Technical Note No. 26*
Options, Futures, and Other Derivatives
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A Binomial Measure of Credit Correlation

A credit correlation measure sometimes used by rating agencies is the *binomial correlation measure*. For two companies, A and B, this is the coefficient of correlation between:

1. A variable that equals 1 if company A defaults between times 0 and T and zero otherwise; and
2. A variable that equals 1 if company B defaults between times 0 and T and zero otherwise

The measure is

$$\beta_{AB}(T) = \frac{P_{AB}(T) - Q_A(T)Q_B(T)}{\sqrt{[Q_A(T) - Q_A(T)^2][Q_B(T) - Q_B(T)^2]}} \tag{1}$$

where $P_{AB}(T)$ is the joint probability of A and B defaulting between time zero and time T, $Q_A(T)$ is the cumulative probability that company A will default by time T, and $Q_B(T)$ is the cumulative probability that company B will default by time T. Typically $\beta_{AB}(T)$ depends on T, the length of the time period considered. Usually it increases as T increases.

From the definition of a Gaussian copula model $P_{AB}(T) = M[x_A(T), x_B(T); \rho_{AB}]$. where $x_A(T) = N^{-1}(Q_A(T))$ and $x_B(T) = N^{-1}(Q_B(T))$ are the transformed times to default for companies A and B, and ρ_{AB} is the Gaussian copula correlation for the times to default for A and B. $M(a, b; \rho)$ is the probability that, in a bivariate normal distribution where the correlation between the variables is ρ, the first variable is less than a and the second variable is less than b.[1] It follows that

$$\beta_{AB}(T) = \frac{M[x_A(T), x_B(T); \rho_{AB}] - Q_A(T)Q_B(T)}{\sqrt{[Q_A(T) - Q_A(T)^2][Q_B(T) - Q_B(T)^2]}} \tag{2}$$

This shows that, if $Q_A(T)$ and $Q_B(T)$ are known, $\beta_{AB}(T)$ can be calculated from ρ_{AB} and vice versa. Usually ρ_{AB} is markedly greater than $\beta_{AB}(T)$. This illustrates the important point that the magnitude of a correlation measure depends on the way it is defined.

Example

Suppose that the probability of company A defaulting in one-year period is 1% and the probability of company B defaulting in a one-year period is also 1%. In this case $x_A(1) = x_B(1) = N^{-1}(0.01) = -2.326$. If ρ_{AB} is 0.20, $M(x_A(1), x_B(1), \rho_{AB}) = 0.000337$ and equation (2) shows that $\beta_{AB}(T) = 0.024$ when $T = 1$.

[1] See Technical Note 5 for the calculation of $M(a, b; \rho)$.