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Capital Cash Flows, APV and Valuation

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Abstract

This paper examines three different methods of valuing companies and projects: the adjusted present value (APV), capital cash flows (CCF) and weighted average cost of capital (WACC) methods. It develops the appropriate WACC and beta leveraging formulae appropriate for each valuation model, so that given a particular valuation model the correct APV and CCF values can be determined from the WACC value and vice versa. Further it goes on to show when the perpetuity formulae give poor estimates of the value of individual cash flows, even though the overall values are correct. The paper cautions that the APV and CCF models require more information than is currently known, such as the value of the corporate use of debt, and consequently can give misleading results, particularly in sensitivity analyses.

Keywords: *capital cash flows*, *APV*, *valuation*

JEL classification: *G*31, *G*32

1. Capital Cash Flows, APV and Valuation

There's an old story in England about the passenger arriving at Euston railway station asking a porter how to get to Leeds only to be told 'you can't get there from here'. Obviously you can get anywhere from anywhere, the correct answer is that there's no direct route or it is *difficult* to get there from here. It seems that choosing between different valuation methods gives rise to the same problem. While conceptually there is one correct value for a firm or project, in practice there are multiple ways of calculating it. As a result the important question is which valuation technique offers the most direct route, that is, the easiest and most accurate implementation and how can you check the resulting value against other models.

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In this respect, Ruback (2002) has recently proposed the Capital Cash Flows (CCF) method with the claim (p. 21) that 'in many instances the CCF method is substantially easier to apply and, as a result, is less prone to error'. In general this claim significantly overstates the advantages of the CCF approach and it is one objective of this paper to show that the CCF method in general offers no advantages over the traditional weighted average cost of capital (WACC) approach. However, more important this paper shows the route from the CCF valuation to the WACC as well as the route from the WACC to the CCF and other adjusted present value (APV) applications.

The outline of this paper is that Section 2 first considers the general valuation problem and what approaches have been used in the literature. Section 3 then develops the standard static tradeoff model (STO) model of capital structure, since both the APV and CCF methods impose a particular view on the value of the capital structure decision. Appendix A has a full discussion of the appropriate beta leveraging formulae implied by these models.[1] Section 4 then considers these alternative models of capital structure and their implications for estimating WACC, that is, the route going from here (APV) to there (WACC) assuming an optimal debt ratio. Section 5 then considers the main advantage suggested for APV and CCF valuations that they are more appropriate for highly leveraged transactions (HLTs) through a worked example offered by Ruback, which was specifically structured to show the advantages of the CCF method. This section then shows how to go from here (APV) to there (WACC) assuming fixed amount of debt. Section 6 adds some conclusions, warnings and suggestions for further research.

2. Consistent Valuation

The starting point for most discussion of different valuation methodologies is the classic work of Modigliani and Miller (1958). M&M showed with their proposition 1 that in a world without taxes, under quite general assumptions the value of the firm was independent of its use of debt, that is,

$$V_L = V_U + \gamma D \tag{1}$$

where V is the value of the firm with, V_L, and without, V_U, debt and γ is the advantage to using debt (D), which under their assumptions was zero.

Equation (1) is the starting point for most valuation models, since it focused valuation on what has come to be called adjusted present value (APV), where the unlevered value of the firm is adjusted for the advantages to using debt. However, in their proposition (2) M&M proved that the cost of equity capital increased with the use of debt financing

$$K_e = K_U + (K_U - K_d)^D/_E \tag{2}$$

where K is the cost of equity capital, with, K_e, and without, K_U, debt financing, K_d is the cost of debt financing and E is the market value of equity. The importance of equation (2) is that it is a corollary of equation (1), that is, it simply follows logically from (1). This means that for the flows to equity method of valuation (FTE), where the equity cash flows are discounted using the cost of equity capital, equation (2) is the correct *linking* equation to give the same FTE value as APV.

[1] See Yagill (1982) for an early development.

Finally in proposition (3) M&M showed that the weighted average cost of capital (WACC) was a constant equal to the unlevered equity cost, that is,

$$WACC = K_e \frac{E}{V} + K_d \frac{D}{V} = K_U \tag{3}$$

Again equation (3) is a corollary of equation (1). If value is determined by discounting the firm's free cash flows using the weighted average cost of capital, equation (3) provides the correct linking equation to ensure that the WACC value is the same as the FTE value and APV.

The motivation for M&M (1958) was to value the use of debt financing and show how it affected the firm's cost of capital. In M&M (1963) they corrected their treatment of corporate taxes to adjust (1) for the tax shield value of debt and in 1977 Miller adjusted the tax shield value again to consider the interaction of personal with corporate taxes. The equivalents of (2) and (3) were then derived as corollaries of the basic valuation equation in terms of the cost of capital. It was left to Taggart (1977) to explicitly show how equations (2) and (3) were the correct linking equations to ensure consistent valuation between WACC, FTE and APV, rather than the implications for the cost of capital. More recently Fernandez (2005) has extended Taggart's results for the three basic valuation models to include seven other formulations of the valuation problem including Ruback's CCF model and Stern Stewart's Economic Value Added (EVA).

However, while the linking equations show that it is possible to get 'there' from 'here', they do not show how easy it is or what information is external to the analysis. Booth (1982) was the first to look at these implementation problems. He looked at a low cost loan provided a multinational company as a subsidy to undertake a project on the basis that this is a standard feature of international capital budgeting. As a subsidy it adds a component of value to the APV so that γ in (1) is positive. Booth (1982) then showed that while it was possible to come up with the same values using FTE, APV and WACC by using different versions of (2) and (3) the values would be different depending on whether an optimal debt ratio versus an optimal amount of debt is assumed. That is, although linking equations can be derived to ensure consistent valuation, Booth (1982) showed that they have different information requirements that make actual implementation of APV and FTE problematic.

In this respect it is striking that most applications of APV and CCF[2] use M&M (1963) as the base valuation model, where 'here', the value of the levered firm is simply the unlevered firm value plus the value of the corporate debt tax shield, that is, the corporate tax rate (T) times the amount of debt. The value of the corporate tax shield is the major advantage to the corporate use of debt, however, determining that value remains controversial. In contrast to M&M (1963) which values the interest tax shields using the cost of debt, Harris and Pringle (1985) discount the interest tax shields using the unlevered equity cost, an approach that Ruback (2002) has dubbed capital cash flow (CCF) valuation. However, Booth (2002) showed that for consistent valuation with WACC the correct APV formula for a growth firm is

$$V_L = V_U + \frac{K_u T D}{(K_u - g)} \tag{4}$$

[2] See for example Myers (1974), Luerhman (1997), Inselberg and Kaufold (1989) and Ruback (2002).

which is in between M&M (1963) and CCF valuation. For the no-growth firm (4) collapses to M&M (1963), but for a growth firm the future additional tax shields are as risky as the future additional EBIT.

Fernandez (2004) also derived (4) in a different context from the present value of the taxes of the unlevered and levered firm, as two separate sets of risky cash flows. However, the result is the same and was recently confirmed by Arzac and Glosten (2005) in this journal[3] and discussed extensively in a forthcoming paper in this journal of Massari *et al*. (forthcoming). What all these papers have in common is that the average and marginal advantage to the use of debt is a constant. Hence, the levering and unlevering formulae are easy to use as they are linear in the debt equity ratio. However, note that (4) as in M&M (1963) is an incomplete model as there is no offset to the corporate use of debt which implies 100% debt financing.

The problem is that as Brennan (1970) pointed out equity income is preferentially taxed, relative to interest income. This reduces the value of the corporate tax shield and as Miller (1977) suggested may create an equilibrium where there is no value to the corporate use of debt. Further even if we ignore taxes, there is the question of distress risk due to the corporate use of debt. Modigliani and Miller (1958) assumed away distress by allowing the firm to issue risk free debt. However, the existence of bankruptcy costs and more subtle value losses due to the loss of financial flexibility, for the reasons discussed in Myers (1974), leads most to believe that there are offsetting, for the want of a better term, I will label distress costs. The interaction of tax advantages and distress disadvantages to the use of debt generates what Myers (1984) dubbed the static tradeoff model which before M&M (1958) was known simply as the traditional model of capital structure.

This leads to a more general form to (1) that

$$V_L = V_U + (\alpha D - b(D) = \gamma(D)) \tag{5}$$

where α is the net tax advantage, b is the distress cost function and γ the overall value added attached to corporate debt.[4] This is perfectly general; it handles the M&M (1958) no tax and bankruptcy model where $\gamma = 0$, the M&M (1963) tax corrected model where $\alpha = \gamma = T$ as well as the static tradeoff or traditional model. It also handles the pecking order model (POM) where there is no optimum capital structure and instead the firm issues securities either based on information asymmetries as in Myers (1977) or managerial self interest as in Donaldson (1963). In either of these cases if the capital structure decision is not based on long run value maximising behaviour, as in the static model, then the implication is that there is no optimal capital structure, in which case $\gamma = 0$.

The importance of (5) is that it is a general APV model. This means that we can derive the linking equations equivalent to (2) and (3) consistent with any general APV

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[3] Cooper and Nyborg (2006) and Fieten *et al*. (2005) also discuss the value of the tax shields in the context of Fernandez derivation.

[4] See Haugen and Senbet (1978) for a discussion of distress costs in the context of the static tradeoff model. They point out that for distress costs to affect capital structure choice they have to be an external value drain and as Higgins and Schall (1975) point out there have to be market imperfections and transactions costs to support this. In the M&M perpetuity model this means that when the firm's EBIT is less than its interest payments and the shareholders have to commit funds to support the debt, they also incur additional contracting and issuing costs that they would not incur in the absence of debt.

model. In this way we can show how to generate consistent values using FTE, APV, CCF and WACC and point out the pitfalls and implementation problems in doing so. This extends the work of Fernandez (2005) and Booth (2002) who both assumed that value was linear in the use of debt due to the absence of any offset to the tax advantage to debt. However, it is important to realise that the equivalents to (2) and (3) are simply corollaries of (5), that is, we are simply finding the correct linking equations and how to go back and forth between APV, FTE, WACC and CCF to get the same value. In this sense, it is an 'accounting' exercise of how to do consistent valuation with different models when valuation is determined by a general APV.

In practice, the net advantage to issuing corporate debt is determined by the firm specific characteristics analysed, for example, by Rajan and Zingales (1995) and Booth *et al*. (2001). These factors, such as the proportion of tangible assets, profitability, size and future growth opportunities determine the marginal distress function. However, it is beyond the scope of this paper to incorporate these into the net advantage to debt, γ, except to note that they exist since firms do not finance 100% with debt.

3. The Static Trade-off Model and the Equity Cost Equation

First note for a debt-free perpetuity[5] the value of the firm is

$$V_U = \frac{\pi(1-T)}{K_U} \tag{6}$$

where π is the firm's expected operating income or EBIT (earnings before interest and tax). The unlevered equity cost, K_U, is also variously referred to as the debt free cost of capital or the expected asset return. We use K simply to denote the investor's required rate of return and use the subscript $_U$ to indicate the absence of debt.

The levered equity cost is the earnings yield or the expected net income divided by the equity market value, E. Using the definition of net income this is

$$K_e = \frac{(\pi - K_d D)(1-T)}{E} \tag{7}$$

where the levered equity cost is subscripted e for equity and the expected debt cost is the investor's required rate of return for debt, $K_{d,}$ times the market value of debt, D.

The key question in all valuation models is how the levered equity cost varies with the firm's use of debt, since this is the key linking equation. This determines how we get 'there' from 'here'. To see this we can rearrange (5) and note that the value of the levered firm is simply the value of its debt plus equity, that is, $V_L = E + D$ to get

$$\pi(1-T) = K_U E + K_U D(1-\gamma) \tag{8}$$

Substituting this definition of $\pi\ (1-T)$ into (7) we get,

$$K_e = K_U + (K_U(1-\gamma) - K_d(1-T))^D/_E \tag{9}$$

Equation (9) is the general form of M&M (1958)'s proposition 2 and encompasses all of the classic results as special cases. The critical question is the coefficient on the debt equity ratio.

[5] Booth (2002) shows that the critical assumption needed in the valuation equations is not the firms' future growth prospects, but the firm's debt policy.

Table 1

Model	Debt Equity Coefficient	Reason
M&M (1958) no taxes or distress costs	$K_U - K_d$	Pure leverage effect
M&M (1963) corporate taxes	$(K_U - K_d)(1 - T)$	Debt increases equity values + pure leverage effect
Miller (1977) corporate + personal taxes	$(K_U - K_d(1 - T))$	No value to leverage but interest still deductible
Static Trade-off Everything	$(K_U(1 - \gamma^* + b\eta) - K_d(1 - T))$	Advantage to debt + increased distress costs

The M&M (1958) equity cost increases by the differential between the unlevered equity and debt costs. This is the 'pure' leverage effect. In M&M (1963) the use of debt increases equity value as well due to the tax shield value, so the increase in the cost of equity is moderated by multiplying by $(1 - T)$. Finally if there is no advantage to corporate leverage, so that $\gamma = 0$, but interest is still tax deductible, the financial leverage effect depends on the extent of arbitrage in the fixed income market, that is, whether the risk-free rate R equals the firm's after tax borrowing cost $(K_d(1 - T))$.

For the static trade-off model (9) is the relationship between the equity cost and the debt equity ratio at the optimal level of debt. It will also hold at every debt equity ratio except γ will not be the optimal advantage to using debt. To show how the equity cost varies with the debt equity ratio, let γ^* be the non-optimal advantage to debt and define the debt equity ratio as θ. If (9) is totally differentiated; after dividing we get

$$\frac{dK_e}{d\theta} = K_U\left(1 - \gamma^* + b\left[\frac{\theta db}{bd\theta}\right]\right) - K_d(1 - T) \tag{10}$$

Here the expected debt cost is assumed constant as bond holders protect themselves. If the new term in (10) is recognised as the elasticity of distress costs with respect to changes in the debt equity ratio, which we can denote by η, then (10) simply states that the marginal impact of changes in the debt equity ratio changes with the distress costs.

There has been little empirical work on distress cost functions, since they are affected by firm specific characteristics as discussed in Rajan and Zingales (1995), but it is reasonable to assume that the distress cost elasticity is not constant. As discussed in the traditional model it is initially likely to be insensitive to the debt equity ratio, but increase rapidly as the firm becomes more highly levered due to the loss of flexibility and debt overhang problems as discussed in Myers (1977). Since the bond holders are assumed to protect themselves these costs are borne by the equity holders in reduced equity values. Table 1 illustrates the coefficient on the debt equity ratio in the various equity cost models.

The most important insight is that with the static tradeoff model, by definition, there is an interior optimum debt ratio, so that the coefficient on the debt equity ratio is non-linear, that is, η is not constant. This means that the equity cost increases in a more complex, firm specific, way than is assumed in M&M (1963) and that which underlies almost all applications of APV, FTE and CCF models.[6]

[6] In turn this means that the firm specific factors also affect the beta leveraging formulae.

We know that firm value does not increase continuously with the debt equity ratio, so it is important that a valuation model reflect this. Consequently, a realistic equity cost unlevering formula is not a simple function of an economy wide constant, such as the corporate tax rate. Instead it will vary from firm to firm with variations in firm specific characteristics in an identical way to the optimum capital structure. This observation has important implications for both the APV and CCF methodologies, since if one believes in the complexity of realistic capital structure decisions, then one can not simultaneously use the APV, FTE and CCF methodologies based on M&M (1963).[7]

4. Weighted Average Cost of Capital (WACC) and Valuation

This discussion of the equity cost equation is a more extended discussion of M&M (1958)'s proposition II, where with the static trade-off model equation (10) is the equivalent of their proposition II. For the equivalent of their proposition III in (10) multiply through by E, rearrange to isolate and solve for K_U, and divide by the firm's market value. In this case, we get

$$K_U(1 - \gamma^D/_V) = K_e\frac{E}{V} + K_d(1 - T)\frac{D}{V} \tag{11}$$

Equation (11) states that the traditional weighted average cost of capital (WACC) is equal to the unlevered equity cost times one minus the debt ratio, times the optimal advantage to debt financing. This means that for each equity cost leveraging formula in Table 1, there is an equivalent formula for how that model affects the WACC, since (11) is just a rearrangement of (10). Consequently in going from APV/CCF to WACC and back we can either estimate the correct equity cost or the equivalent WACC; in either case we simply have to choose the right linking equation consistent with the assumed model.

In M&M (1958) with neither taxes nor distress costs $\gamma = 0$ so the WACC is equal to the unlevered equity cost. However, (11) then shows that to estimate this unlevered equity cost we simply calculate the WACC. With M&M (1963) $\gamma = T$ and the WACC is equal to what is often referred to as the M&M cost of capital

$$WACC = K_U(1 - T^D/_V) \tag{12}$$

A corollary to M&M (1963) and the result that firm value increases with the use of debt, is the fact that the WACC decreases. Without any offset to the tax advantages of debt the optimal debt ratio is 100% and the WACC declines to $K_U(1 - T)$. Of course the fact that M&M (1963) is an incomplete model of capital structure implies that (12) is an incomplete model of the behaviour of the WACC with leverage.

Finally (11) can be rearranged as

$$WACC = K_U(1 - \gamma^D/_V) \tag{13}$$

which is the static trade-off or traditional version of M&M (1958)'s proposition III. In this case the WACC varies with the advantages to using debt and (13) is simply an algebraic form of the graphical representation in most finance textbooks. Initially the

[7] Note that the optimum capital structure is that which minimises the weighted average cost of capital, so if firm value has an interior maximum then the WACC has an interior minimum and the simple Hamada beta leveraging formula is incorrect.

tax advantages outweigh the distress cost until an optimum is reached which determines γ; beyond which distress costs outweigh the tax advantages and the WACC increases.

Note that in deriving (11)–(13) the WACC is consistent with a particular valuation model in the same way as the equity cost formulae in Table 1 and the beta leveraging formulae in the Appendix. They are all corollaries of the assumed valuation model. However, in each of equations (11)–(13) estimating the WACC will in all cases estimate the correct discount rate, since regardless of which model is correct the valuation effects show up in the equity cost and hence the WACC. It follows that an understanding of capital structure theory is an absolute precondition for understanding the different valuation methodologies.

It is in this context that we can understand 'you can't get there from here'. Suppose as in Booth (2002) someone performs a standard WACC analysis and values a firm's perpetuity cash flows

$$V = \frac{\pi(1-T)}{WACC} \tag{14}$$

Under what conditions will this value be consistent with that obtained using the APV or CCF methodologies? For example if the expected EBIT is \$20mm, the tax rate 50%, the debt ratio 50% and the equity and debt costs 15% and 10% respectively, the WACC is 10% and the value \$100mm.

Suppose someone else uses APV to value the firm and is given the exact same information, how easy is it for them to get the same value? First, unlike the WACC valuation they have to know which APV model to use, that is, what is the value added by debt? If they use M&M (1958) the answer is easy, since the value of the levered and unlevered firms are the same. This means that the unlevered equity cost is the same as the WACC at 10% and the value added from debt is zero. Hence, the value of the unlevered cash flows is \$100mm.

Suppose instead we use M&M (1963) so we acknowledge corporate taxes, but assume that all personal income is taxed at the same rate and there are no distress costs. In this case, we know that the value of the firm is an APV equation

$$V = \frac{\pi(1-T)}{K_U} + DT = \frac{\$10}{K_U} + 0.5D \tag{15}$$

The problem is we do not immediately know D and K_U? However, we know the route from here, the APV equation, to there, the \$100mm WACC value, is through Hamada's beta leveraging formula (11) or M&M (1963)'s proposition II with taxes, so we use (12)

$$K_U = \frac{WACC}{(1 - T^D/_V)} \tag{16}$$

If we correctly estimate the WACC at 10% and use the optimal debt ratio of 50% we can estimate the unlevered equity cost at 13.33% and unlevered value at \$75mm.

Apart from the fact that we have to estimate an additional unobservable value, the unlevered equity cost, assume an incomplete model of capital structure and do an additional calculation, deriving the \$75mm is straightforward. However, to derive the value of the debt tax shield requires the optimal amount of debt. However, all we know is that the value of the debt tax shield is 50% and with M&M (1963) the optimal amount of debt is 100%. Hence in practice using APV with M&M (1963) poses significant problems. If we knew that the total value was \$100mm, then we would know that the optimal amount of debt is \$50mm and the tax shield is worth \$25mm.With this knowledge we could also estimate the APV at \$100mm the same as the WACC value.

However, working through this simple example reveals the central problem in moving between the different methods. Conceptually we can get part way to the WACC value by using (12) but we cannot get all the way until we know the answer that the value is \$100mm. This is because APV/CCF is based on absolute dollar values of debt whereas WACC is based on the optimal debt ratio. APV proponents often answer that we can iterate towards the optimal value (of \$100mm). Suppose, for example, we used the purchase cost of the firm or its book value to determine the amount of debt. I have not used this information before because it is not necessary with WACC, but suppose this is \$60mm. In this case the debt might be estimated at \$30mm, 50% of the cost, and the value of the debt tax shield \$15mm, so the APV is \$90mm.

In this case the right route has been taken to get to the WACC valuation, but the answer is wrong. Clearly \$90mm is less than \$100mm since APV is assuming less debt than is optimal. The reason is that the optimal amount of debt in the WACC is determined endogenously by the debt ratio times the implicit firm value. In contrast, in APV we do not know the firm value and instead have to use as a first approximation its cost. Booth (1982) first pointed out this result that to the extent that the NPV is non zero, the two methodologies will give different values. The APV value can be fixed by noting that with a \$90mm value the debt ratio is not 50% but 30/90 or 33.33%, so clearly the firm needs to raise more debt. By repeatedly increasing the amount of debt and checking on the debt ratio we can determine the consistent amount of debt at \$50mm, where the APV will equal the WACC valuation.

However, note what we have had to do with APV. First we have had to assume a particular APV model. Second we have had to use the equity cost equation consistent with that APV model. Finally we have had to iterate towards the optimal amount of debt. However, think about the iteration, suppose we simply specify the debt tax shield as the corporate tax rate times the debt ratio times the unknown firm value, that is,

$$V = \frac{\pi(1-T)}{K_U} + T\theta V \tag{17}$$

Immediately we can see that with firm value on both sides of (17), we can solve for firm value and get

$$V = \frac{\pi(1-T)}{K_U(1 - T^{D}/_{V})} \tag{18}$$

From (18) the denominator is none other than the WACC consistent with the M&M (1963) APV formula. Hence, endogenising the debt ratio in an APV model short-circuits any iteration and causes it to degenerate to WACC.

For the static tradeoff model we can use (13) and suppose we assume that $\gamma = 0.20$ in this case the unlevered equity cost is 11.11%. This means that the unlevered firm value is \$90mm, which is more than the M&M (1963) APV model since distress costs and personal tax reduce the corporate debt tax shield. However we still have to estimate the value to using debt and again knowing the 50% debt ratio we can not get to the \$100mm value unless we either iterate or substitute for the optimal debt ratio, which as before causes APV to degenerate to WACC. Once we know that the firm value is \$100mm we can estimate the optimal debt at \$50mm and its value to the equity holders at \$10mm.

The static tradeoff model has the advantage of generating an interior optimum for the debt level, but at the cost of having to break out the advantages of debt into the net tax advantages as well as the distress cost disadvantages. We have finessed this problem simply by specifying the net advantage at 0.20, but if the APV model were to be properly implemented these extra components would need to be estimated for each firm adding another layer of measurement error.

So far this discussion has been in the context of APV. However, in the CCF model the valuation equation is

$$V = \frac{\pi(1-T) + K_d DT}{K_U} \tag{19}$$

where the interest tax shields are discounted at the unlevered equity cost. We can formulate this as an APV equation simply by separating out the tax shield value,

$$V = \frac{\pi(1-T)}{K_U} + \frac{K_d DT}{K_U} \tag{20}$$

From which it follows that the CCF is just another version of the APV model. However, subtract the tax shield term from both sides and solve for firm value as

$$V = \frac{\pi(1-T)}{K_U\left(1 - \left(T\frac{K_d}{K_u}\right)^D\!/_V\right)} \tag{21}$$

For the CCF version of APV the linking equation to the conventional WACC is

$$K_U = WACC + TK_d{}^D\!/_V \tag{22}$$

Special to the CCF model is that the unlevered equity cost can easily be calculated as the non-tax WACC, that is,

$$K_U = K_e\frac{E}{V} + K_d\frac{D}{V} \tag{23}$$

Note that the non-tax WACC is not generally the same as the unlevered equity cost, it happens to be the same in the CCF model simply because the interest tax shields are discounted at this cost and there are no distress costs.[8] Again like M&M (1963) the CCF model is a special case, since it is not a complete model of capital structure as it implies 100% debt financing.

With our example the unlevered equity cost with the CCF model is now 12.5%, so that the unlevered firm value is \$80mm and again iterating or solving directly for firm value implies optimal debt of \$50mm and a tax shield value of \$20mm. This is expressing the CCF model as another APV model. Expressing it directly as the CCF model means that the \$50mm in debt generates annual interest of \$5mm and annual interest tax shields of \$2.5mm. Hence, the annual expected capital cash flows are \$12.5mm a year, which discounted at 12.5% gives the same \$100mm as WACC. Again the only way that this is consistent with WACC is if the \$50mm in debt is known beforehand, since this determines the \$2.5mm annual interest tax shield.[9]

Which of these APV models is correct? Is it M&M (1958), M&M (1963), Miller (1977), the CCF or the static tradeoff model? I have shown that you can always get there (APV value) from here (WACC) provided you know the route which is the appropriate linking equation (equity cost or WACC equation consistent with the APV model) *and*

[8] This is not clear in Ruback (2002, p. 9) where the derivation of the asset return (unlevered equity cost) precedes the discussion of the CCF model. Further even with CCF valuation this only holds with perpetuities as I discuss later.

[9] Note that even though the value of \$100mm is the same as for WACC, the individual period cash flows are valued differently as will be discussed in Section 4.

you know the optimal amount of debt. Whether any credence can be placed on the different APV components, that is, the unlevered firm value and the value of the debt tax shield is problematic given our current state of knowledge. As an APV model the static trade-off model is the most realistic, but as a result it is also by far the most difficult to implement, since γ varies non-linearly with the debt equity ratio and is firm specific.

Finally are there other APV models? The answer to this is yes. I specified the debt advantage in terms of the net tax advantage, minus the distress costs. Since this is the way that we normally discuss the static tradeoff model. However, there are lots of other advantages to the corporate use of debt. For example, most large companies can access the swap market, both to change their interest rate and foreign exchange exposure, and to lower their borrowing cost relative to that of external investors. Similarly firms are able to access longer term debt markets than most investors.

For the APV equation it means that we can specify γD without talking about taxes or distress costs or anything specific. This is useful since we know that firms used debt prior to the introduction of interest deductibility! This means that *any* relationship between firm value and its use of debt will generate an APV model and a corresponding equity cost or WACC equation that provides the correct route or linking equation back to WACC, provided the amount of debt is known. However, whereas these gains are capitalised into the firm's market values and implicitly valued using WACC, they need to be explicitly valued in APV and incorporated into the relevant equity cost and WACC formulae. Otherwise we will get lost going from here to there.

5. Fixed Debt Levels

The prior discussion has been in the context of a target or optimal debt ratio in valuing perpetuities. This is consistent with Graham and Harvey (2001)'s survey results that indicate that most companies have target debt ratios. As I showed above when there is a target or optimal debt ratio, both APV and CCF collapse to the standard WACC valuation. However, most APV proponents have placed the advantages of APV and CCF in the context of highly leveraged transactions (HLTs), where the debt ratio varies through time.[10] This raises the question can we get from an APV HLT valuation to a WACC valuation with fixed debt levels?

In answering this, first consider the present value of the expected unlevered free cash flow in year t, FCF_t, this free cash flow will generate some tax advantages and some external value drain due to distress, similar to the perpetuity amounts in (1). In this case the static tradeoff version of the APV model will value this free cash flow one year earlier as

$$V_{L,t-1} = \frac{FCF_t}{(1 + K_U)} + \gamma_t D_t \tag{24}$$

[10] Note that in most examples of HLTs designed to illustrate the advantages of APV/CCF there is no recognition that much of the debt is rarely regular 'interest bearing' debt. Willis and Clark (1989) show how payment in kind interest (paper instead of cash), and equity kickers like warrants and convertibles bridge the gap between straight debt and equity. In practice even in LBOs the true debt ratio rarely comes close to the level assumed in examples demonstrating APV/CCF valuations.

where $\gamma_t = \gamma^* K_U/(1 + K_U)$ is the annualised advantage to debt consistent with (5). In this case we can rewrite (24) as

$$V_{L,t-1} = \frac{FCF_t\left(1 + K_U \gamma_t {}^{D_t}/_{FCF_t}\right)}{(1 + K_U)} \tag{25}$$

So that the net advantage from debt is expressed as a 'gross up' (ε_t) of the free cash flow. This gross up can in turn be interpreted as the advantage to debt ($\gamma_t D_t$) expressed as a percentage of the unlevered perpetuity value of that period's expected free cash flow (FCF_t/K_U).

The equivalent WACC valuation for the same free cash flow is

$$V_{L,t-1} = \frac{FCF_t}{(1 + WACC)} \tag{26}$$

Hence, using (25) and (26) to solve for the WACC that gives consistent valuation gives

$$(1 + WACC) = \frac{(1 + K_U)}{(1 + \varepsilon_t)} \tag{27}$$

Equation (27) is quite general. It shows how the WACC varies with the unlevered equity cost and the net advantage to debt or gross up to give consistent values between the WACC and APV models.

Consider, for example, if the net advantage to debt is zero then the gross up is also zero ($\varepsilon_t = 0$) and we are back to M&M (1958) or Miller (1977). As a result, we discount each period's free cash flow with the constant WACC equal to the unlevered equity cost. If there is a net advantage to debt then $\varepsilon_t > 0$ and we have some form of APV model and the WACC declines with the use of debt. Equation (27) then applies to the valuation of every future expected free cash flow when they are discounted back one period.

However, first consider the perpetuity versions of (25) and (26). In this case the parameters are expected to be constant so that the gross up is also a constant. Hence, summing (25) and (26) to infinity we get

$$V_L = \frac{FCF}{K_U} + \gamma^* D = \frac{FCF}{WACC} \tag{28}$$

and we are back to our original models. This simply shows that our per-period valuation models are consistent with the perpetuity models.

Now take the present value of the expected free cash flow for some arbitrary time period t. The equivalent to (27) is

$$(1 + WACC)^t = \frac{(1 + K_U)^t}{(1 + K_U \gamma_t D_t / FCF_t)} \tag{29}$$

where we simply discount back t periods to get the present value. Equation (29) can be used to consider how the WACC should vary with both constant and changing financial risk to ensure consistent valuation with APV.

Consider first the case of constant financial risk, so that the denominator in (29) is constant. This simply means that for each forecast free cash flow the firm has the same proportion of debt outstanding.[11] Consider, the previous CCF example, where the

[11] This case includes, for example, the constant growth case discussed by Booth (2002), where even though the forecast free cash flows are expected to grow, so too is the expected amount of debt so that at any point in time the debt ratio and the gross up, ε_t, are constant.

advantage to debt is the interest tax shield discounted at the unlevered equity cost. As a result the gross up is simply

$$\frac{K_U \gamma D}{FCF} = \frac{K_d TD}{FCF} \tag{30}$$

which is a rearrangement of the interest coverage ratio. With our example numbers the gross up is a constant 25% since the free cash flows are \$10m and the tax advantage to debt is worth \$2.5m per year.

From equation (29) the appropriate WACC to discount period t's free cash flows is

$$WACC = \frac{1.125}{(1.25)^{1/t}} - 1 \tag{31}$$

For example for the free cash flow expected in the first period, the required WACC is −10%, that is, in the WACC valuation a negative discount rate is needed to get the same value as in the CCF valuation. This is because the CCF model discounts the CCF cash flows of \$12.5m, whereas WACC discounts the free cash flows of \$10m. For the free cash flows in period 2 the required WACC increases to 0.62% and keeps increasing until in the limit from (31) the WACC is the unlevered equity cost.[12]

The result is that even with constant financial risk the WACC discount rate has to vary each period to get the same value for each period's free cash flows as the CCF valuation, even though the value of the infinite stream of cash flows is the same.[13] With varying financial risk the gross up in the denominator of (29) also varies requiring a second set of adjustments to the WACC.

To illustrate the practical problems in using CCF consider the example posed by Ruback, which was set up to illustrate the advantages of CCF valuation. Ruback's example is for a simple project with the data in Table 2. The project is actually an extreme HLT where the creditors provide all the financing and the equity holders nothing. As in all APV/CCF examples a simple tax advantage with exogenous debt is assumed so there is no explanation as to why these particular levels of debt are assumed. However, assuming the CCF valuation is correct, the correct WACC from (29) is

$$(1 + WACC)^t = \frac{(1 + K_U)^t}{(1 + K_d TD/FCF_t)} \tag{32}$$

For example, the first year's WACC is simple as $t = 1$, so the unlevered equity cost of 18% is adjusted for the tax shield on debt. For the first year the interest tax shield is \$4224 and the free cash flow \$54,500 so the gross up is 7.75% and from (32) the WACC is 1.18/1.0775) or 9.51%.[14] For year 2 the interest tax shield is \$2,046 and the free cash flow increases to \$61,200 for a gross up of 3.34%. Consequently, the WACC

[12] In this sense the contrast between WACC and APV is the same as between using risk adjusted discount rates and certainty equivalents: APV uses the same approach as certainty equivalents and WACC RADRs.

[13] The reason for this is that implicit in WACC is the assumption of an optimal debt ratio for each free cash flow being valued. In this case with a present value of \$9.09mm and a 50% debt ratio, WACC assumes debt of \$4.55mm, whereas CCF explicitly assumes \$50mm. As you value all the free cash flows in perpetuity, the sum of the debt financing each future cash flow sums to the \$50mm assumed by CCF.

[14] The interest tax shield is only worth 7.75% of the free cash flows due to the very high assumed non-cash receipts of \$43,333.

Table 2
Ruback's example

	1	2	3
EBIT	16,667	26,667	36,667
Expected Interest	12,800	6,200	2,400
FCF	54500	61200	67900
Interest tax shield	4224	2046	792
CCF	58724	63246	68692
Value @ $K_U = 18\%$	49766	45422	41808
Debt	100000	50000	20000
Debt ratio	73.0	56.8	34.4
Gross up	7.75%	3.34%	1.17%
FCF debt ratio	201	111	47.8
Correct WACC	9.51%	16.07%	17.54%
Ruback WACC	14.9%	16.0%	16.6%

from (32) is determined by $1.18^2/1.034$ or 16.07%. Similarly for year 3 the WACC is 17.54%. Discounting the free cash flow by the WACC as calculated from (32) gives the same assumed correct values as the CCF method. However, they are not what Ruback estimates, which is the set of WACCs given in the last row of Table 2.

In Ruback's analysis the value of each individual free cash flow differs between the CCF and WACC valuations even though the totals are the same. For example, in year 1 he uses a higher WACC and consequently his WACC analysis penalises the value of the first year's free cash flow. In contrast his WACC in year 3 is too low and he overestimates the WACC value.[15] Understanding the reason for this difference provides the final reconciliation between the APV versus WACC valuation methods.

First, note that in (30) the gross up is determined by the free cash flow coverage ratio. In the perpetuity model this is determined by the reciprocal of the after tax interest coverage ratio, $K_d D/\pi(1-T)$ and is constant each period as is the debt ratio, since they are both determined by discounting at the unlevered equity cost. As a result, using the debt ratio to make adjustments for financial risk in the WACC is consistent with the annual financial risk attached to the free cash flows. However, this is not the case when the debt ratio varies through time as it does with HLTs.

To see this more explicitly return to (24) and express the net advantage to debt in terms of a per period 'debt ratio'

$$V_{L,t-1} = \frac{FCF_t}{(1+K_U)(1-\gamma_t {}^{D}\!/_{V})} \tag{33}$$

With a perpetuity the debt ratio is the stock of debt divided by the total firm value. However, (33) is derived from a single period's cash flow, so the debt ratio is the amount of debt divided by the market value of *that period's free cash flow*. Equating (33) with (26) gives another way of expressing the WACC as

$$(1+WACC)^t = (1+K_U)^t\left(1-\gamma_t {}^{D_t}\!/_{V}\right) \tag{34}$$

[15] Note that Ruback's present values in his Table 1 for CCF and WACC differ.

Again with CCF this simplifies to

$$(1 + WACC)^t = (1 + K_U)^t - K_{d,t}T^{D_t}/_V \tag{35}$$

For $t = 1$ the WACC is the unlevered equity cost of 18% minus the tax shield on debt times the 'debt ratio' for the first year's free cash flow. However, getting the right debt ratio is not simple.

For example, in year 1 Ruback effectively uses the perpetuity version[16] of (33) with the overall debt ratio of 73%. In this case the 4.24% tax shield times the debt ratio of 73% or 3.1% is subtracted from 18% to get 14.9% (last row in Table 2). This is the non-tax WACC, which in the perpetuity case with the CCF assumptions is also the unlevered equity cost. However, for valuing a single future free cash flow this perpetuity assumption is incorrect, since the financial leverage is not 'in perpetuity' but for a single year. The very essence of HLTs, and the use of CCF and APV in these situations, is that they are *not* perpetuities. The 'correct' WACC, in the sense that it gives the same answer as CCF/APV, is to use the debt ratio appropriate to the first year's free cash flow. If we knew the value of this first year's free cash flow was \$49,766 we could calculate the debt ratio for this year as \$100,000/\$49,766 or 2.0. In this case subtracting the tax shield of 4.24% times 2.0 or 8.49% from 18% gives the correct WACC of 9.51%.[17] Similarly if we use this free cash flow debt ratio (FCF in Table 2) we can get the correct WACCs for years 2 and 3 at 16.07% and 17.54% respectively.

This HLT example highlights the fact that if we want the same valuation for each period's free cash flow, then the WACC has to vary each period. In the case of HLT's this varying WACC will *also* vary due to the changed debt ratio supporting each year's free cash flow. In this sense going from APV/CCF to WACC needs the value to get the debt ratio for the WACC, in the same way as going from WACC to APV/CCF needed the value to get the absolute amount of debt. However, this endogeneity problem is avoided by using the correct WACC equation (29), since this avoids the 'debt' ratio and goes directly to the gross up caused by the net advantage to debt.[18]

6. Conclusions

The key result is that you can always move between different valuation methodologies, that is, you *can* get there from here. However, the route has to be planned out ahead, which means the correct linking equation, either the equity cost or WACC equation has to be consistent with the assumed APV model. Further depending on the problem at hand additional information may be needed, since APV uses absolute dollars while WACC the debt ratio. In terms of 'ease of use' and how 'prone to error' I offer the following conclusions:

- If the firm has an optimal or target debt ratio then APV and CCF add little, if anything, to a conventional WACC valuation. If we start with the WACC valuation there are as

[16] Ruback actually uses a more roundabout route, rather than the direct route through (20). He estimates the firm value to get the debt ratio by iteration.

[17] Note that calling this a debt ratio is somewhat misleading since it exceeds 1.0. This is because the CCF value is based on interest tax shields attached to \$100,000 of debt outstanding while the value of the cash flow is much less. The overall 73% debt ratio is derived from the sum of the present values of the three cash flows.

[18] Care also needs to be taken in any sensitivity analysis for the same reason.

many routes as there are versions of APV. However, provided the correct one is used we can always get to the APV value provided we already know the value. Why we need the value is simply that all APV models specify the debt in terms of absolute dollars, so that knowing the debt ratio is not in itself sufficient.

- In contrast if we start with any APV model we can always get the WACC value, since with an optimal debt ratio every APV model collapses to WACC. This is hidden with 'iterative' processes, but substituting in the optimal debt ratio causes the APV to degenerate to WACC.
- These two conclusions also apply to the CCF model which is simply another variant of APV. Whether interest tax shields are valuable and whether they should be discounted at the debt free equity cost is not the issue, since this is just asking whether the CCF version of APV is better than others. Since it is still an incomplete capital structure model and implies 100% debt, the answer to this question should be obvious.
- If fixed debt levels are assumed there is still a correct route going from APV to WACC but a correct WACC valuation may now require the answer, that is, value beforehand if we are to estimate a WACC as a weighted average of debt and equity costs. This is the reverse of going from WACC to APV. The reason WACC needs the value is that the absolute value of debt from the APV model needs to be converted to a debt ratio in the WACC.
- Finally as Table 2 and (29) show, to generate not just the same WACC value as APV but also the same value for the individual free cash flows requires either the debt ratio appropriate for each of the free cash flows or that the WACC be estimated based on an annual coverage ratio or gross up. In neither case is the overall debt ratio important, since this does not reflect the risk of the debt payments relative to the annual free cash flows. Otherwise, the overall value may be correct but the individual parts are not.

It is a judgment call whether WACC is 'better' than a particular APV model, but knowing one, we can always get the other, although the process is not easy! This paper has shown the mechanical linking equations that connect the two via both the equity cost and the WACC equation. In this sense the results here are generalisations of those in M&M (1958). The critical question is whether there is an optimal or target debt ratio or whether debt levels can be exogenously specified.[19]

This latter question can be interpreted as, is the debt level independent of the result of the valuation? APV may be easier than WACC if the exogenous debt levels are in fact fixed and completely unaffected by the results of the analysis, that is, it does not matter whether the estimated value is $0.5mm or $500mm, the debt level does not change. If on the other hand the valuation changes the debt level, then implicitly there are 'spill over' effects and the firm value affects the amount of debt. This implicitly argues for some form of target or optimal debt ratio. This is clearly the case in conventional capital budgeting where the net present value of any project gets capitalised into the firm's market value and the firm has a target capital structure.

[19] This is also the conclusion of Miles and Ezzel (1980), Booth (1982) and Booth (2002).

Appendix: Implied Beta Adjustment Models

Assume that required rates of return are determined by the capital asset pricing model (CAPM). In this case, the firms' unlevered equity cost is simply

$$K_U = R + \lambda\beta_0 \tag{A1}$$

where R is the risk free rate, λ the market risk premium and β_0 the firms unlevered or asset beta. Inserting (A1) into (5) gives

$$K_e = R(1 + (1-\gamma)^D/_E) + \lambda\beta_0(1 + (1-\gamma)^D/_E) - K_d(1-T)^D/_E \tag{A2}$$

How (A2) is simplified largely depends on the assumptions made about debt markets and the determination of the required return on debt.

There is no question that the promised yield to maturity on debt is affected by the firm's debt equity ratio. However, it is not clear that the expected rate of return on debt exceeds the risk free rate, that is, that there is a risk premium attached to debt. This is due to the unsystematic nature of distress and bankruptcy costs and the clustering of distress events that makes it difficult to empirically estimate debt betas. This difficulty has increased as the tools of monetary policy have changed.[20] For these reasons we can simplify (A2) by assuming as an approximation that $K_d = R$[21] that is, that bondholders protect themselves from the possibility of distress and continue to expect to earn the risk free rate.

With this assumption equation (A2) simplifies to

$$K_e = R + (T-\gamma)^D/_E + \lambda\beta_0(1 + (1-\gamma))^D/_E \tag{A3}$$

The key terms in equation (A3) are the beta leveraging formula and the constant added to the risk free rate.

First, consider the beta leveraging formula,

$$\beta_L = \beta_0(1 + (1-\gamma)^D/_E) \tag{A4}$$

In M&M (1958) where are were neither taxes nor distress risks, so that $\alpha = \gamma = T = b = 0$ and the unlevering formula simply collapses to

$$\beta_L = \beta_0(1 + {}^D/_E) \tag{A5}$$

which is the pure risk magnification effect of leverage.

Now consider M&M (1963) where corporate taxes are introduced so that $\alpha = \gamma = T$, that is, there are no offsets to the value of the corporate tax shield. The beta adjustment formula collapses to

$$\beta_L = \beta_0(1 + (1-T)^D/_E) \tag{A6}$$

which is the standard Hamada (1972) formula. In this case the negative effects of pure financial leverage are offset by the tax advantages of the deductibility of interest. The

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[20] Empirically bond betas were negative prior to the change in monetary policy tools in 1979, subsequently they have at times been equivalent to that on low risk stocks, (see Petit, 1999) for an interesting discussion), and have declined with more stable monetary policy and inflation.

[21] Note this does not assume that debt is risk free as the promised yield can still significantly exceed the risk free rate, i.e., there is a positive default spread.

easiest way to see this is that with perpetuities the value of the tax shield is *TD*, so the true cost of the debt outstanding is $(1 - T)D$ and the true debt equity ratio $(1 - T)D/E$, which is what (11) indicates is the impact of financial leverage. Consequently, the equity cost increases at a decreasing rate with leverage, as compared with M&M (1958).

Further, consider the Miller tax shield, where the tax advantages of debt are changed to

$$\left(1 - \frac{(1-T)(1-t_e)}{(1-t_p)}\right) \tag{A7}$$

where t represents the marginal individual tax rates on equity, e, and personal (interest), p, income respectively. (A7) simply reduces the tax advantages of debt. However, Miller (1977) showed that in equilibrium arbitrage between fixed income instruments can drive (A7) to zero so that the beta leveraging formula reverts to (A5) even in the presence of corporate taxes.

Equations (A5)–(A7) are all special cases of (A4) where the net advantage to debt also reflects distress costs. Consequently, (A4) is fundamentally different since it is not necessarily linear in the debt equity ratio. For example, for each of equations (A5)–(A7) the tax terms are largely economy wide constants. This means that they can be reversed to unlever betas and then used again to relever betas to a new debt equity ratio. This is common advice in textbooks and is fundamental to both the APV and CCF approaches where it is needed to estimate the unlevered equity cost. However, the basic problem with (A4) is that the net advantage to debt (γ) is the equilibrium value at the optimal debt ratio. It can be used to unlever betas, assuming that γ can be estimated, but it can not be used to relever them, since γ varies with the firm's debt ratio.[22]

The second feature of equation (A3) is the constant term

$$(T - \gamma)^{D}/_{E} \tag{A8}$$

In most cases this term is zero. For example, in M&M (1958) $\alpha = \gamma = T = 0$, and in M&M (1963) $\alpha = \gamma = T$. However, in the static tradeoff model this is not the case. Consider the case when $\gamma = 0$ and $T > 0$. Overall, there is no advantage to using debt but corporate interest costs are deductible. Without distress costs this would be the Miller case and (A3) becomes

$$K_e = R(1 + {}^{D}/_{E}) - K_d(1 - T)^{D}/_{E} + \lambda\beta_0(1 + {}^{D}/_{E}) \tag{A9}$$

In understanding (A9) note that it is derived from a CAPM that ignores personal taxes, which is inconsistent with the value of the tax shield in (A7). If the equity tax rate is assumed to be zero while corporate interest payments are fully taxable then (A9) collapses to (A4) if the constant term is zero if $K_d = R/(1 - T)$. This is the Miller equilibrium where arbitrage causes the interest rate on taxable corporate debt to be pushed up until it is held by individuals in the same tax bracket as the corporate tax rate. In this case there is no advantage to the corporate use of debt and the after tax rate on corporate debt equals the non-taxable risk-free rate.

In the Miller equilibrium the constant term is zero. However, if the offset to the corporate deductibility of interest is not met entirely by personal taxes but in part by

[22] The fundamental problem with both the APV and CCF models is that they assume that we know the correct capital structure M&M (1963) model when we do not. Further most people accept the static tradeoff model, while using Hamada's beta leveraging formula without realising they are mutually inconsistent.

distress costs there is imperfect arbitrage that leaves a residual tax advantage to the corporate use of debt ($\alpha > 0$), which is offset by distress costs. The constant term then depends on the extent of arbitrage activity and how closely $K_d = R/(1 - T)$.

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