

Technical Note No. 20*
Options, Futures, and Other Derivatives
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**Changing the Market Price of Risk for Variables
That Are not the Prices of Traded Securities**

This Note shows that the rules for changing the stochastic process when there is a change in the market price of risk are the same, regardless of whether the variable is or is not the price of a traded security.

Suppose that a single variable θ (not the price of a traded security) follows the process

$$d\theta = m\theta dt + s\theta dz$$

and that a traded security f dependent θ follows the process

$$df = \mu f dt + \sigma f dz$$

When the growth rate of θ increases by Δm so that the drift of θ increases by $\theta\Delta m$ we know from Ito's lemma that the growth rate of f increases by $\Delta\mu$ where

$$f\Delta\mu = \theta\Delta m \frac{\partial f}{\partial \theta}$$

Also using Ito's lemma

$$\sigma f = s\theta \frac{\partial f}{\partial \theta}$$

Hence

$$\frac{\Delta\mu}{\sigma} = \frac{\Delta m}{s} \tag{1}$$

Consider the situation where the market price of risk changes from λ to λ^* . We know that

$$\Delta\mu = (\lambda^* - \lambda)\sigma$$

Equation(1) shows that

$$\Delta m = (\lambda^* - \lambda)s$$

This shows that when the market price of risk changes, the stochastic processes of variables that are not traded securities are adjusted in the same way as the stochastic processes of those that are.

This analysis can be extended to the situation where there are several sources of uncertainty so that

$$d\theta = m\theta dt + \sum_{i=1}^n s_i\theta dz_i$$

and

$$df = \mu f dt + \sum_{i=1}^n \sigma_i f dz_i$$

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In this case we still have

$$f\Delta\mu = \theta\Delta m \frac{\partial f}{\partial \theta}$$

Using Ito's lemma

$$\sigma_i f = s_i \theta \frac{\partial f}{\partial \theta}$$

Hence

$$\frac{\Delta\mu}{\sigma_i} = \frac{\Delta m}{s_i} \tag{2}$$

When

$$\Delta\mu = \sum_{i=1}^n (\lambda_i^* - \lambda_i) \sigma_i$$

Equation(2) shows that

$$\Delta m = \sum_{i=1}^n (\lambda^* - \lambda) s_i$$

which is the required result.