# Krugman's Love-of-Variety Model of International Trade ECO 2304: Topics in International Trade 

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Spring 2013

## Love of Variety

- Next week we will review the Melitz model. It is therefore useful to understand its precursor, Krugman's love-of-variety model with identical firms. Melitz took this model and added heterogeneous firms to it.
- Krugman actually has two models:
- In his AER (1980) paper he assumes CES preferences. Let $\sigma$ be the constant elasticity of substitution. It is also equal to the elasticity of demand. Hence, the elasticity of demand is constant.
- In his JIE (1979) model he allows the elasticity of demand to be variable.
- Today we will review the AER version. In week 4 we will review the JIE version.
- To gain familiarity with the AER version, and as a precursor to reading the Melitz model, you must do the problem set.


## Competition and Trade (Krugman, JIE, 1979)

- Consumers value variety:

$$
U=\Sigma_{i=1}^{N}\left(q_{i}^{d}\right)^{(\sigma-1) / \sigma}
$$

where $i$ indexes the $n$ firms (varieties), $q_{i}^{d}$ is consumption, and $\sigma$ is the elasticity of demand $(\sigma>1)$. There are $L$ consumers so that total demand is $L q_{i}^{d}$.

- Labour demand for firm $i$ is given by

$$
l_{i}=f+q_{i} / \varphi
$$

where productivity $\varphi$ is the same for all $i$ and, because all firms are identical, $q_{i}=L q_{i}^{d}$.

- Supply equals demand for labour:

$$
L=\Sigma_{i}\left(f+q_{i} / \varphi\right)
$$

- In the symmetric equilibrium we can ignore the $i$ subscripts.


## Love of Variety (Continued)

- Profit maximization: $M R=M C . M R=p\left(1-\frac{1}{\sigma}\right)=p \frac{\sigma-1}{\sigma} . M C=\omega / \varphi$ :

$$
\begin{equation*}
\frac{p}{w / \varphi}=\frac{\sigma}{\sigma-1} . \tag{1}
\end{equation*}
$$

- Zero profits: $p q=(f+q / \varphi) w$ or, since $q=L q^{d}$ :

$$
\begin{equation*}
\frac{p}{w / \varphi}=1+\frac{f \varphi}{L q^{d}} . \tag{2}
\end{equation*}
$$

Krugman (JIE, 1979)


Zero Profits:
$\frac{p}{w / \varphi}=1+\frac{f \varphi}{L q^{d}}$

Krugman - CES (AER, 1980)
$\frac{p}{w / \varphi}$

Profit max:
$\frac{p}{w / \varphi}=\frac{\sigma}{\sigma-1}$

Zero Profits:
$\frac{p}{w / \varphi}=1+\frac{f \varphi}{L q^{d}}$

## Competition and Trade (Continued)

Krugman - CES (AER, 1980)


- Trade integration means a rise in $L$ : the zero-profit condition shifts in.
- While this reduces per capita consumption of each variety, it raises the number of varieties. Since consumers love variety, the net effect is an increase in welfare.
- Specifically, $d \ln N / d \ln L=1$ and $d \ln U / d \ln L=1 / \sigma>0$.


## Proofs of $d \ln N / d \ln L=1$ and $d \ln U / d \ln L=1 / \sigma>0$

(1) Equating (1) and (2) and simplifying yields

$$
\begin{equation*}
L q^{d} / \varphi=(\sigma-1) f . \tag{3}
\end{equation*}
$$

Labour demand and $q=L q^{d}$ imply $I=f+L q^{d} / \varphi$. Hence $I=f+(\sigma-1) f=\sigma f$.
(2) Total employment $(L)$ equals employment per firm times the number of firms $(I \times N)$. Hence $N=L / I$. But $I=\sigma f$ so that

$$
N=\frac{L}{\sigma f}
$$

and $d \ln N / d \ln L=1$.
(0) From (3), $q^{d}=(\sigma-1) f L / \varphi$. Hence:

$$
U=N\left(q^{d}\right)^{(\sigma-1) / \sigma}=\frac{L}{\sigma f}\left(\frac{(\sigma-1) f \varphi}{L}\right)^{(\sigma-1) / \sigma}=\left(\frac{L \varphi^{\sigma-1}(\sigma-1)^{\sigma-1}}{f \sigma^{\sigma}}\right)^{1 / \sigma}
$$

so that $d \ln U / d \ln L=1 / \sigma$.

