James B. Wiggins
Cornell University

Empirical Tests of the Bias and Efficiency of the Extreme-Value Variance Estimator for Common Stocks*

This article examines the empirical bias and efficiency of Parkinson's extreme-value variance estimator for common stocks using an extensive NYSE/AMEX data base. Bias and efficiency are analyzed as a function of stock price level and trading volume. The results are sensitive to outliers in daily high and low prices. After an outlier screen is applied to the data, the efficiency of the extreme-value estimator significantly exceeds that of the close-close estimator for most price and volume groups.

I. Introduction

Risk estimation is an important issue in empirical financial economics. The Black-Scholes (1973) option-pricing formula requires an estimate of the expected variance on the stock over the option's remaining life as an input. Stochastic volatility option-pricing models (e.g., Hull and White 1987; Wiggins 1987) require estimates of parameters of the distribution of the variance process in addition to an estimate of current volatility in valuation. To evaluate the statistical significance of abnormal stock returns in event studies, an estimate of the standard deviation of abnormal event-period returns is needed.

Stock-return variances have traditionally been estimated by the method of moments, using daily or monthly close-close returns. A different approach, developed by Parkinson (1980) and extended by Garman and Klass (1980), is to employ the high and low prices observed during the day. If it is assumed that trading is continuous and always monitored, these extreme-value estimators are at least five times more efficient than the close-close estimator. Intuitively, extreme-value

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estimators are superior to the close-close estimator because they incorporate the range or dispersion of prices observed over the entire day, not just a "snapshot" price at the end of the day.

Marsh and Rosenfeld (1986) have analyzed and compared the properties of extreme-value and close-close estimators in a discrete-trading model. Discrete trading, both in time and in price, can bias and reduce the efficiency of variance estimators; Garman and Klass (1980) recognized that discrete-time trading imparts a downward bias to the extreme-value estimator because "true" highs and lows often remain unobserved. Marsh and Rosenfeld's model assumes transactions occur only when the true stock price, driven by a continuous Brownian motion, crosses a multiple of an eighth of a dollar, the typical minimum price change for listed stocks. The authors show that "snapshot" estimators such as the close-close are not biased by nontrading itself in their model but are less efficient than in a continuous trading world. They also show that nontrading downward biases and dramatically reduces the efficiency of the extreme-value estimator, particularly for low-price and low-risk stocks.[1]

While the simulation results of Marsh and Rosenfeld (1986) are interesting and informative, they depend on specific assumptions on when and at what price trades occur as a function of an assumed stochastic process for the unobserved "true" stock price. Empirical analysis of the properties of alternative estimators has been largely neglected in the literature.

Beckers (1983) investigated the ability of the close-close and Parkinson (1980) estimators to predict future quarterly volatilities. His study used high-, low-, and closing-price data on 208 stocks with listed options over the January 1973–March 1980 period. Beckers found that Parkinson's estimator was comparable to the close-close estimator in forecasting future close-close variance in a simple linear regression context. After making an adjustment for cross-sectional variation in the relation between the two estimators, Parkinson's estimator outperformed the close-close measure. Despite the theoretical biases and efficiency losses associated with discrete trading, his results suggest that high and low prices contain variance information unavailable in closing prices and can be useful in a variance prediction model.

This article complements the work of Beckers (1983) by explicitly estimating the empirical bias in Parkinson's estimator relative to the close-close estimator as a function of stock price and trading volume. Marsh and Rosenfeld's (1986) model predicts that the downward bias in Parkinson's estimator will be a decreasing function of the stock

1. Gottlieb and Kalay (1985) and Ball (1988) have also analyzed the performance of the traditional estimator when observed prices are discrete. Cho and Frees (1988) develop an intraday variance estimation approach utilizing the time interval between trades.

price. The simulation results of Garman and Klass (1980) suggest that the downward bias in Parkinson's estimator will be inversely related to the number of trades during the day, and volume can serve as a proxy for trading continuity. The empirical work that follows represents the initial test of these hypotheses for common stocks. This article also examines the efficiency of the close-close estimator versus alternative estimators which use high- and low-price data. The efficiency criterion is mean squared error (MSE) in the current period, as opposed to MSE in a future period in the regression framework in Beckers (1983). This analysis is relevant for applications where an efficient variance *estimate* (e.g., event studies) rather than an efficient variance *prediction* (option pricing) is required.

Section II provides a description of the data set, and Section III presents the empirical tests. The results are sensitive to outliers in high and low prices in the data. In both the full sample and in an outlier-screened sample, Parkinson's estimator is generally downward biased relative to the close-close estimator, consistent with the simulation results of Garman and Klass (1980) and Marsh and Rosenfeld (1986). In the full sample, the relative efficiency of Parkinson's estimator is very poor but improves dramatically after application of the outlier screen. In the outlier-screened sample, Parkinson's estimator is significantly more efficient than the close-close estimator for most price and volume groups. The final section offers a brief summary and conclusions.

II. A Description of the Data

This study employs data from the Cornell University Price Volume (CUPV) tapes. The CUPV tapes contain daily open, high, low, and closing stock prices and trading volumes for virtually all New York Stock Exchange (NYSE) and American Stock Exchange (AMEX) listed stocks over the January 1970–January 1986 period, collected by the Francis Emory Fitch Company. The first CUPV tape covers the January 1970–December 1979 period, and the second the January 1980–January 1986 period.[2] Planned updates will add data through the most recent calendar year, in the same manner as updates to the Center for Research in Security Prices (CRSP) tapes.

For each daily price record, the CUPV tapes contain a quality code indicating the compatibility of its closing-price relatives, defined here as "CUPV returns," with daily returns from the CRSP tapes. There are a number of possible causes of mismatches between CRSP and

2. All stocks that were exchange delisted prior to 1979 do not appear on the first CUPV tape. This survivorship bias is not likely to bias the results of this article in one direction or another, and there are plans to include the delisted stocks in future versions of the tapes.

CUPV returns. If a stock goes ex-dividend, cash, or stock, CUPV returns do not represent actual returns. One of the two tapes may contain an error in recording closing prices. Finally, starting in 1976, CRSP used the daily closing price from a composite tape for its returns calculation. The CUPV tapes use NYSE or AMEX closing prices. If a mismatch on date t occurs because the NYSE or AMEX close differs from a later close on another exchange, cumulative CRSP and CUPV returns from date t onward will eventually converge if no distributions are paid in the meantime. For each daily return mismatch, the CUPV quality code indicates whether cumulative CRSP and CUPV returns eventually converge, but it does not indicate the source (e.g., cash dividend) of the incompatibility.

Variances are estimated over quarterly intervals using the daily price records, with a quarter defined as 63 trading days.[3] For a date t price record to be included in a variance estimate, strictly positive trading volume on dates t and $t - 1$, a complete open-, high-, low-, and closing-price record on date t, and a strictly positive closing price on date $t - 1$ are required. Due to the unavailability of the AMEX monthly master tape, it was impossible to adjust all the CUPV data around ex-dates for dividends and splits. It was decided to eliminate all CUPV records with daily return mismatches where cumulative returns did not converge, which eliminates all ex-dates from the sample. Date t records with obvious recording errors, such as an open, low, or closing price exceeding the high price, were eliminated. To be included in the sample for quarter T, a stock must meet the above criteria on at least 60 trading days in the quarter. Variance estimates are expressed on an average daily basis in the tables that follow.[4] Define in logarithms

C = closing price on date t;
C_{-1} = closing price on date $t - 1$;
H = high price on date t, inclusive of C_{-1}; and
L = low price on date t, inclusive of C_{-1}.

The close-close and Parkinson's high-low estimators are

$$\left.\begin{aligned} \sigma^2_{CC} &= (C - C_{-1})^2 \\ \text{and} \qquad & \\ \sigma^2_{HL} &= .3607(H - L)^2. \end{aligned}\right\} \qquad (1)$$

3. A quarterly interval was selected to be consistent with Beckers (1983) and to control the size of the data set.

4. The average daily variances are based on the actual number of daily records included in each quarterly observation, between 60 and 63.

The version of Parkinson's estimator considered here does not explicitly incorporate an estimate of overnight variance, though it does include the previous closing price as a possible high or low during the close-close period. This estimator will be downward-biased relative to σ^2_{CC}, even if trading is continuous during exchange hours, because it does not account for true high or low prices occurring during nighttime hours.[5] This procedure does have two advantages over one that uses the opening price to compute and then combine separate overnight and daytime variances. First, since the opening price is not required, the results for σ^2_{HL} that follow are applicable to future research using the CRSP daily master tapes, which do not currently provide opening prices. Second, close-open and open-close returns for common stocks tend to be serially correlated from the bid-ask effect (see Roll 1984), which creates a bias in the estimated variance if ignored or an efficiency loss if an autocorrelation coefficient needs to be estimated.[6]

Neither of the above estimators include a cross-product term to account for serial correlation in daily close-close returns. Daily returns are autocorrelated over the sample period considered (see French and Roll 1986). This article does not address the serial correlation issue because it is concerned with the *relative* performance of the daily close-close and high-low estimators. Any correction for serial dependence would be the same for the two estimators since each employs data collected over the same close-to-close interval.

In order to assess the empirical bias and efficiency of σ^2_{HL} relative to σ^2_{CC} for stocks classified on their stock price and volume of trade, stocks are sorted into price-volume groups based on their average previous day's closing price P and daily volume V in round lots in each quarter. The three price categories are $P \leq 15$, $15 < P \leq 30$, and $P > 30$, and the four volume categories are $V \leq 40$, $40 < V \leq 125$, $125 < V \leq 500$, and $V > 500$. The resulting 12 price-volume groups have sample sizes ranging from 1,745 for the $P < 15$, $V > 500$ group to 12,107 for the $P < 15$, $40 < V \leq 125$ group. The $V > 500$ groups are generally smaller than the others in the same price range, but separate analysis of these stocks is of interest given the increase in average volume in recent years and the concentration of option trading in high-volume stocks.

5. The degree of bias from this source is an open empirical issue. On a per-unit time basis, French and Roll (1986) found that variances appear to be much higher when the market is open than when it is closed, but nontrading periods are more than three times longer than trading periods.

6. This procedure also avoids bias and efficiency problems caused by recording errors in opening prices whenever the open is not the daily high or low. Extensive experiments were performed using a version of σ^2_{HL} including a separate estimate of the nighttime variance, and the performance of this estimator was relatively poor.

III. Empirical Tests

A. *Full Sample Results*

Table 1 presents summary statistics for σ^2_{CC} and σ^2_{HL} for each price-volume group over the full January 1970–December 1985 sample period. All data are expressed on a daily basis in percent. The notation $\overline{\sigma}^2$ is the grand mean of quarterly variances, SD(σ^2) is the standard deviation of quarterly variances, and CV(σ^2) is the average quarterly coefficient of variation, the ratio of the intraquarter standard deviation of the daily variance to its intraquarter mean. The notation N is the number of quarterly variance observations in each price-volume group.

Table 1 reveals an inverse relationship between $\overline{\sigma}_{CC}$ and average price, consistent with the results of Stoll and Whaley (1983) and Ohlson and Penman (1985).[7] The relatively high close-close variance estimates for low-priced stocks partially reflect their higher bid-ask spreads, inducing a negative autocorrelation effect not included in σ^2_{CC}. There appears to be a weak direct relationship between average trading volume and $\overline{\sigma}_{CC}$; this is consistent with an extensive literature reporting a positive association between abnormal volume and variance for individual stocks (see Karpoff [1987] for a survey). The coefficient of variation statistics for σ^2_{CC} are stable across price-volume groups, falling between 1.74 and 1.89. If trading were continuous and variances constant over quarterly intervals, the expected value of CV(σ^2_{CC}) would be approximately 0.18,[8] suggesting considerable intraquarter variation in true variances is present in the data.

In 10 of the 12 cells, $\overline{\sigma}^2_{CC}$ exceeds $\overline{\sigma}^2_{HL}$, as predicted by the extant literature. The standard deviation of σ^2_{HL} is generally much higher than that of σ^2_{CC}, by an order of magnitude in two of 12 cases. In contrast, CV(σ^2_{HL}) is below CV(σ^2_{CC}) throughout, indicating that σ^2_{HL} exhibits less sampling variability in proportion to its mean than σ^2_{CC}.

Formal statistical tests of estimator bias appear in table 2. Recall that the focus of this study is on the relative performance of σ^2_{CC} and σ^2_{HL}; when the bias of σ^2_{HL} is discussed, it is relative to σ^2_{CC}, not relative to the variance of an underlying continuous price process. Variance ratios $\sigma^2_{HL}/\sigma^2_{CC}$ are calculated for each quarterly observation and then averaged across securities in each price-volume group in each quarter.[9] Time-series means of these (independent) average variance ratios, weighted by the number of observations in each quarter, are illustrated

7. Whether variances actually increase after stock splits as reported in Ohlson and Penman (1985), or whether a bid-ask spread bias drives their results, is still an open empirical issue.

8. Ignoring small higher-order terms in a Taylor expansion, the expected value of CV(σ^2_{CC}) under these assumptions is the square root of $2/n$, where n is the number of daily observations in the quarter.

9. This procedure implicitly assumes that the bias in σ^2_{HL} is proportional to the level of σ^2_{CC}.

TABLE 1 Summary Statistics for Quarterly Close-Close (*CC*) and High-Low (*HL*) Variances—Full Sample

	$V \leq 40$	$40 < V \leq 125$	$125 < V \leq 500$	$500 < V$
$P \leq 15$:				
$\bar{\sigma}^2_{CC}$	11.81	12.71	12.81	13.35
$SD(\sigma^2_{CC})$	11.32	11.51	12.76	16.16
$CV(\sigma^2_{CC})$	1.80	1.85	1.86	1.89
$\bar{\sigma}^2_{HL}$	8.71	10.41	11.63	14.32
$SD(\sigma^2_{HL})$	20.91	29.27	35.04	58.45
$CV(\sigma^2_{HL})$	1.37	1.35	1.29	1.30
N	9,603	12,107	7,183	1,745
$15 < P \leq 30$:				
$\bar{\sigma}^2_{CC}$	3.93	4.97	5.44	5.25
$SD(\sigma^2_{CC})$	3.49	4.20	4.91	6.66
$CV(\sigma^2_{CC})$	1.83	1.83	1.82	1.82
$\bar{\sigma}^2_{HL}$	2.48	3.87	4.82	4.63
$SD(\sigma^2_{HL})$	2.35	16.27	19.12	16.14
$CV(\sigma^2_{HL})$	1.37	1.33	1.27	1.25
N	6,723	11,916	10,942	3,695
$30 < P$:				
$\bar{\sigma}^2_{CC}$	2.69	3.26	3.72	3.78
$SD(\sigma^2_{CC})$	2.79	2.67	3.51	3.71
$CV(\sigma^2_{CC})$	1.89	1.83	1.77	1.74
$\bar{\sigma}^2_{HL}$	4.06	2.80	3.04	3.75
$SD(\sigma^2_{HL})$	79.16	28.90	16.44	22.72
$CV(\sigma^2_{HL})$	1.49	1.37	1.28	1.25
N	2,112	6,570	9,274	4,711

NOTE.—Stocks are placed into quarterly price-volume groups based on their average daily trading volume V in round lots and their average lagged closing stock price P during the quarter. For each price-volume group $\bar{\sigma}^2$ is the average of quarterly variances, and $SD(\sigma^2)$ is the standard deviation of quarterly variances, each expressed on a daily basis in percent; $CV(\sigma^2)$ is the average coefficient of variation, the ratio of the intraquarter standard deviation of the daily variance to its intraquarter mean; N is the number of quarterly variance observations in each price-volume group.

for each price-volume group, with standard errors of the means in parentheses.

Table 2 confirms the general tendency of downward bias in σ^2_{HL} observed in table 1, with variance ratios in five of 12 cells more than two standard errors below unity. For lower-priced stocks, the downward bias is decreasing in average trading volume, consistent with the predictions in Garman and Klass (1980). For the $30 < P$, $V \leq 40$ group, the mean ratio and its standard error are very large, suggesting the presence of data errors.

To evaluate the relative efficiency of the two estimators, a mean squared error criterion is employed. Assume that the close-close estimator σ^2_{CCt} for day t is unbiased and that the expected daily drift in the true variance is zero.[10] By definition,

10. For the desired relative efficiency results that follow to be exact, $E[(\epsilon_{HLt})(\sigma^2_{t+1} - \sigma^2_t)] = 0$, where E is the expectation operator, is required. Given that the average

TABLE 2 **Weighted Average Ratios of High-Low to Close-Close Quarterly Variances—Full Sample**

	$V \leq 40$	$40 < V \leq 125$	$125 < V \leq 500$	$500 < V$
$P \leq 15$	.777	.843	.972	1.072
	(.031)	(.025)	(.046)	(.077)
$15 < P \leq 30$	.670	.817	.944	.938
	(.005)	(.026)	(.053)	(.049)
$30 \leq P$	2.099	1.046	.832	1.091
	(.585)	(.178)	(.030)	(.126)

NOTE.—Variance ratios are formed for each stock in each quarter and then averaged over stocks in each price-volume group in each quarter. These quarterly average variance ratios are then weighted by the number of observations and averaged over the 64 quarters in the sample. Standard errors of the means are in parentheses.

$$\left.\begin{aligned} \sigma^2_{CCt} &= \sigma^2_t + \epsilon_{CCt}, \\ \sigma^2_{CCt+1} &= \sigma^2_{t+1} + \epsilon_{CCt+1}, \\ \text{and} \qquad \sigma^2_{HLt} &= \sigma^2_t + \epsilon_{HLt}. \end{aligned}\right\} \tag{2}$$

Since sampling errors around the true daily variance are serially independent,

$$\frac{E(\sigma^2_{CCt+1} - \sigma^2_{HLt})^2}{E(\sigma^2_{CCt+1} - \sigma^2_{CCt})^2} = \frac{E(\sigma^2_{t+1} - \sigma^2_t)^2 + E(\epsilon_{CCt+1})^2 + E(\epsilon_{HLt})^2}{E(\sigma^2_{t+1} - \sigma^2_t)^2 + E(\epsilon_{CCt+1})^2 + E(\epsilon_{CCt})^2}. \tag{3}$$

If $(\sigma^2_{CCt+1} - \sigma^2_{HLt})^2$ is on average less than $(\sigma^2_{CCt+1} - \sigma^2_{CCt})^2$, then σ^2_{HL} is more efficient than σ^2_{CC}.[11] This same analysis applies to other estimators competing with σ^2_{CCt}.

Simply using σ^2_{HLt} as an estimate of day t variance may not be optimal because the results of table 2 suggest that σ^2_{HL} is downward-biased. Any adjustment for bias must be based on information available at time t for efficiency comparisons with σ^2_{CCt} to be valid. The adjustment factor used in the tests below for dates t in quarter T, denoted m_T, is the average of the variance ratio $\sigma^2_{HL}/\sigma^2_{CC}$ over all records in the relevant price-volume group in the sample up through quarter $T - 1$. This adjustment is premised on the assumption that $\sigma^2_{HL}/\sigma^2_{CC}$ is relatively stable across securities and over time within each price-volume group. The bias-adjusted extreme-value estimator for date t in quarter T is thus σ^2_{HLt}/m_T.

daily drift in stock variances over the sample period is very small, this term will be several orders of magnitude smaller than $E(\epsilon_{HLt})^2$ and $E(\epsilon_{CCt})^2$ and can be ignored for all practical purposes.

11. If true variances are not constant within quarters, as suggested by the coefficient of variation data in table 1, this statement is valid in terms of the average efficiency of σ^2_{HL} vs. σ^2_{CC}.

The final estimator considered is a weighted average of the bias-corrected high-low variance and the close-close variance. Unless sampling errors are perfectly correlated, an efficiency gain from an optimal combination of the two estimators is possible in theory, and Beckers (1983) demonstrated that combining high-low with close-close data can enhance variance forecasting power.[12]

One natural weighting rule uses the historical squared coefficients of variation of the two estimators. Define w_T as the average over all records in the group in each quarter of the sample through $T - 1$ of $CV(\sigma^2_{HL})^2/CV(\sigma^2_{CC})^2$. The weighted estimator is

$$[1/(1 + w_T)] \cdot \sigma^2_{HLt}/m_T + [w_T/(1 + w_T)] \cdot \sigma^2_{CCt}. \qquad (4)$$

Squared errors around σ^2_{CCt+1} are calculated for the close-close and the raw, bias-adjusted, and weighted high-low estimators for each day t in each quarter for each stock, and ratios of the resulting intraquarter mean squared errors are formed as in (3). These MSE ratios are then averaged across securities within each price-volume group in each quarter. Time-series means for each group, weighted by the number of quarterly firm observations, are presented in table 3, with standard errors in parentheses. Recall that each estimator is constructed using information available as of the beginning of quarter T, so the efficiency tests in table 3 are tests of feasible variance estimation rules.

The table 3 results suggest Parkinson's estimator is much less efficient than the close-close estimator using the full sample of CUPV price data. The average ratio of MSEs is never below unity in the table and, in most cases, is extremely large. The price-volume groups with the highest ratios of $SD(\sigma^2_{HL})$ to $\overline{\sigma}^2_{HL}$ in table 1 generally have the highest MSE ratios in table 3.

B. *Outlier-screened Results*

Given the dismal performance of Parkinson's estimator in table 3, as well as the instability in MSE ratios across price-volume groups, the CUPV data was screened for high- and low-price observations thought to represent recording errors. Recording errors can have a dramatic effect on high-low variance estimates. Random recording errors in the true high price add upward bias to σ^2_{HL} even if the errors are bounded from above by the difference between the true high price and the next highest price because the variance calculation is composed of squares of price relatives. This particular problem is common to close-close variance estimation. More significant biases are likely in situations where the true transaction price is within and the miscoded price outside the extreme values that would prevail without the error, or vice

12. Garman and Klass (1980) derive the optimal weighting function for σ^2_{HL} and σ^2_{CC} in a frictionless market setting.

TABLE 3 Weighted Average Ratios of Mean Squared Errors of σ^2_{HLt} versus σ^2_{CCt} around σ^2_{CCt+1}—Full Sample

σ^2_{HL}	$V \leq 40$	$40 < V \leq 125$	$125 < V \leq 500$	$500 < V$
$P \leq 15$:				
Raw	88.64	45.61	160.09	78.85
	(62.10)	(17.88)	(122.32)	(33.73)
Bias-adjusted	159.75	58.80	167.58	75.03
	(119.19)	(25.03)	(130.67)	(28.35)
Weighted	60.62	23.39	70.36	30.37
	(44.99)	(9.73)	(55.23)	(11.55)
$15 < P \leq 30$:				
Raw	1.81	87.53	310.18	144.95
	(1.38)	(39.87)	(206.69)	(86.46)
Bias-adjusted	2.34	103.80	389.15	179.35
	(2.09)	(44.87)	(260.51)	(111.44)
Weighted	1.43	44.11	164.97	80.92
	(.84)	(18.85)	(109.24)	(50.73)
$30 < P$:				
Raw	5,436.33	1,215.64	157.51	1,086.68
	(2,327.39)	(706.36)	(87.31)	(707.26)
Bias-adjusted	4,678.75	506.73	167.86	987.09
	(2,420.39)	(356.32)	(88.35)	(657.27)
Weighted	1,806.45	202.10	69.95	1,086.68
	(934.68)	(139.75)	(36.51)	(222.51)

NOTE.—Squared errors around date $t + 1$ close-close variance are calculated for date t close-close, raw high-low, bias-adjusted high-low, and weighted variance estimates for each security and then averaged within each quarter T. Ratios of these mean squared errors are formed for each security and averaged across securities in price-volume groups. These mean ratios are then averaged, using the number of observations as weights, across the 62 quarters in the sample. Standard errors of the means are in parentheses. Calculation of the bias-adjusted and the weighted variance estimates are described in the text.

versa. Since an error of this type is possible on every trade, errors are much more likely in high or low prices than in closing prices.

The outlier screen is intended to keep the proportion of truly high daily variance records that are removed reasonably small; trimming or eliminating extreme high, low, or closing observations in the process of developing an optimal variance prediction rule is a topic outside the scope of this article. As a first screen, records with high or low prices that differ from the previous closing price by more than a factor of five are dropped. This screen eliminates records where a (lagging) zero is mistakenly added to or deleted from a price record. Out of a total of 5,347,650 daily price records, 242 records are caught by this screen.

A second screen attempts to uncover slightly more subtle errors. Since σ^2_{CC} is inversely related to the average stock price in the sample, this screen is conditioned on the stock price grouping. A daily price record is classified as containing a recording error for $P \leq 15$ if either (1) the high price is more than 40% (continuously compounded) above the previous closing price, and the closing price is within 10% of the previous closing price, or (2) the low price is more than 40% below the previous close, and the close is within 10% of the previous close. For price groupings $15 < P \leq 30$ and $30 < P$, the cutoffs for the high or low versus the previous close are 30% and 25%, respectively, with associated cutoffs for the close versus the previous close of 7.5% and 6.25%, respectively. Roughly speaking, the second screen eliminates records with intraday returns more than 10–15 times greater than the average daily standard deviation for the price group, which then reverse themselves by between 75% and 125% by the end of the day. While a few truly high variance daily records may be eliminated by this screen, logic suggests the vast majority include recording errors. An additional 129 records are dropped as outliers by the second screen.[13] Tables 4–6 replicate the analyses of tables 1–3 for the outlier-screened sample.[14]

Comparison of tables 4 and 1 reveals that implementation of the outlier screen has very little effect on the properties of the close-close estimator. Both the mean and the standard deviation of σ^2_{CC} in each cell are virtually unchanged after removal of the outliers. Estimates of $\overline{\sigma}^2_{HL}$ and $\text{SD}(\sigma^2_{HL})$ are noticeably smaller after outlier removal, particularly in cells where they were relatively high in table 1.

The data in table 5 indicate that Parkinson's estimator is downward-biased relative to σ^2_{CC} throughout and is strongly statistically significant in all 12 cells. Since σ^2_{HL} does not incorporate highs and lows occurring during nontrading hours, some degree of downward bias, independent of price and volume, is expected. The bias in σ^2_{HL} is a decreasing function of volume for each price group, as predicted by the Garman and Klass (1980) analysis. The bias is also indirectly inversely related to average variance in the sample, as predicted by the Marsh and Rosenfeld (1986) model, because volume and variance are positively correlated conditioning on price. But unlike Marsh and Rosenfeld's (1986) results, the percentage bias does not appear to be decreasing in price, controlling for volume. Under their assumption that trades occur only when the true stock price passes an eighth of a dollar, Parkinson's estimator is severely downward-biased for low-priced stocks. Evidently trades occurring at asks (bids) for stocks with true prices above (below) the latest recorded high (low) price mitigates the one-eighth bias, and this bid-ask effect has its largest proportional impact on variance estimates for low-priced stocks.

Efficiency tests for the three variance estimators incorporating extreme price data appear in table 6, and the improvements from table 3 are dramatic. The high-low estimator without a bias adjustment is

13. As expected, the outlier error rate generally increases with average trading volume, though it does not decline with average price. The error rate was less than one in 7,200 for each price-volume group.

14. No quarterly variance observations are dropped after applying the screen, but a few are reclassified based on slightly different average volumes over the quarter.

TABLE 4 **Summary Statistics for Quarterly Close-Close (CC) and High-Low (HL) Variances—Outlier Screened Sample**

	$V \leq 40$	$40 < V \leq 125$	$125 < V \leq 500$	$500 < V$
$P \leq 15$:				
$\bar{\sigma}^2_{CC}$	11.81	12.71	12.81	13.34
$SD(\sigma^2_{CC})$	11.33	11.51	12.76	16.16
$CV(\sigma^2_{CC})$	1.80	1.85	1.86	1.89
$\bar{\sigma}^2_{HL}$	7.81	9.30	10.19	10.86
$SD(\sigma^2_{HL})$	7.33	8.15	9.53	13.28
$CV(\sigma^2_{HL})$	1.35	1.33	1.26	1.26
N	9,605	12,104	7,184	1,745
$15 < P \leq 30$:				
$\bar{\sigma}^2_{CC}$	3.93	4.97	5.44	5.26
$SD(\sigma^2_{CC})$	3.49	4.20	4.91	6.66
$CV(\sigma^2_{CC})$	1.83	1.84	1.82	1.82
$\bar{\sigma}^2_{HL}$	2.45	3.47	4.19	4.12
$SD(\sigma^2_{HL})$	2.04	2.94	4.10	3.86
$CV(\sigma^2_{HL})$	1.36	1.31	1.25	1.22
N	6,723	11,916	10,942	3,695
$30 < P$:				
$\bar{\sigma}^2_{CC}$	2.69	3.26	3.72	3.78
$SD(\sigma^2_{CC})$	2.79	2.67	3.51	3.71
$CV(\sigma^2_{CC})$	1.89	1.83	1.77	1.74
$\bar{\sigma}^2_{HL}$	1.60	2.08	2.66	2.83
$SD(\sigma^2_{HL})$	3.42	1.68	2.41	2.62
$CV(\sigma^2_{HL})$	1.48	1.36	1.26	1.20
N	2,112	6,571	9,273	4,711

NOTE.—Stocks are placed into quarterly price-volume groups based on their average daily trading volume V in round lots and their average lagged closing stock price P during the quarter. For each price-volume group, $\bar{\sigma}^2$ is the average of quarterly variances, and $SD(\sigma^2)$ is the standard deviation of quarterly variances, each expressed on a daily basis in percent; $CV(\sigma^2)$ is the average coefficient of variation, the ratio of the intraquarter standard deviation of the daily variance to its intraquarter mean; N is the number of quarterly variance observations in each price-volume group.

TABLE 5 **Weighted Average Ratios of High-Low to Close-Close Quarterly Variances—Outlier Screened Sample**

	$V \leq 40$	$40 < V \leq 125$	$125 < V \leq 500$	$500 < V$
$P \leq 15$	.683	.761	.837	.854
	(.003)	(.004)	(.004)	(.009)
$15 < P \leq 30$	.665	.734	.804	.836
	(.003)	(.004)	(.005)	(.004)
$30 < P$	.604	.661	.737	.773
	(.019)	(.004)	(.006)	(.003)

NOTE.—Variance ratios are formed for each stock in each quarter and then averaged over stocks in each price-volume group in each quarter. These quarterly average variance ratios are then weighted by the number of observations and averaged over the 64 quarters in the sample. Standard errors of the means are in parentheses.

TABLE 6 **Weighted Average Ratios of Mean Squared Errors of σ^2_{HLt} versus σ^2_{CCt} around σ^2_{CCt+1}—Outlier Screened Sample**

σ^2_{HL}	$V \leq 40$	$40 < V \leq 125$	$125 < V \leq 500$	$500 < V$
$P \leq 15$:				
Raw	.681	.702	.697	.953
	(.005)	(.012)	(.008)	(.371)
Bias-adjusted	.803	.822	.775	1.132
	(.010)	(.025)	(.011)	(.536)
Weighted	.810	.814	.787	.957
	(.004)	(.010)	(.005)	(.261)
$15 < P \leq 30$:				
Raw	.662	.706	.782	.704
	(.003)	(.026)	(.050)	(.008)
Bias-adjusted	.781	.852	.924	.782
	(.006)	(.060)	(.088)	(.011)
Weighted	.807	.828	.857	.791
	(.003)	(.024)	(.039)	(.006)
$30 < P$:				
Raw	2.331	.671	.800	.697
	(2.789)	(.004)	(.117)	(.026)
Bias-adjusted	5.524	.785	.955	.794
	(7.895)	(.008)	(.180)	(.037)
Weighted	2.630	.813	.879	.806
	(3.007)	(.004)	(.081)	(.019)

NOTE.—Squared errors around date $t + 1$ close-close variance are calculated for date t close-close, raw high-low, bias-adjusted high-low, and weighted variance estimates for each security and then averaged within each quarter T. Ratios of these mean squared errors are formed for each security and averaged across securities in price-volume groups. These mean ratios are then averaged, using the number of observations as weights, across the 62 quarters in the sample. Standard errors of the means are in parentheses. Calculation of the bias-adjusted and the weighted variance estimates are described in the text.

most efficient of the three throughout. The efficiency ratio of the raw σ^2_{HL} is more than four standard errors below unity in nine of the 12 cells. The high weighted average ratios in the $P \leq 15$, $500 < V$ and $30 < P$, $V \leq 40$ cells in table 6 are each driven by an extreme average MSE ratio from a single quarter, suggesting the outlier screen does not catch all the recording errors in high and low prices.[15] Given the costs of processing the more than five million daily price records in the sample, development of more refined outlier screens is deferred to future research. An interesting related issue is how unusual high or low prices that are found to represent actual transactions should be treated in a variance prediction rule.[16]

The relatively poor performance of the bias-adjusted estimator is somewhat surprising, given the significant downward bias in σ^2_{HL} for all price-volume groups. The mean adjustment factors described in

15. In the $P \leq 15$, $500 < V$ cell, there is a quarter with an average MSE ratio of 290.72, and in the $30 < P$, $V \leq 40$ cell, a quarter with an average MSE ratio of 35.36.

16. I would like to thank the referee for suggesting this extension to my article.

Section IIIA exhibit relatively little variation over time, as implied by the small standard errors of the mean variance ratios in table 5. But since the distribution of σ^2_{HL} is positively skewed relative to the distribution of σ^2_{CC}, the mean adjustment evidently magnifies the large overestimates of σ^2_{CCt+1} in the upper tail of the σ^2_{HLt} distribution enough to increase the mean squared error. A more detailed analysis of optimal bias adjustment, including the use of firm-specific historical variance data, is left for future work. Relative to the bias-adjusted estimator, there do not appear to be any large efficiency gains or losses on average from weighting σ^2_{CC} and σ^2_{HL} based on their respective historical squared coefficients of variation.

While the data in table 6 provide strong statistical evidence suggesting that the extreme-value estimator is more efficient than the close-close estimator, the magnitude of the improvement in MSE versus σ^2_{CC} is unknown. A lower bound on the ratio $E(\epsilon_{CCt})^2/E(\epsilon_{HLt})^2$ can be determined by assuming that the true variance σ^2 is constant within quarter T. Denote the left-hand side of equation (3) as X_t. Since $E(\sigma^2_{t+1} - \sigma^2_t)^2 = 0$ and $E(\epsilon_{CCt+1})^2 = E(\epsilon_{CCt})^2$ under the constant variance assumption, rearranging (3) yields

$$E(\epsilon_{CCt})^2/E(\epsilon_{HLt})^2 = 1/(2X_t - 1). \tag{5}$$

For $E(\epsilon_{CCt})^2 > E(\epsilon_{HLt})^2$, (5) is a lower bound for the true ratio of mean squared errors.

Returning to the results in table 6, (5) indicates that the efficiency gains from using the extreme-value estimator are substantial. For example, given a MSE ratio of .70, (5) implies that σ^2_{HL} is at least 2.5 times more efficient than σ^2_{CC}. To the extent that the true variance fluctuates from day to day, this figure understates the relative efficiency of the estimators.

IV. Summary

This article has examined the empirical bias and efficiency of Parkinson's (1980) extreme-value variance estimator as a function of average stock price and trading volume. The data sample is developed from the CUPV tapes, which contain daily high, low, and closing prices and volumes for the universe of NYSE and AMEX stocks over a 16-year period.

In terms of the CUPV data base itself, there appear to be a number of cases where recorded high or low prices are significantly "out of line" relative to adjacent closing prices. Without direct observation of actual transactions, it is impossible to know whether these high- and low-price data represent actual trades or recording errors. The performance of Parkinson's estimator was very poor relative to the close-close estimator without applying an outlier screen. Future empirical researchers employing daily highs and lows, either from the CUPV or CRSP daily master tapes, should be aware of this problem.

In a continuous-trading model, the extreme-value estimators of Parkinson (1980) and Garman and Klass (1980) are much more efficient than the close-close estimator. In a discrete-time, discrete-price world, the properties of alternative estimators are much different, and Marsh and Rosenfeld's (1986) model predicts that Parkinson's estimator will be downward-biased and less efficient than the close-close estimator for most stocks. After applying a screen for errors in high and low prices, the statistical tests confirm a downward bias in the data. The proportional bias is inversely related to trading volume, as predicted by Garman and Klass (1980), though not inversely related to price, as in Marsh and Rosenfeld (1986). In terms of efficiency, Parkinson's estimator significantly outperforms the close-close estimator for nine of 12 price-volume groups. The empirical mean squared error of the close-close estimator is shown to be at least 2.5 times that of Parkinson's estimator for these stocks. This last result implies that the extreme-value estimator will be a more precise variance measure than the close-close estimator in event study applications, even though it is downward-biased.

There are several avenues for future research suggested by the results of this article. One is a more detailed analysis of outliers in high and low prices in the CUPV data base. It is likely that recording errors that slipped through the outlier screens here can explain why Parkinson's estimator did not outperform the close-close estimator for all stock groups. Recording errors could also be masking relationships between relative estimator efficiency and stock price and trading volume, as suggested by Marsh and Rosenfeld (1986) and Garman and Klass (1980), respectively. Another important issue is how outliers in extreme prices, whether representing actual transactions or not, should be incorporated into a variance prediction model. Finally, ex ante corrections for the downward bias in Parkinson's estimator were unsuccessful in enhancing its MSE efficiency. Additional research into optimal bias adjustment, including the use of both firm-specific and more general historical volatility information, could provide an explanation for this surprising result.

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