

# A Direct Test of Roll's Conjecture on the Firm Size Effect

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## ABSTRACT

Empirical research indicates that small firms earn higher average rates of return than large firms, even after accounting for beta risk. Roll conjectured that the small firm effect might be attributed to improper estimation of security betas. The evidence shows that while the direction of the bias in beta estimation is consistent with Roll's conjecture, the magnitude of the bias appears to be too small to explain the firm size effect.

EMPIRICAL RESEARCH NOW INDICATES that small firms earn higher average rates of return than large firms, even after accounting for differences in estimated betas (cf. Banz [1] and Reinganum [7]). In a recent study, Roll [8] suggests that the small firm effect might be attributed to improper estimation of security betas. In particular, Roll conjectures that standard beta estimates obtained using daily data seriously understate the actual risk of a small firm portfolio. Unfortunately, Roll did not possess data on market capitalizations of individual firms and thus could not directly test his proposition.

In this paper, Roll's possible explanation of the small firm effect is investigated directly by examining the daily returns of ten portfolios grouped on the basis of firm size. Portfolio betas are estimated using ordinary least squares and the aggregated coefficients method proposed by Dimson [3]. The test results reveal that the average returns of the small firms exceed those of the large firms by more than thirty percent on an annual basis. If one employs Dimson's estimator, the difference between the estimated betas of the small firm and large firm portfolios is about 0.7. The evidence indicates, however, that this difference in estimated betas cannot account for the more than thirty percent difference in average portfolio returns. Thus, while the direction of the bias in beta estimation is consistent with Roll's conjecture, the magnitude of the bias appears to be too small to explain the firm size effect.

## I. The Empirical Tests

The data to test Roll's conjecture are the ten market value portfolios analyzed in Reinganum [6]. In that paper, the securities selected for analysis were a subset of the stocks contained on the December 1978 version of the University of Chicago's Center for Research in Security Prices (CRSP) daily tape files. At the end of

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each calendar year, the market values of the common stocks on the file were calculated, and firms were placed into one of ten portfolios based upon their relative position in the value ranking. In the following year, the daily returns of each market value portfolio were computed by combining with equal weights the daily returns of the component securities within the portfolio. Thus, the compositions of the ten market value portfolios were updated annually. The number of firms that satisfied the data requirements range from 1457 in 1963 to over 2500 in the mid-1970s.

In this paper, the betas for each market value portfolio are estimated using ordinary least squares and the Dimson aggregated coefficients method. With the OLS estimator, betas are estimated by regressing market returns against security returns. Thus,

$$R_{i,t} = \alpha_i + \beta_i R_{M,t} + \epsilon_{i,t} \quad (1)$$

In the aggregated coefficients method proposed by Dimson, lagged, leading, and contemporaneous market returns are regressed on observed security returns. That is,

$$R_{i,t} = \alpha_i + \sum_{k=-n}^{+T} \beta_{i,k} R_{M,t+k} + w_{i,t} \quad (2)$$

A consistent estimate of beta is obtained by aggregating (summing) the slope coefficients from this regression. The relative merits and disadvantages of each estimator are discussed by Dimson.<sup>1</sup> However, when infrequent trading is a serious problem, as might be the case for very small firms, the aggregated coefficient method seems to be the superior technique.

Table I presents the average daily returns and estimated betas of the ten market value portfolios over the entire period from 1964 through 1978. The smallest market value portfolio, MV1, experienced average returns greater than .14% per trading day. Since each year contains about 250 trading days, this represents an annual compounded return of nearly 42%. On the other hand, the portfolio with the largest stocks, MV10, only earned slightly more than 6% on an annual basis during this period. Can differences in estimated betas plausibly explain differences in average returns of nearly 36% per year?

Estimated betas in Table I are calculated with the two estimators discussed above. The CRSP value-weighted NYSE-AMEX market returns are used as the index. For the aggregated coefficients technique, the multiple regressions are run with contemporaneous, twenty lagged, and five leading market returns. The lagged terms represent approximately one month of daily trading days. This approach is virtually identical to Roll's methodology and is consistent with Dimson's observation that the lagged coefficients are those of importance when a value weighted index is employed.

Examination of beta estimates in Table I does indeed reveal differences across estimators. Using ordinary least squares, the portfolio with smallest firms possessed an estimated beta of only .75. The estimated beta of the large firm portfolio equalled .98. Thus, the OLS estimates actually indicate that small firms are less

<sup>1</sup> In a very recent paper, Fowler and Rorke [5] suggest that even the Dimson estimator may not be consistent. However, it is difficult to assess the magnitude or direction of this bias. Further research is needed to judge the potential impact of this problem.

**Table I**  
Mean Daily Returns and Estimated Betas for the Ten Market Value Portfolios

Portfolio	Mean <sup>a</sup> Daily Return	Average <sup>b</sup> Percent On AMEX	Average <sup>c</sup> Median Value	OLS Beta	Aggregated Coefficients Beta
MV1	1.421 (.149)	92.41	4.7	.75	1.69
MV2	.921 (.145)	72.12	11.1	.87	1.64
MV3	.786 (.141)	49.77	19.8	.90	1.55
MV4	.636 (.140)	32.15	31.5	.96	1.50
MV5	.581 (.138)	20.50	48.3	.98	1.46
MV6	.532 (.133)	12.63	75.4	.97	1.39
MV7	.460 (.128)	7.81	120.4	.95	1.31
MV8	.421 (.127)	4.09	213.4	.97	1.24
MV9	.354 (.123)	3.29	436.3	.95	1.13
MV10	.241 (.122)	2.27	1086.0	.98	.97

<sup>a</sup> A mean daily return is calculated using 3756 daily excess returns from 1964 through 1978. The mean returns are multiplied by 1000 for reporting purposes. Standard errors are reported in parentheses. MV1 is the smallest market value portfolio; MV10 is the largest market value portfolio.

<sup>b</sup> The percentage of firms within each portfolio that are listed on the American Stock Exchange averaged over the fifteen years of the study.

<sup>c</sup> The median value of the common stock (in millions of dollars) for firms within each portfolio averaged over the fifteen years of the study.

risky than large firms.<sup>2</sup> On the other hand, the betas estimated with the aggregated coefficients methodology (AC) are consistent with the hypothesis that small firms are more risky than large firms. Furthermore, the relationship between the AC estimated betas and median firm size within a portfolio is a monotonic one; smaller firms are associated with higher estimated betas. While the ordering of this relationship is consistent with Roll's possible explanation of the firm size effect, the magnitude of differences in AC estimated betas does not seem to be great enough to account for the differences in average returns. The AC estimated beta of the small firm portfolio, MV1, equaled 1.69; the large firm portfolio possessed an AC estimated beta of .97. It seems unlikely, however, that a difference in estimated betas of .7 can explain a difference in average returns of nearly 36%. In order for this difference in estimated betas to account for a 36% return differential, the expected market return must exceed the risk-free (zero-beta) return by more than 50% ( $36\%/.7$ ).

Table II presents the regression coefficients of their *t*-values for the contem-

<sup>2</sup> Surprisingly, further tests indicated that a similar conclusion would be reached if one assessed risk on the basis of Scholes-Williams beta estimates [9].

Table II  
 Estimated Coefficients and Their  $T$ -Values ( $H_0 = 0$ ) for the Lagged, Contemporaneous, and Leading Market Returns

LAG/LEAD	Portfolio									
	MV1	MV2	MV3	MV4	MV5	MV6	MV7	MV8	MV9	MV10
-20	0.026 (1.7)	0.021 (1.6)	0.015 (1.3)	0.016 (1.6)	0.015 (1.8)	0.005 (0.7)	0.012 (1.8)	0.006 (1.0)	0.000 (0.0)	-0.003 (-1.2)
-19	0.036 (2.3)	0.037 (2.8)	0.032 (2.8)	0.012 (1.2)	0.017 (1.9)	0.016 (2.0)	0.008 (1.1)	0.011 (1.9)	0.009 (1.8)	0.001 (0.3)
-18	0.019 (1.2)	0.008 (0.6)	0.013 (1.1)	0.020 (2.0)	0.006 (0.7)	0.005 (0.7)	0.004 (0.6)	0.007 (1.2)	-0.004 (-0.9)	0.002 (0.6)
-17	0.003 (0.2)	0.011 (0.8)	0.010 (0.9)	0.002 (0.2)	0.002 (0.2)	0.005 (0.6)	-0.002 (-0.3)	-0.008 (-1.4)	-0.003 (-0.6)	-0.002 (-0.6)
-16	0.045 (2.8)	0.032 (2.4)	0.007 (0.6)	0.001 (0.1)	0.001 (0.1)	0.006 (0.8)	-0.005 (-0.7)	-0.002 (-0.3)	-0.008 (-1.8)	-0.007 (-2.5)
-15	0.032 (2.0)	0.027 (2.0)	0.040 (3.5)	0.026 (2.6)	0.013 (1.4)	0.007 (0.9)	0.016 (2.2)	0.007 (1.2)	0.009 (1.9)	0.002 (0.6)
-14	0.040 (2.5)	0.010 (0.8)	-0.000 (-0.0)	0.002 (0.2)	0.003 (0.3)	-0.003 (0.4)	-0.009 (-1.3)	-0.001 (-0.2)	-0.004 (-0.8)	0.000 (0.0)
-13	0.013 (0.8)	0.015 (1.1)	0.021 (1.8)	0.006 (0.6)	0.014 (1.6)	0.002 (0.3)	0.010 (1.5)	0.002 (0.4)	0.002 (0.4)	-0.002 (-0.5)
-12	-0.010 (-0.6)	0.006 (0.4)	-0.014 (-1.2)	0.003 (0.3)	-0.005 (-0.6)	0.013 (1.6)	0.007 (1.0)	0.004 (0.6)	0.009 (1.8)	0.001 (0.3)
-11	0.027 (1.7)	0.007 (0.5)	0.002 (0.2)	-0.009 (-0.8)	0.005 (0.6)	0.001 (0.1)	-0.003 (-0.4)	0.004 (0.6)	-0.003 (-0.6)	-0.002 (-0.7)
-10	0.059 (3.7)	0.047 (3.6)	0.046 (4.0)	0.031 (3.0)	0.028 (3.1)	0.031 (3.8)	0.024 (3.4)	0.019 (3.1)	0.012 (2.6)	-0.002 (-0.8)
-9	0.055 (3.5)	0.016 (1.2)	0.018 (1.6)	0.015 (1.4)	0.008 (0.9)	0.005 (0.6)	0.003 (0.4)	-0.006 (-1.0)	-0.008 (-1.7)	-0.006 (-2.2)
-8	0.036 (2.3)	0.038 (2.9)	0.028 (2.5)	0.023 (2.3)	0.026 (2.9)	0.010 (1.2)	0.022 (3.2)	0.010 (1.7)	0.004 (0.9)	-0.001 (-0.3)
-7	0.045 (2.8)	0.032 (2.4)	0.028 (2.4)	0.035 (3.5)	0.026 (2.9)	0.031 (3.9)	0.023 (3.3)	0.011 (1.8)	0.011 (2.3)	-0.004 (-1.6)
-6	0.039	0.030	0.031	0.018	0.018	0.014	0.017	0.005	0.005	-0.002

-5	(2.4) 0.092 (5.8)	(2.3) 0.088 (6.7)	(2.7) 0.062 (5.3)	(1.8) 0.061 (6.0)	(2.0) 0.055 (6.1)	(1.8) 0.035 (4.3)	(2.5) 0.026 (3.7)	(0.9) 0.023 (3.9)	(1.0) 0.013 (2.8)	(-0.8) -0.008 (-2.8)
-4	(5.6) 0.089 (5.3)	(6.7) 0.069 (5.3)	(6.1) 0.070 (6.1)	(7.1) 0.073 (7.1)	(6.4) 0.057 (6.4)	(6.9) 0.055 (6.9)	(6.5) 0.046 (6.5)	(5.5) 0.033 (5.5)	(5.9) 0.028 (5.9)	(2.7) 0.008 (2.7)
-3	(5.9) 0.093 (5.9)	(6.5) 0.085 (6.5)	(7.4) 0.085 (7.4)	(5.9) 0.060 (5.9)	(6.1) 0.056 (6.1)	(6.3) 0.051 (6.3)	(7.0) 0.049 (7.0)	(7.0) 0.042 (7.0)	(5.2) 0.025 (5.2)	(1.1) 0.003 (1.1)
-2	(2.0) 0.032 (2.0)	(2.0) 0.026 (2.0)	(0.7) 0.008 (0.7)	(0.9) 0.009 (0.9)	(1.3) 0.012 (1.3)	(0.9) 0.007 (0.9)	(1.0) 0.007 (1.0)	(-0.2) -0.001 (-0.2)	(-2.3) -0.011 (-2.3)	(-1.0) -0.003 (-1.0)
-1	(18.1) 0.286 (18.1)	(18.8) 0.247 (18.8)	(20.5) 0.236 (20.5)	(20.0) 0.204 (20.0)	(19.8) 0.178 (19.8)	(21.0) 0.168 (21.0)	(19.0) 0.132 (19.0)	(21.0) 0.125 (21.0)	(21.8) 0.105 (21.8)	(7.2) 0.020 (7.2)
0	(42.3) 0.668 (42.3)	(61.3) 0.805 (61.3)	(73.3) 0.843 (73.3)	(88.2) 0.899 (88.2)	(103.6) 0.933 (103.6)	(115.3) 0.922 (115.3)	(131.4) 0.912 (131.4)	(156.3) 0.931 (156.3)	(191.8) 0.921 (191.8)	(343.9) 0.970 (343.9)
1	(-1.0) -0.016 (-1.0)	(-2.1) -0.027 (-2.1)	(-2.7) -0.032 (-2.7)	(-1.4) -0.014 (-1.4)	(-2.0) -0.018 (-2.0)	(-1.2) -0.010 (-1.2)	(-0.9) -0.007 (-0.9)	(-0.7) -0.004 (-0.7)	(0.4) 0.002 (0.4)	(1.0) 0.003 (1.0)
2	(-1.6) -0.026 (-1.6)	(-0.1) -0.002 (-0.1)	(-0.6) -0.007 (-0.6)	(-1.5) -0.016 (-1.5)	(-0.7) -0.007 (-0.7)	(-0.5) -0.004 (-0.5)	(0.6) 0.004 (0.6)	(-0.5) -0.003 (-0.5)	(0.4) 0.002 (0.4)	(1.2) 0.003 (1.2)
3	(0.2) 0.004 (0.2)	(-0.2) -0.002 (-0.2)	(-1.1) -0.012 (-1.1)	(-0.0) -0.000 (-0.0)	(-0.1) -0.001 (-0.1)	(-0.7) -0.006 (-0.7)	(0.0) 0.000 (0.0)	(0.10) 0.010 (0.10)	(0.6) 0.003 (0.6)	(0.7) 0.002 (0.7)
4	(-1.1) -0.017 (-1.1)	(-0.4) -0.005 (-0.4)	(1.9) 0.022 (1.9)	(0.9) 0.009 (0.9)	(0.0) 0.000 (0.0)	(1.0) 0.008 (1.0)	(-0.3) -0.002 (-0.3)	(0.3) 0.002 (0.3)	(2.3) 0.011 (2.3)	(-0.6) -0.002 (-0.6)
5	(1.3) 0.019 (1.3)	(1.9) 0.024 (1.9)	(0.2) 0.002 (0.2)	(1.3) 0.013 (1.3)	(1.7) 0.015 (1.7)	(2.0) 0.015 (2.0)	(1.8) 0.012 (1.8)	(2.7) 0.015 (2.7)	(0.6) 0.003 (0.6)	(0.0) 0.000 (0.0)

poraneous, lagged, and leading market returns for each market value portfolio. In general, the coefficients associated with the leading market returns are within two standard errors of zero. The behaviors of the coefficients associated with the lagged market returns differ dramatically. For the largest firm portfolio, the  $t$ -values for the first ten lagged market returns exceed two only four times. On the other hand, each of the first ten lagged coefficients for the small firms portfolio are at least two standard errors from zero; four of the next ten coefficients are also more than two standard errors away from zero. Thus, it seems fair to conclude that nontrading may be a much more serious concern for small firms than for large firms. Furthermore, failure to account for this potential nontrading can lead to an understatement of beta risk. But this understatement seems not sufficiently large enough to explain the firm size effect.

One can more formally test whether differences in estimated betas alone account for differences in portfolio returns by employing a Fama-MacBeth type methodology. Each month the daily returns of the market value portfolios are compounded into monthly returns. These monthly portfolio returns are regressed against their Dimson betas and a market capitalization variable. The time-series of monthly estimated coefficients for each of these independent variables is then analyzed. If the Dimson betas fully explain the differences in average returns, then one would expect that on average the estimated coefficients for the market capitalization variable would be zero. On the other hand, if the average value of these coefficients is statistically different from zero, then a firm size effect would still be present.

To test for a firm size effect, the following regression is run in each of the 180 months from 1964 through 1978:

$$R_{p,t} = \gamma_{0,t} + \gamma_{1,t}\hat{\beta}_{p,y} + \gamma_{2,t}S_{p,y} + \epsilon_{p,t} \quad (3)$$

where

- $R_{p,t}$  = return in month  $t$  on market value portfolio  $p$ ;
- $\hat{\beta}_{p,t}$  = estimated Dimson beta for portfolio  $p$  during year  $y$ ;
- $S_{p,y}$  = logarithm of median firm size in portfolio  $p$  at end of year  $y - 1$ ; and
- $\epsilon_{p,t}$  = disturbance term.

One should note that although a portfolio's return changes from month to month, its estimated beta and capitalization variable change only every twelve months. Since the observed relationship between capitalizations and returns is nonlinear, a log transformation is applied to the market value variables.<sup>3</sup> The Fama-MacBeth methodology is used with both ten and thirty market value portfolios; the thirty portfolios are created by subdividing each of the ten MV portfolios into three portfolios based on firm size. The reason for the subdivision was to increase the dispersion in the independent variables, thereby increasing the precision of the cross-sectional estimates and reducing their sampling error. With thirty portfolios, the standard deviations of the time-series of  $\hat{\gamma}_{1,t}$  and  $\hat{\gamma}_{2,t}$  are smaller than the standard deviations of these variables when only ten portfolios are used.

<sup>3</sup> Banz [1] noted that even a log transformation would not totally eliminate the nonlinearity. Qualitatively, the same results reported in the text are obtained when the market capitalization variable is the portfolio's relative rank (1 . . . 30).

**Table III**  
**Pooled Cross-Sectional Regression**  
**Estimates Using Dimson Betas and**  
**Logarithms of Market Capitalizations**

$$R_{p,t} = \gamma_{0,t} + \gamma_{1,t}\hat{\beta}_{p,y} + \gamma_{2,t}S_{p,y} + \epsilon_{p,t}$$

$$p = 1, \dots, 30$$

Period	Months	$\bar{\gamma}_{0,t}$	$\bar{\gamma}_{1,t}$	$\bar{\gamma}_{2,t}$
1/64-12/78	180	8.502 (2.04)	.039 (.36)	-.911 (.22)
1/64-12/68	60	13.322 (2.74)	.118 (.41)	-1.420 (.30)
1/69-12/73	60	.902 (3.38)	-.940 (.60)	.024 (.35)
1/74-12/78	60	11.283 (4.16)	.941 (.77)	-1.337 (.43)

Monthly portfolio returns are multiplied by 100 before regressions are run. Standard errors of time-series point estimates are in parentheses. Estimated betas and logarithms of the median firms within each portfolio are updated every twelve months (yearly).

However, the point estimates of the means are virtually the same regardless of whether ten or thirty portfolios are used.

Table III presents the values for  $\bar{\gamma}_{1,t}$  and  $\bar{\gamma}_{2,t}$  based on the full 180-month period as well as for three five-year subperiods. Results in this table are constructed with  $\hat{\gamma}_{1,t}$  and  $\hat{\gamma}_{2,t}$  estimates from cross-sectional regressions run with the thirty market value portfolios. For the overall period, market capitalizations exhibit a statistically significant negative relationship to portfolio returns. The average premium on the size variable is  $-.91$ , which is more than four standard errors away from zero. This evidence suggests that a \$10 million dollar firm will experience monthly returns nearly 1% greater than a \$100 million dollar firm with an identical beta. On the other hand, the evidence indicates that during this period the risk premiums associated with betas were very small. The point estimate of the market risk premium is only .04% per month; the standard error associated with this point estimate is .36% per month. Thus, after controlling for size effects, differences in estimated Dimson betas seem to explain only a small portion of the differences in average portfolio returns.<sup>4</sup>

Results within the subperiods seem to be consistent with the overall period findings. In two of the three five-year periods, the size effect was large and statistically significant. However, during 1969-1973, the average premium on the size variable equalled .024 with a standard error of .35. Thus the effect of market

<sup>4</sup> A time-series test conditioned on the Sharpe-Lintner version of the CAPM using T-Bill rates and the CRSP NYSE-AMEX market index corroborated the results derived using the Fama-MacBeth type methodology. For example, the time-series tests revealed that the average return differential between the smallest and largest firm portfolios is about 2.7% per month, even after adjusting portfolio returns for differences in estimated Dimson betas.

capitalization on returns was small during this period. But, this period was unusual in that during several of these years large firms actually outperformed small firms. Furthermore, during this period low beta stocks experienced average returns in excess of high beta firms; the market risk premium,  $\bar{\gamma}_{1,t}$ , equalled  $-.94\%$  per month. Even including this unusual five-year period, the results for the overall period indicate that market capitalizations are strongly related to average portfolio returns.

Table IV contains mean returns and estimated betas computed with monthly and quarterly returns for each market value portfolio; these holding period returns are created by compounding the daily portfolio returns. Betas are estimated using ordinary least squares. The AC estimated beta of the small firm portfolio (1.69) reported in Table I is greater than the beta estimated with monthly returns (1.47) but less than the beta estimated with quarterly returns (2.00); the estimated beta of the large firm portfolio is approximately 1.0 regardless

**Table IV**  
Mean Returns and Estimated Betas for the Ten Market Value Portfolios Based on Daily Returns Compounded Monthly and Quarterly

Portfolio	MONTHLY STATISTICS (Sample Size = 180)			QUARTERLY STATISTICS (Sample Size = 60)		
	Mean <sup>a</sup> Monthly Return	Estimated <sup>b</sup> Beta	$\sigma_p/\sigma_m$ <sup>c</sup>	Mean Quarterly Return	Estimated Beta	$\sigma_p/\sigma_m$
MV1	3.36 (.751)	1.47 (.134)	2.321	11.34 (3.24)	2.00 (.268)	2.863
MV2	2.18 (.619)	1.45 (.094)	1.913	7.15 (2.43)	1.70 (.173)	2.145
MV3	1.84 (.562)	1.39 (.078)	1.737	5.98 (2.13)	1.57 (.138)	1.886
MV4	1.50 (.531)	1.38 (.067)	1.641	4.91 (2.03)	1.52 (.120)	1.772
MV5	1.37 (.506)	1.34 (.060)	1.562	4.47 (1.91)	1.48 (.108)	1.689
MV6	1.24 (.467)	1.29 (.049)	1.443	4.03 (1.75)	1.40 (.088)	1.551
MV7	1.06 (.433)	1.22 (.041)	1.334	3.45 (1.60)	1.31 (.072)	1.450
MV8	0.97 (.411)	1.20 (.031)	1.269	3.14 (1.52)	1.28 (.055)	1.345
MV9	0.81 (.380)	1.13 (.024)	1.174	2.61 (1.38)	1.18 (.043)	1.222
MV10	0.54 (.325)	0.99 (.012)	1.005	1.72 (1.12)	0.98 (.021)	0.988
Market	0.57 (.324)	1.00	1.000	1.82 (1.13)	1.00	1.000

<sup>a</sup> Mean portfolio returns are expressed as percentages. Standard errors are in parentheses. Monthly and quarterly returns are compounded using daily portfolio returns from 1964 through 1978.

<sup>b</sup> Estimated betas are OLS estimates obtained by regressing portfolio returns on the CRSP value-weighted market index. The standard errors of the estimates are in parentheses.

<sup>c</sup> The ratio of the portfolio standard deviation to the standard deviation of the market portfolio.



of whether monthly or quarterly returns are used. Thus, the evidence in Table IV indicates that the risk measures of the small firm portfolio are somewhat sensitive to the estimation interval. Even so, it does not seem likely that misestimation of beta risk can explain the firm size effect. That is, the difference in estimated betas between the small firm and large firm portfolios is 1.0 based on quarterly returns. However, the small firm portfolio experienced average returns that exceeded those of the large firm portfolio by about 9.5% per quarter. Hence, while the small firm beta estimates in Table IV differ from the AC estimates reported in Table I, the differences are not great enough to overturn the major conclusion drawn from those data; namely, that differences in estimated betas do not seem to account for the firm size effect.

## II. Conclusion

This paper explored Roll's conjecture that the "abnormal" returns attributed to small firms are the statistical artifacts of improperly estimated betas. The conjecture is tested using ten portfolios that are constructed using the market capitalizations of individual firms. To be sure, the test results indicate that precise estimates of betas for small firms may be difficult to obtain. Nonetheless, even the highest point estimate for the beta of the small firm portfolio did not seem to account for its superior performance. Roll recognized this possibility: "We must therefore conclude tentatively that perhaps only part of the observed risk-adjusted excess returns related to size can be explained by mis-assessment of risk." The evidence reported in this paper indicates that such a conclusion need not be so tentative. While the OLS estimates seem to understate the betas of small firms, the excess returns not explained by the misestimation could easily exceed twenty percent per year on average. Thus, one can conclude with confidence that the small firm effect is still a significant economic and empirical anomaly.

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