Technical Note No. 13[1]

Options, Futures, and Other Derivatives

John Hull

Efficient Procedure for Valuing American-Style Lookback Options

A number of researchers have suggested an interesting and instructive approach to valuing lookback options.[2] Consider an American-style lookback put. The initial stock price is 50, the stock price volatility is 40%, the risk-free interest rate is 10% and the life of the option is 3 months. We suppose that three steps are used to model the stock price movements.

When exercised, the option provides a payoff equal to the excess of the maximum stock price over the current stock price. We define $G(t)$ as the maximum stock price achieved up to time t and set

$$Y(t) = \frac{G(t)}{S(t)}$$

We next use the Cox, Ross, and Rubinstein tree for the stock price to produce a tree for Y. Initially, Y=1 because $G = S$ at time zero. If there is an up movement in S during the first time step, both G and S increase by a proportional amount u and $Y = 1$. If there is a down movement in S during the first time step, G stays the same, so that $Y = 1/d = u$. Continuing with these types of arguments, we produce the tree shown in Figure 1 for Y. (Note that in this example $\Delta t = 0.08333$, $u = 1.1224$, $d = 0.8909$, $a = 1.0084$, and $p = 0.5073$). The rules defining the geometry of the tree are

1. When Y=1 at time t, it is either u or 1 at time $t+\Delta t$.
2. When $Y=u^m$ at time t for $m \geq 1$, it is either u^{m+1} or u^{m-1} at time $t+\Delta t$.

An up movement in Y corresponds to a down movement in the stock price, and vice versa. The probability of an up movement in Y is, therefore, always $1-p$ and the probability of a down movement in Y is always p.

We use the tree to value the American lookback option in units of the stock price rather than in dollars. In dollars, the payoff from the option is

$$SY-S$$

In stock price units, the payoff from the option, therefore, is

$$Y-1$$

[2] The approach was proposed by Eric Reiner in a lecture at Berkeley. It is also suggested in S. Babbs, ``Binomial Valuation of Lookback Options,'' Working paper, Midland Global Markets, 1992; and T. H. F. Cheuk and T. C. F. Vorst, ``Lookback Options and the Observation Frequency: A Binomial Approach,'' Working Paper, Erasmus University, Rotterdam.

We roll back through the tree in the usual way, valuing a derivative that provides this payoff except that we adjust for the differences in the stock price (i.e., the unit of measurement) at the nodes. If f_{ij} is the value of the lookback at the jth node at time $i\Delta t$ and Y_{ij} is the value of Y at this node, the rollback procedure gives

$$f_{ij} = \max\left(Y_{ij} - 1, e^{-r\Delta t}\left[(1-p) f_{i+1,j+1} d + p f_{i+1,j-1} u\right]\right)$$

when $j \geq 1$.

Note that $f_{i+1,j+1}$ is multiplied by d and $f_{i+1,j-1}$ is multiplied by u in this equation. This takes into account that the stock price at node (i,j) is the unit of measurement. The stock price at node (i+1,j+1), which is the unit of measurement for $f_{i+1,j+1}$ is d times the stock price at node (i,j) and the stock price at node (i+1,j-1), which is the unit of measurement for $f_{i+1,j-1}$, is u times the stock price at node (i,j).

Similarly, when j=0 the rollback procedure gives

$$f_{ij} = \max\left(Y_{ij} - 1, e^{-r\Delta t}\left[(1-p) f_{i+1,j+1} d + p f_{i+1,j} u\right]\right)$$

The calculations for our example are shown in Figure 1. The tree estimates the value of the option at time zero (in stock price units) as 0.1094. This means that the dollar value of the option is 0.1094× 50=5.47.

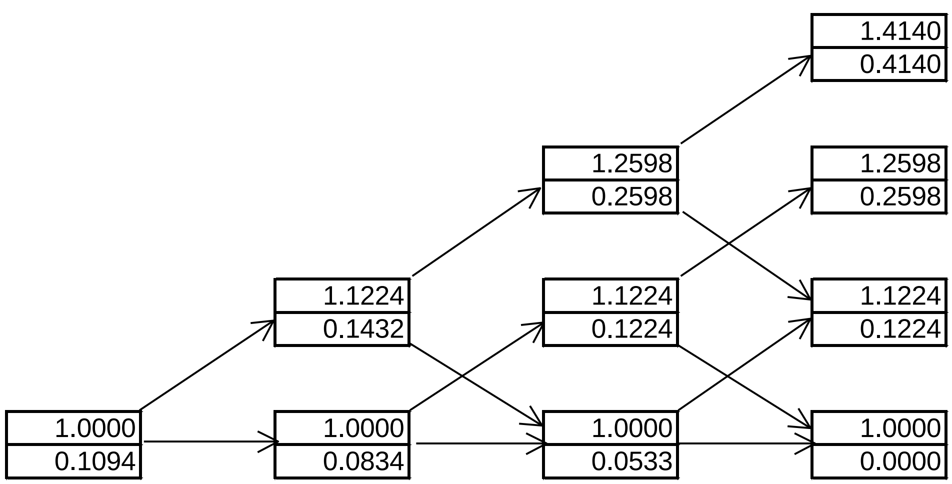

Figure 1 Procedure for valuing an American-style lookback option.