# Capabilities, Wealth, and Trade

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We explore the relation between a country's income and the mix of products it exports. Both are simultaneously determined by countries' capabilities, that is, by countries' productivity and quality levels for each good. Our theoretical setup has two features. (1) Some goods have fewer high-quality producers/countries than others, meaning that there is comparative advantage. (2) Imperfect competition allows high- and low-quality producers to coexist. These two features generate an inverted-U, general equilibrium relationship between a country's export mix and its GDP per capita. We show that this inverted-U permeates the international data on trade and GDP per capita.

### I. Introduction

A country's capability—meaning the set of goods the country is able to produce and its quality and productivity in producing them—drives its per capita income and the sectoral mix of its exports. Aspects of the relationship between quality, income, and the sectoral mix of exports have been analyzed by a number of researchers. Hummels and Klenow (2005)

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estimate the impact of a country's per capita income and size on export quality. Hausmann, Hwang, and Rodrik (2007) explore how the process of "cost discovery" affects the sectoral mix of exports, which in turn affects per capita incomes. Flam and Helpman (1987) and Fajgelbaum, Grossman, and Helpman (2011) examine the codetermination of quality, income, and the sectoral mix of exports in a model in which demandside consumer heterogeneity plays a central role. In contrast, we use a supply-side Ricardian model to show how the general equilibrium logic of comparative advantage provides important theoretical and empirical insights into how quality capabilities simultaneously affect per capita incomes and the sectoral mix of exports (as well as prices, markups, and profits at the firm level).

To bring out these insights as clearly as possible, we focus theoretically and empirically on characterizing the range of countries exporting a specific good, as in Schott (2004), and on characterizing how these countries' market shares vary with their incomes. A standard intuition for the relationship between market shares and incomes runs through quality. Hummels and Klenow (2005) show that rich countries must have highquality exports because, at the aggregate level, rich countries have high prices and high world market shares. A related inference appears in Khandelwal (2010), Baldwin and Harrigan (2011), and Hallak and Schott (2011). We show both theoretically and empirically that this aggregate insight does not carry over in general equilibrium to the sectoral level because of Ricardian comparative advantage. For example, the United States is a high-quality producer of stainless steel, but this cannot be inferred from US stainless steel's high price and small world market share: the United States simply cannot compete with markedly inferior Chinese stainless steel because US wages have been bid up by high demand for military aircraft, virtualization software, and other hard-to-make goods and services that only a handful of rich countries are capable of producing.

Our model has two key elements. (*a*) We make the Ricardian assumption that products can be ordered by the scarcity of quality capabilities. Specifically, if a country has high quality in good k, then it has high quality in all goods ranked below k. This means that low-k goods are ones for which most countries have high quality and are in this sense "easy" to make. In contrast, high-k goods are ones for which few countries have high quality: they are "hard" to make. This assumption captures the notion of relative (in a Ricardian sense) scarcity of quality capabilities.<sup>1</sup>

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<sup>&</sup>lt;sup>1</sup> For concreteness, let there be K goods, K countries, and two quality levels. We are assuming that goods and countries can be ranked such that good k is produced at low quality

(b) We assume that goods are differentiated only by quality (pure vertical differentiation) and are supplied in markets characterized by Nash equilibrium in quantities (Cournot competition). We use this assumption to ensure that differing levels of quality will coexist in equilibrium. Elements a and b generate a correlation between a country's income and its export mix. A country that can produce only a few goods at high quality will survive in only a few markets, and these will be the low-k or easy markets. As a result, derived demand for the country's labor will be low and wages will be low. Thus, low-wage countries will export low-k goods. A country that can produce many goods at high quality will have a high derived demand for its labor and have high wages. High wages will make the country a high-cost producer of low-k goods. Hence, a high-wage country will survive only in high-k markets.

This Ricardian sorting generates an inverted-U relationship between income and market shares at the sectoral level. To understand why, consider a country whose capabilities improve at the sectoral level: that is, the country improves its quality in a k-ranked good until it reaches the world quality frontier, and then it improves its quality in the next, higherranked, good. During this quality improvement process, demand for the country's labor rises, as do its wages. As quality rises in good k, the country's world market share of the good rises initially because quality must rise faster than wage costs. This "direct" or "quality" effect underpins the Hummels and Klenow aggregate correlation. However, as quality then rises in a higher-ranked good, wages continue to rise, thus killing off the country's competitiveness in good k: even though the country is a high-quality producer of good k, its world market share must decline as capabilities rise in higher-ranked, tougher-to-make sectors. This familiar intersectoral, general equilibrium feedback through the labor market is what we call the Ricardian or wage effect. It is the reason for the downward-sloping section of the inverted-U relationship between income and market share. The model generates a large number of other theoretical predictions, which we describe below, but our empirics are concentrated on this inverted-U relationship.

Turning to our empirical work, we investigate the sectoral-level, inverted-U relationship between income and market shares using data for 94 countries in 2005. Data are from COMTRADE (four-digit Standard International Trade Classification [SITC] and six-digit Harmonized System [HS]) and, to a lesser extent, the US imports file (10-digit HS). The

by firms in countries ranked  $1, \ldots, k - 1$  and produced at high quality by firms in countries ranked  $k, \ldots, K$ . Then the number of countries that can produce good k at high quality is decreasing in k; i.e., high-quality capabilities are scarce and this scarcity is relatively greater for higher-ranked goods.

theory states that the inverted-U relationship is driven by labor market spillovers across sectors. We thus confine our attention to country-good pairs for which the good is important in the country's export basket and, by implication, in the country's labor market. For each good separately, we build a "product range," that is, a range of incomes defined by the income levels of the poorest and richest exporters of the good. Product ranges are related to Khandelwal's (2010) quality ladders: the latter describes a range of qualities while the former describes a range of incomes. We will not be estimating quality and hence will have nothing to say about quality ladders.<sup>2</sup> Schott's (2004) work on "overlap" leads us to expect that product ranges will be large, and this is indeed what we find. (The finding is not driven by China.) We then nonparametrically estimate the relationship between income and world market shares and show that it is exactly as predicted by the theory. (1) For those products produced only by the richest countries, the relationship is primarily positive: the direct or quality effect dominates. (2) For those products produced only by the poorest countries, the relationship is primarily negative: the Ricardian or wage effect dominates. (3) For the remaining "middle" products, the relationship is inverted-U, as first the quality effect and then the wage effect kick in. Restated, Ricardian comparative advantage based on relative scarcity of quality capabilities leads to general equilibrium wage effects that are central for understanding the cross-country, cross-sector relationship between quality, per capita income, and the sectoral mix of exports.

Changing subjects, we next turn to motivating our use of a nonstandard trade model, that is, one without perfect competition or constant elasticity of substitution (CES) monopolistic competition. Consider table 1. We ranked all six-digit HS codes by the size of their world exports, chose the top 10 codes, and identified the seven industries to which they belong. For each of these seven industries we then used firm-level data on worldwide production levels to compute four-firm concentration ratios. The second column of table 1 shows that the seven industries are typically highly concentrated at the global level. With the exception of auto parts, just four firms in each industry produce between 21 percent and 70 percent of global output. Thus these industries, which account for a huge 21 percent of global exports, are typically dominated by a small number of very large firms.

To account for global market dominance by a handful of firms we need a model with the following key feature: an infinite sea of low-quality rivals cannot erode the market share of a high-quality incumbent. CES

<sup>&</sup>lt;sup>2</sup> The fact that we are not estimating quality means that our agenda is very different from that of Khandelwal (2010) and Hallak and Schott (2011).

Industry	Share of World Exports (%)	Four-Firm Concentration Ratio (%)	
Passenger cars	6.0	48	
Semiconductors	5.2	35	
Auto parts	3.2	9	
Pharmaceuticals	3.1	21	
Laptops	1.4	57	
Mobile phones	1.3	56	
Aircraft	.9	70	
Aggregate	21.0	37	

TABLE 1						
TOP INDUSTRIAL	Exports A	ARE IN	Concentratei	INDUSTRIES		

NOTE.—This table lists the industries with the largest values of world exports. Export data are from COMTRADE. Four-firm concentration ratios are authors' calculations based on data sources reported in App. table G1. The aggregate four-firm concentration ratio is the exportweighted average of the industry-level concentration ratios.

and other monopolistic competition models do not have this key feature because love of variety ensures that an (infinite) inflow of new entrants reduces (to zero) the market share of any incumbent firm. Thus, such models cannot explain why global markets can be dominated by a handful of firms. In contrast, high concentration in global markets is readily explained by appeal to the above-mentioned key feature that low quality cannot drive out high. Further, the simplest and most analytically tractable model having this property is the Cournot model with quality.<sup>3</sup>

Our paper has four key elements: (1) multiple sectors that are ranked on the basis of Ricardian scarcity of quality capabilities, (2) an imperfectly competitive market structure that supports the coexistence of differing levels of quality, (3) endogenous income so that there can be general equilibrium spillovers across sectors via the labor market (wages), and (4) empirical work relating income to market shares at the sectoral level. With this in mind, we relate our paper to the existing literature.

Our results are driven entirely by supply-side considerations. Demand considerations play no role in our work. Allowing for demand-side heterogeneity and demand for quality that rises with income has yielded important insights for comparative advantage and per capita incomes (e.g., Flam and Helpman 1987; Hallak 2006, 2010; Choi, Hummels, and Xiang 2009; Fajgelbaum et al. 2011). However, the demand side by itself does

<sup>&</sup>lt;sup>3</sup> See Sutton (1998, 71) for a discussion. Cournot competition in international trade models appears in Neary (2003) and Neary and Tharakan (2012). The need to model small numbers of exporters is also taken up by Eaton, Kortum, and Sotelo (2012). The fact that individual exporters account for a large share of a country's exports is documented by, e.g., Bernard et al. (2007). The fact that individual exporters account for a large share of a country's output appears in, e.g., di Giovanni and Levchenko (2012). Our result is about the fact that individual firms account for a large share, not of a country's exports or output, but of the world market of a good. That is, our result is about market structure.

not provide a complete explanation of the cross-country relationship between per capita incomes, export baskets, and quality; one must also look at supply-side capabilities (e.g., Grossman and Helpman 1991).

The endogeneity of income allows us to bring issues of economic development to the forefront of our research. The relationship between per capita income and the mix of exports has been the subject of investigation at least since the discussion of ladders of development by Chenery (1960) and more recently by Leamer (1984, 1987), Michaely (1984), and Schott (2003). In these papers, as in ours, sectors are asymmetric and ordered. For example, Schott orders sectors by labor intensity. However, these papers do not consider quality, which is how we order sectors. Lall, Weiss, and Zhang (2006), Hausmann et al. (2007), and Schott (2008) provide policy-oriented discussions of the thesis that "what you export matters." We do not discuss policy in this paper. Implicitly, however, our work shifts the policy prescription away from getting the right mix of exports and toward raising the quality of what is exported.

This paper builds on a series of papers by Sutton. In Sutton (1991, 1998, 2007a, 2007b), firms optimally invest in building quality capabilities, and once these capabilities are developed, firms engage in Cournot competition. This leads to a world in which the relative scarcity of capabilities is an endogenous equilibrium outcome. We can motivate this idea of relatively scarce capabilities by reference to a key idea in the modern "market structure" literature: if firms must incur fixed and sunk outlays to develop their capabilities, then the number of firms that find it profitable to develop these capabilities will be limited: the greater the elasticity of quality (or productivity) responses to R&D or other fixed outlays, the greater the degree to which firms "escalate" their R&D spending in competing with rivals, and the smaller the number of producers that survive in the market. As a result, capabilities are scarce and scarcer in some markets than in others: relative scarcity emerges endogenously. This argument holds for a broad class of models, of which the Cournot model with quality is the simplest and most tractable example; it does not hold for CES models with atomistic firms. For a concise review, see Sutton (2007a). For a general equilibrium analysis of the mechanism of entry and R&D competition leading to this, see Sutton (2007b). In this paper, we simply take as given that some capabilities are relatively scarce.<sup>4</sup>

<sup>&</sup>lt;sup>4</sup> Sutton's (1991, 1998, 2007a) work is related to the literature on the endogenous choice of quality, e.g., Verhoogen (2008), Khandelwal (2010), and Kugler and Verhoogen (2012). Models with exogenous quality include Baldwin and Harrigan (2011) and Johnson (2012). On a separate note, Sutton (2007b) provides an international trade model with two countries that produce final goods and a third country that produces raw materials. He shows that when there are raw materials that are internationally traded, quality and productivity are not isomorphic and, in particular, that there is a minimum level of quality (independent of productivity or wages) that must be attained if a firm or country is to enter

The paper is structured as follows. Section II sets up the model, Sections III and IV present our two main results (goods are produced by ranges of countries and market shares exhibit an inverted-U), and Section V bridges from the theory to the empirics, which appear in Sections VI–IX.

## II. Setup

## A. Consumer Choice

Each country has L identical workers. All workers in all countries have identical Cobb-Douglas utility functions defined over goods indexed by m,

$$U = \prod_{m} (u_m x_m)^{\delta_m}, \tag{1}$$

where  $\sum_{m} \delta_{m} = 1$ , and  $u_{m}$  and  $x_{m}$  denote the quality and quantity of good *m* consumed. It follows from the form of the utility function that each consumer spends fraction  $\delta_{m}$  of income on good *m*. We assume that all profits accrue to a separate group of individuals, who also have a utility function of the form (1). From this it follows that total global expenditure on good *m*, which we denote as  $S_{m}$ , is a fraction  $\delta_{m}$  of world income.

Note that consumers choose both quantity  $x_m$  and quality  $u_m$ . Looking ahead, each firm is associated with a quality level, so choosing quality is equivalent to choosing a firm. Consumers of good *m* will be indifferent between any two firms that charge the same quality-adjusted price for *m*. That is, there is pure vertical differentiation.

## B. Equilibrium in the Product Markets

We characterize product market competition using the standard "Cournot model with quality" introduced in Sutton (1991). In this model, firms are characterized by a level of capability, consisting of a quality level and a productivity parameter denoting the number of worker hours per unit of output produced, together with a ("local") wage rate specific to the country in which the firm is located.<sup>5</sup> At equilibrium, some subset of

world export markets. Thus, there are limits on how much low wages can offset low quality. Hallak and Sivadasan (2013) also break the quality-productivity isomorphism by postulating the existence of a minimum quality threshold needed for exporting. In our paper, by contrast, quality and productivity are isomorphic.

<sup>&</sup>lt;sup>5</sup> Thus all costs are labor costs, and fixed costs are sunk, and so do not enter the present analysis. Materials costs, though of crucial importance in general, are here ignored in order to keep the analysis as clear as possible. This issue is examined in depth by Sutton (2007b), who shows that the key point is this: in the absence of material cost, low-wage countries can become viable in world markets even at low quality once their wage costs

firms are active in the production of the good. At equilibrium, all active firms offer the same quality-adjusted price. For each active firm, indexed by *i*, its output level is related to the inverse of its productivity  $c_i$ , its quality  $u_i$  and its (local) wage rate  $w_i$ . Solving for a Nash equilibrium in quantities, we obtain the firm's quality-adjusted equilibrium price,

$$\frac{p_i}{u_i} = \frac{1}{N_m - 1} \sum_j \frac{w_j c_j}{u_j},$$
(2)

and its quality-adjusted output level,

$$x_{i}u_{i} = \frac{N_{m} - 1}{\sum_{j} w_{j}c_{j}/u_{j}} \left[ 1 - (N_{m} - 1)\frac{w_{i}c_{i}/u_{i}}{\sum_{j} w_{j}c_{j}/u_{j}} \right] S_{m},$$
(3)

where  $N_m (\geq 2)$  denotes the total number of firms that are active in the global market for good *m* and the sum  $\Sigma_j$  is taken over all active firms. See Appendix A for a derivation of equations (2) and (3). One can see from equation (2) that  $p_i/u_i$  is the same for all active firms. It is useful to plug equation (2) into equation (3) to obtain an alternative expression for output:

$$x_i = \frac{1}{p_i} \left( 1 - \frac{w_i c_i}{p_i} \right) S_m \tag{3'}$$

for  $p_i > w_i c_i$  and  $x_i = 0$  otherwise. Thus, a firm is active in equilibrium if its price  $p_i$  exceeds its marginal cost  $w_i c_i$  or, equivalently, if its qualityadjusted price  $p_i/u_i$  exceeds its effective cost level  $w_i c_i/u_i$ .

Note that the right-hand sides of equations (2) and (3) depend on  $u_i$  and  $c_i$  only through the ratio  $u_i/c_i$ , which we refer to as the "capability" of firm *i*. It follows that some key relationships between capabilities and wages developed below will depend only on  $u_i/c_i$  and not on the absolute levels of  $u_i$  and  $c_i$ . Since our empirical focus is on quality  $u_i$ , without loss of generality we set  $c_i = 1$  for the remainder of the paper and periodically remind the reader that our comments about quality are also germane to productivity.

are sufficiently low: only the ratio of unit costs (wages times labor input) to quality matters to viability, and shortcomings in quality can be offset by a low value of the wage. But once material inputs as well as labor are required, a fall in the wage can reduce only unit costs to the world market value of the material input. This places a floor on price and so establishes a corresponding minimum quality level, independent of local wages, that is required for viability. Deficiencies in productivity can always be compensated for by low wages, but deficiencies in quality cannot. This is an important reason for emphasizing the role of quality in our present discussion.

## C. The Scarcity of Quality Capabilities

Assume for the moment that there are m = 1, ..., K goods and k = 1, ..., K types of countries. Equal numbers of goods and country types are a prelude to assortive matching between goods and country types. A type k country is a country whose firms can produce goods 1 to k but cannot produce goods k + 1 to K. The interpretation is that goods with higher indexes require capabilities that are scarcer. Higher-indexed goods are "harder" to make.

Following up on our introductory comments on the scarcity of capabilities, we assume that each type k country is endowed with a finite number of firms. For simplicity alone, we assume that each type k country has exactly one firm. This firm can potentially produce up to k products, that is, products 1, ...,  $k^{.6}$  Let  $N_k \ge 2$  be the number of type k countries. Since country types 1, ..., k - 1 cannot produce good k, there are  $\sum_{k \ge k} N_k$  firms that can potentially produce good k. Importantly, this sum is decreasing in k. How many firms actually produce in equilibrium is endogenous. As in Chaney (2008), we distinguish between potential firms and firms that are active. It is the potential number of firms that is fixed.

We next generalize this setup slightly by assuming that there are K groups of goods, each with H identical goods. Each good m is now indexed by the pair (h, k), and there is a total of HK goods; see figure 1. Now the above setup applies to each good in this expanded set of goods. Specifically, good (h, k) is potentially produced by one firm from each country of type  $k' \ge k$ , and, as before, there are  $\sum_{k \ge k} N_{k'}$  potential firms per good.

Conditional on being able to produce a good, firms differ in the quality of their goods. We begin with a very simple assumption: all firms that are able to produce a good produce it to common quality u. There is no need to let u vary across goods because quality comparisons are never made across goods. Quality comparisons are, however, made across the many producers of a single good: these quality differences are introduced below, where they play a central role.<sup>7</sup>

## D. Equilibrium

The above assumptions imply that there is symmetry across all countries within a country type. We therefore index only country types (k = 1, ...,

<sup>&</sup>lt;sup>6</sup> Recall that marginal costs are constant so that the firm's profit function is the additively separable sum of k profit functions. Thus, whether we think of there being k firms each potentially producing one good or one firm potentially producing up to k goods is a matter of notation. We choose the latter.

 $<sup>^7</sup>$  See our key propositions 3–5. Our remaining propositions (propositions 1 and 2) also generalize to the case in which there are quality differences across producers. These gen-



FIG. 1.-Types of countries and groups of goods

*K*) and speak of a representative type *k* country. We add one last assumption—preferences are symmetric across all goods within a group of goods—so that there is also symmetry across all goods within a group of goods. There is thus no need for the complicated m = (h, k) notation, which we replace with the following simpler notation. We let *g* denote a representative good in group g = 1, ..., K and use the phrase "a group *g* good" to indicate a representative good from group *g*. Thus, for example,  $S_g$  denotes the share of world expenditure spent on a representative good in group *g* and  $S_gH$  is the share of world expenditure spent on all *H* goods in group *g*. With this setup, the variables in equations (2) and (3) become  $u_i = u$ ,  $c_i = 1$ ,  $S_m = S_g$ ,  $p_i = p_g$  (the common price faced by all producers of group *g* goods),  $w_i = w_k$  (the wage in type *k* countries), and  $x_i = x_{gk}$  (the output of a representative group *g* good produced by a type *k* country).

*Product market equilibrium.*—The price that equates firm supplies with consumer demands is given by equation (2). Substituting our new notation into equation (2) yields the equilibrium price of a representative group g good:

$$p_g = \frac{1}{\sum_k N_k - 1} \sum_k N_k w_k,\tag{4}$$

where the sum  $\Sigma_k$  is over the set of country types that produce *g* in equilibrium.<sup>8</sup>

Labor market equilibrium.—Substituting our new notation into equation (3'), the profit-maximizing supply of a group g good by a firm in a type k country is

$$x_{gk} = \frac{1}{p_g} \left( 1 - \frac{w_k}{p_g} \right) S_g \tag{5}$$

eralizations appear as propositions 6 and 7 in App. E. They are relegated to an appendix because they are not essential for the empirics.

<sup>&</sup>lt;sup>8</sup> That is, over the set  $\mathbb{K}_g \equiv \{k : p_g > w_k \text{ and } k \ge g\}$ . Also, note that we are assuming throughout that there are no trade costs. With trade costs, eq. (4) must be modified.

for  $p_g > w_k$  and  $x_{gk} = 0$  otherwise. Since one unit of labor is needed to produce one unit of output and there are *H* goods in each group of goods,  $H\sum_g x_{gk}$  is the demand for labor in a type *k* country. Each type *k* country is endowed with *L* workers and each worker supplies one unit of labor so that *L* is the supply of labor. Hence the labor market-clearing condition in each type *k* country is

$$L = H \sum_{g} x_{gk}.$$
 (6)

Balanced trade.—To develop an expression for net exports of a type k country, note that sales of a typical group g good are  $p_g x_{gk}$ , total sales of all goods within group g are  $Hp_g x_{gk}$ , national income is  $H\sum_g p_g x_{gk}$ , and GDP per worker is  $y_k \equiv H\sum_g p_g x_{gk}/L$ . For a typical group g good produced by a type k country, output is  $x_{gk}$  and the value of consumption is  $\delta_g H\sum_{g'} p_{g'} x_{g'k}$  so that net exports are  $x_{gk} - (\delta_g H\sum_{g'} p_{g'} x_{g'k})/p_g$ . By Walras's law, product and labor market clearing together imply balanced trade.

An equilibrium is a set of product prices  $\{p_g\}_{g=1}^{K}$  and a set of wages  $\{w_k\}_{k=1}^{K}$  such that when consumers maximize utility and producers maximize profits, product markets clear internationally (eq. [4]), labor markets clear nationally (eq. [6]), and trade is balanced. The proof that an equilibrium exists is standard and appears in online Appendix F.

# III. Characterizing Equilibrium, Part 1: Product Ranges

Our analysis, and the empirical evidence presented later, focus on two equilibrium outcomes. First, higher-ranked country types will be richer. That is, both wages  $w_k$  and GDP per capita  $y_k$  are strictly increasing in k. Second, group g goods are produced by and only by the range of country types  $g, g + 1, ..., k_g$  ( $k_g$  is an endogenous integer, the highest-ranked producer of g). Countries ranked above  $k_g$  have wages that are too high to profitably produce g and countries ranked below g are not capable of producing g. Only countries  $g, ..., k_g$  produce g. Thus, each group of goods is produced by an interval of country types running from the poorest ( $y_g$ ) to the richest ( $y_{k_g}$ ). In this section, we develop a proposition that characterizes these "product range" intervals.

PROPOSITION 1 (Product ranges).

1. A group g good is produced by a type k country if and only if  $k = g, ..., k_g$  for some country type  $k_g$  that is increasing in g. That is, each good is produced by an interval of country types and both boundaries of the interval are increasing in g.

- 2. Wages  $w_k$  and GDP  $y_k$  are strictly increasing in k and  $p_g$  is strictly increasing in g. That is, countries with scarce capabilities have high wages and high GDP per worker while goods for which capabilities are scarce have high prices.
- 3. A type *k* country produces and produces only goods in groups  $g = g_k, \ldots, k$  for some  $g_k \le k$  that is increasing in *k*. That is, each country produces an interval of goods whose boundaries are increasing in *k*.

The proof appears in Appendix B. We note that it is parts 1 and 2 of this proposition that are central to our empirical development below. We describe the equilibria in proposition 1 as "product range" equilibria because each group g good is produced by a set of country types with a range of GDP per capita.<sup>9</sup>

*Remarks.*—First, there are two features of the model that are not amenable to empirical work. (*a*) The number of producers of any group *g* good is  $N_g + N_{g+1} + \cdots + N_{k_z}$ . This will not in general be decreasing in *g* because  $k_g$  is increasing in *g*. Thus, harder-to-make goods need not have fewer producers. It is easy to construct equilibria in which the number of producers is nonmonotonic or even increasing in *g*. We will thus have nothing to say empirically about the number of producers. (*b*) Proposition 1 places no restrictions on the length of product range intervals  $[g, k_g]$  because both boundaries are increasing in *g*. Further,  $k_g$  is determined as part of a general equilibrium solution that is necessarily complex. We will thus have nothing to say empirically about the length of product ranges.

Second, proposition 1 does not rely on any assumptions about the size of markets for each group of goods, that is, on the Cobb-Douglas parameters  $\delta_g$  or, equivalently, on the  $S_g$ . Further, proposition 1 holds for CES preferences.<sup>10</sup>

Third, we have assumed that quality (u) and labor forces (L) are the same for all countries. If we allow u and L to vary across countries, it is

<sup>9</sup> Schott (2004) pioneered research on product ranges. Proposition 1 deepens his analysis in that income is endogenous and there are multiple sectors, with the result that there are cross-sector spillovers via general equilibrium wage effects. It is these spillovers that determine the size of product ranges (i.e., the size of the  $k_{g}$ ) or, in Schott's terminology, the degree of "product overlap."

<sup>10</sup> Assume that utility is given by  $U = (\sum_m x_m^{\rho})^{1/\rho}$  (Cobb-Douglas is  $\rho = 0$ ). Then the eq. (5) expression for  $x_{gk}$  becomes

$$x_{gk} = \frac{1-\rho}{\left(p_g\right)^{1-\rho}} \left(1-\frac{w_k}{p_g}\right) S_g.$$

This does not result in any changes to proposition 1. Further, it leads to only minor changes in proposition 2 below. Specifically, in proposition 2 the term  $N_k/(N_k - 1)$  is replaced by  $N_k/(N_k - 1 + \rho)$  and the term  $N_k/S_k$  is replaced by  $N_k^{1-\rho}/S_k$ . Cobb-Douglas is used in propositions 3–5 below in order to derive closed-form solutions for all variables.

possible to construct examples in which the product range property of part 1 of proposition 1 fails. The simplest counterexample relates to heterogeneity in country sizes. Suppose there are three country types and three groups of goods. Countries of types 1 and 3 are very large while countries of type 2 are very small. Countries of type 1 can produce only group 1 goods. Countries of type 2 produce group 2 goods, demand for which absorbs all of their small labor force, so they do not produce group 1 goods. Countries of type 3, with their large labor force, produce all three groups of goods. In short, group 1 goods are produced by country types 1 and 3 but not 2, thus violating the product range property.<sup>11</sup>

Finally, in Appendix E we extend proposition 1 (and proposition 2 below) to the case in which qualities differ by country type  $(u_k)$ . There we assume that  $u_k \ge u_{k-1}$  and show that propositions 1 and 2 continue to hold; in addition, there will also be an equilibrium range of qualities and prices for each good. Such quality differences will be introduced explicitly in the next section, where they play a central role for our core testable propositions 3–5.

# IV. Characterizing Equilibrium, Part 2: The Inverted-U Relationship

Having identified product ranges, we now introduce cross-country differences in quality capabilities and show that product ranges are characterized by an inverted-U relationship between market shares and income. We do so in two steps. First, we initially abstract from quality differences and describe a benchmark perfect-sorting equilibrium in which group k goods are produced by and only by type k countries. Second, we then allow one type k "developing" country to improve its quality capabilities, first for group k goods and then for group (k + 1) goods. The increase in group k quality increases the developing country's wage and its group k exports. This is the upward-sloping portion of the inverted-U. The subsequent increase in group (k + 1) quality increases the developing country's wage and thus reduces its group k exports. This is the downward-sloping portion of the inverted-U. The next theorem describes the first-step perfect-sorting equilibrium.

PROPOSITION 2 (Perfect-sorting equilibria).

1. An equilibrium displays perfect sorting if and only if

(PSC) 
$$\frac{N_{k-1}}{S_{k-1}} \ge \left(\frac{N_k}{N_k - 1}\right) \frac{N_k}{S_k} \quad \text{for } k = 2, \dots, K.$$

 $^{\scriptscriptstyle 11}$  We will return to the consequences of country size heterogeneity in the empirical Sec. VIII below.

2. If the perfect-sorting condition (PSC) holds, then there is a unique equilibrium set of product prices and wages given by

$$p_k = \frac{HS_k}{LN_k} \quad \text{and} \quad w_k = \frac{N_k - 1}{N_k} \frac{HS_k}{LN_k} \quad \forall k.$$
(7)

Further, the markup is  $N_k/(N_k - 1)$ , GDP per worker is  $y_k = (HS_k)/(LN_k)$ , total profits are  $H\pi_{kk} = HS_k/N_k^2$ , output is  $x_{kk} = L/H$ , and net exports of a group g good are  $(1 - \delta_g H)L/H$  if the good is exported and  $-\delta_g L$  if the good is imported.

Note that from proposition 1,  $p_k$ ,  $w_k$ , and  $y_k$  are strictly increasing in k. The proof appears in Appendix C.<sup>12</sup>

It may help in interpreting the PSC to note that when the  $S_k$  are the same across countries, then the PSC is equivalent to  $N_{k-1} \ge N_k + 2$ . See Appendix D for a proof. This says that as we move to higher-ranked products, the number of firms capable of producing these products is smaller. In what follows we set all  $S_k = S$  in order to simplify notation and assume that we are starting from a perfect-sorting equilibrium.

Against this background of perfect sorting, we take one type (k - 1) country and allow it to produce goods in group k at a quality level  $v_k$ , which rises over time from zero to u. We will refer to this country as the "developing" country. As  $v_k$  rises, the developing country's mix of output will gradually shift from the production of group (k - 1) goods to group k goods. This change will, in general, affect the equilibrium wage rate of all countries of adjacent types and the total income (and expenditure) in each country and market. For a typical type k country, let  $w_k$  be its wage, let  $\overline{p}_k$  be its price, let  $\overline{x}_k$  be its quantity, and let u be its quality. Let  $w_{k-1}$ ,  $\overline{p}_{k-1}$ ,  $\overline{x}_{k-1}$ , and u be the corresponding variables for a typical type (k - 1) country. For the developing country, which may be producing both group (k - 1) and group k goods, let w be its wage, let  $p_{k-1}$  and  $p_k$  be its quantities, let u be its quality in group (k - 1) goods, and let  $v_k$  be its quality in group k goods.

Income or GDP per worker for our three types of countries is

$$y_{k-1} = H\overline{p}_{k-1}\overline{x}_{k-1}/L,$$
  

$$y_{k} = H\overline{p}_{k}\overline{x}_{k}/L,$$
  

$$y = H(p_{k-1}x_{k-1} + p_{k}x_{k})/L,$$
  
(8)

<sup>&</sup>lt;sup>12</sup> Part 2 of the proposition is trivial to prove because in a perfect-sorting equilibrium, eqq. (4) and (6) become  $p_k = w_k N_k / (N_k - 1)$  and  $L = H x_{kk} = H(1/p_k)(1 - w_k/p_k)S_k$ , which is trivially solved for  $p_k$  and  $w_k$ . The markup  $p_k / w_k$  follows from eq. (7);  $x_{kk}$  follows from  $L = H x_{kk}$ ; and  $y_k$  follows from  $y_k = H p_k x_{kk} / L$ . Profits are  $\pi_{kk} = (p_k - w_k) x_{kk}$  and total profits are  $H \pi_{kk}$ . At the end of Sec. II.D, we showed that net exports are  $x_{kk} - (\delta_k H p_k x_{kk})/p_k$ , which simplifies to  $(1 - \delta_k H) x_{kk}$ .

that is, revenue per good times the number of goods (H) divided by the workforce (L).

Consider the situation in which  $v_k$  has risen to the point where the developing country is producing both groups of goods. All producers of a good charge the same quality-adjusted price so that

$$\frac{p_{k-1}}{u} = \frac{\overline{p}_{k-1}}{u},\tag{9}$$

$$\frac{\underline{p}_k}{v_k} = \frac{\overline{p}_k}{u}.$$
(10)

For each group (k - 1) good there are  $N_{k-1} - 1$  typical producers with wage  $w_{k-1}$  and the one developing country producer with wage w. Hence multiplying equation (2) through by the common u,

$$\overline{p}_{k-1} = \frac{1}{N_{k-1} - 1} [(N_{k-1} - 1)w_{k-1} + w].$$
(11)

The developing country's presence in both the k - 1 and k industries means that it now has 2H firms, the original H firms producing each of the group (k - 1) goods and the H new firms producing each of the group k goods.<sup>13</sup> The total number of firms producing each type k good is therefore  $N_k + 1$ :  $N_k$  firms have wage  $w_k$  and quality u while the one developing country firm has wage w and quality  $v_k$ . Hence,

$$\frac{\overline{p}_k}{u} = \frac{1}{N_k} \left( N_k \frac{w_k}{u} + \frac{w}{v_k} \right). \tag{12}$$

To determine prices and wages, we need simply look at the labor marketclearing conditions. Recalling that labor supply is given by *L* and that labor demand is given by total output, labor market clearing for a typical type (k-1) country is  $L = H\overline{x}_{k-1}$ . Plugging the equation (3') expression for  $\overline{x}_{k-1}$  into  $L = H\overline{x}_{k-1}$  yields

$$L = H \frac{1}{\bar{p}_{k-1}} \left( 1 - \frac{w_{k-1}}{\bar{p}_{k-1}} \right) S, \tag{13}$$

where we have used the fact that every firm in a typical type (k - 1) country charges price  $\overline{p}_{k-1}$  and has wage  $w_{k-1}$ .

<sup>&</sup>lt;sup>13</sup> An alternative interpretation is as follows: the group k good is produced by the same firm that produces the corresponding group (k-1) good. Each firm now has two independent businesses, making k and k-1. Given Cobb-Douglas preferences, constant marginal costs, and the fact that the firm takes S as given, it follows that the firm's profit function is additively separable in its activities in the k-1 and k markets so that the firm sets the Cournot output in each of these markets. (Since there are no fixed costs, a firm will be active in any market where its quality supports a price that exceeds its marginal cost of production.)

For a typical type *k* country, labor market clearing is likewise given by

$$L = H \frac{1}{\overline{p}_k} \left( 1 - \frac{w_k}{\overline{p}_k} \right) S.$$
(14)

For the developing country, labor market clearing is  $L = H\overline{x}_{k-1} + H\overline{x}_k$  or

$$L = H \frac{1}{p_{k-1}} \left( 1 - \frac{w}{p_{k-1}} \right) S + H \frac{1}{p_k} \left( 1 - \frac{w}{p_k} \right) S.$$
(15)

Equations (9)–(15) are seven equations in the four prices and three wages. It is very easy to solve explicitly for these seven variables in terms of a numeraire, and this is done in Appendix F, Section A, where the numeraire is expenditures per good S. This establishes that for  $v_k$  in the range where the developing country produces both groups of goods, there is a unique equilibrium, and in this equilibrium there are closed-form expressions for all the equilibrium prices. Since our focus is comparative statics with  $v_k$ , we write all equilibrium outcomes as functions of  $v_{k}$  for example,  $w(v_k)$ ,  $p_k(v_k)$ ,  $x_k(v_k)$ , and  $S(v_k)$ .

We now explore the effect of raising the exogenous variable  $v_k$  from the critical value at which  $x_k(v_k)$  becomes positive, past the critical value at which  $x_{k-1}(v_k)$  becomes zero, and through to the value  $v_k = u$  at which the developing country becomes identical to a type k country. At the point where  $v_k = u$ , we are back in a world of perfect sorting, but with  $N_{k-1}$  and  $N_k$  replaced by  $N_{k-1} - 1$  and  $N_k + 1$ , respectively. To ensure this, for the remainder of the paper we assume that the PSC holds with these substitutions.<sup>14</sup>

Lemma 1. There are constants  $v_k^L$  and  $v_k^H$  with  $0 < v_k^L < v_k^H < u$  such that

$$egin{aligned} &v_k^L < v_k < v_k^H \Leftrightarrow x_{k-1}(v_k) > 0 & ext{ and } & x_k(v_k) > 0; \ &v_k^H < v_k < u \Leftrightarrow x_{k-1}(v_k) = 0 & ext{ and } & x_k(v_k) > 0. \end{aligned}$$

We refer to the situation in which the developing country is producing both  $x_{k-1}(v_k)$  and  $x_k(v_k)$  as *phase I*. We refer to the situation in which the developing country is producing only  $x_k(v_k)$  as *phase II*. A third phase, where the developing country has gained the capability of producing goods in group k + 1, will be described later. These three phases appear at the top of figure 2. Note that when  $v_k < v_k^L$ , the developing country's

<sup>14</sup> Specifically,

$$N_{k-1} - 1 \ge \left(\frac{N_k + 1}{N_k + 1 - 1}\right)(N_k + 1) = (N_k + 1)^2/N_k.$$

quality is so low that it specializes in group (k - 1) goods; that is, we are in the perfect sorting case of the previous section.

# A. Phase I

We begin by characterizing the equilibrium properties of phase I. These are illustrated in figure 2 and stated in the following proposition.

**PROPOSITION 3** (Phase I). Consider  $v_k \in (v_k^L, v_k^H)$  so that  $x_{k-1}(v_k) > 0$ and  $x_k(v_k) > 0$ . Then as  $v_k$  rises from  $v_k^L$  to  $v_k^H$ :

- 1.  $x_{k-1}(v_k)$  falls to zero and  $x_k(v_k)$  rises from zero; that is, the developing country reduces output of group (k 1) goods and increases output of group k goods;
- 2.  $p_{k-1}(v_k)/w(v_k)$  falls and  $p_k(v_k)/w(v_k)$  rises; that is, markups charged by developing country firms fall for group (k-1) goods and rise for group k goods;
- 3.  $w(v_k)/S(v_k)$  rises; that is, the wage in the developing country rises relative to the numeraire; and
- 4.  $y(v_k)/S(v_k)$  rises; that is, GDP per worker in the developing country rises relative to the numeraire.

The core phase I insight is that as the developing country's quality capabilities improve in group k goods, the developing country experiences rising demand for its output and labor and hence rising wages. This creates two effects: (1) Improved quality improves the country's competitiveness in group k goods. This is the direct or "quality" effect. (2) Higher wages reduce the country's competitiveness in group (k - 1) goods. This is the general equilibrium Ricardian or "wage" effect.

While not used for our empirics, we note that there are cross-country general equilibrium impacts. For example, as China improves its quality capabilities in autos, we would expect that clothing manufacturers in Bangladesh would be better off and auto manufacturers in the United States would be worse off. The next proposition makes this point.

**PROPOSITION 4** (Phase I). As  $v_k$  rises from  $v_k^L$  to  $v_k^H$ , (1)  $w_{k-1}(v_k)/S(v_k)$  rises and  $w_k(v_k)/S(v_k)$  falls; that is, relative to the numeraire, the wage rises in type (k - 1) countries and falls in type k countries; and (2)  $w(v_k)/w_{k-1}(v_k)$  rises; that is, the wage rises faster in the developing country than in type (k - 1) countries.

## B. Phase II

When quality capabilities rise above  $v_k^H$ , our developing country's wage becomes so high that its firms are no longer viable (profitable) in the



FIG. 2.—Advancing quality. The top panel shows the developing country's quality in group k goods advancing from  $v_k = 0$  to  $v_k = u$ , and then its quality in group (k + 1) goods advancing from  $v_{k+1} = 0$  to  $v_{k+1} = u$ . The critical value of  $v_k$  at which production of group k goods becomes viable is labeled  $v_k^L$  and is marked by the left-most dashed vertical line. The critical value of  $v_k$  at which production of group (k - 1) goods becomes unviable is labeled  $v_k^H$  and is marked by the left-most dashed vertical line. The critical value of  $v_k$  at which production of group (k - 1) goods becomes unviable is labeled  $v_k^H$  and is marked by the dashed vertical line separating phases I and II. The second and third panels show how equilibrium income (wages and GDP per worker) and the markup rise as qualities rise. The bottom two panels show how the output of goods in groups k - 1, k, and k + 1 change.

markets for group (k - 1) goods. Equations (10)–(15) continue to hold with only two modifications. The labor market equilibrium condition for the developing country, equation (15), now becomes

$$L = Hx_k = H\frac{1}{p_k} \left(1 - \frac{w}{p_k}\right) S.$$
(15-II)

Also, the price of group (k - 1) goods, equation (11), must be modified because there are now only  $N_{k-1} - 1$  producers, all with wage  $w_{k-1}$  and quality *u*. Hence, from equation (2) with  $N_g = N_{k-1} - 1$ ,

$$\overline{p}_{k-1} = \frac{N_{k-1} - 1}{N_{k-1} - 2} w_{k-1}.$$
(11-II)

Equations (10), (11-II), (12), (13), (14), and (15-II) are six equations in the three prices and three wages. It is very easy to solve explicitly for these six variables in terms of the numeraire *S*, and this is done in Appendix F, Section B. This establishes existence and uniqueness and provides a complete characterization of equilibrium in phase II.

The impact of  $v_k$  rising to u appears in figure 2 as the early part of phase II (where  $v_{k+1}$  is still zero) and is stated in the next proposition.

PROPOSITION 5 (Early phase II). Consider  $v_k \in (v_k^H, u)$  so that  $x_{k-1}(v_k) = 0$  and  $x_k(v_k) > 0$ . Then as  $v_k$  rises from  $v_k^H$  to u:

- 1.  $p_k(v_k)/w(v_k)$  rises; that is, the markup charged by developing country firms rises;
- 2.  $w(v_k)/S(v_k)$  rises; that is, the wage in the developing country rises relative to the numeraire; and
- 3.  $y(v_k)/S(v_k)$  rises; that is, GDP per worker in the developing country rises relative to the numeraire.

# C. Phase III

We now allow the developing country's quality capabilities in group (k + 1) goods, which we denote by  $v_{k+1}$ , to rise from zero to u (fig. 2, top panel). All of our lemma 1 and proposition 3 results relating changes in  $v_k$  to changes in variables subscripted by k - 1 and k now apply when relating changes in  $v_{k+1}$  to changes in variables subscripted by k and k + 1, respectively. In particular, there is a critical value  $v_{k+1}^L$  at which the developing country begins producing goods in group k + 1 and a critical value  $v_{k+1}^H$  at which the developing country stops producing goods in group k. During this process markups rise for group (k + 1) goods (the direct or quality effect). Further, wages and GDP per worker rise relative to the numeraire, and this reduces the markups for group k goods (the Ricardian or wage effect); see figure 2.

## V. Toward Empirics: Implications for Exports

To examine these predictions empirically, we will use international data and therefore need expressions for exports. We start by noting that, in equilibrium, all producers charge the same quality-adjusted price so that consumers do not care which producer they buy from. We therefore assume that in equilibrium there is no cross-hauling of goods across international borders; that is, a good is either imported or exported, but not both.

Let  $\overline{X}_{k-1}(v_k)$  and  $\overline{X}_k(v_k)$  be the value of exports for typical countries of type k-1 and k, respectively. Note that a lowercase x is the quantity of output and an uppercase X is the value of exports. Also note that these are values (price times quantity) since that is what we observe in the data. The developing country's value of exports for a good in group k-1 and a good in group k is denoted by  $X_{k-1}(v_k)$  and  $X_k(v_k)$ , respectively.<sup>15</sup> The next lemma states that exports behave like production.

LEMMA 2 (Exports).

- 1. As  $v_k$  rises in phase I,  $X_{k-1}(v_k)/S(v_k)$  and  $X_{k-1}(v_k)/\overline{X}_{k-1}(v_k)$  fall to zero.
- 2. As  $v_k$  rises in phases I and II,  $X_k(v_k)/S(v_k)$  and  $X_k(v_k)/\overline{X}_k(v_k)$  rise from zero.

In our empirics we will examine the value of exports of group (k - 1) and group k goods as a share of world exports. For the developing country these shares are given by

$$\theta_{k-1}^{X}(v_{k}) \equiv \frac{X_{k-1}(v_{k})}{X_{k-1}(v_{k}) + (N_{k-1} - 1)\overline{X}_{k-1}(v_{k})},$$

$$\theta_{k}^{X}(v_{k}) \equiv \frac{X_{k}(v_{k})}{X_{k}(v_{k}) + N_{k}\overline{X}_{k}(v_{k})};$$
(16)

 $\theta_{k+1}^X$  is the same as  $\theta_k^X$  but with the *k* index incremented by 1.

The evolution of these world export shares follows immediately from lemma 2. In phase I, the developing country is shifting out of group (k - 1) production and into group k production. This benefits type (k - 1) countries and hurts type k countries so that  $\theta_{k-1}^{X}$  falls and  $\theta_{k}^{X}$  rises. In phase II, the developing country is specialized in group k production and getting better at it, which hurts type k countries. Thus,  $\theta_{k}^{X}$  rises.

<sup>&</sup>lt;sup>15</sup> Typical type (k - 1) countries produce and export a single good so that gross and net exports are equal. For some values of  $v_{k_3}$  the developing country produces two goods, and one of these is produced in such small amounts that the good is imported. Thus, for the developing country, exports are gross rather than net.

We now make the transition from theory to empirics. We do not observe capabilities, but changes in capabilities induce observable changes in GDP per worker and the value of exports. These are illustrated in figure 3, which plots export shares against GDP per worker for the developing country. In phase I,  $y(v_k)$  increases,  $\theta_{k-1}^X(v_k)$  falls to zero, and  $\theta_k^X(v_k)$  rises from zero. Note that  $\theta_k^X(v_k)$  does not start rising as soon as phase I is entered because at this point production of group *k* goods is so small that these goods are still imported. The point at which exporting of group *k* goods starts is indicated in figure 3 by  $y_{\min,k}$ . This  $y_{\min,k}$  is very important for our empirics.

In the first part of phase II, where  $v_k$  is rising to u, both  $y(v_k)$  and  $\theta_k^X(v_k)$  rise. In the second part of phase II, where capabilities in k + 1 are rising but have not yet reached a level at which the developing country can enter group (k + 1) markets, nothing happens; see figure 2. It follows that the system is "stuck" at the point where y = y(u).

In phase III, the developing country enters group (k + 1) markets and grabs world market share; that is,  $\theta_{k+1}^X$  rises. This drives up wages and makes the developing country less competitive in group k markets. As a result  $\theta_k^X(v_k)$  falls. As the process continues, the developing country reaches the point where it consumes the small amount of  $x_k$  that it is still producing, that is,  $\theta_k^X(v_k) = 0$ . By definition, phase III ends when  $x_k = 0$ , so  $\theta_k^X(v_k)$  goes to zero just before the end of the phase. This is indicated



FIG. 3.—Empirics: Quality and the income-export nexus. The figure illustrates two of the central empirical predictions of the model. First, a single good can be produced both by low-wage, low-quality countries and by high-wage, high-quality countries. The points  $y_{\min,k}$  and  $y_{\max,k}$  are the GDP per worker of the poorest and richest countries producing a group k good. Second, world market shares will be an inverted-U-shaped function of GDP per worker.

on figure 3 by the point  $y_{\max,k}$ , which is also important for our empirics. It is straightforward to calculate closed-form solutions for  $y_{\min,k}$  and  $y_{\max,k}$ .<sup>16</sup>

Two key empirical points emerge from figure 3. First, poorer, lowquality countries can produce the same good as richer, higher-quality countries. Low-quality countries compete because in equilibrium they have low wages. This implies that there are "product ranges," that is, ranges of GDP per worker compatible with viability in the market. In figure 3, the product range for group *k* goods is  $(y_{\min,k}, y_{\max,k})$ . Second, whereas in a single-sector model, world market shares increase with quality, in a multisector world there are Ricardian forces that lead to inverted-U-shaped world market shares. As quality rises, wages rise, and this leads to a loss of competitiveness in low-*k* goods. As a result, export shares eventually "turn down." These are the two main empirical predictions that we will examine. A third prediction, that prices rise with quality and hence with income, is an important implication of the model; however, we explore it empirically only in Appendix H because it has been examined elsewhere (e.g., in Schott 2004).<sup>17</sup>

We can restate this in a way that makes one of the key points of our thesis crystal clear. A poor country can advance out of low-ranked goods and still remain poor: this happens when the country enters as a lowquality producer into goods with wide product ranges. Since, as we shall

<sup>16</sup> There are two minor points in the figure that are not used in what follows. (1) At the start of phase I, all countries are identical so that each of the type (k - 1) countries has a world market share of  $\theta_{k-1}^x = 1/N_{k-1}$ . At the end of phase II, all countries are again identical so that each type k country has a world market share of  $\theta_k^x = 1/(N_k + 1)$ . At the end of phase III, it is  $\theta_{k+1}^x = 1/(N_{k+1} + 1)$ . (2) Production always goes to zero at the end of a phase. Since gross exports go to zero before production does (i.e., at the point where all domestic production is consumed domestically), gross exports always go to zero before the end of a phase. Likewise, at the start of a phase, production starts and is consumed domestically so that exporting starts just after the start of a phase.

<sup>17</sup> Additionally, we note that our model, which has endogenous wages, delivers a clear statement about how prices rise both because of quality improvements and because of rising marginal costs (wages). Many models of international trade and quality treat wages as exogenous and so cannot give such a clear statement. On a separate note, some readers will have noticed that the output-quality or output-income relationship in fig. 3 looks like the Heckscher-Ohlin output-capital or cones of diversification relationship (see Learner 1984; Schott 2003). One might wonder, then, why was our theory needed? For one, improvements in quality are very different empirically from capital deepening. More importantly, this prediction is just one of several predictions that arise in our model, and all these predictions flow from the fact that there is imperfect competition. Imperfect competition is needed to ensure that firms are large and markets concentrated (as in table 1), that different qualities coexist, that prices are a nonconstant markup over marginal cost, that prices are correlated with quality, and that export shares are correlated with income in ways that reflect quality. In short, our output-quality relationship is just one of several predictions. The output prediction in isolation can be modeled more simply; however, we are interested in a bundle of predictions that require us to append an imperfectly competitive market structure onto a trade model. Thus, a cones of diversification Heckscher-Ohlin model delivers at best only a small part of what is needed and a more natural trade model in our setting is the Ricardian model, which emphasizes the role of technological capability for delivering quality.

show empirically, most goods have wide product ranges, we might expect this type of no-growth shift in product mix to be common. By the same token, a country may move from being poor to being rich without changing its product mix: this happens when it improves the quality of the wide product range goods that it already exports. In short, GDP per worker depends not just on what a country produces (as in Hausmann et al. 2007) but on the quality of what is produced.

# VI. Data

Trade data are from COMTRADE for 2005. We use the four-digit SITC revision 2 classification (henceforth SITC4).<sup>18</sup> To verify that all of our cross-sectional results hold for more detailed commodity breakdowns, we also use the 2005 COMTRADE data at the six-digit HS level (1996 revision; henceforth HS6) and the 2005 US import data at the 10-digit HS level (henceforth HS10). We exclude countries whose population was less than 2 million in 2005 or whose territorial integrity changed substantially between 1980 and 2005, for example, the USSR. (The exception is Germany, which we include.) This leaves us with the 94 countries listed in Appendix G. GDP per capita and population data are from the United Nations. We do not use a purchasing power parity adjustment because we are interested in nominal price competition in world product markets.

## VII. Product Ranges

A key prediction of our theory is that in general, equilibrium countries with different quality capabilities may nevertheless export the same good. See the product range in figure 3. While we do not observe quality, an observable implication is that at least some goods will be produced by both rich and poor countries. To investigate, we build on Schott's (2004) earlier and highly influential observation about "product overlap." For each product *g* we identify the poorest and richest exporters of the product. Denote these by  $y_{min,g}$  and  $y_{max,g}$ , respectively. In constructing these we avoid "noise" associated with small reported export values, a problem to which trade data are notoriously prone, by looking only at the set of countries for which good *g* is a "significant" export. A good is a significant export for a country if the value of exports of that good constitutes at least 1 percent of the value of exports of the country's principal export good.<sup>19</sup>

<sup>&</sup>lt;sup>18</sup> This allows us to go back to 1980 for a large number of countries and check that our results hold for these earlier years.

<sup>&</sup>lt;sup>19</sup> More formally, let  $\ell$  index countries, let g index goods, let  $X_{g\ell}$  be the value of country  $\ell$ 's exports of good g, and let  $\sum_{g} X_{g\ell}$  be country  $\ell$ 's total exports. Identify the good that ac-

ensures that the good is sufficiently important to the exporter to generate the general equilibrium wage impacts on which our theory rests.

Product ranges are displayed in figure 4. Each point corresponds to a unique SITC4 good (*g*), and the figure plots ( $y_{\min,g}$ ,  $y_{\max,g}$ ). A point therefore shows the range of income levels of countries for which *g* is a significant export. All the points necessarily lie above the 45-degree line. For reference, along the axes we show the log GDP per capita of Nepal, China, Poland, and the United States.

The striking feature of figure 4 is the preponderance of points in the top-left corner, that is, the preponderance of products for which the income range is very wide. To get a clearer sense of magnitudes, consider goods with product ranges for which  $y_{\max,g} - y_{\min,g} > 4$ . For such goods the richest significant exporter is at least 55 times richer than its poorest significant exporter ( $e^4 = 55$ ). These are huge differences. And there are a lot of goods in this region: the region contains 50 percent of all products displayed in the figure and accounts for 73 percent of world trade in our 547 products.

We will shortly show the reader that this observation about wide product ranges is robust and holds even in the most detailed trade data (HS10). However, we first draw three economic insights from the wideness of product ranges. The first deals with Hausmann et al. (2007). Their exercise uses all goods in a country's export basket even though products with wide ranges are "uninformative" about a country's income in the sense that knowing that a wide range product is a significant contributor to a country's export basket tells us little about the country's income. Figure 4 shows that such "uninformativeness" is the norm rather than the exception.

Second, our theory emphasizes that for each product, multiple quality levels can coexist in equilibrium. One can therefore interpret the wide ranges as support for the theory provided that one is willing to accept that product ranges are the result of quality differences. As is well known, quality is difficult to identify without detailed data about product characteristics. Since we do not have this information, we refer to the ranges as product ranges rather than as quality ranges and take the weaker position that wide product ranges are implied by the theory but do not imply the theory.

Third, there are two distinct groups of points that lie far from the topleft corner in figure 4. These are "informative" products. The first group lies to the bottom left and consists of those goods exported only by low-

counts for the largest share of country  $\ell$ 's exports, i.e., the good with the largest  $X_{g\ell}/X_{\ell}$ . Call this good  $g(\ell)$ . Then good g is a significant export of country  $\ell$  if  $X_{g\ell}/X_{\ell} > 0.01X_{g(\ell),\ell}/X_{\ell}$ . Next, let K(g) be the set of countries for which g is a significant export. Then  $y_{\min,g}$  is the poorest country in K(g) and  $y_{\max,g}$  is the richest country in K(g).



FIG. 4.—Product ranges. Each point represents an SITC4 product. The horizontal axis is  $\ln y_{\min,g}$ , the poorest country for which the product is a significant export. The vertical axis is  $\ln y_{\max,g}$ , the richest country for which the product is a significant export.

income and middle-income countries. The second group lies to the top right and consists of those goods exported only by high-income countries. On our present interpretation, low- and middle-income goods are not produced by high-income countries because their wage costs are too high, whereas high-income goods are not produced by low- and middleincome countries because their quality capabilities are too low.

The reader will and should be skeptical about the wide product ranges in figure 4. For the remainder of this section we anticipate five possible objections to the figure.

1. It is all aggregation bias.—One would expect that the large product ranges displayed in figure 4 would become much narrower with finer product-level data. This is not the case. In figure 5 we repeat the exercise using HS6 data (world trade data from COMTRADE) and using HS10 data (US import data). The distributions of product ranges in figures 4



FIG. 5.—Product ranges: Insensitivity to aggregation. Each panel in this figure is constructed in the same way as figure 4 but with different data. Figure 4 used the SITC4 classification and COMTRADE (world) data. The top panel of the current figure uses the HS6 classification and COMTRADE data. The bottom panel uses the HS10 classification and US import data.

and 5 are very similar. In particular, product ranges remain large, and about half of the world exports of the plotted goods are accounted for by uninformative products in the top left (ln  $y_{max,g} > \ln y_{min,g} + 4$ ).<sup>20</sup>

2. Finer disaggregation is always better.—The fact that nothing changes when moving to finer levels of product disaggregation may seem puzzling, since if the move to a finer level of aggregation involved the breaking up of technologically disparate subindustries into individual industries, we might expect the range to narrow as we move to this new level of aggregation. An examination of the way in which industries are broken up in the HS6 and HS10 data throws light on why disaggregation beyond SITC4 does not alter the distribution of ranges. In some cases the SITC4 industry is as disaggregated as the HS6 and even the HS10 industries; for example, new tires for motor cars is a single category in both SITC4 and HS6. In other cases, the disaggregation is based only on size or value, without any reference to capabilities; for example, new tires for motor cars feeds into seven HS10 codes that distinguish between technologyambiguous differences in the diameter of the tire. In yet other cases the SITC4 code is disaggregated only by introducing a capability-irrelevant "parts of" HS6 or HS10 code. This is pervasive; for example, see the HS6 categories associated with SITC4 7817, nuclear reactors. Finally, in those cases in which a technology-based disaggregation of products is introduced, it is often unclear whether this disaggregation conveys any information about differences in required capabilities; for example, SITC4 7252, machinery for making paper pulp, paper, paperboard; cutting machines, is disaggregated in HS10 into a number of industries, including machines for making paper bags etc. and machines for making paper cartons etc. Thus, finer disaggregation is typically not more informative about quality capabilities. Were an ideal disaggregation of industries to be constructed on the basis of the quality capabilities required, this would doubtless lead to some narrowing in the relevant ranges. However, the limitations of the published data are quite serious even at the most disaggregated level.21

3. *Estimation error.*—Another possible objection to our wide product ranges is that we have not reported standard errors. Let  $N_x^{\text{sig}}$  be the num-

 $<sup>^{20}</sup>$  In fig. 5 there are thousands of points, many of which lie on top of each other. To make the figure clearer, instead of plotting ln  $y_{\max,g}$  on the vertical axis we have plotted ln  $y_{\max,g} + \epsilon$ , where  $\epsilon$  is a uniformly distributed random variable on (-0.05, 0.05). This adds a tiny random vertical shift to the data, which helps the reader see where the bulk of points are located. Likewise, we have added a tiny random horizontal shift to ln  $y_{\min,g}$ .

<sup>&</sup>lt;sup>21</sup> For what we are doing, the relevant market is never equatable with an item in a government commodity classification, be it SITC4, HS6, or HS10. Sometimes the relevant market is more detailed than HS10 (as in many electronic parts) and sometimes the relevant market is less detailed than SITC4 (as in many apparel products). Thus, all of our conclusions must be thought of relative to a definition of the market that is determined by the commodity classification, not the actual product producers.

ber of countries for which *g* is a significant export.<sup>22</sup> It is possible that products with wide ranges are products for which  $N_g^{\text{sig}}$  is small, that is, for which there are very few observations and hence large standard errors. This is not the case; indeed, the opposite is true. The correlation between  $N_g^{\text{sig}}$  and the product range  $\ln y_{\max,g} - \ln y_{\min,g}$  is positive (.57), and, for example, products with  $N_g^{\text{sig}} \ge 20$  (one-quarter of all products) all have large product ranges. However, to deal with this objection in the simplest way possible, in figures 4 and 5 we have displayed only those products for which there are at least three significant exporters ( $N_g^{\text{sig}} \ge 3$ ). That is, we displayed only 547 of the possible 746 SITC4 goods. These 547 products account for 98.3 percent of world trade so that we are excluding only very minor products. We conclude from this that wide product ranges are not an artifact of statistical uncertainty. To be safe though, we will continue throughout this paper to restrict attention only to products for which  $N_g^{\text{sig}} \ge 3$ .

4. Wide product ranges are an artifact of using a 1 percent cutoff for "significant exporters."—Again, this is not the case. Online appendix figure B1 shows that the inference we have drawn from figures 4 and 5 is insensitive to the choice of cutoffs. It repeats figure 4 for a low percentage cutoff (0.1 percent), a high percentage cutoff (10 percent), and cutoffs based on mixtures of percentages and dollar values ( $x_{gk} >$ \$5 million or  $x_{gk} >$ \$50 million). In every case the pattern displayed in figures 4 and 5 is repeated.<sup>23</sup>

5. Wide product ranges are driven by China.—Omitting China does not alter the impression that product ranges are wide. Indeed, the reader can omit China from these figures simply by deleting all points for which either  $\ln y_{\min,g} = 7.5$  or  $\ln y_{\max,g} = 7.5$  (China's log GDP per capita is 7.5).

Having established the robustness of figure 4, we can now restate our conclusion. Our theory implies that there will be product ranges: the empirical surprise is that product ranges are often so large.

## VIII. Market Share Predictions

Figure 3 presented our predictions about a country's share of world exports. Underlying that figure is a comparative static in which a country

<sup>&</sup>lt;sup>22</sup> The number  $N_g^{\text{sig}}$  is the dimension of K(g) in fn. 19.

<sup>&</sup>lt;sup>23</sup> There is a minor technical point about fig. 5 that should be reviewed. Since the United States is far from most countries and since trade costs increase in distance, we expect that countries' exports to the United States will be more concentrated on a few goods than their exports to the world. This is indeed the case. Therefore, for the HS10 panel of fig. 5, which is based on US data, we use a 0.1 percent cutoff instead of a 1 percent cutoff. This results in far more points in the figure but does not alter the distribution of points in the figure. See online app. fig. B1 for the HS10 figure using a 1 percent cutoff.

that previously specialized in producing good k-1 first sees its quality in good k rise up from a very low level to that of the world standard and then sees its quality in good k + 1 rise up from a very low level to that of the world standard. This comparative static highlighted two mechanisms affecting world export shares. First, as capabilities rise in good k, the country produces more of k and gains an increasing share of world exports. This is the direct or quality effect. Second, as quality rises for good k + 1 wages are pushed up, which erodes the country's competitiveness in good k. This is the general equilibrium Ricardian or wage effect. These two mechanisms lead to the world export share predictions in figures 2 and 3. For middle-capability goods (k), world export shares display an inverted-U-shaped relationship with income as first the quality effect and then the wage effect come into play. For low-capability goods (k-1), the wage effect is dominant and world export shares tend to decline in income. For high-capability goods (k + 1), the quality effect is dominant and world export shares tend to increase in income; see figure 3.

We operationalize these distinct export share predictions of goods k - 1, k, and k + 1 as follows. Consider figure 4. In our baseline method we draw two vertical lines on the figure, one at some income  $\underline{c}$  and another at some higher income  $\overline{c}$ . This divides all points in the figure into three groups. Good  $\underline{g}$  is in group k - 1 if  $\ln y_{\min,g} \leq \underline{c}$ , in group k if  $\underline{c} < \ln y_{\min,g} \leq \overline{c}$ , and in group k + 1 if  $\ln y_{\min,g} > \overline{c}$ . In our baseline specification we choose the  $\underline{c}$  and  $\overline{c}$  so that one-third of countries are in each group  $(\underline{c} = 6.81 \text{ and } \overline{c} = 8.5)$ . In our alternative method, we draw two horizontal lines on figure 4, which divides goods into three groups based on  $y_{\max,g}$  rather than  $y_{\min,g}$ . As we shall see, the two methods yield almost identical results. Further, we will show that our results are not sensitive to the choices of  $\underline{c}$  and  $\overline{c}$ .

In defining groups, we must eliminate the "uninformative" goods to the top left of figure 4. We do so in two ways. First, we exclude goods with the widest product ranges, that is, goods *g* for which  $\ln y_{\max,g} - \ln y_{\min,g} > d$ . In our baseline specification we use d = 4 and show below that our results are not very sensitive to the choice of *d*. Second, in terms of the theory, the low and middle groups should consist of goods that are produced only by low- and middle-income countries, not high-income countries such as Germany that are famous for high quality. We therefore exclude from the low and middle groups those goods with  $\ln y_{\max,g} \ge \ln y_{Germany} = 10.4$ .

Since we will be pooling product ranges that have different ranges of incomes (both mean and variance), it is essential that we recenter and normalize incomes within product ranges. Letting  $\ell$  index countries, define normalized GDP per capita, normalized within product range g, as

$$\frac{\ln y_{\ell} - \mu_g}{\sigma_g}$$

We consider three alternative normalizations, that is, three alternative definitions of  $(\mu_g, \sigma_g)$ .

• Baseline normalization: In our baseline specification we consider the set of countries with positive exports of g and define  $\mu_g$  and  $\sigma_g$ as the median and interquartile range, respectively, of the ln  $y_\ell$  in this set.

Using medians and interquartile ranges has the advantage of being robust to outliers.

- Alternative normalization 1: Again consider the set of countries with positive exports of g and let  $\ln \underline{y}_g$  and  $\ln \overline{y}_g$  be the minimum and maximum, respectively, of the  $\ln y_t$  in this set. Then  $\mu_g = (\ln \underline{y}_g + \ln \overline{y}_g)/2$  and  $\sigma_g = \ln \overline{y}_g \ln y_g$ .
- and  $\sigma_g = \ln \overline{y}_g \ln \underline{y}_g$ . • Alternative normalization 2:  $\mu_g = (\ln y_{\min,g} + \ln y_{\max,g})/2$  and  $\sigma_g = \ln y_{\max,g} - \ln y_{\min,g}$ .<sup>24</sup>

All three normalizations are centered on 0 and yield similar results.

We also need a normalization for the level of exports. This will be affected, as the theory indicates, by product market size and country size. The global market size for product g is given by  $S_g$  (or, equivalently,  $\delta_g$ ) in the theory; see Section II.A. To control for  $S_g$ , we scale country  $\ell$ 's exports of good g,  $X_{g\ell}$ , by world exports of g,  $X_g \equiv \Sigma_\ell X_{g\ell}$ . To control for country size we scale  $X_{g\ell}/X_g$  by its average  $\Sigma_g (X_{g\ell}/X_g)/n_\ell$ , where  $n_\ell$  is the number of goods exported by country  $\ell$ .<sup>25</sup> Summarizing, we plot

(Normalized GDP per Capita)<sub>gl</sub> = 
$$\frac{\ln y_l - \mu_g}{\sigma_g}$$
 (17)

<sup>&</sup>lt;sup>24</sup> Note that  $y_{g}$  differs from  $y_{min,g}$ . The former is the minimum across all countries that export any positive amount of good g while the latter is the minimum across all countries for which g is a significant export; likewise for the difference between  $\overline{y}_{g}$  and  $y_{max,g}$ .

<sup>&</sup>lt;sup>25</sup> This normalization comes from a simple extension of our theory. In the theory, all countries have the same size *L*. When country sizes are allowed to differ so that labor forces become  $L_6$  then the eq. (6) labor market-clearing condition becomes  $L_{\ell} = H \sum_{g \in \Gamma_{\ell}} x_{g\ell}$ , where  $\Gamma_{\ell}$  is the set of goods produced by country  $\ell$ . Thus, conditional on the set of goods a country produces, larger countries will, on average, have higher output per good produced. It follows that larger countries will, on average, have higher exports per good exported, i.e., higher  $\Sigma_g(X_{g\ell}/X_g)/n_{\ell}$ .

against

(Normalized World Export Share)<sub>gl</sub> 
$$\equiv \frac{(X_{gl}/X_g)}{(1/n_l)\Sigma_g(X_{gl}/X_g)},$$
 (18)

where the numerator is country l's share of world exports of g and the denominator is country l's average share of world exports.

Figure 6 plots normalized world export shares against normalized GDP per capita for our three groups of goods. The first thing to note about the plots is the preponderance of points on or very near the horizontal axis. This reflects the fact that smaller countries have zero exports of many goods. Empirically, a group of goods requiring a given level of manufacturing capability may be very large, and a small country that specializes in the production of goods in this group will typically not produce all of the goods in the group. This point has been emphasized by Eaton and Kortum (2002).<sup>26</sup> Hence in interpreting these scatters, our focus of interest lies not on means—which tend to be dominated by the many (*g*, *l*) pairs with zero and near-zero exports—but on the upper bound of the scatter. With this in mind, we estimate a quantile regression (the 90th quantile). This appears as the curve shown in each of the panels of figure 6.<sup>27</sup>

The upper, middle, and lower panels correspond to low-group goods (k - 1), middle-group goods (k), and high-group goods (k + 1), respectively. The panels bear out the inverted-U predictions of the model. World export shares are decreasing in income for the low group, are increasing in income for the high group, and display an inverted-U relationship for the middle group. This is exactly as predicted in figure 3 and is a key take-away of this paper.<sup>28</sup>

Specification searches.—Underlying figure 6 are a large number of specification choices: the choice of normalization, the choice of grouping goods vertically using  $y_{\min,g}$  or horizontally using  $y_{\max,g}$ , the choice of group boundaries ( $\underline{c}$  and  $\overline{c}$ ), the choice in defining "uninformative" goods (d in  $y_{\max,g} - y_{\min,g} > d$ ), the choice between SITC4 and HS6, and

<sup>27</sup> We use the SAS 9.2 QUANTREG procedure with a polynomial of order n, where n is determined using standard 1 percent significance tests.

<sup>28</sup> There are a few extreme "vertical" outliers that would "squash" fig. 6 down to the horizontal axis if displayed. Rather than leave them off the figure entirely, we shrink them toward the horizontal axis as follows. In the top panel, if a vertical point *y* exceeds 5, then it is replaced by  $5 + f(\Delta)$ , where  $\Delta = y - 5$  and  $f(\Delta) = \ln(1 + \Delta)/5$  so that f(0) = 0 and f' > 0; likewise for the middle and bottom panels, but with 5 replaced by 3 and 4.5, respectively. This does not have any effect on the position of the quantile regressions.

<sup>&</sup>lt;sup>26</sup> One feature that we do not model but that captures this is that producing any product involves some fixed cost. A large country has the labor force to produce all products within the group or groups in which it specializes. A small country has the labor force to produce only a small number of products within some single group. See the closely related point regarding heterogeneous country sizes in the second-last paragraph of Sec. III.



FIG. 6.—Normalized world market shares. Each point in the plot corresponds to a product-country  $(g, \ell)$  pair. The vertical axis is country  $\ell$ 's share of world exports of good g, normalized as in equation (18). The horizontal axis is country  $\ell$ 's income, normalized as in equation (17). The figure uses the baseline specification:  $\underline{c} = 6.81$ ,  $\overline{c} = 8.50$ , d = 4,  $\mu_g$  is the median GDP per capita for exporters of g,  $\sigma_g$  is the corresponding interquartile range, and the curves are 90th quantile regressions.

some less explicit choices surrounding quantile estimation. The only feature of figure 6 that is sensitive to these specification choices is the monotonicity displayed in the top panel. In some specifications the top panel displays an inverted-U shape, though, reassuringly, with a maximum to the far left of the panel. This far-left maximum is consistent both with having defined the low group too broadly (so that it includes some middle goods) and with imprecise estimation caused by too few observations at the extreme left of the panel.

For the remainder of the section we briefly review a large number of alternative specifications that hit on each of the specification choices listed above. In each case, the results are identical to those of figure 6 (with the exception of the above-mentioned far-left maximum in the top panel). Online appendix figure B2 uses y<sub>max,g</sub> to group goods. Online appendix figure B3 varies the group boundaries c and  $\overline{c}$ . In the first column of panels, the low-group results are presented for the cases in which the baseline boundary c = 6.81 takes on the values 5.6, 6.0, 6.4, and 6.8. In the third column of panels, the middle-group results are presented for the cases in which the baseline boundary  $\underline{c} = 6.81$  takes on the values 6.0, 6.4, 6.81, and 7.2. In the middle column of panels, the middle-group results are presented for the cases in which the baseline boundary  $\overline{c}$  = 8.5 takes on the values 7.0, 8.0, 8.5, and 9.0. Even stronger results hold for the high group (not reported). Online appendix figure B4 reports the results for the two alternative normalizations of GDP per capita. Online appendix figure B5 reports results for six-digit HS goods. Because there are many more goods, we are able to divide the sample into four groups, each with one-quarter of the 94 countries in the sample. Online figure B6 examines the role of the optimal polynomial fitted to the quantiles. Online figure B7 examines the impact of changing the definition of uninformative goods (d in the expression  $y_{max,q}$   $y_{\min,g} > d$ ; d takes on the values 2, 3, 4, and  $\infty$ . From online figures B2-B7, it is apparent that figure 6 and our inverted-U results are not sensitive to our specification choices.29

# IX. Development Ladders in the Cross Section: The Role of Product Ranges

Appendix H shows that at the HS10 level, unit value import indexes (prices) are correlated with income within each product range (as in

<sup>&</sup>lt;sup>29</sup> There are a series of more minor specification choices that also do not matter. In eq. (18) we normalized world export shares  $X_{g\ell}/X_g$  by average world export shares  $n_l^{-1}\Sigma_g(X_{g\ell}/X_g)$ . We have experimented extensively with alternative normalizations of  $X_{g\ell}/X_g$ , including the 50th, 75th, 90th, and 99th percentiles of each country's world export shares. (For each country  $\ell$  these are percentiles of the set  $\{X_{1\ell}/X_1, \dots, X_{g\ell}/X_g, \dots, X_{d\ell}/X_G\}$ .) All of these normalizations produce curves with the same shapes as those in fig. 6. Also, we reported quantile regressions based on the 90th quantile. The curves do not change when, e.g., we drop down to the 80th quantile or rise up to the 99th quantile.

Schott 2004). The appendix also shows that wider product ranges have wider ranges of unit value import indexes. Wide product ranges, their correlation with unit values, and, most importantly, our inverted-U relationship all suggest that a country's wealth depends not just on what goods it produces but also on the quality of the goods produced.

To investigate further, let L goods denote the low- and middle-group goods of figure 6 and let H goods denote the high-group goods in figure 6. Figure 7 shows the relationship of a country's GDP per capita with (a) the share of L goods in its export basket (left panel) and (b) the share of H goods in its export basket (right panel). We see a clear fall in the share of L goods and a rise in the share of H goods as GDP per capita increases. But an important feature of figure 7 lies in the fact that the relation between the product mix and income is not bidirectional: while significant exporters of H goods are necessarily rich, it is not the case that rich countries are necessarily significant exporters of H goods. A very low contribution of H goods is consistent with a relatively high level of GDP per capita (see Malaysia in the right-hand panel). Similarly, while a high share of L goods necessarily implies that a country is poor, many poor countries have a low share of L goods (see Burundi or Zimbabwe in the left-hand panel).

We can restate this in a way that makes one of the key points of our thesis crystal clear. A poor country can advance out of L goods and still remain poor: this happens when the country enters as a low-quality pro-



FIG. 7.—The share of *L* and *H* goods in each country's export basket. Each point represents a country (there are 94 points in each panel). The horizontal axis is log GDP per capita in 2005. The vertical axis is a country's exports of *L* goods (right panel) or *H* goods (left panel) as a share of the country's total exports. The *L* goods are goods in the low and middle groups of figure 6; *H* goods are goods in the high group of figure 6.

ducer into uninformative goods, that is, goods with wide quality ranges. Since most goods are uninformative, we might expect this type of nogrowth shift in product mix to be common. By the same token, a country may move from being poor to being rich without changing its product mix: this happens when it improves the quality of the uninformative, wide quality range goods that it already exports.

# X. Conclusions

The aim of this paper has been to explore the way in which advances in wealth are associated both with changes in the product mix and with changes in quality (and productivity) within a given set of industries. The central point relates to the fact that the range of income levels of significant exporters of most products is very wide. At a theoretical level, one reason for the wide product ranges lies in aggregation of disparate subindustries (though the result is not sensitive to the level of aggregation); another reason lies in the fact that within any industry, in a general equilibrium multicountry setting, there will be a product range, that is, a coexistence of producers from countries with different income levels. In this product range, poor low-quality exporters compete with rich high-quality exporters.

The central property of product ranges is that, in a multimarket general equilibrium setting, the relation between quality and price on the one hand and output and global market share on the other is nonmonotonic. There is at equilibrium a range of producer qualities (and so wealth levels) that are viable in a given industry. As quality rises, the country moves into the production of higher-ranked goods, and its equilibrium wage (and GDP per capita) rises. But this means that its output and global market share both exhibit an inverted-U relationship with quality, and so with GDP per capita. As quality rises, market share rises, and wages rise also. As the country advances into the production of higher-ranked products, the rise in wage causes its effective cost level to rise and its global market share in this industry to fall. It is this inverted-U relation that is the basis of the selection effect that links a country's wealth to its product mix.

## Appendix A

## The Cournot Equilibrium and Derivation of Equations (2) and (3)

Consider good *m*. Since there is no horizontal differentiation, all firms charge the same quality-adjusted price  $p_m^*$  (=  $p_i/u_i$ ). Let  $x_i^* \equiv x_i u_i$  and  $w_i^* = w_i c_i/u_i$  be quality-adjusted quantity and effective cost, respectively. Cobb-Douglas demand implies  $S_m = \sum_i p_m^* x_i^*$  so that

$$p_m^* = \frac{S_m}{\sum_j x_j^*} \quad \text{and} \quad \frac{\partial p_m^*}{\partial x_i^*} = -\frac{S_m}{(\sum_j x_j^*)^2} = -\frac{p_m^{*2}}{S_m}.$$
 (A1)

Firms choose quality-adjusted output  $x_i^*$  to maximize profits  $\pi_i = (p_m^* - w_i^*)x_i^*$ . The first-order condition together with equation (A1) imply that the qualityadjusted optimal output is  $x_i^* = (p_m^* - w_i^*)S_m/p_m^{*2}$ . Summing over active firms  $i = 1, ..., N_m$  yields

$$\sum_j x_j^* = N_m S_m p_m^* - \left(\sum_j w_j^*\right) S_m / p_m^{*2}.$$

Plugging  $\sum_{j} x_{j}^{*} = S_{m}/p_{m}^{*}$  (from eq. [A1]) into this equation and simplifying yields  $p_{m}^{*} = (\sum_{j} w_{j}^{*})/(N_{m} - 1)$ , that is, equation (2). Plugging this back into

$$\begin{aligned} x_i^* &= (p_m^* - w_i^*) S_m / p_m^{*2} \\ &= (p_i / u_i - w_i c_i / u_i) S_m / (p_i / u_i)^2 \\ &= u_i (1 / p_i) (1 - w_i c_i / p_i) S_m \end{aligned}$$

yields equation (3).

## Appendix B

## **Proof of Proposition 1**

Lemma 3. Let  $\mathbb{K}_g$  and  $\mathbb{K}_{g'}$  be the sets of country types that produce goods in groups g and g', respectively. If  $\mathbb{K}_{g'}$  is a proper subset of  $\mathbb{K}_g$  ( $\mathbb{K}_{g'} \subseteq \mathbb{K}_g$ ), then  $p_{g'} > p_g$ .

\* *Proof.* Since  $\mathbb{K}_{g'}$  is a proper subset of  $\mathbb{K}_{g}$ , we can write  $\mathbb{K}_{g}$  as  $\mathbb{K}_{g} = \mathbb{K}_{g'} \cup A$  for some set *A*. From equation (4),

$$p_{\mathbf{g}'} = \frac{\sum_{\mathbf{k} \in \mathbb{K}_{\mathbf{g}'}} N_{\mathbf{k}} w_{\mathbf{k}}}{\sum_{\mathbf{k} \in \mathbb{K}_{\mathbf{g}'}} N_{\mathbf{k}} - 1} \text{ and } p_{\mathbf{g}} = \frac{\sum_{\mathbf{k} \in \mathbb{K}_{\mathbf{g}'}} N_{\mathbf{k}} w_{\mathbf{k}} + \sum_{\mathbf{k} \in \Lambda} N_{\mathbf{k}} w_{\mathbf{k}}}{\sum_{\mathbf{k} \in \mathbb{K}_{\mathbf{g}'}} N_{\mathbf{k}} + \sum_{\mathbf{k} \in \Lambda} N_{\mathbf{k}} - 1}.$$

In equilibrium, *k* produces *g* if and only if  $w_k < p_{g^*}$ . Hence

$$p_g < \frac{\sum_{k \in \mathbb{K}_{\mathscr{C}}} N_k w_k + \sum_{k \in A} N_k p_g}{\sum_{k \in \mathbb{K}_{\mathscr{C}}} N_k + \sum_{k \in A} N_k - 1}.$$

Cross-multiplying yields

$$\left(\sum_{k\in\mathbb{K}_{g'}}N_k+\sum_{k\in A}N_k-1\right)p_g<\sum_{k\in\mathbb{K}_{g'}}N_kw_k+\sum_{k\in A}N_kp_g$$

 $\left(\sum_{k\in\mathbb{K}_{\nu}}N_{k}-1\right)p_{g} < \sum_{k\in\mathbb{K}_{\nu}}N_{k}w_{k}$ 

or

$$p_g < \left(\sum_{k \in \mathbb{K}_{g'}} N_k w_k\right) / \left(\sum_{k \in \mathbb{K}_{g'}} N_k - 1\right) = p_{g'}.$$

QED

LEMMA 4. Each type *k* country produces and produces only goods  $g = g_k, \ldots, k$  for some group of goods  $g_k \leq k$ .

*Proof.* Recall that  $g_k$  is the lowest-ranked group of goods produced by a type k country so that the lemma states that each country type produces and produces only goods in an interval  $g_k, \ldots, k$ . Suppose that the lemma is false. Then there is some type k' country that produces goods in some group g but not group g + 1 where  $g + 1 \le k'$ . Equivalently, suppose that  $p_{g+1} \le w_{k'} < p_g$  for  $g + 1 \le k'$ . The set of producers of groups of goods g and g + 1 are

$$\mathbb{K}_{g} \equiv \{ k : w_{k} < p_{g}, k \ge g \},$$
$$\mathbb{K}_{g+1} \equiv \{ k : w_{k} < p_{g+1}, k \ge g + 1 \}.$$

Since  $p_{g^{+1}} \le w_k$ , the set  $A = \{k : w_k \le w_k, k \ge g + 1\}$  contains  $\mathbb{K}_{g^{+1}}$ . Mathematically,  $\mathbb{K}_{g^{+1}} \le A$ . Since  $w_{k'} \le p_g$ , the set  $B = \{k : w_k \le w_k, k \ge g\}$  is contained in  $\mathbb{K}_g$ . Mathematically,  $B \subseteq \mathbb{K}_g$ . But  $A \subseteq B$ . Hence  $\mathbb{K}_{g^{+1}} \subseteq A \subseteq B \subseteq \mathbb{K}_g$  or  $\mathbb{K}_{g^{+1}} \subseteq \mathbb{K}_g$ . If  $\mathbb{K}_{g^{+1}} \subseteq \mathbb{K}_g$ , then, by lemma 3,  $p_{g^{+1}} \ge p_g$ . If  $\mathbb{K}_{g^{+1}} = \mathbb{K}_g$ , then  $p_{g^{+1}} = p_g$ . Hence  $p_{g^{+1}} \ge p_g$ , which contradicts  $p_{g^{+1}} \le w_{k'} \le p_g$ . QED

LEMMA 5.  $p_g$  is strictly increasing in g.

*Proof.* Group g goods are produced by some country type *k*. If *k* does not produce g - 1, then  $p_{g^{-1}} \le w_k < p_g$ , as required. If *k* does produce g - 1, then it must be that  $p_{g^{-1}} < p_g$ . For suppose not, that  $p_{g^{-1}} \ge p_g$ . Since  $p_g > w_k \Rightarrow p_{g^{-1}} > w_k$ , all producers of *g* profitably produce g - 1. In addition, by lemma 4, type (g - 1) countries also produce g - 1. (They of course cannot produce *g*.) Thus, the set of producers of *g* is a proper subset of the set of producers of g - 1. It follows from lemma 3 that  $p_g > p_{g^{-1}}$ . This contradicts our maintained assumption of  $p_{g^{-1}} \ge p_g$ , as required. QED

LEMMA 6. Suppose  $w_k \ge w_{k-1}$ , k = 2, ..., K. Then for each g = 1, ..., K, there exists a  $k_g$  such that group g goods are produced by and only by country types  $k = g, ..., k_g$ .

*Proof.* Group g goods are produced by some country. Let  $k_g$  be the highest-ranked country that produces good g. Then  $w_{k_z} < p_g$ . By the hypothesis of the lemma,  $w_k \le w_{k_x}$  for all  $k < k_g$  so that  $w_k < p_g$  for all  $k < k_g$ . Hence g is produced by all country types  $k \le k_g$  provided that those types are capable of producing them, that is, provided that  $k \ge g$ . QED

LEMMA 7. In equilibrium,  $w_k > w_{k-1}$ ,  $k = 2, \ldots, K$ .

*Proof.* Suppose not, that  $w_k \le w_{k-1}$  for some k. Since all countries have the same labor supply L, it suffices to show that  $w_k \le w_{k-1}$  implies that labor demand is higher in k than in k - 1. From equation (5),  $x_{jk} = (1/p_j)(1 - w_k/p_j)S_j$ , so that  $x_{j,k-1} \le x_{jk}$ . This implies

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862 or

$$L = H \sum_{j \in \mathbb{K}_{k-1}} x_{j,k-1}$$
$$\leq H \sum_{j \in \mathbb{K}_{k-1}} x_{jk}$$
$$\leq H \sum_{j \in \mathbb{K}_{k}} x_{jk}$$
$$= L.$$

where  $\mathbb{K}_{k-1}$  and  $\mathbb{K}_k$  are the set of goods produced by country types k - 1 and k, respectively. The first line is the labor market-clearing condition for a type (k - 1) country (eq. [5]); the second line follows from  $x_{jk} \leq x_{j,k-1}$ ; the third line follows from the fact that  $w_{k-1} < p_j$ , and  $w_k \leq w_{k-1}$  imply  $w_k < p_j$  so that  $j \in \mathbb{K}_{k-1}$  implies  $j \in \mathbb{K}_k$ ; and the fourth line follows from labor market clearing in a type k country. If either of the above inequalities holds strictly, then we have the contradiction L < L. Suppose  $w_k < w_{k-1}$ . Then  $x_{jk} < x_{j,k-1}$  for all  $j \in \mathbb{K}_{k-1}$  and the first inequality holds strictly. Suppose  $w_k = w_{k-1}$ . Then the conditions of lemma 6 are satisfied so that type k countries produce group k goods. Thus, type (k - 1) countries produce a proper subset of the goods produced by type k countries so that  $\sum_{j \in \mathbb{K}_{k-1}} x_{jk} < \sum_{j \in \mathbb{K}_k} x_{jk}$  and the second inequality holds strictly. QED

LEMMA 8.  $k_g$  is nondecreasing in g.

*Proof.* Suppose not, that  $k_{g+1} < k_g$ . Then by lemmas 7 and 6, group (g + 1) and group g goods are produced by the set of country types in the sets  $\mathbb{K}_{g+1} \equiv \{k : g + 1 \le k \le k_{g+1}\}$  and  $\mathbb{K}_g \equiv \{k : g \le k \le k_g\}$ , respectively. Since  $g < g + 1 \le k_{g+1} < k_g$ , we have  $\mathbb{K}_{g+1} \subset \mathbb{K}_g$  and, by lemma 3,  $p_g < p_{g+1}$ . Since  $p_g < p_{g+1}$  and country type  $k_g$  produces  $g(w_{k_z} < p_g)$ , it follows that  $k_g$  also produces g + 1 ( $w_{k_z} < p_{g+1}$ ). But then the highest-ranked producer of g + 1 is at least  $k_g$ , that is,  $k_{g+1} \ge k_g$ . This is a contradiction. QED

LEMMA 9.  $g_k$  is nondecreasing in k.

*Proof.* Suppose not, that  $g_k < g_{k-1}$  for some  $k \ge 2$ . Lemma 4 implies that good  $g_k$  is produced by country types k and  $g_k$ . Good  $g_{k-1}$  is the lowest-ranked good produced by country k - 1 so that good  $g_k$  ( $\leq g_{k-1}$ ) is not produced by k - 1. It follows that good  $g_k$  is produced by countries  $g_k$  and k, but not country k - 1. Further, the definition of  $g_{k-1}$  implies that  $g_{k-1} \le k - 1$  so that  $g_k < g_{k-1} \le k - 1 < k$ . It follows that good  $g_k$  is not produced by an interval of countries. But lemma 6 implies that all goods are produced by intervals of countries, a contradiction. QED

LEMMA 10. For a type k country let  $f_{gk} \equiv Hx_{gk}/L$  be the share of the labor force employed in group g goods. Let  $F_k(g) = \sum_{j=1}^g f_{jk}$  be the share of the labor force employed in groups ranked g or less. Then  $F_k(g) < F_{k-1}(g)$  for  $g = g_{k-1}, \ldots, k-1$ and  $F_k(g) = F_{k-1}(g) = 1$  for g = k; that is,  $F_k$  first-order stochastic dominates  $F_{k-1}$ . (Economically, employment in k is skewed toward higher-ranked goods.)

*Proof.* Recall that  $g_{k-1}$  and  $g_k$  are defined as the lowest-ranked goods produced by countries of type k - 1 and k, respectively. Lemma 4 states that a type (k - 1) country produces and produces goods only in groups  $g = g_{k-1}, ..., k - 1$  and a type k country produces and produces goods only in groups  $g = g_k, ..., k - 1$  and a type k country produces and produces goods only in groups  $g = g_k, ..., k - 1$  and the type k country produces and produces goods only in groups  $g = g_k, ..., k - 1$ . Lemma 9 states that  $g_{k-1} \leq g_k$ . Consider goods in the region  $g_{k-1}, ..., g_k - 1$ . If the region exists, then goods in the region are produced by country type k - 1.

but not *k*. Hence  $f_{gk} = 0 < f_{g,k-1}$  so that  $F_k(g) < F_{k-1}(g)$  for  $g = g_{k-1}, \dots, g_k - 1$ , as required. Consider goods in the region  $g_k, \dots, k-1$ . Goods in this region are produced by country types k - 1 and *k*. Hence,  $f_{gk}$  and  $f_{g,k-1}$  are strictly positive and

$$\begin{split} f_{gk} - f_{g,k-1} &= \frac{H x_{gk}}{L} - \frac{H x_{g,k-1}}{L} \\ &= \frac{H}{p_g} \left( 1 - \frac{w_k}{p_g} \right) \frac{S_g}{L} - \frac{H}{p_g} \left( 1 - \frac{w_{k-1}}{p_g} \right) \frac{S_g}{L} \\ &= \frac{H}{L(p_g)^2} (w_{k-1} - w_k) S_g < 0. \end{split}$$

It follows that  $f_{gk} < f_{g,k-1}$  for  $g = g_{k-1}, ..., k-1$ . Since good *k* is produced by country *k* but not country k-1,  $F_k(k-1) < F_{k-1}(k-1) = 1$ . Thus,  $F_k(g) < F_{k-1}(g)$  for  $g = g_{k-1}, ..., k-1$  as required. QED

*Proof of proposition 1.* Part 1 follows immediately from lemmas 6 and 8. Part 3 follows immediately from lemmas 4 and 9. Consider part 2. By lemma 7,  $w_k$  is strictly increasing in k. By lemma 5,  $p_g$  is strictly increasing in g. It remains to show that  $y_k$  is strictly increasing in k. We can write  $y_k$  as

$$y_k = H \sum_{g=g_k}^k p_g x_{gk} / L = \sum_{g=g_k}^k p_g f_{gk}$$

From lemma 5,  $p_g$  is strictly increasing in g. From lemma 10,  $F_k(g)$  first-order stochastic dominates  $F_{k-1}(g)$  (and strictly so for some g). Hence  $y_k > y_{k-1}$ .<sup>30</sup> QED

## Appendix C

#### **Proof of Proposition 2**

From footnote 12,  $(p_g, w_k)$  simultaneously solves equation (4) or  $p_k = w_k N_k / (N_k - 1)$  and equation (6) or  $L = H x_{kk} = H(1/p_k)(1 - w_k/p_k)S_k$ . Solving these trivially yields equation (7). Footnote 12 also contains the derivation of the expressions for the markup,  $y_k$ ,  $H\pi_{kk}$ , and net exports. It follows from the derivation and the definition of equilibrium that  $\{w_k, p_k\}_{k=1}^K$  is an equilibrium if and if only each firm in each type *k* country finds it unprofitable to produce each group *g* good for which g < k. A necessary and sufficient condition for such unprofitability is derived next.

If a firm from a type k country produces a group g good, then the price of the group g good is

$$p'_{g} = rac{1}{(N_{g}+1)-1}(N_{g}w_{g}+w_{k}),$$

where we have used equation (4) and the facts that there are  $N_g$  firms from type g countries and one firm from a type k country. We require equilibrium wages and prices to be such that

<sup>30</sup> See online app. E.1 for a step-by-step proof of this argument.

or

$$egin{aligned} w_k &\geq p_g' = rac{1}{N_g} \left( N_g w_g + w_k 
ight) \ &rac{N_g - 1}{N_g} w_k \geq w_g. \end{aligned}$$

Plugging in the equation (7) expression for wages yields

$$\frac{N_g - 1}{N_g} \left( \frac{N_k - 1}{N_k} \frac{HS_k}{LN_k} \right) \ge \left( \frac{N_g - 1}{N_g} \frac{HS_g}{LN_g} \right).$$

Hence, no firm from a type *k* country will produce a group *g* good for which  $g \le k$  if and only if

$$\frac{N_g}{S_g} \ge \frac{N_k}{N_k - 1} \frac{N_k}{S_k} \quad \text{for } g = 1, \dots, \ k - 1 \text{ and } k = 2, \dots, K.$$
(C1)

To summarize, equation (C1) is necessary and sufficient for a perfect-sorting equilibrium.

Finally, we show that equation (C1) holds if and only if the PSC holds. Equation (C1) obviously implies the PSC. Conversely, the PSC together with  $N_k \ge 2$  imply

$$\frac{N_g}{S_g} \ge \frac{N_{g+1}}{N_{g+1} - 1} \frac{N_{g+1}}{S_{g+1}} > \frac{N_{g+1}}{S_{g+1}} \ge \dots > \frac{N_{k-1}}{S_{k-1}} \ge \frac{N_k}{N_k - 1} \frac{N_k}{S_k};$$

that is, they imply equation (C1). It follows that the PSC is necessary and sufficient for all equilibria to display perfect sorting. QED

## Appendix D

## Equivalence of the PSC and $N_{k-1} \ge N_k + 2$

When  $S_k = S$  for all k, the PSC can be written as  $N_{k-1} \ge \phi_k$ , where  $\phi_k \equiv N_k^2/(N_k - 1)$ . Then  $\phi_k$  can be rewritten as  $\phi_k = N_k + 1 + [1/(N_k - 1)]$ , where, since  $N_k \ge 2$ ,  $1/(N_k - 1) \in (0, 1]$ . Thus, we have that  $\phi_k \in (N_k + 1, N_k + 2]$ . Hence, the PSC  $N_{k-1} \ge \phi_k$  implies  $N_{k-1} > N_k + 1$ , which, given that the  $N_k$  are integers, implies  $N_{k-1} \ge N_k + 2$ . Conversely, failure of the PSC  $(N_{k-1} < \phi_k)$  implies  $N_{k-1} < N_k + 2$ .

## Appendix E

## Allowing for Quality Differences across Producers

In the main text we made three symmetry assumptions: (1) the number of workers is the same for all country types  $(L_k = L)$ ; (2) if a country type is able to produce a good, then it does so at a standard level of quality that is independent of its country type  $(u_k = u)$ ;<sup>31</sup> and (3) inverse productivity is normalized to unity

 $^{\rm 31}$  Recall that quality comparisons are never made across goods, so we do not subscript quality  $u_{\rm k}$  by g.

 $(c_k = 1)$ . The latter two points imply  $u_k/c_k = u$ . In this appendix we allow firms to produce at different levels of quality. Specifically, we assume that

$$u_k / c_k \ge u_{k-1} / c_{k-1}$$
 for  $k \ge 2$ .

That is, higher-ranked countries produce at a higher level of quality or productivity. This means that at equilibrium, when a good is produced by firms from several different country types, there will be a range of different quality levels offered on the market.

This generalization turns out to be a trivial extension of the model provided that we replace our usual assumption that the number of workers in a country  $(L_k)$  is the same across countries  $(L_k = L)$  with the assumption that the number of effective workers in a country  $L^* \equiv L_k u_k/c_k$  is the same across countries  $(L_k^* = L^*)$ . We maintain these assumptions throughout this appendix.

Recall that  $w_k^* \equiv w_k c_k / u_k$ , which we earlier referred to as "effective costs," is the cost of a unit of effective labor. Also, define quality-adjusted outputs  $x_{gk}^* \equiv x_{gk} u_k$  and quality-adjusted prices

$$p_g^* = \frac{1}{\sum_k N_k - 1} \sum_k N_k w_k^*, \tag{4'}$$

where the sums are over the set of countries that produce group g goods. Equation (4') follows from equation (2) just as equation (4) followed from equation (2). (As elsewhere in this paper, quality-adjusted prices are the same for all producers of a good.) Because different producers of a good have different qualities  $u_{k}$ , they face different prices  $p_{gk} \equiv p_g^* u_k$ . We can now restate our propositions 1 and 2 for the case in which qualities differ across producers.

**PROPOSITION 6** (Product ranges). Assume that  $u_k/c_k \ge u_{k-1}/c_{k-1}$  for  $k \ge 2$ .

- 1. A group g good is produced by a type k country if and only if  $k = g, ..., k_g$  for some country type  $k_g$  that is increasing in g. That is, each good is produced by an interval of country types and both boundaries of the interval are increasing in g.
- 2. Each type *k* country produces and produces only goods  $g = g_k, ..., k$  for some group of goods  $g_k \leq k$ . That is, each country produces an interval of goods that is bounded above by the country's most difficult-to-make group of goods.
- 3. Wage  $w_k$ ,  $w_k^* \equiv w_k c_k/u_k$ , and  $y_k$  are all strictly increasing in k; that is, countries with scarce capabilities have high wages, high effective costs, and high GDP per worker.
- 4. Prices  $p_{gk}$  and  $p_g^*$  are strictly increasing in g; that is, hard-to-make goods have high prices and high quality-adjusted prices.

*Proof.* Just as equation (3') became equation (5), equation (3') now becomes

$$x_{gk}^{*} = \frac{1}{p_{g}^{*}} \left( 1 - \frac{w_{k}^{*}}{p_{g}^{*}} \right) S_{g}$$
(5')

for  $p_g^* > w_k^*$  and  $x_{gk}^* = 0$  otherwise. Because  $x_{gk}$  units of output require  $x_{gk}c_k$  units of labor to produce, the labor market-clearing condition (eq. [6]) with  $c_k \neq 1$  is  $L_k = H\sum_g x_{gk}c_k$ . Multiplying this through by  $u_k/c_k$  yields

$$L^* = H \sum_g x_{gk}^*.$$
 (6')

Equations (4')-(6') establish that equations (4)-(6) continue to hold with quality differences, but with  $(p_g^*, w_k^*, x_{gk}^*, L^*)$  replacing  $(p_g, w_k, x_{gk}, L)$ . Since proposition 1 of the main text was based solely on equations (4)-(6), it follows that the proposition holds with  $(p_g^*, w_k^*, x_{gk}^*, L^*)$  replacing  $(p_g, w_k, x_{gk}, L)$ . This establishes parts 1 and 2 of proposition 6. It also establishes that  $w_k^*$  is strictly increasing in k, that  $y_k^* \equiv H \sum_g p_g^* x_{gk}^* / L^*$  is strictly increasing in k, and that  $p_g^*$  is strictly increasing in g.

Consider the remainder of parts 3 and 4. From the definition of  $w_k^*$  we have  $w_k = w_k^*(u_k/c_k)$ . Since  $w_k^*$  is strictly increasing in k and  $u_k/c_k$  is weakly increasing in k,  $w_k$  is strictly increasing in k. Turning to GDP per worker, recall that  $y_k \equiv H\sum_g p_{gk} x_{gk}/L_k$ ,  $y_k^* \equiv H\sum_g p_g^* x_{gk}^*/L^*$ , and  $L^* \equiv L_k u_k/c_k$  so that  $y_k = (u_k/c_k)y_k^*$ . Since  $y_k^*$  is strictly increasing in k and  $u_k/c_k$  is weakly increasing in k,  $y_k = u_k/c_k$ ,  $y_k = H\sum_g p_g^* x_g^*/L^*$ , and  $L^* = L_k u_k/c_k$  so that  $y_k = (u_k/c_k)y_k^*$ . Since  $y_k^*$  is strictly increasing in k and  $u_k/c_k$  is weakly increasing in k,  $y_k$  is strictly increasing in k. Turning to  $p_{gk} = p_g^* u_k$ , since  $p_g^*$  is increasing in g, so is  $p_{gk}$ . QED

PROPOSITION 7 (Perfect-sorting equilibria). Assume that  $u_k/c_k \ge u_{k-1}/c_{k-1}$  for  $k \ge 2$ .

- 1. An equilibrium displays perfect sorting if and only if the PSC of proposition 2 holds.
- 2. If the PSC holds, then there is a unique equilibrium set of product prices and wages given by

$$p_{kk} = u_k \left(\frac{HS_k}{L^*N_k}\right)$$
 and  $u_k = \frac{u_k}{c_k} \left(\frac{N_k - 1}{N_k}\frac{HS_k}{L^*N_k}\right) \quad \forall k.$  (E1)

3. Markups, GDP per worker, total profits, and output per good are given by, respectively,

$$egin{aligned} rac{p_{kk}}{w_kc_k} &= rac{N_k}{N_k - 1}, \quad y_k &= rac{u_k}{c_k}\left(rac{HS_k}{L^*N_k}
ight), \ H\pi_{kk} &= rac{HS_k}{N_k^2}, \quad x_{kk} &= rac{1}{u_k}\left(rac{L^*}{H}
ight). \end{aligned}$$

Further, net exports of a group g good are  $(1 - \delta_k H) x_{kk}$  if the good is exported (g = k) and  $-\delta_g H x_{kk}$  if the good is imported  $(g \neq k)$ .

*Proof.* Picking up from the proof of proposition 6, equations (4)–(6) hold, but with  $(p_g, w_k, x_{gk}, L)$  replaced by  $(p_g^*, w_k^*, x_{gk}^*, L^*)$ . Since proposition 2 of the main text was based solely on equations (4)–(6), it follows that proposition 2

holds with  $(p_g, x_{gk}, w_k, L)$  replaced by  $(p_g^*, x_{gk}^*, w_k^*, L^*)$ . Proposition 7 follows from rewriting proposition 2 using  $(p_{\sigma}^*, x_{gk}^*, \hat{w}_k^*, L^*)$  and  $y_k^*$  (defined in the proof of proposition 6) and then substituting in  $p_g = p_{kk} = u_k p_k^*$ ,  $w_k = (u_k/c_k) w_k^*$ ,  $p_{kk}/(w_kc_k) = p_k^*/w_k^*, \quad y_k = (u_k/c_k)y_k^*, \quad H\pi_{kk} = H(p_{kk} - w_kc_k)x_{kk} = H(p_k^* - w_k^*)x_{kk}^*,$ and  $x_{kk} = (1/u_k) x_{kk}^*$ . QED

## Appendix F

### Proofs of Propositions 3-5 and Lemma 2

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## A. Equilibrium Conditions and Outcomes in Phase I

A typical type (k-1) country producing a group (k-1) good has wage  $w_{k-1}$  and price  $\overline{p}_{k-1}$ . Rearranging equation (11) as  $1 - w_{k-1}/\overline{p}_{k-1} = w/[\overline{p}_{k-1}(N_{k-1}-1)]$  and plugging this into equation (13) yields

or

$$L = H \frac{w}{\overline{p}_{k-1}^{2}(N_{k-1} - 1)} S$$
$$\overline{p}_{k-1} = \left(\frac{HS}{L} \frac{w}{N_{k-1} - 1}\right)^{1/2}.$$
(13')

Likewise, rearranging equation (12) as  $1 - w_k/\overline{p}_k = (wu/v_k)/(\overline{p}_k N_k)$  and plugging this into equation (14) yields

$$L = H \frac{w}{(v_k/u)\overline{p}_k^2 N_k} S$$

or

$$\overline{p}_{k} = \left[\frac{HS}{L}\frac{w}{(v_{k}/u)N_{k}}\right]^{1/2}.$$
(14')

For the developing country, equations (9) and (10) imply  $p_{k-1} = \overline{p}_{k-1}$  and  $p_k =$  $(v_k/u)\overline{p}_k$ . Plugging these into equation (15) yields

$$\frac{L}{HS} = \frac{1}{\overline{p}_{k-1}} \left( 1 - \frac{w}{\overline{p}_{k-1}} \right) + \frac{1}{(v_k/u)\overline{p}_k} \left[ 1 - \frac{w}{(v_k/u)\overline{p}_k} \right].$$
(15')

Plugging equations (13') and (14') into equation (15') establishes that the equilibrium wage in the developing country is

$$\frac{w(v_k)}{S(v_k)} = \frac{H}{L} \left[ \frac{(N_{k-1} - 1)^{1/2} + N_k^{1/2} / (v_k/u)^{1/2}}{N_{k-1} + N_k / (v_k/u)} \right]^2.$$
(F1)

Plugging this back into equations (13') and (14') yields closed-form solutions for the equilibrium prices  $\overline{p}_{k-1}(v_k)/S(v_k)$  and  $\overline{p}_k(v_k)/S(v_k)$ . (Recall that all solutions are normalized by the numeraire  $S(v_k)$ .) Plugging  $w(v_k)$ ,  $\overline{p}_{k-1}(v_k)$ , and  $\overline{p}_k(v_k)$  into equations (9)-(12) yields closed-form solutions for the remaining equilibrium wages and prices, namely,  $p_{k-1}(v_k)$ ,  $p_k(v_k)$ ,  $w_{k-1}(v_k)$ , and  $w_k(v_k)$ , all relative to the numeraire  $S(v_k)$ .

#### B. Equilibrium Conditions and Outcomes in Phase II

Plugging  $\overline{p}_{k-1}$  from equation (11-II) into equation (13) and rearranging yields

$$\frac{w_{k-1}(v_k)}{S(v_k)} = \frac{H}{L} \frac{N_{k-1} - 2}{(N_{k-1} - 1)^2}.$$
(13-II)

For closed-form solutions to all of the equilibrium prices and wages in phase II, proceed as follows. Plugging equation (13-II) into (11-II) yields a closed-form solution for  $\overline{p}_{k-1}(v_k)/S(v_k)$ . The equation (14') expression for  $\overline{p}_k$  holds in phase II, as does the equation (10) expression  $p_k = (v_k/u)\overline{p}_k$ . Hence,

$$p_k = (v_k/u)\overline{p}_k = (v_k/u) \left[ \frac{HS}{L} \frac{w}{(v_k/u)N_k} \right]^{1/2}$$

Plugging this into equation (15-II) and rearranging yields

$$\frac{w(v_k)}{S(v_k)} = \frac{H}{L} \frac{\alpha}{(1+\alpha)^2}, \quad \text{where } \alpha \equiv \frac{v_k/u}{N_k} < 1.$$
 (F2)

Plugging equation (F2) into equation (14') yields a closed-form equilibrium solution for  $\overline{p}_k(v_k)/S(v_k)$ . Plugging this and w/S from equation (F2) into (12) yields a closed-form equilibrium solution for  $w_k(v_k)/S(v_K)$ . Thus, we have provided closed-form equilibrium solutions for all of the endogenous prices in phase II.

## C. Proof of Lemma 1

Let  $v_k^L$  be the value of  $v_k$  at which the developing country is about to produce a type k good but has not yet started. At this point its wage  $w(v_k^L)$  is that of a typical type (k - 1) country, that is,  $w(v_k^L) = w_{k-1}$ , where  $w_{k-1}$  is given in equation (7). Equating the equation (7) expression for  $w_{k-1}$  with the equation (F1) expression for  $w(v_k^L)$  and solving for  $v_k^L$  yields

$$v_k^L = \frac{(N_{k-1} - 1)N_k}{N_{k-1}^2} u.$$
 (F3)

Let  $v_k^H$  be the value of  $v_k$  at which the developing country stops producing goods of group (k-1), that is,  $x_{k-1}(v_k^H) = 0$ . At  $v_k = u$ , we are in a perfect-sorting world with  $N_{k-1}$  and  $N_k$  replaced by  $N_{k-1} - 1$  and  $N_k + 1$ , respectively. See the discussion immediately preceding lemma 1. Thus, at  $v_k = u$  the developing country produces only group k goods and, specifically, does not produce group (k-1)goods. Thus, there is some  $v_k^H$  that is strictly less than u at which the developing country stops producing group (k-1) goods. Further, since at  $v_k$  just above  $v_k^L$ the developing country produces group (k-1) goods,  $v_k^H > v_k^L$ .

## D. Monotonicity of Wages in Phases I and II

LEMMA 11. (1)  $w(v_k)/S(v_k)$  is increasing in  $v_k$  for  $v_k$  in the range  $v_k^L < v_k < u$ . (2)  $[w(v_k)/S(v_k)]/(v_k/u)$  is decreasing in  $v_k$  for  $v_k$  in the range  $v_k^L < v_k < v_k^H$ .

*Proof of part 1.* Consider phase I, where  $v_k^L \le v_k \le v_k^H$ . Differentiating equation (F1) with respect to  $v_k$  yields

$$\frac{\partial w(v_k)/S(v_k)}{\partial v_k} = \alpha_k (\xi^+ - \sqrt{v_k})(\sqrt{v_k} - \xi^-), \tag{F4}$$

where  $\alpha_k(v_k) > 0$ , and

$$\xi^{-} = \sqrt{u} \, \frac{N_{k}^{1/2}}{N_{k-1}} \left[ (N_{k-1} - 1)^{1/2} - (2N_{k-1} - 1)^{1/2} \right] < 0, \tag{F5}$$

$$\xi^{+} = \sqrt{u} \, \frac{N_{k}^{1/2}}{N_{k-1}} [(N_{k-1} - 1)^{1/2} + (2N_{k-1} - 1)^{1/2}] > 0.$$
 (F6)

It is tedious but straightforward to show that the PSC (see fn. 14) implies  $\sqrt{v_k^H} \le \xi^+$ . See online appendix E.3 for a step-by-step derivation. Since  $\xi^- \le 0$ , it must be that  $\xi^- \le \sqrt{v_k^L}$ . Hence the interval  $(\xi^-, \xi^+)$  contains the interval  $(\sqrt{v_k^L}, \sqrt{v_k^H})$ . Hence,  $\xi^- \le \sqrt{v_k} \le \xi^+$  in phase I. Hence the right-hand side of equation (F4) is positive and  $w(v_k)/S(v_k)$  is increasing in  $v_k$ .

Next consider phase II, where  $v_k^H < v_k < u$ . From equation (F2),  $w(v_k)/S(v_k)$  is increasing in  $\alpha$  and hence in  $v_k$ .

*Proof of part 2*  $(v_k^L \le v_k \le v_k^H)$ . Dividing equation (F1) by  $v_k/u$  yields

$$\frac{w(v_k)/S(v_k)}{v_k/u} = \frac{H}{L} \left[ \frac{(N_{k-1}-1)^{1/2} + N_k^{1/2}/(v_k/u)^{1/2}}{N_{k-1}(v_k/u)^{1/2} + N_k/(v_k/u)^{1/2}} \right]^2$$
$$= \frac{H}{L} \left[ \frac{(N_{k-1}-1)^{1/2}(v_k/u)^{1/2} + N_k^{1/2}}{N_{k-1}(v_k/u) + N_k} \right]^2.$$

Differentiating,

$$rac{\partial rac{w(v_k)/S(v_k)}{v_k/u}}{\partial v_k} = -lpha_k(v_k) \Bigg[ rac{v_k}{u} + 2 \Bigg(rac{N_k}{N_{k-1}-1} \Bigg)^{1/2} \Bigg(rac{v_k}{u} \Bigg)^{1/2} - rac{N_k}{N_{k-1}} \Bigg]$$

for some  $\alpha_k(v_k) > 0.^{32}$  The term in brackets is quadratic in  $\sqrt{v_k/u}$  and its largest root is

$$\xi = [(2N_{k-1}-1)^{1/2}/(N_{k-1})^{1/2}-1](N_k)^{1/2}/(N_{k-1}-1)^{1/2}.$$

It follows that the quadratic term is positive when  $\sqrt{v_k/u} > \xi$ . Using equation (F3), it is easy to show that  $\sqrt{v_k^L/u} \ge \xi$  if and only if  $N_{k-1} \ge 1$ . Hence the quadratic is positive for  $v_k > v_k^L$  and the derivative is negative for  $v_k > v_k^L$ .

32 The term

$$lpha_k(v_k) \equiv (H/L)^{1/2} (w/S)^{1/2} (v_k)^{-1} N_{k-1} (N_{k-1}-1)^{1/2} / (N_{k-1}v_k/u + N_k)^2 > 0.$$

## E. Proof of Proposition 4

Part 1 ( $w_{k-1}/S$  increasing in  $v_k$ ): Equating the right-hand sides of equations (11) and (13') yields

$$\frac{w_{k-1}(v_k)}{S(v_k)} = \left(\frac{H}{L}\frac{1}{N_{k-1}-1}\right)^{1/2} \left[\frac{w(v_k)}{S(v_k)}\right]^{1/2} - \frac{1}{N_{k-1}-1}\frac{w(v_k)}{S(v_k)}.$$
 (F7)

The right-hand side of equation (F7) is quadratic in  $\sqrt{w/S}$  and thus increasing in  $\sqrt{w/S}$  for  $\sqrt{w/S} \le \sqrt{H/L}\sqrt{N_{k-1}-1}/2$ . It thus suffices to show that this last inequality is satisfied in phase I. Since w/S is increasing in  $v_k$  (lemma 11), it suffices to show that the inequality holds at  $v_k = v_k^H$ . Recall that  $v_k^H$  is the value of  $v_k$ at which the developing country stops producing goods of type (k-1), that is,  $x_{k-1}(v_k^H) = 0$ . From equation (3'),

$$x_{k-1}(v_{k}^{H}) = \frac{1}{\overline{p}_{k-1}(v_{k}^{H})} \left[ 1 - \frac{w(v_{k}^{H})}{\overline{p}_{k-1}(v_{k}^{H})} \right] S(v_{k}^{H})$$

so that  $x_{k-1}(v_k^H) = 0$  implies  $\overline{p}_{k-1}(v_k^H) = w(v_k^H)$ . Plugging this into equation (13') yields

$$\frac{w(v_k^H)}{S(v_k^H)} = \frac{H}{L} \frac{1}{N_{k-1} - 1}.$$

Hence

$$\sqrt{w(v_k^H)/S(v_k^H)} \le \sqrt{H/L}\sqrt{N_{k-1}-1}/2$$

implies

$$\left(\frac{H}{L}\frac{1}{N_{k-1}-1}\right)^{1/2} \le \left[\frac{H}{L}(N_{k-1}-1)\right]^{1/2} / 2$$

or  $4 \le (N_{k-1} - 1)^2$  or  $N_{k-1} \ge 3$ . But  $N_k \ge 2$  and  $N_{k-1} \ge N_k + 2$  (the PSC) together imply  $N_{k-1} \ge 3$ . Thus, the inequality holds at  $v_k^H$ .

Part 1 ( $w_k/S$  decreasing in  $v_k$ ): We start with a preliminary result. At  $v_k = v_k^L$  the developing country does not produce  $x_k$ . Hence we are in the perfect sorting equilibrium of proposition 2:  $w(v_k^L) = w_{k-1}(v_k^L)$ , which, from equation (7), gives us

$$w(v_k^L)/S(v_k^L) = rac{N_{k-1}-1}{N_{k-1}^2}rac{H}{L}.$$

Further, from equation (F3),

$$\frac{w(v_k^L)/S(v_k^L)}{v_k^L/u} = \frac{N_{k-1}-1}{N_{k-1}^2} \frac{H}{L} \left/ \frac{(N_{k-1}-1)N_k}{N_{k-1}^2} = \frac{1}{N_k} \frac{H}{L} \right.$$
(F8)

We now turn to the core of the proof. Equating the right-hand sides of equations (12) and (14') yields

$$rac{w_k(v_k)}{S(v_k)} = \left[rac{H}{L}rac{1}{N_k}rac{w(v_k)/S(v_k)}{v_k/u}
ight]^{1/2} - rac{1}{N_k}rac{w(v_k)/S(v_k)}{v_k/u}\,.$$

This is just equation (F7) with  $N_k$  replacing  $N_{k-1} - 1$  and  $(w/S)/(v_k/u)$  replacing w/S. Thus,  $w_k/S$  is increasing in  $(w/S)/(v_k/u)$  if  $\sqrt{(w/S)/(v_k/u)} \le \sqrt{(H/L)N_k}/2$ . Since  $(w/S)/(v_k/u)$  is decreasing in  $v_k$  (part 2 of lemma 11), the inequality holds throughout phase I if it holds at  $v_k = v_k^L$ . But from equation (F8), this means

$$\frac{1}{N_k}\frac{H}{L} \le \frac{N_k}{4}\frac{H}{L}$$

or  $4 \le N_k^2$ , which always holds. Hence  $w_k/S$  is increasing in  $(w/S)/(v_k/u)$ . But  $(w/S)/(v_k/u)$  is decreasing in  $v_k$  (part 2 of lemma 11). Hence  $w_k(v_k)/S(v_k)$  is decreasing in  $v_k$ .

Part 2: Dividing both sides of equation (F7) by  $w(u_k)/S(u_k)$  establishes that  $w_{k-1}/w$  is decreasing in w/S and hence, by lemma 11, decreasing in  $v_k$ .

## F. Proof of Proposition 3

Part 1: From equation (3'),  $x_{k-1} = (S/\overline{p}_{k-1})(1 - w/\overline{p}_{k-1})$ . From equation (13'),  $\overline{p}_{k-1}/S$  is increasing in w/S. Dividing equation (13') by  $w(v_k)$ ,

$$\frac{\overline{p}_{k-1}(v_k)}{w(v_k)} = \left[\frac{H}{L}\frac{1}{N_{k-1}-1}\frac{S(v_k)}{w(v_k)}\right]^{1/2},$$
(F9)

which is decreasing in w/S. Hence  $x_{k-1}(v_k)$  is decreasing in w/S and, by lemma 11, decreasing in  $v_k$ . By the definition of  $v_k^H$ ,  $x_{k-1}(v_k^H) = 0$  so that  $x_{k-1}(v_k)$  falls to zero. By labor market clearing,  $Hx_{k-1} + Hx_k = L$  so that  $x_k$  is increasing in  $v_k$ . By the definition of  $v_k^L$ ,  $x_k(v_k^L) = 0$  so that  $x_k(v_k)$  rises from zero.

Part 2: From equations (9) and (F9),  $p_{k-1}(v_k)/w(v_k)$  is decreasing in w/S, and hence, by lemma 11,  $p_{k-1}(v_k)/w(v_k)$  is decreasing in  $v_k$ .

From equation (14'),

$$\frac{\overline{p}_{k}(v_{k})}{w(v_{k})} = \left[\frac{H}{L}\frac{1}{N_{k}}\frac{1}{v_{k}/u}\frac{1}{w(v_{k})/S(v_{k})}\right]^{1/2}.$$
(F10)

From equation (10),  $p_k = (v_k/u)\overline{p}_k$ . Hence

$$\frac{p_k(v_k)}{w(v_k)} = \frac{(v_k/u)\overline{p}_k(v_k)}{w(v_k)} = \left(\frac{H}{L}\frac{1}{N_k}\right)^{1/2} \left[\frac{w(v_k)/S(v_k)}{v_k/u}\right]^{-1/2},$$
 (F11)

which, by lemma 11, is increasing in  $v_k$ .

Part 3: See lemma 11.

Part 4: Revenue for any firm *i* is  $R_i = p_i x_i$ . Multiplying the expression for  $x_i$  in equation (3') by  $p_i$  yields  $R_i/S = p_i x_i/S = 1 - w_i/p_i$ . Hence, for a developing country producing a type (k - 1) good,

$$\frac{R_{k-1}(v_k)}{S(v_k)} = 1 - \frac{w(v_k)}{p_{k-1}(v_k)} = 1 - \left[\frac{L}{H}(N_{k-1} - 1)\frac{w(v_k)}{S(v_k)}\right]^{1/2},$$
 (F12)

where the second equality follows from equation (9)  $(p_{k-1} = \overline{p}_{k-1})$  and the equation (13') expression for  $\overline{p}_{k-1}(v_k)$ . For a developing country firm producing a type k good,

$$\frac{R_k(v_k)}{S(v_k)} = 1 - \frac{w(v_k)}{p_k(v_k)} = 1 - \left[\frac{L}{H}N_k\frac{w(v_k)/S(v_k)}{v_k/u}\right]^{1/2},$$
(F13)

where the second equality follows from equation (10)  $(p_k = (v_k/u)\overline{p}_k)$  and the equation (14') expression for  $\overline{p}_{k-1}(v_k)$ .

GDP per capita of the developing country is given by  $y = (HR_{k-1} + HR_k)/L$  (see eq. [8]), so

$$\frac{y(v_k)}{S(v_k)} = \frac{H}{L} \left[ \frac{R_{k-1}(v_k)}{S(v_k)} + \frac{R_k(v_k)}{S(v_k)} \right]$$
$$= 2\frac{H}{L} - \left(\frac{H}{L}\right)^{1/2} \left(\sqrt{N_{k-1} - 1} + \frac{\sqrt{N_k}}{\sqrt{v_k/u}}\right) \left[\frac{w(v_k)}{S(v_k)}\right]^{1/2}$$

Substituting into this the equation (F1) expression for  $w(v_k)/S(v_k)$  and simplifying yields

$$\frac{\mathbf{y}(v_k)}{S(v_k)} = \frac{H}{L} + \frac{H}{L} \cdot \frac{1 - 2\sqrt{N_{k-1} - 1}\sqrt{N_k}\sqrt{u/v_k}}{N_{k-1} + N_k(u/v_k)}.$$

Differentiating and simplifying yields

$$\frac{\partial y(v_k)/S(v_k)}{\partial v_k} = \alpha'_k(v_k) \left[ \frac{v_k}{u} + \frac{N_k^{1/2}}{(N_{k-1} - 1)^{1/2} N_{k-1}} \left( \frac{v_k}{u} \right)^{1/2} - \frac{N_k}{N_{k-1}} \right]$$

for some  $\alpha'_k(v_k) > 0$ . The term in brackets is quadratic in  $(v_k/u)^{1/2}$  with roots  $-[N_k/(N_{k-1}-1)]^{1/2} < 0$  and  $(v_k^L/u)^{1/2} > 0$  (see eq. [F3]). Hence the term in brackets is positive for  $v_k > v_k^L$  and the derivative is positive for  $v_k > v_k^L$ . This completes the proof of proposition 3.

## G. Proof of Proposition 5

Part 2 follows immediately from part 1 of lemma 11. Since equations (9) and (14') hold in phase II, so do equations (F11) and (F13). Part 1 follows from equa-

tion (F11) and part 2 of lemma 11. For part 3, start by observing that  $R_k(v_k)/S(v_k)$  rises in  $v_k$ , which follows from equation (F13) and part 2 of lemma 11. In phase II all income comes from production of type k goods so that  $y(v_k)/S(v_k) = (H/L)R_k(v_k)/S(v_k)$ , which is also rising in  $v_k$ .

## H. Proof of Lemma 2

Consumers spend a fraction  $\delta$  of income on each good. It follows that the value of exports of a typical type *k* country is  $\overline{X}_k = \overline{p}_k \overline{x}_k - \delta H \overline{p}_k \overline{x}_k$ , where  $H \overline{p}_k \overline{x}_k$  is income or GDP. Labor market equilibrium is  $L = H \overline{x}_k$ . It follows that

$$\frac{\overline{X}_{k}(v_{k})}{S(v_{k})} = (1 - \delta H) \frac{L}{H} \frac{\overline{p}_{k}(v_{k})}{S(v_{k})},$$

$$\frac{\overline{X}_{k-1}(v_{k})}{S(v_{k})} = (1 - \delta H) \frac{L}{H} \frac{\overline{p}_{k-1}(v_{k})}{S(v_{k})},$$
(F14)

where the right-hand equation follows from the left-hand equation by symmetry.

Turning to the developing country, consider phase II, where only group k goods are produced and exported. By the logic of equation (F14),  $X_k/S = (1 - \delta H)(L/H)(p_k/S)$ , which is increasing in  $p_k/S$  and hence, by parts 1 and 2 of proposition 5, in  $v_k$ . Further,  $X_k/\overline{X}_k = (X_k/S)/(\overline{X}_k/S) = p_k/\overline{p}_k = v_k/u$ , where the last equality follows from equation (10). Hence  $X_k/\overline{X}_k$  is increasing in  $v_k$ . Next consider phase I, where both goods are produced. Let  $R_{k-1} \equiv p_{k-1}x_{k-1}$  and  $R_k \equiv p_k x_k$  be revenues so that income or GDP is  $H(R_{k-1} + R_k)$ . It follows that the values of the developing country's exports of group (k - 1) and group k goods are given by

$$\begin{aligned} X_{k-1}(v_k) &= \max\{0, R_{k-1}(v_k) - \delta H[R_{k-1}(v_k) + R_k(v_k)]\} \\ &= \max\{0, (1 - \delta H)R_{k-1}(v_k) - \delta HR_k(v_k)\}, \end{aligned}$$
(F15)  
$$X_k(v_k) &= \max\{0, (1 - \delta H)R_k(v_k) - \delta HR_{k-1}(v_k)\}. \end{aligned}$$

(The max operator is needed because it is possible that one of the two goods is imported.) Revenue for any firm *i* is  $R_i = p_i x_i$ . Multiplying the expression for  $x_i$  in equation (3') by  $p_i$  yields  $R_i = p_i x_i = (1 - w_i/p_i)S$ . Hence for the developing country producing group (k - 1) and group k goods,  $R_{k-1}/S = (1 - w/p_{k-1})$  and  $R_k/S = (1 - w/p_k)$ . From part 2 of proposition 3, a phase I rise in  $v_k$  raises  $w/p_{k-1}$  and lowers  $w/p_k$  so that  $R_{k-1}/S$  falls and  $R_k/S$  rises. Hence from equation (F15),  $X_{k-1}(v_k)/S(v_k)$  falls and  $X_k(v_k)/S(v_k)$  rises in phase I.

Consider  $\overline{X}_{k-1}/S$ . Equation (11) and proposition 4 imply that  $\overline{p}_{k-1}/S$  is increasing in  $v_k$  in phase I. Hence from equation (F14),  $\overline{X}_{k-1}/S$  is increasing in  $v_k$  in phase I, as required. But we have already seen that  $X_{k-1}/S$  falls. Hence  $X_{k-1}/\overline{X}_{k-1}$  falls. Finally, in phase I,  $R_{k-1}/S$  is falling so that by equation (F15),  $X_k/S$  is rising faster than  $(1 - \delta H)p_k x_k/S$ . Compare this to  $\overline{X}_k/S = (1 - \delta H)(\overline{p}_k/S)(L/H)$  from equation (F14). By equation (10),  $p_k/S$  is rising faster than  $\overline{p}_k/S$ . Also,  $x_k$  is rising faster than the constant L/H. Hence  $X_k/\overline{X}_k$  is rising.

## Appendix G

#### **Trade Data**

COMTRADE reports each bilateral transaction twice, once by the importer and once by the exporter. We always use the importer's data as they are known to be more reliable for most countries.

The countries in our sample are (using International Organization for Standardization codes for brevity)<sup>33</sup> AFG, AGO, ALB, ARG, AUS, AUT, BDI, BEN, BFA, BGD, BGR, BOL, BRA, CAN, CHE, CHL, CHN, CMR, COL, CRI, CUB, DEU, DNK, DOM, DZA, ECU, EGY, ESP, FIN, FRA, GBR, GHA, GIN, GRC, GTM, HND, HTI, HUN, IDN, IND, IRL, IRQ, ISR, ITA, JAM, JOR, JPN, KEN, KHM, LBN, LKA, MAR, MDG, MEX, MLI, MMR, MOZ, MWI, MYS, NER, NGA, NIC, NLD, NOR, NPL, NZL, PAK, PER, PHL, PNG, POL, PRT, PRY, ROU, RWA, SAU, SDN, SEN, SGP, SLE, SLV, SOM, SWE, TCD, TGO, THA, TUN, TUR, UGA, URY, USA, VEN, ZMB, and ZWE. The only major countries not included in our list are Taiwan and Hong Kong. Taiwan is excluded because there are no 1980 data. Hong Kong is excluded because, for our purposes, it should be merged with China in 2005 and be by itself in 1980. None of our 2005 cross-section results are affected by the inclusion of Taiwan and Hong Kong (the latter either by itself or merged with China).

We exclude live animals, meat, fish, and dairy. These goods account for only 2.1 percent of trade, and including them does not affect our results at all; however, it is hard to relate trade in these goods to the issues raised in this paper.

Price data  $p_{gt}$  are from the US historical imports CD, 2001–5. This CD reports only what is called the "first quantity" and "first value" so that all observations within an HS10 product have the same quantity units. We sum US imports and quantities by HS10 product and trading partner (exporter to the United States). We calculate unit values with the summed data. In addition, we winsorize the unit values below the 10th within-HS10 percentile and above the 90th within-HS10 percentile. Winsorizing makes virtually no difference to our results.

Table 1 in the text was constructed as follows. HS6 codes were ranked on the basis of world exports. The top 10 codes were chosen. Only industrial products were ranked; that is, we excluded animal and vegetable products, mineral products, precious stones, gold, and oil (two-digit HS codes 16–24, 28–70, and 72–96). We then assigned an industry to each HS6 code. There are only seven industries because passenger cars includes three of the top 10 codes (870323, 870332, 870324, ranked 1, 5, and 6, respectively) and semiconductors includes two of the top 10 codes (854231 and 854239, ranked 7 and 10). Export data are from COMTRADE. Four-firm concentration ratios are authors' calculations based on data sources reported in table G1.

<sup>33</sup> See online app. table B1 for a full list of country names and GDPs per capita.

Industry	Year	Source	Units			
Passenger cars	2012	Org. Internat. des Constructeurs d'Automobiles (OICA)	Production units			
Semiconductors	2010	Market Share Reporter 2012	Revenue			
Pharmaceuticals	2011	MarketLine Industry Profile: Global Pharmaceuticals	Market value			
Laptops	2010	Market Share Reporter 2012	Shipments			
Mobile phones	2010	Market Share Reporter 2012	Units shipped			
Aircraft	2010	IBISWorld: Global Commercial Aircraft Manufacturing 2011	Revenue			
Auto parts	2010	IBISWorld: Global Auto Parts & Accessories Manufacturing 2011	Revenue			

TABLE G1Data Sources for Table 1

## Appendix H

## **Price Ranges**

The theory predicts that, for a single good, all producers of the good will share the same price-quality ratio. Since richer countries have higher quality, they should have higher prices. That is, prices should be increasing in the income of the exporter. Since price data are not available, we follow Schott (2004) in proxying for prices using HS10 unit values from the 2005 US import file. We emphasize that unit values are extremely noisy so that caution must be exercised in interpreting them as prices. See Appendix G for a discussion of the data.

Let  $p_{gk}$  be the unit value of good *g* exported by country *k* to the United States. We are interested in how the  $p_{gk}$  vary as we move through product ranges. The most familiar way of doing this is Schott's (2004, table V) famous regression  $\ln p_{gk} = \alpha_g + \beta \ln y_k$ , where  $\alpha_g$  is an HS10 product fixed effect. Reestimating Schott's regression using 2005 US imports from our 94 exporters (187,363 observations), the ordinary least squares estimate of  $\beta$  is 0.29 (clustered t = 8.05) so that, as in Schott, there is indeed a statistically significant positive correlation between unit values and exporter incomes.

A sharper prediction of our theory is as follows. Consider a single HS10 good *g*. Recall that in the HS10 panel of figure 5 we plotted the income of the poorest and richest countries that had significant exports of *g*; that is, we plotted  $(\ln y_{\min,g}, \ln y_{\max,g})$ . Since for each *g* we know the identity of the poorest and richest countries, we know these countries' unit values. We denote them in obvious fashion by  $p_{\min,g}$  and  $p_{\max,g}$ . We expect that

$$\Delta_g \equiv \ln p_{\max,\sigma} - \ln p_{\min,\sigma} > 0.$$

This inequality is sharp in that it is directly related to our product ranges, that is, to the poorest (min) and richest (max) exporters that define the boundaries of our product ranges. It is also an inequality that is unlikely to hold because we are examining two specific unit values ( $p_{\max,g}$  and  $p_{\min,g}$ ) even though we know that such unit values are extremely noisy.

The term  $\Delta_g > 0$  defines one inequality for each product range in the HS10 panel of figure 5. A nonparametric test of  $\Delta_g > 0$  is the sign test, which easily rejects the null hypothesis that the signs of the  $\Delta_g$  are random (*p*-value of less than .0001). The mean value of  $\Delta_g$  is 0.63 (t = 27.23) and, more robustly with noisy data, the median value of  $\Delta_g$  is 0.45. Since  $e^{0.45} - 1 = 0.57$ , this implies that the richest significant exporter of the median product has a unit value that is 57 percent higher than the corresponding unit value of the poorest significant exporter. Cautiously interpreting unit values as prices, this means that prices are increasing as one moves through a product range in figure 5.

It is tempting to examine an even stronger prediction, namely, that unit value ranges  $\Delta_g$  are large when product ranges  $\ln y_{\max,g} - \ln y_{\min,g}$  are large. While this is not a prediction of the model, it can be generated by adding more restrictions on how scarcity varies across countries and products. To examine this prediction we estimate the following regression:

$$\ln p_{\max,\sigma} - \ln p_{\min,\sigma} = 0.15 + 0.18(\ln y_{\max,\sigma} - \ln y_{\min,\sigma})$$

(clustered t = 11.23). Thus, large product ranges in figure 5 are associated with large unit value ranges.

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