A TIME SERIES ANALYSIS OF REPRESENTATIVE AGENT MODELS OF CONSUMPTION AND LEISURE CHOICE UNDER UNCERTAINTY*

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This paper investigates empirically a model of aggregate consumption and leisure decisions in which utility from goods and leisure is nontime-separable. The nonseparability of preferences accommodates intertemporal substitution or complementarity of leisure and thereby affects the comovements in aggregate compensation and hours worked. These cross-relations are examined empirically using postwar monthly U. S. data on quantities, real wages, and the real return on the one-month Treasury bill. The estimated values of the parameters governing preferences differ significantly from the values assumed in several studies of real business models. Several possible explanations of these discrepancies are discussed.

I. INTRODUCTION

Two important characteristics of the time series of aggregate wages and hours worked (see, for example, Ashenfelter and Card [1982] and Kydland and Prescott [1982]) are wages are smooth relative to hours worked and hours worked are procyclical. Barro and King [1984], Clark and Summers [1982], Kennan [1987], Kydland [1984], and Kydland and Prescott [1982] argue that temporal nonseparabilities in preferences may be an important ingredient in explaining these findings.

More precisely, Barro and King [1984] compared the theoretical implications of time separable and nontime-separable utility functions for responses of consumption and hours worked to technology shocks. Using a perfect certainty model, they considered once and for all shifts in the production technology that left unaffected the marginal productivity of labor. When preferences are time separable and consumption and leisure are normal goods, they showed that consumption and hours worked move in opposite directions. On the other hand, when preferences are not time separable, they argued that it is possible for consumption and hours

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worked to move in the same direction consistent with the observed procyclicality of aggregate hours worked.

Kennan [1987] analyzed an exactly identified equilibrium model of the aggregate labor market, allowing preferences of the consumer to be time nonseparable. His model accommodates uncertainty in the form of random taste and technology shocks. He found nontime-separable preferences to be an important ingredient for explaining the observed time series behavior of hours worked and compensation.

In this paper we explore further the empirical properties of representative agent models with nontime-separable preferences using data on aggregate consumption, hours worked, compensation, and interest rates. The nonseparability of preferences is modeled in terms of an intertemporal household technology that converts consumption and leisure into consumption and leisure services. The nonseparability of preferences is modeled as being time separable over services. The intertemporal mapping of leisure into leisure services can be interpreted in several ways. Barro and King [1984] interpret this mapping as reflecting the effect of fatigue from working hard in previous time periods. Kydland [1984] interprets this mapping as arising because leisure time is used in augmenting an unobservable stock of household capital. Under both of these interpretations, past quantities of leisure increase current leisure services. In addition to considering models with this positive dependence, Kennan [1987] also considered models in which there was some notion of habit persistence so that current leisure services depend negatively on past quantities of leisure. We also allow for this effect in our empirical analysis.

Although there are similarities between our analysis and the analyses by Altug [1985], Kennan [1987], and Kydland and Prescott [1982] of models with nonseparable preferences, there are also some important differences. For instance, while Kennan admits preference shocks in his model, he only considers preference specifications that are linear in consumption. As a consequence, the equilibrium real interest rate in terms of a consumption numeraire is implied to be constant. We exclude preference shocks and in effect follow Kydland and Prescott [1982] by assuming that the technology shocks are the most prevalent exogenous forcing process. In addition, consumption does not enter linearly into preferences in our model so that equilibrium real interest rates are not restricted to be constant.

In contrast to all three papers, we do not consider a complete equilibrium model. Instead we use the no-preference-shock restriction to identify and estimate the parameters of preferences and the household technology. Altug [1985] and Kydland and Prescott [1982] study implications of their models by deducing implications from the equilibrium law of motion for quantity variables as calculated from an approximate social planners problem. They do not, however, investigate empirically the cross relations between prices and quantities that are implied by their model. The focal points of our empirical analysis are the cross relations between prices and quantities implied by our specifications of preferences and household technology. The assumption of nontime-separable preferences, among other things, differentiates our analysis of price-quantity relations from that of Mankiw, Rotemberg, and Summers [1985].

A representative consumer framework is used in this study because it provides an analytically tractable way of deducing implications of consumption and leisure choice under uncertainty for the joint behavior of asset returns and other aggregates. Representative agent models of aggregate labor supply have been used by Lucas and Rapping [1969]; Hall [1980]; Kydland and Prescott [1982]; and Mankiw, Rotemberg, and Summers [1985], among others. We recognize that the assumptions commonly used to rationalize a representative agent model in the presence of heterogeneous consumers (e.g., see Rubinstein [1974] and Eichenbaum, Hansen, and Richard [1985]) are not very compelling in the case of aggregate labor supply. For instance, the common assumption of complete securities markets implies that the implicit price of leisure for all consumers be identical.

For the particular specifications of preferences that we use, time-invariant efficiency units could be introduced, and after rescaling, the rationalization for a representative consumer would be preserved (see Muellbauer [1981] and Appendix A). However, this introduces only a very limited amount of diversity in skills among workers and still imposes restrictions that are not supported by the microeconomic evidence (e.g., see Sattinger [1978]). Further, the assumption that consumers choose optimally to be at interior points in their respective commodity spaces rules out consumers moving in and out of the labor force over time. Hence, the behavior of the fictitious representative agent confounds movements of some consumers into and out of the labor force with movements in hours worked by other consumers who are in the labor force. In fact, there is substantial evidence that much of the variation in aggregate hours worked can be attributed to movements in and out of employment (e.g., see Coleman [1984]). In spite of these well-known criticisms of the representative consumer paradigm, we still use it in this paper to help document its ability or inability to explain the aggregate time series.

The empirical methodology used is an extended version of the Euler equation methods suggested by Hansen and Singleton [1982]. They show how to exploit shock exclusion restrictions from preferences to estimate and test representative consumer models using generalized method of moments estimators. Eichenbaum and Hansen [1985] and Dunn and Singleton [1986] show how their methodology can be extended in a straightforward manner to apply to the more general specifications of preferences considered in this paper. In addition to applying this methodology, we illustrate how to test a subset of the implied moment restrictions (in this case, the moment restrictions implied by the intratemporal Euler equation).

The paper is organized as follows. In Section II the preferences of the representative consumer are described, and then, using this specification, relations among consumption, hours worked, compensation, and asset returns are deduced. In Section III we describe the data used in our empirical analysis. In Section IV we show how to obtain estimates of preference parameters and test the relations derived in Section II. The empirical results are presented and discussed in Section V. Finally, concluding remarks are presented in Section VI.

II. Preferences of the Representative Consumer

The representative consumer is assumed to have preferences defined over the services provided by the acquisitions of consumption goods and leisure time. Accordingly, we introduce two hypothetical services that are linear functions of current and past values of consumption and leisure, respectively:

$$c_t^* = A(L)c_t, \tag{1}$$

$$l_t^* = B(L)l_t, \tag{2}$$

where c_t is the amount of the consumption good purchased at date t and l_t denotes hours of leisure at date t.[1] The polynomial in the lag

1. A more general specification of this technology would allow $c_t^*(l_t^*)$ to depend also upon current and lagged values of $l_t(c_t)$. However, for reasons of empirical tractability, we consider the specifications given by (1) and (2).

operator $A(L)$ is given by

$$A(L) = 1 + \alpha L, \tag{3}$$

and $B(L)$ is given by either

$$B_1(L) = 1 + \delta L/(1 - \eta L) \tag{4}$$

or

$$B_2(L) = 1 + bL. \tag{5}$$

The time t leisure and consumption decisions are constrained to be in an exogenously specified information set I_t of the representative agent.

Expression (1) and the assumed form of $A(L)$ imply that the service flow from consumption goods at date t, c_t^*, depends linearly on consumption acquisitions at dates t and $t - 1$. The coefficient α is assumed to be nonnegative so that consumption acquisitions at time t contribute consumption services (and not disservices) in the current and one future time period.

In (2), l_t^* denotes a leisure service that depends linearly on current and lagged values of leisure time. The case in which $B(L) = B_1(L)$ corresponds to the leisure service specification suggested by Kydland and Prescott [1982]. They assume that δ is greater than or equal to zero and that η is between zero and one. In contrast, we do not restrict the sign of δ in our empirical analysis. Under this service technology, one unit of leisure time at date t contributes $\delta\eta^{\tau-1}$ units of leisure services at date $t + \tau$. Therefore, the sign of δ determines whether leisure time today provides leisure services or disservices in future time periods. Leisure time today augments leisure services in future time periods when δ is positive, diminishes leisure services in future time periods when δ is negative, and has no impact on leisure services in future time periods when δ is zero. The impact of current leisure time on future leisure services decays geometrically at the rate η.

When $B(L) = B_2(L)$, leisure time today provides leisure services today and either leisure services or disservices one period in the future depending on whether b is positive or negative.

The representative agent is assumed to rank alternative streams of consumption and leisure services using the time and state separable utility function,

$$E \sum_{t=0}^{\infty} \beta^t \frac{(c_t^{*\gamma} l_t^{*(1-\gamma)})^\theta - 1}{\theta}, \tag{6}$$

where β and γ are preference parameters between zero and one, θ is a preference parameter that is less than one, and E denotes the mathematical expectation. When θ is equal to zero, we interpret (6) to be the logarithmic specification,

$$E \sum_{t=0}^{\infty} \beta^t \{\gamma \log c_t^* + (1 - \gamma) \log l_t^*\}, \tag{7}$$

which is separable across consumption and leisure services.

There are several reasons for selecting this specification of preferences. First, it has received considerable attention in the literature following the analysis of Kydland and Prescott [1982]. Second, it accommodates preferences that are separable across consumption and leisure as a special case (i.e., (7)). Third, our analysis of a representative consumer extends immediately to an environment with many consumers who have identical preferences but possibly heterogeneous initial endowments of capital. Very similar econometric relations to those implied by (6) can also be derived in an environment in which consumers' marginal products of labor are distinct as long as there is a time-invariant transformation in terms of efficiency units that makes consumers' labor perfectly substitutable. In this latter case efficiency units are priced, and their relative price can be inferred from the aggregate compensation data after correction by a time-invariant translation factor (see Appendix A). There are some additional econometric advantages to the preference specification (6) that are discussed in Section III.

The service technologies (4) and (5) allow for either intertemporal substitutability or complementarity of leisure. To see this, notice that the marginal utilities of services implied by (6) are

$$MC_t^* = \beta^t \gamma c_t^{*\gamma\theta-1} l_t^{*(1-\gamma)\theta} \tag{8}$$

$$ML_t^* = \beta^t (1 - \gamma) c_t^{*\gamma\theta} l_t^{*(1-\gamma)\theta-1} \tag{9}$$

The joint specification of an intertemporal service technology and preferences defined over services can be viewed as inducing an indirect set of preferences defined over leisure time and consumption acquisitions. Letting MC_t and ML_t denote the indirect marginal utilities of consumption acquisitions and leisure at time t, it follows from (1) and (2) that[2]

$$MC_t = E[A(L^{-1}) MC_t^* | I_t], \tag{10}$$

2. Relations (10) and (11) ignore any nonnegativity constraints on c_t and l_t.

(11) $$ML_t = E[B(L^{-1})ML_t^* \mid I_t].$$

Holdings fixed expectations about future consumptions and leisures (or examining the corresponding perfect foresight marginal utilities), the sign of $\partial ML_t/\partial l_{t-1}$ is determined by the signs of δ and b in (4) and (5), respectively. If $b > 0$ or $\delta > 0$, then $\partial ML_t/\partial l_{t-1} < 0$, and preferences exhibit intertemporal substitution. On the other hand, if $b < 0$ or $\delta < 0$, then l_t and l_{t-1} are considered complements. When $b = 0$ or $\delta = 0$, then $ML_t = ML_t^*$ and preferences are time separable along the leisure dimension.

The first-order conditions of the representative agent choosing optimally to allocate consumption and leisure over time imply that

$$w_t MC_t = ML_t,$$

where w_t is the real wage. Substituting from (8)–(11) and rearranging terms gives

(12) $$E[w_t\{A(\beta L^{-1})\{\gamma[A(L)c_t]^{\gamma\theta-1}[B(L)l_t]^{(1-\gamma)\theta}\}\} - B(\beta L^{-1})\{(1-\gamma)[A(L)c_t]^{\gamma\theta}[B(L)l_t]^{(1-\gamma)\theta-1}\} \mid I_t] = 0.$$

Note that when $A(L)$ and $B(L)$ are the identity operators, relation (12) holds without taking conditional expectations. In this case, (12) implies an exact relation among current wages, consumption, and leisures: $c_t/w_t l_t = \gamma/(1-\gamma)$.

If the consumer can trade a one-period asset with a price of one unit of c_t and with a random payoff of r_{t+1} units of c_{t+1} at date $t+1$, then a second necessary condition for utility maximization is that

(13) $$E[r_{t+1}MC_{t+1}] = MC_t.$$

Substituting from (8) and (10) gives

(14) $$E[r_{t+1}\beta\{A(\beta L^{-1})\{[A(L)c_{t+1}]^{\gamma\theta-1}[B(L)l_{t+1}]^{(1-\gamma)\theta}\}\} - A(\beta L^{-1})\{[A(L)c_t]^{\gamma\theta-1}[B(L)l_t]^{(1-\gamma)\theta}\} \mid I_t] = 0.$$

Expressions (12) and (14) are used in Section IV to deduce a set of estimation equations.

III. Description of the Data and Analysis of Trends

The formal justification of the econometric procedures described in Section IV and implemented in Section V rely on the assumption that the variables entering the estimation equations are stationary (see Hansen [1982]). In fact, some of the time series considered exhibited pronounced trends during the sample period.

Consequently, a stationary-inducing transformation of the data is required. The choice of detrending procedure is restricted in our context by the requirement that the transformed series satisfy the stochastic Euler equations (12) and (14). Therefore, after briefly describing the data used in the empirical analysis, we discuss in detail a model of nonstationarity that rationalizes the particular transformation involved here. This transformation does not require a priori or simultaneous estimation of parameters governing the nonstationarities because of the particular parameterization of preferences adopted in our analysis.

The monthly, seasonally adjusted observations on aggregate real consumption of nondurables and services were obtained from the Citibank Economic Database. The per capita consumption series was constructed by dividing each observation of the aforementioned measure of aggregate real consumption by the corresponding observation on the total adult (age sixteen and over) population, published by the Bureau of the Census. The asset return considered is the ex post real return on one-month Treasury bills.[3] Nominal returns reported in Ibbotson and Sinquefield [1979] were converted to ex post real returns using the implicit price deflator for nondurables and services. Nominal wages were measured by the seasonally adjusted average hourly compensation for all employees on nonagricultural payrolls, obtained from the Citibank Economic Database. Real wages were constructed by dividing each observation on nominal wages by the implicit price deflator associated with our measure of consumption.

We constructed a measure of hours worked, h_t, by forming the ratio of total hours worked by the civilian labor force and our measure of population. Like our compensation measure, this measure of hours averages across members of the population who were and were not employed, a point to which we shall return subsequently. The representative consumer was given a time endowment of 112 hours a week and 4.25 weeks per month, which gives a monthly time endowment (h_0) of 476 hours. The leisure series (l_t) was then calculated by subtracting hours worked from the monthly time endowment. All data covered the period 1959:1 to 1978:12.

For the equilibrium relations (12) and (14) to be consistent with these data, certain relations among the respective growth rates of the series must be satisfied. The most desirable way to model

3. We also considered the value-weighted average of returns on the New York Stock Exchange. The results of the empirical analysis were qualitatively the same as those reported in this paper.

nonstationarities in consumption and hours worked is to specify technologies for capital accumulation and the production of new consumption goods that include temporal shifts in the productivity of labor or capital. By combining such a specification of technology with a preference specification, one could in principle construct a stochastic growth model with the nonstationarities in consumption and hours worked modeled endogenously.

In our analysis, we assume that the following vector,

$$x_t = (c_t/c_{t-1}, l_t, w_t l_t/c_t, r_t - 1), \tag{15}$$

forms a strictly stationary stochastic process. Notice that the assumption that l_t and $w_t l_t/c_t$ are stationary implies that lnw_t and lnc_t have a common trend. This assumption is consistent with Altug's modification of the Kydland-Prescott model in which there is a geometric trend in the technology. It is also consistent with Christiano's [1986] growth model in which the technology shock can have a random walk component with drift.[4]

It is possible to derive relations from (12) and (14), respectively, that involve only current, past, and future values of x_t. We illustrate this point for the case in which $B(L) = B_2(L) = 1 + bL$. Let $\sigma_0 = (\beta,\theta,\gamma,\alpha,\beta)$,

$$H_c[c_t,c_{t-1},l_t,l_{t-1},\sigma_0] = \{\gamma[c_t + \alpha c_{t-1}]^{\gamma\theta-1}[l_t + bl_{t-1}]^{(1-\gamma)\theta}\}; \tag{16}$$

$$H_l[c_t,c_{t-1},l_t,l_{t-1},\sigma_0] = \{(1-\gamma)[c_t + \alpha c_{t-1}]^{\gamma\theta}[l_t + bl_{t-1}]^{(1-\gamma)\theta-1}\}. \tag{17}$$

The expressions given in (16) and (17) are in the information set at time t. Therefore, (12) implies that

$$E[H_w(x_t,x_{t+1},x_{t-1},\sigma_0) \mid [I_t] = 0, \tag{18}$$

where

$$H_w(x_t,x_{t+1},x_{t-1},\sigma_0) = \frac{w_t(1+\alpha\beta L^{-1})H_c[c_t,c_{t-1},l_t,l_{t-1},\sigma_0] - (1+b\beta L^{-1})H_l(c_t,c_{t-1}l_t,l_{t-1},\sigma_0)}{H_l(c_t,c_{t-1},l_t,l_{t-1},\sigma_0)}. \tag{19}$$

Even though $H_c(\bullet)$ and $H_l(\bullet)$ depend on c_t, c_{t-1}, l_t, and l_{t-1} separately, $H_w(\bullet)$ depends only on x_t, x_{t-1}, and x_{t+1}, where x_t is defined in (15). A similar strategy can be employed in transforming equation (14) to obtain

$$E[H_r(x_t,x_{t+1},x_{t+2},x_{t-1},\sigma_0) \mid I_t] = 0, \tag{20}$$

4. To obtain this result, Christiano [1986] assumes that preferences are logarithmically separable in consumption and leisure and time separable in consumption.

where

(21) $H_r(x_t, x_{t+1}, x_{t+2}, x_{t-1}, \sigma_0)$

$$= \frac{\beta r_{t+1}\{(1 + \alpha\beta L^{-1})H_c(c_{t+1}, c_t, l_{t+1}, l_t, \sigma_0)\} - (1 + \alpha\beta L^{-1})H_c(c_t, c_{t-1}, l_t, l_{t-1}, \sigma_0)}{H_c(c_t, c_{t-1}, l_t, l_{t-1}, \sigma_0)}.$$

Relations (18) and (20) are used in Section IV to derive the estimation equations.

IV. Estimation and Inference

Our approach to estimation and inference follows closely that of Eichenbaum and Hansen [1985] and Dunn and Singleton [1986]. These authors show how to modify the analysis of Hansen and Singleton [1982] to allow for multiple consumption goods and preferences that are not separable over time.

First, we consider the case in which $B(L) = B_2(L) = 1 + bL$. Using the notation from Section III, consider the following two estimation equations:

$$d_{t+2} = \begin{bmatrix} H_w(x_t, x_{t+1}, x_{t-1}, \sigma_0) \\ H_r(x_t, x_{t+1}, x_{t+2}, x_{t-1}, \sigma_0) \end{bmatrix}. \tag{22}$$

Relations (18) and (20) imply that the $E[d_{t+2}|I_t] = 0$. Consequently, d_{t+2} is orthogonal to all variables in I_t. Let z_t be an R-dimensional vector of variables in I_t, where $2R$ is greater than or equal to five. Using the components of z_t as instruments, the $2R$-dimensional function g_T,

$$g_T(\sigma) = \left(\frac{1}{T}\right) \sum_{j=1}^{T} z_t \otimes d_{t+2}(\sigma), \tag{23}$$

can be formed from the sample information. The vector $g_T(\sigma)$ is a consistent estimator of $Ez_t \otimes d_{t+2}(\sigma)$, and the expectation $Ez_t \otimes d_{t+2}(\sigma)$ is, in general, nonzero except at the point $\sigma = \sigma_0$. Therefore, we estimate σ_0 by the choice of σ, say σ_T, in an admissible parameter space that makes $g_T(\sigma)$ close to zero in the sense of minimizing the quadratic form,

$$g_T(\sigma)' W_T g_T(\sigma), \tag{24}$$

where W_T is a symmetric positive definite distance matrix that can depend on sample information.

Hansen [1982] shows that the choice of W_T that minimizes the asymptotic covariance matrix of σ_T, depends on the autocovariance structure of the disturbance vector d_{t+2}. Although this vector is serially correlated, it is in the information set at time $t + 2$. Hence the theory implies the restrictions,

$$E\{(z_{t+k} \otimes d_{t+k+2})(z_t \otimes d_{t+2})'\} = 0, \qquad \text{for } |k| \geq 2. \tag{25}$$

It follows that the optimal estimator is obtained by choosing W_T^{-1} to be a consistent estimator of

$$S_0 = \sum_{k=-1}^{1} E(z_{t+k} \otimes d_{t+k+2})(z_t \otimes d_{t+2})'. \tag{26}$$

Hansen [1982] discusses a candidate estimator of S_0. In Appendix B we describe an alternative estimator that, unlike the estimator suggested by Hansen, is constrained to be positive definite in finite samples.

The corresponding sequence of minimized values of (24), denoted $\{J_T{:}T \geq 1\}$, converges in distribution to a chi-square random variable with degrees of freedom equal to the difference between the total number of unconditional moment restrictions and the number of coordinates of σ. Hence J_T can be used to test the overidentifying restrictions.

Recall from the discussion of (12) that if the induced preferences defined over consumption acquisitions and leisure are time separable, then there is an exact relationship between hours worked, consumption acquisitions, and wages. In this case, the first component of d_{t+2} is actually in I_t and hence is zero. An analogous observation applies to any specification of time-separable preferences that like ours exclude unobservable shocks to preferences. Hence, temporal nonseparabilities in preferences are necessary in our analysis in order for one of the disturbance terms to be different from zero.

The introduction of unobservable shocks to preferences or measurement errors does not lead to additive error terms for the specification of preferences given in Section II. Consequently, in the presence of such unobservables, our approach to estimation cannot be used. Accommodation of these unobservables seems to require explicit or numerical solutions to the stochastic general equilibrium model, while the approach adopted here avoids the need for such solutions.

Two additional problems arise in estimating the parameters of the model with $B(L) = B_1(L)$. First, for hypothetical values of the parameters, the leisure service at any point in time depends on the entire infinite past of the consumption of leisure time. For instance, in the first time period we have that the leisure service is given by

(27) $$l_1^* = l_1 + \delta \sum_{j=0}^{\infty} \eta^j l_{-j}.$$

Since we do not have observations on values of leisure time prior to time period 1, we approximate the infinite sum,

(28) $$\sum_{j=0}^{\infty} \eta^j l_{-j},$$

by the average of the consumption of leisure time in our sample divided by $(1 - \eta)$ for each hypothetical value of η.[5] Then, given an initial value of leisure services, the remaining values of leisure services for our sample can be calculated using the sample observations on leisure time consumption and hypothetical values of η and δ. In this manner we are able to calculate values of MC_t^* and ML_t^* for hypothetical values of the preference parameters.[6]

The second problem that occurs is that ML_t as given by (11) now depends on the current and expected infinite future of ML_t^*. However, following Holtz, Kydland, and Sedlacek [1985], the relation,

(29) $$w_t MC_t = ML_t,$$

also implies that

(30) $$E\{(1 - \eta L^{-1})\{w_t(1 - \alpha L^{-1})MC_t^*\}|I_t\} = E\{[1 + (\delta - \eta)L^{-1}]ML_t^* | I_t\},$$

for $B(L) = B_1(L)$. A virtue of the expression in (30) is that it depends only on terms involving MC_t^*, MC_{t+1}^*, MC_{t+2}^*, ML_t^* and ML_{t+1}^*.

Relation (30) can be used in deriving an expression analogous to (12) by substituting in for MC_t^* and ML_t^* from (28) and (29). This expression together with (14) then can be used to define two estimation equations with disturbance terms arising from expectational errors. The stationary-inducing transformation described in Section III can be modified appropriately to convert these relations to relations among variables that are assumed to be components of a strictly stationary stochastic process. Estimation then proceeds in the same fashion as in the case in which $B(L) = B_2(L)$.

5. Under our assumption that the l_t process is stationary, $E\Sigma_{j=0}^{\infty} \eta^j l_{-j} = El_t/(1 - \eta)$. Thus, our procedure amounts to replacing (28) with the sample estimate of its unconditional mean.

6. It can be shown that neither the consistency of our estimators nor the relevant asymptotic distribution theory is affected by the fact that our measure of the initial condition is undoubtedly incorrect.

V. Empirical Results

Estimates for the Kydland and Prescott specification of $B(L)$ were obtained using the following orthogonality conditions:

$$(31) \quad E[d_{1t+2}(1,V_t')] = 0 \quad \text{and} \quad E[d_{2t+2}(1,V_t',V_{t-1}')] = 0,$$

where

$$V_t' = \left[\frac{c_t - c_{t-1}}{c_{t-1}}, \frac{l_t - l_{t-1}}{l_{t-1}}, \frac{w_t - w_{t-1}}{w_{t-1}}, r_t - 1\right].$$

Thus, fourteen orthogonality conditions were imposed. The results are displayed in Table I.

The estimates displayed under the heading "Wage 1" were obtained using the data described in Section III. All of the parameter estimates are economically meaningful except for $\hat{\beta}$, which is slightly larger than unity. The latter finding is common to several recent empirical studies of intertemporal Euler equations using

TABLE I[a]
$A(L) = 1 + \alpha LB(L) = 1 + \delta L/(1 - \eta L)$

Parameters	Wage 1[b]	Wage 2[b]	Tax-adjusted[b]
β	1.0012 (0.0002)	1.0009 (0.0002)	1.0013 (0.0003)
θ	0.85585 (0.0827)	0.8014 (0.1880)	−0.1690 (0.4337)
γ	0.14299 (0.0237)	0.1676 (0.0213)	0.14390 (0.1062)
α	0.30744 (0.0741)	0.3520 (0.0655)	0.3654 (0.0592)
η	0.98302 (0.0146)	0.9816 (0.0177)	0.9884 (0.0123)
δ	−0.02394 (0.0212)	−0.0269 (0.0272)	−0.0169 (0.0203)
J_T	25.102 (0.9985)	20.30 (0.9907)	16.55 (0.9648)
C_T test	23.529 (0.9997)	12.48 (0.8689)	65.12 (1.000)

a. Standard errors of the estimates and probability values of the test statistics are given in parentheses.
b. The estimates under the heading Wage 1 were obtained using the data described in Section III. The estimates under Wage 2 were obtained with nominal wages measured as the ratio of aggregate employee compensation (from the National Income and Product Accounts) divided by our constructed measure of aggregate hours worked. The Tax-adjusted run is identical to the Wage 2 run, except that wages and asset returns are calculated on an after-tax basis.

treasury bill returns (see Singleton [1988]). The estimates of θ and γ imply that the representative consumer's utility function is concave. In contrast, the estimates obtained by Mankiw, Rotemberg, and Summers [1985] for a time-separable specification were typically in the nonconcave region of the parameter space. Potential reasons for the economically more plausible results obtained here include the specifications of period utility functions are different; consumption goods and leisure provide services over time in our formulation but not in theirs; and they do not detrend consumption, hours worked, and the real wage, whereas our econometric procedure accommodates geometric growth or stochastic trends in the logarithms of these variables.

Next, consider the parameters that govern the intertemporal aspects of the service technologies. In all cases the estimate of α is both positive and large relative to its estimated standard error.[7] This implies that consumption good acquisitions today give rise to consumption services both today and one period in the future. The estimates of η and δ raise some interesting quandaries. The estimate of δ is negative implying that current leisure acquisitions give rise to future leisure disservices. The estimate of δ, however, is small relative to its estimated standard error. When δ is zero, η ceases to be identified if the model is specified correctly, because relation (30) simplifies to

$$E\{w_t(1 - \alpha L^{-1})MC_t^* \mid I_t\} = E\{ML_t^* \mid I_t\}, \tag{30'}$$

which does not include the parameter η. The results in Table I indicate that η is estimated quite accurately, even though δ is estimated imprecisely. This suggests that the model is fundamentally misspecified for the following reason. Under the null hypothesis that the model is correct, the disturbance term associated with (30′) should be serially uncorrelated. In fact, the forward filtering is apparently being exploited to improve the fit of the model, which would imply that the autocorrelation function of the disturbance terms in our econometric model is inconsistent with the implications of the theory.

We also studied a specification of the mapping from leisure to leisure services that does not require forward filtering. We esti-

7. Interestingly, Eichenbaum and Hansen [1985] and Dunn and Singleton [1986] in their analysis of purchases of nondurable and durable consumption goods also present evidence of intertemporal nonseparabilities in the mapping from nondurable consumption goods to nondurable consumption good services.

TABLE II[a]
$A(L) = 1 + \alpha L \quad B(L) = 1 + bL$

Parameters	Wage 1[b]	Wage 2[b]	Tax-adjusted[b]
β	31.0013 (0.0002)	1.0009 (0.0002)	1.0020 (0.0002)
θ	0.0061 (0.0680)	−0.0761 (0.0681)	−0.0009 (0.0352)
γ	0.1158 (0.0002)	0.1459 (0.0002)	0.1832 (0.0006)
α	0.7304 (0.1471)	0.4032 (0.0820)	0.4405 (0.0778)
b	−0.6824 (0.0386)	−0.7562 (0.0429)	−0.8321 (0.0216)
J_T	56.067 (1.000)	25.46 (0.9975)	35.15 (0.9999)
C_T test	48.119 (1.000)	17.52 (0.9749)	23.61 (1.000)

a. Standard errors of the estimates and probability values of the test statistics are given in parentheses.
b. The estimates under the heading Wage 1 were obtained using the data described in Section III. The estimates under Wage 2 were obtained with nominal wages measured as the ratio of aggregate employee compensation (from the National Income and Product Accounts) divided by our constructed measure of aggregate hours worked. The Tax-adjusted run is identical to the Wage 2 run, except that wages and asset returns are calculated on an after-tax basis.

mated the model using the parsimonious representation of $B(L)$ given by (5) and fourteen orthogonality conditions. The results are reported in the first column of Table II. Notice that the estimated values of θ are closer to zero than those reported in Table I. Also, there is little evidence against the hypothesis that preferences are logarithmically separable. Perhaps more importantly, the point estimates again imply that current and future leisure decisions are intertemporal complements. Unlike the estimates of δ, the estimates of b are large in absolute value relative to their standard errors.

The representative consumer always chooses positive values of l_t^*. Therefore, when b is negative, he always must choose enough leisure to offset the negative impact of past leisure choices on the level of current leisure services. For example, if $B(L) = B_2(L)$ and $b < 0$, then it must be the case than $l_t > |bl_{t-1}|$ for all t. Thus, based on the estimates of b reported in Table II, the representative consumer will always choose a value of l_t that is greater than approximately ⅔ of l_{t-1}. It follows that increases in hours worked will be accomplished in a relatively gradual way, while decreases in hours worked are unrestricted.[8]

For comparison, estimates were also obtained using the ratio of aggregate total employee compensation from the National Income and Product Accounts to our measure of aggregate hours as the nominal wage rate. These results are displayed in Tables I and II under the heading "Wage 2." The estimated parameters are similar to those obtained using "Wage 1."

We next discuss the estimates of γ. Kydland and Prescott [1982] argue that γ should be approximately ⅓. Their rationale for the choice is "motivated by the fact that households' allocation of time to nonmarket activities is about twice as large as the allocation to market activities" [p. 1352]. Since our estimates of γ are considerably smaller than ⅓, it is of interest to understand why. One rough set of calculations involves abstracting from uncertainty as well as dynamics and conducting a steady state analysis. The steady state that we consider treats leisure, and the valuation of leisure relative to consumption as constants, but accommodates geometric growth in consumption and wages. Letting $[c/wl]$ be the steady state ratio of consumption to the valuation of leisure, it follows from (12) that

$$\gamma = \frac{c/(wl)}{1 + [c/(wl)]} . \tag{32}$$

Relation (32) is the standard relation between γ and expenditure shares for Cobb-Douglas preferences.

Recall that relation (12) was also used to construct relation (18) which is utilized in our econometric analysis. One of the orthogonality conditions that we imposed in our estimation procedure amounts to scaling (12) in order to induce stationarity and then taking unconditional expectations ($Ed_{1t+2} = 0$). This orthogonality condition imposes the stochastic counterpart of the steady state relation (32). Substituting time averages of consumption relative to the valuation of leisure for $c/(wl)$ in (32) gives values of $\gamma = 0.13$ and $\gamma = 0.16$ for the "Wage 1" and "Wage 2" measures of compensation, respectively. These values are very similar to the point estimates reported in columns 1 and 2 of Tables I and II, respectively, suggesting that (32) is a useful guide for interpreting $\hat{\gamma}$.

8. There is a literature that models temporally nonseparable preferences defined over consumption goods as reflecting the presence of "habit-formation." Negative estimated values of b and δ are consistent with this interpretation. See Pollak [1970] for an overview of habit-formation models.

From (32) it follows that γ depends on the assumed value of the number of leisure hours. For our choice of total time endowment and measure of hours worked, the ratio of average hours worked to leisure is about 0.20, which is considerably less than one half, the number assumed by Kydland and Prescott [1982]. Increasing the percentage of total hours allocated to leisure could drop γ closer to the value assumed by Kydland and Prescott. We have chosen to include all individuals age 16 and over in our sample when calculating leisure time. Hence our sample includes unemployed adults. This approach seems sensible, because the representative consumer model confounds the behavior of employed and non-employed individuals.

Formula (32) also suggests that the value of γ will be sensitive to the measure of compensation. One possible problem is that wages should be measured in efficiency units. Interpreting the model as applying to efficiency units of labor in an environment where consumers have distinct marginal products of labor complicates the relation between observed total compensation and efficiency unit wages (see Appendix A). A second possible problem is that the measure of compensation used in obtaining the results reported in columns 1 and 2 of Tables I and II is not corrected for taxes. For the sake of comparison we also estimated the model using after-tax wages and returns. Our results are displayed in the last columns of Tables I and II. The time series on annual marginal tax rates was taken from Seater [1985]. The annual rates were interpolated linearly to obtain monthly rates. The adjustment for taxes lowers the average real wage. Equation (32) implies that this should result in a *larger* value of γ. The estimated values of γ in Tables I and II are larger for the tax-adjusted data than the corresponding estimates from the unadjusted data. In fact, for the specification $B_1(L)$, the estimates of γ are within one standard error of the value of one third that was imposed by Kydland and Prescott [1982]. The estimates of γ are less precise when tax adjustments are made, however.

Our discussion of the point estimates must be qualified by the fact that the J_T statistics reported in Tables I and II are large relative to the degrees of freedom. These large test statistics may occur because of model misspecification or measurement errors in some of the time series. Of course, test statistics are more revealing when more specific alternative hypotheses are considered. One particular alternative hypothesis is straightforward for us to consider. Suppose that the aggregate compensation series does not reflect the appropriate marginal valuation of time. This may be true because of measurement errors or because compensation arrangements other than the payment of spot market wages are used to implement the competitive equilibrium. In these cases, the estimation equation obtained from the intratemporal Euler equation (12) is misspecified, while the equation obtained from the intertemporal Euler equation is still valid. We take this as the alternative hypothesis and test the null hypothesis that equation (12) is valid as well. We examined these hypotheses using a statistical test that is analogous to a likelihood ratio test. A formula for the test statistic is presented formally in Appendix C, and its asymptotic properties are discussed. In Tables I and II the value of this test statistic is denoted by C_T. The values of C_T do suggest that the large J_T statistics are indicative of the failure of the orthogonality conditions associated with the Euler equation relating ML_t, MC_t, and w_t to hold in the sample.

To explore this possibility further, we reestimated the parameters using only the orthogonality conditions associated with the intertemporal relation (14). In conducting this exercise, it was necessary to fix the values of γ and η in the model with $B(L) = B_1(L)$ and the value of γ in the specification of the model with $B(L) = B_2(L)$ in order to obtain convergence of the minimization algorithm. (Recall that $\hat{\gamma}$ seems to be determined largely by the intratemporal Euler equation.) The results are displayed in Table III for the second measure of wages (Wage 2). Notice first that the probability values of the J_T statistics are substantially smaller than the probability values for the corresponding statistics in Tables I and II. For

TABLE III
ESTIMATES BASED ON INTERTEMPORAL EULER EQUATION[a]

	$B(L) = (1 + \delta L/(1 - \eta L))$	$\gamma = 0.14$ $\eta = 0.98$	
β	θ	α	δ
1.00164	−0.02867	0.33049	−0.01564
(0.0006)	(1.9831)	(0.0626)	(0.3048)
	$J_T^{**} = 8.663$	(0.8767)	
	$B(L) = (1 + bL)$ $\gamma = 0.14$	$\eta = 0.98$	
β	θ	α	b
1.00143	0.69126	0.31175	0.70621
(0.0003)	(0.7944)	(0.0754)	(0.6234)
	$J_T^{**} = 8.1206$	(0.8503)	

a. Standard errors of the estimates and probability values of the test statistics are given in parentheses.

both models the estimates of α remain positive and are estimated precisely. Second, with $B(L) = B_1(L)$ the point estimates are qualitatively similar to the corresponding estimates reported in Table I. The primary difference is the loss of precision when only the intertemporal Euler equation is used in the empirical analysis. On the other hand, for the model with $B(L) = B_2(L)$, the sign of b changes from negative to positive when the intratemporal Euler equation is omitted from the analysis, though the standard error is large relative to $\hat{b}$. Clearly, the precision in estimating b in Table II is due to the inclusion of the moment conditions associated with the intratemporal equation (12), in which case $\hat{b}$ is negative. Thus, the results in Table III convey little information about the nature of nonseparabilities in preferences.

VI. Conclusion

In this paper we estimated and tested a representative consumer model that relates aggregate consumption, hours worked, compensation, and interest rates. We found substantial evidence for nontime-separable preferences, both with respect to consumption and leisure. In the case of leisure, we found that leisure today decreased leisure services in the subsequent time period. Kennan [1987] found a similar effect in his empirical analysis using a different preference specification, a different set of identifying restrictions, and a different method of estimation. Hence, both Kennan's empirical analysis and ours failed to find empirical evidence in support of one of the sources of endogenous dynamics in the Kydland-Prescott [1982] real business cycle model.

We also found substantial evidence against the overidentifying restrictions implied by our model. There was, however, substantially less evidence against an alternative hypothesis that maintained only the intertemporal Euler equation relating aggregate consumption and hours worked to the interest rate. Under this alternative hypothesis, the statistical evidence against our original model is attributable to discrepancies between measured real wages and consumers' marginal rates of substitution between consumption and leisure.

Appendix A

In this appendix we consider the implications for our econometric analysis of consumers having distinct marginal products of labor. We consider only the special case in which individual labor supply can be converted into efficiency units that are comparable across consumers. Consumers are presumed to be compensated for the quantities of efficiency units of labor they supply. Muellbauer [1981] studies this problem in a single-period context and obtains necessary and sufficient conditions for aggregation. Here we allow for multiple time periods but restrict our attention to the class of preferences used in our empirical analysis.

First, we introduce some notation. Let c_t^j denote the consumption of person j at time t and l_t^j denote the leisure of person j at time t. We assume that hours worked at time t by person j can be converted to efficiency units by multiplying the hours worked by e^j, where e^j is a positive number not indexed by time. Hence the efficiency units of leisure of person j at time t are $e^j l_t^j$. Similarly, the efficiency units of leisure services are given by $e^j l_t^{*j}$, where $l_t^{*j} = B(L) l_t^j$.

Suppose that all J consumers have identical preferences given by (6). These preferences could equivalently be expressed in terms of efficiency units of leisure services. The conversion to efficiency units simply scales the utility function. Since preferences are homothetic, in a competitive equilibrium with complete markets in consumption and leisure services;

$$\text{(A.1)} \qquad \begin{aligned} c_t^{*j} &= \omega^j [c_t^{*1} + c_t^{*2} + \ldots + c_t^{*J}]/J; \\ e^j l_t^{*j} &= \omega^j [e^1 l_t^{*1} + e^2 l_t^{*2} + \ldots + e^J l_t^{*J}]/J, \end{aligned}$$

where ω^j is strictly positive and $[\omega^1 + \omega^2 + \ldots = \omega^J]/J = 1$. The proportionality relations in (A.1) do not imply corresponding proportional relations for acquisitions of consumption goods or efficiency units of leisure because the initial period lags in consumption and leisure do not satisfy these proportionality restrictions. The impact of the initial conditions, however, vanishes over time as long as $A(Z)$ and $B(Z)$ have stable zeroes. That is, proportionality will be obtained for appropriately defined stochastic steady states. Therefore, we strengthen (A.1) to be

$$\text{(A.2)} \qquad \begin{aligned} c_t^j &= \omega^j [c_t^1 + c_t^2 + \ldots + c_t^J]/J \\ e^j l_t^j &= \omega^j [e^1 l_t^1 + e^2 l_t^2 + \ldots + e^J l_t^J]/J, \end{aligned}$$

although we shall not address formally the approximation involved.

We define the efficiency units so that

$$\text{(A.3)} \qquad \left(\frac{\omega^1}{e^1} + \frac{\omega^2}{e^2} + \ldots + \frac{\omega^J}{e^J}\right) \Big/ J = 1.$$

Then

(A.4) $$(1/J)(l_t^1 + l_t^2 + \ldots + l_t^J) = (1/J)(e^1 l_t^1 + e^2 l_t^2 + \ldots + e^J l_t^J),$$

so that the average amount of leisure is equal to the average amount of efficiency units of leisure.

Since consumers are compensated in terms of efficiency units, person j receives $w_t^* e^j [h - l_t^j]$ units of the consumption good at time t, where w_t^* is the wage rate in terms of efficiency units and h is the total time endowment. Average compensation w_t^a is then equal to

(A.5) $$w_t^a = w_t^*(h^* - l_t^a),$$

where

(A.6) $$h^* = (e^1 + e^2 + \ldots + e^J)h/J.$$

Solving for w_t^* gives

(A.7) $$w_t^* = w_t^a/(h^* - l_t^a).$$

The efficiency wage w_t^* is equal to average compensation divided by the number of efficiency units worked. The parameter h^* depends on both h and the efficiency units correction. In the special case in which the e^j are one for all, j, $h^* = h$ as is assumed in our empirical analysis. Otherwise, it could be treated as a free parameter to be estimated. This describes one possible source of measurement error in our wage series that could in principle be accommodated by augmenting the parameter vector to include h^*.

APPENDIX B: ESTIMATING THE ASYMPTOTIC COVARIANCE MATRICES

In this appendix we describe the procedure used to estimate the distance matrix in our IV criterion function and the asymptotic covariance matrix of σ_T, the minimizer of (24).

Suppose that the $K \times 1$ vector of disturbances in the estimation equations is observed by agents at date $t + q$ and satisfies $E_t d_{t+q}(\sigma_0) = 0$, for some finite integer $q \geq 1$. Also, let

$$g_T(\sigma) = \frac{1}{T}\sum_{t=1}^{T} z_t \otimes d_{t+q}(\sigma),$$

where z_t is an $R \times 1$ vector of elements of I_t, and suppose that the estimator of σ_0 is chosen from the admissible parameter space to minimize $g_T(\sigma)' W_T g_T(\sigma)$, where W_T is a consistent estimator of the inverse of the matrix

(B.1) $$S_0 = \sum_{i=-q}^{q} E(z_t \otimes d_{t+q})(z_{t-i} \otimes d_{t+q-i})'.$$

Finally, let

(B.2) $$D_0 = E\left[z_t \otimes \frac{\partial d_{t+q}}{\partial \sigma_0}\right].$$

Then Hansen [1982] shows under certain regularity conditions that the limiting distribution of $\{\sqrt{T}\sigma_T\colon T \geq 1\}$ is normal with mean vector zero and covariance matrix $(D_0' S_0^{-1} D_0)^{-1}$. To implement this estimator and conduct inference about σ_0 requires consistent estimators of S_0 and D_0. Here we describe such estimators for the case of arbitrary q. The results can be applied to study (23), for example, by setting $q = 2$.

Hansen supplies sufficient conditions to guarantee that if $\{\sigma_T\colon T \geq 1\}$ converges in probability to σ_0, then $\{\partial g_T/\partial\sigma(\sigma_T)\colon T \geq 1\}$ converges in probability to D_0. Therefore, in our empirical analysis we use $D_T = \partial g_T/\partial\sigma\ (\sigma_T)$ as our estimator of D_0. Estimation of S_0 is somewhat more involved. The matrix S_0 is a covariance matrix and is therefore positive semidefinite. In this paper we impose the stronger requirement that it be positive definite. Hansen [1982] suggests estimating S_0 by replacing the population moments in (B.1) by their sample counterparts evaluated at σ_T. Although the resulting estimator converges almost surely to S_0, it is not constrained algorithmically to be positive definite in finite samples. There have been several empirical applications in which this estimator has turned out to be positive definite, but we encountered cases in which it was not positive definite.[9] For this reason we consider an alternative estimator of S_0 that is constrained to be positive definite in finite samples.[10]

Specifically, we estimate the coefficients of a Wold decomposition of the process $\{u_{t+q} = z_t \otimes d_{t+q}\colon -\infty < t < +\infty\}$, and then use these coefficient estimates in estimating the covariance matrix of

9. Brown and Maital [1981] and Hansen and Hodrick [1980, 1983] have used the estimator proposed by Hansen without encountering any problem.

10. A third alternative is to estimate S_0 using procedures developed for estimating spectral density matrices. While this method gives rise to a positive definite estimate of S_0, it ignores the implication of the theory that all but a finite number of the autocovariances of $\{z_t \otimes d_{t+q}\ -\infty < t < +\infty\}$ are zero. Under the alternative hypotheses considered in SIV, the zero restrictions in the autocovariance function may not hold. In conducting tests with respect to these alternatives, it is not clear for power considerations whether one should or should not impose these zero restrictions. Under the null hypotheses the asymptotic distribution of the test statistics are likely to approximate more accurately their finite sample distributions if the zero restrictions are imposed.

the one-step-ahead linear least squares forecast errors and S_0. The zero restrictions on the autocovariances imply that the Wold decomposition can be represented as

$$u_t = e_t + B_1 e_{t-1} + \ldots + B_q e_{t-q}, \tag{B.3}$$

where e_t is the one-step ahead forecast error in forecasting $z_{t-q} \otimes d_t$ from linear combinations of past values of $z_{t-q} \otimes d_t$ and $B_1, \ldots, B_q$ are $RK \times RK$ dimensional matrices. The matrix S_0 is related to the B_j's via the formula,

$$S_0 = (I + B_1 + \ldots + B_q)\Omega_0(I + B_1' + \ldots + B_q'), \tag{B.4}$$

where $\Omega_0 = Ee_te_t'$. Once we obtain consistent estimators of $B_1, \ldots, B_q$ and a consistent estimator of Ω_0 that is constrained to be positive semidefinite in finite samples, we can use formula (B.4) to obtain a consistent estimator of S_0 that will be positive semidefinite.[11]

To estimate the moving average coefficients $B_1, B_2, \ldots, B_q$, we use a procedure suggested by Durbin [1960] with some minor modifications. A virtue of Durbin's procedure is that it provides estimators of the moving average coefficients without resorting to numerical search procedures. Numerical search procedures become intractable in our application because of the large number of elements in the B_j matrices that have to be estimated simultaneously.

The first step of our modified Durbin procedure is to use the Yule-Walker equations to obtain estimates of $A_1, A_2, \ldots, A_{NLAG}$ in the finite order autoregression,

$$u_t = A_1 u_{t-1} + \ldots + A_{NLAG} u_{t-NLAG} + \tilde{e}_t. \tag{B.5}$$

These estimates are then used to construct estimates $\tilde{e}_t^T$ of the one-step-ahead forecast errors of the finite order vector autoregression. The sample forecast errors $\{\tilde{e}_t^T: t = NLAG + 1, \ldots, T\}$ are used subsequently as estimates of the forecast errors $\{e_t: t = NLAG + 1, \ldots, T\}$ in (B.4). Since the autoregressive representation of the process $\{u_t: -\infty < t < +\infty\}$ has infinite order when q is greater than zero, the choice of $NLAG$ should be an increasing function of sample size in order that sample forecast errors will converge to the true forecast errors. Recall that in our applications there is a priori information that all but a finite number of the autocovariances are zero. Therefore, the number of nonzero sample

11. As long as Ω_0 is nonsingular, this approach will, in general, give rise to a nonsingular estimate of S_0 in finite samples.

autocovariances used in estimation of (B.5) does not need to increase with sample size, even though $NLAG$ does.[12]

The second step is to estimate the regression equation,

$$z_{t-q} \otimes d_t(\sigma_T) = \tilde{B}_1 \tilde{e}_{t-1}^T + \ldots + \tilde{B}_q \tilde{e}_{t-q}^T + \nu_t,$$

where ν_t is the vector disturbance term. Let $\tilde{B}_1^T, \ldots, \tilde{B}_q^T$ denote the resulting estimators of $\tilde{B}_1, \ldots, \tilde{B}_q$, respectively, and let

$$\Omega_T = \frac{1}{T - NLAG - q} \sum_{t=NLAG+q+1}^{T} \nu_t^T \nu_t^{T\prime},$$

where

$$\nu_t^T = z_{t-q} \otimes d_t(\sigma_T) - \tilde{B}_1^T \tilde{e}_{t-1}^T - \ldots - \tilde{B}_q^T \tilde{e}_{t-q}^T.$$

As an estimator of S_0 in our empirical work we use[13]

$$S_T = (I + \tilde{B}_1^T + \ldots + \tilde{B}_q^T)\tilde{\Omega}_T(I + \tilde{B}_I^{T\prime} + \ldots + \tilde{B}_1^{T\prime}).$$

Appendix C: Testing Subsets of Orthogonality Conditions

In this appendix we consider the problem of testing whether a subset of the orthogonality conditions holds (see Appendix B for notation). More precisely, partition the vector $u_{t+q} = z_t \otimes d_{t+q}(\sigma_0)$ into two subvectors u_{t+q}^1 and u_{t+q}^2, where u_{t+q}^1 depends on the Q_1, parameters, σ_{10}, and u_{t+q}^2 depends on (possibly a subset of) these Q_1 parameters plus an additional Q_2 parameters σ_{20} that do not enter the expressions for u_{t+1}^q ($\sigma_0' = (\sigma_{10}', \sigma_{20}')$ and $Q_1 + Q_2 = Q$). u_{t+q}^1 is J_1 dimensional with J_1 greater than or equal to Q_1, and u_{t+q}^2 is a J_2 dimensional vector, $J_2 = RK - J_1$. Let the assumptions that $E[u_{t+q}^1] = 0$ and $E[\partial u_{t+1}^1(\sigma_0)/\partial\sigma] = D_0^1$ has rank Q_1 be maintained as true. Suppose that a researcher wishes to test the null hypothesis that $E[u_{t+q}^2] = 0$. The elements of the vector u_{t+q}^1 may be chosen, for example, to be the orthogonality conditions associated with a particular disturbance.

Throughout this discussion we shall assume that the matrices S_0 and D_0 can be consistently estimated by $\{S_T\colon T \geq 1\}$ and $\{D_T\colon$

12. Durbin's [1960] procedure is designed to handle mixed autoregressive moving average models that do not, in general, have only a finite number of nonzero autovariances.

13. Cumby, Huizinga, and Obstfeld [1982] propose a related method for estimating S_0. They use a Yule-Walker equation to obtain estimates of the autoregressive parameters, invert the autoregressive polynomial, and then use the resulting first q moving average coefficient matrices to estimate $B_1, \ldots, B_q$. Durbin [1960] suggests a third step in the procedure described here that increases the asymptotic efficiency of $\tilde{B}^I, \ldots, \tilde{B}_q^T$ when the underlying time series process is linear.

$T \geq 1\}$, and that S_0 is nonsingular. Partitioning W_0, S_0, S_0^{-1} and D_0 in accord with the two sets of orthogonality conditions, gives

$$D_0 = \begin{vmatrix} D_0^1 \\ D_0^2 \end{vmatrix} \qquad W_0 = \begin{vmatrix} W_0^{11} & W_0^{12} \\ W_0^{21} & W_0^{22} \end{vmatrix}$$

$$S_0 = \begin{vmatrix} S_0^{11} & S_0^{12} \\ S_0^{21} & S_0^{22} \end{vmatrix} \quad S_0^{-1} = \begin{vmatrix} \tilde{S}_0^{11} & \tilde{S}_0^{12} \\ \tilde{S}_0^{21} & \tilde{S}_0^{22} \end{vmatrix}.$$

Similarly, $g_T(\sigma)'$ is partitioned as $[g_{1T}(\sigma_1)'g_{2T}(\sigma_2)']$, where

$$g_{1T}(\sigma_1) = \frac{1}{T}\sum_{t=1}^{T} u_{t+q}^1(\sigma_1) \quad \text{and} \quad g_{2T}(\sigma_2) = \frac{1}{T}\sum_{t=1}^{T} u_{t+q}^2(\sigma_2).$$

The test which we consider exploits the fact that the sample orthogonality conditions $\{g_T(\sigma_T)\colon T > 1\}$ converge in distribution to a normally distributed random vector with mean zero and covariance matrix V_0, where $V_0 = S_0 - D_0(D_0'S_0D_0)^{-1}D_0'$ (see Hansen [1982]).

Gallant and Jorgenson [1979] have proposed a procedure for testing nonlinear restrictions on the parameter vector using instrumental variable estimators that is analogous to the likelihood ratio test. While they assumed that the disturbance terms were serially independent and conditionally homoskedastic, their procedure is easily modified to apply to the inference problem considered here for subsets of orthogonality conditions. To implement this test, first, one obtains an estimator $\{\sigma_T\colon T \geq 1\}$ of σ_0 by minimizing the objective function $g_T(\sigma)'S_T^{-1}g_T(\sigma)$ by choice of σ. This estimator exploits all of the orthogonality conditions appropriate under the null hypothesis. Next the estimator $\{\sigma_{1T}\colon T \geq 1\}$ of σ_{10} is formed using only the first J_1 orthogonality conditions that are presumed to hold under the alternative hypotheses, and the weighting matrix $(S_T^{11})^{-1}$. Using both estimators,

$$\text{(C.1)} \qquad C_T = Tg_T(\sigma_T)'S_T^{-1}g_T(\sigma_T) - Tg_{1T}(\sigma_{1T})'(S_T^{11})^{-1}g_{1T}(\sigma_{1T})$$

is then calculated. Under the null hypothesis the asymptotic distribution of $\{C_T\colon T \geq 1\}$ is chi square with $[RK - Q - (J_1 - Q_1)] = (J_2 - Q_2)$ degrees of freedom. To see this factor S_0^{-1} and $(S_0^{11})^{-1}$ as $P_0'P_0$ and $P_1'P_1$, respectively. In proving Theorem 3.1, Hansen [1982] shows that $\{\sqrt{T}P_0g_T(\sigma_0)\colon T \geq 1\}$ and $\{\sqrt{T}P_1g_{1T}(\sigma_{10})\colon T \geq 1\}$ have limiting distributions under the null hypothesis that are normals with zero means and covariance matrices I_{RK} and I_{J_1}, respectively. These results, together with Lemma 4.1 in Hansen, imply that

$\{\sqrt{T}P_{0T}g_T(\sigma_T): T \geq 1\}$ has the same limiting distribution as

$$\{\sqrt{T}(I_{RK} - P_0D_0(D_0'\, S_0^{-1}D_0)^{-1}D_0'P_0')P_0g_T(\sigma_0): T \geq 1\}$$
$$\equiv \{\sqrt{T}\, NP_0g_T(\sigma_0): T \geq 1\}$$

and $\{\sqrt{T}P_{1T}g_{1T}(\sigma_{1T}): T \geq 1\}$ has the same limiting distribution as

$$\{\sqrt{T}(I_{T_{J_1}} - P_1D_0^1(D_0^{1\prime}(S_0^{11})^{-1}D_0^1)^{-1}D_0^{1\prime}P_1')P_1g_{1T}(\sigma_0): T \geq 1\}$$
$$\equiv \{\sqrt{T}\, MP_1g_{1T}(\sigma_{10}): T \geq 1\}.$$

Thus, under the null hypothesis, C_T has the same asymptotic distribution as the statistic

$$\text{(C.2)} \quad Tg_T(\sigma_0)'P_0'\{N - (P_0')^{-1}\begin{vmatrix} I_{J_1} \\ 0 \end{vmatrix} P_1'MP_1[I_{J_1}0]P_0^{-1}\}P_0g_T(\sigma_0).$$

Now the matrix in brackets in (C.2) is idempotent with rank equal to $(J_2 - Q_2)$ and, therefore, C_T is distributed asymptotically as chi square with $(J_2 - Q_2)$ degrees of freedom.

To conclude the discussion, note that the test procedure is easily modified to handle restrictions on parameters of the form,

$$\text{(C.3)} \qquad f_2(\sigma_2) = 0,$$

where f_2 has J_2 coordinates and where J_2 is less than Q. We simply view (C.3) as being a set of orthogonality conditions that we wish to test just as above. However, now there is no randomness in the orthogonality conditions that we wish to test so the S_0 matrix has the partitioned form,

$$S_0 = \begin{vmatrix} S_0^{11} & 0 \\ 0 & 0 \end{vmatrix},$$

and is therefore singular. Subject to this modification, the analysis above carries over immediately to testing restrictions on the unknown parameters.

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