# Online Appendix for <br> "Stock Return Autocorrelations and Expected Option Returns" 

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March 12th, 2024
This document supplements the paper "Stock Return Autocorrelations and Expected Option Returns." It provides additional results and robustness analyses which are not displayed in the published text.

## OA. 1 Proof of Proposition 1

Consider a general European derivative with maturity $t+\tau$ and a payoff at maturity given by

$$
\begin{equation*}
H_{t+\tau}=h\left(S_{t+\tau}\right) \tag{1}
\end{equation*}
$$

where $h\left(S_{t+\tau}\right)$ is a deterministic function of the underlying stock price at $t+\tau$. As noted in Grundy (1991) and Lo and Wang (1995), the drift of the stock price process is irrelevant for determining the price of the derivative today, and we can use the risk-neutralized process of the stock price to determine the price of the European derivative today. Under the risk neutral measure, the continuously compounded return of $r_{t+\tau}=\log \left(S_{t+\tau}\right)-\log \left(S_{t}\right)$ is normally distributed with a mean of $\tau\left(r-\frac{\sigma^{2}}{2}\right)$ and variance of $\tau \sigma^{2}$. It follows that the current price of the European derivative is given by

$$
\begin{align*}
H_{t}\left(S_{t}, \sigma\right) & =e^{-r \tau} E_{t}^{\mathbb{Q}}\left[h\left(S_{t+\tau}\right)\right] \\
& =e^{-r \tau} \int_{-\infty}^{\infty} h\left(S_{t} e^{\left(r-\frac{1}{2} \sigma^{2}\right) \tau+\sigma \sqrt{\tau} v}\right) \phi(v) \mathrm{d} v . \tag{2}
\end{align*}
$$

Similarly, the price of the derivative at time $t+k$, where $0 \leq k \leq \tau$, can be obtained as

$$
\begin{align*}
H_{t+k}\left(S_{t+k}, \sigma\right) & =e^{-r(\tau-k)} E_{t+k}^{\mathbb{Q}}\left[h\left(S_{t+\tau}\right)\right] \\
& =e^{-r(\tau-k)} \int_{-\infty}^{\infty} h\left(S_{t+k} e^{\left(r-\frac{1}{2} \sigma^{2}\right)(\tau-k)+\sigma \sqrt{\tau-k} v}\right) \phi(v) \mathrm{d} v \tag{3}
\end{align*}
$$

Under the physical measure, the stock price follows a trending O-U process and its $k$-period continuously compounded return $r_{k}=\log \left(S_{t+k}\right)-\log \left(S_{t}\right)$ is normally distributed with mean $k \mu$ and variance $k \sigma_{k}^{2}$. As a result, we can write $S_{t+k}$ as

$$
\begin{equation*}
S_{t+k}=S_{t} e^{\mu k+\sigma_{k} \sqrt{k} w} \tag{4}
\end{equation*}
$$

where $w$ is a standard normal random variable. Then, we can compute the expected price of the derivative at time $t+k$ as

$$
\begin{aligned}
E_{t}\left[H_{t+k}\right] & =\int_{-\infty}^{\infty} H_{t+k}\left(S_{t} e^{\mu k+\sigma_{k} \sqrt{k} w}, \sigma\right) \phi(w) \mathrm{d} w \\
& =\int_{-\infty}^{\infty}\left[e^{-r(\tau-k)} \int_{-\infty}^{\infty} h\left(S_{t} e^{\mu k+\sigma_{k} \sqrt{k} w} e^{\left(r-\frac{\sigma^{2}}{2}\right)(\tau-k)+\sigma \sqrt{\tau-k} v}\right) \phi(v) \mathrm{d} v\right] \phi(w) \mathrm{d} w \\
& =e^{-r(\tau-k)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h\left(S_{t} e^{\mu k+\left(r-\frac{\sigma^{2}}{2}\right)(\tau-k)} e^{\sqrt{\sigma_{k}^{2} k+\sigma^{2}(\tau-k)} u}\right) \phi_{2}\left(u, w ; \frac{\sigma_{k} \sqrt{k}}{\sigma^{*} \sqrt{\tau}}\right) \mathrm{d} w \mathrm{~d} u
\end{aligned}
$$

$$
\begin{align*}
& =e^{-r(\tau-k)} \int_{-\infty}^{\infty} h\left(S_{t} e^{\mu k+\left(r-\frac{\sigma^{2}}{2}\right)(\tau-k)+\sigma^{*} \sqrt{\tau} u}\right) \phi(u) \mathrm{d} u \\
& =e^{r k} H_{t}\left(S_{t} e^{\mu k+\left(r-\frac{\sigma^{2}}{2}\right)(\tau-k)-\left(r-\frac{\sigma^{* 2}}{2}\right) \tau}, \sigma^{*}\right) \\
& =e^{r k} H_{t}\left(S_{t}^{*}, \sigma^{*}\right), \tag{5}
\end{align*}
$$

where

$$
\begin{equation*}
S_{t}^{*}=S_{t} e^{\mu k+\left(r-\frac{\sigma^{2}}{2}\right)(\tau-k)-\left(r-\frac{\sigma^{* 2}}{2}\right) \tau}=S_{t} e^{\left(\mu-r+\frac{\sigma_{k}^{2}}{2}\right) k} \tag{6}
\end{equation*}
$$

and $\phi_{2}(\cdot, \cdot ; \rho)$ stands for the density function of a standard bivariate normal random variable with correlation $\rho$. In the above derivation, we make a change of variable of

$$
\begin{equation*}
u=\frac{\sigma_{k} \sqrt{k} w+\sigma \sqrt{\tau-k} v}{\sqrt{\sigma_{k}^{2} k+\sigma^{2}(\tau-k)}}=\frac{\sigma_{k} \sqrt{k} w+\sigma \sqrt{\tau-k} v}{\sigma^{*} \sqrt{\tau}} \sim N(0,1) \tag{7}
\end{equation*}
$$

and we have

$$
\begin{equation*}
\operatorname{Corr}[u, w]=\operatorname{Cov}[u, w]=\frac{\sigma_{k} \sqrt{k}}{\sigma^{*} \sqrt{\tau}} . \tag{8}
\end{equation*}
$$

This completes the proof.

## OA. 2 Proof of Corollary 1.1

This is a special case of Proposition 1 with $k=\tau$, thus $\sigma^{*}=\sigma_{\tau}$ and $S_{t}^{*}=S_{t} e^{\left(\mu-r+\frac{\sigma_{\tau}^{2}}{2}\right) \tau}=\tilde{S}_{t}$. This completes the proof.

## OA. 3 Proof of Proposition 2

From the Black-Scholes formula, it is easy to show that

$$
\begin{align*}
& \frac{\partial C^{B S}\left(S_{t}^{*}, K, r, \tau, \sigma^{*}\right)}{\partial S_{t}^{*}}=\Phi\left(d_{1}^{*}\right)  \tag{9}\\
& \frac{\partial P^{B S}\left(S_{t}^{*}, K, r, \tau, \sigma^{*}\right)}{\partial S_{t}^{*}}=-\Phi\left(-d_{1}^{*}\right)  \tag{10}\\
& \frac{\partial C^{B S}\left(S_{t}^{*}, K, r, \tau, \sigma^{*}\right)}{\partial \sigma^{*}}=S_{t}^{*} \phi\left(d_{1}^{*}\right) \sqrt{\tau}  \tag{11}\\
& \frac{\partial P^{B S}\left(S_{t}^{*}, K, r, \tau, \sigma^{*}\right)}{\partial \sigma^{*}}=S_{t}^{*} \phi\left(d_{1}^{*}\right) \sqrt{\tau} \tag{12}
\end{align*}
$$

where

$$
\begin{equation*}
d_{1}^{*}=\frac{\log \left(\frac{S_{t}^{*}}{K}\right)+\left(r+\frac{\sigma^{* 2}}{2}\right) \tau}{\sigma^{*} \sqrt{\tau}} . \tag{13}
\end{equation*}
$$

It follows that

$$
\begin{align*}
\frac{\partial E_{t}\left[C_{t+k}\right]}{\partial \sigma_{k}} & =e^{r k}\left[\frac{\partial C^{B S}\left(S_{t}^{*}, K, r, \tau, \sigma^{*}\right)}{\partial S_{t}^{*}} \frac{\partial S_{t}^{*}}{\partial \sigma_{k}}+\frac{\partial C^{B S}\left(S_{t}^{*}, K, r, \tau, \sigma^{*}\right)}{\partial \sigma^{*}} \frac{\partial \sigma^{*}}{\partial \sigma_{k}}\right] \\
& =e^{r k}\left[\Phi\left(d_{1}^{*}\right) S_{t}^{*} k \sigma_{k}+S_{t}^{*} \phi\left(d_{1}^{*}\right) \sqrt{\tau} \frac{k \sigma_{k}}{\tau \sigma^{*}}\right] \\
& =e^{r k} S_{t}^{*} k \sigma_{k}\left[\Phi\left(d_{1}^{*}\right)+\frac{\phi\left(d_{1}^{*}\right)}{\sigma^{*} \sqrt{\tau}}\right],  \tag{14}\\
\frac{\partial E_{t}\left[P_{t+k}\right]}{\partial \sigma_{k}} & =e^{r k}\left[\frac{\partial P^{B S}\left(S_{t}^{*}, K, r, \tau, \sigma^{*}\right)}{\partial S_{t}^{*}} \frac{\partial S_{t}^{*}}{\partial \sigma_{k}}+\frac{\partial P^{B S}\left(S_{t}^{*}, K, r, \tau, \sigma^{*}\right)}{\partial \sigma^{*}} \frac{\partial \sigma^{*}}{\partial \sigma_{k}}\right] \\
& =e^{r k}\left[-\Phi\left(-d_{1}^{*}\right) S_{t}^{*} k \sigma_{k}+S_{t}^{*} \phi\left(d_{1}^{*}\right) \sqrt{\tau} \frac{k \sigma_{k}}{\tau \sigma^{*}}\right] \\
& =e^{r k} S_{t}^{*} k \sigma_{k}\left[-\Phi\left(-d_{1}^{*}\right)+\frac{\phi\left(d_{1}^{*}\right)}{\sigma^{*} \sqrt{\tau}}\right] . \tag{15}
\end{align*}
$$

We now show that when $k=\tau$ and $\mu>0, \partial E_{t}\left[P_{t+k}\right] / \partial \sigma_{k}>0$ for at-the-money and out-of-the-money put options. Note that when $k=\tau$ and $\mu>0$,

$$
\begin{equation*}
S_{t} \geq K \Rightarrow S_{t}^{*} \geq K e^{\left(\mu-r+\frac{\sigma_{\tau}^{2}}{2}\right) \tau} \Rightarrow d_{1}^{*} \geq \sigma^{*} \sqrt{\tau} \tag{16}
\end{equation*}
$$

It follows that

$$
\begin{equation*}
-\Phi\left(-d_{1}^{*}\right)+\frac{\phi\left(d_{1}^{*}\right)}{\sigma^{*} \sqrt{\tau}} \geq-\Phi\left(-d_{1}^{*}\right)+\frac{\phi\left(d_{1}^{*}\right)}{d_{1}^{*}}=\frac{\Phi\left(-d_{1}^{*}\right)}{d_{1}^{*}}\left[-d_{1}^{*}+\frac{\phi\left(d_{1}^{*}\right)}{\Phi\left(-d_{1}^{*}\right)}\right]>0 \tag{17}
\end{equation*}
$$

The last inequality follows from the result of Gordon (1941) regarding inverse Mill's ratio for normal random variable that states for $d_{1}^{*} \geq 0$,

$$
\begin{equation*}
\frac{\phi\left(d_{1}^{*}\right)}{1-\Phi\left(d_{1}^{*}\right)}>d_{1}^{*} \tag{18}
\end{equation*}
$$

This completes the proof.

## OA. 4 Proof of Monotonicity of $\sigma_{k}^{2}$ in $\gamma$

We show that under the bivariate O-U process, $\sigma_{k}^{2}$ is a monotonically decreasing function of $\gamma$. The expression of $\sigma_{k}^{2}$ is given by

$$
\begin{equation*}
\sigma_{k}^{2}=\frac{1}{k \gamma}\left[\sigma^{2}+\frac{\lambda^{2} \sigma_{x}^{2}}{\delta(\gamma+\delta)}\right]\left[\left(1-e^{-k \gamma}\right)-\frac{\lambda}{\gamma-\delta} \beta_{q x}\left(e^{-k \delta}-e^{-k \gamma}\right)\right], \tag{19}
\end{equation*}
$$

where

$$
\begin{equation*}
\beta_{q x}=\frac{\gamma \lambda \sigma_{x}^{2}}{\delta(\delta+\gamma) \sigma^{2}+\lambda^{2} \sigma_{x}^{2}} \tag{20}
\end{equation*}
$$

Taking derivative of $\sigma_{k}^{2}$ with respect to $\gamma$, we obtain

$$
\begin{equation*}
\frac{\partial \sigma_{k}^{2}}{\partial \gamma}=\frac{e^{-k(\delta+\gamma)}}{\delta \gamma^{2}\left(\delta^{2}-\gamma^{2}\right)^{2} k}\left[f_{1}+\lambda^{2} \sigma_{x}^{2} f_{2}\right] \tag{21}
\end{equation*}
$$

where

$$
\begin{align*}
& f_{1}=-\delta e^{k \delta}\left(\delta^{2}-\gamma^{2}\right)^{2}\left(e^{k \gamma}-1-k \gamma\right) \sigma^{2}  \tag{22}\\
& f_{2}=2 \gamma^{3} e^{k \gamma}-(\delta-\gamma)^{2}(\delta+2 \gamma) e^{k(\delta+\gamma)}+\left[\delta^{2}(1+k \gamma)-\gamma^{2}(3+k \gamma)\right] \delta e^{k \delta} \tag{23}
\end{align*}
$$

Since $e^{k \gamma}>1+k \gamma$, it is obvious that $f_{1} \leq 0$. It suffices to show that $f_{2} \leq 0$. Let $a=k \gamma$ and $b=k \delta$. We can re-write $f_{2}$ as a function of $a$ and $b$ as follows

$$
\begin{align*}
f_{2} / k^{3}=f(a, b) & =2 a^{3} e^{a}-(a-b)^{2}(b+2 a) e^{a+b}+\left(b^{2}(1+a)-a^{2}(3+a)\right) b e^{b} \\
& =2 a^{3} e^{a}+b\left[(1+a) b^{2}-(3+a) a^{2}\right] e^{b}-(a-b)^{2}(2 a+b) e^{a+b} \tag{24}
\end{align*}
$$

We first show that $f(a, a+d) \leq f(a, a)=0$ for $d>0$. We have

$$
\begin{equation*}
f(a, a+d)=e^{a}\left\{2 a^{3}+(a+d)\left[2 a^{2}(d-1)+d^{2}+a d(2+d)\right] e^{d}-d^{2}(3 a+d) e^{a+d}\right\} \tag{25}
\end{equation*}
$$

Since $e^{a}>0$, it suffices to show that the expression within the braces is non-positive. This follows because

$$
\begin{aligned}
& 2 a^{3}+(a+d)\left[2 a^{2}(d-1)+d^{2}+a d(2+d)\right] e^{d}-d^{2}(3 a+d) e^{a+d} \\
\leq & 2 a^{3}+(a+d)\left[2 a^{2}(d-1)+d^{2}+a d(2+d)\right] e^{d}-d^{2}(3 a+d) e^{d}\left(1+a+\frac{a^{2}}{2}\right) \\
= & -\frac{a^{2}}{2}\left[d^{3} e^{d}+\left(-4+4 e^{d}-4 d e^{d}+3 d^{2} e^{d}\right) a\right]
\end{aligned}
$$

$$
\begin{equation*}
=-\frac{a^{2}}{2}\left(d^{3} e^{d}+a \sum_{n=2}^{\infty} \frac{3 n^{2}-7 n+4}{n!} d^{n}\right) \leq 0 . \tag{26}
\end{equation*}
$$

We next show that $f(b+c, b) \leq f(b, b)=0$ for $c>0$. We have

$$
\begin{equation*}
f(b+c, b)=e^{b}\left\{b^{3}(1+b+c)-b(b+c)^{2}(3+b+c)+2(b+c)^{3} e^{c}-c^{2}(3 b+2 c) e^{b+c}\right\} . \tag{27}
\end{equation*}
$$

Since $e^{b}>0$, it suffices to show that the expression within the braces is non-positive. This follows because

$$
\begin{align*}
& b^{3}(1+b+c)-b(b+c)^{2}(3+b+c)+2(b+c)^{3} e^{c}-c^{2}(3 b+2 c) e^{b+c} \\
\leq & b^{3}(1+b+c)-b(b+c)^{2}(3+b+c)+2(b+c)^{3} e^{c}-c^{2}(3 b+2 c) e^{c}\left(1+b+\frac{b^{2}}{2}\right) \\
\equiv & -\frac{b}{2}\left(d_{0}+d_{1} b+d_{2} b^{2}\right) \tag{28}
\end{align*}
$$

Hence, it suffices to show that $d_{0}, d_{1}$, and $d_{2}$ are all non-negative. Using power series expansion around 0 , we observe that

$$
\begin{align*}
& d_{0}=2 c^{2}\left[3+c+(2 c-3) e^{c}\right]=2 c^{2} \sum_{n=2}^{\infty} \frac{2 n-3}{n!} c^{n} \geq 0 \\
& d_{1}=6 c(2+c)+2 c[c(3+c)-6] e^{c}=2 c \sum_{n=2}^{\infty} \frac{n^{2}+2 n-6}{n!} c^{n} \geq 0, \\
& d_{2}=4(1+c)+\left(3 c^{2}-4\right) e^{c}=\sum_{n=2}^{\infty} \frac{3 n^{2}-3 n-4}{n!} c^{n} \geq 0 . \tag{29}
\end{align*}
$$

This completes the proof.

## OA. 5 Proof of Monotonicity of $\rho_{k}(1)$ in $\gamma$

We show that under the bivariate O-U process, $\rho_{k}(1)$ is a monotonically decreasing function of $\gamma$. Given the expression of $\rho_{k}(1)$, it can be shown that

$$
\begin{equation*}
\frac{\partial \rho_{k}(1)}{\partial \gamma}=\frac{\sigma^{4} f_{1}+\sigma^{2} \lambda^{2} \sigma_{x}^{2} f_{2}+\lambda^{4} \sigma_{x}^{4} f_{3}}{c} \tag{30}
\end{equation*}
$$

where

$$
\begin{equation*}
c=2 e^{k \gamma}\left[e^{k \gamma}\left(e^{k \delta}-1\right) \gamma \lambda^{2} \sigma_{x}^{2}+\delta e^{k \delta}\left(e^{k \gamma}-1\right)\left(\gamma^{2} \sigma^{2}-\lambda^{2} \sigma_{x}^{2}\right)-\delta^{3} e^{k \delta}\left(e^{k \gamma}-1\right) \sigma^{2}\right]^{2}>0 \tag{31}
\end{equation*}
$$

and

$$
\begin{align*}
f_{1}= & -\delta^{2} e^{2 k \delta}\left(e^{k \gamma}-1\right)^{2}\left(\delta^{2}-\gamma^{2}\right)^{2} k,  \tag{32}\\
f_{2}= & \delta\left(\delta^{2}+\gamma^{2}\right) e^{k \gamma}\left(e^{k \delta}-1\right)\left(e^{k \gamma}-1\right)\left(e^{k \delta}-e^{k \gamma}\right) \\
& -\delta\left(\delta^{2}-\gamma^{2}\right)\left[2 k \delta e^{2 k \delta}\left(e^{k \gamma}-1\right)^{2}-k \gamma\left(e^{k \delta}-1\right) e^{k \gamma}\left(e^{k \delta+k \gamma}+e^{k \gamma}-2 e^{k \delta}\right)\right],  \tag{33}\\
f_{3}= & -\delta\left[e^{k \delta+3 k \gamma}-e^{3 k \gamma}+k \delta e^{2 k \delta}\left(e^{k \gamma}-1\right)^{2}+(1+k \gamma) e^{2 k \gamma}\right. \\
& \left.-(1+k \gamma) e^{2 k \delta+2 k \gamma}-(1+2 k \gamma)\left(e^{k \delta+k \gamma}-e^{2 k \delta+k \gamma}\right)\right] . \tag{34}
\end{align*}
$$

To prove $\partial \rho_{k}(1) / \partial \gamma \leq 0$, we need to prove that $f_{1} \leq 0, f_{2} \leq 0$, and $f_{3} \leq 0$. It is obvious that $f_{1} \leq 0$.
Proof of $f_{2} \leq 0$ :
Let $a=k \gamma$ and $b=k \delta$. Dividing $f_{2}$ by $\delta / k^{2}$, which preserves the sign of $f_{2}$, we obtain a function of $a$ and $b$

$$
\begin{align*}
g(a, b)= & \left(a^{2}+b^{2}\right) e^{a}\left(e^{a}-1\right)\left(e^{b}-1\right)\left(e^{b}-e^{a}\right) \\
& +\left(a^{2}-b^{2}\right)\left[2 b e^{2 b}\left(e^{a}-1\right)^{2}-a e^{a}\left(e^{b}-1\right)\left(e^{a+b}+e^{a}-2 e^{b}\right)\right] \tag{35}
\end{align*}
$$

First, consider the case $\delta>\gamma$ so that $b>a$. We want to show that $g(a, a+d)<g(a, a)=0$ for $d>0$, hence $g$ is a decreasing function of $b$ for $b>a$ when fixing $a$. Taking a partial derivative of $g(a, a+d)$ with respect to $d$, we get $\partial g(a, a+d) / \partial d=-e^{2 a} g_{1}(a, d)$, where

$$
\begin{align*}
g_{1}(a, d)= & -\left(2 a^{2}+2 a d+d^{2}\right) e^{a+d}\left(e^{a}-1\right)\left(e^{d}-1\right)-\left[a^{2}+(a+d)^{2}\right] e^{d}\left(e^{a}-1\right)\left(e^{a+d}-1\right) \\
& -2(a+d)\left(e^{a}-1\right)\left(e^{d}-1\right)\left(e^{a+d}-1\right) \\
& +2(a+d)\left[2(a+d) e^{2 d}\left(e^{a}-1\right)^{2}-a\left(e^{a+d}-1\right)\left(e^{a+d}-2 e^{d}+1\right)\right] \\
& +2 d(2 a+d) e^{d}\left[(1+2 d) e^{d}\left(e^{a}-1\right)^{2}+a\left(e^{2 a+d}-2 e^{a+d}+2 e^{d}-1\right)\right] \tag{36}
\end{align*}
$$

We now show that $g_{1}(a, d) \geq 0$. Observing that $g_{1}(0, d)=0$, it suffices to show that $\frac{\partial g_{1}}{\partial a} \geq 0$. Repeating similar argument, we have $\frac{\partial g_{1}}{\partial a}(0, d)=0$, hence it reduces to showing that $\frac{\partial^{2} g_{1}}{\partial a^{2}} \geq 0$. We then have

$$
\frac{\partial^{2} g_{1}}{\partial a^{2}}(0, d)=-2 d+8 d e^{d}+4 d^{2} e^{d}-6 d e^{2 d}+6 d^{2} e^{2 d}+8 d^{3} e^{2 d}
$$

$$
\begin{equation*}
=\sum_{n=1}^{\infty} \frac{(n+2)\left[(2 n-3) 2^{n}+4\right]}{n!} d^{n+1} \geq 0 . \tag{37}
\end{equation*}
$$

Therefore, it now suffices to show that $\frac{\partial^{3} g_{1}}{\partial a^{3}}$ is non-negative. We have the following functional form

$$
\begin{equation*}
\frac{\partial^{3} g_{1}}{\partial a^{3}}=2 e^{a} d_{0}(a, d)+2 e^{a} d_{1}(a, d) a+8 e^{a+d} d_{2}(a, d) a^{2} \tag{38}
\end{equation*}
$$

where

$$
\begin{align*}
d_{0}(a, d)= & 4\left(4 d^{3}+19 d^{2}+13 d-6\right) e^{a+2 d}+4(d+2)(d+3) e^{a+d} \\
& -\left(4 d^{3}+35 d^{2}+47 d-3\right) e^{2 d}-d-3  \tag{39}\\
d_{1}(a, d)= & 8\left(5 d^{2}+9 d-4\right) e^{a+2 d}+8(d+4) e^{a+d}-\left(10 d^{2}+32 d-1\right) e^{2 d}-1,  \tag{40}\\
d_{2}(a, d)= & (4 d-2) e^{a+d}+2 e^{a}-d e^{d} . \tag{41}
\end{align*}
$$

We now show that $d_{0}, d_{1}$, and $d_{2}$ are all non-negative. Since $d_{0}(a, 0)=0$, taking a derivative with respect to $d$ we obtain

$$
\begin{align*}
\frac{\partial d_{0}}{\partial d}= & 4\left(d^{2}+7 d+11\right) e^{a+d}+4\left(8 d^{3}+50 d^{2}+64 d+1\right) e^{a+2 d} \\
& -\left(8 d^{3}+82 d^{2}+164 d+41\right) e^{2 d}-1 \\
\geq & 4\left(d^{2}+7 d+11\right) e^{d}+4\left(8 d^{3}+50 d^{2}+64 d+1\right) e^{2 d}-\left(8 d^{3}+82 d^{2}+164 d+41\right) e^{2 d}-1 \\
= & 4\left(d^{2}+7 d+11\right) e^{d}+\left(24 d^{3}+118 d^{2}+92 d-37\right) e^{2 d}-1 \geq 6 \tag{42}
\end{align*}
$$

where the last inequality can be shown by power series expansion in $d$ around 0 , which we omit the expression for brevity. Next, since $d_{1}(a, 0)=0$, taking a derivative with respect to $d$ we obtain

$$
\begin{align*}
\frac{\partial d_{1}}{\partial d} & =2 e^{d}\left[4\left(10 d^{2}+28 d+1\right) e^{a+d}+4(d+5) e^{a}-\left(10 d^{2}+42 d+15\right) e^{d}\right] \\
& \geq 2 e^{d}\left[4\left(10 d^{2}+28 d+1\right) e^{d}+4(d+5)-\left(10 d^{2}+42 d+15\right) e^{d}\right] \\
& =2 e^{d}\left[4(5+d)+\left(30 d^{2}+70 d-11\right) e^{d}\right] \geq 18 e^{d} \tag{43}
\end{align*}
$$

where the last inequality can be shown by power series expansion in $d$ around 0 , which we omit the expression for brevity. Lastly, since $d_{2}(a, 0)=0$, taking a derivative with respect to $d$ we obtain

$$
\begin{equation*}
\frac{\partial d_{2}}{\partial d}=e^{d}\left[2 e^{a}-1+\left(4 e^{a}-1\right) d\right]>0 \tag{44}
\end{equation*}
$$

This completes the proof that $g(a, a+d)<g(a, a)=0$ for $d>0$.
Similarly, we now need to show that $g(b+c, b)<g(b, b)=0$ for $c>0$. Taking a partial derivative of $g(b+c, b)$ with respect to $c$, we get $\partial g(b+c, b) / \partial c=-e^{2 b} g_{2}(b, c)$, where

$$
\begin{align*}
g_{2}(b, c)= & \left(2 b^{2}+2 b c+c^{2}\right) e^{c}\left(e^{b}-1\right)\left(3 e^{b+2 c}-2 e^{b+c}-2 e^{c}+1\right) \\
& +2(b+c) e^{c}\left(e^{b}-1\right)\left(e^{c}-1\right)\left(e^{b+c}-1\right) \\
& -2(b+c)\left[2 b\left(e^{b+c}-1\right)^{2}-(b+c) e^{c}\left(e^{b}-1\right)\left(e^{b+c}+e^{c}-2\right)\right] \\
& -c(2 b+c) e^{c}\left\{2 b\left(e^{2 b+c}-e^{b}+e^{c}-1\right)-\left(e^{b}-1\right)\left[(2 c+1)\left(e^{b+c}+e^{c}-1\right)-1\right]\right\} . \tag{45}
\end{align*}
$$

The proof of $g_{2}(b, c) \geq 0$ is similar to the proof of $g_{1}(a, d) \geq 0$, so we only sketch it here. Since $g_{2}(0, c)=0$, it suffices to show that $\frac{\partial g_{2}}{\partial b} \geq 0$. We then show that $\frac{\partial g_{2}}{\partial b}(0, c) \geq 0$, which suggests that it suffices to show that $\frac{\partial^{2} g_{2}}{\partial b^{2}} \geq 0$. Repeating similar argument, we have $\frac{\partial^{2} g_{2}}{\partial c^{2}}(0, c)=0$ and it suffices to show $\frac{\partial^{3} g_{2}}{\partial b^{3}} \geq 0$. After simplification, we obtain

$$
\begin{equation*}
\frac{\partial^{3} g_{2}}{\partial b^{3}}=e^{b+c} e_{0}(b, c)+2 e^{b+c} e_{1}(b, c) b+2 e^{b+c} e_{2}(b, c) b^{2} \tag{46}
\end{equation*}
$$

where

$$
\begin{align*}
e_{0}(b, c)= & 2 c^{3}\left(8 e^{b+c}-1\right)+6\left(e^{c}-1\right)\left(16 e^{b+c}-7 e^{c}-7\right)+4 c\left(e^{c}-1\right)\left(22 e^{b+c}-5 e^{c}-5\right) \\
& +c^{2}\left(24 e^{b+2 c}+32 e^{b+c}-3 e^{2 c}-11\right)  \tag{47}\\
e_{1}(b, c)= & c^{2}\left(8 e^{b+c}-1\right)+\left(e^{c}-1\right)\left(80 e^{b+c}-19 e^{c}-19\right) \\
& +c\left(24 e^{b+2 c}-56 e^{b+c}-3 e^{2 c}+11\right)  \tag{48}\\
e_{2}(b, c)= & c\left(2-16 e^{b+c}\right)+3\left(e^{c}-1\right)\left(8 e^{b+c}-e^{c}-1\right) . \tag{49}
\end{align*}
$$

The proof that $e_{0}, e_{1}$, and $e_{2}$ are positive is similar to the proof of of positivity of $d_{0}, d_{1}$, and $d_{2}$, so we do not repeat it here. This completes the proof that $g(b+c, b)<0$, hence $g(a, b)$ is negative for all $a$ and $b$.
Proof of $f_{3} \leq 0$ :
We now show that $f_{3} \leq 0$. Let $a=k \gamma$ and $b=k \delta$, we need to show that

$$
\begin{equation*}
f(a, b)=e^{3 a+b}-e^{3 a}+b e^{2 b}\left(e^{a}-1\right)^{2}-(1+a) e^{2 a}\left(e^{2 b}-1\right)+(1+2 a)\left(e^{a+2 b}-e^{a+b}\right) \geq 0 \tag{50}
\end{equation*}
$$

We first show that $f(a, a+d) \geq f(a, a)=0$ for $d>0$. We have $f(a, a+d)=e^{2 a} g(a, d)$,
where

$$
\begin{equation*}
g(a, d)=a\left(e^{d}-1\right)^{2}+\left(e^{a}-1\right)\left[e^{d}-1-d e^{2 d}+e^{a+d}+(d-1) e^{a+2 d}\right] \tag{51}
\end{equation*}
$$

As $g(0, d)=0$, it suffices to show that $\frac{\partial g}{\partial a} \geq 0$. Using the inequality

$$
\begin{equation*}
1+(d-1) e^{d}=\sum_{n=2}^{\infty} \frac{n-1}{n!} d^{n} \geq 0 \tag{52}
\end{equation*}
$$

we obtain

$$
\begin{align*}
\frac{\partial g}{\partial a} & =\left(e^{a}-1\right)\left\{2 e^{a+d}\left[1+(d-1) e^{d}\right]-\left(e^{d}-1\right)^{2}\right\} \\
& \geq\left(e^{a}-1\right)\left\{2 e^{d}\left[1+(d-1) e^{d}\right]-\left(e^{d}-1\right)^{2}\right\} \\
& =\left(e^{a}-1\right)\left[-1+4 e^{d}+(2 d-3) e^{2 d}\right] \\
& =\left(e^{a}-1\right) \sum_{n=2}^{\infty} \frac{4+(n-3) 2^{n}}{n!} d^{n} \geq 0 \tag{53}
\end{align*}
$$

We next show that $f(b+c, b) \geq f(b, b)=0$ for $c>0$. We have $f(b+c, b)=e^{2 b} h(b, c)$, where

$$
\begin{equation*}
h(b, c)=b\left(e^{c}-1\right)^{2}+e^{c}\left(e^{b}-1\right)\left[\left(e^{c}-1\right)\left(e^{b+c}-1\right)-c\left(-2+e^{c}+e^{b+c}\right)\right] . \tag{54}
\end{equation*}
$$

As $h(0, c)=0$, it suffices to show that $\frac{\partial h}{\partial b} \geq 0$. We have

$$
\begin{align*}
\frac{\partial h}{\partial b} & =\left(e^{b+c}-1\right)\left[2 e^{b+c}\left(e^{c}-1-c\right)-\left(e^{c}-1\right)^{2}\right] \\
& \geq\left(e^{b+c}-1\right)\left[2 e^{c}\left(e^{c}-1-c\right)-\left(e^{c}-1\right)^{2}\right] \\
& =\left(e^{b+c}-1\right)\left(e^{2 c}-2 c e^{c}-1\right) \\
& =\left(e^{b+c}-1\right) \sum_{n=2}^{\infty} \frac{2^{n}-2 n}{n!} c^{n} \geq 0 . \tag{55}
\end{align*}
$$

This completes the proof that $f_{3} \leq 0$.

## OA. 6 Delta-hedged Option Return

As in Goyal and Saretto (2009), we consider static delta-hedged call option gain held-toexpiration given by

$$
\begin{equation*}
\Pi_{t, T}^{C}=C_{T}-\Delta_{t}^{C} S_{T}-\left(C_{t}-\Delta_{t}^{C} S_{t}\right) e^{r \tau} \tag{56}
\end{equation*}
$$

Hence, the expected Delta-hedged call option gain held-to-expiration can be computed as

$$
\begin{align*}
E_{t}\left[\Pi_{t, T}^{C}\right] & =E_{t}\left[C_{T}-\Delta_{t}^{C} S_{T}-\left(C_{t}-\Delta_{t}^{C} S_{t}\right) e^{r \tau}\right] \\
& =E_{t}\left[C_{T}\right]-\Delta_{t}^{C} E_{t}\left[S_{T}\right]-\left(C_{t}-\Delta_{t}^{C} S_{t}\right) e^{r \tau} \\
& =E_{t}\left[C_{T}\right]-\Delta_{t}^{C} S_{t} e^{\tau \mu+\frac{\tau \sigma_{\tau}^{2}}{2}}-\left(C_{t}-\Delta_{t}^{C} S_{t}\right) e^{r \tau} . \tag{57}
\end{align*}
$$

Now, if we take the partial derivative of the above expected gain with respect to $\sigma_{\tau}$, which has equivalent sign as taking partial derivative with respect to the first-order autocorrelation, using (15) we get

$$
\begin{align*}
\frac{\partial E_{t}\left[\Pi_{t, T}^{C}\right]}{\partial \sigma_{\tau}} & =\frac{\partial E_{t}\left[C_{T}\right]}{\partial \sigma_{\tau}}-\Delta_{t}^{C} S_{t} e^{\tau \mu+\frac{\tau \sigma_{\tau}^{2}}{2}} \tau \sigma_{\tau} \\
& =e^{r \tau} S_{t}^{*} \tau \sigma_{\tau}\left[\left(\Phi\left(d_{1}^{*}\right)-\Phi\left(d_{1}\right)\right)+\frac{\phi\left(d_{1}^{*}\right)}{\sigma_{\tau} \sqrt{\tau}}\right] \tag{58}
\end{align*}
$$

For put option, we have

$$
\begin{equation*}
E_{t}\left[\Pi_{t, T}^{P}\right]=E_{t}\left[P_{T}\right]-\Delta_{t}^{P} S_{t} e^{\tau \mu+\frac{\tau \sigma_{\tau}^{2}}{2}}-\left(P_{t}-\Delta_{t}^{P} S_{t}\right) e^{r \tau} \tag{59}
\end{equation*}
$$

Using (16), we get

$$
\begin{align*}
\frac{\partial E_{t}\left[\Pi_{t, T}^{P}\right]}{\partial \sigma_{\tau}} & =\frac{\partial E_{t}\left[P_{T}\right]}{\partial \sigma_{\tau}}-\Delta_{t}^{P} S_{t} e^{\tau \mu+\frac{\tau \sigma_{\tau}^{2}}{2}} \tau \sigma_{\tau} \\
& =e^{r \tau} S_{t}^{*} \tau \sigma_{\tau}\left[-\Phi\left(-d_{1}^{*}\right)+\frac{\phi\left(d_{1}^{*}\right)}{\sigma_{\tau} \sqrt{\tau}}+\Phi\left(-d_{1}\right)\right] \\
& =e^{r \tau} S_{t}^{*} \tau \sigma_{\tau}\left[\left(\Phi\left(d_{1}^{*}\right)-\Phi\left(d_{1}\right)\right)+\frac{\phi\left(d_{1}^{*}\right)}{\sigma_{\tau} \sqrt{\tau}}\right] \\
& =\frac{\partial E_{t}\left[\Pi_{t, T}^{C}\right]}{\partial \sigma_{\tau}} \tag{60}
\end{align*}
$$

The last equality also follows from the put-call parity. Since $\Phi\left(-d_{1}\right)$ is always positive, if $\partial E_{t}\left[P_{T}\right] / \partial \sigma_{\tau}$ was positive, then the delta-hedged put option gain has also positive partial derivative with respect to the first-order autocorrelation. Hence, we conclude that deltahedged option gain also follows the same pattern as the raw return in our case.

## OA. 7 Expected Option Return under the Stochastic Volatility Model

In this section, we provide analytical expressions for computing expected option returns under the stochastic volatility model discussed in Section 4.3. Expected held-to-expiration call option return is defined by

$$
\begin{equation*}
\frac{E_{t}\left[\max \left(S_{T}-K, 0\right)\right]}{E_{t}^{\mathbb{Q}}\left[\max \left(S_{T}-K, 0\right)\right]}-1 \tag{61}
\end{equation*}
$$

Since the model is cast in affine form, we can apply the standard result to obtain conditional characteristic function of log terminal stock price. The characteristic function of the log-spot price under the physical measure is then given by

$$
\begin{align*}
\left.E_{t}\left[\exp \left(i u \log \left(S_{t+\tau}\right)\right]\right)\right] & =f\left(u, \tau, \log \left(S_{t}\right), V_{t}, X_{t}\right) \\
& =\exp \left(A(u, \tau)+B_{0}(u, \tau) \log \left(S_{t}\right)+B_{1}(u, \tau) V_{t}+B_{2}(u, \tau) X_{t}\right) \tag{62}
\end{align*}
$$

where $A, B_{1}$, and $B_{2}$ are given as the solution to the following Ricatti ODE with the initial conditions $A(0)=B_{1}(0)=B_{2}(0)=0$,

$$
\begin{align*}
\frac{d A}{d \tau} & =(\mu+\gamma \mu(T-\tau)) i u e^{-\gamma(T-\tau)}+\kappa \theta B_{1}+\frac{1}{2} \sigma_{x}^{2} B_{2} \\
\frac{d B_{1}}{d \tau} & =-\frac{1}{2} u^{2} e^{-2 \gamma(T-\tau)}+\left(\xi \rho i u e^{-\gamma(T-\tau)}-\kappa\right) B_{1}+\frac{1}{2} \xi^{2} B_{1}^{2} \\
\frac{d B_{2}}{d \tau} & =\lambda i u e^{-\gamma(T-\tau)}-\delta B_{2} \tag{63}
\end{align*}
$$

and $B_{0}(u, \tau)=i u e^{-\gamma(T-\tau)}$. The above system of Ricatti equations can be solved numerically using standard techniques such as the fourth-order Runge-Kutta method. Once the characteristic function is available in a closed-form, expected call option payoff can be valued using the formula following Heston (1993)

$$
\begin{equation*}
C_{t}=S_{t} P_{1}-K e^{-r \tau} P_{2}, \tag{64}
\end{equation*}
$$

where the $P_{1}$ and $P_{2}$ probabilities are computed using Fourier inversion

$$
P_{1}=\frac{1}{2}+\frac{1}{\pi} \int_{0}^{\infty} \operatorname{Re}\left[\frac{e^{i u \log \left(\frac{S_{t}}{K}\right)} f\left(u+1, \tau, \log \left(S_{t}\right), V_{t}, X_{t}\right)}{i u S_{t} e^{r \tau}}\right] d u,
$$

$$
\begin{equation*}
P_{2}=\frac{1}{2}+\frac{1}{\pi} \int_{0}^{\infty} \operatorname{Re}\left[\frac{e^{i u \log \left(\frac{S_{t}}{K}\right)} f\left(u, \tau, \log \left(S_{t}\right), V_{t}, X_{t}\right)}{i u}\right] d u \tag{65}
\end{equation*}
$$

The integrands in the above expressions vanish quickly and can be computed effectively using a numerical integration scheme such as quadrature.

On the other hand, the model is identical to the Heston (1993)'s stochastic volatility model under the risk-neutral measure. Therefore, the denominator term, which is simply the option price, can be computed following Heston (1993). Since we know the functional form of the conditional characteristic function, a similar method can be applied to compute expected put option return as well.

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Table OA. 1
Description of Control Variables in the Fama-MacBeth Regression

| Variable | Description |
| :--- | :--- |
| skew | skew is the physical skewness calculated using the past 22 -day daily returns for each <br> stock. |
| ex-ante_skew | ex-ante_skew is the risk-neutral skewness calculated based on Boyer and Vorkink (2014). |
| bm_ratio | bm_ratio for June of year $t-1$ to May of year $t$ is computed as the ratio of the book value <br> of common equity in fiscal year $t-1$ to the market value of equity (size) in December <br> of year $t-1$. Book equity is the book value of stockholders' equity, plus balance sheet <br> deferred taxes and investment tax credit (if available), minus the book value of preferred <br> stock. |
| size | size is the natural logarithm of a firm's market cap at the end of each month, and market <br> cap is defined as the product of the closing price and the number of shares outstanding <br> (in millions of dollars). |
| beta is the beta coefficient of each underlying stock based on the CAPM. |  |

This table lists predictors used as control variables in Table 4 of the Fama-MacBeth regression.

Table OA. 2
Portfolio Sorted by Stock Return Autocorrelation

| Panel A. Equal-weighted Portfolio |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Call Option | Put Option | Delta-hedged Call | Delta-hedged Put | Straddle | Underlying Stock |
| Low | 0.050 | -0.143 | -0.0055 | -0.0068 | -0.041 | 0.0107 |
| 2 | 0.076 | -0.129 | $-0.0036$ | -0.0048 | -0.023 | 0.0114 |
| 3 | 0.090 | -0.124 | -0.0034 | -0.0047 | -0.015 | 0.0117 |
| 4 | 0.084 | -0.097 | -0.0021 | -0.0042 | -0.011 | 0.0113 |
| High | 0.088 | -0.079 | -0.0017 | -0.0040 | 0.002 | 0.0105 |
| High-Low | 0.037 | 0.064 | 0.0038 | 0.0028 | 0.043 | -0.0002 |
| $t$-stat | (2.60) | (4.61) | (4.07) | (3.46) | (5.72) | (-0.10) |
| Two Option-factor alpha | 0.043 | 0.058 | 0.0050 | 0.0038 | 0.043 | -0.0011 |
| $t$-stat | (2.90) | (4.12) | (6.04) | (5.05) | (5.58) | (-0.57) |
| Panel B: Security Price-weighted Portfolio |  |  |  |  |  |  |
|  | Call Option | Put Option | Delta-hedged Call | Delta-hedged Put | Straddle | Underlying Stock |
| Low | 0.040 | -0.133 | -0.0041 | -0.0056 | -0.045 | 0.0098 |
| 2 | 0.070 | -0.135 | -0.0024 | -0.0045 | -0.028 | 0.0116 |
| 3 | 0.090 | -0.132 | -0.0020 | -0.0037 | -0.017 | 0.0123 |
| 4 | 0.082 | -0.116 | -0.0018 | -0.0039 | -0.018 | 0.0125 |
| High | 0.076 | -0.083 | $-0.0007$ | $-0.0028$ | -0.001 | 0.0118 |
| High-Low | 0.036 | 0.051 | 0.0034 | 0.0028 | 0.043 | 0.0020 |
| $t$-stat | (2.35) | (3.31) | (4.11) | $(3.54)$ | $(4.96)$ | (1.16) |
| Two Option-factor alpha | 0.043 | 0.046 | 0.0044 | 0.0036 | 0.045 | 0.0013 |
| $t$-stat | (2.78) | (2.92) | (5.74) | (4.86) | (5.05) | (0.74) |

This table summarizes the average returns in monthly frequencies for portfolios sorted by the stock return autocorrelation and hold for one month. Panel A reports the equal-weighted average returns, while Panel B reports the security price-weighted average returns assuming we invest equal shares for all firms in the portfolio. In Panel B, for call option, put option, and stock portfolios, the weights are based on the corresponding security prices. For delta-hedged call, delta-hedged put, and straddle, the weights are based on the initial investment for each firm in the portfolio. We follow Zhan, Han, Cao, and Tong (2022) to construct a two option-factor model: illiquidity and idiosyncratic volatility. The factor realizations in each month are obtained as the high-minus-low spread returns of stock value-weighted portfolios of writing delta-neutral calls sorted on the idiosyncratic volatility or the Amihud illiquidity measure of the underlying stock. The alpha is calculated based on the two option-factor model. The sample period is from January 1996 to December 2020.

Table OA. 3
Option Portfolios Double Sorted by Stock Return Autocorrelation and Other Stock Characteristics

| Panel A: Double Sorting Call Option Return |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Sorted by Realized Volatility | Low | 2 | 3 | 4 | High |
| $\alpha \text { of High } \rho-\text { Low } \rho$ $t \text {-stat of } \alpha$ | $\begin{aligned} & 0.061 \\ & (2.60) \end{aligned}$ | $\begin{aligned} & 0.086 \\ & (3.66) \end{aligned}$ | $\begin{aligned} & 0.060 \\ & (2.75) \end{aligned}$ | $\begin{aligned} & 0.035 \\ & (1.50) \end{aligned}$ | $\begin{aligned} & 0.031 \\ & (1.18) \end{aligned}$ |
| Sorted by Idiosyncratic Volatility | Low | 2 | 3 | 4 | High |
| $\alpha \text { of High } \rho-\operatorname{Low} \rho$ $t \text {-stat of } \alpha$ | $\begin{aligned} & 0.071 \\ & (2.88) \end{aligned}$ | $\begin{aligned} & 0.071 \\ & (3.13) \end{aligned}$ | $\begin{aligned} & 0.070 \\ & (3.14) \end{aligned}$ | $\begin{aligned} & 0.041 \\ & (1.75) \end{aligned}$ | $\begin{aligned} & 0.002 \\ & (0.06) \end{aligned}$ |
| Sorted by Variance Risk Premium | Low | 2 | 3 | 4 | High |
| $\begin{gathered} \alpha \text { of High } \rho-\text { Low } \rho \\ t \text {-stat of } \alpha \end{gathered}$ | $\begin{aligned} & 0.038 \\ & (1.63) \end{aligned}$ | $\begin{aligned} & 0.032 \\ & (1.40) \end{aligned}$ | $\begin{aligned} & 0.062 \\ & (2.65) \end{aligned}$ | $\begin{aligned} & 0.037 \\ & (1.55) \end{aligned}$ | $\begin{aligned} & 0.064 \\ & (2.85) \end{aligned}$ |
| Sorted by ILIQ | Low | 2 | 3 | 4 | High |
| $\alpha$ of High $\rho-$ Low $\rho$ $t$-stat of $\alpha$ | $\begin{aligned} & 0.051 \\ & (2.23) \end{aligned}$ | $\begin{aligned} & 0.034 \\ & (1.45) \end{aligned}$ | $\begin{aligned} & 0.033 \\ & (1.39) \end{aligned}$ | $\begin{aligned} & 0.023 \\ & (1.02) \end{aligned}$ | $\begin{aligned} & 0.053 \\ & (1.95) \end{aligned}$ |
| Sorted by IVTS | Low | 2 | 3 | 4 | High |
| $\alpha \text { of High } \rho-\operatorname{Low} \rho$ $t$-stat of $\alpha$ | $\begin{aligned} & 0.012 \\ & (0.54) \end{aligned}$ | $\begin{aligned} & 0.053 \\ & (2.31) \end{aligned}$ | $\begin{aligned} & 0.040 \\ & (1.70) \end{aligned}$ | $\begin{aligned} & 0.074 \\ & (3.37) \end{aligned}$ | $\begin{aligned} & 0.025 \\ & (1.05) \end{aligned}$ |
| Panel B: Double Sorting Put Option Return |  |  |  |  |  |
| Sorted by Realized Volatility | Low | 2 | 3 | 4 | High |
| $\alpha$ of High $\rho-$ Low $\rho$ $t$-stat of $\alpha$ | $\begin{aligned} & 0.012 \\ & (0.45) \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.054 \\ & (2.53) \\ & \hline \end{aligned}$ | $\begin{array}{r} 0.064 \\ (3.27) \\ \hline \end{array}$ | $\begin{aligned} & 0.094 \\ & (4.12) \\ & \hline \end{aligned}$ | $\begin{array}{r} 0.022 \\ (0.93) \\ \hline \end{array}$ |
| Sorted by Idiosyncratic Volatility | Low | 2 | 3 | 4 | High |
| $\alpha$ of High $\rho-$ Low $\rho$ $t$-stat of $\alpha$ | $\begin{aligned} & 0.028 \\ & (1.01) \end{aligned}$ | $\begin{aligned} & 0.040 \\ & (1.92) \end{aligned}$ | $\begin{aligned} & 0.086 \\ & (4.12) \end{aligned}$ | $\begin{aligned} & 0.029 \\ & (1.24) \end{aligned}$ | $\begin{aligned} & 0.062 \\ & (2.76) \end{aligned}$ |
| Sorted by Variance Risk Premium | Low | 2 | 3 | 4 | High |
| $\alpha \text { of High } \rho-\operatorname{Low} \rho$ $t$-stat of $\alpha$ | $\begin{aligned} & 0.089 \\ & (4.19) \end{aligned}$ | $\begin{aligned} & 0.048 \\ & (1.96) \end{aligned}$ | $\begin{aligned} & 0.065 \\ & (3.00) \end{aligned}$ | $\begin{aligned} & 0.051 \\ & (2.26) \end{aligned}$ | $\begin{aligned} & 0.059 \\ & (2.73) \end{aligned}$ |
| Sorted by ILIQ | Low | 2 | 3 | 4 | High |
| $\alpha$ of High $\rho-$ Low $\rho$ $t$-stat of $\alpha$ | $\begin{aligned} & 0.055 \\ & (2.46) \end{aligned}$ | $\begin{aligned} & 0.075 \\ & (3.24) \end{aligned}$ | $\begin{aligned} & 0.058 \\ & (2.51) \end{aligned}$ | $\begin{aligned} & 0.039 \\ & (1.98) \end{aligned}$ | $\begin{aligned} & 0.073 \\ & (3.01) \end{aligned}$ |
| Sorted by IVTS | Low | 2 | 3 | 4 | High |
| $\alpha \text { of High } \rho-\text { Low } \rho$ $t \text {-stat of } \alpha$ | $\begin{aligned} & 0.087 \\ & (4.52) \end{aligned}$ | $\begin{aligned} & 0.058 \\ & (2.62) \end{aligned}$ | $\begin{aligned} & 0.049 \\ & (2.11) \end{aligned}$ | $\begin{aligned} & 0.049 \\ & (2.40) \end{aligned}$ | $\begin{aligned} & 0.061 \\ & (2.71) \end{aligned}$ |

In this table, we conduct an unconditional sorting based on a certain stock characteristic and stock return autocorrelation, in total twenty-five bins in two dimensions. We classify a certain security into each bin based on the cutoffs of the sorted characteristic and stock return autocorrelation. ILIQ stands for the stock illiquidity computed following Amihud (2002) and IVTS denotes the implied volatility term structure defined in Section 3. Within each bin we compute the difference of average returns between the high and low stock return autocorrelation quintile. We follow Zhan et al. (2022) to construct a two option-factor model: illiquidity and idiosyncratic volatility. The factor realizations in each month are obtained as the high-minus-low spread returns of stock-value-weighted portfolios of writing delta-neutral calls sorted on the idiosyncratic volatility or the Amihud illiquidity measure of the underlying stock. We report the alpha and the corresponding t-stat based on the two option-factor model for equal-weighted call and put option portfolios.

Table OA. 4
Option Portfolios Double Sorted by Stock Return Autocorrelation and Other Stock Characteristics

| Panel A: Double Sorting Delta-hedged Call Option Return |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Sorted by Realized Volatility | Low | 2 | 3 | 4 | High |
| High $\rho$ - Low $\rho$ $t$-stat | $\begin{aligned} & 0.002 \\ & (2.89) \end{aligned}$ | $\begin{aligned} & 0.005 \\ & (5.06) \end{aligned}$ | $\begin{aligned} & 0.005 \\ & (4.54) \end{aligned}$ | $\begin{aligned} & 0.007 \\ & (3.91) \end{aligned}$ | $\begin{aligned} & 0.008 \\ & (3.36) \end{aligned}$ |
| Sorted by Idiosyncratic Volatility | Low | 2 | 3 | 4 | High |
| High $\rho$ - Low $\rho$ $t$-stat | $\begin{aligned} & 0.004 \\ & (3.93) \end{aligned}$ | $\begin{aligned} & 0.005 \\ & (4.90) \end{aligned}$ | $\begin{aligned} & 0.006 \\ & (4.83) \end{aligned}$ | $\begin{aligned} & 0.006 \\ & (3.82) \end{aligned}$ | $\begin{aligned} & 0.006 \\ & (2.67) \end{aligned}$ |
| Sorted by Variance Risk Premium | Low | 2 | 3 | 4 | High |
| High $\rho$ - Low $\rho$ <br> $t$-stat | $\begin{aligned} & \hline 0.006 \\ & (3.94) \end{aligned}$ | $\begin{aligned} & \hline 0.003 \\ & (2.56) \end{aligned}$ | $\begin{aligned} & \hline 0.004 \\ & (3.71) \end{aligned}$ | $\begin{aligned} & 0.005 \\ & (3.76) \end{aligned}$ | $\begin{aligned} & 0.004 \\ & (2.49) \end{aligned}$ |
| Sorted by ILIQ | Low | 2 | 3 | 4 | High |
| High $\rho$ - Low $\rho$ <br> $t$-stat | $\begin{aligned} & 0.003 \\ & (2.85) \end{aligned}$ | $\begin{aligned} & \hline 0.004 \\ & (2.96) \end{aligned}$ | $\begin{aligned} & 0.003 \\ & (2.22) \end{aligned}$ | $\begin{aligned} & 0.004 \\ & (2.93) \end{aligned}$ | $\begin{aligned} & 0.006 \\ & (2.97) \end{aligned}$ |
| Sorted by IVTS | Low | 2 | 3 | 4 | High |
| High $\rho$ - Low $\rho$ $t$-stat | $\begin{array}{r} 0.004 \\ (2.22) \\ \hline \end{array}$ | $\begin{aligned} & 0.005 \\ & (3.58) \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.003 \\ & (2.72) \\ & \hline \end{aligned}$ | $\begin{array}{r} 0.005 \\ (4.43) \\ \hline \end{array}$ | $\begin{array}{r} 0.005 \\ (3.36) \\ \hline \end{array}$ |
| Panel B: Double Sorting Delta-hedged Put Option Return |  |  |  |  |  |
| Sorted by Realized Volatility | Low | 2 | 3 | 4 | High |
| High $\rho$ - Low $\rho$ $t$-stat | $\begin{aligned} & 0.002 \\ & (2.16) \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.004 \\ & (3.99) \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.005 \\ & (4.20) \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.007 \\ & (4.43) \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.003 \\ & (1.56) \\ & \hline \end{aligned}$ |
| Sorted by Idiosyncratic Volatility | Low | 2 | 3 | 4 | High |
| High $\rho$ - Low $\rho$ <br> $t$-stat | $\begin{aligned} & 0.003 \\ & (2.83) \end{aligned}$ | $\begin{aligned} & 0.004 \\ & (4.59) \end{aligned}$ | $\begin{aligned} & \hline 0.005 \\ & (4.62) \end{aligned}$ | $\begin{aligned} & 0.005 \\ & (3.19) \end{aligned}$ | $\begin{aligned} & 0.002 \\ & (1.22) \end{aligned}$ |
| Sorted by Variance Risk Premium | Low | 2 | 3 | 4 | High |
| High $\rho$ - Low $\rho$ $t$-stat | $\begin{aligned} & 0.004 \\ & (2.98) \end{aligned}$ | $\begin{aligned} & 0.002 \\ & (1.46) \end{aligned}$ | $\begin{aligned} & 0.004 \\ & (3.09) \end{aligned}$ | $\begin{aligned} & 0.004 \\ & (3.23) \end{aligned}$ | $\begin{aligned} & 0.004 \\ & (2.52) \end{aligned}$ |
| Sorted by ILIQ | Low | 2 | 3 | 4 | High |
| High $\rho$ - Low $\rho$ $t$-stat | $\begin{aligned} & 0.003 \\ & (2.94) \end{aligned}$ | $\begin{aligned} & 0.003 \\ & (2.46) \end{aligned}$ | $\begin{aligned} & 0.003 \\ & (2.33) \end{aligned}$ | $\begin{aligned} & 0.002 \\ & (1.59) \end{aligned}$ | $\begin{aligned} & 0.004 \\ & (2.03) \end{aligned}$ |
| Sorted by IVTS | Low | 2 | 3 | 4 | High |
| High $\rho$ - Low $\rho$ $t$-stat | $\begin{aligned} & 0.003 \\ & (2.12) \end{aligned}$ | $\begin{aligned} & 0.004 \\ & (2.87) \end{aligned}$ | $\begin{aligned} & 0.003 \\ & (2.18) \end{aligned}$ | $\begin{aligned} & 0.004 \\ & (3.90) \end{aligned}$ | $\begin{aligned} & 0.003 \\ & (2.08) \end{aligned}$ |

In this table, we conduct an unconditional sorting based on a certain stock characteristic and stock return autocorrelation, in total twenty-five bins in two dimensions. We classify a certain security into each bin based on the cutoffs of the sorted characteristic and stock return autocorrelation. ILIQ stands for the stock illiquidity computed following Amihud (2002) and IVTS denotes the implied volatility term structure defined in Section 3. Within each bin we compute the difference of average returns between the high and low stock return autocorrelation quintile. We show the results for equal-weighted delta-hedged call and delta-hedged put portfolios.

Table OA. 5

## Option Portfolios Double Sorted by Stock Return Autocorrelation and Other Stock Characteristics

| Panel A: Double Sorting Straddle Return |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Sorted by Realized Volatility | Low | 2 | 3 | 4 | High |
| High $\rho$ - Low $\rho$ | 0.049 | 0.058 | 0.042 | 0.055 | 0.026 |
| $t$-stat | (3.51) | (4.40) | (3.31) | (4.16) | (1.66) |
| Sorted by Idiosyncratic Volatility | Low | 2 | 3 | 4 | High |
| High $\rho$ - Low $\rho$ | $0.056$ | 0.048 | 0.057 | 0.038 | 0.025 |
| $t$-stat | (3.81) | $(3.85)$ | (4.79) | (2.83) | $(1.66)$ |
| Sorted by Variance Risk Premium | Low | 2 | 3 | 4 | High |
| High $\rho$ - Low $\rho$ | 0.049 | 0.033 | 0.055 | 0.052 | 0.050 |
| $t$-stat | (3.88) | (2.66) | (4.49) | (3.74) | (3.78) |
| Sorted by ILIQ | Low | 2 | 3 | 4 | High |
| High $\rho$ - Low $\rho$ | 0.049 | 0.049 | 0.042 | 0.037 | 0.022 |
| $t$-stat | (4.40) | (3.78) | (3.37) | (2.71) | (1.15) |
| Sorted by IVTS | Low | 2 | 3 | 4 | High |
| High $\rho$ - Low $\rho$ | 0.041 | 0.053 | 0.040 | 0.052 | 0.040 |
| $t$-stat | $(3.00)$ | (4.28) | (2.93) | (4.21) | (3.09) |
| Panel B: Double Sorting Straddle Return (Alpha) |  |  |  |  |  |
| Sorted by Realized Volatility | Low | 2 | 3 | 4 | High |
| $\alpha$ of High $\rho$ - Low $\rho$ | 0.045 | 0.056 | 0.041 | 0.053 | 0.022 |
| $t$-stat of $\alpha$ | (3.15) | (4.21) | (3.15) | (3.91) | (1.37) |
| Sorted by Idiosyncratic Volatility | Low | 2 | 3 | 4 | High |
| $\alpha$ of High $\rho-$ Low $\rho$ | 0.058 | 0.042 | 0.057 | 0.035 | 0.024 |
| $t$-stat of $\alpha$ | (3.81) | (3.30) | (4.65) | (2.53) | (1.53) |
| Sorted by Variance Risk Premium | Low | 2 | 3 | 4 | High |
| $\alpha$ of High $\rho$ - Low $\rho$ | 0.049 | 0.032 | 0.061 | 0.048 | 0.046 |
| $t$-stat of $\alpha$ | $(3.84)$ | (2.48) |  | (3.38) | (3.41) |
| Sorted by ILIQ | Low | 2 | 3 | 4 | High |
| $\alpha$ of High $\rho-$ Low $\rho$ | $0.049$ | $0.050$ | 0.038 | 0.032 | 0.010 |
| $t$-stat of $\alpha$ | (4.40) | $(3.83)$ |  |  |  |
| Sorted by IVTS | Low | 2 | 3 | 4 | High |
| $\alpha$ of High $\rho-$ Low $\rho$ | 0.040 | 0.049 | 0.039 | 0.054 | 0.042 |
| $t$-stat of $\alpha$ | (2.83) | (3.88) | (2.77) | (4.22) | (3.20) |

In this table, we conduct an unconditional sorting based on a certain stock characteristic and stock return autocorrelation, in total twenty-five bins in two dimensions. We classify a certain security into each bin based on the cutoffs of the sorted characteristic and stock return autocorrelation. ILIQ stands for the stock illiquidity computed following Amihud (2002) and IVTS denotes the implied volatility term structure defined in Section 3. Within each bin we compute the difference of average returns between the high and low stock return autocorrelation quintile. In Panel A, we show the results for equal-weighted straddle portfolios. In Panel B, we follow Zhan et al. (2022) to construct a two option-factor model: illiquidity and idiosyncratic volatility. The factor realizations in each month are obtained as the high-minus-low spread returns of stock-value-weighted portfolios of writing delta-neutral calls sorted on the idiosyncratic volatility or the Amihud illiquidity measure of the underlying stock. We report the alpha and the corresponding t-stat based on the two option-factor model for straddle portfolios.

Table OA. 6
Fama-MacBeth Regressions with Stock Return Autocorrelation

|  | Call Option | Put Option | Delta-hedged <br> Call | Delta-hedged <br> Put | Straddle | Underlying <br> Stock |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Intercept | 7.761 | -11.415 | -0.326 | -0.490 | -1.766 | 1.113 |
| $t$-stat | $(2.42)$ | $(-2.28)$ | $(-1.70)$ | $(-2.74)$ | $(-0.98)$ | $(2.74)$ |
| Autocorrelation | 1.200 | 2.284 | 0.125 | 0.083 | 1.363 | -0.009 |
| $t$-stat | $(2.48)$ | $(4.87)$ | $(3.86)$ | $(2.97)$ | $(5.67)$ | $(-0.12)$ |
| Average adj. $R^{2}(\%)$ | 0.29 | 0.32 | 0.25 | 0.26 | 0.28 | 0.54 |

This table reports the Fama-MacBeth regressions for each dependent variable that is the return of different securities specified at the top of each column. The independent variable is stock return autocorrelation. All predictors are normalized to have mean zero and standard deviation of one at each month. The detailed cross-sectional regression and time-series test are specified in Section 3.3. All dependent and independent variables are expressed as monthly values and the coefficients are multiplied by 100 . The coefficients in the table are calculated by taking the time-series average of the cross-sectional regressions over time. The t-stat reported is the t-test with Newey-West one-lag correction. The sample period is from January 1996 to December 2020.


Figure OA. 1
Expected Delta-hedged Call Option Gain under the Trending O-U Process
This figure plots the expected hold-to-expiration call option gain as a function of first-order autocorrelation of stock returns under the trending O-U process. All options are at-the-money options with the following parameters: $\mu=0.10, r=0.05, \tau=1 / 12$, and $\sigma=0.2$.

