Valuing Derivatives: Funding Value Adjustments and Fair Value*

John Hull and Alan White

Joseph L. Rotman School of Management
University of Toronto

April 2013

ABSTRACT

Derivatives models are used by dealers for two purposes. They are used to calculate the fair value of the derivatives book for accounting purposes and they are used by traders to choose trades that improve the profitability of the derivatives group, as measured by senior management. Given the way that the performance of the derivatives group is typically measured, it is not possible to develop a single model that serves both purposes. Traders want to incorporate a funding value adjustment (FVA) in valuations to reflect the funding costs they are charged, but this can lead to prices that are different from fair value. This paper concludes that, if an FVA is made, the best practice is to use two models, one for determining breakeven trading prices, the other for hedging and marking to market.

*A very preliminary version of this paper was titled “Should a Derivatives Dealer make a Funding Value Adjustment?”

Valuing Derivatives: Funding Value Adjustments and Fair Value

One of the most controversial issues for a derivatives dealer in the last few years has been whether or not to make what is known as a "funding value adjustment" (FVA). This is an adjustment to the value of a derivative or a derivatives portfolio designed to reflect the dealer's average funding costs. Theoretical arguments indicate that the adjustment should not be made. But, in practice, many dealers find these theoretical arguments unconvincing and choose to make the adjustment anyway.[1] This paper examines the issues raised by FVA. It is more comprehensive and has a more managerial focus than Hull and White (2012b)

The theoretical valuation of a derivative nearly always involves an application of risk neutral valuation. In this, expected cash flows are estimated in a risk-neutral world and discounted using a "risk-free" interest rate. The interest rate serves two purposes. It is used as the discount rate and it enables risk-neutral growth rates to be calculated. The economic rationale for using the risk-free rate is the riskless hedging argument presented in Black and Scholes (1973) and Merton (1973). This argument is reproduced in the appendix with general assumptions about the funding rate for a) the derivative and b) the instrument used to hedge the derivative.[2] It is sometimes assumed that risk-neutral valuation can be used only when delta hedging of the underlying risk is possible.[3] This is not so. Equilibrium arguments similar to those in the original Black and Scholes paper show that the Black-Scholes-Merton differential equation, and therefore risk-neutral valuation, applies whether or not hedging is possible. Indeed, many applications of derivatives pricing (for example, the real options approach to capital budgeting) are based on risk neutral valuation when the underlying risks cannot be hedged.

Prior to the credit crisis that started in 2007, dealers used interest rates calculated from LIBOR and LIBOR-swap rates to value derivatives. Post-crisis, most dealers have used overnight-indexed swap (OIS) rates to determine the applicable interest rates for fully collateralized

[1] See for example Ernst and Young (2012).
[2] See also Piterbarg (2010) for this type of extension of Black-Scholes-Merton.
[3] See for example Burgard and Kjaer (2011)

transactions.[4] This means that in practice OIS rates are used for most interdealer transactions because these transactions are almost invariably fully collateralized.

It is tempting to say that the interest rates used in the valuation of derivatives reflects the dealers' best estimate of the risk-free rate. In fact, this has never been the case. The "risk-free" rates used in derivatives valuation have always reflected funding costs. As mentioned earlier, prior to the crisis LIBOR and LIBOR-swap rates were used as "risk-free" rates. The reason was that LIBOR was a good approximation to the dealers' short-term funding costs and LIBOR-swap rates are the longer-term rates corresponding to continually refreshed LIBOR rates.[5] The current use of OIS discounting for fully collateralized transactions is based on the argument that fully collateralized transactions are funded by collateral. If this collateral is cash, the interest rate paid is usually the overnight federal funds rate and the OIS rate is a longer-term rate corresponding to a continually refreshed overnight federal funds rate.[6]

Consistent with their view that the rates used to value derivatives should reflect funding costs, derivatives dealers are advancing the argument that they should incorporate their funding costs into the valuation of non-collateralized transactions. This leads to what is referred to as a "funding value adjustment" (FVA).[7] The purpose of FVA is to move valuations from those given by the valuation models in a dealer's systems to those that incorporate the dealer's average funding costs. The precise definition of FVA therefore depends on a) the model in the dealer's systems and b) the model that incorporates funding costs. FVA is the difference between the valuations given by these two models.

[4] See Hull and White (2012a) for a discussion of OIS rates.

[5] See Collin Dufresne and Solnik (2001) for a discussion of the continually refreshed argument. For example, the five-year LIBOR-swap rate has a risk corresponding to 20 three-month loans made to qualifying borrowers, not to a single five-year loan.

[6] See Hull and White (2012a) for a discussion of this.

[7] This paper focuses on the funding required to finance an uncollateralized hedged derivatives position. As discussed in Hull and White (2012a), in the case of collateralized derivatives, the collateral provides the funding necessary for hedged derivative only under ideal circumstances where collateral is exchanged continuously and there is no threshold or independent amount. Initial margin requirements (i.e., independent amounts) are becoming more prevalent in collateralized trading and this leads to another potential source of FVA. As we will see, the FVA associated with the funding to finance an uncollateralized hedged position changes the trader's mid-market price. By contrast, the FVA associated with initial margins does not change the mid-market price, but does increase the bid-offer spread.

The model for non-collateralized transactions in the dealer's systems is likely to use either LIBOR or OIS as the "risk-free" rate. The model incorporating funding costs depends on how a) derivatives and b) the instruments used to hedge derivatives are assumed to be funded. One possibility is that they are both assumed to be funded at the dealer's average funding costs. Another possibility is that the instruments used for hedging are assumed to be subject to repo agreements and are therefore funded at the repo rate (which in practice is close to the OIS rate) and the derivative is assumed to be funded at the dealer's average funding cost.[8] The appendix shows how FVA-adjusted valuations depend on the rates assumed. When risk-neutral valuation is implemented to calculate the FVA-adjusted value of a derivative, the expected return on the underlying asset in a risk-neutral world is the funding costs of the asset underlying the derivative and the discount rate applied to the expected payoff is the funding cost of the derivative.

In recent years dealers have implemented a number of adjustments when calculating the value of a derivatives portfolio with a counterparty. The first step in valuing a derivative is to calculate the total no-default value (NDV) of the derivatives in the portfolio. This is the value assuming that neither party will default. A credit value adjustment (CVA) is then made to reflect the possibility that the counterparty will default. This is followed by a debit (or debt) value adjustment (DVA) to reflect the possibility that the dealer will default. A further adjustment may be necessary if transactions are collateralized and the interest rate paid on cash collateral is different from the assumed risk-free rate. This leads to the valuation of the portfolio being

$$\text{NDV} - \text{CVA} + \text{DVA} - \text{CRA} \qquad (1)$$

where CRA is a collateral rate adjustment reflecting the cost to the dealer arising from the interest paid on cash collateral being different from the discount rate.

The FVA adjustment is a further potential adjustment to NDV. If this adjustment is made, the value of the portfolio becomes

$$\text{NDV} - \text{CVA} + \text{DVA} - \text{CRA} - \text{FVA} \qquad (2)$$

[8] Derivatives cannot in general be repoed.

A key point here is that the FVA adjustment is not related to credit issues as these are taken into account in CVA and DVA.[9]

There is no theoretical basis for FVA. Making this point was the main purpose of Hull and White (2012b). Corporate finance theory, stretching back as far as Modigliani and Miller (1958), shows that the way a project is funded should not affect the way it is valued. It is the riskiness of the project that matters. This result applies to all projects including those that involve decisions to enter into hedged derivatives trades. The reason for the result, as we will explain in more detail in Section 3, is that the incremental impact of a project on the funding costs of company should always correspond to the riskiness of the project. The historical average funding costs of the company are irrelevant.

The purpose of this paper is to move a little away from the theory. The paper considers the tensions that exist between fair value accounting and dealer valuations and how they can be resolved. This paper shows that the motivation for the funding value adjustment is the way the performance of the derivatives desk is assessed by senior management. This is different from the way the performance of the derivatives desk is assessed for accounting purposes. The difference between the two values creates a number of problems and has a number of unintended consequences. An FVA has no theoretical basis. But we conclude that, if a dealer wants to make an FVA, the best practice is to use FVA-adjusted valuations only as a guide to calculating breakeven trading prices. It should not be used for marking to market or hedging purposes.

1. Performance Measurement

The funding value adjustment arises from a difference between the way derivatives are valued in the market and the way the activities of a derivatives desk are assessed by dealers. This section explains this point.

Much of finance theory assumes that decisions are driven by valuation models. If a new project has a positive net present value, it is undertaken; if it does not, it is not undertaken. But in

[9] Note that, because of netting agreements, CVA and DVA must be calculated for the whole portfolio of a dealer with a counterparty. NDV, CRA, and FVA can be calculated separately for each transaction.

practice for a financial institution, return on capital, the annual profit from the investment divided by the capital allocated to the project, is often the key metric when projects are considered.

In particular, the return-on-capital metric is usually used to measure performance of the derivatives activities of a financial institution. In this calculation the profitability of derivatives trading is measured as trading profits less expenses. The expenses include funding costs and other costs that are considered relevant. The funding costs are calculated by applying the dealer's average funding rate to the average funding used in derivatives trading.

An important distinction is between costs that change the mid-market price and costs that change the bid-offer spread while keeping the mid-market price the same. The costs of funding a derivative and costs associated with regulatory capital requirements change the trader's mid-market price. Administrative costs such as those associated with computer systems and trader salaries change bid-offer spreads.

We now present some simple examples illustrating how the funding cost affects the way traders calculate derivative prices.

Examples

Suppose that a dealer's client wants to enter a forward contract to buy a non-dividend paying stock in one year's time. Consider how a trader views this transaction. If she enters into a contract to sell forward, she will hedge the forward contract by buying the stock now so that she has it available to deliver one year from now. Her profit at the end of the year will be the delivery price less the current stock price compounded forward at the funding rate for a stock purchase. If the current stock price is 100 and her funding rate for equity purchases is 4%, the delivery price must be higher than 104 in order for the trader to earn a profit. The delivery price at which the trader is willing to sell in one year's time reflects the rate at which a position in the underlying can be funded.

The trader's funding cost also affects the discount rate that she uses. Suppose that the delivery price is set at 106. If the trader pays the counterparty X in order to enter into this forward contract and the funding rate for this payment is 5%, the year-end profit is $106 - 104 - 1.05X$. As a result the trader will pay no more than 2/1.05 = 1.905 to enter into this contract. The present value of the forward contract, the maximum amount the trader will pay, is determined by discounting the payoff on the contract at the rate at which a position in the derivative can be funded.

Putting this in standard derivative pricing terms, the interest rate that is used to determine the expected future payoff on the derivative is the rate at which positions in the underlying can be funded, and the discount rate is the rate at which a position in the derivative can be funded. In practice, it is usual to assume that the former is the repo rate[10] and the latter is the bank's cost of funds. If the derivative is bought or sold at this price the hedged portfolio earns enough to exactly pay all the funding costs.

The use of funding costs in determining prices applies to more complex transactions as well. Suppose that at the beginning of a year a trader buys a one-year uncollateralized European call option on a non-dividend paying stock with a strike price of 100. The stock price is 100 and the stock price volatility is 30%. Suppose further that the bank's funding cost is 5%, and the repo rate at which the stock can be financed is 2%, both quoted with continuous compounding. The trader delta hedges the option perfectly for one year. At the end of the year, the trader liquidates the remaining hedge portfolio (if any) and receives the payoff (if any) on the option. The proceeds of these two terminal transactions are exactly equal in size but of opposite sign.

The price of the FVA-adjusted option price, 12.44, is based on using the repo rate to calculate the expected return on the stock and then discounting the option payoff at the bank's cost of funds.[11] (With the notation in the appendix, r_s and r_d are 2% and 5% respectively.) If the trader buys the option for this price and earns 2% on the short stock position used to delta hedge the option while paying 5% on the funds used to purchase the option, the net profit on the trade will be zero. (We are making the idealized assumption that the pricing model assumptions are true, delta-hedging is implemented in such a way that it works perfectly, and assuming that the 30%

[10] The haircut applied to equity repos is between 5% and 10%. As a result the actual cost of financing the stock will be a weighted average of the repo rate and the bank's cost of funds. In order for FVA prices to seem reasonable it is necessary that it be possible to fund the underlying asset at an interest rate that is close to the riskless rate. The assumption that equity can be repoed achieves this objective. However, there will be cases, for example swap options, in which it is not possible to repo the underlying asset.

[11] In this example and all subsequent option pricing examples we are assuming that all the assumptions underlying the Black-Scholes-Merton model are correct. This ensures that all our hedging arguments work. In reality none of these assumptions is true. However, if the hedging arguments underlying FVA do not work in our idealized setting, it does not seem possible that they can work in a more realistic setting.

volatility is the actual stock price volatility.) If the purchase price is higher than 12.44, the trader's net profit will be positive. Note that the FVA-adjusted price is less than the price of a fully collateralized transaction. For a fully collateralized transaction (such as that typically seen in the interdealer market), both interest rates used in the option valuation are 2% and the resulting option price is 12.82.

Note that 12.44 is also the dealer's price if she is selling the option. In this case the option is being sold at a relatively low price. The dealer's justification for this is that the sale of the option provides funding. The amount of funds that have to be raised at 5% to accommodate other transactions is reduced as a result of this transaction.

If it is assumed that neither the underlying stock nor the derivative can be repoed, both r_s and r_d in the appendix are 5%. The option price then rises to 14.23. This shows that the FVA-adjusted option price is quite sensitive to funding assumptions.

There are a number of consequences of making a funding value adjustment. FVA causes dealers with different funding costs to have different prices. If all dealers can use the repo market to fund the hedge position at the same rate, then dealers with higher funding costs will always calculate lower prices (closer to zero) than dealers with lower funding costs. Dealers with low funding costs (high FVA-adjusted prices) will tend to be buyers of options and dealers with high funding costs (low FVA-adjusted prices) will usually be sellers of options.

Normally we might expect that, when 'production' costs are higher, a higher price must be charged for the product. However, a dealer with higher funding costs is willing to sell an option for a lower price than his competitor who has lower funding costs. The reason for this strange result is that the sale of the option is a source of funds and the two dealers disagree on the rate of return earned when these funds are invested. In each case the dealer assumes that the funds are invested to reduce their own outstanding debt, and the interest rate paid on the debt is different for the two dealers. It appears that the low funding cost dealer would be better off by investing the funds in the debt of the high funding cost dealer.[12]

[12] This raises many questions about the role of credit risk in all these calculations. Do you really expect to earn the promised rate of interest? If you expect to earn less than the promised rate then, based on the symmetry between borrower and lender, it seems that the borrower must be expecting to pay less than the promised rate, his funding

Regulations now require most transactions between dealers to be collateralized whereas those with end users need not be collateralized. Given this, FVA adjustments are likely to lead to the situation where end users when they want to buy options will get the best pricing from high-funding-cost dealers and, when they want to sell options, they will get the best pricing from low-funding-cost dealers. This may be a cause for concern because a dealer's derivatives book with end users will not be balanced between long and short positions.

A dealer's uncollateralized transactions with end users are sometimes hedged by entering into collateralized transactions with other dealers. Dealers who make funding value adjustments will not value the two transactions in the same way and will not regard them as perfect offsets for each other. If the extra funding costs for the non-collateralized transactions are believed to be an artifact of the performance measures used for traders, this is an undesirable by-product of FVA. But if the extra funding costs are believed to be real, the view that there is not a perfect offset is justifiable.

In spite of its impact on a firm's competitive position, FVA can be seen as nothing more than a rational response by traders to the incentives provided by their employers. It produces the trader's private valuation, the price that must be charged at inception in order for a hedged portfolio to earn enough to cover the assumed funding costs.

The private valuation can be calculated over the whole life of the transaction as well as at inception. However, the private valuation is in general different from the fair value required by accounting standards. We now explore this point.

2. Fair Value

SFAS 157 and IFRS 13 define the fair value as "the price that would be received to sell an asset or paid to transfer a liability in an orderly transaction between market participants at the measurement date."[13] Should two entities with different funding costs have different fair value estimates for the same asset? The answer is no. Consider two individuals, A and B. A can

rate. If the borrower is expecting to pay less than the promised rate is it appropriate for the derivatives desk to assume that the proceeds from the sale of the option earns the promised rate, the funding rate?

[13] See Financial Accounting Standards Board (2006) and International Accounting Standards Board (2011)

borrow money at 2% to buy IBM shares and B can borrow money at 6% to do the same thing. It is quite possible that borrowing costs will influence their decisions on whether to buy the shares. But A and B should agree that the fair value of the shares is their market price. This fair value may be different from the private values of A or B for the shares either because of their funding costs or for other reasons.

The fair value of an IBM share is the market price. This is the price that balances supply and demand. Similarly, the fair value of a derivative is the price that balances supply and demand. It is the price for which the number of market participants wanting to buy equals the number of market participants wanting to sell. Those market participants who want to buy at the market price presumably have a private value for the derivative that is higher than the market price. Those who want to sell at the market price presumably have a private value that is lower than the market price.

In practice, there are many reasons why different market participants might have different private values for the same derivative transaction. Funding costs (rightly or wrongly) may be one reason. Other potential considerations are liquidity constraints, regulatory capital constraints, the bank's ability to hedge the underlying risks, other similar transactions in the dealer's portfolio, and so on. The key point here is that there can be a number of different private values for a derivative, but there is only one market price or fair value.

Calculation of Fair Value

Accounting standards distinguish between Level 1, Level 2, and Level 3 estimates of fair value. Level 1 estimates are based on quoted prices for identical assets or liabilities in active markets. Level 2 estimates are usually based on quoted prices for similar assets or liabilities in active markets. Level 3 estimates usually involve situations where no similar assets or liabilities trade. Level 1 valuations do not require a model. Level 2 valuations require a model to interpolate between the prices of actively traded instruments to determine the value of an instrument that is not actively traded. Level 3 valuations require a model to incorporate the underlying assumptions on which the valuation is based.

Nearly all derivatives valuations are Level 2. For example JPMorgan Chase reported that about 98% of its derivatives valuations were Level 2 in 2011. Derivatives pricing models are therefore nearly always used to determine a price for a derivative that is consistent with other instruments that trade actively.

If the price of a derivative cannot be observed in the market, because the valuation models are calibrated to market prices, two different dealers who have the same information and use the same models should agree about the fair value. In the case of options, the Black-Scholes-Merton model (or one of its extensions) is used to calculate the fair price using the following procedure:

1. In the calibration stage, the model is used to calculate implied volatilities from other similar options that trade actively on the same asset.
2. Interpolation procedures are then used to estimate the implied volatility corresponding to the strike price and maturity of the option under consideration.
3. The model is then used to price this option using the interpolated volatilities.

It does not matter whether the Black-Scholes-Merton model reflects the behavior of the underlying asset price because the model is used as nothing more than a sophisticated interpolation tool. It is used first to calculate implied volatilities and then to value the option. For other derivatives, the calibration process might be different but the overall approach to calculating fair market values is the same.

When CVA, DVA and CRA are calculated for equation (1), dealers who have the same information and same models should agree fair market prices. In particular, this is true of the two parties to the transaction since a) the no-default value of the portfolio to one party is the negative of its no-default value to the other, b) the counterparty's CVA is the dealer's DVA and vice versa, and c) the collateral interest paid by the dealer is the interest received by the counterparty so that the counterparty's CRA is equal in magnitude and opposite in sign to the dealer's CRA.

In this case, although everyone agrees on the valuation, the valuation is specific to the pair of entities involved in the transaction because different entities have different CVAs and DVAs. It may appear that this is a violation of the accounting standards' definition of fair value since, if one party novates a transaction to a new counterparty, the price at which the transaction is done will reflect the credit risk of the new counterparty. However, this is not so when all costs and benefits are taken into account.

Consider the situation where there is a single transaction between A and B. From A's point of view the no default value is 100, CVA is 5, DVA is 10 and CRA is zero. The value to A is 105 and the value to B is –105. Now suppose that A novates the transaction to C, a firm with no credit risk. C's DVA is zero so the value of the transaction to C is 95. It appears 10 of value has been lost. After novation, the value to A is 95 and B is –95. However, because B makes a gain of 10, it should at least in principle, be willing to pay A 10 when A announces that it is novating the transaction to C. Assuming this payment happens, A receives 95 from C and 10 from B. B pays 10 to A and ends up with a transaction worth –95. The original valuations (105 to A and –105 to B) are preserved.

In passing, we note that a troubling aspect of FVA is that it results in different market participants having different estimates of the fair value, even when they are using the same models and the same market data. Consider equation (2) which includes FVA. If the dealer and the counterparty have the same funding costs, the dealer's FVA is equal in magnitude and opposite in sign to the counterparty's FVA and both continue to agree on the fair value. However, if the dealer and the counterparty have different funding costs they no longer agree on the fair value.

3. Theoretical Arguments

The heart of the FVA debate flows from the procedures used to measure the performance of the derivatives desk. Theoretical arguments, which are covered in more detail in Hull and White (2012b, 2012c), show that funding costs should not influence estimates of market value.

The evaluation of an investment should depend on the risk of the investment, not how it is financed. This can be quite difficult to accept. Suppose a bank is financing itself at an average rate of 4.5% and the risk-free rate is 3%. Should the bank undertake a risk-free investment earning 4%? The answer is that of course it should accept the investment. Because the investment is risk-free, its cash flows should be discounted at the risk-free rate and when this is done the investment has a positive value.

It appears that the bank is earning a negative spread of 50 basis points on the investment. However, the incremental cost of funding the investment should be the risk-free rate of 3%. As the bank enters into projects that are risk-free (or nearly risk-free) its funding costs should come down. Let us take an extreme example and assume that the bank we are considering doubles in size by undertaking entirely risk-free projects. This should lead to the bank's funding cost changing to 3.75% (an average of 4.5% for the old projects and 3% for the new projects), showing that the incremental funding cost associated with the new projects is 3%.

This argument does not usually cut much ice with practitioners because it seems far removed from reality.[14] They argue that because of the opaqueness of banks, investors cannot correctly assess the risk of the institution or how it changes as a result of new investment. This is probably correct, but the requirements underlying the theory may be much weaker than practitioners believe. Investors may be wrong in their assessment of the risk. However, all that is required is that investors do not systematically over- or under-estimate the risk and that managers cannot distinguish between the situations in which the investors over- and under-estimate the risk. In this case the investors are correct on average and management does not know when the investors are getting it wrong. As a result, management should assume that the investors are getting it right.

In practice, what usually happens is that its average cost of funding remains approximately the same through time. This average cost of funding is, in the opinion of its investors, presumably matched by the average risk of the projects the bank undertakes. Risk-free projects enable riskier-than-average projects to be taken elsewhere in the bank so that the overall risk of the bank's portfolio remains approximately the same. Why not then use the same discount rate for all projects? The answer is that this is liable to have dysfunctional consequences. It makes riskier-than-average projects seem more attractive than they should be and risk-free (or almost risk-free) projects unattractive.

[14] A related argument is that in the absence of taxes and bankruptcy the amount of capital that a bank has should not matter to the shareholders. Shareholders in a bank with more capital will be happy with a lower rate of return because the risk they bear is also lower. See Admati (2010). This argument is also not well received by practitioners

The argument we have just given seems to be at least partially accepted by financial institutions. Banks and other financial institutions do undertake very-low-risk investments such as those in government securities even though they know the return is less than their average funding costs. For our purpose, the key point about this argument is that it shows that funding costs should in theory be irrelevant in the valuation of any investment, risk-free or otherwise. To make the point that funding costs are irrelevant to the pricing of derivatives, we can use this result in conjunction with the fact that the hedged transactions are riskless to show that the correct incremental funding rate for hedged derivatives should be the risk-free rate.

4. Complications in Option Pricing Arising from FVA

In Section 1 we discussed some of the consequences of making a funding value adjustment. The basic point being made there is that when dealers have different private valuations for a derivative it leads to particular patterns in trading and may lead to risk concentrations. In this section we consider some other complications that apply specifically to option trading.

The main reference point for valuing derivatives is the prices of actively traded instruments in the interdealer (fully collateralized) market. As we have explained, there seems to be general agreement that the "risk-free" rate used to determine valuations in this market should be the OIS rate. Furthermore CVA and DVA are very small in this market so that the market provides good direct estimates of the no-default value of derivatives. Implied volatilities can be calculated from these no-default values. When fully collateralized transactions that do not trade actively are valued the interpolation procedure described in Section 2 can be used.

When the no-default values of non-collateralized transactions are calculated, it is usual to use implied volatilities from the interdealer (fully collateralized) market. If the no-default value of a non-collateralized transaction is calculated in the same as that of a similar collateralized transaction there is no problem in doing this. However, if funding value adjustments are made, the models used to value non-collateralized transaction involves different interest rates from the models used to value collateralized transactions. Using implied volatilities from the collateralized market to value options in the non-collateralized market is then valid only if it can be argued that implied volatilities are the same in the two markets.

Consider two options with the same strike price and time to maturity, one collateralized and the other non-collateralized. Suppose first that both options are European and that the funding cost of the asset underlying the options is assumed to be the same for both options. The expected risk-neutral payoff for both options is then the same and it can be shown that, regardless of whether the Black-Scholes-Merton assumptions apply, both options will have the same implied volatility.[15] Using volatilities implied from collateralized options to price a non-collateralized option is then a valid procedure. However, if the options are American the procedure is imperfect because the early exercise decision does depend on the interest rate. Furthermore, if the underlying asset cannot be repoed so that the expected risk-neutral payoff is not the same for both options, the procedure is liable to work quite badly.

5. Best Practice

The central issue in the FVA debate is that a single price cannot serve two purposes. It cannot both reflect the traders funding costs and be consistent with market prices. One solution is for traders to try and convince accountants that the FVA-adjusted prices should be used as fair values. But they are unlikely to be successful. In general, the FVA-adjusted prices of any given dealer will be markedly different from prices in the market.

Many banks seem content to use two models, one for calculating the value of a portfolio to a dealer and another for calculating fair values. This is may be a viable approach, but it has the disadvantage that it is liable to lead to the situation where the internal performance measure is out of line with the results reported in the company's financial statements.

One way of resolving the difference between trader prices and fair values is to change the way the performance of the derivatives desk is assessed. If funds were charged to the derivatives desk at the OIS rate rather than at the average funding costs, traders would have no incentive to make an FVA. However, this is likely to be viewed as unsatisfactory because it involves treating the derivatives group differently from other groups with a bank. Also the treasury department (which is often a profit center) is likely to complain that it incurs a loss by raising funds at one rate and passing them on to the derivatives group at a lower rate.

[15] Increasing the discount factor multiplies both a Black-Scholes price and the market price model by the same factor so that the implied volatility remains the same.

The best-practice solution may well be the following. A dealer should use two models when trading on an uncollateralized basis. Model A should be used at inception for calculating trading prices. Model B should be used for calculating mark-to-market prices and hedging. Model A should include all the costs the trader needs to cover. These costs might include average funding costs, administrative costs, and regulatory capital costs. Model B should be the market model. Assuming that fully collateralized interdealer prices determine market prices, the interest rates in Model B will be calculated from OIS rates.

Although the trader uses Model B for hedging, the overall profit made by the trader on a transaction if it is held to maturity will reflect Model A prices. The trader will make an inception profit or loss reflecting to the difference between Model A and Model B prices. This profit or loss covers the impact of the dealer's average costs over the life of the trade. For the purposes of calculating the trader's performance, it may be desirable to amortize the profit or loss over the life of the transaction.

6. Conclusions

The funding value adjustment is really a transfer pricing problem caused by charging the derivatives desk a funding rate that is different from the funding rate implied by market prices. In determining the price at which she is willing to trade the trader considers many things. Among them are the various costs that will be allocated to the trade including operational expenses. In most markets these costs lead to a bid-offer spread. However, the funding costs in uncollateralized markets are potentially different from most other costs because they change the level of the price.

An FVA incentivises the derivatives desk to sell derivatives at lower prices if the funding cost is high. This means that the internal measure of the value of derivatives is nearly always different from the fair market value. Also, the effectiveness of hedging in an environment in which the dealer chronically assumes that the value of the derivative is different from its actual fair market value is open to question.

Much of the discussion around funding value adjustment has been driven by technical analyses designed to show that a price adjustment is necessary if the trading desk is to earn some target funding rate. However, the real issues are managerial. To quote KPMG, funding related valuation is "….a topic that evokes questions about transfer pricing, steering of risk and, most importantly, the business model of each bank."[16]

It is important for management to choose incentives which encourage the derivatives desk to trade in the best interests of their shareholders. The best alternative would seem to be to determine trading prices using a model that incorporates all costs that are considered relevant for trading and to use the market model for both marking to market and hedging.

[16] See KPMG (2011).

References

Admati, Anat, Peter DeMarzo, Martin Hellwig, and Paul Pfleiderer (2010), "Fallacies, Irrelevant Facts, and Myths in Capital Regulation: Why Bank Equity is *Not* Expensive" Working Paper, Stanford University.

Black, Fischer and Myron Scholes (1973), "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy*, Vol. 81, No. 3, May-June, pp. 637–654

Burgard, Christoph and Mats Kjaer (2011), "In the Balance," *Risk*, November, 72–75.

Collin-Dufresne, Pierre and Bruno Solnik (2001), "On the Term Structure of Default Premia in the Swap and Libor Market," *The Journal of Finance*, 56, 3 (June), 1095-1115.

Ernst and Young (2012) "Reflecting Credit and Funding Value Adjustments in Fair Value. Insight into Practices: A Survey"

Financial Accounting Standards Board (2006), "Fair Value Measurements," (September).

Hull, John and Alan White (2012a), "LIBOR vs. OIS: the Derivatives Discounting Dilemma," forthcoming in *Journal of Investment Management*.

Hull, John and Alan White (2012b), "The FVA Debate" *Risk*, 25th anniversary edition, July, 83-85

Hull, John and Alan White (2012c), "Collateral and Credit Issues in Derivatives Pricing" Working Paper, University of Toronto.

International Accounting Standards Board (2011) "IFRS 13 Fair Value Measurement," (May).

KPMG, "New Valuation and Pricing Approaches for Derivatives in the Wake of the Financial Crisis," October 2011.

Merton, Robert (1973), "Theory of Rational Option Pricing," *Bell Journal of Economics and Management Science*, 4, Spring, pp. 141–183.

Modigliani, F. and M. Miller, "The Cost of Capital, Corporation Finance and the Theory of Investment," *American Economic Review*, 48, 3, (1958), 261-297.

Piterbarg (2010) ‘Funding beyond Discounting: Collateral Agreements and Derivatives Pricing” *Risk*, February, 97-102.

Appendix

Black-Scholes with Funding Costs

This appendix presents the Black and Scholes (1973) and Merton (1973) arguments for valuing a derivative dependent on a non-dividend-paying stock when interest rates are constant and funding costs are considered. We allow the interest rate at which the derivative is funded to be different from that at which the stock is funded. This is necessary because assets such as a stock can often be funded using repo agreements whereas derivatives cannot normally be repoed.

The process assumed for the stock price, S, is

$$dS = \mu S dt + \sigma S dz \tag{A1}$$

where μ is the expected return on the stock, σ is its volatility, and dz is a Wiener process. An application of Ito's lemma shows that the price of the derivative satisfies

$$\begin{aligned} df &= \mu_f f dt + \sigma S \frac{\partial f}{\partial S} dz \\ \mu_f &= \frac{1}{f}\left[\frac{\partial f}{\partial t} + \mu S \frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2}\right] \end{aligned} \tag{A2}$$

We now consider a portfolio consisting of a short position in the derivative and a position of $\partial f / \partial S$ in the stock. The portfolio value is Π

$$\Pi = -f + \frac{\partial f}{\partial S} S$$

Suppose that the derivative is funded at r_d and the stock is funded at r_s (where both rates are continuously compounded). Because the portfolio is risk-free the change in the value of the portfolio in time Δt is

$$\Delta \Pi = -r_d f + r_s \frac{\partial f}{\partial S} S \tag{A3}$$

Using equations (A1) and (A2) the change in the portfolio value is also

$$\Delta\Pi = -\left[\frac{\partial f}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2}\right]\Delta t \tag{A4}$$

Combining equations (A3) and (A4) leads to the differential equation

$$\frac{\partial f}{\partial t} + r_s S \frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = r_d f$$

The risk-neutral valuation argument shows that the solution of this differential equation is obtained by assuming that the expected return on the stock is r_s and then discounting the expected payoff at r_d . For a derivative that provides a payoff only at time T

$$f_0 = e^{-r_d T}\hat{E}(P)$$

where f_0 is the derivative price today (time zero), P is the payoff at time T, and $\hat{E}$ denotes expectations in a world where the expected growth rate of the stock price is r_s. The value of a European call option on the stock with strike price K and time to maturity T is

$$S_0 N(d_1) e^{(r_s - r_d)T} - K e^{-r_d T} N(d_2)$$

where

$$d_1 = \frac{\ln(S_0 / K) + (r_s + \sigma^2/2)T}{\sigma\sqrt{T}}$$
$$d_2 = d_1 - \sigma\sqrt{T}$$

These results can be extended to derivatives dependent on assets other than non-dividend-paying stocks.