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The structure of factor content predictions

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ABSTRACT

The last decade witnessed an explosion of research into the impact of international technology differences on the factor content of trade. Yet the literature has failed to confront two pivotal issues. First, with international technology differences and traded intermediate inputs, there is no existing definition of the factor content of trade that is compatible with Vanek's factor content prediction. We fill this gap. Second, as Helpman and Krugman (1985) showed, many models beyond Heckscher-Ohlin imply the Vanek prediction. Thus, absent a complete list of these models, we do not fully know what models are being tested when the Vanek prediction is tested. We completely characterize the class of models being tested by providing a familiar consumption similarity condition that is necessary and sufficient for a robust Vanek prediction. Finally, we reassess the performance of the prediction using the correct factor content definition and input–output tables for 41 countries. We find that the prediction performs well except for the presence of missing trade. Further, missing trade is not pervasive: it is associated entirely with 'home bias' in the consumption of agricultural goods, government services and construction.

Vanek's (1968) factor content of trade prediction states that each country is a net exporter of the services of its abundant factors. More specifically, a country's factor content of trade vector (F) is predicted by a linear function of the country's endowment vector (V) and its share of world consumption (s):

$$F = V - sV_w \tag{1}$$

where V_w is the world endowment vector. Assuming that data are measured without error, there are only two reasons why empirical tests would reject the Vanek prediction. First, there are many ways of defining the factor content of trade and the particular definition of F used may not be Vanek-relevant i.e., may not satisfy Eq. (1). Second, the set of assumptions used to derive the prediction (i.e., the model) may be 'false.' Most researchers would agree that we thus have a complete understanding of how to implement and interpret empirical tests of the Vanek prediction. In contrast, we will argue that our understanding of both reasons for rejection is far from complete. First, we do not know what the Vanek-relevant definition of F is for the case where there are traded intermediate inputs and international differences in the choice of techniques. This is exactly the case that has received the most intense empirical scrutiny. See Trefler (1993, 1995), Davis et al. (1997), Davis and Weinstein (2001), Hakura (2001), Antweiler and Trefler (2002), Conway (2002), Debaere (2003), Reimer (2006), and Maskus and Nishioka (2009). We provide the correct, Vanek-relevant, definition. This definition is what Deardorff (1982) calls the 'actual' factor content of trade. Second, given that the Vanek prediction is implied by many models (e.g., Helpman and Krugman, 1985), we do not know which model or models are being rejected. We provide a complete characterization of the class of models implying and implied by the Vanek prediction. This tells us exactly which models are being rejected.

Consider the first of the above reasons for rejecting the Vanek prediction. Researchers routinely and mistakenly use factor content definitions that are not Vanek-relevant. For example, Davis and Weinstein (2001) allow for international choice-of-technique differences and traded intermediates, yet they use a definition of F that only equals $V - sV_w$ when there are no traded intermediates. A definitional mistake is also made in Trefler and Zhu (2000). Section 6 shows that definitional mistakes are endemic.[1]

To understand the problem consider the factor content of Chinese imports of U.S. machinery built with Brazilian steel. In a world of international choice-of-technique differences, assessing the factor content of Chinese machinery imports requires one to keep track of the factor content of U.S. steel imports from Brazil. The more countries

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[1] As Davis and Weinstein (2003, page 129) note, "understanding how to incorporate traded intermediates into factor content studies remains an important area for future research." Harrigan (1997, page 492) and Feenstra (2004, page 55) echo this call for more research. The debate about theory-relevant definitions of the factor content of trade originates with Deardorff (1982), Hamilton and Svensson (1983) and Staiger (1986).

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there are, the more there is to track. Reimer (2006) provides a tracking algorithm for the case of two countries. We provide a completely general algorithm. This algorithm generates what Deardorff (1982) calls the 'actual' factor content of trade. We then show that this is the Vanek-relevant definition of the factor content of trade.

The algorithm we propose has important implications for studying vertical production networks, as in Feenstra and Hanson (1996, 1999), Hummels et al. (2001) and Yi (2003). The algorithm explains exactly how to calculate trade in intermediates and how to evaluate value-added in intermediate trade in a completely general setting. It is thus useful for the empirical literature on trade in vertical production networks.

The second reason why empirical tests might lead to a rejection of the Vanek prediction is that the model is wrong. There was a time when 'the' model in question was the Heckscher-Ohlin model. We now know that there are a very large number of models that imply the Vanek prediction. In 1979, Deardorff opened up the possibility of a factor content prediction without factor price equalization and by 1985 Helpman and Krugman were able to derive the Vanek prediction under a variety of assumptions about increasing returns and imperfect competition. This leads to an important question. When one is testing the Vanek prediction, just how large a class of models is one examining? A primary contribution of this paper is to completely characterize the class of models that imply and are implied by the Vanek prediction. We do this using a familiar 'consumption similarity' condition.

Let C_{gij} be country i's consumption of final good g produced in country j and let C_{gwj} be world consumption of this good. Consumption similarity is said to hold if there are a set of scalars s_i such that

$$C_{gij} = s_i C_{gwj} \text{ for all } g, \ i, \text{ and } j. \tag{2}$$

Consumption similarity means that each country consumes a proportion s_i of all the final goods produced by all countries. This will be familiar from models with taste for variety (e.g., a variety is a good g produced in a location j), and models with production specialization (g is produced by only one country).[2]

Our core theorem is as follows. We consider the class of models with the following common properties. (i) Each country is endowed with a fixed supply of at least two factors. (ii) Factors are mobile across firms and industries within a country, but immobile across countries. (iii) Factor markets are perfectly competitive. (iv) Factor supply equals factor demand. (v) There are arbitrary international differences in choice of techniques. Our core theorem states that for models with these properties, consumption similarity is necessary and sufficient for the Vanek prediction. Restated, Eqs. (1) and (2) are equivalent.

There are two aspects of our core theorem that are potentially confusing. First, the theorem applies to the case of arbitrary international differences in choice of techniques. If only limited international differences are allowed (e.g., Trefler 1993 or Davis and Weinstein 2001) then Eq. (2) must be replaced by a weaker form of consumption similarity. For example, if all countries use the same choice of techniques, as in the standard Heckscher-Ohlin model, then Eq. (2) must be replaced by $\Sigma_j C_{gij} = s_i \Sigma_j C_{gwj}$ for all g and i. That is, it must be replaced by the standard Heckscher-Ohlin notion of consumption similarity. More generally, in Section 3.1 we show how easy it is to extend our core theorem to cases with only limited forms of international choice-of-technique differences.

The second potentially confusing point about our core theorem is that Eq. (2) is not quite necessary for Vanek predictions. Rather, it is necessary for 'locally robust' Vanek predictions. These are predictions that are not sensitive to minor perturbations of technology. For example, suppose good g in country j is produced with labour and capital using Cobb-Douglas production functions with α_{gj} as labour's cost share. A Vanek prediction is locally robust if when it holds for some α_{gj}^0, it also holds for all α_{gj} in the arbitrarily small interval $(\alpha_{gj}^0 - \varepsilon, \alpha_{gj}^0 + \varepsilon)$. Local robustness is implicit in all of the literature.[3]

A powerful and unusual feature of our core theorem is that it makes no assumptions about preferences or market structure in product markets (e.g., perfect competition vs. monopolistic competition). The proof thus holds for all types of product market structure without us having to handle each type on a case-by-case basis. To our knowledge, this kind of proof has not previously appeared in the international trade literature.

We conclude this paper by reassessing the Vanek prediction using the Vanek-relevant factor content definition and input–output tables for 41 countries. We find that the prediction performs well. However, missing trade continues to be a problem. Digging deeper, we find that missing trade is not pervasive across goods. Rather, it is associated with departures from consumption similarity for agricultural goods, processed food, government services and construction. We show that these four industries alone explain all of the remaining missing trade. This is related to results by Davis and Weinstein (2001). They 'fix' the missing-trade problem by inflating trade flows using a gravity equation. We inflate trade flows by using our theory i.e., by imposing consumption similarity. However, unlike Davis and Weinstein who impose the gravity equation on *all* industries, we impose consumption similarity on only four industries.

The paper is organized as follows. Sections 1–2 provide a Vanek-relevant definition of the factor content of trade. Sections 3–4 completely characterize the class of models implying and implied by a robust Vanek prediction. Sections 5–6 review previous empirical work in light of our theoretical findings and section 7 presents new empirical results.

1. Setup

Let $g = 1,...,G$ index goods, let i and $j = 1,...,N$ index countries, and let $f = 1,...,K$ index factors.[4] Let V_i be the $K \times 1$ vector of country i endowments, let $V_w \equiv \Sigma_i V_i$ be the world endowment vector, and let F_i be the $K \times 1$ vector giving the factor content of trade for country i . Let s_i be country i's share of world consumption, where $s_i > 0$ for all i and $\Sigma_i s_i = 1$.[5] The object of analysis is the Vanek factor content of trade prediction, $F_i = V_i - s_i V_w$. F_i will be fully defined in the next section.

Every good is consumed as a final product and/or used as an intermediate input. Let C_{ij} be a $G \times 1$ vector denoting country i consumption of goods produced in country j. Let Y_{ij} be a $G \times 1$ vector denoting i's usage of intermediate inputs produced in country j. Country j's output Q_j is split between consumption and intermediate inputs:[6]

$$Q_j \equiv \Sigma_i \left(C_{ij} + Y_{ij} \right). \tag{3}$$

World consumption of goods produced in country j is

$$C_{wj} \equiv \Sigma_i C_{ij}. \tag{4}$$

Let $B_{ij}(g,h)$ be the amount of intermediate input g used to produce one unit of good h, where g is made in country i and h is made in

[2] As will become clear, we are not interested in deriving Eq. (2) from assumptions on primitives. This paper is only about observable variables such as consumption C_{gij}. However, if we were to derive Eq. (2) from primitives we would need internationally identical and homothetic preferences as well as internationally identical product prices. The last requires free and frictionless trade in all goods.

[3] Local robustness is defined more precisely in sections 3.2 and 4. It is actually a much weaker requirement than the already weak requirement just described.

[4] We do not use $f = 1,...,F$ in order to reserve F for the factor content of trade.

[5] s_i is measured in the usual way as gross domestic product (GDP) less the value of the trade surplus, all divided by world gross GDP. This is the only adjustment for trade imbalances that is needed. We do not define s_i more precisely since it will not be necessary. A more precise definition would require a statement about which prices are used to evaluate GDP. See also footnote 2 above.

[6] We are assuming that there is no investment. This is dealt with theoretically in Trefler (1996) and empirically in Trefler and Zhu (2000, page 146).

country j.[7] Let $Q_j(h)$ be a typical element of Q_j. Then $B_{ij}(g,h)Q_j(h)$ is the amount of input g used to produce $Q_j(h)$ and $\Sigma_h B_{ij}(g,h)Q_j(h)$ is the amount of input g used by country j. Restated, $\Sigma_h B_{ij}(g,h)Q_j(h)$ is the gth element of Y_{ji}. In matrix notation,

$$Y_{ji} = B_{ij}Q_j \tag{5}$$

where B_{ij} is the $G\times G$ matrix with typical element $B_{ij}(g,h)$.

Let D_i be the $K\times G$ matrix whose (f,g) element gives the amount of factor f used directly to produce one unit of good g in country i. To ensure that factors are fully employed, we assume that D_i satisfies

$$D_iQ_i = V_i. \tag{6}$$

Eqs. (5) and (6) are best viewed as data identities that (partly) define B_{ij} and D_i.

Country i's vector of imports from country j is $M_{ij} \equiv Y_{ij} + C_{ij}$ for $j\neq i$. From Eq. (5), M_{ij} may alternatively be defined as

$$M_{ij} \equiv B_{ji}Q_i + C_{ij} \text{ for } j\neq i. \tag{7}$$

Country i's vector of exports to the world is $X_i \equiv \Sigma_{j\neq i}M_{ji} = \Sigma_{j\neq i}(Y_{ji} + C_{ji}) = \Sigma_j(Y_{ji} + C_{ji}) - Y_{ii} - C_{ii}$. Hence, from Eqs. (3) and (5), X_i may alternatively be defined as

$$X_i \equiv Q_i - B_{ii}Q_i - C_{ii}. \tag{8}$$

This completes the definition of the variables that we will use.[8]

2. The factor content of trade

In this section we define the factor content of trade in a way that satisfies $F_i = V_i - s_iV_w$. Verbally, F_i will be the factors employed *worldwide* to produce country i's net trade flows, as in Deardorff's (1982) 'actual' factor content of trade. Mathematically, we need to construct a regional input–output model of the world economy where each region is a country. This will allow us to track the movement of intermediate inputs across countries. At the heart of the regional input–output model is the $NG\times NG$ matrix

$$B \equiv \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1N} \\ B_{21} & B_{22} & \cdots & B_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ B_{N1} & B_{N2} & \cdots & B_{NN} \end{bmatrix}.$$

The off-diagonal sub-matrices track the requirements for foreign intermediate inputs.

We will need the $NG\times N$ matrices

$$Q \equiv \begin{bmatrix} Q_1 & 0 & \cdots & 0 \\ 0 & Q_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & Q_N \end{bmatrix}, \quad C \equiv \begin{bmatrix} C_{11} & C_{21} & \cdots & C_{N1} \\ C_{12} & C_{22} & \cdots & C_{N2} \\ \vdots & \vdots & \ddots & \vdots \\ C_{1N} & C_{2N} & \cdots & C_{NN} \end{bmatrix}, \quad \text{and}$$

$$T \equiv \begin{bmatrix} X_1 & -M_{21} & \cdots & -M_{N1} \\ -M_{12} & X_2 & \cdots & -M_{N2} \\ \vdots & \vdots & \ddots & \vdots \\ -M_{1N} & -M_{2N} & \cdots & X_N \end{bmatrix}.$$

Then Eqs. (7) and (8) can be written compactly as

$$T = Q - BQ - C. \tag{9}$$

This is just the fundamental input–output identity, usually written as $Q = BQ + C + T$ or $Q = (I - B)^{-1}(C + T)$. This famous identify has the following interpretation. Q is referred to as 'gross' output and $C + T$ as 'net' output (or final demand). Let Z_i be an arbitrary net output vector for some country i. Z_i is $NG\times 1$, reflecting the fact that we must keep track not only of what intermediate inputs were used to produce Z_i, but also of where these inputs came from. BZ_i is the vector of intermediate inputs directly needed to produce Z_i. Further, $B(BZ_i) = B^2Z_i$ is the intermediate inputs directly needed to produce BZ_i. Less abstractly, a sports car consumed in country i (an element of Z_i) requires steel (an element of BZ_i) which requires iron (an element of B^2Z_i) and so on. Thus, $\Sigma_{n=1}^{\infty}B^nZ_i$ is the matrix of intermediate inputs directly and indirectly needed to produce Z_i. Turning from intermediate requirements to total requirements, delivering net output Z_i requires gross output of $Z_i + \Sigma_{n=1}^{\infty}B^nZ_i = \Sigma_{n=0}^{\infty}B^nZ_i = (I - B)^{-1}Z_i$. This is Leontief's famous contribution.

Armed with Leontief's insight, we can now define the factor content of any vector Z_i. Let

$$D \equiv [D_1 \quad D_2 \quad \cdots \quad D_N]$$

be the $K\times NG$ matrix of direct factor requirements. Also, define $A \equiv D(I - B)^{-1}$ where I is the $NG\times NG$ identify matrix. Then AZ_i is the factor content of Z_i i.e., the amount of factors employed worldwide to produce any net output vector Z_i. Finally, let T_i be the ith column of T. Then $F_i \equiv AT_i$ is the factor content of country i's trade.

Theorem 1. *Assume that $(I - B)$ is invertible and define $A \equiv D(I - B)^{-1}$. Then*

$$F_i \equiv AT_i \tag{10}$$

is the factor content of country i's trade. Specifically, F_i gives the amount of factors employed worldwide to produce country i's net trade vector T_i.

With the exception of Reimer (2006) for the two-country case, no other empirical researcher has ever defined the factor content of trade as in theorem 1, i.e., using *worldwide* factor usage. Remarkably, Deardorff (1982) proposed this definition long ago. As will be seen, F_i is the Vanek-consistent definition of the factor content of trade i.e., $F_i = V_i - s_iV_w$. Section 6 explains precisely how our definition improves on those used by previous researchers.

3. Sufficiency, necessity and local robustness

We next turn to completely characterizing the class of models that imply and are implied by the Vanek prediction $F_i = V_i - s_iV_w$. We begin with a preliminary lemma. We partition A as $A = [A_1 \ A_2 \cdots A_N]$ where A_i is a $K\times G$ matrix.

Lemma 1. *$F_i = (V_i - s_iV_w) - \Sigma_jA_j(C_{ij} - s_iC_{wj})$ $\forall i$. Restated, $\Sigma_jA_j(C_{ij} - s_iC_{wj}) = 0$ is necessary and sufficient for a Vanek prediction.*

[7] For all variables in this paper with two subscripts, the first is the user and the second is the producer. The exception is B_{ij} where the first subscript is the producer (of the input) and the second is the user. This is confusing, but will help later with the matrix notation.

[8] For readers who prefer to work with factors measured in productivity-equivalent units as in Trefler (1993), there is a trivial transformation of the model. Let Λ_i be a $K\times K$ diagonal matrix whose typical diagonal element is the productivity of factor f in country i. Let $D_i^* \equiv \Lambda_iD_i$ and $V_i^* \equiv \Lambda_iV_i$ be factor inputs and endowments, respectively, measured in productivity-equivalent units. In our set-up the only variables involving factors and factor endowments are D_i and V_i. Further, these variables only appear in Eq. (6). Pre-multiplying Eq. (6) by Λ_i yields $D_i^*Q_i = V_i^*$. Thus, in Trefler's productivity equivalence case, (D_i^*, V_i^*) replaces (D_i, V_i) throughout our analysis and the Λ_i are never seen again. Thus, there is no point in carrying the Λ_i around in this paper.

Lemma 1 establishes a relationship between the Vanek prediction $F_i = (V_i - s_i V_w)$ and the factor content of consumption patterns $\Sigma_j A_j (C_{ij} - s_i C_{wj})$.

The proof of Lemma 1 is straightforward. $\Sigma_j A_j C_{ij}$ is another way of writing AC_i where C_i is country i's $NG \times 1$ consumption vector (column i of C). Likewise, $\Sigma_j A_j C_{wj}$ is just another way of writing AC_w where $C_w \equiv \Sigma_i C_i$. This is useful because AC_i and AC_w are the factor contents of consumption for country i and the world, respectively. With this fact in hand, re-write the equation in Lemma 1 as

$$F_i - (V_i - AC_i) = s_i (AC_w - V_w). \tag{11}$$

We will prove that this equation must always hold by proving that each side equals zero. Consider the left-hand side. The factor content of trade (F_i) is always equal to the difference between the factor content of production (V_i) and the factor content of consumption (AC_i). Thus, the left-hand side of Eq. (11) always equals zero. That the right-hand side of Eq. (11) equals zero follows from the fact that the factor content of world production (V_w) must equal the factor content of world consumption (AC_w). We have now established that each side of Eq. (11) must always equal zero. This proves Eq. (11) and Lemma 1.[9]

3.1. Sufficiency

We now address the question of which models imply the Vanek prediction $F_i = V_i - s_i V_w$. An immediate consequence of Lemma 1 is the following.

Theorem 2. (*Sufficiency*): $C_{ij} = s_i C_{wj}\ \forall\ i$ and $j \Rightarrow F_i = V_i - s_i V_w\ \forall i$.

To interpret this sufficient condition, introduce g subscripts to denote elements of the $G \times 1$ vectors C_{ij} and C_{wj}. The condition states that $C_{gij}/C_{gwj} = s_i$ for all g, i, and j. This means that country i consumes a proportion s_i of the final goods produced by every country. We will sometimes refer to $C_{ij} = s_i C_{wj}$ as 'strong' consumption similarity. It makes the Vanek prediction hold even though choice of techniques vary across countries. Theorem 2 is a generalization of Helpman and Krugman's (1985) result that the Vanek prediction holds for many more models than just the standard Heckscher-Ohlin model.[10]

Strong consumption similarity appears in the monopolistic competition models of trade that feature taste for variety or ideal varieties (e.g., Helpman and Krugman, 1985). It also appears in models with homothetic preferences and complete production specialization. Production specialization is associated with scale returns (Helpman and Krugman, 1985), failure of factor price equalization (Deardorff, 1979) or both (Markusen and Venables, 1998).

Lemma 1 and theorem 2 also provide an algorithm for deriving sufficient conditions for the Vanek prediction in cases where restrictions are imposed on the form of international choice-of-technique differences. We consider two examples before making the general point. In Acemoglu and Zilibotti (2001) new technologies are developed in the North, but are imperfectly adapted to Southern conditions and hence are less efficient in the South. In this case, choice of techniques differ across regions N and S, but are the same within regions. That is, $A_i = A_S$ for all Southern countries and $A_i = A_N$ for all Northern countries. Then $\Sigma_j A_j (C_{ij} - s_i C_{wj}) = A_S \Sigma_{j \in S} (C_{ij} - s_i C_{wj}) + A_N \Sigma_{j \in N} (C_{ij} - s_i C_{wj})$. Thus, a sufficient condition for the Vanek prediction is $\Sigma_{j \in R} C_{ij} = s_i \Sigma_{j \in R} C_{wj}$ for $R = N, S$.

Our next example deals with the extreme case of internationally identical choice of techniques. In this example $A_i = A_{US}$ for all i where A_{US} is the U.S. choice-of-techniques matrix. Then $\Sigma_j A_j (C_{ij} - s_i C_{wj}) = 0$ becomes $A_{US} [\Sigma_j C_{ij} - s_i \Sigma_j C_{wj}] = 0$ and a sufficient condition is $\Sigma_j C_{ij} = s_i \Sigma_j C_{wj}$. This condition is the notion of consumption similarity used in the standard Heckscher-Ohlin-Vanek model. It states that what country i consumes of a good (regardless of where the good is produced) is proportional to what the world consumes of the good. We sometimes refer to $\Sigma_j C_{ij} = s_i \Sigma_j C_{wj}$ as 'weak' consumption similarity.

There is an important general point here. Strong consumption similarity is sufficient when no restrictions are placed on the form of international choice-of-technique differences. However, the more restrictions that are placed on the form of these differences, the weaker is the form of consumption similarity needed for sufficiency. Specifically, if good g' is produced by countries j' and j'' using identical choice of techniques then strong consumption similarity for pairs (g', j') and (g', j'') is replaced by $(C_{g'ij'} + C_{g'ij''}) = s_i (C_{g'wj'} + C_{g'wj''})$ for all i.

Finally, what models do not imply $C_{ij} = s_i C_{wj}$? There are three possibilities. The first is models with international differences in preferences. The second is models with income effects associated with non-homothetic preferences e.g., Hunter and Markusen (1988). This occurs when richer countries spend disproportionately more on certain types of goods such as health or better-quality goods. The third possibility is that consumers in different countries face different product prices. If consumers face different prices, they will not make choices consistent with $C_{ij} = s_i C_{wj}$. Tariffs and transportation costs are an important source of international differences in product prices. Product price differences also appear in Balassa-Samuelson models where nontraded consumption goods such as haircuts are cheaper in poor countries. Thus, nontradeable final goods pose a serious challenge to the Vanek prediction. Summarizing, preference differences, income effects and price differences all lead to models with $C_{ij} \neq s_i C_{wj}$.[11, 12]

3.2. Necessity and local robustness

We have shown that consumption similarity implies the Vanek prediction. Does the Vanek prediction imply consumption similarity? The answer is: almost. It is not difficult to construct specialized examples in which the Vanek prediction holds without $C_{ij} = s_i C_{wj}$. One can already see this in the Lemma 1 condition $\Sigma_j A_j (C_{ij} - s_i C_{wj}) = 0$. It is possible that separate terms in the summation may be non-zero but cancel each other out i.e., $\Sigma_j A_j (C_{ij} - s_i C_{wj}) = 0$ without $C_{ij} = s_i C_{wj}$. In such examples, however, small perturbations of A_j will lead to deviations from the Vanek prediction, i.e., to $\Sigma_j A_j (C_{ij} - s_i C_{wj}) \neq 0$. Our aim in this section is to show that such examples in which the Vanek prediction holds without consumption similarity are too special to be

[9] Here is a more formal proof. Pre-multiply $T = (I - B)Q - C$ (Eq. 9) by $A = D(I - B)^{-1}$ to obtain $AT = DQ - AC$. Consider column i of this equation. Column i of AT is $AT_i = F_i$. From the definitions of D and Q, $DQ = [D_1 Q_1 \cdots D_N Q_N]$ so that column i of DQ is V_i. Hence column i of $AT = DQ - AC$ is $F_i = AT_i = V_i - AC_i$. Restated, $F_i - (V_i - AC_i) = 0$. This establishes that the left-hand side of Eq. (11) is a zero vector. Summing $AT_i = V_i - AC_i$ over i yields $A\Sigma_i T_i = V_w - AC_w$. What one country exports another imports ($X_i = \Sigma_{j \neq i} M_{ji}$) so that $\Sigma_i T_i$ is a zero vector. Thus, $V_w - AC_w$ is a zero vector and hence so is the right-hand side of Eq. (11).

[10] Helpman has described this as one of the key results in Helpman and Krugman (1985). See Trefler's (1999) interview of Helpman.

[11] There is one last minor point related to production indeterminacy when there are more goods than factors. As Melvin (1968) and Helpman and Krugman (1985) show, neither C_{gwj} nor the factor content of trade suffer from indeterminacy. Hence production indeterminacy is not an issue for our analysis of the Vanek (and Melvin) factor content prediction. See Deardorff (1999) for a discussion of a related point.

[12] A referee suggested a tantalizing example. Consider a $2 \times 2 \times 2$ Heckscher-Ohlin model with country 1 being capital abundant and good 1 being capital-intensive. There are international technology differences and in equilibrium each country specializes. The Vanek condition holds and we expect the Heckscher-Ohlin theorem to hold as well (country 1 exports good 1). However, suppose country 1 has such a Hicks-neutral productivity advantage in good 2 that the pattern of specialization is reversed (country 1 produces and exports good 2). Does this mean that we have broken the link between the Vanek and Heckscher-Ohlin predictions? No. Since each country specializes in a single good, the capital–labour ratio in endowments must equal the capital–labour ratio in production of the single good. To sustain this, the price of capital will have to be lower in country 1, so much so that good 2 has the higher capital–labour ratio in production. While country 1 is not exporting the capital-intensive good (good 1), it is exporting the good that has the higher equilibrium capital–labour ratio. This is clearly in the spirit of the Heckscher-Ohlin model.

anything but theoretical curiosities. We will show that they are not robust to small perturbations of the exogenous technology parameters of the model.

The starting point of our new approach is the notion of local robustness. Before providing a formal definition we illustrate it using a very familiar example: the Heckscher-Ohlin model with two factors, G goods and internationally identical, Cobb-Douglas, constant-returns-to-scale technologies. Let $\alpha_g \in (0,1)$ be the Cobb-Douglas exponent on labour and let $(\alpha_g - \varepsilon, \alpha_g + \varepsilon)$ be an interval around α_g that is strictly within the unit interval. Let $\pi = (\alpha_1, \ldots, \alpha_G)$ collect all the primitive technology parameters of the model and let $N(\pi) \equiv (\alpha_1 - \varepsilon, \alpha_1 + \varepsilon) \times \ldots \times (\alpha_G - \varepsilon, \alpha_G + \varepsilon)$ be a neighbourhood of π. If the Vanek prediction held for some π, but failed to hold elsewhere on $N(\pi)$ no matter how small $\varepsilon > 0$, then we would all agree that the Vanek prediction at π is more of a curiosity than an important finding. Restated, a minimal requirement for an interesting Vanek prediction is that, if it holds for some value π in the interior of the technology space $[0,1] \times \ldots \times [0,1]$, then it holds everywhere on $N(\pi)$. We will say that such a Vanek prediction is 'robust at π' or 'locally robust.' Local robustness is clearly a feature of the Vanek predictions derived, for example, throughout Helpman and Krugman's book.

Our core theorem states that consumption similarity is necessary for a locally robust Vanek prediction. That is, if the Vanek prediction is robust at π then consumption similarity holds at π. To explain why, we return to our Heckscher-Ohlin example with its internationally identical choice of techniques. Specifically, $D_i = D_{US}$ for all i. To keep things simple for this sketch and this sketch only, assume that there are no intermediate goods so that $A_i = D_i = D_{US}$. From Lemma 1 with $A_j = D_{US}$ for all j, the necessary condition for a Vanek prediction is $D_{US}\Sigma_j(C_{ij} - s_i C_{wj}) = 0$. Since each term in this equation is an equilibrium outcome that depends on the underlying technology, write it as

$$D_{US}(\pi')\left[\Sigma_j C_{ij}(\pi') - s_i(\pi')\Sigma_j C_{wj}(\pi')\right] = 0 \tag{12}$$

where π' is an arbitrarily chosen member of $N(\pi)$. Suppose that the Vanek prediction is robust at π so that Eq. (12) holds for all π' on $N(\pi)$. We will construct a subset of $N(\pi)$ – call it $\Pi(\pi)$ – with the following properties: (1) equilibrium outcomes of factor prices, income, output, and consumption are constant on $\Pi(\pi)$ and (2) D_{US} is *not* constant on $\Pi(\pi)$. Variations in the D_{US} capture changes in firm-level and industry-level factor demands that are offsetting at the national level. Such changes do not affect national-level factor demands and hence do not affect equilibrium factor prices or product–market outcomes. In terms of Eq. (12), as we vary π' on $\Pi(\pi)$, the term in brackets remains constant, but $D_{US}(\pi')$ does not. It can then be shown that the only way Eq. (12) can hold for all π' on $\Pi(\pi)$ is if the term in brackets is zero.

This sketches out the core insight underlying why consumption similarity is necessary for a locally robust Vanek prediction. Why is necessity important or interesting? For one, in conjunction with sufficiency it provides a full characterization of the Vanek prediction in terms of observables used in the empirical literature. For another, our results are independent of the product market equilibrium concept. Previous research invariably starts by specifying product market equilibrium conditions. This need to look at each case separately (e.g., perfect competition, monopolistic competition) is completely circumvented here.

4. Necessity: a formal proof

We turn now to formalizing the discussion of the previous section. We do this for the most general case in which no restrictions are imposed on the form of international choice-of-technique differences. In an earlier version of the paper (Trefler and Zhu, 2005) that is available on request, we extended our result to the case with restrictions.

To ensure that the set of perturbations $\Pi(\pi)$ contains more than just a single point π, we assume throughout that every country is positively endowed with at least two factors and every country produces at least two goods.[13] Finally, in the spirit of the second goal of this paper — which is about how to properly deal with traded intermediate inputs — we assume that for each country, at least one of the goods it produces is a traded intermediate input.[14] The reader who is not interested in the details should jump straight to definition 1 or even to theorem 3.

4.1. Technology primitives π and factor market equilibrium

We consider the class of models with the following common features.

Assumption 1. (i) Factor markets are perfectly competitive: factors are mobile across firms within a country and firms are price takers in factor markets. (ii) There is no joint production. (iii) Cost functions are differentiable.

All but the factor mobility assumption can be eliminated. However, this would make the proof of appendix Lemma 2 far more complex.

By technology π we mean the set of cost functions used by each firm in each country. For any industry, we make no assumptions about how cost functions vary across countries or even across firms within a country. See Appendix A for details. Perturbations of π mean perturbations of cost functions. From Assumption 1(*ii*) to (*iii*) and Shephard's lemma, these perturbations generate perturbations in firm-level factor demands. These in turn generate perturbations in industry-level factor demands D. All this is spelled out in Appendix A.

4.2. Product market equilibrium outcomes

We next turn to the problem of characterizing product market equilibrium *outcomes* without fully specifying the equilibrium concept. To this end, consider an economy with the following features. (*i*) Consumers maximize utilities. (*ii*) Country i producers minimize costs and maximize profits, taking the $K \times 1$ factor price vector ω_i as given. (*iii*) Factor markets clear (Eq. 6).

The *exogenous* parameters of the economy are preferences, endowments V_i, and technology π. π is best thought of as a complete description of the cost functions for all goods in all countries. See Appendix A. The endogenous variables include $D \equiv (D_1, \ldots, D_N)$, $\omega \equiv (\omega_1, \ldots, \omega_N)$, and $E \equiv \{p_k, q_k, s_i, C_{ij}, C_{wj}, Y_{ij}, Q_i, B\}_{\forall i,j,k}$ where for firm k, q_k is the vector of outputs produced by firm k and p_k is the firm's vector of prices. E collects all the endogenous variables explicitly referred to below that relate to the markets for final goods and intermediate inputs. These endogenous variables are all functions of π.

4.3. Defining the local robustness set Π(π, ε)

We have already informally defined the set of technology perturbations $\Pi(\pi)$. We now rename it $\Pi(\pi,\varepsilon)$ and define it more formally.

Definition 1. $\Pi(\pi,\varepsilon)$ *is the set of* π' *satisfying the following: (1)* $||D(\pi') - D(\pi)|| < \varepsilon$ *where* $||\cdot||$ *is the Euclidean norm. (2)* $E(\pi') = E(\pi)$*: equilibrium outcomes in the markets for final goods and intermediate inputs are constant on* Π*. (3)* $D_i(\pi')Q_i(\pi') = V_i$ $\forall i$*: the economy-wide demand for factors is constant on* Π*. (4)* $\omega_i(\pi') = \omega_i(\pi)$ $\forall i$*: factor prices are constant on* Π*. (5)* $\omega_i(\pi')D_i(\pi') = \omega_i(\pi)D_i(\pi)$ $\forall i$*: industry-level factor costs are constant on* Π*.*

[13] Our theorems hold trivially with just one good provided it is either nontradable or an intermediate input.

[14] The assumption is only used once in the paper and then only at the very end of the proof of theorem 3 in the Appendix. It can be dispensed with when more structure is placed on the form of the international choice-of-technique differences.

$\Pi(\pi,\varepsilon)$ has been defined so that almost the only thing to vary on Π is the industry-level factor demands $D(\pi')$. In this sense $\Pi(\pi,\varepsilon)$ is so small that local robustness is a weak requirement of the Vanek prediction. Restated, a Vanek prediction that holds for π, but not for all $\pi'\in\Pi(\pi,\varepsilon)$ is really nothing more than a curiosity.

It would be nice to know what the set of $D(\pi')$ satisfying definition 1 looks like. This set is completely characterized by Eq. (25) and Lemma 3 of Appendix A.

4.4. The necessity theorem

The next theorem is a key result of this paper. We say that the Vanek prediction is *locally robust at* π if $F_i(\pi') = V_i - s_i(\pi')V_w$ holds for all π' in $\Pi(\pi,\varepsilon)$.

Theorem 3. *(Necessity)*: *Suppose assumption 1 holds.* Then

$$\{F_i(\pi') = V_i - s_i(\pi')V_w\}_{i=1}^{N} \text{ for all } \pi' \text{ in } \Pi(\pi,\varepsilon) \\ \Longrightarrow \\ C_{ij}(\pi) = s_i(\pi)C_{wj}(\pi)\forall i \text{ and } j.$$

That is, strong consumption similarity is necessary and sufficient for the Vanek prediction to be locally robust at π.

The proof appears in the Appendix. This concludes our discussion of the necessary conditions for a locally robust Vanek prediction in a setting with no restrictions on the form of international choice-of-technique differences. Research from Deardorff (1979) to Helpman and Krugman (1985) showed us that many models imply the Vanek prediction. Our paper shows that the consumption similarity condition completely characterizes the set of models featuring a robust Vanek prediction.

5. Empirical counterpart of F_i

The factor content of trade F_i is a function of B, the world input–output table. National statistical agencies only report domestic input–output matrices. That is, they report

$$\overline{B}_i \equiv \Sigma_j B_{ji}.$$

$\overline{B}i$ is country i's input requirements summed over both national and international sources of supply. Fortunately, there is a standard 'proportionality' technique for imputing B using the $\overline{B}_i$. To quote from the OECD:

> "This technique assumes that an industry uses an import of a particular product in proportion to its total use of that product. For example if an industry such as motor vehicles uses steel in its production processes and 10 per cent of all steel is imported, it is assumed that 10 per cent of the steel used by the motor vehicle industry is imported." (Organisation for Economic Co-operation and Development, 2002, page 12)

To formalize the proportionality assumption, let $Q_i(g), X_i(g), M_{ij}(g)$, and $M_i(g)$ be the gth elements of the vectors Q_i, X_i, M_{ij}, and $\Sigma_{j\neq i}M_{ij}$, respectively. For good g, $Q_i(g) + M_i(g) - X_i(g)$ is domestic absorption i.e., the amount of g used by country i for both intermediate use and final consumption. Define

$$\theta_{ij}(g) \equiv \frac{M_{ij}(g)}{Q_i(g) + M_i(g) - X_i(g)} \qquad \text{for } j\neq i. \tag{13}$$

$\theta_{ij}(g)$ is the share of domestic absorption that is sourced from country j. Also define

$$\theta_{ii}(g) \equiv 1 - \Sigma_{j\neq i}\theta_{ij}(g) \tag{14}$$

which is the share of domestic absorption that is sourced locally. Finally, let $B_{ji}(g,h)$ and $\overline{B}_i(g,h)$ be elements of B_{ji} and $\overline{B}_i \equiv \Sigma_j B_{ji}$, respectively. Then the proportionality assumption is

$$\Sigma_{j\neq i}B_{ji}(g,h) = \overline{B}_i(g,h)\Sigma_{j\neq i}\theta_{ij}(g)(\text{imported intermediates}) \tag{15}$$
$$B_{ii}(g,h) = \overline{B}_i(g,h)\theta_{ii}(g)(\text{local intermediates})$$

This is how the OECD and GTAP break out domestic and foreign purchases. It is one of the assumptions that allows Hummels et al. (2001) and Yi (2003) to estimate the growth in world trade in intermediate inputs and in inputs used in vertical production networks. (See Eqs. 2–3 in Hummels et al..) It is also the assumption used by Feenstra and Hanson (1996, 1999) to develop their broad measure of outsourcing.[15]

An obvious extension of the proportionality assumption in Eq. (15) is

$$B_{ji}(g,h) = \overline{B}_i(g,h)\theta_{ij}(g) \text{ for all } i \text{ and } j. \tag{16}$$

Eq. (16) allows one to recover the B matrix from available data in a way that is consistent with the efforts of Feenstra and Hanson (1996, 1999), Hummels et al. (2001) and Yi (2003). In the empirical section below, we will use Eq. (16) to calculate B.

6. Previous definitions of the factor content of trade

In the literature on the Vanek prediction with international choice-of-technique differences and traded intermediates we count at least five different and mutually incompatible definitions of the factor content of trade. Here we reconsider this literature in light of our definition of the factor content of trade as the amount of factors used *worldwide* to produce a country's trade flows. In reviewing the literature, it is best to have a narrative or story line. In our view, this narrative has been the on-going challenge to come up with a definition of the factor content of trade that satisfies three criteria: (*1*) The definition is Vanek-relevant i.e., under the maintained hypothesis that the Vanek prediction is correct, F_i is defined so as to equal $V_i - s_iV_w$. (*2*) The definition has a clear and useful economic interpretation. (*3*) The definition does not require restrictions on the form of international choice-of-technique differences.

We begin by defining

$$\overline{A}_i \equiv D_i(I - \overline{B}_i)^{-1}$$

where, as before, $\overline{B}_i \equiv \Sigma_j B_{ji}$ is the standard national input–output table i.e., the input requirements summed over both national and international sources of supply. With the exception of Reimer (2006), all previous work on the Vanek prediction has used $\overline{A}_i$ rather than our A.

Trefler (1993) assumes that choice-of-technique differences take the form $\overline{B}_i = \overline{B}_{US}$ and $D_i = \Lambda_i^{-1}D_{US}$ where Λ_i is a diagonal matrix whose typical diagonal element gives the productivity of factor f in country i relative to the United States. Under Trefler's assumption, the full employment condition $D_iQ_i = V_i$ can be re-written as $D_{US}Q_i = V_i^*$ where $V_i^* \equiv \Lambda_iV_i$ is country i's endowments measured in productivity-equivalent units. This transforms the model into the standard Heckscher-Ohlin-Vanek model with internationally identical choice of techniques, but with factors measured in productivity-equivalent units. In particular, the Vanek prediction becomes $\overline{A}_{US}(X_i - M_i) = V_i^* - s_i\Sigma_jV_j^*$ where $\overline{A}_{US}(X_i - M_i)$ is the factor content of trade measured in productivity-equivalent units. Variants of this approach are used by Trefler (1995, hypothesis T1), Davis and Weinstein (2001, hypothesis T3), Conway (2002), and Debaere (2003). This approach satisfies our first and second criteria above, but not our third.

[15] Feenstra and Hanson care about outsourcing, but not about which intermediates g are outsourced. They thus sum Eq. (15) over intermediates g to obtain $\Sigma_g\overline{B}_i(g,h)\Sigma_{j\neq i}\theta_{ij}(g)$. This multiplied by $Q_i(h)$ is their measure of outsourcing.

When choice of techniques are allowed to differ internationally in more general ways, coming up with a sensible definition of the factor content of trade has proved far more difficult. For example, Davis et al. (1997) is a major contribution that improves on Trefler (1993) by relaxing all restrictions on the form of the international choice-of-technique differences. Absent such restrictions however, it is clear that their dependent variable $\overline{A}_{JAPAN}(X_i - M_i)$ is *not* the Vanek-relevant factor content of trade. After all, it evaluates goods produced in country i using Japan's choice of techniques.[16] Likewise for Hakura (2001) who moves from using a single country's input–output table to using the input–output tables of 4 OECD countries. Contrary to what Hakura claims, her dependent variable $\overline{A}_i(X_i - M_i)$ is not Vanek-relevant. It evaluates the factor content of i's imports using i's choice of techniques rather than using the producing country's choice of techniques.

There are only two serious factor content definitions that allow for international choice-of-technique differences (criterion 3). The Davis and Weinstein (2001, hypothesis T4-T7) definition is economically clear and somewhat useful (criterion 2), but it is not Vanek-relevant (criterion 1). The Trefler and Zhu (2000) and Antweiler and Trefler (2002) definition is Vanek-relevant, but it is not economically clear or useful. This requires some explanation.

Davis and Weinstein (2001), in their core hypothesis T4, define the factor content of trade as[17]

$$F_i^{DW} \equiv \overline{A}_i X_i - \Sigma_{j \neq i} \overline{A}_j M_{ij}. \tag{17}$$

This definition first appeared in Helpman and Krugman (1985, equation 1.11) and is very intuitive in the sense that it appears to evaluate the output of country j using country j's choice of techniques. That is, it evaluates M_{ij} using $\overline{A}_j$. Further, the definition looks a lot like our F_i. To see this, partition our A as $[A_1 \ \ A_2 \cdots A_N]$. Then F_i can be written as

$$F_i = A_i X_i - \Sigma_{j \neq i} A_j M_{ij}.$$

It follows that $F_i^{DW} = F_i$ when $\overline{A}_i = A_i$. Restated, F_i^{DW} is the factor content of trade when $\overline{A}_i = A_i$. When is $\overline{A}_i = A_i$? Without additional restrictions on B, a necessary and sufficient condition for $\overline{A}_i = A_i$ is $B_{ji} = 0$ for all $j \neq i$.[18]

$B_{ji} = 0$ means that country i does not import any intermediate inputs from country j. Thus, without additional restrictions on B, F_i^{DW} is the factor content of trade only when there is no trade in intermediate inputs. Clearly, this is an uncomfortable assumption in light of the enormous interest in global vertical production networks e.g., Feenstra and Hanson (1996, 1999), Hummels et al. (2001) and Yi (2003).

What is wrong with the Davis and Weinstein definition? The problem is that $\overline{A}_i$ shares with $\overline{B}_i$ a failure to distinguish intermediate inputs that are produced domestically from intermediate inputs that are produced abroad. $\overline{A}_i$ can therefore not be used in any simple way to evaluate the factor content of trade.

Trefler and Zhu (2000) and Antweiler and Trefler (2002) get around this problem, but at a cost. They define the factor content of trade as

$$F_i^T \equiv \overline{A}_i X_i^c - \Sigma_{j \neq i} \overline{A}_j M_{ij}^c + \overline{A}_i (X_i^y - M_i^y) - s_i \Sigma_j \overline{A}_j \left(X_j^y - M_j^y \right)$$

where X_i^c is i's exports of consumption goods, M_{ij}^c is i's imports of consumption goods produced in country j, X_i^y is i's exports of intermediate inputs, and M_i^y is i's imports of intermediate inputs. These authors show that F_i^T is Vanek-relevant i.e., under the Vanek null, $F_i^T = V_i - s_i V_w$ (criterion 1). Unfortunately, F_i^T is economically meaningless (criterion 2). Specifically, unless the Vanek prediction is true so that F_i^T can be equated with the interpretable expression $V_i - s_i V_w$, it is unclear how to interpret F_i^T.[19]

This places the literature at an impasse. F_i^{DW} is a factor content definition that is economically meaningful, but not Vanek-relevant. F_i^T is a definition that is Vanek-relevant, but economically difficult to interpret. One contribution of this paper is that it provides a factor content definition F_i that moves the discipline beyond this impasse. F_i is both economically meaningful and Vanek-relevant.

7. A new empirical test of the Vanek prediction

In this section we assess the Vanek prediction for one factor, aggregate labour, in 41 developed and developing countries.[20] Input–output tables are from GTAP (version 5) and are documented in Dimaranan and McDougall (2002). We use GTAP data together with Eq. (16) to compute the world B matrix. We construct D for labour as described in Appendix C. Data are for 1997 whenever possible. Note that all the set-up definitions (Eqs. 3–8) are satisfied in the data.

Since we have only one factor (labour), we abuse notation slightly by treating F_i, V_i, and V_w as scalars and A_i as a $1 \times G$ vector. In order to control for country size, we scale observation i of $F_i = V_i - s_i V_w$ by $s_i^{1/2}$.[21]

Table 1 reports some standard statistics about the performance of the Vanek prediction. Column 1 evaluates country i's factor content of trade using common (U.S.) choice of techniques: $\overline{A}_{US}(X_i - M_i)$. As is well known, the Vanek prediction does horribly with this factor-content definition. Row 1 of Table 1 is the share of observations for which F_i has the same sign as $V_i - s_i V_w$. Only 34 percent have the right sign. This is even lower than the success rate of Trefler's 'coin-toss' model (Trefler, 1995, p. 1029). Row 2 is the Spearman (or rank)

[16] This statement should not be misunderstood to mean that the equations estimated by Davis et al. (1997) contain mathematical errors. The equations are correct. It is the interpretation of the dependent variable that we are questioning. This caveat applies to all the papers reviewed below.

[17] Their definition is actually more complicated, but these complications only obscure our main point without altering it. In particular, see Davis and Weinstein (2001, page 1425–26) and their hypotheses T5, T6, and T7.

[18] To see this, first consider the case of 2 countries. To keep the expression for F_1 manageable we assume that intermediate inputs flow only in one direction, from country 2 to country 1, so that $B_{21} = 0$. Then it is straightforward to show that our Eq. (10) definition of the factor content of trade reduces to

$$F_1 = D_1 (I - B_{11})^{-1} X_1 - D_2 (I - B_{22})^{-1} M_{12} - D_1 (I - B_{11})^{-1} B_{12} (I - B_{22})^{-1} M_{12}$$

while Davis and Weinstein's definition reduces to

$$F_i^{DW} = \overline{A}_1 X_1 - \overline{A}_2 M_{12} = D_1 (I - B_{11})^{-1} X_1 - D_2 (I - B_{22} - B_{12})^{-1} M_{12}.$$

Clearly, these definitions are equivalent only in the special case where there is no intermediate trade i.e., where $B_{12} = 0$. More generally, consider the definitions of $\overline{A}_i$ and A as well as the definition of B at the start of section 2. Then $\overline{A}_i = A_i$ when $(I - B)^{-1}$ is a block diagonal matrix with typical diagonal matrix $(I - \overline{B}_i)^{-1}$. Without further restrictions on B, a necessary and sufficient condition for this block-diagonality is that the off-diagonal elements of B equal 0 i.e., $B_{ji} = 0$ for all $j \neq i$. To see this, note that $B_{ji} = 0$ for all $j \neq i$ implies two things. First, $(I - B)^{-1}$ is block diagonal with typical diagonal element $(I - B_{ii})^{-1}$. Second, $\overline{B}_i \equiv \Sigma_j B_{ji} = B_{ii}$. Hence, $(I - B)^{-1}$ is block diagonal with typical diagonal element $(I - \overline{B}_i)^{-1}$, as required.

[19] The reader should understand that the Antweiler and Trefler (2002) results based on F_i^T are correct. The fact that F_i^T is not the factor content of trade when the maintained assumption of consumption similarity is relaxed is irrelevant to Antweiler and Trefler: they never relax the assumption. Their null hypothesis is consumption similarity plus constant returns to scale and their alternative hypothesis is consumption similarity plus increasing returns to scale.

[20] The 41 countries (ranked by per capita GDP in 1996) are the United States, Hong Kong, Singapore, Switzerland, Denmark, Japan, Canada, Austria, the Netherlands, Australia, Germany, Belgium, Sweden, Italy, France, the United Kingdom, Finland, Ireland, New Zealand, Taiwan, Spain, South Korea, Portugal, Greece, Argentina, Uruguay, Malaysia, Chile, Hungary, Poland, Mexico, Thailand, Venezuela, Brazil, Turkey, Colombia, Peru, Indonesia, Sri Lanka, the Philippines, and China.

[21] More precisely, we scale by $\sigma_i \equiv s_i^{\mu} \sigma$ where σ^2 is the cross-country variance of $(F_i - V_i + s_i V_w) / s_i^{\mu}$ and $\mu = 0.5$. Very similar results obtain for all choices of μ between 0.1 and 2.0. Scaling by σ is irrelevant and serves only to normalize the variance of the residuals $(F_i - V_i + s_i V_w) / s_i^{\mu}$ to unity.

Table 1
The Vanek prediction for labour.

	All Observations		Trimmed Sample
	$\overline{A}_{f,US}(X_i-M_i)$ $=V_{fi}-s_iV_{fw}$ (1)	F_{fi} $=V_{fi}-s_iV_{fw}$ (2)	F_{fi} $=V_{fi}-s_iV_{fw}$ (3)
1. Sign Test	.34	.95	.91
2. Rank Correlation	−.13	.89	.86
(p-value)	(0.40)	(0.00)	(0.00)
3. Missing Trade	.001	.12	.27
4. Slope Coefficient	.002	.32	.44
(t-statistic)	(.48)	(14.58)	(7.34)
5. R^2	.01	.85	.74
Observations	41	41	21

Notes: The trimmed sample excludes all the 'extreme' observations in Fig. 1 i.e., those with $|V_{fi}-s_iV_{fw}|$ <0.6 as well as Hong Kong and Singapore.

correlation between F_i and $V_i-s_iV_w$. The correlation of −0.13 has the wrong sign and is statistically insignificant ($p=0.40$). Row 3 reports Trefler's (1995) 'missing trade' statistic, which is the variance of F_i divided by the variance of $V_i-s_iV_w$. It is a tiny 0.001. An equivalent way of thinking about missing trade and the fit of the Vanek prediction is the slope and R^2 from the regression $F_i=\alpha+\beta(V_i-s_iV_w)+\varepsilon_i$. These are reported in rows 4 and 5 and are disturbingly small, as has been found in many previous studies.

We turn next to our Vanek-relevant definition $F_i=AT_i$. Column 2 of Table 1 reports results for $F_i=V_i-s_iV_w$. This prediction performs dramatically better than the standard column 1 prediction. 95 percent of the observations have matching signs, the rank correlation of F_i with $V_i-s_iV_w$ is 0.89 ($p=0.00$), the missing trade statistic has grown substantially (from 0.001 to 0.12), and the slope coefficient has also grown (from 0.002 to 0.32). This is a significant improvement on the standard HOV model. However, there remains a substantial amount of missing trade.

Fig. 1 plots F_i against $V_i-s_iV_w$ for labour. The origin is shown as a large cross. Panel (a) shows that the correlation is strong and that the sign test is meaningful i.e., many observations are squarely in orthants 1 and 3. The dashed line is the fitted line from the regression of F_i on $V_i-s_iV_w$. With the notable exceptions of Hong Kong, Singapore and Japan, all the observations lie close to the regression line.

Since it is hard to see what is happening near the origin of Fig. 1, panel (b) blows up this area: it excludes all observations with $V_i-s_iV_w$ greater than 0.6 in absolute value.[22] It also excludes the outliers Hong Kong and Singapore. 0.6 was chosen so that half the observations (21 of 41) are left in this trimmed sample. The figure shows that the model fits well even close to the origin. This can also be seen from column 3 of Table 1, which reports statistics for these 21 observations. Interestingly, the missing trade statistic improves from 0.12 to 0.26 and the slope coefficient grows from 0.32 to 0.44.

We have also calculated the Table 1 statistics using the Davis and Weinstein definition of the factor content of trade i.e., $|F_i^{DW}|$ of Eq. (17). The results for the sign test, rank correlation, and R^2 are very similar to ours (0.93, 0.89, and 0.80, respectively). However, there are large differences in the missing trade statistics (0.20 versus our 0.12) and the slope coefficient (0.40 versus our 0.32). That is, F_i^{DW} is upward-biased, thus masking the extent of missing trade. At first glance this is surprising. However, Reimer shows that the factor content of intermediate inputs evaluated using $|F_i^{DW}|$ can in theory be too large in absolute value (Reimer, 2006, pp. 391–392). He also shows empirically that F_i^{DW} *is* biased upwards (Reimer, 2006, figures 3–4). Reimer does this for the case where the world is divided into two regions, the United States and 'Rest of World'. We confirm his finding in our 41 country sample.

[22] As described in footnote 21, $V_i-s_iV_w$ is scaled by something that is very close to the standard deviation of $V_i-s_iV_w$. Hence, 0.6 is in standard-deviation units.

We interpret Table 1 and Fig. 1 as evidence that the HOV model does a reasonable job of explaining why the factor content of trade varies dramatically across rich and poor countries. As good as the model is however, it is clearly not perfect. The theorems of this paper show exactly what is wrong: either there is measurement error or the factor content of a country's consumption (AC_i) is not proportional to the world factor content of consumption (s_iAC_w). We turn next to documenting whether the failure of consumption similarity is systematic or whether it is confined to particular industries and countries.

7.1. Deviations from consumption similarity

Let A_{gj} be the amount of labour used to produce one unit of good g in country j. Let e:italic>C_{gij} be country i's consumption of good g produced in j and let $C_{gwj}\equiv\Sigma_iC_{gij}$ be the corresponding world consumption. From Lemma 1, the Vanek residual for labour in country i is $F_i-(V_i-s_iV_w)=\Sigma_j\Sigma_g\varepsilon_{gij}$ where

$$\varepsilon_{gij}\equiv A_{gj}\left(C_{gij}-s_iC_{gwj}\right). \tag{18}$$

In this section we examine the ε_{gij}.[23] Theorem 3 shows that the necessary condition for a robust Vanek prediction is consumption similarity ($C_{gij}=s_iC_{gwj}$), which is equivalent to $\varepsilon_{gij}=0$ for all g, i and j.

With 24 industries and 41 countries there are $24\times41\times41=40,344$ ε_{gij} so we need some way of organizing them. Davis and Weinstein (2001) provide a good starting point. They argue that departures from the HOV model are in large part explained by the nontradeability of services. If good g produced in country j is perfectly nontradeable then it is only consumed in country j i.e., $C_{gij}=0$ for $i\neq j$ and $C_{gjj}=C_{gwj}$. Plugging these into Eq. (18) yields $\varepsilon_{gij}=-s_iA_{gj}C_{gjj}$ for $i\neq j$ and $\varepsilon_{gjj}=(1-s_j)A_{gj}C_{gjj}$. Since the s_i are small, we would expect the ε_{gij} to be negative and close to zero and the ε_{gjj} to be positive and large. This is born out in the data.

The left-hand panel of Table 2 displays the ε_{gjj} that are largest in absolute value. They are all positive. This generalizes to all of the 24×41 ε_{gjj}: 97% are positive and the remaining 3% are close to zero. The right-hand panel of Table 2 displays the ε_{gij} ($i\neq j$) that are largest in absolute value. They are all negative and relatively close to zero. Large positive ε_{gjj} and near-zero negative ε_{gij} ($i\neq j$) are consistent with the Davis and Weinstein (2001) claim about nontradeability as a major cause for the poor performance of the Vanek prediction.

Table 2 also displays the names of the industries. The fact that many of these are Government and Construction lends further support to the Davis and Weinstein (2001) nontradeability claim. However, the repeated appearance of Agriculture and Food in Table 2 suggests that nontradeability is only part of the story. Legislated barriers to trade such as agricultural subsidies and technical barriers to trade (discriminatory national food standards) are also important sources of deviations from consumption similarity. As we shall see, such restrictive trade policies are more important than nontradeability for explaining departures from the Vanek prediction.[24]

The Table 2 results generalize to all of the ε_{gij}. The best way to see this is to look at the residuals separately by industry. Table 3 reports the variance of the residuals by industry (σ_g^2). Just four industries have large variances. These are Agriculture, Government, Construction, and Processed food. Further, the variances in these industries are large because of the $i=j$ observations. This is shown in columns 2 and 4 which report the proportion of σ_g^2 that is due to $i=j$ observations. On average, 91 percent of the variance is explained by $i=j$ observations, which means that consumption is 'biased' towards domestically produced goods.

[23] To measure the ε_{gij} we need the C_{gij}. C_{gij} and C_{gii} are calculated from Eqs. (7) and (8), respectively. Note that the ε_{gij} are scaled as described in footnote 21 above.

[24] The impact of restrictive trade policies on factor contents is explored in a general context by Staiger et al. (1988) and in an HOV context by Staiger et al. (1987). The latter find that protection explains very little of the departures from the HOV model.

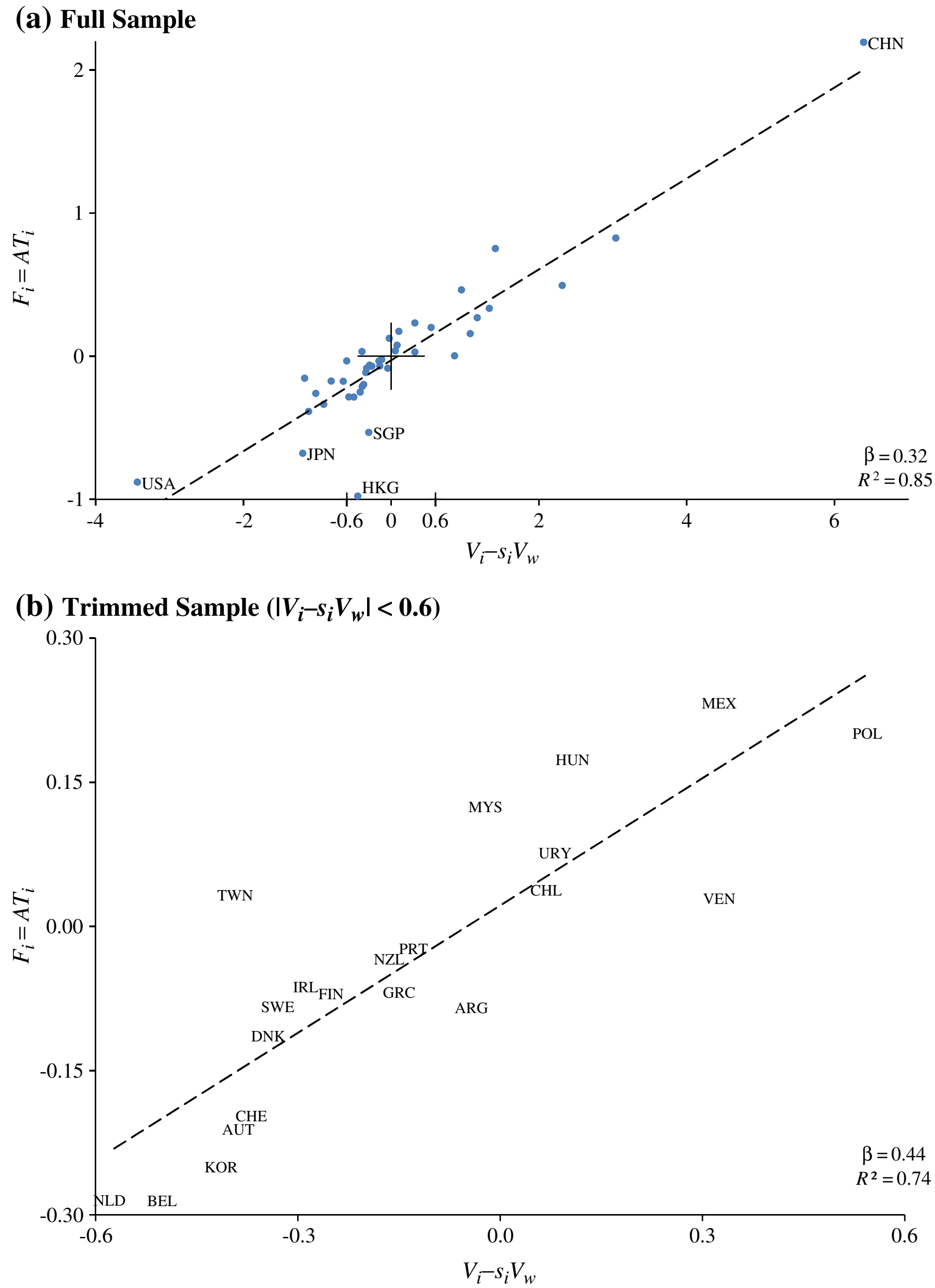

Fig. 1. The Vanek prediction for labour.

Table 2
Largest error components $\varepsilon_{gij} \equiv A_{gj}(C_{gij} - s_i C_{gwj})$.

Consumer (i) = Producer (j)			Consumer $(i) \neq$ Producer (j)			
$i = j$	Industry (g)	ε_{gij}	i	j	Industry (g)	ε_{gij}
China	Agriculture	2.68	China	USA	Agriculture	−0.57
China	Construction	1.58	China	Japan	Agriculture	−0.33
Brazil	Government	1.25	China	USA	Construction	−0.33
China	Food	0.94	China	Germany	Agriculture	−0.26
Philippines	Government	0.89	USA	China	Government	−0.22
USA	Government	0.88	China	Great Britain	Agriculture	−0.22
Indonesia	Agriculture	0.80	China	France	Agriculture	−0.22
China	Government	0.79	China	Italy	Agriculture	−0.22
Indonesia	Food	0.78	China	Brazil	Agriculture	−0.21
Japan	Government	0.77	China	USA	Food	−0.21

Having established that the residuals are large for the $i = j$ observations of just a few industries, we can ask what impact lack of consumption similarity in these industries has on the performance of the HOV model. To this end we pick an industry g from our list of four industries and impose consumption similarity i.e., we choose C_{gij} to equal $s_i C_{gwj}$. Since this will expand trade in good g, we do so in the unique way that minimizes trade; that is, we assume that there is no intermediate input trade for good g. Restated, for good g all trade is assumed to be final goods trade so that $M_{gij} = C_{gij}$ for all i and j.[25]

[25] In this footnote we trace out all the implications of this assumption for our set-up equations. Let a g subscript on a $G \times 1$ vector denote the gth element of the vector. Assume $C_{gij} = s_i C_{gwj}$. Then $M_{gij} = C_{gij}$ for $i \neq j$ is equivalent to each of the following. (a) $Y_{gij} = 0$ for $i \neq j$. That is, there is no intermediate trade in good g. (b) The gth row of B_{ji} is a zero vector for $i \neq j$. (c) $\theta_{ij}(g) = 0$ for $i \neq j$ and $\theta_{jj}(g) = 1$. (d) $C_{gwj} = Q_{gj} - Y_{gjj}$ for all j. This last point is derived by substituting Eq. (4) into Eq. (3).

Table 3
Within-industry variance of errors $\varepsilon_{gij} \equiv A_{gj}(C_{gij} - s_i C_{gwj})$.

ISIC Industries	σ_g^2 (1)	$i=j$ (2)		σ_g^2 (3)	$i=j$ (4)
Agriculture	8.08	.89	Textiles	0.02	.23
Government	7.66	.93	Other Manufacturing	0.01	.88
Construction	4.28	.91	Leather	0.01	.48
Food	2.32	.93	Paper and Publishing	0.00	.93
Wholesale-Retail	1.06	.95	Wood Products	0.00	.92
Machinery	0.13	.86	Fabricated Metal Manuf.	0.00	.95
Beverages	0.12	.91	Electricity and Water	0.00	.91
Distribution	0.10	.95	Petroleum Products	0.00	.90
Transport equipment	0.08	.85	Non-Metal Manufacturing	0.00	.91
FIRE (Fin., Insur., Real Est.)	0.05	.91	Non-Ferrous Metal Manuf.	0.00	.16
Apparel	0.04	.62	Mining	0.00	.94
Chemicals	0.03	.94	Basic Metal Manufacturing	0.00	.97

Notes: This table reports variances of the $\varepsilon_{gij} \equiv A_{gj}(C_{gij} - s_i C_{gwj})$ for each of our 24 industries. Columns 1 and 3 report $\sigma_g^2 \equiv \Sigma_{ij}(\varepsilon_{gij} - \bar{\varepsilon}_g)^2 / N^2$ where $\bar{\varepsilon}_g = \Sigma_{ij}\varepsilon_{gij} / N^2$. Columns 2 and 4 report the proportion of σ_g^2 accounted for by $i=j$ observations. That is, it reports $\Sigma_j(\varepsilon_{gjj} - \bar{\varepsilon}_g)^2 / N^2$ divided by σ_g^2.

Table 4
The Vanek prediction with consumption similarity.

	None (1)	Agr. (2)	Govt. (3)	Agr. Govt. (4)	Agr. Food (5)	Govt. Constr. (6)	All 4 (7)
Sign Test	0.95	0.90	0.95	0.98	0.93	0.98	0.98
Rank Correlation	0.89	0.93	0.92	0.95	0.96	0.95	0.98
Missing Trade	0.12	0.39	0.17	0.45	0.55	0.29	0.89
Slope Coefficient	0.32	0.60	0.38	0.66	0.72	0.53	0.94
(t-statistic)	(14.58)	(19.30)	(14.62)	(29.08)	(27.75)	(26.99)	(52.57)
R^2	0.85	0.91	0.85	0.96	0.95	0.95	0.99

Notes: Column 1 is the same as column 2 of Table 1. In column 2, we impose that $C_{gij} = s_i C_{gwj}$ where g is Agriculture. Columns 3–7 imposes these assumptions, respectively, for $g=$ Government, $g=$ Agriculture and Government, $g=$ Agriculture and Food, $g=$ Government and Construction, and $g=$ Agriculture, Food, Government, and Construction. There are 41 observations, one for each country.

Table 4 reports the results of imposing consumption similarity plus no intermediates trade for good g. Column 1 reports the baseline result that already appeared as column 2 of Table 1. From Table 3, the obvious industries to consider first are Agriculture and Government. Column 2 reports what happens when we impose consumption similarity on Agriculture i.e., $g=$ Agriculture. Remarkably, with this change to just a single industry, the missing trade statistic triples (to 0.39) and the slope coefficient doubles (to 0.60). When plotting the corresponding F_i against $V_i - s_i V_w$, the observations line up in an almost perfect line — there are no outliers whatsoever. Column 3 imposes consumption similarity on Government. While there is an improvement in the missing trade statistic and the slope coefficient, the improvement is modest. When we impose consumption similarity on both Agriculture and Government (see column 4) the improvement is not much more than what we obtain from imposing consumption similarity only on Agriculture. Columns 5 and 6 compare what happens when we impose consumption similarity on industries subject to trade restrictions (Agriculture plus Food) versus on industries subject to nontradeability (Government plus Construction). As is apparent, we get more of an improvement from industries with trade restrictions than from nontradeable industries. Finally, when we impose consumption similarity on all four industries we essentially get a perfect fit. Thus, lack of consumption similarity in just a few industries – especially two highly protected agricultural industries – explains virtually all of the missing trade.

8. Conclusions

After years of theoretical and empirical investigation, most researchers are confident that they completely understand the implications of empirical tests of the Vanek prediction. We have shown that this confidence is misplaced. Assuming the data are measured without error, rejection of the Vanek prediction either means that the factor content definition used is not Vanek-relevant or that the underlying assumptions (the model) are 'false.' This paper investigated both sources of rejection. First, we provided an expression for the Vanek-relevant factor content of trade in a world with arbitrary international choice-of-technique differences and traded intermediate inputs. As suggested by Deardorff (1982), this definition involves the worldwide factors used to produce a country's trade flows. We also showed that several prominent empirical examinations of the Vanek prediction have failed to use a Vanek-relevant factor content definition.

Second, the Vanek prediction is implied by a very large number of models e.g., Heckscher-Ohlin and CES monopolistic competition. This raises the question of what model or models are being tested when the Vanek prediction is under investigation. We completely characterized the class of models being tested. Specifically, we showed that consumption similarity is necessary and sufficient for a locally robust Vanek prediction. The proof is complex. For one, it uses methods that hold without any restrictions on the form of product market competition: yet we were able to avoid a pitfall in the literature in which trade propositions must be proved on a case-by-case basis for each type of product market structure. For another, we were able to show that all cases in which the Vanek prediction holds without consumption similarity are cases that are not robust to small changes in the underlying technology.

These two contributions of the paper – a Vanek-relevant factor content definition and a complete characterization of all models that imply and are implied by the Vanek prediction – make it clear exactly what is being tested when the Vanek prediction is investigated.

With this theoretical machinery we revisited the failure of the Vanek prediction. One source of failure is the use of U.S. input–output tables. There is a large improvement when using input–output tables from many countries, but not much is gained from moving to our Vanek-consistent definition of the factor content of trade. The remaining failure is entirely due to lack of consumption similarity in industries that are heavily protected (Agriculture and Processed food) or nontradeable (Government and Construction).

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Appendix A. The perturbations of D implied by perturbations of π' on $\Pi(\pi,\varepsilon)$

Before we can establish theorem 3 we will need to know more about how perturbations of technology π translate into perturbations of factor requirements D.

A.1. Technology primitives π

With imperfect competition, which is one possibility that we allow for, technology primitives may include firm-specific costs such as fixed costs. We must therefore drill down to the firm level. Let k index firms. Let q_{kgi} be the amount of good g that firm k produces in country i. The cost of producing q_{kgi} is $c_{kgi}(\omega_i, q_{kgi})$ where ω_i is a vector of factor prices.[26] Let π be the underlying technology that generates the cost functions $\{c_{kgi}\}_{\forall kgi}$. π is a set of firm-level technology parameters. We will write $c_{kgi}(\omega_i, q_{kgi}|\pi)$ as a function of π in order to indicate that c_{kgi} is parameterized by π.

A.2. Factor demands $D(\Pi)$

Under Assumption 1, a firm's vector of cost-minimizing average factor inputs is given by

$$d_{kgi} \equiv \left(\frac{1}{q_{kgi}}\right) \frac{\partial c_{kgi}\left(\omega_i, q_{kgi}|\pi\right)}{\partial \omega_i} \tag{19}$$

for $q_{kgi}>0$ and $d_{kgi}\equiv 0$ for $q_{kgi}=0$.

Let Q_{gi} be the gth element of Q_i and let D_{gi} be the gth column of D_i. Industry g output Q_{gi} is the sum of firm-level outputs q_{kgi}: $Q_{gi}=\sum_k q_{kgi}$. Industry factor demands $D_{gi}Q_{gi}$ are the sum of firm-level factor demands $d_{kgi}q_{kgi}$:

$$D_{gi}Q_{gi} = \sum_k d_{kgi}q_{kgi}. \tag{20}$$

A.3. Technical lemma

Define $R \equiv \left\{(\omega_i, q_{kgi}) : \omega_i \geq 0, ||\omega_i|| = 1, \underline{q} \leq q_{kgi} \leq \overline{q}\right\}$ for finite constants $\underline{q} > 0$ and $\overline{q}$. Let $\mathcal{K}(g,i)$ be the set of firms producing good g in country i. For notational ease we drop the (g,i) subscripts on d_{kgi}, c_{kgi} and q_{kgi} of Eq. (19) above.

Lemma 2. Assume Assumption 1. *Fix* $\delta>0$ and $k\in\mathcal{K}(g,i)$. For each $K\times 1$ vector of constants d'_k satisfying $\omega_i(\pi)d'_k = \omega_i(\pi)d_k(\pi), d'_k > 0$, and $||d'_k - d_k(\pi)||<\delta$, there exists a π' (i.e., a $c_k(\cdot|\pi')$ on R) such that

$$d'_k = (1/q_k(\pi))\partial c_k(\omega_i, q_k(\pi)|\pi')/\partial\omega_i \quad \textit{evaluated at } \omega_i = \omega_i(\pi) \tag{21}$$

$$c_k(\omega_i(\pi), \cdot|\pi') = c_k(\omega_i(\pi), \cdot|\pi). \tag{22}$$

Proof. Define

$$c_k(\omega_i, q_k|\pi') \equiv c_k(\omega_i, q_k|\pi) + \omega_i(d'_k - d_k(\pi))q_k(\pi) \quad \forall(\omega_i, q_k)\in R. \tag{23}$$

We first show that since $c_k(\cdot|\pi)$ is a cost function on R, so is $c_k(\cdot|\pi')$. $c_k(\cdot|\pi)$ and hence $c_k(\cdot|\pi')$ are differentiable (Assumption 1 (*iii*)), increasing in q_k, concave in ω_i, and linearly homogeneous in ω_i. Differentiating Eq. (23),

$$\frac{\partial c_k(\omega_i, q_k|\pi')}{\partial\omega_i} = \frac{\partial c_k(\omega_i, q_k|\pi)}{\partial\omega_i} + (d'_k - d_k(\pi))q_k(\pi). \tag{24}$$

Since $c_k(\cdot|\pi)$ is increasing in ω_i, $\partial c_k(\cdot|\pi)/\partial\omega_i$ is bounded away from zero on the compact set R. Since $||d'_k - d_k(\pi)||<\delta$ one can choose δ such that the right-hand side of Eq. (24) is positive. Thus, $c_k(\cdot|\pi')$ is increasing in ω_i. From Diewert (1982, theorem 2 and corollary 1.1), this establishes that $c_k(\cdot|\pi')$ is a cost function on R.[27] Eq. (21) follows from Eqs. (19) and (24) evaluated at $(\omega_i(\pi), q_k(\pi))$. Further, by hypothesis, $\omega_i(\pi)(d'_k - d_k(\pi)) = 0$. Hence, Eq. (22) follows from Eq. (23) with $\omega_i = \omega_i(\pi)$. Eq. (22) implies that total and marginal costs are constant on $\Pi(\pi, \varepsilon)$. □

A.4. The perturbations of D implied by perturbations of π' on $\Pi(\pi,\varepsilon)$ — Main Result

We can now characterize the set of perturbations of $D(\pi')$ implied by perturbations π' on Π. Specifically, define

$$\begin{aligned}\Delta(\pi,\varepsilon) \equiv \{D' : D' > 0, ||D' - D(\pi)|| < \varepsilon, \\ D'_i Q_i(\pi) = V_i \forall i, \\ \omega_i(\pi)D'_i = \omega_i(\pi)D_i(\pi)\forall i\}\end{aligned} \tag{25}$$

where $D' \equiv (D'_1, \ldots, D'_N)$ and $D'>0$ means that D' is non-negative with at least one positive element. Let $vec(D')$ be the $KGN\times 1$ vector built from the $K\times GN$ matrix D'. We say that two matrices D' and D'' are linearly independent if $vec(D')$ and $vec(D'')$ are linearly independent.

Lemma 3. Under Assumption 1, (i) $D(\Pi(\pi,\varepsilon)) = \Delta(\pi,\varepsilon)$ and (ii) $\Delta(\pi,\varepsilon)$ is a convex set that contains at least N linearly independent matrices.

Part (i) of Lemma 3 characterizes $\Pi(\pi,\varepsilon)$ in terms of the set $D(\Pi)$ it generates. That is, if $\pi'\in\Pi$ then $D'\equiv D(\pi')$ clears factor markets, generates the same factor costs per unit of output as $D(\pi)$, and is arbitrarily close to $D(\pi)$. Conversely, if $D'\in\Delta(\pi,\varepsilon)$ then there exists a $\pi'\in\Pi$ such that $D' = D(\pi')$. Hence, perturbing π is equivalent to perturbing D. This is helpful because D, but not π, is observable. Part (ii) of Lemma 3 shows that $\Delta(\pi,\varepsilon)$ consists of much more than just the singleton $D(\pi)$. It contains a continuum of elements. This implies that $\Pi(\pi,\varepsilon)$ also contains a continuum of elements.

Proof of Lemma 3.

Proof. Recall that Q_{gi} is the gth element of Q_i and D_{gi} is the gth column of D_i. Industry g output Q_{gi} is the sum of firm-level outputs q_k: $Q_{gi}=\sum_{k\in\mathcal{K}(g,i)} q_k$. Industry factor demands $D_{gi}Q_{gi}$ are the sum of firm-level factor demands $d_k q_k$:

$$D_{gi}Q_{gi} = \sum_{k\in\mathcal{K}(g,i)} d_k q_k.$$

For part (i) consider a $D'\in\Delta$. For each column D'_{gi} of D' it is tedious but straightforward to verify the following. There exists a $d'\equiv\{d'_k\}_{k\in\mathcal{K}(g,i)}$ satisfying the conditions of Lemma 2 and

$$\sum_{k\in\mathcal{K}(g,i)} d'_k q_k(\pi) = D'_{gi}Q_{gi}(\pi)\forall k, g \text{ and } i. \tag{26}$$

This equation states that the industry-level D'_{gi} are derivable from the firm-level d'_k.[28]

An outcome is a list O of all the endogenous variables. We next show that outcome $O'\equiv(d', D', \omega(\pi), E(\pi))$ satisfies Eqs. (6), (19) and (20) when the equations are evaluated at $(\pi', E(\pi))$ i.e., O' is consistent with competitive factor market clearing. Recall that E is a list that includes p_k, q_k as well as Q_i and its gth element Q_{gi}. Eq. (6) follows from $D'\in\Delta$ and the definition of Δ i.e., competitive factor demand $D'_i Q_i(\pi)$ equals exogenous supply V_i. Eq. (19) follows from Eq. (21)

[26] Cost functions also depend on intermediate input prices. Since these prices will be fixed throughout, we do not include them as arguments.

[27] Diewert lists four other regularity conditions on c_k that are easily verified. One can allow for $c_k(\cdot|\pi')$ to be non-decreasing and also deal with $q_k = 0$ (Diewert's II(ii)) by allowing d'_k to be a function on R rather than a constant.

[28] The case where $\mathcal{K}(g,i)$ has only one firm and the case where every firm in $\mathcal{K}(g,i)$ has a $d_k(\pi)$ with only one positive element must be treated separately from the general case because of the degeneracy of one or more of the conditions $\omega_i(\pi)d'_k = \omega_i(\pi)d_k(\pi)$, $\omega_i(\pi)D'_{gi} = \omega_i(\pi)D_{gi}(\pi)$, and $\sum_{k\in\mathcal{K}(g,i)} d_k(\pi)q_k(\pi) = D_{gi}(\pi)Q_{gi}(\pi)$.

evaluated at $E(\pi)$ i.e., d_k is cost minimizing. Eq. (20) follows from Eq. (26).

This result together with Eq. (22) imply that $D' = D(\pi')$. From the definitions of Δ and Π, this establishes that if $D' \in \Delta$ then there is a $\pi' \in \Pi(\pi, \varepsilon)$ such that $D' = D(\pi')$. Restated, $\Delta \subseteq D(\pi)$. The definitions of Π and Δ imply that if $\pi' \in \Pi$ then $D(\pi') \in \Delta$ i.e., $D(\Pi) \subseteq \Delta$. This establishes $\Delta = D(\Pi)$ and part (i) of Lemma 3.

Turning to part (ii), let K_i be the number of factors available in country i (i.e., non-zero elements of V_i) and let G_i be the number of goods produced in country i. Consider the equation systems $D_i'Q_i = V_i$ and $\omega_i D_i' = \omega_i D_i \; \forall i$. The unknowns $\{D_i'\}_{i=1}^N$ have $\Sigma_i K_i G_i$ elements that need not be zero. Post-multiplying $\omega_i D_i' = \omega_i D_i$ by Q_i and using $D_i'Q_i = V_i$ yields $\omega_i V_i = \omega_i V_i$ so that there is at least one linearly dependent equation per country or at most $\Sigma_i(K_i + G_i - 1)$ linearly independent equations. Since the solution set is non-empty ($D_i' = D_i \; \forall i$ is a solution), the solution set has at least $\Sigma_{i=1}^N K_i G_i - \Sigma_{i=1}^N (K_i + G_i - 1) = \Sigma_{i=1}^N (K_i - 1)(G_i - 1)$ linearly independent solutions. Since we have assumed that every country has at least two factors and produces at least two goods ($K_i \geq 2$ and $G_i \geq 2$), $\Sigma_{i=1}^N (K_i - 1)(G_i - 1) \geq N$ and there are at least N linearly independent solutions. Further, convex combinations of these linearly independent solutions are themselves solutions so that Δ is convex. □

Appendix B. Sketch of the proof of necessity (Theorem 3)

Theorem 3 can be written as follows. Suppose

$$D'(I-B)^{-1}(C_i - s_i C_w) = 0_K \forall i \tag{27}$$

holds for all D' in $\Delta(\pi, \varepsilon)$ i.e., for $D' > 0$ close to D such that

$$D_i' Q_i = V_i \forall i \tag{28}$$

$$\omega_i D_i' = \omega_i D_i \forall i. \tag{29}$$

Then

$$C_i = s_i C_w \forall i.$$

To see that theorem 3 can be written in this way, note that Eq. (27) follows from $F_i = (V_i - s_i V_w) - (AC_i - s_i AC_w)$ (Eq. 11) and $A \equiv D(I-B)^{-1}$ while Eqs. (28)–(29) follow from Lemma 3 and Eq. (25). In our notation, $D' \equiv [D_1' \cdots D_N']$, ω_i is a $1 \times K$ vector of factor prices and 0_K is a $K \times 1$ vector of zeros.

Eqs. (27)–(29) form a system of equations that is linear in D'. Also, for all D' on $\Delta(\pi, \varepsilon)$ or equivalently, for all π' on $\Pi(\pi, \varepsilon)$, we have that $(I-B)^{-1}(C_i - s_i C_w)$, Q_i, V_i, and ω_i are fixed. ($D_i = D_i(\pi)$ is also fixed because π is fixed.) Thus, we may interpret Eqs. (27)–(29) as a system of $KN + KN + GN$ equations in the KGN 'unknown' elements of D'. That is, the system can be re-written as $Mx' = m$ where the $KGN \times 1$ vector x' collects the elements of D', the $(KN + KN + GN) \times KGN$ matrix M collects the coefficients on D' and the $(KN + KN + GN) \times 1$ vector m collects the remaining terms.

The 'suppose' part of Lemma 3 states that every solution of the $KN + GN$ Eqs. (28) and (29) is also a solution of Eq. (27). Thus, the row rank of M is at most $KN + GN$. Further, because Eqs. (28)–(29) have at least one linear dependency for each i,[29] the row rank of M is at most $KN + GN - N$. The assumptions $K \geq 2$ and $G \geq 2$ imply that this row rank is at least one less than the number of columns KGN. Hence, the rank of M is at least one less than its number of rows and columns. This in turn implies that every one of the many $KN + GN - N + 1$ square sub-matrices that can be formed from the rows and columns of M has a zero determinant. These zero determinants place restrictions on the elements of M and, in particular, on the $C_i - s_i C_w$. Lemmas 4–6 in Trefler and Zhu (2005) establish the zero-determinant restrictions place $(G-1)N^2$ linear restrictions on the GN^2 elements of $C_i - s_i C_w$ $(i = 1, \ldots N)$. Finally, as assumed in the main text, suppose that each country j produces at least one traded intermediate good $g(j)$. Since $g(j)$ is not consumed, $C_{g(j),i,j} = 0 \; \forall i,j$. These additional N^2 restrictions together with the previously noted $(G-1)N^2$ restrictions imply GN^2 restrictions of the form $C_i - s_i C_w = 0$ $(i = 1, \ldots N)$. See Lemma 6 in Trefler and Zhu (2005). (The proof of the lemma in Trefler and Zhu (2005) is not quite right in that it does not assume at least one traded intermediate. The relevant modification is trivial.) This completes the sketch of the proof of necessity.

[29] To see this, pre-multiply Eq. (28) by ω_i and post-multiply Eq. (29) by Q_i to obtain the same equation.

Appendix C. Data description

Data on labour endowments V_i and direct labour usage by industry D_i are from various sources. Employment data by industry L_{gi} are from the OECD STAN database for OECD countries, the UNIDO data base for manufacturing in non-OECD countries and from the ILO for non-manufacturing in non-OECD countries. The endowment of labour, $V_{Li} \equiv \Sigma_g L_{gi}$, is scaled so that it sums to the PWT 6.1 workforce totals in 1997. Direct usage of labour by industry (D_{fgi}) is calculated as L_{gi}/Q_{gi} where Q_{gi} is output of industry g in country i. Q_{gi} is from GTAP. World consumption shares s_i are defined as $(GDP_i - TB_i) / \Sigma_j GDP_j$ where GDP_i is country i's real GDP in 1997 and TB_i is i's trade surplus. Data on GDP_i come from the PWT 6.1. Using PWT notation, $GDP_i \equiv RGDPC_i \times POP_i$ where $RGDPC_i$ is country i's real GDP per capita using the chain index (in 1996 international price) and POP_i is i's population.

In order to match the classification of industries in D with those in B we aggregated industries 3-digit ISIC (rev. 2) industries up to the 24 industries that appear in Table 3.[30]

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[30] The industries are: 110–130 (Agriculture), 200 (Mining); 311 + 312 (Food); 313 + 314 (Beverages); 321 (Textiles); 322 (Apparel); 323 + 324 (Leather); 331 + 332 (Wood products); 341 + 342 (Paper and publishing); 353 + 354 (Petroleum products); 351 + 352 + 355 + 356 (Chemicals); 361 + 362 + 369 (Non-metal manufacturing); 371 (Basic metal manufacturing); 372 (Non-ferrous metal manufacturing); 381 (Basic metal manufacturing); 384 (Transport equipment); 382 + 383 + 385 (Machinery); 390 (Other manufacturing); 400 (Electricity and water); 500 (Construction); 600 (Wholesale–retail); 700 (Distribution); 800 (FIRE); and 900 (Government).

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