

## **MEAN REVERSION IN STOCK PRICES Evidence and Implications\***

**James M. POTERBA**

*Massachusetts Institute of Technology, Cambridge, MA 02139, USA  
National Bureau of Economic Research, Cambridge, MA 02138, USA*

**Lawrence H. SUMMERS**

*Harvard University, Boston, MA 02138, USA  
National Bureau of Economic Research, Cambridge, MA 02138, USA*

Received August 1987, final version received March 1988

**This paper investigates transitory components in stock prices. After showing that statistical tests have little power to detect persistent deviations between market prices and fundamental values, we consider whether prices are mean-reverting, using data from the United States and 17 other countries. Our point estimates imply positive autocorrelation in returns over short horizons and negative autocorrelation over longer horizons, although random-walk price behavior cannot be rejected at conventional statistical levels. Substantial movements in required returns are needed to account for these correlation patterns. Persistent, but transitory, disparities between prices and fundamental values could also explain our findings.**

### **1. Introduction**

**The extent to which stock prices exhibit mean-reverting behavior is crucial in assessing assertions such as Keynes' (1936) that 'all sorts of considerations enter into market valuation which are in no way relevant to the prospective yield' (p. 152). If market and fundamental values diverge, but beyond some range the differences are eliminated by speculative forces, then stock prices will revert to their mean. Returns must be negatively serially correlated at**

**\*We are grateful to Changyong Khee, Jeff Zweibel and, especially, David Cutler for excellent research assistance, to Ben Bernanke, Fischer Black, Olivier Blanchard, John Campbell, Robert Engle, Eugene Fama, Terence Gorman, Pete Kyle, Andrew Lo, Greg Mankiw, Robert Merton, Julio Rotemberg, Kenneth Singleton, Mark Watson, an anonymous referee, and the editor, William Schwert, for helpful comments, and to James Darcel and Matthew Shapiro for data assistance. This research was supported by the National Science Foundation and a Batterymarch Financial Fellowship to the first author, and is part of the NBER Programs in Economic Fluctuations and Financial Markets. A data appendix is on file with the ICPSR in Ann Arbor, Michigan.**

some frequency if 'erroneous' market moves are eventually corrected.<sup>1</sup> Merton (1987) notes that reasoning of this type has been used to draw conclusions about market valuation from failure to reject the absence of negative serial correlation in returns. Conversely, the presence of negative autocorrelation may signal departures from fundamental values, although it could also arise from variation in risk factors over time.

Our investigation of mean reversion in stock prices is organized as follows. Section 2 evaluates alternative statistical tests for transitory price components. We find that variance-ratio tests of the type used by Fama and French (1986a) and Lo and MacKinlay (1988) are close to the most powerful tests of the null hypothesis of market efficiency with constant required returns against plausible alternative hypotheses such as the fads model suggested by Shiller (1984) and Summers (1986). These tests nevertheless have little power, even with monthly data for a 60-year period. We conclude that a sensible balancing of Type I and Type II errors suggests using critical values above the conventional 0.05 level.

Section 3 examines the extent of mean reversion in stock prices. For the U.S. we analyze monthly data on real and excess New York Stock Exchange (NYSE) returns since 1926, as well as annual returns data for the 1871–1985 period. We also analyze 17 other equity markets and study the mean-reverting behavior of individual corporate securities in the U.S. The results consistently suggest the presence of transitory components in stock prices, with returns showing positive autocorrelation over short periods but negative autocorrelation over longer periods.

Section 4 uses our variance-ratio estimates to gauge the significance of transitory price components. For the U.S. we find the standard deviation of the transitory price component varies between 15% and 25% of value, depending on our assumption about its persistence. The point estimates imply that transitory components account for more than half of the monthly return variance, a finding confirmed by international evidence.

Section 5 investigates whether observed patterns of mean reversion and the associated movements in *ex ante* returns are better explained by shifts in required returns due to changes in interest rates or market volatility or as byproducts of noise trading.<sup>2</sup> We argue that it is difficult to account for observed transitory components on the basis of changes in discount rates. The

<sup>1</sup>Stochastic speculative bubbles, considered by Blanchard and Watson (1982), could create deviations between market prices and fundamental values without negative serial correlation in returns. In the presence of any limits on valuation errors set by speculators or real investment opportunities, however, such bubbles could not exist.

<sup>2</sup>Noise traders are investors whose demands for securities are best treated as exogenous, rather than the result of maximizing a conventional utility function using rational expectations of the return distribution. Black (1986), Campbell and Kyle (1986), DeLong et al. (1987), and Shiller (1984) discuss a variety of possible models for noise trader behavior.

conclusion discusses some implications of our results and directions for future research.

## **2. Methodological issues involved in testing for transitory components**

A vast literature dating at least to Kendall (1953) has tested the efficient-markets/constant-required-returns model by examining individual autocorrelations in security returns. The early literature, surveyed in Fama (1970), found little evidence of patterns in security returns and is frequently adduced in support of the efficient-markets hypothesis. Recent work by Shiller and Perron (1985) and Summers (1986) has shown that such tests have relatively little power against interesting alternatives to the null hypothesis of market efficiency with constant required returns. Several recent studies using new tests for serial dependence have nonetheless rejected the random-walk model.<sup>3</sup>

This section begins by describing several possible tests for the presence of stationary stock-price components, including those used in recent studies. We then present Monte Carlo evidence on each test's power against plausible alternatives to the null hypothesis of serially independent returns. Even the most powerful tests have little power against these alternatives to the random walk when we specify the conventional size of 0.05. We conclude with a discussion of test design when the data can only weakly differentiate alternative hypotheses, addressing in particular the degree of presumption that should be accorded to our null hypothesis of serially independent returns.

### **2.1. Test methods**

Recent studies use different but related tests for mean reversion. Fama and French (1986a) and Lo and MacKinlay (1988) compare the relative variability of returns over different horizons using variance-ratio tests. Fama and French (1988b) use regression tests that also involve studying the serial correlation in multiperiod returns. Campbell and Mankiw (1987) study the importance of transitory components in real output using parametric ARMA models. Each of these approaches involves using a particular function of the sample autocorrelations to test the hypothesis that all autocorrelations equal zero.

The variance-ratio test exploits the fact that if the logarithm of the stock price, including cumulated dividends, follows a random walk, the return

<sup>3</sup>Fama (1976) acknowledges the difficulty of distinguishing the random-walk model from some alternative specifications. In addition to the recent work of Fama and French (1988b) and Lo and MacKinlay (1988), O'Brien (1987) demonstrates the presence of negative serial correlation at very long (up to twenty-year) horizons. Huizinga (1987) provides a spectral interpretation of the variance-ratio estimator and reports evidence that exchange rates also show long-horizon deviations from random-walk behavior.

variance should be proportional to the return horizon.<sup>4</sup> We study the variability of returns at different horizons, in relation to the variation over a one-year period.<sup>5</sup> For monthly returns, the variance-ratio statistic is therefore

$$VR(k) = \frac{\text{var}(R_t^k)}{k} \bigg/ \frac{\text{var}(R_t^{12})}{12}, \quad (1)$$

where

$$R_t^k = \sum_{i=0}^{k-1} R_{t-i},$$

$R_t$  denoting the total return in month  $t$ . This statistic converges to unity if returns are uncorrelated through time. If some of the price variation is due to transitory factors, however, autocorrelations at some lags will be negative and the variance ratio will fall below one. The statistics reported below are corrected for small-sample bias by dividing by  $E[VR(k)]$ .<sup>6</sup>

The variance ratio is closely related to earlier tests based on estimated autocorrelations. Using Cochrane's (1988) result that the ratio of the  $k$ -month return variance to  $k$  times the one-month return variance is approximately equal to a linear combination of sample autocorrelations, (1) can be written

$$VR(k) \cong 1 + 2 \sum_{j=1}^{k-1} \left( \frac{k-j}{k} \right) \hat{\rho}_j - 2 \sum_{j=1}^{11} \left( \frac{12-j}{12} \right) \hat{\rho}_j. \quad (2)$$

The variance ratio places increasing positive weight on autocorrelations up to and including lag 11, with declining positive weight thereafter. Our variance ratios for  $k$ -period annual returns place declining weight on all autocorrelations up to order  $k$ .

A second test for mean reversion, used by Fama and French (1988b), regresses multiperiod returns on lagged multiperiod returns. If  $\tilde{R}_t^k$  denotes the

<sup>4</sup>Testing the relationship between the variability of returns at different horizons has a long tradition: see Osborne (1959) and Alexander (1961).

<sup>5</sup>We use twelve-month returns in the denominator of the variance ratio to permit comparability with our results using annual returns data. With annual data, the variance-ratio denominator is  $\text{var}(R_t)$ .

<sup>6</sup>Kendall and Stuart (1976) show that under weak restrictions, the expected value of the  $j$ th sample autocorrelation is  $-1/(T-j)$ . Using this result, we compute  $E[VR(k)]$ . When the horizon of the variance ratio is large in relation to the sample size, this can be substantially less than unity. For example, with  $T = 720$  and  $k = 60$ , the bias is  $-0.069$ . It rises to  $-0.160$  if  $k = 120$ . Detailed Monte Carlo analysis of the variance-ratio statistic may be found in Lo and MacKinlay (1988).

de-meaned  $k$ -period return, the regression coefficient is

$$\hat{\beta}_k = \frac{\sum_{t=2k}^T (\bar{R}_t^k \bar{R}_{t-k}^k)}{\sum_{t=2k}^T (\bar{R}_{t-k}^k)^2}. \quad (3)$$

This statistic applies negative weight to autocorrelations up to order  $2k/3$ , followed by increasing positive weight up to lag  $k$ , followed by decaying positive weights.<sup>7</sup> Fama and French (1988b) report regression tests because they reject the null hypothesis of serially independent returns more strongly than the variance-ratio test. This is the result of the actual properties of the returns data, not a general rule about the relative power of the two tests. We show below that returns display positive, then negative, serial correlation as the horizon lengthens. In this case the regression test, by virtue of its negative, then positive, weights on sample autocorrelations, will reject the null hypothesis of serial independence more often than the variance-ratio test.

A third method of detecting mean reversion involves estimating parametric time-series models for returns, or computing likelihood-ratio tests of the null hypothesis of serial independence against particular parametric alternatives. Because returns are nearly white noise under both the null hypothesis and the alternatives we consider, standard ARMA techniques often fail.<sup>8</sup> When they are feasible, however, the Neyman-Pearson lemma dictates that the likelihood-ratio test is the most powerful test of the null of serial independence against the particular alternative that generated the data, so its Type II error rate is a lower bound on the error rates that other tests with the same size could achieve. In practice, this bound is unlikely to be achieved, since we do not know the precise data-generation process.

## 2.2. Power calculations

We analyze the power of tests for transitory components against the alternative hypotheses that Summers (1986) suggests, where the logarithm of stock prices ( $p_t$ ) embodies both a permanent ( $p_t^*$ ) and a transitory ( $u_t$ )

<sup>7</sup>Further details on the relationship between regression tests and the sample autocorrelogram are presented in an earlier draft, available on request.

<sup>8</sup>We tried estimating ARMA models for the pseudo-returns generated in our Monte Carlo study. Although these data were generated by an ARMA(1,1) model with first-order autoregressive and moving-average coefficients of roughly equal but opposite signs, standard ARMA estimation packages (i.e., RATS) had difficulty recovering this process. For example, with three-quarters of the variation in returns due to transitory factors, the estimation package encountered noninvertibilities in the moving-average polynomial and therefore broke down in more than a third of all Monte Carlo runs. Less than 10% of the cases led to well-estimated parameters that were close to those from the data-generation process.

component. We assume that  $p_t = p_t^* + u_t$ . If the stationary component is an AR(1) process

$$u_t = \rho_1 u_{t-1} + v_t, \quad (4)$$

and  $\varepsilon_t = p_t^* - p_{t-1}^*$  denotes the innovation to the nonstationary component, then

$$\Delta p_t = \varepsilon_t + (1 - L)(1 - \rho_1 L)^{-1} v_t. \quad (5)$$

If  $v_t$  and  $\varepsilon_t$  are independent,  $\Delta p_t$  follows an ARMA (1,1) process.<sup>9</sup> This description of returns allows us to capture in a simple way the possibility that stock prices contain transitory, but persistent, components. The parameter  $\rho_1$  determines the persistence of the transitory component, and the share of return variation due to transitory factors is determined by the relative size of  $\sigma_v^2$  and  $\sigma_\varepsilon^2$ .

We perform Monte Carlo experiments by generating 25,000 sequences of 720 returns, the number of monthly observations in the Center for Research in Securities Prices (CRSP) data base.<sup>10</sup> We set  $\sigma_\varepsilon^2 = 1$  so that the variance of returns ( $\Delta p_t$ ) equals  $1 + 2\sigma_v^2/(1 + \rho_1)$  and set parameters for the return-generating process by choosing  $\rho_1$  and  $\delta = 2\sigma_v^2/(1 + \rho_1 + 2\sigma_v^2)$ . The parameter  $\delta$  denotes the share of return variance accounted for by the stationary component;  $\delta$  and  $\rho_1$  determine  $\sigma_v^2$ . We consider cases where  $\delta$  equals 0.25 and 0.75. We set  $\rho_1$  equal to 0.98 for both cases, implying that innovations in the transitory price component have a half-life of 2.9 years.

In evaluating Type II error rates, the probability of failing to reject the null hypothesis when it is false, we use the empirical distribution of the test statistic generated with  $\delta = 0$  to determine the critical region for a one-sided 0.05 test of the random-walk null against the mean-reverting alternative. The panels of table 1 report Type II error rates for each test when the data are generated by the process indicated at the column head. The mean value of the test statistic under the alternative hypothesis is also reported.

The first row in table 1 analyzes a test based on the first-order autocorrelation coefficient. As Shiller and Perron (1985) and Summers (1986) observe, this

<sup>9</sup>The parameters of the ARMA(1,1) model  $(1 - \phi L)\Delta p_t = (1 + \theta L)w_t$  are

$$\phi = \rho_1,$$

$$\theta = \left\{ -(1 + \rho_1^2) - 2\sigma_v^2 + (1 - \rho_1) \left[ 4\sigma_v^2 + (1 + \rho_1)^2 \right]^{1/2} \right\} / (2\sigma_v^2 + 2\rho_1),$$

$$\sigma_w^2 = -(\rho_1 + \sigma_v^2)/\theta.$$

<sup>10</sup>In practice we draw 720 pairs of random variables, associate them with  $(\varepsilon_t, v_t)$ , and then construct  $\Delta p_t$ .

Table 1

Simulated Type II error rates of alternative tests for transitory components in security returns.

Each row describes the statistical properties of a particular test for mean reversion. All tabulations are based on one set of 25,000 Monte Carlo experiments using 720 monthly returns generated by the process described at the column heading. Both underlying processes are ARMA(1,1), with parameters set by  $\delta$ , the share of return variation due to transitory components, and  $\rho_1$ , the monthly serial correlation of the transitory component. Each test we analyze has size 0.05.

Test statistic and return measurement interval	Parameters of return-generating process			
	$\rho_1 = 0.98$ $\delta = 0.25$		$\rho_1 = 0.98$ $\delta = 0.75$	
	Type II error rate	Mean value of test statistic	Type II error rate	Mean value of test statistic
First-order autocorrelation	0.941	-0.002	0.924	-0.007
Variance ratio				
24 months	0.933	0.973	0.863	0.927
36 months	0.931	0.952	0.844	0.867
48 months	0.929	0.935	0.839	0.815
60 months	0.927	0.920	0.820	0.771
72 months	0.925	0.906	0.814	0.733
84 months	0.927	0.894	0.814	0.700
96 months	0.929	0.884	0.813	0.670
Return regression				
12 months	0.933	-0.044	0.863	-0.089
24 months	0.929	-0.080	0.842	-0.158
36 months	0.929	-0.112	0.841	-0.210
48 months	0.934	-0.141	0.856	-0.250
60 months	0.934	-0.167	0.868	-0.282
72 months	0.941	-0.194	0.887	-0.308
84 months	0.941	-0.221	0.903	-0.332
96 months	0.943	-0.250	0.914	-0.354
LR test	0.924	1.244	0.760	4.497

test has minimal power against the alternative hypotheses we consider. The Type II error rate for a size 0.05 test is 0.941 (0.924) when one-quarter (three-quarters) of the variation in returns is from the stationary component (i.e.,  $\delta = 0.25$  and  $\delta = 0.75$ ).

The next panel in table 1 considers variance-ratio tests comparing return variances for several different horizons, indexed by  $k$ , with one-period return variances. The variance-ratio tests are more powerful than tests based on first-order autocorrelation coefficients, but they still have little power to detect persistent, but transitory, return components. When one-quarter of the return variation is due to transitory factors ( $\delta = 0.25$ ), the Type II error rate never falls below 0.81. It is useful in considering the empirical results below to note

that when the transitory component in prices has a half-life of less than three years and accounts for three-quarters of the variation in returns ( $\delta = 0.75$ ), the variance ratio at 96 months is 0.67.

The next panel in table 1 shows Type II error rates for the long-horizon regression tests. The results are similar to those for variance ratios, although the regression tests appear to be somewhat less powerful against our alternative hypotheses. For example, the best variance-ratio test against the  $\delta = 0.25$  case has a Type II error rate of 0.925, compared with 0.929 for the most powerful regression test.

The final panel of the table presents results on likelihood-ratio tests.<sup>11</sup> Although these are more powerful than the variance-ratio tests, with Type II error rates of 0.922 in the  $\delta = 0.25$  case and 0.760 in the  $\delta = 0.75$  case, the error rates are still high. Even the best possible tests therefore have little power to distinguish the random-walk model of stock prices from alternatives that imply highly persistent, yet transitory, price components.

One potential shortcoming of our Monte Carlo analysis is our assumption of homoskedasticity in the return-generating process. To investigate its importance, we fit a first-order autoregressive model to monthly data on the logarithm of volatility.<sup>12</sup> We expand our Monte Carlo experiments to allow  $\sigma_e^2$  to vary through time according to this process. The Type II error calculations from the resulting simulations are similar to those in table 1. Fig. 1 illustrates this, showing the empirical distribution function for the 96-month variance ratio in both the homoskedastic and heteroskedastic cases.

### 2.3. Evaluating statistical significance

For most of the tests described above, the Type II error rate would be between 0.85 and 0.95 if the Type I error rate were set at the conventional 0.05 level. Leamer (1978) echoes a point made in most statistics courses when he writes that 'the [popular] rule of thumb, setting the significance level arbitrarily at 0.05, is... deficient in the sense that from every reasonable viewpoint the significance level should be a decreasing function of sample size' (p. 92). For the case where three-quarters of the return variation is due to transitory

<sup>11</sup>The likelihood value under each hypothesis is evaluated using Harvey's (1981) exact maximum likelihood method. Because estimating the mean induces a small-sample bias toward negative autocorrelations, even under the null hypothesis of serial independence the mean likelihood ratios for each alternative hypothesis are above one.

<sup>12</sup>The estimated volatility process that we use for our simulations is

$$\log(\sigma_t^2) = -2.243 + 0.7689 \cdot \log(\sigma_{t-1}^2) + \omega_t,$$

where  $\omega_t$  has a normal distribution with mean zero and standard deviation 0.691. The monthly volatility data are described in French, Schwert, and Stambaugh (1987).



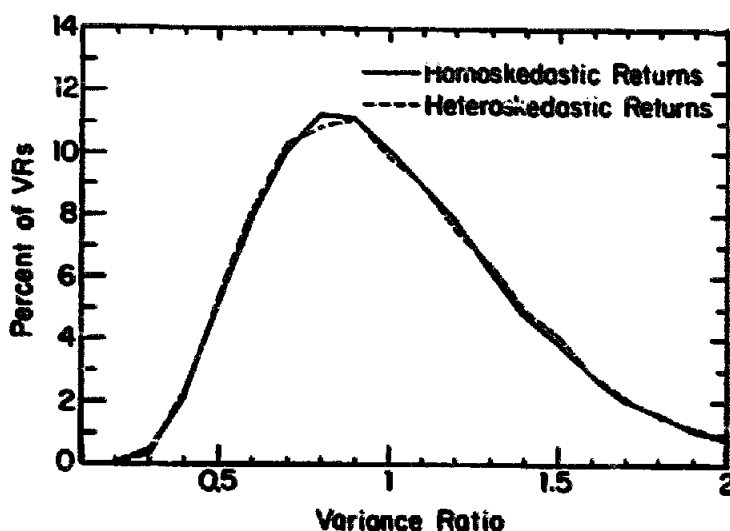


Fig. 1. Empirical distribution of 96-month variance-ratio statistic with homoskedastic and heteroskedastic returns.

The solid curve shows the empirical distribution of the 96-month variance-ratio statistic, calculated from 25,000 replications of 720-observation time series under the null hypothesis of serially independent draws from an identical distribution. The broken curve presents a similar empirical distribution calculated from the same number of Monte Carlo draws, but allowing for heteroskedasticity in the simulated returns. The logarithm of the simulated return variance evolves through time as noted in footnote 12.

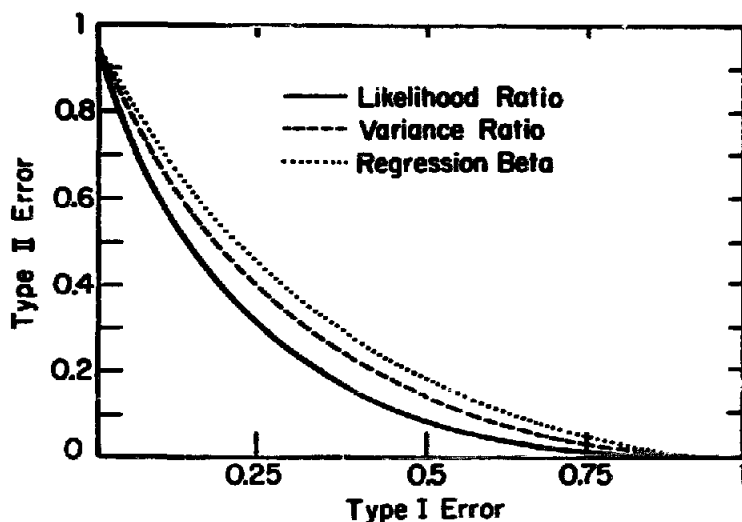


Fig. 2. Type II versus Type I error rates for three alternative tests of mean reversion.

Each curve displays the tradeoff between Type I and Type II error rates for a particular test of mean reversion in stock returns. Critical regions for each test are found using simulated empirical distributions for the variance-ratio, regression-beta, and likelihood-ratio tests under the null hypothesis of serially independent, homoskedastic returns. The Type II error rate for each test under the alternative hypothesis of  $\delta = 0.75$ ,  $\rho_1 = 0.98$  is calculated using another set of simulated empirical distributions. Under both the null and the alternative hypothesis, the empirical distributions are calculated using 25,000 replications of 720-observation time series for synthetic returns. For variance-ratio, regression-beta, and likelihood-ratio tests with given Type I error rates shown along the horizontal axis, the figure shows the associated Type II error rate against the alternative hypothesis.

factors, fig. 2 depicts the attainable tradeoff between Type I and Type II errors for the most powerful variance-ratio and regression tests, as well as for the likelihood-ratio test. The Type II error curve for the variance-ratio test lies between the frontiers attainable using regression and likelihood-ratio tests. For the variance-ratio test, a 0.40 significance level is appropriate if the goal is to minimize the sum of Type I and Type II errors. To justify using the conventional 0.05 test, one would have to assign three times as great a cost to Type I as to Type II errors.

Since there is little theoretical basis for strong attachment to the null hypothesis that stock prices follow a random walk, significance levels in excess of 0.05 seem appropriate in evaluating the importance of transitory components in stock prices. Many asset-pricing models, involving rational and irrational behavior, suggest the presence of transitory components and time-varying returns. Furthermore, the same problems of statistical power that plague our search for transitory components complicate investors' lives, so it may be difficult for speculative behavior to eliminate these components. The only solution to the problem of low power is the collection of more data. In the next section, we bring to bear as much data as possible in evaluating the importance of transitory components.

### **3. Statistical evidence on mean reversion**

This section uses variance-ratio tests to analyze the importance of stationary components in stock prices. We analyze excess and real returns using four major data sets: monthly returns on the NYSE for the period since 1926, annual returns on the Standard and Poor's-Cowles stock price indices for the period since 1871, post-World War II monthly stock returns for 17 stock markets outside the U.S., and returns on individual firms in the U.S. for the post-1926 period.

#### *3.1. Monthly NYSE returns, 1926-1985*

We begin by analyzing monthly returns on both the value-weighted and equal-weighted NYSE indices from the CRSP data base for the 1926-1985 period. We consider excess returns with the risk-free rate measured as the Treasury bill yield, as well as real returns measured using the Consumer Price Index (CPI) inflation rate. The variance-ratio statistics for these series are shown in table 2. We confirm the Fama and French (1988b) finding that both real and excess returns at long horizons show negative serial correlation. Eight-year returns are about four rather than eight times as variable as one-year returns. Despite the low power of our tests, the null hypothesis of serial independence is rejected at the 0.08 level for value-weighted excess

Table 2

Variance ratios for U.S. monthly data, 1926–1985.

Calculations are based on the monthly returns for the value-weighted and equal-weighted NYSE portfolios, as reported in the CRSP monthly returns file. The variance-ratio statistic is defined as  $VR(k) = (12/k) * \text{var}(R^k) / \text{var}(R^{12})$ , where  $R^j$  denotes returns over a  $j$ -period measurement interval. Values in parentheses are Monte Carlo estimates of the standard error of the variance ratio, based on 25,000 replications under the null hypothesis of serially independent returns. Each variance ratio is corrected for small-sample bias by dividing by the mean value from Monte Carlo experiments under the null hypothesis of no serial correlation.

Data series	Annual return standard deviation	Return measurement interval							
		1 month	24 months	36 months	48 months	60 months	72 months	84 months	96 months
Value-weighted real returns	20.6%	0.797 (0.150)	0.973 (0.108)	0.873 (0.177)	0.747 (0.232)	0.667 (0.278)	0.610 (0.320)	0.565 (0.358)	0.575 (0.394)
Value-weighted excess returns	20.7%	0.764 (0.150)	1.036 (0.108)	0.989 (0.177)	0.917 (0.232)	0.855 (0.278)	0.781 (0.320)	0.689 (0.358)	0.677 (0.394)
Equal-weighted real returns	29.6%	0.809 (0.150)	0.963 (0.108)	0.835 (0.177)	0.745 (0.232)	0.642 (0.278)	0.522 (0.320)	0.400 (0.358)	0.353 (0.394)
Equal-weighted excess returns	29.6%	0.785 (0.150)	1.010 (0.108)	0.925 (0.177)	0.878 (0.232)	0.786 (0.278)	0.649 (0.320)	0.487 (0.358)	0.425 (0.394)

returns and at the 0.005 level for equal-weighted excess returns.<sup>13</sup> Mean reversion is more pronounced for the equal-weighted than for the value-weighted returns, but the variance ratios at long horizons are well below unity for both.

The variance ratios also suggest positive return autocorrelation at horizons shorter than one year. The variance of the one-month return on the equal-weighted index is only 0.79 times as large as the variability of twelve-month returns implies it should be. A similar conclusion applies to the value-weighted index. This finding of first positive then negative serial correlation parallels Lo and MacKinlay's (1988) result that variance ratios exceed unity in their weekly data, whereas variance ratios fall below one in other studies concerned with longer horizons.<sup>14</sup>

One potential difficulty in interpreting our finding of positive serial correlation at short horizons concerns nontrading effects. If some of the securities in

<sup>13</sup>These  $p$ -values are calculated from the empirical distribution of our test statistic, based on Monte Carlo results. They permit rejection at lower levels than would be possible using the normal approximation to the distribution of the variance ratio, along with the Monte Carlo estimates of the standard deviation of the variance ratio. Further details are available on request.

<sup>14</sup>French and Roll (1986) apply variance-ratio tests to daily returns for a sample of NYSE and AMEX stocks for the period 1963–1982. They find evidence of negative serial correlation, especially among smaller securities. The divergence between their findings and those of Lo and MacKinlay (1988) is presumably due to differences in the two data sets.

the market index trade infrequently, returns will show positive serial correlation. We doubt this explanation of our results since we are analyzing monthly returns. Nontrading at this frequency is likely to affect only a small fraction of securities, whereas accounting for the degree of positive correlation we observe would require that one security in ten typically did not trade in a given month. We also investigated the incidence of nontrading in a portfolio similar to the value-weighted index by analyzing daily returns on the Standard and Poor's Index [see Poterba and Summers (1986)] for the period 1928–1986. The first-order autocorrelation coefficient for daily returns is only 0.064, and grouping returns into nonoverlapping five-day periods yields a first-order autocorrelation coefficient of  $-0.009$ . This suggests that autocorrelation patterns in monthly returns are not likely to be due to infrequent trading.

A second issue that arises in analyzing the post-1926 data is the sensitivity of the findings to inclusion or exclusion of the Depression years. A number of previous studies, such as Officer (1973), have documented the unusual behavior of stock price volatility during the early 1930s. One could argue for excluding these years from analyses designed to shed light on current conditions, although the sharp increase in market volatility in the last quarter of 1987 undercuts this view. The counterargument suggesting inclusion of this period is that the 1930s, by virtue of the large movements in prices, contain a great deal of information about the persistence of price shocks. We explored the robustness of our findings by truncating the sample period at both the beginning and the end. Excluding the first ten years weakens the evidence for mean reversion at long horizons. The results for both equal-weighted real and excess returns are robust to the sample choice, with variance ratios of 0.587 and 0.736 at the 96-month horizon, but the long-horizon variance ratios on the value-weighted index rise to 0.97 and 1.10, respectively. The one-month variance ratios are not substantially changed by treatment of the early years. For the post-1936 period, the one-month variance ratios are 0.782 and 0.825 for value- and equal-weighted real returns and 0.833 and 0.851 for value- and equal-weighted excess returns.<sup>15</sup> Truncating the sample to exclude the last ten years of data strengthens the evidence for mean reversion.

### *3.2. Historical data for the United States*

The CRSP data are the best available for analyzing recent U.S. experience, but the low power of available statistical tests and data-mining risks stressed by Merton (1987) suggest the value of examining other data as well. We

<sup>15</sup>We also experimented with crude techniques for accounting for time-varying stock market volatility in estimating variance ratios. Estimating sample autocorrelations with a heteroskedasticity correction based on French, Schwert, and Stambaugh's (1987) estimate of the previous month's return volatility effectively reduces the weight of the early Depression years, yielding variance-ratio estimates closer to unity.

Table 3

Variance ratios for U.S. data, 1871–1985.

Each entry is a bias-adjusted variance ratio with a mean of unity under the null hypothesis. The variance-ratio statistic is defined as  $VR(k) = (12/k) * \text{var}(R^k) / \text{var}(R^{12})$ , where  $R^j$  denotes the return measured over a  $j$ -month interval. Values in parentheses are Monte Carlo standard deviations of the variance ratio, based on 25,000 replications under the null hypothesis of serial independence. The underlying data are annual returns on the Standard and Poor's composite stock index, backdated to 1871 using the Cowles data as reported in Wilson and Jones (1987).

Data series	Annual return standard deviation	Return measurement interval						
		24 months	36 months	48 months	60 months	72 months	84 months	96 months
Excess returns 1871–1925	16.2%	0.915 (0.140)	0.612 (0.210)	0.591 (0.265)	0.601 (0.313)	0.464 (0.358)	0.425 (0.398)	0.441 (0.436)
Real returns 1871–1925	17.2%	0.996 (0.140)	0.767 (0.210)	0.806 (0.265)	0.847 (0.313)	0.737 (0.358)	0.737 (0.398)	0.807 (0.436)
Excess returns 1871–1985	18.9%	1.047 (0.095)	0.922 (0.143)	0.929 (0.179)	0.913 (0.211)	0.856 (0.240)	0.821 (0.266)	0.833 (0.290)
Real returns 1871–1985	19.0%	1.035 (0.095)	0.880 (0.143)	0.876 (0.179)	0.855 (0.211)	0.797 (0.240)	0.769 (0.266)	0.781 (0.290)

therefore consider real and excess returns based on the Standard and Poor's–Cowles Commission stock price indices, revised by Wilson and Jones (1987), which are available beginning in 1871. These data have rarely been used in studies of the serial correlation properties of stock returns, although they have been used in some studies of stock market volatility, such as Shiller (1981).

The results are presented in table 3. For the pre-1925 period, excess returns display negative serial correlation at long horizons. For real returns, however, the pattern is weaker. Although the explanation for this phenomenon is unclear, it appears to result from the volatility of the CPI inflation rate in the years before 1900. This may make the *ex post* inflation rate an unreliable measure of expected inflation during this period. The two lower rows in table 3 present results for the full 1871–1985 sample period. Both series show negative serial correlation at long lags, but real and excess returns provide less evidence of mean reversion than the monthly post-1925 CRSP data.<sup>16</sup>

### 3.3. Equity markets outside the United States

Additional evidence on mean reversion can be obtained by analyzing the behavior of equity markets outside the U.S. We analyze returns in Canada for

<sup>16</sup>The variance ratio for the full sample (1871–1985) period is not a simple weighted average of the variance ratios for the two subperiods, pre- and post-1926. The 96-month variance ratios for the post-1926 period excess and real S&P data, for example, are 0.463 and 0.731, respectively.

Table 4

Variance ratios for international data on real monthly returns.

Each entry reports the variance-ratio statistic (or in the bottom panel, average of variance-ratio statistics) for a particular nation and return horizon. The variance ratio is defined by  $\bar{v}R(k) = (12/k) \cdot \text{var}(R^k) / \text{var}(R^1)$ , where  $R^k$  denotes the real return over a  $k$ -month measurement interval. Data underlying the variance ratios are real dividend-exclusive returns, calculated from share price indices in the International Monetary Funds' *International Financial Statistics*. For most countries the monthly IMF data span the period 1957:1-1986:12; other data ranges are noted. Values in parentheses are Monte Carlo standard deviations of the variance-ratio statistics (and standard deviations for the averages in the last three rows). For the averages, these are computed allowing for a constant correlation across countries. If this correlation was estimated to be negative, we assume it is zero. In all cases except those marked with an asterisk (\*) the data are monthly averages of daily or weekly values. The U.K. data are point-sampled but only at the end of each year. The variance ratios are corrected for the time aggregation induced by averaging closing values of the index within each month.

Return series Country/sample	Annual return standard deviation	Return measurement interval								
		1 month	24 months	36 months	48 months	60 months	72 months	84 months	96 months	
Canada/1919-1986 (capital gains only)	20.1%	0.711* (0.141)	1.055 (0.102)	0.998 (0.166)	0.912 (0.218)	0.799 (0.261)	0.692 (0.301)	0.617 (0.336)	0.575 (0.370)	
U.K./1939-1986	20.9%	0.832 (0.167)	0.987 (0.125)	0.868 (0.198)	0.740 (0.259)	0.752 (0.317)	0.807 (0.358)	0.806 (0.400)	0.794 (0.440)	
Austria/1957-1986 (capital gains only)	21.4%	0.663 (0.214)	1.205 (0.156)	1.206 (0.254)	1.132 (0.334)	0.864 (0.403)	0.666 (0.464)	0.582 (0.518)	0.502 (0.566)	
Belgium/1957-1986 (capital gains only)	17.0%	0.718* (0.214)	1.054 (0.156)	1.137 (0.254)	1.121 (0.334)	1.060 (0.403)	0.876 (0.464)	0.807 (0.518)	0.776 (0.566)	
Colombia/1959-1983 (capital gains only)	21.4%	1.223 (0.214)	0.822 (0.156)	0.743 (0.254)	0.724 (0.334)	0.583 (0.403)	0.477 (0.464)	0.386 (0.518)	0.180 (0.566)	
Germany/1957-1986 (capital gains only)	23.8%	0.610 (0.214)	1.309 (0.156)	1.251 (0.254)	0.987 (0.334)	0.747 (0.403)	0.687 (0.464)	0.581 (0.518)	0.462 (0.566)	
Finland/1957-1986 (capital gains only)	22.1%	0.504 (0.214)	1.141 (0.156)	1.262 (0.254)	1.396 (0.334)	1.463 (0.403)	1.381 (0.464)	1.215 (0.518)	1.014 (0.566)	

France/1957-1986 (capital gains only)	23.6%	0.874* (0.214)	0.961 (0.156)	0.914 (0.254)	0.865 (0.334)	0.607 (0.403)	0.460 (0.464)	0.433 (0.518)	0.438 (0.566)
India/1957-1986 (capital gains only)	15.6%	0.752 (0.214)	0.985 (0.156)	0.974 (0.254)	0.942 (0.334)	0.823 (0.403)	0.767 (0.464)	0.619 (0.518)	0.596 (0.566)
Japan/1957-1986 (capital gains only)	20.0%	0.870 (0.214)	1.135 (0.155)	1.015 (0.254)	0.927 (0.334)	0.803 (0.403)	0.691 (0.464)	0.595 (0.518)	0.538 (0.567)
Netherlands/1957-1986 (capital gains only)	20.0%	0.710 (0.214)	1.238 (0.155)	1.263 (0.254)	1.217 (0.334)	1.083 (0.403)	1.047 (0.464)	0.894 (0.518)	0.741 (0.567)
Norway/1957-1986 (capital gains only)	24.2%	0.601 (0.214)	1.033 (0.155)	0.961 (0.254)	0.926 (0.334)	0.844 (0.403)	0.825 (0.464)	0.840 (0.518)	0.784 (0.567)
Philippines/1957-1986 (capital gains only)	29.7%	0.910 (0.214)	0.908 (0.155)	0.749 (0.254)	0.707 (0.334)	0.703 (0.403)	0.839 (0.464)	0.898 (0.518)	0.887 (0.567)
South Africa/1957-1986 (capital gains only)	23.2%	0.767 (0.214)	1.515 (0.155)	1.063 (0.254)	0.963 (0.334)	0.980 (0.403)	1.090 (0.464)	1.131 (0.518)	1.151 (0.567)
Spain/1961-1986 (capital gains only)	27.7%	0.603 (0.230)	1.289 (0.166)	1.584 (0.273)	1.831 (0.359)	2.008 (0.433)	2.246 (0.498)	2.347 (0.5556)	2.373 (0.609)
Sweden/1957-1986 (capital gains only)	21.1%	0.728 (0.214)	0.898 (0.155)	0.822 (0.254)	0.901 (0.334)	0.885 (0.403)	0.916 (0.464)	0.760 (0.518)	0.629 (0.567)
Switzerland/1957-1986 (capital gains only)	21.5%	0.789 (0.214)	1.343 (0.155)	1.395 (0.254)	1.300 (0.334)	1.034 (0.403)	0.749 (0.464)	0.489 (0.518)	0.382 (0.567)
U.S./1957-1986 (capital gains only)	16.6%	0.813 (0.214)	0.814 (0.155)	0.653 (0.254)	0.656 (0.334)	0.696 (0.403)	0.804 (0.464)	0.803 (0.518)	0.800 (0.567)
Average value		0.760 (0.140)	1.074 (0.036)	1.048 (0.062)	1.014 (0.164)	0.930 (0.212)	0.890 (0.228)	0.822 (0.266)	0.757 (0.312)
Average value (excluding U.S.)		0.757 (0.155)	1.089 (0.024)	1.071 (0.100)	1.035 (0.173)	0.943 (0.204)	0.895 (0.267)	0.824 (0.243)	0.754 (0.290)
Average value (excluding U.S., Spain)		0.766 (0.135)	1.077 (0.042)	1.039 (0.155)	0.985 (0.259)	0.877 (0.331)	0.811 (0.392)	0.728 (0.447)	0.653 (0.494)

the period since 1919, in Britain since 1939, and in 15 other nations for a shorter postwar period.

The Canadian data consist of monthly capital gains on the Toronto Stock Exchange. The British data are monthly returns, inclusive of dividends, on the *Financial Times*-Actuaries Share Price Index. The first two rows of table 4 show that both markets display mean reversion at long horizons. The 96-month variance ratio for the Canadian data is 0.585, while for the British data it is 0.794. Both markets also display statistically significant positive serial correlation at lags of less than 12 months. For Canada, the one-month variance is 0.718 times the value that would be predicted on the basis of the 12-month variance. For Britain, the comparable value is 0.832.

The variance ratios for the 15 other stock markets are calculated from monthly returns based on stock price indices in the International Monetary Fund's *International Financial Statistics*. The IMF does not tabulate dividend yields, so the reported returns correspond to capital gains alone. To assess the importance of this omission, we reestimated the variance ratios for dividend-exclusive CRSP and British stock market returns. The results, available from the authors on request, show only minor differences as a result of dividend omission. For example, the 96-month variance ratio for real value-weighted CRSP returns inclusive of dividends is 0.575 and that for dividend-exclusive returns is 0.545. We suspect that yield-inclusive data, although superior to the returns we use, would affect our results in only minor ways.<sup>17</sup>

Table 4 presents the variance ratios for individual countries, based typically on data starting in 1957. Most of the countries display negative serial correlation at long horizons. In Germany, for example, the 96-month variance ratio is 0.462; in France it is 0.438. Only three of the fifteen countries have 96-month variance ratios that exceed unity, and many are substantially below one. Evidence of positive serial correlation at short horizons is also pervasive. Only one country, Colombia, has a one-month variance ratio greater than unity. The short data samples, and associated large standard errors, make it difficult to reject the null hypothesis of serial independence for any individual country. The similarity of the results across nations nevertheless supports our earlier finding of substantial transitory price components.

Average variance ratios are shown in the last three rows of the table for all countries, all countries except the U.S., and all countries except the U.S. and Spain. The mean 96-month variance ratio is 0.754 when all countries are

<sup>17</sup>In some cases, the monthly stock index data from the IFS are time averages of daily or weekly index values. Working (1960) showed that the first difference of a time-averaged random walk would exhibit positive serial correlation, with a first order autocorrelation coefficient of 0.25 as the number of observations in the average becomes large. This will bias our estimated variance ratios. For the countries with time aggregated data we therefore modify our small-sample bias correction. Instead of taking the expected value of the first-order autocorrelation to be  $-1/(T-1)$  when evaluating  $E[VR(k)]$  we use  $0.25 - 1/(T-1)$ . The reported variance ratios have been bias-adjusted by dividing by the resulting expected value.



aggregated and 0.653 when we exclude Spain, an outlier because of the unusual pattern of hyperinflation followed by deflation that it experienced during our sample period. By averaging across many countries, we also obtain a more precise estimate of the long-horizon variance ratio, although the efficiency gain is attenuated because the results for different countries are not independent.<sup>18</sup>

### 3.4. Individual firm data

Arbitrageurs should be better at trading in individual securities to correct mispricing than at taking positions in the entire market to offset persistent misvaluations. Although we expect transitory components to be less likely in the relative prices of individual stocks than in the market as a whole, some previous work has suggested that individual stock returns may show negative serial correlation over some horizons [Lehmann (1987), DeBoadt and Thaler (1985)]. We examine the 82 firms in the CRSP monthly master file that have no missing return information between 1926 and 1985. This is a biased sample, weighted toward large firms that have been traded actively over the entire period. Firms that went bankrupt or began trading during the sample period are necessarily excluded.

We compute variance ratios using both real and excess returns for these 82 firms. Because the returns for different firms are not independent, we also examine the returns on portfolios formed by buying one dollar of each firm and short-selling \$82 of the aggregate market. That is, we examine properties of the time series  $R_{it} - R_{mt}$ , where  $R_{mt}$  is the value-weighted NYSE return. Table 5 reports the mean values of the individual-firm variance ratios, along with standard errors that take account of cross-firm correlation. The results suggest some long-horizon mean reversion for individual stock prices in relation to the overall market or a risk-free asset. The point estimates suggest that 12% of the eight-year variance in excess returns is due to stationary factors, and the increased precision gained by studying returns for many independent firms enables us to reject the null hypothesis that all of the price variation arises from nonstationary factors. The last row, which reports variance-ratio calculations using the residuals from market-model equations estimated for each firm (assuming a constant  $\beta$  for the entire period), shows

<sup>18</sup>The standard errors for the cross-country averages allow for correlation between the variance ratios for different countries. If all nations have a constant pairwise correlation  $\tau$  between their variance ratios and these variance ratios have constant variance  $\sigma_x^2$ , then the expected value of the sample variance of the variance-ratio statistics is  $E(s_n^2) = \sigma_x^2(1 - \tau)$ . Replacing the expected sample variance with the actual value, we estimate  $\tau$  as  $1 - s_n^2/\sigma_x^2$ . The variance of the sample mean for  $N$  observations, each with the same variance  $\sigma_x^2$  but constant cross-correlation  $\tau$ , is  $\sigma_x^2[1 + (N - 1)\tau]/N$ . We use our estimate of  $\tau$  to evaluate this expression, generalized to allow for different sampling variances for different variance ratios on the basis of our Monte Carlo standard errors from table 4.

Table 5

Average variance ratios for individual company monthly returns, 1926-1985.

Each entry reports the average of variance ratios calculated for the 82 firms on the monthly CRSP returns file with continuous data between 1926 and 1985. The variance-ratio statistic is defined as  $VR(k) = (12/k) \cdot \text{var}(R^k) / \text{var}(R^1)$ , where  $R^k$  denotes returns over a  $k$ -period measurement interval. Values in parentheses are Monte Carlo estimates of the standard error on the variance ratio, based on 25,000 replications under the null hypothesis of serially independent returns. Each variance ratio is correlated for small-sample bias by dividing the average Monte Carlo value under the null hypothesis of no serial correlation. For the returns in relation to the risk-free rate the standard errors take account of estimated contemporaneous correlation among variance ratios, using the techniques described in footnote 18.

Return concept	Return measurement interval									
	1 month	24 months	36 months	48 months	60 months	72 months	84 months	96 months		
Excess returns in relation to risk-free rate	0.942 (0.063)	1.035 (0.063)	1.000 (0.100)	0.950 (0.135)	0.888 (0.166)	0.820 (0.198)	0.755 (0.236)	0.739 (0.258)		
Excess returns in relation to value-weighted NYSE	1.088 (0.017)	1.034 (0.012)	1.019 (0.020)	1.002 (0.026)	0.968 (0.031)	0.928 (0.035)	0.898 (0.040)	0.886 (0.044)		
Residuals from market model	1.107 (0.017)	1.055 (0.012)	1.065 (0.020)	1.057 (0.026)	1.034 (0.031)	1.008 (0.035)	0.995 (0.040)	0.985 (0.044)		

less evidence of serial correlation than the results that subtract the market return. These results suggest that transitory factors account for a smaller share of the variance in relative returns for individual stocks than for the market as a whole.

### **3.5. Summary**

Our point estimates generally suggest that over long horizons return variance increases less than proportionally with time, and in many cases they imply more mean reversion than our examples in the last section, where transitory factors accounted for three-fourths of the variation in returns. Many of the results reject the null hypothesis of serial independence at the 0.15 level, a level that may be appropriate given our previous discussion of size versus power tradeoffs. Furthermore, each of the different types of data we analyze provides evidence of departure from serial independence in stock returns. Taken together, the results are stronger than any individual finding, although not by as much as they would be if the various data sets were independent.

There is some tendency for more mean reversion in less broad-based and sophisticated equity markets. The U.S. data before 1925 show greater evidence of mean reversion than the post-1926 data. The equal-weighted portfolio of NYSE stocks shows more mean reversion than the value-weighted portfolio.<sup>19</sup> In recent years, mean reversion is more pronounced in smaller foreign equity markets than in the U.S.

## **4. The substantive importance of transitory components in stock prices**

This section assesses the substantive importance of mean reversion in stock prices. One possible approach would involve calibrating models of the class considered in the first section. We do not follow this strategy because our finding of positive autocorrelation over short intervals implies that the AR(1) specification of the transitory component is inappropriate and because of our difficulties in estimating the ARMA(1,1) models implied by this approach. Instead, we use an approach that does not require us to specify a process for the transitory component, but nevertheless allows us to focus on its standard deviation and the fraction of the one-period return variance that can be attributed to it.

<sup>19</sup>We conjectured that the greater mean reversion in the equal-weighted than the value-weighted portfolio might be because the less heavily traded equal-weighted portfolio experienced larger swings in required returns or fluctuated more in relation to fundamental values than the value-weighted portfolio. Assuming similar-sized movements in the permanent component of the two indices, this conjecture can be tested by analyzing the degree of mean reversion in the *relative* returns on the two indices. These returns show positive serial correlation at all lags, contrary to our conjecture.

We treat the logarithm of the stock price as the sum of a permanent and a transitory component. The permanent component evolves as a random walk and the transitory component follows a stationary process. This decomposition may be given two (not necessarily exclusive) interpretations. The transitory component may reflect fads – speculation-induced deviations of prices from fundamental values – or it may be a consequence of changes in required returns. In either case, describing the stochastic properties of the stationary price component is a way of characterizing the part of stock price movements that cannot be explained by changing expectations about future cash flows.

Given our assumptions, the variance of  $T$ -period returns is

$$\sigma_T^2 = T\sigma_e^2 + 2(1 - \rho_T)\sigma_u^2, \quad (6)$$

where  $\sigma_e^2$  is the variance of innovations to the permanent price component,  $\sigma_u^2$  is the variance of the stationary component, and  $\rho_T$  is the  $T$ -period autocorrelation of the stationary component. Given data on the variance of returns over two horizons  $T$  and  $T'$  and assumptions about  $\rho_T$  and  $\rho_{T'}$ , a pair of equations with the form (6) can be solved to yield estimates of  $\sigma_e^2$  and  $\sigma_u^2$ . Using  $\sigma_R^2$  for the variance of one-period returns, and  $VR(T)$  for the  $T$ -period variance ratio in relation to one-period returns, estimates of  $\sigma_e^2$  and  $\sigma_u^2$  are given by

$$\sigma_e^2 = \frac{\sigma_R^2 [VR(T)(1 - \rho_{T'})T - VR(T')(1 - \rho_T)T']}{(1 - \rho_{T'})T - (1 - \rho_T)T'}, \quad (7a)$$

$$\sigma_u^2 = \frac{\sigma_R^2 T' [VR(T) - VR(T')]T}{2[(1 - \rho_T)T' - (1 - \rho_{T'})T]}. \quad (7b)$$

Many pairs of variance ratios and assumptions about the serial correlation properties of  $u_t$  could be analyzed by using (7a)–(7b). We begin by postulating that  $u_t$  is serially uncorrelated at the horizon of 96 months. For various degrees of serial correlation at other horizons, we can then estimate the variance of the transitory component,  $\sigma_u^2$ , and the share of the return variation due to transitory components,  $1 - \sigma_e^2/\sigma_R^2$ . We present estimates based on values of 0, 0.35, and 0.70 for  $\rho_{12}$ , the twelve-month autocorrelation in  $u_t$ . The findings are insensitive to our choice of  $\rho_{96}$ ; we report values of 0, 0.15, and 0.30.

Table 6 presents estimates of the standard deviation of the transitory component in stock prices for the value-weighted and equal-weighted NYSE portfolios over the period 1926–1985 for various values of  $\rho_{12}$ , assuming  $\rho_{96} = 0$ . For the equal-weighted portfolio, the transitory component accounts

Table 6

## Permanent and transitory return components, U.S. monthly data.

Each entry reports the standard deviation of the transitory component of prices, measured at annual rates ( $\sigma_u$ ), as well as the share of return variation due to transitory factors, calculated from eqs. (7a) and (7b) to match the observed pattern of variances in long- and short-horizon returns. The variance-ratio estimates that underlie this table are drawn from the entries for 96-month variance ratios for excess returns in table 2. The different cases of  $\rho_{12}$  ( $\rho_{96}$ ) correspond to different assumptions about the 12-month (96-month) autocorrelation in the transitory price component.

	$\rho_{12} = 0.0$		$\rho_{12} = 0.35$		$\rho_{12} = 0.70$	
	$\sigma_u$	$1 - \sigma_e^2/\sigma_R^2$	$\sigma_u$	$1 - \sigma_e^2/\sigma_R^2$	$\sigma_u$	$1 - \sigma_e^2/\sigma_R^2$
Value-weighted excess returns						
$\rho_{96} = 0.00$	9.7%	0.369	12.5%	0.400	21.6%	0.554
$\rho_{96} = 0.15$	-	-	12.3%	0.386	20.5%	0.500
$\rho_{96} = 0.30$	-	-	12.1%	0.373	19.6%	0.456
Equal-weighted excess returns						
$\rho_{96} = 0.00$	16.8%	0.657	21.7%	0.712	37.7%	0.986
$\rho_{96} = 0.15$	-	-	21.4%	0.687	35.8%	0.890
$\rho_{96} = 0.30$	-	-	21.0%	0.664	34.2%	0.812

for between 43% and 99% of the variance in equal-weighted monthly returns, depending on our serial correlation assumption, and it has a standard deviation of between 14% and 37%. Results for value-weighted returns also suggest a substantial, though smaller, transitory component. Since other nations and historical periods show patterns of variance-ratio decline similar to those in U.S. data, we do not present parallel calculations for them. As one would expect, nations with 96-month variance ratios lower than those for the U.S. have larger transitory components.

Table 6 indicates that increasing the assumed persistence of the transitory component raises both its standard deviation and its contribution to the return variance. More persistent transitory components are less able to account for declining variance ratios at long horizons. To rationalize a given long-horizon variance ratio, increasing the transitory component's persistence requires increasing the weight on the transitory component in relation to the permanent component. Sufficiently persistent transitory components will be unable to account for low long-horizon variance ratios, even if they account for all of the return variation. A transitory component that is almost as persistent as a random walk, for example, will be unable to explain very much long-horizon mean reversion.

Which cases in table 6 are most relevant? As an *a priori* matter, it is difficult to argue for assuming that transitory components should die out rapidly.

Previous claims that there are fads in stock prices have typically suggested half-lives of several years, implying that the elements in the table corresponding to  $\rho_{12} = 0.70$  are most relevant. With geometric decay, this suggests a half-life of two years. One other consideration supports large values for  $\rho_{12}$ . For given values of  $\sigma_e^2$  and  $\sigma_u^2$ , eq. (6) permits us to calculate  $\rho_T$  over any horizon. A reasonable restriction, that  $\rho_T$  not be very negative over periods of up to 96 months, is satisfied only for cases where  $\rho_{12}$  is large. For example, with  $\rho_{96} = 0$ , imposing  $\rho_{12} = 0.35$  yields an implied autocorrelation for the stationary component of  $-0.744$  at 36 months,  $-1.27$  at 60 months, and  $-0.274$  at 84 months. In contrast, when  $\rho_{12} = 0.70$  and  $\rho_{96} = 0$ , the implied values of  $\rho_{36}$  and  $\rho_{60}$  are  $0.168$  and  $-0.173$ , respectively. Similar results obtain for other large values of  $\rho_{12}$ . This is because actual variance ratios decline between long and longer horizons, and as eq. (6) demonstrates, rationalizing this requires declining values of  $\rho_T$ . If  $\rho_T$  starts small, it must become negative to account for the observed pattern. Larger autocorrelations at short horizons do not necessitate such patterns.

Insofar as the evidence in the last section and in Fama and French (1988b) is persuasive in suggesting the presence of transitory components in stock prices, this section's results confirm Shiller's (1981) conclusion that models assuming constant *ex ante* returns cannot account for all of the variance in stock market returns. Since our analysis does not rely on the present-value relation between stock prices and expected future dividends, it does not suffer from some of the problems that have been highlighted in the volatility-test debate.<sup>20</sup>

## 5. The source of the transitory component in stock prices

Transitory components in stock prices imply variation in *ex ante* returns.<sup>21</sup> Any stochastic process for the transitory price component can be mapped into a stochastic process for *ex ante* returns, and any pattern for *ex ante* returns can be represented by describing the associated transitory price component. The central issue is whether variations in *ex ante* returns are better explained

<sup>20</sup>Shiller's conclusion that market returns are too volatile to be reconciled with valuation models assuming constant required returns is controversial; see West (1988) for a survey of recent work.

<sup>21</sup>Several recent studies have considered the extent to which equity returns can be predicted using various information sets. Keim and Stambaugh (1986) find that between 8% and 13% of the variation in returns for a portfolio of stocks in the bottom quintile of the NYSE can be predicted using lagged information. A much smaller share of the variation in returns to larger companies can be accounted for in this way. Campbell (1987) finds that approximately 11% of the variation in excess returns can be explained on the basis of lagged information derived from the term structure. Fama and French (1988a) find that lagged dividend yields can predict a much higher fraction of returns over longer horizons.

by changes in interest rates and volatility, or indeed as byproducts of price deviations caused by noise traders.<sup>22</sup> This section notes two considerations that incline us toward the latter view.

First, we calibrate the variation in expected returns that risk factors would have to generate to account for the observed transitory components in stock prices. We assume for simplicity that the transitory component follows an AR(1) process as postulated in Summers (1986). This has the virtue of tractability, although it is inconsistent with the observation that actual returns show positive, then negative, serial correlation. If required returns show positive autocorrelation, then an innovation that raises required returns will reduce share prices. This will induce a holding period loss, followed by higher returns. The appendix shows that when required returns follow an AR(1) process,<sup>23</sup> *ex post* returns ( $R_t$ ) are given by

$$R_t - \bar{R} \cong \frac{1 + \bar{g}}{1 + \bar{r} - \rho_1(1 + \bar{g})} (r_t - \bar{r}) - \frac{(1 + \bar{r})^{-1}(1 + \bar{g})^2}{1 + \bar{r} - \rho_1(1 + \bar{g})} (r_{t+1} - \bar{r}) + \zeta_t, \quad (8)$$

where  $\zeta_t$ , a serially uncorrelated innovation that is orthogonal to innovations about the future path of required returns ( $\xi_t$ ), reflects revisions in expected future dividends. The average dividend yield and dividend growth rate are  $\bar{d}$  and  $\bar{g}$ , respectively; in steady state,  $\bar{r} = \bar{d} + \bar{g}$ .

If changes in required returns and profits are positively correlated, then the assumption that  $\xi_t$  and  $\zeta_t$  are orthogonal will *understate* the variance in *ex ante* returns needed to rationalize mean reversion in stock prices. It is possible to construct theoretical examples in which profits and interest rates are negatively related, as in Campbell (1986), but the empirical finding of weak

<sup>22</sup>Lucas (1978) and Cox, Ingersoll, and Ross (1985) study the pricing of assets with time-varying required returns. Several recent papers, including Black (1986), Campbell and Kyle (1986), DeLong et al. (1987), and Shiller (1984), have discussed the possible influence of noise traders on security prices and required returns. Fama and French (1986b) show that the negative serial correlation in different stocks may be attributable to a common factor, and interpret this finding as support for the time-varying returns view of mean reversion.

<sup>23</sup>The possibility of negative expected excess returns is an unattractive feature of the simple model we have analyzed. In principle the analysis could be repeated using Merton's (1980) model, which requires the expected excess return to be positive. The exact parallel between the time-varying returns model and the fads model would not hold in this case, however.

positive correlation between bond and stock returns suggests either positive or weak negative correlation between shocks to cash flows and required returns.<sup>24</sup>

Our assumption that required returns are given by  $r_t - \bar{r} = (1 - \rho_1 L)^{-1} \xi_t$ , enables us to rewrite (8), defining

$$\tilde{\xi}_t \equiv -\xi_{t+1}(1 + \bar{r})^{-1}(1 + \bar{g})^2 / [1 + \bar{r} - \rho_1(1 + \bar{g})],$$

as

$$(1 - \rho_1 L)(R_t - \bar{R}) \equiv \tilde{\xi}_t + \zeta_t - (1 + \bar{d})\tilde{\xi}_{t-1} - \rho_1 \zeta_{t-1}. \quad (9)$$

The first-order autocovariance of the expression on the right-hand side of (9) is nonzero, but all higher-order autocovariances equal zero.<sup>25</sup> Provided  $\sigma_\xi^2 > 0$ , returns follow an ARMA(1,1) process; if  $\sigma_\xi^2 = 0$ , then returns are white noise.

The simple model of stationary and nonstationary price components summarized in eq. (5) also yields an ARMA(1,1) representation for returns. This allows us to calculate the variation in required returns that is needed to generate the same time-series process for observed returns as fads of various sizes. In the appendix we show that the required return variance corresponding to a given fad variance is

$$\sigma_r^2 = \frac{[1 + \bar{r} - \rho_1(1 + \bar{g})]^2(1 - \rho_1)^2(1 + \bar{r})^2}{\{(1 + \bar{d})(1 + \rho_1^2) - \rho_1[1 + (1 + \bar{d})^2]\}(1 + \bar{g})^2} \sigma_u^2. \quad (10)$$

Table 7 reports calculations based on (10). It shows the standard deviation of required excess returns, measured on an annual basis, implied by a variety of fad models. We calibrate the calculations using the average excess return (8.9% per year) on the NYSE equal-weighted share price index over the 1926–1985 period. The dividend yield on these shares averages 4.5%, implying an average dividend growth rate of 4.4%. We use estimates of the variance ratio at 96 months to calibrate the degree of mean reversion.

Substantial variability in required returns is needed to explain mean reversion in prices. For example, if we postulate that the standard deviation of the transitory price component is 20%, then even when required return shocks have a half-life of 2.9 years, the standard deviation of *ex ante* returns must be 5.8% per annum. Even larger amounts of required return variation are needed

<sup>24</sup>Campbell (1987) estimates that the correlation between excess returns on long-term bonds and corporate equities was 0.22 for the 1959–1979 period and 0.36 for the more recent 1979–1983 period.

<sup>25</sup>Ansley, Spivey, and Wroblewski (1977) prove that an autocorrelogram with zero entries beyond order  $k$  implies an MA( $k$ ) process.



Table 7

Amount of variation in required returns needed to account for mean reversion in stock prices.

Each entry answers the question: 'If both required returns and price fads follow first-order autoregressions with half-lives indicated in the row margin, and the amount of mean reversion in observed returns is consistent with a price fad with a standard deviation ( $\sigma_p$ ) given in the column heading, what would the standard deviation of required returns need to be to generate the same time-series process for *ex post* returns?' Our calculations employ the fact that with AR(1) required returns, the *ex post* returns process is given by eq. (8). Similarly the price fad is assumed to follow an AR(1) that yields a process like (5) for *ex post* returns. We then ask what value of  $\sigma_r$  (or implicitly  $\sigma_\epsilon$ ) is needed to generate a given size transitory price pattern implied by  $\sigma_p$ . The calculations are calibrated using data on excess returns for the equal-weighted NYSE index over the 1926–1985 period and are based on eq. (10) in the text. The average excess return for this period is 8.9% per year, with a dividend yield of 4.5%.

Half-life	Standard deviation of transitory component			
	15.0%	20.0%	25.0%	30.0%
1.4 years	7.9%	10.6%	13.2%	15.8%
1.9 years	6.1%	8.2%	10.2%	12.3%
2.9 years	4.4%	5.8%	7.3%	8.7%

to explain the same size price fads when required return shocks are less persistent. These estimates of the standard deviation of required returns are large in relation to the mean of *ex post* excess returns and imply that if *ex ante* returns are never negative they must frequently exceed 20%.

It is difficult to think of risk factors that could account for such variation in required returns. Campbell and Shiller (1987), using data on real interest rates and market volatilities, find no evidence that stock prices help to forecast future movements in discount rates, as they should if stock price movements are caused by fluctuations in these factors.<sup>26</sup> Although they show that stock prices do forecast consumption fluctuations, the sign is counter to the theory's prediction. On the other hand, if the transitory components are viewed as a reflection of mispricing, they are also large in relation to traditional views of market efficiency.

The second difficulty in explaining the observed correlation patterns with models of time-varying returns arises from our finding of positive followed by negative serial correlation. Models with first-order autoregressive transitory components can rationalize the second but not the first of these observations. It is instructive to consider what type of expected returns behavior is necessary to account for both observations.

<sup>26</sup>Contrary evidence suggesting that stock returns do predict future volatility patterns is provided by French, Schwert, and Stambaugh (1987).

There are two potential explanations for the positive autocorrelation in observed returns at short lags. First, contrary to our maintained specification, shocks to required returns and to prospective dividends may be positively correlated. This could lead to positive autocorrelation at short horizons because increases in expected dividends, which would raise share prices, would be followed by higher *ex ante* returns. We explored this possibility by forming monthly 'dividend innovations' ( $IDIV_t$ ) for the 1926–1985 period as the residuals from a regression of real dividends (on the value-weighted NYSE portfolio) on twelve lagged values of real dividends, a time trend, and a set of monthly dummy variables. We then regressed real returns on the value-weighted index on lagged values of  $IDIV_t$ . A representative equation, including six lagged values, is shown below.  $R_t$  is measured in percentage points and standard errors are given in parentheses:

$$\begin{aligned}
 R_t = & 1.568 + 0.844 * IDIV_{t-1} - 0.109 * IDIV_{t-2} \\
 & (0.040) \quad (1.380) \quad (1.380) \\
 & - 3.667 * IDIV_{t-3} - 0.904 * IDIV_{t-4} \\
 & (1.380) \quad (1.380) \\
 & - 1.061 * IDIV_{t-5} - 1.769 * IDIV_{t-6}, \\
 & (1.377) \quad (1.374)
 \end{aligned}$$

$$R^2 = 0.037, \quad 1927:7-1985:12.$$

The coefficients on lagged values of  $IDIV$  should be positive if required returns and prospective dividends are positively correlated, but the results provide no support for this view. If anything, they suggest a negative but statistically insignificant relationship between dividend innovations and subsequent returns. This would suggest that positive dividend news is followed by lower required returns, a pattern that should be reflected in negative autocorrelation of *ex post* returns over short horizons.

The second potential explanation for positive serial correlation is that the autocorrelogram of *ex post* returns reflects the dynamics of required returns. Some required-return processes could generate positive, followed by negative, return autocorrelation. The required-return processes with this feature that we have identified all show increasing coefficients in some part of their moving average representation.<sup>27</sup> We are unaware of evidence suggesting that observ-

<sup>27</sup>Two examples of required return processes are twelve-order moving-average processes with the following coefficients: 1, -1.5, -0.75, -0.5, -0.5, 0.75, 0.75, 0.75, 0.75, 0.75, 0.75, 0.75 and 1, 1.3, 2, 2.5, 3, 3.5, 4, 4.5, 5, 4, 3, 2, 1. The autocorrelogram of the former process displays positive, then negative, correlation in required returns, while the second process exhibits positive autocorrelation at all lags. Both processes generate positive, then negative, autocorrelation in *ex post* returns.

able proxies for required returns display such stochastic properties. Studies of volatility such as French, Schwert, and Stambaugh (1987) or Poterba and Summers (1986) suggest that shocks are persistent but that their moving-average representations show declining coefficients. An alternative possibility is that movements in required returns are due to changes in the equity demands of noise traders. For example, assume that the required return of sophisticated traders is equal to  $\alpha + \beta S_t$ , where  $S_t$  is the fraction of the outstanding common stock that these investors must hold. Equity demands of noise traders (which in equilibrium must equal  $1 - S_t$ ) that follow a moving-average process similar to one of those for required returns that generate positive, then negative, autocorrelation in *ex post* returns will also generate this pattern in *ex post* returns. The notion that noise trading impulses intensify and then decline comports with qualitative discussions of fads, but further work is clearly necessary to evaluate this conjecture.

## 6. Conclusions

Our results suggest that stock returns show positive serial correlation over short periods and negative correlation over longer intervals. This conclusion emerges from data on equal-weighted and value-weighted NYSE returns over the 1926–1985 period, and is corroborated by data from other nations and time periods. Although individual data sets do not consistently permit rejection of the random-walk hypothesis at high significance levels, the various data sets together strengthen the case against its validity. Our point estimates suggest that transitory price components account for a substantial part of the variance in returns.

Our finding of significant transitory price components has potentially important implications for financial practice. If stock price movements contain large transitory components, then for long-horizon investors the stock market may be less risky than it appears to be when the variance of single-period returns is extrapolated using the random-walk model. Samuelson (1988) demonstrates that in the presence of mean reversion, an investor's horizon will influence his portfolio decisions. If the investor's relative risk aversion is greater (less) than unity, as his horizon lengthens he will invest more (less) in equities than he would with serially independent returns. The presence of transitory price components also suggests the desirability of investment strategies, such as those considered by DeBondt and Thaler (1985), involving the purchase of securities that have recently declined in value. It may also justify some institutions' practice of spending on the basis of a weighted average of their past endowment values, rather than current market value.

Although the temptation to apply more sophisticated statistical techniques to stock return data in an effort to extract more information about the magnitude and structure of transitory components is ever present, we doubt

that a great deal can be learned in this way. Even the broad characteristics of the data examined in this paper cannot be estimated precisely. As the debate over volatility tests has illustrated, sophisticated statistical results are often very sensitive to maintained assumptions that are difficult to evaluate. We have validated the statistical procedures in this paper by applying them to pseudo data conforming to the random-walk model. Our suspicion, supported by Kleidon's (1986) results, is that such Monte Carlo analysis of much of the more elaborate work on stock-price volatility would reveal poor statistical properties.

We suggest in the paper's final section that noise trading, trading by investors whose demand for shares is determined by factors other than their expected return, provides a plausible explanation for the transitory components in stock prices.<sup>28</sup> Pursuing this will involve constructing and testing theories of noise trading, as well as theories of changing risk factors, that could account for the characteristic stock return autocorrelogram documented here. Evaluating such theories is likely to require information other than stock returns, such as data on fundamental values, proxies for noise trading such as the net purchases by odd-lot traders, turnover, or the level of participation in investment clubs, and indicators of risk factors such as *ex ante* volatilities implied by stock options. Only by comparing models based on the presence of noise traders with models based on changing risk factors can we judge whether financial markets are efficient in the sense of rationally valuing assets, as well as precluding the generation of excess profits.

## Appendix

### *Derivation of ex post return process when required returns are AR(1)*

The price of a common stock,  $P_t$ , equals

$$P_t = E_t \left\{ \sum_{j=0}^{\infty} \left[ \prod_{i=0}^{j-1} (1 + r_{t+i})^{-1} (1 + g_{t+i}) \right] D_{t+j} \right\}, \quad (\text{A.1})$$

where  $r_{t+i}$  denotes the required real return in period  $t+i$ ,  $D_t$  is the dividend paid in period  $t$ ,  $g_{t+i}$  is the real dividend growth rate between periods  $t+i$  and  $t+i+1$ , and  $E_t\{\cdot\}$  designates expectations formed using information available as of period  $t$ . We linearize inside the expectation operator in  $r_{t+i}$

<sup>28</sup>Cutler, Poterba, and Summers (1988) document the difficulty of explaining a significant fraction of return variation on the basis of observable news about future cash flows or discount rates.

and  $g_{t+j}$ :

$$\begin{aligned}
 P_t &\equiv E_t \left\{ \sum_{j=1}^{\infty} \left( \frac{1+\bar{g}}{1+\bar{r}} \right)^{j-1} D_t + \sum_{j=0}^{\infty} \frac{\partial P_t}{\partial r_{t+j}} [r_{t+j} - \bar{r}] \right. \\
 &\quad \left. + \sum_{j=0}^{\infty} \frac{\partial P_t}{\partial g_{t+j}} [g_{t+j} - \bar{g}] \right\} \\
 &= \frac{D_t(1+\bar{r})}{\bar{r}-\bar{g}} - \frac{D_t(1+\bar{g})}{(1+\bar{r})(\bar{r}-\bar{g})} E_t \left\{ \sum_{j=0}^{\infty} \beta^j [r_{t+j} - \bar{r}] \right\} \\
 &\quad + \frac{D_t}{\bar{r}-\bar{g}} E_t \left\{ \sum_{j=0}^{\infty} \beta^j [g_{t+j} - \bar{g}] \right\},
 \end{aligned} \tag{A.2}$$

where  $\beta = (1+\bar{g})/(1+\bar{r})$ . We denote  $D_t(1+\bar{r})/(\bar{r}-\bar{g})$  as  $\bar{P}_t$ . In the special case of

$$(r_t - \bar{r}) = \rho_1(r_{t-1} - \bar{r}) + \xi_t, \tag{A.3}$$

we can simplify the second term in (A.2) to obtain

$$\begin{aligned}
 P_t - \bar{P}_t &\equiv \frac{-D_t(1+\bar{g})}{(1+\bar{r})(\bar{r}-\bar{g})} \sum_{j=0}^{\infty} \beta^j \rho_1^j [r_t - \bar{r}] \\
 &\quad + \frac{D_t}{\bar{r}-\bar{g}} E_t \left\{ \sum_{j=0}^{\infty} \beta^j [g_{t+j} - \bar{g}] \right\}, \\
 &= \frac{-D_t(1+\bar{g})(r_t - \bar{r})}{(\bar{r}-\bar{g})(1+\bar{r}-\rho_1(1+\bar{g}))} + \frac{D_t}{\bar{r}-\bar{g}} E_t \left\{ \sum_{j=0}^{\infty} \beta^j [g_{t+j} - \bar{g}] \right\}.
 \end{aligned} \tag{A.4}$$

Now recall that the holding period return,  $R_t$ , is given by

$$R_t = \frac{P_{t+1} + D_t}{P_t} - 1. \tag{A.5}$$

It can be linearized around  $P_t$  and  $P_{t+1}$  as follows:

$$R_t \cong \bar{R} + \frac{P_{t+1} - \bar{P}_{t+1}}{\bar{P}_t} \dots \frac{(\bar{P}_{t+1} + D_t)}{\bar{P}_t^2} (P_t - \bar{P}_t), \quad (\text{A.6})$$

where  $\bar{P}_{t+1} = (1 + \bar{g})\bar{P}_t$  and  $\bar{R} = \bar{r}(1 + \bar{g})/(1 + \bar{r})$ . Substituting (A.4) into (A.6) yields

$$\begin{aligned} R_t - \bar{R} &\cong \frac{-D_{t+1}(1 + \bar{g})}{\bar{P}_t(\bar{r} - \bar{g})[1 + \bar{r} - \rho_1(1 + \bar{g})]} (r_{t+1} - \bar{r}) \\ &+ \frac{D_{t+1}}{\bar{P}_t(\bar{r} - \bar{g})} \sum_{j=0}^{\infty} \beta^j E_{t+1} \{g_{t+1+j} - \bar{g}\} \\ &+ \frac{D_t(1 + \bar{r})(1 + \bar{g})}{\bar{P}_t(\bar{r} - \bar{g})[1 + \bar{r} - \rho_1(1 + \bar{g})]} (r_t - \bar{r}) \\ &- \frac{D_t(1 + \bar{r})}{\bar{P}_t(\bar{r} - \bar{g})} \sum_{j=0}^{\infty} \beta^j E_t \{g_{t+j} - \bar{g}\}. \end{aligned} \quad (\text{A.7})$$

This can be rewritten as

$$\begin{aligned} R_t - \bar{R} &\cong - \frac{D_t(1 + \bar{g})^2}{\bar{P}_t(\bar{r} - \bar{g})(1 + \bar{r} - \rho_1(1 + \bar{g}))} \\ &\times [(r_{t+1} - \bar{r}) - \beta^{-1}(r_t - \bar{r})] + \zeta_t \\ &= - \frac{\beta(1 + \bar{g})}{1 + \bar{r} - \rho_1(1 + \bar{g})} \\ &\times [(1 - \rho_1 L)^{-1} \xi_{t+1} - \beta(1 - \rho_1 L)^{-1} \xi_t] + \zeta_t, \end{aligned} \quad (\text{A.8})$$

where  $\zeta_t$  reflects changes in expected future dividend growth rates between  $t$  and  $t + 1$ , and the last expression exploits the fact that  $(1 - \rho_1 L)(r_t - \bar{r}) = \xi_t$ . Now defining  $\{\beta(1 + \bar{g})/[1 + \bar{r} - \rho_1(1 + \bar{g})]\} \xi_{t+1} = \tilde{\xi}_t$ , we can multiply through by  $(1 - \rho_1 L)$  so that

$$(1 - \rho_1 L)(R_t - \bar{R}) \cong \tilde{\xi}_t - \left( \frac{1 + \bar{r}}{1 + \bar{g}} \right) \tilde{\xi}_{t-1} + \xi_t - \rho_1 \xi_{t-1}. \quad (\text{A.9})$$

This yields an ARMA(1, 1) representation of returns. Since  $(1 + \bar{r})/(1 + \bar{g}) = (1 + \bar{d})$ , this is eq. (8) in the text.

We now explore the parallel between the time-varying returns model and the fad model, which postulates that returns evolve according to

$$(1 - \rho_1 L)(R_t - \bar{R}) \equiv \varepsilon_t - \rho_1 \varepsilon_{t-1} + \nu_t - \rho_1 \nu_{t-1}. \quad (\text{A.10})$$

For this ARMA(1, 1) process to be the same as (A.9), two restrictions must be satisfied. We find them by equating the variances and first-order autocovariance of the right-hand sides of (A.9) and (A.10):

$$\left[1 + (1 + \bar{d})^2\right] \sigma_\xi^2 + (1 + \rho_1^2) \sigma_r^2 = 2\sigma_r^2 + (1 + \rho_1^2) \sigma_v^2, \quad (\text{A.11})$$

$$(1 + \bar{d}) \sigma_\xi^2 + \rho_1 \sigma_r^2 = \sigma_r^2 + \rho_1 \sigma_v^2. \quad (\text{A.12})$$

Using (A.12) to eliminate  $\sigma_r^2$  from (A.11) we find

$$\sigma_\xi^2 = \frac{(1 - \rho_1)^2}{(1 + \bar{d})(1 + \rho_1^2) - \rho_1 [1 + (1 + \bar{d})^2]} \sigma_v^2. \quad (\text{A.13})$$

Recall that the variance of the fad,  $\sigma_v^2$ , equals  $\sigma_u^2/(1 - \rho_1^2)$ . Using this and the definition of  $\tilde{\xi}_t$ , we find from (A.13) that the variance of required returns corresponding to a given fad variance is

$$\sigma_r^2 = \frac{[1 + \bar{r} - \rho_1(1 + \bar{g})]^2 (1 - \rho_1)^2 (1 + \bar{r})^2}{\left\{ (1 + \bar{d})(1 + \rho_1^2) - \rho_1 [1 + (1 + \bar{d})^2] \right\} (1 + \bar{g})^2} \sigma_u^2. \quad (\text{A.14})$$

This leads immediately to (10) in the text.

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