"The Equity Premium Puzzle"

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equity premium puzzle. The term 'equity premium' refers to the difference between the real rate of return on equity and the real rate of return on a riskless asset. The phrase 'equity premium puzzle' refers to the apparent inability of the standard asset-pricing paradigm to explain the average size of the equity premium in US data. In order to describe this failure it is useful to outline the essential features of the basic intertemporal equilibrium model which has been used to price assets. We then consider various extensions of that model, as well as some econometric issues, in the context of potential explanations for the 'puzzle'.

THE LUCAS MODEL. The fundamental valuation equation for assets has been derived in several different contexts, for example, Harrison and Kreps (1979). However, discussion of the equity premium puzzle has largely been based on the Lucas (1978) model of asset-pricing in an exchange economy. This model is one in which there is an infinitely lived representative agent who in each period receives an endowment of a single nonstorable good. In such an economy, shares in equity are claims to the ownership of the production technology. In this world, dividend income is identically equal to endowment income, which as long as the marginal utility of consumption is everywhere positive will equal equilibrium consumption. Asset-pricing models of this variety are often referred to as being consumption-based, and can be generalized to include money, as in Lucas (1982), and to make dividends endogenous in a production economy, as in Brock (1982).

In the simple economy without money or production, straightforward asset-pricing equations can be derived. For example, the valuation equation can be expressed as

$$p_t = E_t m_{t+1} x_{t+1}, \tag{1}$$

where p_t is the price of some asset at time t, E_t is the mathematical expectations operator conditional on information known at time t, m_{t+1} is the marginal rate of substitution between units of consumption at time t and time $t+1$, and x_{t+1} is the payoff at time $t+1$ for holding some asset for the previous period. Note that all these variables are measured in terms of units of the single consumption good.

In the case of equity, the payoff is $x_{t+1} = p_{t+1} + d_{t+1}$ where d_{t+1} is the dividend paid on each share held between t and $t+1$. Thus the price of equity is given by

$$p_t = E_t m_{t+1}(p_{t+1} + d_{t+1}). \tag{2}$$

If there exists a riskless asset which when purchased at time t pays a unit of the consumption good at time $t+1$ and nothing in any subsequent period, then its price q_t will be given by

$$q_t = E_t m_{t+1}. \tag{3}$$

The gross real rate of return on equity is given by

$$R^e_{t+1} = \frac{p_{t+1} + d_{t+1}}{p_t}$$

while the gross real return on the riskless asset is given by

$$R^f_{t+1} = \frac{1}{q_t}.$$

The equity premium is $R^e_{t+1} - R^f_{t+1}$.

MEHRA AND PRESCOTT (1985) CALIBRATION RESULTS. Mehra and Prescott (1985) solve equations (2) and (3) by making particular assumptions about preferences and forcing processes of the model. First they assume that consumers have preferences of the constant relative risk aversion (CRRA) class, and that these preferences are separable with respect to time. That is, there is a single-period utility function of the form

$$U(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma},$$

with $U(c_t) = \log(c_t)$ when $\gamma = 1$. Consumers are assumed to order stochastic consumption paths according to the value of

$$E_0\left[\sum_{t=0}^{\infty} \beta^t U(c_t)\right].$$

With this type of preferences the intertemporal marginal rate of substitution, m_{t+1}, is given by

$$m_{t+1} = \beta\frac{U'(c_{t+1})}{U'(c_t)} = \beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}.$$

Mehra and Prescott further assume that the total payout of dividends is equal to total production in their economy, so that, in equilibrium, $c_t = d_t$, for all t. This allows them to treat consumption as the exogenous forcing process. The growth rate of consumption is assumed to follow a two-state ergodic Markov chain. They calibrate the model by choosing parameters so that their Markov chain yields the same mean, standard deviation and first-order serial correlation coefficient for the annual growth rate of per capita consumption, as in the US economy between 1889 and 1978.

Arguing that reasonable values of the preference parameters are $0<\beta<1$ and $0<\gamma<10$, Mehra and Prescott show that, in their version of the model, the mean equity premium would be at most 0.35 percent with average riskless rates of between 0 and 4 percent. In US data over the period 1889–1978 the mean real rate of return on the S&P 500 Composite Stock Price Index was 6.98 percent while the real rate of return on relatively riskless short-term securities was 0.8 percent for an equity premium of 6.18 percent. Although neither of these actual time series corresponds directly to the return on the theoretical assets in the asset-pricing model (for example, Labadie (1989) discusses some potential implications of proxying the riskless asset by a bond which is riskless in nominal as opposed to real terms), these results have been interpreted to imply that the consumption-based asset-pricing model cannot predict a mean equity premium of the magnitude observed in US data.

STATISTICAL EVALUATION AND ATTEMPTS TO RESOLVE THE 'PUZZLE'. Naturally a question arises as to how sensitive Mehra and Prescott's results are to changes in the specification. We review, in turn, the implications of: alternative specifications of preferences; enlarging the parameter space; a statistical quantification of the model evaluation; and robustness to changes in assumptions about the forcing processes and market completeness.

Generalization of the specification of preferences. Considerable research has concentrated on explaining the equity premium puzzle and other empirical failures of the model in terms of the stochastic properties of the intertemporal marginal rate of substitution. The essential difficulty with representative agent consumption-based models is that aggregate consumption is an extremely smooth series relative to asset returns. If the growth rate of consumption is constant then there is no equity premium. Although the properties of the equity premium vary depending on assumptions made about the forcing processes, it is generally increasing in the variability of the marginal rate of substitution. Since the time series on consumption is smooth, the marginal rate of substitution can be made variable only by increasing γ when CRRA time-separable utility is used. This insight is supported by the evidence presented in Hansen and Jagannathan (1991) who show that only parameterizations of the model for which γ is large are consistent with a variance bound implied by the model. Although reasonable equity premia can be generated from the CRRA model using $\gamma > 10$, the implied rates of return are generally too high thereby requiring corresponding increases in the value of β.

With a broader class of preferences it might be possible to make the intertemporal marginal rate of substitution more sensitive to small movements in consumption without having to increase risk aversion. Epstein and Zin (1990) use preferences, based on Yaari's (1987) dual theory of choice under uncertainty, which exhibit 'first-order risk aversion'. In this case, small gambles have risk premia which are proportional to standard deviations rather than variances. This modification increases the sensitivity of equilibrium premia to small changes in the forcing process and generates higher equity premia than the standard formulation. However, the risk-free rate remains high.

Preferences may also be generalized by introducing non-separabilities in the time dimension. Constantinides (1990) argues that the equity premium puzzle can be explained if one allows consumers to have preferences which exhibit habit formation. That is, current consumption enters negatively into future utility. Epstein and Zin (1989) and Weil (1989) separate the intertemporal elasticity of substitution from the coefficient of relative risk aversion using Kreps and Porteus (1978) non-expected-utility preferences which allow both temporal and state dependencies. However, these latter extensions have not been successful in resolving the equity premium puzzle.

Enlarging the 'reasonable' parameter space. Using a generalization of the basic model which assumes a jointly normal bivariate vector autoregression for consumption and dividends, Kocherlakota (1990b) finds results similar to Mehra and Prescott's. However, he shows that if one allows γ to be greater than 10 and β to be greater than 1, one can generate a risk-free rate and equity premium which match those found in the US data. In support of his assignment of parameter values which are outside the region designated as 'reasonable' by Mehra and Prescott (1985), Kocherlakota (1990a) shows that in an economy with steady-state growth, discount factors greater than 1 are consistent with finite expected utility if γ is sufficiently large. Furthermore, Kocherlakota (1990b) argues that Friend and Blume's (1975) analysis, which was used by Mehra and Prescott (1985) to argue that values of γ near 2 are reasonable, does not apply directly to these asset-pricing models. Disagreement about what range of parameter settings is 'reasonable' highlights the importance of accounting for parameter uncertainty in evaluating the model.

Statistical evaluation of the 'puzzle'. Several authors have attempted to provide a statistical quantification of the Mehra and Prescott (1985) results. Although the disparity between equity premia of 6.18 and 0.35 percent seems large it is not clear whether this disparity is statistically significant. Define W_T as a vector of moments for the equity premium estimated from an historical sample of size T. Analogously, let $\tilde{W}(\theta, \phi)$ be the corresponding vector of population moments implied by a fully-parameterized model (an artificial economy) with parameter vectors θ and ϕ corresponding to preferences and technology respectively.

As noted above, the calibration approach used by Mehra and Prescott (1985) estimates the parameters ϕ such that the forcing process for the artificial economy matches certain features of the observed process, and then computes $\tilde{W}$ for reasonable values of θ. Inference concerning whether the resulting values of $\tilde{W}$ are close to W_T involves uncertainty associated with the parameters of the forcing process as well as that due to sampling variability associated with the historical moment itself. Asymptotic tests, such as those proposed by Cecchetti, Lam and Mark (1991), can be used to evaluate the Mehra and Prescott (1985) results by incorporating such uncertainty. Standard asymptotic methods also could be used to estimate the preference parameters θ rather than fixing them by calibration.

Since an artificial economy is fully parameterized, repeated simulation can be used to obtain the empirical density implied by the model for the sample moments. Gregory and Smith (1991) use this Monte Carlo technique to compute the size of the test implicit in Mehra and Prescott's rejection of the model.

Assumptions about forcing processes and market completeness. Other research has attempted to determine the robustness of Mehra and Prescott's results to changes in assumptions about the forcing processes of the model. Rietz (1988) modifies Mehra and Prescott's model by adding a third state which occurs with very small probability, but in which a drastic decline in equilibrium consumption occurs. This third state is meant to capture the notion of stock market crashes, which are rare events but may have significant effects on the mean rate of return on equity. In some of his parameterizations, Rietz manages to generate equity premia and risk-free rates consistent with US data. Mehra and Prescott (1988) question whether the size and probability of the crashes in these cases are reasonable from an historical perspective.

Another recent modification to the specification of the forcing processes has been to allow leverage so that consumption and dividends need not be equal in equilibrium. Early applications of this generalization, for example, Benninga and Protopapadakis (1990) and Kandel and Stambaugh (1990), use the share of dividends to consumption as a free parameter which can be varied to help match moments. One way that Cecchetti, Lam and Mark (1991) model leverage is to calibrate the aggregate dividend to consumption ratio to the average value in the data and price equity as the residual claim on the univariate consumption endowment after payment is made on one-period real bonds. While there exist leverage ratios which allow these models to match jointly the first and second moments of the equity premium and the risk-free rate, this is not possible for reasonable leverage ratios.

An incomplete markets approach has been pursued by Lucas (1990), Marcet and Singleton (1990) and Telmer (1991). Given the smoothness of aggregate consumption, this extension is appealing since it relaxes the representative agent assumption and introduces nondiversifiable individual consumption risk to the Lucas model. However, the initial results with respect to matching even the mean of the equity premium are not encouraging.

The fundamental valuation equation (1) is consistent with various asset-pricing models which can be distinguished by their specification of the pricing kernel m_{t+1}. We have summarized some attempts to resolve the equity premium puzzle by generalizing the specification of one or more of the components of the Mehra and Prescott (1985) parameterization of the consumption-based asset-pricing model. Although this literature provides no clear consensus on resolving the equity premium puzzle, there is some agreement that this puzzle is only one aspect of the time-series behaviour of asset returns which needs to be explained.

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See also CAPITAL ASSET PRICING MODEL; CONSUMPTION FUNCTION; EQUITY PREMIUM; NONEXPECTED UTILITY THEORY; RISK AVERSION; RISKLESS ASSET.

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