14) with the above restrictions imposed, using a restricted least squares (RLS) trace minimization estimation procedure. In a footnote they assert that a generalized RLS estimator did not lead to any major changes in the conclusions. We are unable to determine exactly what alternative generalized RLS estimator was employed; in the next section, however, we report estimates using a maximum likelihood (ML) estimator. Parkin claims that all of his published RLS estimates are unbiased ((1970), p. 487, fn. 1). While this may be true for coefficient estimates, it is *not* true for estimates of the covariance matrix of the coefficient estimates. The covariance matrix estimates are of course extremely important in the context of hypothesis testing; hence Parkin's test results for the significance of individual parameters can be called into question.

3. EMPIRICAL RESULTS: DISTURBANCES ASSUMED FREE OF AUTOCORRELATION

We now report results based on the Parkin *et al.* (1970) quarterly data for U.K. commercial banks, 1954(I)–1967(I). As noted above, Parkin *et al.* estimated the parameters of the equation system (14) using a restricted least squares (RLS) trace minimization procedure. Using the same method, we have re-estimated the model and have obtained results very similar to those reported by Parkin *et al.*

There are two principal problems with the RLS trace minimization procedure. The first is that the portfolio demand system (14) is inherently singular due to the portfolio balance constraints. Singularity implies that only $n-1$ of the n equation disturbances are linearly independent; this linear dependence is not taken into account when the Parkin and Parkin *et al.* RLS trace minimization estimation procedure is used. Second, singularity of the disturbance covariance matrix implies that the matrix is non-diagonal; failure to account for this non-diagonality implies that the RLS trace minimization estimator will yield inefficient estimates of the parameters and an inconsistent estimate of the disturbance covariance matrix. The procedure taken here is to allow for the singularity and non-diagonality of the disturbance covariance matrix by using a maximum likelihood (ML) estimation procedure for $n-1$ of the n portfolio equations.[10] The ML estimates are of course numerically invariant to the choice of which $n-1$ equations are estimated directly.[11]

Since there are four choice assets in the Parkin *et al.* study, we specify that the three-element disturbance vector ε is independently multivariate normally distributed with mean vector zero and constant nonsingular covariance matrix; the ML estimates are then consistent, asymptotically efficient, and asymptotically normally distributed.

The original results of Parkin *et al.* are reproduced in the top panel of Table I. These estimates can be compared with our ML results, based on 53 observations, which are presented in the bottom panel of Table I. Several significant changes occur. Using individual t-tests, Parkin *et al.* report that six of the ten interest rate coefficients are significant; we obtain only two significant estimates. Two sign changes also occur when ML is employed: G_{12}, formerly positive and significant, becomes negative and insignificant, while G_{23}, formerly negative and significant, now becomes positive and insignificant. Results for the H and J coefficients are essentially the same as those of Parkin *et al.* Whether these variations constitute " major changes in conclusions " is of course a matter of judgment. In general, our estimated asymptotic standard errors based on ML estimation are larger than the Parkin *et al.* RLS trace minimization estimates.[12]

We now turn to a discussion of the empirical validity of the theoretical restrictions which result from the utility maximization paradigm employed by Parkin *et al.* Two alternative models are examined. In Model II we use four seasonal dummies with no constraints on their coefficients in the estimated equations. This is equivalent econometrically to an intercept plus three unconstrained seasonal dummies. The model also has an intercept in the sense that the sums of the intercepts across the seasons in each equation are not constrained to zero (which, as we have noted, is inconsistent with the basic theory). Hence we sometimes refer to Model II as the model with an intercept. Model I has the same structure, except that in each equation estimated the sum of the seasonal dummy coefficients is constrained to zero. Model I is, therefore, a testable special case of Model II which imposes (at least on an annual basis) the requirements of the Parkin *et al.* theory.[13] Maximized log-likelihood values for the no autocorrelation specification are presented in Table II. When the symmetry and homogeneity restrictions are not imposed, the test results suggest that Model I must be rejected as an empirically valid special case of Model II. The likelihood ratio test statistic (computed as twice the difference in the sample log-likelihood) is 29·936, while the 0·01 chi-square critical value is 11·345. For purposes of comparison with Courakis, we now test for homogeneity and symmetry using Models I

TABLE I

Parkin et al. *and ML estimates—theoretical restrictions imposed*

	G_{i1}	G_{i2}	G_{i3}	G_{i4}	H_{i5}	H_{i6}	H_{i7}	H_{i8}	J_{i1}	J_{i2}	J_{i3}	J_{i4}	J_{i5}
Parkin *et al.* estimates													
$i = 1$	71·12	64·48	−40·41	−95·19	−0·21	0·14	−0·44	−0·40	−89·82	−7·21	51·53	45·50	236·28
	(3·24)	(3·10)	(2·37)	(3·63)	(6·09)	(1·22)	(18·90)	(25·50)	(8·99)	(0·72)	(5·16)	(4·51)	(8·45)
2	64·48	8·27	−65·82	−6·93	−0·02	−0·10	0·11	0·04	−2·41	−2·35	−2·35	7·12	−41·52
	(3·10)	(0·36)	(4·17)	(0·36)	(0·51)	(0·85)	(4·85)	(2·87)	(0·24)	(0·23)	(0·24)	(0·71)	(1·50)
3	−40·41	−65·82	35·49	70·74	−0·64	−1·35	−0·75	−0·65	89·27	35·43	−17·08	−107·60	−92·47
	(2·37)	(4·17)	(1·86)	(4·01)	(18·90)	(11·60)	(32·40)	(41·30)	(8·94)	(3·51)	(1·71)	(10·70)	(3·27)
4	−95·19	−6·93	70·73	31·39	−0·14	0·31	0·08	0·01	2·97	−25·87	−32·10	54·99	−102·30
	(3·63)	(0·36)	(4·01)	(1·03)	(3·97)	(2·65)	(3·35)	(0·32)	(0·30)	(2·56)	(3·22)	(5·44)	(3·63)
Maximum likelihood estimates													
$i = 1$	107·39	−11·59	−55·56	−40·24	−0·23	0·05	−0·44	−0·42	−104·05	−5·94	57·69	52·29	268·48
	(2·47)	(0·20)	(2·11)	(1·33)	(3·70)	(0·22)	(9·95)	(14·44)	(5·63)	(−0·18)	(3·05)	(2·73)	(5·23)
2	−11·59	13·30	0·64	−2·36	−0·01	−0·04	0·11	0·04	0·50	−6·81	−4·59	10·91	−78·13
	(0·20)	(1·10)	(0·01)	(0·17)	(0·55)	(0·61)	(9·48)	(5·45)	(0·10)	(1·34)	(0·52)	(2·10)	(5·48)
3	−55·56	0·64	16·02	38·91	−0·60	−1·37	−0·75	−0·64	90·88	33·81	−19·84	−104·85	−89·89
	(2·11)	(0·01)	(0·64)	(0·54)	(11·16)	(7·27)	(19·22)	(24·92)	(5·53)	(2·02)	(1·18)	(3·64)	(1·96)
4	−40·24	−2·36	38·91	3·69	−0·16	0·36	0·08	0·02	12·67	−21·06	−33·26	41·65	−100·46
	(1·33)	(0·17)	(0·54)										

Notes: 1. Ratio of estimated coefficient to asymptotic standard error in parentheses.

2. Subscripts refer as follows: 1 = treasury bills, 2 = commercial bills, 3 = government bonds, 4 = call loans. For the non-choice asset coefficients H_{i5} to H_{i8}, 5 = time deposits, 6 = special deposits, 7 = advances, and 8 = a composite of notes, coins, and demand deposits. J_{i1} to J_{i4} refer to quarterly intercepts for quarters 1 to 4, respectively, while J_{i5} is the coefficient on the data reclassification dummy in the ith equation.

3. Some t-ratios for the call loans equation are not provided because this equation was not directly estimated in our ML approach. The ratios reported are provided by the symmetry restrictions.

TABLE II

Sample values of the maximized log-likelihood function assuming no autocorrelation ($\bar{R} = 0$)

	Model I Sum of seasonal dummies constrained to zero	Model II Sum of seasonal dummies unconstrained
Unrestricted	−775·614 (36)	−760·646 (39)
Homogeneity	−782·302 (33)	−773·178 (36)
Homogeneity and symmetry	−797·262 (30)	−781·788 (33)

Notes: 1. Number of free parameters in parentheses.
2. See Table III for Chi-square critical values.

TABLE III

Sample values of the maximized log-likelihood function assuming $\bar{R}$ = diag and $\bar{R}$ = full

	Model I Sum of seasonal dummies constrained to zero		Model II Sum of seasonal dummies unconstrained	
	$\bar{R}$ = diag	$\bar{R}$ = full	$\bar{R}$ = diag	$\bar{R}$ = full
Unrestricted	−741·660 (37)	−732·802 (45)	−739·018 (40)	−729·260 (48)
Homogeneity	−751·353 (34)	−738·553 (42)	−749·590 (37)	−738·194 (45)
Homogeneity and symmetry	−752·765 (31)	−740·933 (39)	−751·428 (34)	−740·423 (42)

Chi-square critical values

Number of independent restrictions	Significance level 0·050	0·025	0·010	0·005
1	3·841	5·024	6·635	7·879
3	7·815	9·348	11·345	12·838
6	12·592	14·449	16·812	18·548
8	15·507	17·535	20·090	21·955
9	16·919	19·023	21·666	23·589
11	19·675	21·920	24·725	26·757
14	23·685	26·119	29·141	31·319
17	27·587	30·191	33·409	35·718

Notes: 1. Number of free parameters in parentheses.

and II. When these restrictions are tested, either separately or jointly, we find that the corresponding null hypotheses must be rejected at various reasonable significance levels. Thus, although we use a different body of data, when we employ an equivalent stochastic specification and estimation procedure it appears that we must concur with Courakis that the theoretical restrictions of no intercept, homogeneity, and symmetry are not consistent with the data.[14]

4. EMPIRICAL RESULTS: DISTURBANCES ASSUMED TO EXHIBIT FIRST ORDER VECTOR AUTOCORRELATION

The above empirical results were based on the assumption of temporal independence of the disturbance vector. In the Appendix A of Parkin (1970), Durbin–Watson test statistics are presented which indicate that autocorrelation may be present. Such statistics are not reported by Parkin *et al.* or by Courakis.

To test whether first order vector autocorrelation is present, we now specify that the 4×1 disturbance vector ε_t^* follows the stationary autoregressive scheme

$$\varepsilon_t^* = R\varepsilon_{t-1}^* + w_t^*, \quad t = 2, 3, \ldots, 54 \qquad \ldots(15)$$

where w_t^* is an independently identically distributed random vector and where R is a 4×4 matrix of unknown autoregressive coefficients. Since the portfolio " adding-up " restrictions still hold, the equation system (14) is singular, and only three of the four disturbances in ε_t^* of (15) are linearly independent. In turn, this singularity implies restrictions on R—the parameter matrix of autoregressive coefficients. Berndt and Savin (1975) have shown that the singularity of equation systems (14) and (15) implies that all of the columns of R must sum to a common constant. This column sum restriction is a strong one. Note that if R is specified as diagonal, then all the diagonal coefficients must be equal.

To circumvent the singularity problem, we again drop one of the four portfolio demand equations—the call loans equation—from our model. We then specify that the 3×1 disturbance vector ε_t follows the stationary autoregressive scheme

$$\varepsilon_t = \bar{R}\varepsilon_{t-1} + w_t, \quad t = 2, 3, \ldots, 54 \qquad \ldots(16)$$

where the sequence $w_2, w_3, \ldots, w_{54}$ consists of independently identically normally distributed random vectors with mean vector zero and nonsingular covariance matrix Ω, and where $\bar{R}$ is a 3×3 matrix of unknown autoregressive coefficients.[15]

We have re-estimated the model by conditional maximum likelihood under two alternative specifications: (i) $\bar{R}$ is a full matrix ($\bar{R}$ = full), and (ii) $\bar{R}$ is a diagonal matrix ($\bar{R}$ = diag).[16] Nested hypothesis tests can of course be carried out on these models in numerous ways. Since likelihood ratio test statistics are computed as twice the difference in sample maximized log-likelihoods, to allow for flexibility we report in the columns of Table III sample log-likelihood values for all models. Critical values at various levels of significance are presented in the bottom portion of Table III. We now discuss several of the more important test results. Interested readers can perform additional tests by using the information provided in Tables II and III.

Based on the information presented in Tables II and III, we reject the null hypothesis of $\bar{R}$ = zero both for Model I and for Model II; this decisive rejection occurs regardless of whether the alternative hypothesis is $\bar{R}$ = diag or $\bar{R}$ = full, and holds for alternative reasonable levels of significance. However, in both of the unrestricted Models I and II, we are unable to reject the null hypothesis of $\bar{R}$ = diag against the alternative hypothesis $\bar{R}$ = full; this test result is not as decisive, for the likelihood ratio test statistics of 17·716 (Model I) and 19·514 (Model II) are only slightly less than the 0·01 chi-square critical value of 20·090. We conclude, then, that autocorrelation appears to be present in both Models I and II, and that the autoregressive process can be represented with some marginal loss of fit by an $\bar{R}$ = diag specification.

We now turn to a discussion of test results for the theoretical restrictions when disturbances are assumed to follow a first order vector autoregressive process. We begin with a test of the " no intercept " restriction, i.e. we first test the null hypothesis of an otherwise unrestricted Model I against the alternative hypothesis of an otherwise unrestricted Model II. The likelihood ratio test statistics are 5·284 ($\bar{R}$ = diag) and 7·082 ($\bar{R}$ = full), while the 0·01 chi-square critical value is 11·345. Hence we are unable to reject the " no intercept " restrictions, and hereafter we maintain the otherwise unrestricted Model I ($\bar{R}$ = full and $\bar{R}$ = diag) as " empirically valid ".

The next set of theoretical restrictions we test are the homogeneity and symmetry constraints, maintaining $\bar{R}$ = diag. If these restrictions are tested jointly, the homogeneity–symmetry restrictions are rejected handily; the test statistic is 22·210 while the 0·01 (0·025) chi-square critical value is 16·812 (14·449). We conclude that when $\bar{R}$ = diag, the theoretical

TABLE IV

Maximum likelihood estimates—$\bar{R}$ = Full and theoretical restrictions imposed (Model I)

	G_{i1}	G_{i2}	G_{i3}	G_{i4}	H_{i5}	H_{i6}	H_{i7}	H_{i8}	J_{i1}	J_{i2}	J_{i3}	J_{i4}	J_{i5}
$i = 1$	56·394	−6·251	−44·529	−5·614	−0·269	−0·359	−0·827	−0·841	−45·739	13·805	35·762	−3·827	147·264
	(1·50)	(0·13)	(2·65)	(0·21)	(3·02)	(1·37)	(10·25)	(12·16)	(3·97)	(0·68)	(3·99)	(0·28)	(2·87)
2	−6·251	1·713	8·715	−4·178	−0·005	0·071	0·014	−0·052	12·105	−1·086	−12·142	1·122	−2·565
	(0·13)	(0·17)	(0·17)	(0·36)	(0·22)	(0·89)	(0·71)	(3·38)	(3·88)	(0·39)	(2·24)	(0·33)	(0·16)
3	−44·529	8·715	20·593	15·221	−0·677	−1·079	−0·336	−0·118	35·137	12·451	5·659	−53·247	−157·135
	(2·65)	(0·17)	(1·21)	(0·27)	(8·64)	(4·84)	(4·45)	(1·84)	(3·11)	(1·30)	(0·63)	(3·08)	(3·50)
4	−5·614	−4·178	15·221	−5·429	−0·049	0·367	0·149	0·011	−1·503	−25·170	−29·279	55·952	12·436
	(0·21)	(0·36)	(0·27)										

Maximum likelihood estimates of autoregressive parameters in $\bar{R}$

$\bar{R}_{ij}$	$j = 1$	2	3
$i = 1$	1·378	0·744	0·512
	(11·29)	(2·29)	(3·29)
2	0·087	1·007	0·090
	(2·36)	(10·30)	(1·92)
3	−0·671	−1·274	0·134
	(6·53)	(4·66)	(1·03)

Notes: 1. Ratio of estimated coefficient to asymptotic standard error in parentheses.
2. The subscripts are defined in the note to Table I.

homogeneity and symmetry restrictions are not consistent with the Parkin *et al.* data; this negative result is consistent with rejections reported in Section 3 under the assumption $\bar{R}$ = zero.

The test results are more ambiguous, however, if we assume $\bar{R}$ = full. When tested jointly, the test statistic of the homogeneity–symmetry restrictions is 16·262, slightly less than the 0·01 chi-square critical value of 16·812, but greater than the 0·025 (0·05) critical values of 14·449 (12·592). If tested separately at one-half the joint significance level, the homogeneity and symmetry restrictions would not be rejected separately at 0·005, but homogeneity would be rejected at the 0·01 level of significance.

We conclude, then, that the extent of the rejection of the theoretical restrictions varies considerably depending on the generality of the autoregressive specification. If one were religiously zealous in obtaining empirical " support " for the theoretical restrictions, one could specify Model II with $\bar{R}$ = full, then test and not reject the no intercept restriction to Model I, and finally jointly test and not reject the homogeneity–symmetry restrictions in Model I. For the benefit of such individuals and other interested agnostics, we present in Table IV parameter estimates with these restrictions imposed.[17] However, it is worth noting that the choice of another nested testing path (such as along $\bar{R}$ = diag) would at some point lead to a rejection of the theoretical restrictions at the appropriate levels of significance.[18, 19]

A few features of the Table IV coefficient estimates deserve comment. First, one more parameter, the treasury bill own rate coefficient, ceases to be significant. This leaves only the negative coefficient relating treasury bills (government bonds) to the government bond rate (treasury bill rate) as significant. Furthermore, the call loan own rate coefficient changes sign, becoming negative.

It is also quite noticeable that the coefficients on the exogenous variables and the seasonal dummies are much more sensitive to the autocorrelation modelling than to the introduction of ML estimation, *per se*, as represented by the $\bar{R}$ = zero ML results.

In Table V we report ratios of our elasticity estimates to those of Parkin *et al.* Generally speaking, our interest rate elasticity estimates are lower than those of Parkin *et al.*, often substantially so, and, as noted above, we have three changes of sign. The exogenous variable elasticities do not change so systematically. Some of our estimates are substantially higher in modulus, and some are substantially lower. There are three changes of sign. Two of these involve the response to special deposit requirements. One of these

TABLE V

Ratios of elasticities, evaluated at the series means, based on our ML results (Table IV) over Parkin et al. *estimates*

	1	2	3	4
1	0·79			
2	−0·10	0·20		
3	1·10	−0·13	0·43	
4	0·06	0·61	0·21	−0·17
5	1·28	0·28	1·07	0·36
6	−2·51	−0·72	0·80	1·20
7	1·86	0·12	0·45	1·91
8	2·09	−1·16	0·18	2·20

Note: 1. Columns represent equations in the same order as rows of Tables I and IV. Rows represent variables, in the same order as the columns of Tables I and IV. Thus, for example, entry (8,1) above is the ratio of our elasticity estimate to that of Parkin *et al.* for the response of treasury bill demand to changes in required cash reserves.

is particularly noteworthy. Where Parkin *et al.* found that treasury bill holdings would increase with special deposits (they called this surprising and we agree), we find *strong* evidence of substitution. When more special deposits are required treasury bills are sold, according to our estimates. A similar effect through government bonds, found by Parkin *et al.*, is also apparent in our results.

Considering a change in the composite variable 8 to be a change in required reserves, we see that banks would largely accommodate such changes by selling treasury bills, and to a lesser extent by selling government bonds and commercial bills. Two changes are noteworthy here. First, we have a reversal of the relative magnitude of the treasury bill and government bond responses. Parkin *et al.* found the major response to be in government bonds. We find our estimate much more plausible based on liquidity arguments. Second, we have a sign reversal (significant positive to significant negative) for commercial bills. Parkin *et al.* found that banks would provide short term liquidity by increasing commercial bills. We find this effect in call loans. Again, since call loans are the most liquid of the assets, our result has intuitive appeal.

5. CONCLUDING REMARKS

Our concluding remarks can be brief. The main point we wish to make is that when testing for the statistical significance of individual parameters or for the empirical validity of more general theoretical restrictions, one must be aware of the fact that test results may depend critically on the generality and reliability of the maintained hypothesis. In this paper we have focussed attention on the reliability of the stochastic specification, but have not amended the basic static Parkin portfolio model. In this context, we have shown that when the stochastic specification is generalized to a first order vector autoregressive process, test results are affected considerably.

In particular, Courakis' comments on Parkin (1970) carry over to the Parkin, Gray and Barrett (1970) model when one estimates the system assuming no autocorrelation. However, we cannot concur with Courakis who concludes that this rejection makes the model " empirically unconvincing ", since we reject the no autocorrelation maintained hypothesis of Courakis.[20] This is not to say that we find the Parkin model convincing. The presence of autocorrelation in the residuals may simply reflect fundamental deficiencies in the specification of the static model.[21] However, since at this point dynamic considerations cannot be incorporated readily into the rigorous utility maximization framework, we have not investigated this possibility and leave that for future research. Given that our data is quarterly, it would have been desirable to include up to a fourth order general autoregressive process in the stochastic specification. Such a generalization is precluded at this time due to the prohibitive computational costs. Indeed, even for our relatively simple first order specification the attainment of conditional maximum likelihood estimates for the $\bar{R}$ = full model was very costly. Improvement in the numerical analysis, estimation theory or the pure econometric theory of such models would be very useful and beneficial.

Finally, this exercise has provided interesting changes in the qualitative conclusions from earlier econometric work on portfolio behaviour. One of the conspicuous features of the Parkin (1970) and Parkin, Gray and Barrett (1970) work was their " success " at capturing significant interest rate response estimates. Our work calls this into question and provides different estimates of responses to exogenous variables in ways we find more plausible.

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NOTES

1. While institutional differences, variations in the treatment of expectations, and other minor details distinguish the two studies, the structural models are essentially the same. Portfolios of the U.K. discount houses are considered in Parkin (1970), while U.K. commercial bank portfolios are examined in Parkin, Gray and Barrett (1970).

2. Implicit in such a formulation is the additional assumption that costs do not depend on the asset composition of the portfolio. Assets and liabilities are defined in " real " terms by dividing by a price index. Uncertainty about future prices is not permitted in this model.

3. Neither Parkin (1970) nor Parkin *et al.* (1970) discuss necessary and sufficient restrictions on the u_m and u_v distributions for π to be normally distributed.

4. No attempt is made to relate eventual estimates of these parameters to the " objective " evidence of actual covariances.

5. Although Parkin (1970) suggests a specification like $\hat{m} = D+e$ and notes the resulting estimation difficulties, he does not comment on the problem of additional distributional assumptions required for consistency in the model.

6. On page 235, Parkin *et al. define* u_{v_1} to have zeros everywhere except in the call loans position. They derive $\hat{V}_1 = b^{y*}G\bar{m}_1 + H^{y*}V_2$ (their 9.15), using the assumptions we have noted. However, since $\hat{V}_1 \equiv V_1 + \Sigma_{i=1}^{n} u_{v_{1i}}$, by definition, and since $\Sigma_{i=1}^{n} u_{v_{1i}}$ has zeros everywhere except in the call loans equation, the transfer to 9.16 with full ε is not consistent with their earlier definition, or makes it totally redundant.

7. The data dummy is introduced to capture the impact of a change in the classification of an item in the portfolios. This dummy variable turns out to have a considerable impact on empirical results.

8. In Parkin (1970) it is noted that seasonality is formally inconsistent with the model as stated. He adds that " casual inspection of the time series graph[s] . . . reveals marked seasonal fluctuations in some of the variables " (p. 482), and that this observation induced him to specify the seasonal dummies in an *ad hoc* fashion.

9. The error component approach originally was introduced in the context of a single equation; an extension to systems of equations has been discussed briefly by Melvyn Fuss (1977).

10. The computer program used is the full information maximum likelihood package coded by Douglas Chapman and Ray Fair (1972) and adapted by Keith Wales for use on an IBM 370 Model 168 machine at the University of British Columbia.

11. We delete the call loans equation.

12. Although Courakis does not address the statistical significance issue explicitly, his ML estimates differ considerably from those reported by Parkin (1970).

13. At an early stage of the analysis we considered another form of restriction on Model II, a restriction to remove seasonality by constraining the dummies in each equation to have a common coefficient (equivalent to an intercept but no seasonals). These restrictions were overwhelmingly rejected by the data and we therefore undertook no further analysis of this specification.

14. The log-likelihood values reported by Courakis in his Table V were difficult to interpret. It appears that instead of representing log-likelihoods, they express logarithms of the determinant of the estimated disturbance covariance matrix plus a constant.

15. Berndt and Savin also show that, unless additional restrictions are imposed on R, it is not possible to identify all of the elements of R based on estimates of the elements in $\bar{R}$. However, identification of the parameters G, J, J_5, and H^* is still possible.

16. The estimates are conditional ML, since the initial lagged observation is added to the sample.

17. Other coefficient tables are available from the authors on request.

18. We note in passing that the conditions for stability of the estimated autoregressive process are satisfied for all models estimated.

19. When estimating the $\bar{R}$ = zero and $\bar{R}$ = diag models, we used a convergence criterion which accepted estimates when no structural or autoregressive parameter changed by more than 0·1 per cent. However, in the $\bar{R}$ = full cases, computational cost considerations led us to increase this tolerance to 0·5 per cent. In either case the log-likelihood function was stable in at least the sixth significant figure from one iteration to the next, indicating that the likelihood function was " flat " although some (usually one or two) highly insignificant parameters were numerically less stable.

20. Since we are using different data, we cannot demonstrate that Courakis' maintained hypothesis is rejected with his " Discount House " data, but we can reject it for the " Banks " data and we would not be surprised if similar findings were obtained with the " Discount House " data.

21. For further discussion and examples, see Mizon (1977).

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