Technical Note No. 4*
Options, Futures, and Other Derivatives
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Exact Procedure for Valuing American Calls on Dividend-Paying Stocks

The Roll, Geske, and Whaley formula for the value of an American call option on a stock paying a single dividend D_1 at time t_1 is

$$\begin{aligned} C =&(S_0 - D_1 e^{-rt_1})N(b_1) + (S_0 - D_1 e^{-rt_1})M\left(a_1, -b_1; -\sqrt{\frac{t_1}{T}}\right) \\ &- Ke^{-rT}M\left(a_2, -b_2; -\sqrt{\frac{t_1}{T}}\right) - (K - D_1)e^{-rt_1}N(b_2) \end{aligned} \quad (1)$$

where

$$\begin{aligned} a_1 &= \frac{\ln[(S_0 - D_1 e^{-rt_1})/K] + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \\ a_2 &= a_1 - \sigma\sqrt{T} \\ b_1 &= \frac{\ln[(S_0 - D_1 e^{-rt_1})/S^*] + (r + \sigma^2/2)t_1}{\sigma\sqrt{t_1}} \\ b_2 &= b_1 - \sigma\sqrt{t_1} \end{aligned}$$

The variable σ is the volatility of the stock price net of the present value of the dividend. The function, $M(a,\, b;\, \rho)$, is the cumulative probability, in a standardized bivariate normal distribution, that the first variable is less than a and the second variable is less than b, when the coefficient of correlation between the variables is ρ. A procedure for calculating the M function is given in Technical Note 5. The variable S^* is the solution to

$$c(S^*) = S^* + D_1 - K$$

where $c(S^*)$ is the Black–Scholes–Merton option price when the stock price is S^* and the time to maturity is $T - t_1$. When early exercise is never optimal, $S^* = \infty$. In this case $b_1 = b_2 = -\infty$ and equation (1) reduces to the Black–Scholes–Merton equation with S_0 replaced by $S_0 - D_1 e^{-rt_1}$. In other situations, $S^* < \infty$ and the option should be exercised at time t_1 when $S(t_1) > S^* + D_1$.

When several dividends are anticipated, early exercise is normally optimal only on the final ex-dividend date as explained in the text. It follows that the Roll, Geske, and Whaley formula can be used with S_0 reduced by the present value of all dividends except the final one. The variable, D_1, should be set equal to the final dividend and t_1 should be set equal to the final ex-dividend date.