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Valuing Derivatives: Funding Value Adjustments and Fair Value

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ABSTRACT

The authors examine whether a bank should make a funding value adjustment (FVA) when valuing derivatives. They conclude that an FVA is justifiable only for the part of a company's credit spread that does not reflect default risk. They show that an FVA can lead to conflicts between traders and accountants. The types of transactions a bank enters into with end users will depend on how high its funding costs are. Furthermore, an FVA can give rise to arbitrage opportunities for end users.

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Valuing Derivatives: Funding Value Adjustments and Fair Value

One of the most controversial issues for derivatives dealers in the last few years has been whether to make what is known as a *funding value adjustment* (FVA). An FVA is an adjustment to the value of a derivative or a derivatives portfolio that is designed to ensure that a dealer recovers its average funding costs when it trades and hedges derivatives. Theoretical arguments indicate that the dealer's valuation should not recover the whole of its funding costs. In practice, however, many dealers find these theoretical arguments unpersuasive and choose to make the adjustment anyway.[2] The arguments in the FVA debate can be summarized as follows:

The trader. The derivatives desk of a bank is charged the bank's average funding cost by the funding desk. The derivatives desk must make a funding value adjustment for uncollateralized trades. If it does not do so, it will show a loss on trades that require funding. For trades that generate funding, such as the sale of options, an FVA is a benefit because such trades reduce the external funding requirements of a bank. Banks with high funding costs should be able to provide pricing favorable to end users on such trades.

The accountant. All derivatives, including those that are uncollateralized, should be valued at exit prices. Exit prices depend on how other market participants price a transaction; they do not depend on the funding costs of the bank conducting the transaction. IFRS 13, Fair Value Measurement, (2011) page 6 states "fair value is a market-based measurement, not an entity-specific measurement." This is clearly not supportive of FVA as FVA is designed to reflect a bank's funding costs and is therefore "entity-specific." There should be only one price that clears the market for any given transaction. Accountants are concerned that funding value adjustments lead to different banks pricing the same transaction differently.

The theoretician. Finance theory holds that the risk of a project should determine the discount rate used by a company for the project's cash flows. The company's funding costs are

[2] See, for example, Ernst & Young (2012) and KPMG (2013).

irrelevant. Would Goldman Sachs value a US Treasury bond by using its average funding cost to discount cash flows? In the case of derivatives, risk-neutral valuation arguments suggest that we should project cash flows in a risk-neutral world and use a risk-free discount rate for the expected cash flows. There is no theoretical basis for replacing the risk-free discount rate with a (higher) funding cost.

The theoretician's arguments and the accountant's arguments, though quite different, should lead to similar valuations because the derivatives models used by theoreticians are usually calibrated to the market and thus produce prices that are consistent with market prices. The trader's viewpoint can lead to markedly different valuations.

The issues involved in the FVA debate are important to more than just the derivatives industry. The question of whether products should be valued at cost or at market prices is important to many of the activities of all corporations, including financial institutions. The issues raised by FVAs are relevant to any company, financial or nonfinancial, when it evaluates potential investments. Virtually every introductory finance textbook argues that the risk of a project, not the way it is funded, should determine the discount rate used for the project's expected cash flows. Nevertheless, many companies use a single hurdle rate equal to their weighted average cost of capital when valuing any project. Finance theory argues that this practice makes risky projects seem relatively more attractive and projects with very little risk seem relatively less attractive. If a company used its average funding cost to value a Treasury bond or any other low-risk bond, it would never buy it.

Credit risk has become increasingly important to derivatives traders in recent years. Such pricing models as the Black–Scholes–Merton model (Black and Scholes 1973; Merton 1973) provide what we refer to as the no-default value (NDV) of a derivatives transaction. The NDV, which assumes that both sides will live up to their obligations, depends on the discount rate that is used. If the risk-free interest rate is used, the resulting value is consistent with theory; it is also consistent with market prices in the interdealer market, where full collateralization is required.

For bilaterally cleared transactions, a derivatives dealer makes a *credit value adjustment* (CVA) to reflect the possibility that the counterparty will default and then makes a *debit* (or *debt*) *value adjustment* (DVA) to reflect the possibility that the dealer will default. The netting of transactions is a complication in the calculation of the CVA and DVA, which means that they must be calculated for the portfolio of transactions a dealer has with a counterparty, not on a transaction-by-transaction basis. After adjusting for credit risk, we obtain

$$\text{Portfolio Value} = \text{NDV} - \text{CVA} + \text{DVA} \tag{1}$$

The FVA is a further adjustment designed to incorporate the dealer's average funding costs for uncollateralized transactions. It is the difference between the NDV obtained when the risk-free rate is used for discounting and the NDV based on discounting at the dealer's cost of funds.[3] The FVA leads to Equation 1 becoming

$$\text{Portfolio Value} = \text{NDV} - \text{CVA} + \text{DVA} - \text{FVA} \tag{2}$$

An important point is that the FVA is not an adjustment for credit risk. In fact, as Equation 2 indicates, credit risk is taken into account by the CVA and the DVA. To again take credit risk into account would be double counting.

Since the credit crisis of 2008, there has been a change in the way derivatives are valued. Before the credit crisis, zero rates calculated from LIBOR and LIBOR-for-fixed swap rates were assumed to be the "risk-free" discount rates in valuation models for both collateralized and uncollateralized transactions. As indicated earlier, since the crisis many derivatives dealers have made FVAs for uncollateralized transactions. This has the effect of increasing the discount rate to their average funding cost. For collateralized transactions, they have reduced the discount rate to the OIS (overnight index swap) rate.[4] In normal market conditions the OIS rate is about 10 basis points below LIBOR, but in stressed market conditions the difference between the two can be considerably greater than this. This is illustrated by Figure 1 which shows the three-month LIBOR–OIS spread over 2002–2013. During the crisis, the spread rose sharply reaching a record 364 bps in October 2008. A year later, it returned to more normal levels. Since then, however, it

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[3] This is the definition of FVA we will use, but a confusing aspect of arguments concerning FVA is that different dealers define it in different ways. For example, FVA is sometimes defined so that it also reflects the impact of the interest rate on cash collateral being different from the fed funds rate. We consider this to be a separate adjustment and discuss it in Hull and White (2012a).

[4] For a discussion of derivatives discounting and the OIS rate, see Hull and White (2013).

has been more volatile in response to stresses and uncertainties in financial markets, such as concerns about the economies of some European countries.

Theoreticians argue that the interest rates used in the valuation of derivatives should reflect the dealer's best estimate of a truly riskless rate. Dealers, however, have never accepted this argument. Before the crisis, LIBOR was used as a discount rate because LIBOR was a good approximation to a dealer's short-term funding costs, not because it was a good approximation to the risk-free rate. It is no longer the case that all banks can fund themselves at LIBOR. (Indeed, some banks' funding costs are several hundred basis points above LIBOR.) This is what has led banks to make FVAs.

The OIS rate is a reasonable proxy for the risk-free rate.[5] But this is not the reason dealers use this as the discount rate for fully collateralized transactions. It is argued that these transactions are funded by the collateral. If the collateral is cash, the interest rate paid is usually the overnight federal funds rate and the OIS rate is a longer-term rate corresponding to a continually refreshed overnight federal funds rate.[6]

Funding Costs and Performance Measurement

The funding value adjustment arises from a difference between the way derivatives are valued in the market and the way the activities of a derivatives desk are assessed. In practice, for a financial institution, return on capital (annual profit divided by allocated capital) is often the key metric when projects are being considered. In particular, return on capital is usually used to measure the performance of the derivatives activities of a financial institution. In this calculation, the profitability of derivatives trading is measured as trading profits less expenses, which include funding costs and other relevant costs. The funding costs are often calculated by applying the dealer's average funding rate to the average funding used in derivatives trading. The following simple examples illustrate how the funding costs might affect the way traders calculate the prices of derivatives.

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[5] We argue this in Hull and White (2013).

[6] The relationship between the OIS rate and the overnight rate is similar to the relationship between the LIBOR-for-fixed swap rate and the LIBOR rate.

Suppose that a dealer's client wants to enter into a forward contract to buy a non-dividend-paying stock in one year's time. Consider how a trader might view this transaction in an FVA world. If she enters into a contract to sell forward, she will hedge the forward contract by buying the stock now so that she will have it available to deliver one year from now. Her profit at the end of the year will be the delivery price less the value of the current stock price when it is compounded forward at the funding rate for a stock purchase. If the current stock price is 100 and her funding rate for equity purchases is 4%, the delivery price must be higher than 104 for the trader to earn a profit. The delivery price at which the trader is willing to sell in one year's time reflects the rate at which a position in the underlying asset can be funded.

The trader's funding costs also affect the discount rate that she uses. Suppose that the delivery price is set at 106 and the trader pays the counterparty X to enter into this forward contract. This expense is another cash outflow that must be funded by borrowing. If the funding rate for this payment is 5%, the year-end profit is $106 - 104 - 1.05X$. Thus, the trader will pay no more than 2/1.05 = 1.905 to enter into this contract. The present value of the forward contract—the maximum amount the trader will pay—is determined by discounting the payoff on the contract at the rate at which a position in the derivative can be funded.

This example shows that the valuation of a derivative depends on two interest rates: the rate at which a position in the underlying asset can be funded and the rate at which a position in the derivative can be funded. If the derivative is bought or sold at the calculated price, the hedged portfolio earns exactly enough to pay all the funding costs. Typically, dealers assume that the underlying asset can be funded through a sale and repurchase (repo) agreement at the repo rate, which is close to the OIS rate. Derivatives cannot be funded through a repo agreement, and in an FVA world, they are assumed to be funded at the dealer's average funding cost.

Appendix A shows how the arguments in Black and Scholes (1973) and Merton (1973) can be extended if we assume that it is correct to apply different funding costs to the derivative and the underlying asset.[7] Suppose that at the beginning of the year, a trader buys a one-year European call option on a non-dividend-paying stock with a strike price of 100. The stock price

[7]For this type of extension of the Black–Scholes–Merton model, see also Piterbarg (2010, 2012) and Burgard and Kjaer (2011a, 2011b, 2012).

is 100, and the stock price volatility is 30%. Suppose further that the relevant funding cost for derivative positions is assumed to be 5% (equal to the bank's average funding cost) and that the interest rate at which positions in the stock can be funded (using a repo agreement) is assumed to be 2%—both quoted with continuous compounding.

The FVA option price, 12.44, is calculated as shown in Appendix A (this option price is the FVA no-default value before the CVA and DVA have been made). The expected return on the stock is the funding rate for the stock, and the payoff is discounted by using the funding rate for the derivative. In terms of the notation in Appendix A, r_s and r_d are 2% and 5%, respectively. If the trader buys the option for 12.44, hedges delta by taking a short stock position, and earns 2% on the proceeds from the short position while paying 5% on the funds used to purchase the option, the net profit on the trade will be zero. (We are making the idealized assumption that the pricing model assumptions are true, the delta hedging works perfectly, and the 30% volatility is the actual stock price volatility. We are also ignoring the CVA and the DVA.)[8]

The FVA price at which the trader would sell to an end user is less than the price that would be offered to another dealer when (as is normal in the interdealer market) the transaction is fully collateralized. As explained earlier, the OIS rate, which is close to the repo rate, would be used for discounting. Thus, $r_s = r_d = 2\%$, and so the price in the interdealer market would be 12.82 (see Appendix A). It is interesting to note that this would also be the price in the uncollateralized market if options could be used in a repo agreement.

FVA, DVA, and Double Counting

The FVA and the DVA concern different aspects of an uncollateralized derivatives portfolio. The FVA concerns funding; the DVA concerns a market participant's own credit risk. In this section, we explore the relationship between the FVA and the DVA.

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[8]The position in the underlying asset used to hedge the derivative is funded at the repo rate, which we assume to be the risk-free rate. If the hedge is funded at a higher rate, the expected future payoffs will change. Higher rates lead to higher expected future stock prices, which increase the price of call options and decrease the price of put options. In many cases, the resulting changes in value are large. In our example, suppose that the funding cost for both the stock and the derivative is 5%: $r_s = r_d = 5\%$ (see Appendix A). The trader's breakeven price before the CVA and DVA are made would then be 14.23. This amount is 15% higher than the price when the underlying hedge is funded at the repo rate.

We refer to the value to a bank that arises because it might default on its derivative obligations as DVA1 and the value to a bank that arises because it might default on the funding required for the derivatives portfolio as DVA2. Accounting bodies have approved the inclusion of both these two quite separate components of the DVA in a bank's financial statements.

Assume for the moment that the whole of the credit spread is compensation for default risk. The FVA then equals DVA2 for a derivative (or a derivatives portfolio) because the present value of the expected excess of the bank's funding for the derivative over the risk-free rate equals the FVA—which also equals the compensation the bank is providing to lenders for the possibility that the bank might default and is thus equal to the expected benefit to the bank from defaulting on its funding. Therefore, FVA and DVA2 cancel each other out. (When a derivative requires funding, FVA is a cost and DVA2 is a benefit. When it provides funding, FVA is a benefit and DVA2 is a cost.)

Equation 2 becomes

$$\text{Portfolio Value} = \text{NDV} - \text{CVA} + \text{DVA1} + \text{DVA2} - \text{FVA} = \text{NDV} - \text{CVA} + \text{DVA1} \quad (3)$$

Note that for transactions that are totally uncollateralized, both FVA and DVA2 are additive across the transactions. For a derivative, a dealer's FVA and DVA2 are thus independent of other transactions entered into by the dealer.

Using our earlier example, we can see that both FVA and DVA2 equal 12.82 − 12.44, or 0.38, for a long option position. Similarly, they both equal − 0.38 for a short option position. If the dealer buys the option, FVA reduces the value of the dealer's portfolio by 0.38 (because the option must be funded) and DVA2 increases the value by 0.38 (because the dealer might default on the funding for the option). If the dealer sells the option, FVA increases the value of the dealer's portfolio by 0.38 (because the option provides funding) and DVA2 reduces the value by 0.38 (because the benefits of a possible dealer default are reduced).

Now let us examine DVA1. As indicated by Equation 3, it is correct to calculate this for a derivatives portfolio that a dealer has with a counterparty. Consider the simple situation where the option is the only derivatives transaction entered into by the dealer and the counterparty. If the dealer who funds at 5% buys the option, DVA1 = 0 because the option is always an asset to the dealer.

Appendix B shows that if the dealer sells the option, DVA1 = FVA* where FVA* = − FVA measures the benefit rather than the cost of the funding adjustment. In this case, DVA1 increases the value of the dealer's position by 0.38 because the possibility of a dealer default on the option she has sold is a benefit to the dealer (note that increasing the value of the position means making it less negative).

In the case of a sold option, both FVA and DVA1 benefit the dealer, reducing her liability. This outcome has led some practitioners to refer to the inclusion of both FVA and DVA1 in valuations as double counting. Because of concerns about this apparent double counting, some dealers choose to include FVA, but not DVA1, in their pricing. The reality is that the choice between FVA and DVA1 is a false one. DVA2 negates FVA, and thus only DVA1 should be included in pricing, as indicated by Equation 3. Except in the case where all derivatives have a negative value to the dealer, including FVA and excluding DVA1 creates incorrect price adjustments.

Appendix B presents our results from comparing DVA1 with FVA*. It shows that the incremental effect of a new derivative on DVA1 can be greater than or less than that on FVA*. For the whole portfolio, DVA1 > FVA*.

The Liquidity Component of Credit Spreads

The analysis we have just given assumes that the whole of the credit spread is compensation for default risk. In practice, there is usually a liquidity risk component to a bond's credit spread. This compensates the bondholder for a possible lack of liquidity in the market for the bond when she wants to sell. Suppose that the credit spread on a bank's bonds is 100 basis points, of which 80 basis points is compensation to the bondholder for default risk and 20 basis points is compensation for liquidity risk. As explained in the previous section, DVA2 and FVA cancel each other as far as the 80 basis points is concerned. However, the 20 basis points is a funding cost for which the banks gets no benefits. Making an FVA for this component of the bank's funding costs is therefore defensible.[9]

The liquidity component of the credit spread is sometimes estimated from the CDS-bond basis. This is defined as the credit default swap (CDS) spread minus the bond yield spread. If the CDS spread is assumed to be a "pure credit spread" reflecting only default risk, then the liquidity component of the credit spread equals the negative of the CDS-bond basis. In our example, we might expect the CDS spread to be 80 basis points so that the CDS-bond basis is −20 basis points. We discuss the liquidity component of the credit spread in more detail in Hull and White (2012a).

For ease of exposition, we assume that the liquidity component of the credit spread is zero in the rest of this article.

Implications of FVA

The use of funding costs in determining the price of a derivative can lead to some unexpected results. In our earlier option example, the repo rate of interest is 2%, which is close to the risk-free rate. The Black–Scholes–Merton option price is 12.82. A trader who could fund at or near the riskless rate would be willing to buy or sell this option for 12.82.

Let us first consider a dealer who makes both adjustments—FVA and DVA1. If the dealer is approached by a new end user (i.e., one with whom she has no outstanding transactions) who wants to buy the option we have been considering, she will quote a price of 12.82 − 0.38 − 0.38 = 12.06. The end user's CVA equals the dealer's DVA1 of 0.38. The net cost to the end user after she has hedged the CVA is thus 12.44, which is less than the price of 12.82 that she would pay in the fully collateralized market. Therefore, the dealer's pricing for an end user who wants to buy options is favorable to the end user. The dealer's pricing will be uncompetitive for a new end user who wants to sell options because in that case, the dealer's FVA is positive (reducing the price she is prepared to pay) whereas her DVA1 is zero. (This scenario is particularly true for dealers with high funding costs. The dealer's price is NDV − FVA − CVA. The CVA is a

[9] The liquidity component of the credit spread is liable to vary from bank to bank and is therefore entity-specific. As indicated earlier, accounting standards state that fair value should not be entity-specific. This suggests that even the liquidity component of the credit spread is not relevant to the computation of fair value.

measure of the end user's credit risk and is the same for all dealers. Dealers who do not incorporate FVA into their pricing will price at NDV − CVA. The prices of dealers with high funding costs who do incorporate FVA into their pricing will be lower than those of dealers with low funding costs who do the same.)

The same result is true for an end user who already has a portfolio of derivatives with the dealer. In this case, the incremental effect of a sold option on DVA1 will be less than 0.38. But including both DVA1 and FVA will lead to the quoted prices being favorable by 0.38 for end users who want to buy the option and uncompetitive by 0.38 for end users who want to sell the option.

The situation for dealers who take FVA, but not DVA1, into account is more complex (see Appendix B). For a dealer who only sells options to an end user, DVA1 = FVA*. Pricing will then be consistent with that of other dealers (but not more favorable to end users). In other situations, Appendix B shows that a dealer will tend to offer competitive prices when selling options and uncompetitive prices when buying options.

We can conclude from these results that dealers with high funding costs who make FVAs (whether or not they also make DVA1s) tend to sell options to end users, not buy options from them—which means that their books are not balanced between long and short positions. Furthermore, they are unable to offer end users a full range of services in derivatives markets competitively.

Arbitrage

These results lead to arbitrage opportunities for end users. If dealers make both FVAs and DVA1s, the arbitrage is straightforward. Let us consider two banks. Bank A funds itself at the risk-free rate of 2%, whereas Bank B funds itself at 5%. If Bank B sells the option we considered earlier and this option is its only derivatives transaction with the counterparty, it will value the option at 12.06. If Bank A buys the option, its price will be 12.82 minus the CVA for the counterparty.

Bank A and Bank B would be happy if Bank B were allowed to sell the uncollateralized option to Bank A for, say, 12.25. In this case, both banks report a profit of 0.19. (After taking

CVA into account, Bank A's price is 12.82 − 0.38 = 12.44.) The new regulations do not allow this outcome because transactions between financial institutions must be fully collateralized. However, a high-credit-quality end user could easily be brought in to arbitrage FVA.[10] The end user buys the option from Bank B and sells a similar option to Bank A. If the end user is of sufficiently high credit quality, Bank A will be willing to pay up to 12.82 for the option bought from the end user.

Suppose the end user buys the option from Bank B for 12.18 and sells a similar option to Bank A for 12.70. The end user then hedges his CVA, which is Bank B's DVA1 (by, for example, buying CDS protection with Bank B as the reference entity or short selling some of Bank B's debt) at a net cost of 0.38 and makes a profit of 12.70 – 12.18 – 0.38 = 0.14; both banks show a profit of 0.12. For Bank A and the end user, the profit is real. Bank B's profit is illusory, a product of its incorrect pricing; Bank B actually suffers a loss of 0.26.

Who could do the arbitrage? To avoid the problem of Bank A's credit risk when buying from the end user and to take advantage of Bank B's FVA, it would have to be a highly creditworthy end user who could trade on an uncollateralized basis. A sovereign wealth fund is one possibility. A creditworthy company with a sophisticated understanding of derivatives is another possibility (e.g., Microsoft, Google, and Apple). Or a hedge fund could collaborate with a company with the specific intention of using the company to trade on an uncollateralized basis.

Regulations may prevent end users from trading on an uncollateralized basis if they are not hedging existing exposures and may limit the amount of trading the end user can do on an uncollateralized basis. Whether this would prevent arbitrage is unclear. Derivatives traders have traditionally been quite successful at finding ways around regulations that prevent profitable trading when pricing is different in two markets. But even if regulations do deny access to arbitrage for all market participants, the theoretical existence of arbitrage is a symptom of serious problems with industry practice. It is hardly reassuring to argue that the derivatives market is being protected from its own mispricing by regulation!

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[10]If the end user has a perceptible probability of default, he can voluntarily agree to enter into a collateralized transaction with Bank A. Alternatively, both Bank A and the end user will have to hedge the end user's credit risk. Either course of action will lead to the same result but make the argument more complex.

The scenario where Bank B incorporates FVA but not DVA1 into its pricing is more complex (see Appendix B). Sometimes the incremental DVA1 is greater than the incremental FVA*, and sometimes it is less. There are potential arbitrage opportunities in the latter case and a tendency for the dealer to be uncompetitive in the former.

We conclude that there are straightforward arbitrage opportunities when dealers include both FVA and DVA1 in their pricing. When dealers make FVAs but not DVA1s, arbitrage is more difficult but still possible. Such arbitrages are simpler than many of the arbitrages that are attempted by hedge funds and banks in derivatives markets. The potential profits from arbitrage increase as the life of the option is increased. When the life of the option in our example is increased from 1 to 10 years, Bank A's breakeven price is 42.91 and Bank B's breakeven price is 31.79 (for both banks, these prices incorporate the FVA but not the CVA and DVA). Over the years, the derivatives market has proved very adept at finding profitable trading activities. It will likely find a way to take advantage of the opportunities created by FVA.

Theoretical Arguments

The FVA debate flows from the procedures used to measure the performance of the derivatives desk. Theoretical arguments (covered in more detail in some of our prior research; see Hull and White 2012a, 2012b) show that funding costs should not influence estimates of market value.

The valuation of an investment should depend on the risk of the investment, not on how it is financed. This precept can be quite difficult to accept. Suppose a bank is financing itself at an average rate of 4.5% and the risk-free rate is 3%. Should the bank undertake a risk-free investment earning 4%? The answer is that of course it should. Because the investment is risk free, its cash flows should be discounted at the risk-free rate, which will give the investment a positive value.

The bank appears to be earning a negative spread of 50 bps on the investment. However, the incremental cost of funding the investment should be the risk-free rate of 3%. As the bank enters into projects that are risk free (or nearly risk free), its funding costs should come down. Let us take an extreme example and assume that the bank doubles in size by undertaking entirely risk-free projects. This course of action should lead to the bank's funding costs changing to

3.75% (an average of 4.5% for the old projects and 3% for the new projects), showing that the incremental funding cost for the new projects is 3%.

This argument does not usually cut much ice with practitioners because it seems far removed from reality. They argue that because of the opaqueness of banks, investors cannot accurately assess the risk of the institution or how it changes as a result of new investment. Their argument is probably correct, but the requirements underlying the theory may be much weaker than practitioners believe. Although investors may be wrong in their assessment of the risk, all that is required is that they not systematically over- or underestimate the risk and that managers be unable to distinguish the situations in which investors over- or underestimate the risk. The investors are correct, on average, and management does not know when the investors are getting it wrong. Thus, management should assume that the investors are getting it right.

In practice, what usually happens is that the average cost of funding remains approximately the same over time. In the opinion of the investors, this average cost of funding is presumably matched by the average risk of the projects the bank undertakes. Risk-free projects enable riskier-than-average projects to be undertaken elsewhere in the bank, and so the overall risk of the bank's portfolio remains approximately the same.

Why not then use the same discount rate for all projects? The answer is that doing so is liable to have dysfunctional consequences, making riskier-than-average projects seem relatively attractive and risk-free (or almost risk-free) projects unattractive.

This argument seems to be at least partly accepted by financial institutions. Banks and other financial institutions do undertake very low-risk investments, such as those involving the purchase of government securities, even though they know the return is less than their average funding costs. For our purposes, the key point of this argument is that it shows that funding costs should, in theory, be irrelevant in the valuation of any investment, risk free or otherwise. To stress that funding costs are irrelevant to the pricing of derivatives, we can use this argument—together with the fact that the hedged transactions are riskless—to show that the correct incremental funding rate for hedged derivatives is the risk-free rate.

Normally, we would expect that when "production" costs increase, a higher price must be charged for the product. However, a bank with high funding costs is willing to sell an option at a lower price than its competitor with lower funding costs because the two banks disagree on the rate of return earned when the funds generated by selling derivatives are invested. The funds generated by the sale are assumed to be invested to reduce outstanding debt, and the interest rate paid on the debt is different for the two banks.

The bank with low funding costs would apparently be better off investing funds in the debt of the bank with high funding costs, rather than reducing its own debt. Of course, the apparent gains from investing in the debt of the high funding cost bank are illusory because of the possibility of a default by that bank. However, this does illustrate the theoretical problem with FVA. The expected return from a bank retiring its own debt is not its funding costs; rather, it is the expected rate of return on its debt after the default risk has been factored in. If the whole of the bank's credit spread is compensation for its default risk, the expected return from a bank retiring its own debt is the risk-free rate and FVA is zero.

Fair Value

Statement of Financial Accounting Standards No. 157 and IFRS 13 define *fair value* as the "price that would be received to sell an asset or paid to transfer a liability in an orderly transaction between market participants at the measurement date."[11] Should two entities with different funding costs have different fair value estimates for the same asset? The answer is no. Let us consider two individuals, A and B. A can borrow money at 2% to buy IBM shares and B can borrow money at 6% to do the same thing. Borrowing costs will quite possibly influence their decisions on whether to buy the shares. But A and B should agree that the fair value of the shares is their market price. This fair value may be different from the private value that A and B place on the shares, because of funding costs or for other reasons.

The fair value of an IBM share is the market price—the price that balances supply and demand. Similarly, the fair value of a derivative is the price that balances supply and demand—the price at which the number of market participants wanting to buy equals the number of market participants wanting to sell. Presumably, those market participants who want to buy at the market price put a private value on the derivative that is higher than the market price, and those who

[11] See Financial Accounting Standards Board (2006) page 2 and International Accounting Standards Board (2011) page 6.

want to sell at the market price put a private value on the derivative that is lower than the market price.

In practice, there are many reasons why different market participants might have different private values for the same derivatives transaction. Funding costs (rightly or wrongly) may be one reason. Other potential considerations are liquidity constraints, regulatory capital constraints, the bank's ability to hedge the underlying risks, other similar transactions in the dealer's portfolio, and so on. The key point is that although there can be a number of different private values for a derivative, there is only one market price or fair value.

Some argue that funding costs have moved markets away from the "law of one price." We do not agree. Only one price clears the market for any given product. One of the features of derivatives (and of many other financial products) is that they can easily be both bought and sold. If the price is different in two markets, they will be bought in one market and sold in the other, which is why accountants and derivatives dealers should value derivatives at market price, not at cost. Indeed, we have shown how end users and dealers can arbitrage FVA very simply.

Equations 1 and 3 show how adjustments should be made for credit risk in derivatives markets. Dealers who have the same information and the same models should agree on the fair market price for a portfolio of derivatives transactions entered into by two parties. In particular, the two parties themselves should agree because (1) the no-default value of the portfolio to one party is the negative of its no-default value to the other and (2) one party's CVA is the other party's DVA1, and vice versa.[12]

In this case, although everyone agrees on the valuation, it is specific to the pair of entities involved in the transaction because different entities have different CVAs and DVA1s. This case may appear to be a violation of the accounting standards' definition of fair value because if one party novates a transaction to a new counterparty, the price at which the transaction is done will reflect the credit risk of the new counterparty. However, when all costs and benefits are taken into account, we can see that no violation occurs.

[12]With collateralized transactions, an adjustment may be necessary if the interest rate paid on cash collateral is different from the risk-free rate. However, the interest received by one party is the interest paid by the other party, and so the adjustments for the two parties are equal in magnitude and opposite in sign.

Let us consider a single transaction between A and B. From A's point of view, the NDV is 100, the CVA is 5, and the DVA1 is 10. The value to A is 105 and the value to B is –105. Now let us suppose that A novates the transaction to C, a firm with no credit risk. C's DVA1 is zero, so the value of the transaction to C is 95. If C takes over the transaction from A by paying A 95 the value to C is 95 and the value to B is –95. It appears that 10 units of value have been lost and A has suffered a loss of 10. However, because B makes a gain of 10, it should, at least in principle, be willing to pay 10 to A to facilitate novating the transaction to C. Assuming this payment happens, A receives 95 from C and 10 from B. B pays 10 to A and ends up with a transaction worth –95. The original valuations (105 to A and –105 to B) are preserved.

A troubling aspect of FVA is that it results in different market participants having different estimates of the fair value, even when they are using the same models and the same market data. Consider Equation 2, which includes FVA. If the dealer and the counterparty have the same funding costs, the dealer's FVA is equal in magnitude and opposite in sign to the counterparty's FVA and both continue to agree on the fair value. If the dealer and the counterparty have different funding costs, however, they no longer agree on the fair value.

Conclusion

There are no simple solutions to the FVA conundrum. A single price cannot serve two purposes. It cannot both reflect the trader's funding costs and be consistent with market prices. Advocates of FVA would like to convince accountants that FVA prices should be used as fair values, but they are unlikely to be successful. Perhaps the most compelling reason for accountants not to endorse FVA is that FVA prices, as we have shown, can be arbitraged. A highly creditworthy end user can buy options from a bank with high funding costs and sell the options to a bank with low funding costs in such a way that all three parties appear to be making a profit. The possibility of accounting systems being gamed in this way will clearly not be tolerated. There is also a danger that traders will use FVA to inflate their profits on long-dated deals in which options or similar derivatives are sold. (The scope for inflating profits increases with the life of the underlying option.)

Many banks seem content to use two parallel systems—one for calculating trader (FVA) values, the other for calculating fair values. The obvious problem is that the derivatives desk's internal performance measure can be out of line with the results reported in the company's financial statements. In addition, traders are likely to ignore the results in financial statements if their bonuses depend on the internal performance measure. As FVAs become more widespread, the scope for disagreements between the bank's internal performance measure and its financial statements will increase.

The only viable solution is to require traders to incorporate CVA and DVA1, but not FVA, into their pricing. Some traders choose to incorporate CVA and FVA, but not DVA1—no doubt a response to the bank's internal performance measure. However, that approach leads to incorrect pricing. The pricing on some transactions will be favorable to the end user and lead to arbitrage; the pricing on other transactions will be uncompetitive. As a result, the dealer will be unable to offer a full range of derivatives products to end users competitively. It is important that trades be marked to market prices as soon as they have been executed. Earlier, we considered an option that traded in the interdealer market at 12.82 but had a price after FVA and DVA1 of 12.06. The correct price for the bank's systems to use is 12.44 (incorporating DVA1 but not FVA). If a dealer sold this option for, say, 12.30, the bank's systems should record a loss of 0.14, not a gain of 0.24.

Should the Treasury desk charge the derivatives desk a funding cost? We believe that the answer is no. We realize that this response flies in the face of how the performance of bank divisions is assessed—and possibly means that derivatives trading is treated differently than other bank activities. As we have explained, however, FVA has the serious disadvantage of creating potential arbitrage opportunities.

The solution for the Treasury group lies in recognizing that the bank's credit spread is compensation for the fact that the bank might default. The possibility of default is a benefit to banks, which are allowed to quantify this benefit under accounting rules. This form of DVA, which we refer to as DVA2, is the adjustment to the value of the bank's debt to reflect the possibility that it may not be repaid. Banks should quantify the DVA2 associated with the trading of derivatives, and the costs or benefits should accrue to the Treasury group. It can then charge (credit) the derivatives desk for the funds it uses (generates) at the risk-free rate without showing a loss itself. We understand that some banks are considering a separation of the funding of the derivatives desk from other funding activities. Such an action might make this proposal easier to implement.

Much of the discussion around funding value adjustments has been driven by technical analyses designed to show that a price adjustment is necessary if the trading desk is to earn some target funding rate. However, the real issues are managerial. To quote KPMG (2011, page 10), funding-related valuation is a “topic that evokes questions about transfer pricing, steering of risk and, most importantly, the business model of each bank.” It is important for management to choose incentives that encourage the derivatives desk to trade in the best interests of shareholders. The best alternative is to use a single system based on market prices adjusted for CVA and DVA1.

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APPENDIX A

Black-Scholes with Funding Costs

In this appendix, we present the arguments by Black and Scholes (1973) and Merton (1973) for valuing a derivative dependent on a non-dividend-paying stock when interest rates are constant and funding costs are considered. We allow the interest rate at which the derivative is funded to be different from the rate at which the stock is funded. Some analysts think that this allowance is necessary because assets such as stocks can often be funded by using repo agreements, whereas derivatives cannot normally be funded in the repo market and are thus funded at a higher rate in the uncollateralized market.

The process assumed for the stock price, S, is

$$dS = \mu S dt + \sigma S dz \tag{A1}$$

where μ is the expected return on the stock, σ is its volatility, and dz is a Wiener process. An application of Ito's lemma shows that the price of the derivative satisfies

$$\begin{aligned} df &= \mu_f f dt + \sigma S \frac{\partial f}{\partial S} dz \\ \mu_f &= \frac{1}{f}\left[\frac{\partial f}{\partial t} + \mu S \frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2}\right] \end{aligned} \tag{A2}$$

We now consider a portfolio consisting of a short position in the derivative and a position of $\partial f / \partial S$ in the stock. The portfolio's value is

$$\Pi = -f + \frac{\partial f}{\partial S} S$$

Suppose that the derivative is funded at r_d and the stock is funded at r_s (where both rates are continuously compounded). Because the portfolio is risk-free, the change in the value of the portfolio in time Δt is

$$\Delta\Pi = -r_d f + r_s \frac{\partial f}{\partial S} S \tag{A3}$$

Using equations (A1) and (A2) the change in the portfolio value is also

$$\Delta\Pi = -\left[\frac{\partial f}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2}\right]\Delta t \tag{A4}$$

Combining equations (A3) and (A4) leads to the differential equation

$$\frac{\partial f}{\partial t} + r_s S \frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = r_d f$$

The risk-neutral valuation argument shows that the solution of this differential equation is obtained by assuming that the expected return on the stock is r_s and then discounting the expected payoff at r_d. For a derivative that provides a payoff only at time T

$$f_0 = e^{-r_d T}\hat{E}(P)$$

where f_0 is the derivative price today (time zero), P is the payoff at time T, and $\hat{E}$ denotes expectations in a world where the expected growth rate of the stock price is r_s. The value of a European call option on the stock with strike price K and time to maturity T is

$$S_0 N(d_1) e^{(r_s - r_d)T} - K e^{-r_d T} N(d_2)$$

where

$$d_1 = \frac{\ln(S_0/K) + (r_s + \sigma^2/2)T}{\sigma\sqrt{T}}$$
$$d_2 = d_1 - \sigma\sqrt{T}$$

These results can be extended to derivatives dependent on assets other than non-dividend-paying stocks.

The notion that risk-neutral valuation can be used only when delta hedging of the underlying risk is possible is not valid. Equilibrium arguments similar to those in the original Black–Scholes paper (1973) show that the Black–Scholes–Merton differential equation—and thus risk-neutral valuation—applies whether or not hedging is possible. Indeed, many applications of derivatives pricing (e.g., the real options approach to capital budgeting) are based on risk-neutral valuation when the underlying risks cannot be hedged.

APPENDIX B

The Relation Between FVA and DVA1

Let us consider a dealer who has m uncollateralized transactions with an end user; T is the life of the longest transaction. We define the value to the end user of the jth transaction at time t as $v_j(t)$ and the dealer's unconditional default rate at time t as $q(t)$.[13] Then,

$$\text{DVA1} = \int_{t=0}^{T} w(t)q(t)[1 - R(t)]E\left[\max\left(\sum_{j=1}^{m} v_j(t),0\right)\right]dt$$

where $R(t)$ is the recovery rate at time t, $w(t)$ is the value of \$1 received at time t, and E denotes risk-neutral expectation. Because $q(t)[1-R(t)]$ is the instantaneous forward credit spread at time t,

$$\text{FVA}^* = \int_{t=0}^{T} w(t)q(t)[1 - R(t)]E\left(\sum_{j=1}^{m} v_j(t)\right)dt$$

where $\text{FVA}^* = -\text{FVA}$ is the benefit provided by FVA. It is always true that

$$\max\left(\sum_{j=1}^{m} v_j(t),0\right) \geq \sum_{j=1}^{m} v_j(t)$$

so $\text{DVA1} \geq \text{FVA}^*$. When the v's are always positive (e.g., when the end user buys options from the dealer), DVA1 = FVA* and DVA1 has the same effect as FVA. In this case, a dealer who includes FVA but not DVA1 in his pricing will be pricing correctly. When the v's are not always positive, DVA1 > FVA*. Consider the situation where a dealer is pricing a transaction with a new end user (i.e., an end user with whom she has no existing transactions). If the transaction does not always have a positive value to the end user the dealer will be uncompetitive because the dealer's price will not fully reflect her DVA1 (i.e., the end user's CVA).

Let us now consider the impact of adding a new transaction worth $u(t)$ at time t to the existing portfolio. The incremental FVA* is

[13] If $h(t)$ is the hazard rate at time t, $q(t) = h(t)\exp\left[-\int_0^t h(\tau)\,d\tau\right]$.

$$\Delta(\text{FVA}^*) = \int_{t=0}^{T} w(t)q(t)\left[1 - R(t)\right]E[u(t)]dt$$

The incremental DVA1 is

$$\Delta(\text{DVA1}) = \int_{t=0}^{T} w(t)q(t)\left[1 - R(t)\right]\left\{E\left[\max\left(\sum_{j=1}^{m} v_j(t) + u(t), 0\right)\right] - E\left[\max\left(\sum_{j=1}^{m} v_j(t), 0\right)\right]\right\}dt$$

We have already shown that the total DVA1 is greater than or equal to the total FVA. The incremental DVA1, however, can be less than or greater than the incremental FVA*. Suppose a dealer has a portfolio with an end user that that can become positively or negatively valued. When a sold option is added to the dealer's portfolio, $\Delta(\text{FVA*}) > \Delta(\text{DVA1})$. When a bought option is added to the dealer's portfolio, $\Delta(\text{FVA*}) < \Delta(\text{DVA1})$. Thus, for a dealer who includes FVA but not DVA1, prices tend to be favorable when options are sold and unfavorable when they are bought. When prices are favorable, there is an arbitrage opportunity. The end user enters into a transaction with a bank with high funding costs and enters into an offsetting transaction with a bank with low funding costs.

Figure 1: The Three-month LIBOR-OIS Spread Between 2002 and 2013