Krugman’s Love-of-Variety Model of International Trade

ECO 2304: Topics in International Trade

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Next week we will review the Melitz model. It is therefore useful to understand its precursor, Krugman’s love-of-variety model with identical firms. Melitz took this model and added heterogeneous firms to it.

Krugman actually has two models:

- In his AER (1980) paper he assumes CES preferences. Let $\sigma$ be the constant elasticity of substitution. It is also equal to the elasticity of demand. Hence, the elasticity of demand is constant.
- In his JIE (1979) model he allows the elasticity of demand to be variable.

Today we will review the AER version. In week 4 we will review the JIE version.

To gain familiarity with the AER version, and as a precursor to reading the Melitz model, you must do the problem set.
Competition and Trade (Krugman, JIE, 1979)

- Consumers value variety:

\[ U = \sum_{i=1}^{N} (q_i^d)^{(\sigma-1)/\sigma} \]

where \( i \) indexes the \( n \) firms (varieties), \( q_i^d \) is consumption, and \( \sigma \) is the elasticity of demand (\( \sigma > 1 \)). There are \( L \) consumers so that total demand is \( Lq_i^d \).

- Labour demand for firm \( i \) is given by

\[ l_i = f + q_i / \varphi \]

where productivity \( \varphi \) is the same for all \( i \) and, because all firms are identical, \( q_i = Lq_i^d \).

- Supply equals demand for labour:

\[ L = \sum_i (f + q_i / \varphi) \]

- In the symmetric equilibrium we can ignore the \( i \) subscripts.
Profit maximization: \( MR = MC \). \( MR = p \left(1 - \frac{1}{\sigma}\right) = p^{\frac{\sigma - 1}{\sigma}} \). \( MC = \frac{w}{\varphi} \):

\[
\frac{p}{w/\varphi} = \frac{\sigma}{\sigma - 1}.
\]  

(1)

Zero profits: \( pq = (f + q/\varphi)w \) or, since \( q = Lq^d \):

\[
\frac{p}{w/\varphi} = 1 + \frac{f\varphi}{Lq^d}.
\]  

(2)

Krugman (JIE, 1979)  
Krugman - CES (AER, 1980)
Trade integration means a rise in $L$: the zero-profit condition shifts in.

While this reduces per capita consumption of each variety, it raises the number of varieties. Since consumers love variety, the net effect is an increase in welfare.

Specifically, $d \ln N / d \ln L = 1$ and $d \ln U / d \ln L = 1 / \sigma > 0$. 
Proofs of $d \ln N / d \ln L = 1$ and $d \ln U / d \ln L = 1/\sigma > 0$

1. Equating (1) and (2) and simplifying yields

$$Lq^d/\varphi = (\sigma - 1)f.$$  \hfill (3)

Labour demand and $q = Lq^d$ imply $l = f + Lq^d/\varphi$. Hence $l = f + (\sigma - 1)f = \sigma f$.

2. Total employment ($L$) equals employment per firm times the number of firms ($l \times N$). Hence $N = L/l$. But $l = \sigma f$ so that

$$N = \frac{L}{\sigma f}.$$ 

and $d \ln N / d \ln L = 1$.

3. From (3), $q^d = (\sigma - 1)fL/\varphi$. Hence:

$$U = N(q^d)^{(\sigma-1)/\sigma} = \frac{L}{\sigma f} \left(\frac{(\sigma - 1)f\varphi}{L}\right)^{(\sigma-1)/\sigma} = \left(\frac{L\varphi^{\sigma-1}(\sigma - 1)^{\sigma-1}}{f\sigma^\sigma}\right)^{1/\sigma}$$

so that $d \ln U / d \ln L = 1/\sigma$. 

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