

# Strategic Consumers, Revenue Management, and the Design of Loyalty Programs

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We study the interaction between the design of a premium-status loyalty program, revenue management, and strategic consumer behavior. Specifically, we consider a contemporaneous change where firms across several industries switch their loyalty programs from quantity-based toward spending-based designs. This change has been met with fierce opposition from the media and consumers. Building on the micro-foundations of strategic, forward-looking, and status-seeking consumer behavior, we endogenize strategic consumer response to firms' pricing and loyalty program design decisions, and we characterize conditions, under which, by coordinating these decisions, firms can benefit from strategic consumer behavior. We further show that by switching to a spending-based design, firms can benefit from strategic behavior even more, under broader conditions, and in a Pareto-improving way. Finally, we also analyze combined designs, which utilize a combination of quantity and/or spending requirements, and show how they can be used to better manage the transition toward spending-based designs, possibly minimizing negative consumer reactions.

*Key words:* operations and marketing interface, strategic consumer behavior, pricing, loyalty (reward) programs, premium status, revenue management

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## 1. Introduction

Loyalty programs are widely used in many industries, including retail, travel, hospitality, and financial services. Originally designed as marketing tools, such programs award customers various types of rewards for their purchases. One particular kind of reward that we will focus on in this paper is the *premium status* reward, under which a select group of the firm's most "loyal" customers, often labeled as Gold, Elite, VIP, or the like, enjoy the exclusive benefits, including free products and services, access to pre-sales, free shipping, and so on.

There are two common mechanisms underlying how a premium status is awarded. OpenTable, for example, awards VIP status<sup>1</sup> to those who "have honored 12 or more OpenTable reservations

<sup>1</sup> <https://community.opentable.com/t5/My-Account/What-is-VIP-Status/ta-p/121>

within one calendar year” – a *quantity*-based, *Q*-design. In contrast, Starbucks<sup>2</sup> awards a Gold status to those who spend \$150 – a *spending*-based, *S*-design. Combinations of the two are also common. For example, some airlines (United<sup>3</sup>) require that consumers fly 50,000 miles (or 60 segments) and also spend 6,000 dollars to qualify for Gold status – a *quantity-and-spending*, *QaS*-design. In contrast, some hotels (Hyatt<sup>4</sup>) allow customers to qualify for premium status either by staying 30 nights, or by spending 10,000 dollars – a *quantity-or-spending*, *QoS*-design.

Not all of these designs came to prominence at once. Starbucks used to award one star per transaction and required 30 stars for Gold status (i.e., ran a quantity-based design) and changed to the current spending-based program in just 2016; the program now awards 2 stars per dollar spent and Gold status now requires 300 stars (i.e., \$150 spending). Likewise, in the airline industry, the “frequent-flyer” or “mileage programs” such as the Delta Airlines’ SkyMiles, or United Airlines’ MileagePlus started as quantity-based programs, as their names suggest. Delta and United added spending-based requirements only in 2014, with American Airlines and Air Canada following in 2017. This trend has been observed in the hospitality industry as well: Hyatt, for instance, started offering a spending option only in 2017. In other words, while the exact mechanisms vary (e.g., Starbucks changed from *Q* to *S*, Delta/United changed from *Q* to *QaS*, while Hyatt changed from *Q* to *QoS*), the overall trend is clear: firms are changing their programs from quantity-based toward spending-based-like designs.

The media and public reactions to these changes have been rather negative. CNN reports a “customer outcry,” E!online states that “people are so pissed,” Fortune writes that “customers are furious,” and social media is full of posts from angry customers who feel “NOT happy” and call the changes to spending-based programs “ill-starred” and “awful” and announce a “boycott” (Cohen 2015, Martin 2016, Vasel 2016, Mullins 2016, Snyder 2016).

The goal of this paper is to analyze these different loyalty program design choices, and to assess their impact on firms and consumers in light of the recent changes from quantity- to spending-based designs. A central phenomenon captured in our analysis is that consumers are forward-looking and strategic (i.e., status-seeking); rather than viewing each transaction in isolation, they look forward to possibly obtaining/maintaining premium status and optimize their purchasing so as to maximize their total surplus over a period of time, taking into account both

<sup>2</sup> <https://www.starbucks.ca/card/rewards/rewards-program-ts-and-cs>

<sup>3</sup> <https://www.united.com/web/en-US/content/mileageplus/premier/qualify.aspx>

<sup>4</sup> <https://world.hyatt.com/content/gp/en/member-benefits/discoverist.html>

the prices they pay and the loyalty status they may obtain as a result. In the airline industry, such behavior under quantity-based designs manifests in so-called “mileage runs”<sup>5</sup> (“mattress runs” in hotels); a survey reported in Del Nero (2014) shows that 60% of frequent travelers engaged in a mileage run in December 2013, and 25% did a mattress run. A New Yorker article by Sernovitz (2016) mentions that “frequent fliers sometimes go to great lengths to keep their airline elite status,” providing an example of a customer who took three flights during the same day between different airports in the same city to accumulate the requisite number of flight segments. Beyond the airline industry, a Fortune magazine article by Wahba (2016) states that prior to the 2016 program change, “Starbucks was finding that many customers were asking baristas to ring up one item at a time to collect more points” – clearly an unnecessary and inconvenient activity that is, in spirit, quite similar to a mileage run.

While such strategic behavior increases demand, its impact on the firm’s profitability is not obvious because the firms incur status-related costs to serve premium customers<sup>6</sup>. Imagine a situation where the premium qualification threshold is 60 purchases, and consider a consumer who needs to make 59 purchases per year. Without being strategic, this consumer would not obtain premium status, and the firm would incur no status-related costs. If, however, this customer becomes strategic and “runs” for one additional purchase every year, then (s)he will maintain premium status. Consequently, every year, the firm will obtain additional revenue from one purchase, but it will incur status-related costs for all the 60 purchases this customer will make. Therefore, even though demand increases, it is unclear whether profit increases as well.

This discussion poses multiple research questions. First, how to characterize consumers’ strategic behavior under different program designs? Second, does such behavior benefit or hurt firms, and whether and how will that depend on program design and firm’s characteristics? Third, how do the different program designs compare in terms of profit? Finally, how does the change toward spending-based designs affect consumers; specifically, how can firms better manage the transition toward spending-based designs to minimize the aforementioned negative reactions?

To address these questions we consider a monopolistic<sup>7</sup> firm that sets prices and determines the loyalty program design and qualification thresholds. In response to the firm’s prices and loyalty

<sup>5</sup> A mileage run is an airline trip designed and taken solely to gain maximum frequent-flyer miles, points, or elite status, [https://en.wikipedia.org/wiki/Frequent-flyer\\_program#Mileage\\_runs](https://en.wikipedia.org/wiki/Frequent-flyer_program#Mileage_runs).

<sup>6</sup> For example, with Delta, United and many other airlines, Gold customers can check their bags for free, while the airlines incur costs for handling bags; <https://www.united.com/web/en-US/content/travel/baggage/chasebag.aspx>. With Neiman Marcus, Hudson’s Bay, and some other retailers, premium-status customers qualify for free expedited (e.g., 2-day) shipping on online orders which, again, results in additional costs; <http://www.incircle.com/category.jsp?itemId=cat103405&parentId=cat103411&masterId=cat000001>.

<sup>7</sup> While many industries are clearly competitive, in many cases they are also highly concentrated. For instance, in the airline industry, many large airports are dominated by a single airline. According to [www.transtats.bts.gov](http://www.transtats.bts.gov), in

decisions, consumers *strategically* decide on how much to purchase and at what prices to maximize their total surplus over the status qualification time period. To describe this complex behavior we present a novel model that captures multi-purchase consumer response to both prices and loyalty program decisions. This allows us to characterize rich patterns of consumer behaviour, analytically solve the customers' problems under different loyalty program designs, and endogenously derive the demand faced by the firm as a function of its pricing and loyalty decisions. We then formulate models for how the firm decides on prices and loyalty program design decisions, factoring in the consumer response. We solve the firm's profit maximization problems under different loyalty program designs and discuss managerial implications.

Our paper is the first to consider the impact of strategic consumer behavior on joint pricing and loyalty program decisions. For this reason, we mainly focus on a single aspect of the loyalty program design where strategic behavior is critical: premium status reward. We then present extensions to our model and robustness checks to consider combined thresholds for status qualification, redeemable points, the dynamics of acquiring status, and various consumer type distributions.

### 1.1. Summary of findings

The insights from our work can be classified into three categories: strategic consumer response, impact of strategic consumer behavior, and comparison of loyalty program designs.

**Strategic Consumer Response to Loyalty Programs.** Our paper is the first to characterize strategic consumers' optimal response to the firm's pricing and loyalty program decisions. Specifically, we show that under the quantity-based design, "buying up" (i.e., purchasing more solely to qualify for status, or "mileage-running") is indeed optimal for a segment of consumers. Under the spending-based design, "spending up" (i.e., paying more for the same product or service to qualify for status) is also an optimal strategic choice for some consumers. Spending-up options include paying up (strategically purchasing the same products/services at higher prices, even when lower prices are available), buying up (buying more than needed at regular prices), or mixing the two. That is, a switch to a spending-based program could reduce buying up, but may not eliminate it completely as one way to spend more is simply to buy more.

January 2016, Delta's share at its hub airport in Atlanta was 73.44%. The second-largest major carrier in Atlanta, American, had a share of only 2.54%. Similarly, American's share at its hub in Dallas/Fort Worth was 67.39% while Delta's was under 3%. As a result, a frequent traveller residing in the Atlanta metro area would be inconvenienced by being loyal to any airline other than Delta, while one residing in Dallas would have a hard time flying anything but American. This pattern holds for many major cities around the world: residents of Paris (hub of AirFrance) have considerably fewer choices on other airlines, such as Lufthansa, while those who live in Frankfurt (hub of Lufthansa) have much fewer choices with AirFrance. Supporting this logic [Sernovitz \(2016\)](#) writes that customers are extremely loyal to United, either because they like it "... or, more probably, because the airline has a hub near your home." In other words, for many frequent travellers – who are the primary object of our study – the choice of airline is mainly determined by where they live and not by the airlines' competitive behaviors.

**The Impact of Strategic Consumer Behavior.** By comparing the firm’s optimal profit in the instances when consumers are strategic with the profit when consumers are myopic, we characterize sets of model parameters – “strategic regions” that describe which firms can *benefit* from strategic consumer behavior. The quantity-based program’s strategic region,  $QSR$ , corresponds to the cases when the firm’s cost to provide the status is relatively small; yet we also show that the firm can benefit even when the value of status is smaller than the firm’s cost to provide them. With the spending-based design the firm benefits *more* from strategic consumer behavior, and these benefits are of two distinct kinds. First, within  $QSR$ , the firm’s profit increases further under the spending-based design. Second, and more importantly, the strategic region under the spending-based program expands beyond  $QSR$ , meaning that under the spending-based program, the firm may benefit from strategic consumers even when it cannot do so under the quantity-based program.

That the firm can benefit from strategic consumers is particularly interesting because most of the literature established that strategic consumers negatively impact the firm (e.g., Cachon and Swinney 2009, Levin et al. 2009, Ovchinnikov and Milner 2012). Only a few studies demonstrate that the profit when consumers are strategic can be higher than when they are myopic: Marinesi et al. (2017) in the context of group buying, Zhang and Zhang (2017) in the context of trade-in remanufacturing, and Wang et al. (2018) in the context of season/flight passes. We thus identify another important element that can reverse conventional wisdom about the impact of strategic consumers on the firm’s profitability<sup>8</sup>. Our work also provides a new insight into the sources of a loyalty program’s value: while traditionally viewed as means of softening competition (e.g., Kim and Srinivasan 2001), we show that even without competition, loyalty programs can be valuable due to how they affect strategic consumer behavior.

**Comparison of Loyalty Program Designs.** By comparing quantity-based and spending-based designs, we first show that the firm’s profit is higher under the spending-based design. One might intuit that additional profit comes mainly from an increase in revenue from consumers who spend more. We show, however, that an equally important driver is the decrease in cost. The spending-based program expands the segment that strategically qualifies for the premium status and simultaneously reduces the segment of consumers who buy up. Replacing buying up with spending up could result in the same revenue, but from fewer purchases, which *decreases* costs,

<sup>8</sup> It is important to understand a subtle, yet conceptual, difference between the overall profit increase from strategic consumer behavior (our result) and a partial profit increase from some customers/activities in the presence of strategic consumer behavior. The findings of Su (2007) (that strategic behavior of one consumer segment increases profit from another strategic segment), Cachon and Swinney (2009) (that an incremental profit from quick response is higher when consumers are strategic), Li et al. (2014) (that it may not be profitable to price so as to eliminate strategic behavior), etc., are all examples of the latter, and are *not* similar to our result.

rendering a change to the spending-based program profitable. This result may thus provide the rationale for switching to spending-based programs that firms implemented in practice.

Speaking directly to the customer outcry generated by switching to spending-based programs, we show that some consumers may indeed suffer: the optimal spending-based solution makes buying up harder and some consumers may be worse off as a result. However, we also show that a switch to a spending-based program could be Pareto-improving: the firm *can* increase its profit without making any consumers worse off. To achieve a Pareto-improving solution, however, the firm might need to sacrifice a portion of its additional profit – i.e., settle with a profit that is below the optimal spending-based solution, but still above the optimal quantity-based solution. While it is not clear if all firms would do that, some firms could consider such an approach; e.g., as [Wahba \(2016\)](#) writes, “Starbucks... is being careful not to irritate customers or make them feel the new program is less generous.” In view of this, we further characterize the conditions under which Pareto-improving solutions are possible, and thus explain which kinds of firms can achieve such win–win solutions and which cannot.

As an extension, we investigate combined designs where the quantity and spending requirements are combined for status qualification. Recall that  $QaS$  is more stringent, as it requires both quantity and spending requirements to be satisfied, while  $QoS$  is more generous, as it allows consumers to choose which requirement to satisfy. One might thus intuit that  $QaS$  should achieve a higher profit due to its ability to select more profitable customers to whom status is awarded, but would question whether  $QoS$  can improve profit over the  $Q$ -design at all. Surprisingly, we show that both can improve profit from the  $Q$ -design and result in the same profit at optimality; both can achieve the optimal  $S$ -design profit, but more interestingly, even  $QaS$  cannot achieve profits higher than those of the  $S$ -design. We also confirm that win–win outcomes are not achievable under  $QaS$  since obtaining status becomes more difficult for consumers – the aforementioned negative reactions are therefore warranted. In contrast, with  $QoS$  the firm can improve profit without making any consumer worse-off; consistent with this, we are not aware of negative customer reactions to  $QoS$  designs. This suggests how firms can gradually transition toward spending-based programs to minimize the negative customer reactions.

Finally, we perform a number of robustness checks to verify that the insights derived from our model hold under broader settings. First, we consider a program with both redeemable points and status, either of which can be managed in a quantity- or spending-based manner. Beyond confirming the robustness of the main results, we also observe that the benefit of changing the redeemable points component from a quantity- to a spending-based design is larger when status is also managed with a spending-based design. This might explain how some firms implemented changes in practice: major airlines (United, Delta), for example, added a spending requirement

to status in 2014, but changed the redeemable miles component later in 2016. Second, we also consider the detailed dynamics of acquiring status, capturing the possible distinction between the qualification and benefit periods. We confirm that our main (steady-state) model is a good approximation of the dynamic setting, and provide additional insights on why some firms extend the benefits period – i.e., while their policies state that the status obtained in one year is valid for the next year, these firms “by courtesy” allow consumers to enjoy the benefits immediately after they qualify. Lastly, we examine the impact of different consumer type / willingness-to-pay (WTP) distributions. We consider a variety of skewed, centered, and bimodal distributions, confirming the robustness of the analytical findings derived for a uniform distribution. In addition, we observe that firms with a larger mass of medium/low WTP customers stand to benefit more from changing to a spending-based design, in terms of the relative profit increase.

To conclude, our paper rigorously examines premium-status loyalty program designs in the presence of strategic consumers and provides insights that are relevant for researchers and industry professionals as they redesign programs to improve their firms’ profits and customer satisfaction.

## 2. Related Literature

Our paper lies at the intersection between marketing and operations, and builds on prior findings from both fields. Marketing researchers examined consumer purchase behavior induced by loyalty programs. For example, [Lewis \(2004\)](#) empirically found that loyalty programs encourage consumers to shift from myopic (single-period) decision making to forward-looking (multiple-period) decision making, so as to maximize the total surplus over a certain time period. [Liu \(2007\)](#) further observed that different consumers react differently to the loyalty program, suggesting a need to consider consumer heterogeneity when studying loyalty programs. [Kivetz et al. \(2006\)](#) demonstrated the “points pressure” effect – i.e., that consumers purchase more as they get closer to the reward threshold in a frequency reward program, and [Kopalle et al. \(2012\)](#) confirmed that point pressure exists in premium-status programs. Beyond the marketing literature, strategic forward-looking behavior has also been established with respect to prices, e.g., [Li et al. \(2014\)](#). Building on these empirical findings, we model forward-looking, strategic, and heterogeneous consumer behavior in response to both loyalty program parameters and prices.

Strategic consumer behavior received significant attention from researchers who studied how a firm should anticipate such behavior and integrate it into revenue management decisions (e.g., [Su 2007](#), [Zhang and Cooper 2008](#), [Liu and van Ryzin 2008](#), [Sahin et al. 2008](#), [Cachon and Swinney 2009](#), [Levin et al. 2009](#), [Su and Zhang 2009](#), [Osadchiy and Vulcano 2010](#), [Jerath et al. 2010](#)). These studies comprehensively address the pricing problem, but do not consider loyalty programs. Our model is less detailed than these studies in dealing with the revenue management problem, as we

consider a menu of prices that remain static throughout the time period. However, in our model, the firm also decides on the premium-status qualification requirement, and consumers strategically take it into account, alongside prices, when making their purchasing decisions.

Combining consumer response to loyalty programs and pricing, our work is related to the existing studies in operations, marketing and economics, which have studied the impact of loyalty programs. These include the implications of loyalty programs on customer retention and switching costs (Carlsson and Löfgren 2006), brand loyalty and attitudinal loyalty (Shugan 2005), demand and market share (Meyer-Waarden 2007), and competition (Kim and Srinivasan 2001). Our paper contributes to this literature by identifying another important element that makes loyalty programs beneficial even in the absence of competition: strategic consumer behavior with respect to the interaction between loyalty programs and pricing. Our paper is also related to the stream of research on the design of loyalty programs; see Chapters 22 and 23 in Blattberg et al. 2008 and Dorotic et al. 2012 for reviews. While different design components of loyalty programs have been considered, e.g., reward types (Keh and Lee 2006), reward timing (Zhang et al. 2000), and frequency and magnitude (Nunes and Drèze 2006), these papers do not consider a change from quantity- to spending-based loyalty programs, which is the main focus of our paper.

Finally, our paper also contributes to the growing body of literature on the interface between marketing and operations – e.g., Afaki and Popescu 2014, Ovchinnikov et al. 2014 and, for general reviews, see Coughlan and Shulman 2010 and Tang 2010. Specific to loyalty programs, Chung et al. (2014) examine how the terms of reward point redemption affect the firm’s pricing and inventory control, Sun and Zhang (2015) study the tradeoff between the long and the short expiration terms of loyalty program rewards, and Chun et al. (2015) study the problem of optimally setting the monetary value of loyalty points in view of inherent liabilities that points create. These papers, however, do not investigate different loyalty program designs. Finally, Lu and Su (2015) compare quantity- and spending-based designs, but different from ours, they study the redeemable points component, while we focus on premium status and the role of strategic consumers. By linking the design characteristics of loyalty program to certain mechanisms, Chun and Hamilton (2017) investigate how different loyalty program design characteristics affect consumers’ decisions to spend points or money for a specific purchase

### 3. The Model

We consider a monopolistic firm with an infinite capacity that sells a single product and decides on the menu of prices,  $p$  and  $p^+ \geq p$ . The firm also decides on the structure of its premium-status loyalty program, either quantity-based, or spending-based, and sets the premium status qualification thresholds,  $Q$  and  $S$ , respectively. Under a quantity-based program, a consumer who

purchases  $Q$  items/times qualifies for a premium status, which we refer to as “Gold.” Under the spending-based program, a consumer who spends  $S$  or more dollars qualifies for Gold. For both programs, the firm incurs cost  $c$  per unit of Gold customers demand, which reflects the cost to provide status benefits as discussed in the Introduction; non-status-related costs are normalized to zero.

Motivated by the empirical observations introduced earlier (Lewis 2004, Liu 2007, Kopalle et al. 2012, Li et al. 2014), we model the behavior of consumers who are forward-looking (strategic) and therefore optimize their purchasing activity so as to maximize the total utility from purchases over a certain time period. In consumers’ decision-making processes, both the prices they pay and the loyalty program status they obtain play a role. Next, we discuss two critical components of our model: consumer heterogeneity and the utility of premium status.

### 3.1. Model of Consumer Heterogeneity

We first restrict our attention to the case of purchasing without a premium status; the impact of Gold status on utility is considered in the next subsection. In our setting, forward-looking behavior implies that a consumer decides on the number of items to purchase in order to maximize the total utility over a period of time<sup>9</sup>. Intuitively, the marginal utility of a purchase decreases in the number of items already purchased, leading to satiation. The satiation point, however, is naturally different among consumers due to their varying needs and the importance of purchases.

To capture this logic, we assume that consumers are heterogeneous with respect to the satiation parameter,  $a_i$ , so that consumer- $i$ ’s marginal utility of the  $(n + 1)^{th}$  unit is  $a_i - n$ : the marginal utility decreases in the number of units purchased and at  $n = a_i$  becomes zero. Note that parameter  $a_i$  can be also interpreted as consumer- $i$ ’s WTP for the first (most important) purchase. The total utility of consumer- $i$  from purchasing  $n$  items equals:

$$U_i(n) = \int_0^n (a_i - \omega) d\omega = a_i n - \frac{n^2}{2}. \quad (1)$$

We note that such a quadratic utility structure is widely used when modeling multiple product uses, and it has been empirically calibrated/verified in various applications (see, e.g., Lambrecht et al. 2007, Economides et al. 2008, Kim et al. 2010, Gilbert and Jonnalagedda 2011).

If the firm charges price  $p$  per item, then consumer- $i$ ’s surplus from purchasing  $n$  items equals:

$$Surplus_{i,R}(n) = U_i(n) - pn, \quad (2)$$

<sup>9</sup> For simplicity, we assume this period equals the status qualification period, typically one year. One might argue that consumers are not looking that far ahead; in that case, our model can be modified to account for purchases made in a shorter period (e.g., in the last 3 months of a year) with the qualification threshold adjusted accordingly.

where subscript ‘ $R$ ’ refers to purchasing “*as needed*” with regular status. Consumer- $i$  determines the optimal number of items to purchase,  $n_{i,R}^*$ , by solving  $\max_n \{Surplus_{i,R}(n)\}$ , which leads to:

$$n_{i,R}^* = \max[0, (a_i - p)]. \quad (3)$$

### 3.2. Model of Premium Status (Gold) Utility

With the loyalty program, the firm rewards some consumers with a premium (Gold) status. Status comes with benefits that provide additional utility, which we capture by a parameter  $g \geq 0$ . We assume that  $g$  is added to  $a_i$ , and thus increases<sup>10</sup> Gold consumers’ WTP to  $a_i + g$ . The surplus of consumer- $i$  who purchases with the Gold status (denoted by a subscript ‘ $G$ ’) then becomes:

$$Surplus_{i,G}(n) = (a_i + g - p)n - \frac{n^2}{2}. \quad (4)$$

As before, a Gold consumer determines the number of items to purchase,  $n_{i,G}^*$ , by solving the optimization problem  $\max_n \{Surplus_{i,G}(n)\}$ , which leads to:

$$n_{i,G}^* = \max[0, a_i - p + g]. \quad (5)$$

Comparing this equation with (3), notice that the optimal number of items to purchase increases if the consumer obtains Gold status – an important observation driven by the increased WTP.

**Remark.** In practice, qualifying for status is dynamic in most cases: meeting the qualification requirement in one period/year (qualification period) results in being awarded a status and the associated benefits in the following period (benefit period). However, data from an industrial partner (a large, established loyalty program) shows that the majority, over 70%, of consumers with status *maintain* their status year-over-year, and thus a distinction between the two periods (qualification and benefit) is not critical; the management of that firm also believes that such majority is observed in other established programs as well. Our model, therefore, considers a steady-state situation, where if it is optimal for a consumer to purchase enough to qualify for status “in one period”, then it is implicitly assumed that the same was true “in the previous period” as well, implying that Gold consumers enjoy status (this period) as they qualify for the status (for the next period) and this situation of “maintaining status” continues in perpetuity. In Section 7.2 we consider a dynamic extension to our model that captures the difference between the qualification and benefit periods; we establish that our model is a good approximation of dynamic reality and confirm the robustness of our results.

<sup>10</sup> This is consistent with abundant empirical evidence that consumers with status are willing to pay more. For example, in the airline context, Mathies and Gudergan (2012) (using lab experiments), and McCaughey and Behrens (2011) and Brunger (2013) (using secondary data) demonstrated that premium-status consumers have a higher WTP for flights, even after controlling for the effects of trip purpose and time of booking.

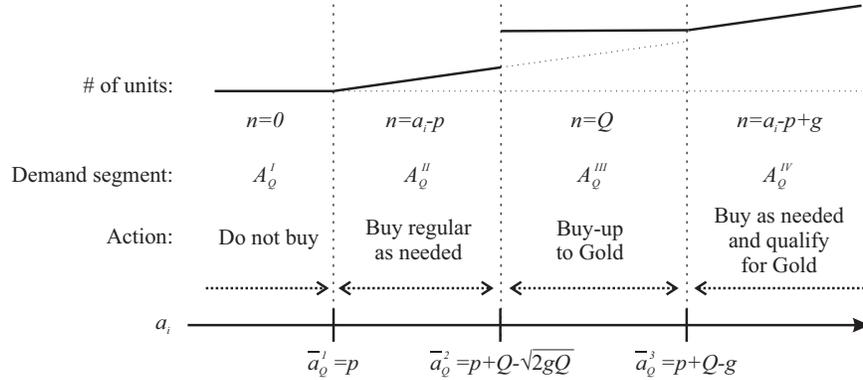


Figure 1 Strategic consumer behavior under the quantity-based loyalty program.

## 4. Consumer Behavior Under Different Program Designs

In this section, we formulate and solve the consumers' and the firm's problems for the quantity-based and spending-based programs, compare the resultant solutions, and discuss managerial intuition.

### 4.1. Quantity-based Program

Under a quantity-based program design, there is no benefit for a consumer to pay a higher price  $p^+$  because the high price lowers the surplus without contributing to the Gold utility. By definition, a customer who purchases  $Q$  or more items (at the regular price,  $p$ ) qualifies for Gold status.

If consumers were myopic, only those with  $n_{i,G}^* \geq Q$  would qualify for the Gold, and they would do so "automatically", simply by purchasing as needed. However, because consumers are strategic, some of those whose "need",  $n_{i,R}^*$ , is below the qualification threshold, may "buy up" to Gold by purchasing  $Q > n_{i,R}^*$  items. The additional  $Q - n_{i,R}^*$  items will have a negative surplus, but the total surplus from purchasing  $Q$  items with the Gold status may exceed the total surplus of purchasing  $n_{i,R}^*$  items with regular status. This strategic behavior is characterized by Lemma 1 and is illustrated in Figure 1 (all proofs are in the Appendix):

LEMMA 1. *Under a quantity-based program, there exist up to three thresholds,  $\bar{a}_Q^1 \leq \bar{a}_Q^2 \leq \bar{a}_Q^3$ , which lead to up to four consumer segments ( $A_Q^j$ ,  $j = I, \dots, IV$ ):*

$A_Q^I$ : *If  $0 \leq a_i \leq \bar{a}_Q^1$ , then it is optimal not to purchase.*

$A_Q^{II}$ : *If  $\bar{a}_Q^1 \leq a_i \leq \bar{a}_Q^2$ , then it is optimal to purchase as needed, without Gold status.*

$A_Q^{III}$ : *If  $\bar{a}_Q^2 \leq a_i \leq \bar{a}_Q^3$ , then it is optimal to strategically buy up to Gold.*

$A_Q^{IV}$ : *If  $\bar{a}_Q^3 \leq a_i$ , then it is optimal to purchase as needed (and automatically qualify for Gold).*

This Lemma implies that depending on  $a_i$  there could be up to three different optimal customer behaviors: do not purchase, purchase as needed (some will qualify for Gold) and, most importantly, buy up to Gold. The latter precisely matches the evidence of strategic behavior (e.g., mileage-running) described in the Introduction. We emphasize that buying up is optimal for the consumers

who do so; because of status benefits, their total surplus is higher with buy up, even though some purchases have a negative surplus.

Next, we examine consumers' response to changes in Gold qualification threshold and benefit.

**COROLLARY 1.** *If all segments  $A_Q^I, \dots, A_Q^{IV}$  are non-empty (as depicted in Figure 1), then:*

- (i)  $\bar{a}_Q^2$  and  $\bar{a}_Q^3$  increase in  $Q$ , and  $\bar{a}_Q^3$  increases faster than  $\bar{a}_Q^2$ . That is, segments  $A_Q^{II}$  and  $A_Q^{III}$  expand and segment  $A_Q^{IV}$  shrinks in  $Q$ .
- (ii)  $\bar{a}_Q^2, \bar{a}_Q^3$  decrease in  $g$ ; more so,  $\bar{a}_Q^2$  decreases faster. That is, segment  $A_Q^{II}$  shrinks and segments  $A_Q^{III}$  and  $A_Q^{IV}$  expand in  $g$ .

This result shows that the total number of consumers who qualify for Gold decreases when the status qualification requirement becomes more stringent, and increases when the Gold status becomes more valuable. Since  $A_Q^{III}$  expands while  $A_Q^{IV}$  shrinks, the proportion of buy-up customers increases in  $Q$ . In other words, if the firm makes it harder to qualify for Gold, then the proportion of consumers who buy up increases. This points to a distinctive feature of the quantity-based design; later, we will show that this is not the case under the spending-based design.

The firm's problem is to select the qualification requirement,  $Q$ , and the regular price,  $p_Q$  (as argued above, there is no demand at the high price  $p^+$ ), that maximize its profit:

$$\max_{p_Q, Q} \pi_M(p_Q, Q) = \max_{p_Q, Q} \left[ p_Q \sum_{j=II, \dots, IV} D_Q^j - c(D_Q^{III} + D_Q^{IV}) \right], \quad (6)$$

where  $D_Q^{II} = \int_{A_Q^{II}} n_{i,R}^* dF(a_i)$  is the demand from the purchase-regular segment,  $D_Q^{III} = \int_{A_Q^{III}} Q dF(a_i)$  is the demand from the strategic Gold (buy-up) segment,  $D_Q^{IV} = \int_{A_Q^{IV}} n_{i,G}^* dF(a_i)$  is the demand from auto-Golds per Lemma 1, and  $F(a_i)$  denotes the CDF of the distribution of  $a_i$ s. Note that the second term excludes  $D_Q^{II}$  since the consumers in that segment purchase with regular status and thus the firm does not incur the status-related cost for servicing them.

## 4.2. Spending-based Program

Under the spending-based loyalty program, consumers who spend  $S$  or more dollars with the firm qualify for Gold status. Similar to the quantity-based program, there are two kinds of such consumers: those with  $n_{i,G}^* p \geq S$  qualify automatically by purchasing as needed, and others who strategically spend more than their purchase need to maintain/obtain Gold status.

However, in contrast to the quantity-based program, the spending-based program offers three different ways to spend up. The first is to simply purchase more at the regular price,  $p$ ; this is analogous to buying up under the quantity-based program. The second way, which might be the main premise of the spending-based loyalty program design, is to choose to pay  $p^+$  for fewer items instead of purchasing more items at  $p$ . We emphasize that the product/service a consumer receives

is identical: that is, paying  $p^+$  might seem irrational – why would someone pay more for exactly the same product/service? Below, we discuss how such pay-up behavior could indeed be optimal for some consumers. The third option is to mix the two.

Let  $n, n^+$  denote the number of items purchased at prices  $p, p^+$ . From (4), these solve:

$$\max_{n, n^+ \in \mathbb{R}^+} \left[ (a_i + g)(n + n^+) - \frac{(n + n^+)^2}{2} - (pn + p^+n^+) \right], \text{ s.t. } pn + p^+n^+ \geq S, \quad (7)$$

which is maximized at:

$$n_{i,S}^* = \max\left[0, \frac{-S + (a_i + g)p^+}{p^+ - p}\right], \text{ and } n_{i,S}^{+*} = \max\left[0, \frac{S - (a_i + g)p}{p^+ - p}\right], \quad (8)$$

where  $n_{i,S}^* \geq 0$  if  $a_i \geq S/p^+ - g$  and  $n_{i,S}^{+*} \geq 0$  if  $a_i \leq S/p - g$ ; if only one of these two conditions is satisfied, the optimal solution is  $(0, S/p^+)$  and  $(S/p, 0)$ , respectively.

Observe that the number of items purchased at a high (regular) price,  $n_{i,S}^{+*}$  ( $n_{i,S}^*$ ), increases (decreases) in  $S$  and decreases (increases) in  $a_i$  and  $g$ . For those consumers who attempt to strategically qualify for Gold, a higher qualification spending requirement makes purchasing higher priced items more attractive, and makes purchasing regular priced items less attractive. This is because in order to spend up to Gold, consumers can purchase less if they purchase higher-priced items; however, they need to purchase more if they purchase regular-priced items. Since these additional purchases have an increasingly smaller surplus, the intuition for this result follows.

As in the case with the quantity-based program, consumers solve (7) and compare the optimal surplus with that of purchasing regular and determine whether to spend up to Gold. The following Lemma summarizes strategic consumer behavior under the spending-based program:

LEMMA 2. *Under a spending-based loyalty program, there exist up to five thresholds,  $\bar{a}_S^1 \leq \bar{a}_S^2 \leq \bar{a}_S^3 \leq \bar{a}_S^4 \leq \bar{a}_S^5$ , which lead to up to six consumer segments ( $A_S^j$ ,  $j = I, II \dots, VI$ ):*

$A_S^I$ : *For  $0 \leq a_i \leq \bar{a}_S^1$ , it is optimal not to purchase.*

$A_S^{II}$ : *For  $\bar{a}_S^1 \leq a_i \leq \bar{a}_S^2$ , it is optimal to purchase as needed at price  $p$  without Gold status.*

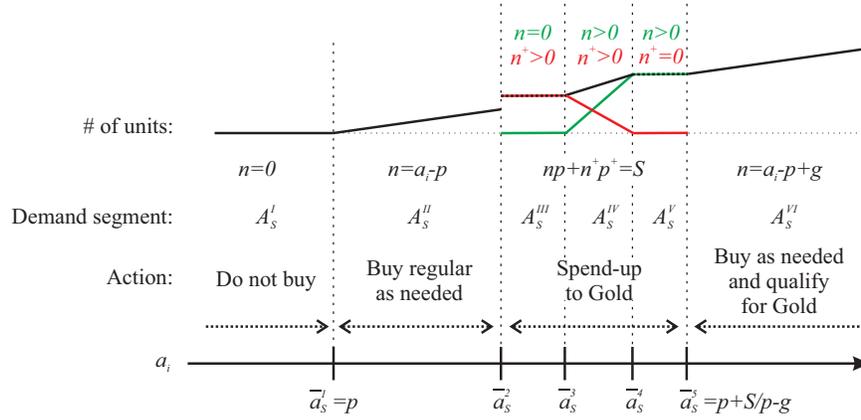
$A_S^{III}$ : *For  $\bar{a}_S^2 \leq a_i \leq \bar{a}_S^3$ , it is optimal to spend up to Gold with  $n_{i,S}^* = 0, n_{i,S}^{+*} > 0$  (pay up).*

$A_S^{IV}$ : *For  $\bar{a}_S^3 \leq a_i \leq \bar{a}_S^4$ , it is optimal to spend up to Gold with  $n_{i,S}^* > 0, n_{i,S}^{+*} > 0$  (mix).*

$A_S^V$ : *For  $\bar{a}_S^4 \leq a_i \leq \bar{a}_S^5$ , it is optimal to spend up to Gold with  $n_{i,S}^* > 0, n_{i,S}^{+*} = 0$  (buy up).*

$A_S^{VI}$ : *For  $\bar{a}_S^5 \leq a_i$ , it is optimal to purchase as needed at price  $p$  and automatically qualify for Gold.*

Figure 2 illustrates this Lemma. Segments  $A_S^I, A_S^{II}$ , and  $A_S^{VI}$  are identical to the equivalent segments  $A_Q^I, A_Q^{II}$ , and  $A_Q^{IV}$  under a quantity-based program. However, segments  $A_S^{III} - A_S^V$  are new and reflect three different ways to spend up: “pay up” – purchase  $n^{+*}$  items at a high price  $p^+$ ; “buy up” – purchase  $n^*$  items at a regular price  $p$ ; or “mix”  $n^*$  at price  $p$  and  $n^{+*}$  at  $p^+$ .



**Figure 2** Strategic consumer behavior under the spending-based loyalty program.

Quite surprisingly, from Lemma 2 and Figure 2, consumers who pay up are not the high-WTP consumers; rather they have *the lowest*  $a_i$ s among all Golds. Should these consumers decide to buy up, they need to purchase the largest number of “unnecessary” items; hence, the option to pay up is most valuable for them.

Next, we examine consumers’ response to changes in Gold qualification threshold and benefit.

**COROLLARY 2.** *If all segments  $A_S^I, \dots, A_S^{VI}$  are non-empty (as depicted in Figure 2), then:*

- (i)  $\bar{a}_S^4$  and  $\bar{a}_S^5$  increase in  $S$  at the same rate;  $\bar{a}_S^3$  increases slower, and  $\bar{a}_S^2$  even slower. That is, segment  $A_S^{VI}$  shrinks, segment  $A_S^V$  is unaffected, while segments  $A_S^{II} \dots A_S^{IV}$  expand in  $S$ .
- (ii)  $\bar{a}_S^3, \bar{a}_S^4$ , and  $\bar{a}_S^5$  decrease in  $g$  at the same rate, and  $\bar{a}_S^2$  decreases faster. That is, segment  $A_S^{II}$  shrinks while  $A_S^{III}$  and  $A_S^{VI}$  expand in  $g$ .  $A_S^{IV}$  and  $A_S^V$  are unaffected.

Similar to the quantity-based program, the total number of consumers who qualify for Gold decreases when the status qualification requirement becomes more stringent and increases when Gold status becomes more valuable. Different from the quantity-based program, however, under the spending-based program, an increase in  $S$  translates into high-price purchases and not in buying up: the proportion of those who purchase at least some items at high price  $p^+$  increases and the proportion of those who purchase *all* items at  $p^+$  increases the fastest, while the number of buy up consumers is not affected. This relates to the important difference between the quantity- and spending-based programs that we explore in more detail in Section 5.2.

The firm’s problem under the spending-based design is to select the regular price,  $p_S$ , the high price,  $p_S^+$ , and the spending requirement,  $S$ , to maximize its profit:

$$\max_{p_S, p_S^+, S} \pi_S(p_S, p_S^+, S) = \max_{p_S, p_S^+, S} \left[ p_S (D_S^{II} + D_S^{VI}) + S \int_{A_S^{III} \dots V} dF(a_i) - c \sum_{j=III, \dots, VI} D_S^j \right], \quad (9)$$

where  $D_S^{II} = \int_{A_S^{II}} n_{i,R}^* dF(a_i)$  is the demand from the purchase-regular segment,  $D_S^{III} = \frac{S}{p_S^+} \int_{A_S^{III}} dF(a_i)$ ,  $D_S^{IV} = \int_{A_S^{IV}} (n_{i,S}^* + n_{i,S}^{*+}) dF(a_i)$  and  $D_S^V = \frac{S}{p_S} \int_{A_S^V} dF(a_i)$  are the demands from

the three strategic Gold segments, and  $D_S^{VI} = \int_{A_S^{VI}} n_{i,G}^* dF(a_i)$  is the demand from auto-Golds as per Lemma 2. Note that the second term in (9) reflects the fact that each customer in segments  $A_S^{III} \dots A_S^V$  spends  $S$  with the firm, and the third term reflects that consumers in segment  $A_S^{II}$  are purchasing regular and thus the firm does not incur status-related costs for servicing them.

## 5. Comparison of Loyalty Program Designs

Having established how consumers strategically respond to the firm's decisions, we next turn our attention to how the firm decides on price(s) and the Gold qualification threshold. We seek to understand which program design the firm might prefer and why. To address these questions, we explicitly consider two important factors that affect the firm's optimal decisions: the firm's cost of offering the Gold status,  $c$ , and the consumer's benefit from the Gold status,  $g$ . To facilitate the analysis, for the remainder of the paper, we assume that consumer's WTP is uniformly distributed between zero and one, i.e.,  $a_i \in U[0, 1]$ . In light of this assumption, we naturally consider  $g, c \in [0, 1]$ .

### 5.1. Benchmark: RM and Loyalty Programs With Myopic Consumers

To study the impact of strategic consumers, we first consider a benchmark case in which consumers are not strategic; those consumers optimize instantaneous surplus and do not look forward to the benefits of Gold status, nor do they alter their behavior to obtain status – i.e., they act myopically.

Myopic behavior has two implications. First, only consumers with  $a_i - p + g \geq Q$  qualify for Gold. Second, quantity-based and spending-based designs become equivalent: under the spending-based design, no consumer will purchase at  $p^+$  as doing so decreases instantaneous surplus.

The firm's problem with myopic (M) consumers is to select the qualification requirement  $Q_M$  and the regular price  $p_M$  that maximize its profit:

$$\max_{p_M, Q_M} \pi_M(p_M, Q_M) = \max_{p_M, Q_M} [p_M (D_M^{II} + D_M^{III}) - c (D_M^{III})], \quad (10)$$

where  $D_M^{II} = \int_p^{Q+p-g} n_{i,R}^* dF(a_i)$  is the demand from the regular segment and  $D_M^{III} = \int_{Q+p-g}^1 n_{i,G}^* dF(a_i)$  is the demand from the Gold segment. Let  $p_M^*$  and  $Q_M^*$  be the optimal price and Gold qualification thresholds, and  $\pi_M^*$  be the optimal profit in problem (10).

PROPOSITION 1. *With myopic consumers there exists two “myopic regions”, MR1,2, as follows:*

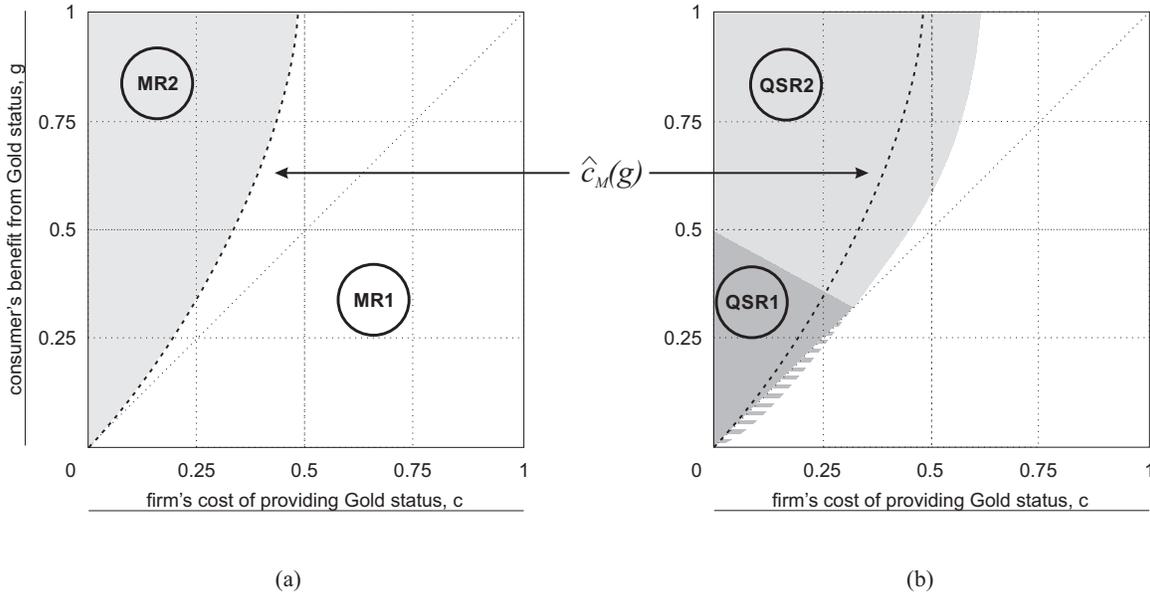
MR1: *If  $c \geq \hat{c}_M(g)$  (where function  $\hat{c}_M$  is presented in the proof,  $\hat{c}_M(0) = 0$ ,  $\hat{c}_M(1) \approx 0.48$ ) then*

$$p_M^* = 1/3, \quad Q_M^* \geq g + 2/3, \quad \text{and} \quad \pi_M^* = 2/27;$$

MR2: *Otherwise,  $p_M^* = \frac{1}{3} (2 + c + 2g - \sqrt{c^2 - 2cg - 2c + 4g^2 + 2g + 1})$ ,  $Q_M^* = g$ , and  $\pi_M^* \geq 2/27$ ,*

$$\text{where } \pi_M^* = \frac{1}{54} \left( \sqrt{c^2 - 2c(g+1) + 4g^2 + 2g + 1} - c - 2g + 1 \right) \\ \times \left( 2 - 2c + 2g - \sqrt{c^2 - 2c(g+1) + 4g^2 + 2g + 1} \right) \left( \sqrt{c^2 - 2c(g+1) + 4g^2 + 2g + 1} - c + 4g + 1 \right).$$

Figure 3a illustrates the solution. Interpreting it, observe that that when providing status benefits is costly, the firm sets the Gold qualification threshold so high that even the highest-WTP consumer ( $a_i = 1$ ) is indifferent between purchasing regular or Gold; otherwise, the firm sets the low threshold so that the lowest-WTP consumer ( $a_i = p$ ) is indifferent. In other words, when consumers are myopic, the firm cannot benefit from discriminating consumers with status: either (almost) all have it, or (almost) none; here, “almost” refers to the marginal consumers who are indifferent. We will next see that this is not the case when consumers are strategic.



**Figure 3** Optimal solution with myopic consumers (a) and quantity-based program (b), as a function of  $c$  and  $g$ .

## 5.2. Analysis of Profit

We first consider the quantity-based program. Let  $p_Q^*$  and  $Q_Q^*$  be the optimal price and the Gold qualification threshold under the quantity-based program with strategic consumers, and let  $\pi_Q^*$  be the resultant optimal profit in the optimization problem (6).

**PROPOSITION 2.** *Under the quantity-based program with strategic consumers, there exist two “Strategic Regions”, QSR1, 2, as follows:*

*QSR1: If  $g \leq 1/2 - c/2$  and  $c \leq \hat{c}_{Q1}(g)$  (function  $\hat{c}_{Q1}$  is presented in the proof,  $\hat{c}_{Q1}(0) = 0$ ,  $\hat{c}_{Q1}(1/3) = 1/3$ ), then  $p_Q^* = 1/3(2c + g + 1)$ ,  $Q_Q^* = 2/3(1 + g - c)$ , and  $\pi_Q^* = 2/27(c - g - 1) \left( 2c^2 + c \left( 6\sqrt{3}\sqrt{g(-c + g + 1)} - 10g - 1 \right) - (g + 1)^2 \right) \geq \pi_M^*$ ;*

*QSR2: If  $g > 1/2 - c/2$  and  $c \leq \hat{c}_{Q2}(g)$  (function  $\hat{c}_{Q2}$  is presented in the proof,  $\hat{c}_{Q2}(1/3) = 1/3$ ,  $\hat{c}_{Q2}(1) \approx 0.61$ ), then  $p_Q^* = \frac{1+c}{2}$ ,  $Q_Q^* = 2g$ , and  $\pi_Q^* = \frac{1}{2}(c - 1)^2 g \geq \pi_M^*$ .*

*Outside these regions,  $p_Q^* = p_M^*$ ,  $Q_Q^* = Q_M^*$ , and thus  $\pi_Q^* = \pi_M^*$ .*

Figure 3b illustrates the solution, with the dashed curve plotting the function  $\hat{c}_M$  from Proposition 1. Proposition 2 has three important implications. First and foremost, the results show that when the firm coordinates its pricing and loyalty program it can, in fact, *benefit* from strategic consumer behavior. As argued in the Introduction, it is not obvious that an increase in demand from strategic consumers will result in increased profit, as strategic behavior also increases costs. Our results, however, characterize the strategic regions where the additional revenue is larger than the additional cost, leading to higher profit. This may explain why some firms proactively remind consumers to be strategic; for example, in the Fall of 2017, Hertz sent the following message to its program members: "Don't let your status slip away, [name] you're only 1 rental away from earning Five Star status for 2018."

This result is also interesting because in many studies, strategic consumer behavior hurts the firm's revenue, e.g., Aviv and Pazgal (2008), Levin et al. (2009), Ovchinnikov and Milner (2012). Note that in the majority of the literature strategic behavior is with respect to waiting for a lower price, e.g., a markdown. When offering a markdown, the firm balances two effects: the (positive) demand effect (lower price attracts additional demand) and the (negative) cost effect (instead of purchasing at full price, strategic consumers purchase at markdown, i.e., the same quantity is purchased, just at a lower price); see Baucells et al. (2017) for a recent analysis of this tradeoff. Demand effect is present with both myopic and strategic consumers, while the cost effect is absent if consumers are myopic: it is thus not surprising that strategic behavior hurts profits in that case.

In our setting, both these effects are also present, but in different ways. The demand effect comes from strategic consumers anticipating the future utility of status and hence increasing their consumption in a multi-unit purchase setting: to qualify for status, strategic consumers purchase more units, including some units that have negative marginal utility. The more additional units strategic consumers buy, the higher is the additional revenue over myopic consumers who do not do so. However, having consumers who qualify for status also increases cost, and – importantly – does so in a non-linear, step-wise way, as the cost to provide status benefits applies to all the units purchased by such consumers, not only the additional units purchased over the myopic quantity. This again results in a tradeoff between the additional revenue and the additional cost, but because of multi-unit purchasing, step-wise costs, and most importantly, because the demand effect is present only when consumers are strategic, such strategic behavior benefits the firm under certain conditions. We emphasize that this constitutes a major difference – a qualitative reversal of the directional impact of strategic consumers from much of the existing literature.

It is interesting to note that the few other papers that show a result similar to ours possess a subset of the features that appear in our model as well. For instance, the result of Marinesi et al. (2017) relies on the step-wise cost effect: in the group buying context, if the number of waiting

consumers is small, the firm closes operation in the low season and incurs no fixed cost. The result in Zhang and Zhang (2017) relies on multi-unit purchasing: in the trade-in remanufacturing context strategic consumers return used “generation<sub>*i*</sub>” products and purchase new “generation<sub>*i+1*</sub>” products. The result in Wang et al. (2018) also relies on multi-unit purchasing in the context where consumers are strategic with respect to uncertain future valuations. Our context is clearly different (no fixed costs, identical products and constant valuations), and therefore our result expands the understanding of situations in which strategic consumer behavior can increase profits. Our result also provides new insights into the sources of loyalty programs’ value: while traditionally viewed as a means of softening competition (e.g., Kim and Srinivasan 2001), we show that even without competition, loyalty programs can be valuable because of how they affect strategic consumer behavior.

At the same time, Proposition 2 also shows that the range of  $(g, c)$ , over which the firm benefits from strategic consumer behavior under the quantity-based program, is limited. In particular, recall that the rightmost point of QSR2 has  $(c \approx 0.61, g = 1)$ . Thus, no matter how much consumers value status, when providing it becomes too costly, the firm opts out by setting the status threshold so high that (almost) no consumer qualifies. As argued in the Introduction, buying up to Gold increases both revenue and cost. When the cost is very high, the latter can dominate. However, when the cost is (very) small we observe the opposite effect: the firm can benefit from strategic consumer behavior and offer a quantity-based loyalty program *even when*  $c > g$ , i.e., when the marginal cost of providing benefits is higher than the value consumers place on them. This happens over a sub-region that is highlighted with stripes in Figure 3b. While this sub-region is small, its existence is still surprising; intuitively, a firm should not be able to profitably sell products that cost more than consumers are willing to pay for them. This intuition, however, misses a discontinuous jump in purchase quantity and the additional revenue from the customers who buy up to status, which may offset the additional cost because the size of the jump is non-linear in  $(c, g)$ .

We next examine these effects for the spending-based program. Before proceeding with the discussion, we note that the model under the spending-based design is significantly more complex than quantity-based or myopic problems. This is not only because there are more decision variables, but also because, with more consumer segments, there are many more potential solution “archetypes” to consider<sup>11</sup>. For these reasons, the complete characterization of the optimal solution, as in Propositions 1 and 2, is analytically intractable. Nevertheless, several insightful results can still be derived, as we discuss next. Let  $\pi_g^*$  be the optimal profits under a spending-based program.

<sup>11</sup> The complication comes from the fact that some segments may be empty at optimality (e.g., there may be no consumers who buy all items at  $p^+$ ). To account for this we need to insert the min and max statements into the boundaries of integration, which makes the problem non-differentiable and effectively implies that we need to consider numerous special cases, many of which have “messy” solutions that, unfortunately, cannot be compared analytically.

- PROPOSITION 3. *If the firm switches from a quantity-based to a spending-based program, then:*
- Over regions  $QSR1,2$ ,  $\pi_S^* \geq \pi_Q^*$ , and*
  - The region of  $(c,g)$  over which the firm benefits from strategic consumer behavior expands.*

This proposition suggests that if the firm switches from a quantity-based to a spending-based program, then the firm benefits in two distinct ways. From part (a), whenever the firm is able to benefit from strategic consumers under the quantity-based program, it will *benefit more* under the spending-based design. From part (b), however, the firm can also benefit from a spending-based design in situations when strategic consumers hurt the firm under a quantity-based program.

The profit increase is driven by the underlying strategic behavior of consumers. To simplify the comparison, assume  $p_Q = p_S$  and  $S = Q_Q p_Q$  (i.e., the regular prices and effective Gold qualifications are the same). In this case, it is not difficult to verify from the proof of Corollaries 1 and 2 that  $\bar{a}_Q^3 = \bar{a}_S^5$ ,  $\bar{a}_Q^2 \leq \bar{a}_S^4$  and  $\bar{a}_Q^2 \geq \bar{a}_S^2$ , as depicted in Figure 4.

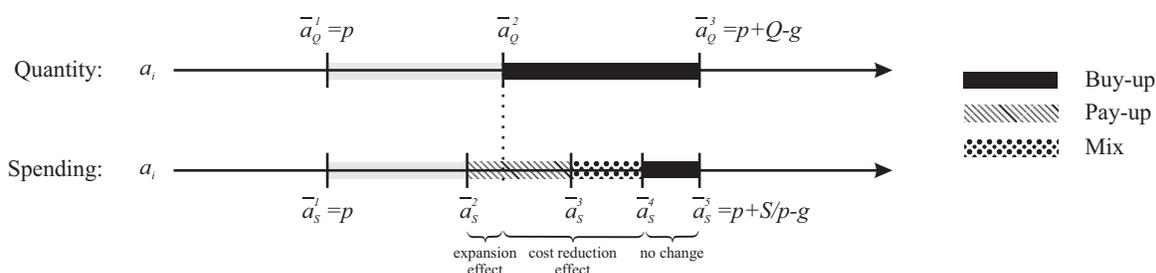


Figure 4 Schematic comparison of strategic consumer behavior under quantity- versus spending-based designs.

Two effects can be observed. First, compared to the quantity-based program, the spending-based design reduces the number of consumers who buy up. This is profitable because buying-up increases the firm's revenue, but also cost, while paying up results in the same revenue (since  $S = Qp$  in this comparison), but from fewer purchases, which decreases cost. Second, the spending-based design increases the number of customers who strategically qualify for Gold status. Because these newly-added Gold customers only make the high-price purchases, the firm incurs the lowest cost for servicing them (out of all strategic Golds). Since all strategic Golds bring the same revenue,  $S$ , such customers are, therefore, the most profitable ones. The two effects (cost reduction and Gold segment expansion) both contribute to the increased profitability of the spending-based program.

To summarize the analysis of profits, by properly coordinating its pricing and loyalty functions the firm is able to benefit from strategic consumers under both designs, with bigger benefits occurring under the spending-based design and under broader conditions. It is thus not surprising that the firms mentioned in the Introduction change their quantity-based programs: if they change to spending-based designs, the firm's profit will increase. However, as we also discussed in the

Introduction, such changes generated significant attention in the media and invited negative public reactions – arguably because the increase in profit might come at the expense of consumers’ surplus. We discuss this issue next.

### 5.3. Analysis of Consumer Surplus

We first note that a switch from a quantity-based to a spending-based design may hurt some consumers. To see why, consider a spending-based problem with  $p_S = p_Q^*$  and  $S = Q_Q^* p_Q^*$ , so that the regular prices and the “effective” qualifications are the same. In this case, for any  $p^+ > p$ , no consumer will be worse off under the spending-based program than under the quantity-based one. However, if  $p_S < p_Q^*$ , then the regular consumers will benefit, but some Gold consumers may suffer. This is because the number of items they must purchase to qualify for Gold,  $S/p_S$ , will exceed  $Q_Q^*$ . In other words, some consumers who otherwise would qualify for Gold automatically have to buy up, and others would have to buy up more. Since some of the items purchase by consumers who buy up have a negative surplus, a lower  $p_S$  makes buying up harder, which may decrease surplus for some consumers. Thus, the negative public reactions mentioned in the Introduction could be partially justified.

A logical question, therefore, is whether the firm can increase its profit by only making design changes that maintain the regular price at  $p_Q^*$  and the effective qualification at  $Q_Q^*$ . As explained earlier, in this case, no consumers will be worse off, and if the firm can be better off, it will clearly lead to a win–win situation. We refer to such situations as “*status-quo Pareto-improving*.”

The following Proposition characterizes the existence of such solutions under the spending-based design. The result is illustrated in Figure 5, where the dashed curve refers to function  $\hat{c}_{Q2}(g)$  from Proposition 2 that defines the right boundary of QSR2 (recall the strategic regions under a quantity-based program (QSR1,2) given in Proposition 2 and illustrated in Figure 3b).

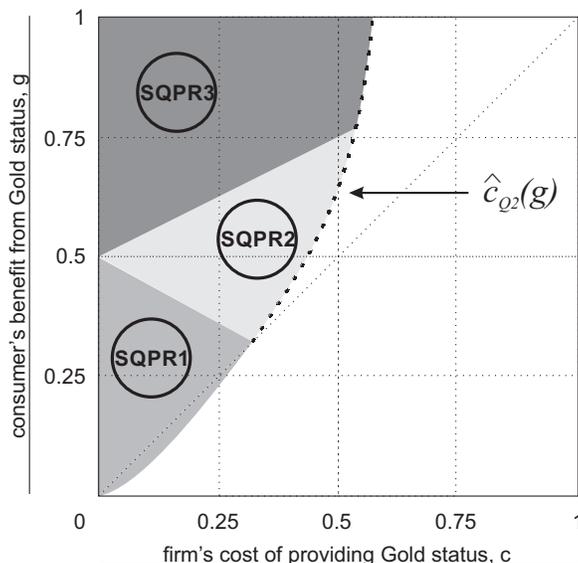
**PROPOSITION 4.** *Under the spending-based program there exist three “status-quo Pareto-improvement regions” of  $c, g$ , SQPR1,2,3, which define the existence of the status-quo Pareto-improving solutions as follows:*

*SQPR1 (equivalent to QSR1): status-quo Pareto-improving solutions never exist;*

*SQPR2 (the part of QSR2 with  $g \leq 1/2 + c/2$ ): status-quo Pareto-improving solutions never exist;*

*SQPR3 (the part of QSR2 with  $g > 1/2 + c/2$ ): status-quo Pareto-improving solutions always exist.*

This Proposition has two fundamental implications. First, a spending-based design could lead to a win–win situation for the firm and its consumers: the firm is able to increase its profit while maintaining the surplus of each and every consumer at the same level as under the optimal quantity-based program, or improving it. To achieve such a Pareto-improving solution, the firm might need



**Figure 5** Schematic of the status-quo Pareto-improving solutions under the spending-based loyalty program.

to sacrifice some of its profit improvement potential, but that profit will still be above the quantity-based optimum and no consumer will be worse off. While it is not clear whether all firms would do that, some firms might find it worthwhile to forgo this incremental profit to improve customer satisfaction. At the same time, the Proposition also implies that such solutions do not exist for all  $(c, g)$ ; outside of SQPR3, the firm has no choice<sup>12</sup> but to reduce some of its consumers' surplus to increase profit by switching to the spending-based program. More specifically, our results indicate that for firms with (relatively) high  $c$  and low  $g$  (SQPR1,2), a switch to a spending-based program might necessarily imply that some consumers will be hurt, and thus customer outcry and negative media reactions are warranted. On the other hand, for firms with relatively low  $c$  and high  $g$ , such change could be a win-win, but it is not automatically attained. The firm could still be “greedy,” i.e., not willing to sacrifice a portion of the profit that is needed to achieve such an outcome.

## 6. Extension: Combined Designs

So far, we have considered “singular” designs where the status requirement is based solely on quantity or solely on spending. As discussed in the Introduction, however, “combined” designs also exist: some firms implement a Quantity *and* Spending (*QaS*) design where consumers must satisfy both quantity and spending requirements to qualify for status; other firms implement a Quantity *or* Spending (*QoS*) design where consumers can satisfy either the quantity or spending requirement. In this section, we analyze such combinations.

<sup>12</sup>As a word of caution, while the above result points to a potential limitation of the spending-based design, we emphasize that other Pareto-improving solutions (that are not status-quo improving) may still exist.

### 6.1. Quantity and Spending (QaS) Design

By adding a spending requirement to the  $Q$ -design, i.e., with  $QaS$ , the firm would be able to more effectively control less profitable consumers, which, intuitively, may improve profit beyond that of the  $S$ -design. Interestingly, this is not the case as shown below. Let  $\pi_{QaS}^*$  be the optimal profit under the  $QaS$  design.

PROPOSITION 5. *When the firm switches from a  $Q$ -design to a  $QaS$ -design, then:*

- a) *It can improve profit. However, the improvement cannot be bigger than the improvement from a switch to an  $S$ -design. That is,  $\pi_Q^* \leq \pi_{QaS}^* = \pi_S^*$ .*
- b) *The status-quo Pareto-improving solutions never exist.*

This Proposition confirms the intuition that the firm can increase its profit over the  $Q$ -design by making the status qualification requirements more stringent. That is, the firm benefits from a switch to a  $QaS$ -design. However, the benefit from a switch to a  $QaS$ -design cannot exceed the benefit from a switch to an  $S$ -design, which is somewhat surprising. This is because, under the optimal  $QaS$ -design, the firm sets the spending requirement to  $S_S^*$  (the  $S$ -design optimum) and sets the quantity threshold below  $S_S^*/p_S^*$ , making the quantity threshold effectively redundant.

On the other hand, the impact of a switch to a  $QaS$ -design on consumers is rather clear:  $QaS$  makes it harder for consumers to obtain status, as they have to meet two requirements rather than one, and therefore status-quo Pareto improvement is not achievable under  $QaS$ -designs. In summary, the  $QaS$ -design is no better than the  $S$ -design for the firm and is worse than the  $S$ -design for consumers.

### 6.2. Quantity or Spending (QoS) Design

Compared with  $QaS$ , the  $QoS$ -design makes obtaining status easier. While it is rather clear that  $QoS$ -designs could be beneficial for consumers, it is not obvious whether the firm can improve profit. Let  $\pi_{QoS}^*$  be the optimal profit under the  $QoS$  design.

PROPOSITION 6. *When the firm switches from a  $Q$ -design to a  $QoS$  design, then:*

- a) *It can still improve profit and the improvement is as good as the improvement from a switch to an  $S$ -design. That is,  $\pi_Q^* \leq \pi_{QoS}^* = \pi_S^*$ .*
- b) *The status-quo Pareto-improving solutions always exist in QSR2 if  $g > 1/2 + c/2$ .*

Surprisingly, this Proposition first shows that the firm can improve profits even if it offers consumers a choice for how to meet the qualification requirement; furthermore, the profit can reach that of the optimal  $S$ -design. As was evident in the  $QaS$  case, under the optimal  $QoS$ -design, the firm sets the spending requirement to the  $S$ -design optimum,  $S_S^*$ . However, different from the  $QaS$ -design case, the firm chooses the quantity threshold above  $S_S^*/p_S^{*+}$  so that consumers self-select to

satisfy the spending requirement rather than the quantity requirement, although consumers can choose which threshold to satisfy. That is, the firm makes it easier for consumers to satisfy the spending requirement than the quantity requirement.

The Proposition also indicates that switching to a  $QoS$  design can be status-quo Pareto-improving, i.e., the firm can improve its profit without changing the  $Q$ -design price and qualification requirement, and by setting the spending requirement to be effectively equivalent to the quantity requirement ( $S = p_Q^* Q_Q^*$ ). In this way, the firm offers some consumers the option to pay up, which results in the same revenue, but smaller cost. It is also interesting to note that the region of  $(c, g)$ , where such status-quo Pareto-improving  $QoS$ -solutions exist, is the same as the region where the status-quo Pareto-improving spending-based solution exists (recall the results given in Proposition 4).

In sum, compared to  $QaS$ ,  $QoS$  is a much better policy for managing the transition toward a spending-based program and reducing fierce consumer backlash. Thus, the firm might want to orchestrate this transition in two steps. First, it can emphasize that the new  $QoS$  program “generously” provides another option to qualify for status without taking away or altering the existing quantity based option, and when the consumers get used to the idea of the spending requirement, if the firm wants to, it could further alter the threshold requirements and achieve the  $S$ -design’s optimal profit.

## 7. Robustness Checks

To conclude our analyses we perform a number of robustness checks to verify that the insights derived from our model hold in broader settings.

### 7.1. Incorporating Redeemable Points

Since our paper focuses on the premium-status component of loyalty programs, thus far we only considered the points that count toward status qualification. In practice, however, loyalty points can often be redeemed for additional products, services, or cash. In some programs, the two types of points (those that count toward status and those that can be redeemed) are effectively different rewards that cannot be exchanged. Both are valuable for consumers, and both can be awarded based on the quantity purchased or on the amount spent. In this section, we consider a firm that awards both “status points” and “redeemable points,” and seek to understand whether the qualitative insights derived thus far continue to hold.

In this situation there are four possible combinations of loyalty program designs that we label as  $sQrQ$ ,  $sQrS$ ,  $sSrQ$ , and  $sSrS$ . Here the small  $s$ ,  $r$  designate the status and redeemable points component of the program and the capital  $Q$ ,  $S$  refer to whether the respective points are awarded

based on quantity of spending. For instance, National Car awards 1 redeemable point for each rental with 12 rentals required to reach Executive status<sup>13</sup> – an sQrQ design. Marriott awards 10 redeemable points per dollar spent with 50 nights required to reach Gold Elite status<sup>14</sup> – an sQrS design. Neiman Markus runs an sSrS design: two redeemable points are awarded per dollar, and a \$2,500 spend is required for Circle Three status<sup>15</sup>, etc. We also use the notation s\*rQ to denote the two designs that award redeemable miles based on quantity, sSr\* for the two designs that award status miles based on spending, and so on.

Let  $x$  be the number of redeemable points awarded (per unit) under the s\*rQ designs, let  $y$  be the number of redeemable points awarded (per dollar) under the s\*rS designs, and let  $\nu$  denote the value of a redeemable point to the consumer. Table 1 summarizes the four designs:

**Table 1 Program designs with status and redeemable points**

Design	Status points	Redeemable points	Threshold for status	Consumer surplus
sQrQ	Quantity	Quantity	$n + n^+ \geq Q$	$(a_i + g_{1\{\text{Gold}\}})(n + n^+) - \frac{(n+n^+)^2}{2} - (np + n^+p^+) + \nu x(n + n^+)$
sQrS	Quantity	Spending	$n + n^+ \geq Q$	$(a_i + g_{1\{\text{Gold}\}})(n + n^+) - \frac{(n+n^+)^2}{2} - (np + n^+p^+) + \nu y(pn + p^+n^+)$
sSrQ	Spending	Quantity	$pn + p^+n^+ \geq S$	$(a_i + g_{1\{\text{Gold}\}})(n + n^+) - \frac{(n+n^+)^2}{2} - (np + n^+p^+) + \nu x(n + n^+)$
sSrS	Spending	Spending	$pn + p^+n^+ \geq S$	$(a_i + g_{1\{\text{Gold}\}})(n + n^+) - \frac{(n+n^+)^2}{2} - (np + n^+p^+) + \nu y(pn + p^+n^+)$

**7.1.1. Consumer Behavior** Under each design, consumers strategically decide the number of purchases  $n$  and  $n^+$  to maximize total surplus which, from Table 1, is a sum of surpluses from status and redeemable points. The following Lemma characterizes consumer behavior:

LEMMA 3. *Adding the redeemable point component to the status loyalty program does not change the consumer behavior segments. That is,*

- (a) *Under the sQr\* designs, as in Lemma 1, there exist up to four consumer segments: no-purchase, purchase as needed (regular), buy up to Gold, and auto-Gold.*
- (b) *Under the sSr\* designs, as in Lemma 2, there exist up to six consumer segments: no-purchase, purchase as needed (regular), spend up to Gold (pay up, mix, buy up), and auto-Gold.*
- (c) *For the same prices, qualification threshold, and the “effective” value of a redeemable point ( $x = py$ ), thresholds for consumer behavior segments are identical under the sQrQ and sQrS designs, but they are different under the sSrQ and sSrS designs; specifically, compared to sSrQ, under sSrS there are more pay up consumers and fewer buy up consumers.*

<sup>13</sup> [https://www.nationalcar.com/en\\_US/car-rental/loyalty/enrollment/benefits.html](https://www.nationalcar.com/en_US/car-rental/loyalty/enrollment/benefits.html)

<sup>14</sup> <http://www.marriott.com/marriott-rewards/member-benefits.mi>

<sup>15</sup> <http://www.incircle.com/category.jsp?masterId=cat000000&itemId=cat103411&parentId=cat000001>

In other words, adding redeemable points changes the values for the  $\bar{a}_Q^j$  and  $\bar{a}_S^j$  thresholds derived in Lemmas 1, 2, but does not affect the structure of consumer behavior. Therefore, we can derive the demands,  $D^j$ , from each segment,  $A_Q^j, A_S^j$ , under each design, as we did earlier, and use these demands in solving the firm's problems below. The interesting element of Lemma 3 is that in the sQrS design, although the redeemable points component is spending-based, the pay up/mix segments do not emerge. This is because in the sQrS design, the status and redeemable components of the program are "in conflict"; to obtain status, the customer needs to purchase more units at the lowest price, but that does not provide many redeemable points. Hence, if the value of additional points is larger than  $p^+ - p$ , i.e.,  $y\nu > 1$ , then paying  $p^+$  is more beneficial than  $p$ ; otherwise paying  $p$  is better. In either case, only one price emerges, which means that the model effectively collapses to that of sQrQ with buy up segment alone. Because of that, the consumer behavior thresholds in the sQr\* cases are the same under the same set of parameters. In contrast, with the sSrS design the status and redeemable components are "in sync" providing stronger incentives for paying-up, and the thresholds are therefore different from those under sSrQ. This explains part c) of the Lemma.

**7.1.2. Firm's Problem** Let  $\kappa$  be the firm's cost of a redeemable point. Note that the  $\kappa$  and  $\nu$  parameters of the redeemable points component are analogous to the  $c, g$  parameters of the status component of the program, therefore, while the notation below dropped this for conciseness, demands  $D^j$  are functions of  $g, \nu$ . The firm's profit maximization problems under the four designs are as follows:

$$\begin{aligned} sQrQ: \pi_{sQrQ}^* &= \max_{p, Q, x} \left[ (p - x\kappa) \sum_{j=II, \dots, IV} D^j - c(D^{III} + D^{IV}) \right]; \\ sQrS: \pi_{sQrS}^* &= \max_{p, Q, y} \left[ (p - py\kappa) \sum_{j=II, \dots, IV} D^j - c(D^{III} + D^{IV}) \right]; \\ sSrQ: \pi_{sSrQ}^* &= \max_{p, p^+, S, x} \left[ p(D^{II} + D^{VI}) + S \int_{A_S^{III \dots V}} dF(a_i) - c \sum_{j=III, \dots, VI} D^j \right] - x\kappa \sum_{j=II, \dots, VI} D^j; \\ sSrS: \pi_{sSrS}^* &= \max_{p, p^+, S, y} \left[ (p - py\nu)(D^{II} + D^{VI}) + (S - Sy\kappa) \int_{A_S^{III}} dF(a_i) - c \sum_{j=III} D^j \right]. \end{aligned}$$

Fully solving these problems is intractable, and therefore only limited results can be obtained analytically. To verify the robustness of our results, we thus performed a series of numerical simulations with  $a_i \in U[0, 1]$  and  $c, g, \kappa, \nu \in [0, 1]$ . The simulation results confirm the robustness of our main findings (labeled by Rs below):

- (R1) The profit under the spending-based status program is higher than under the quantity-based status program even with a redeemable points component. That is, we observed  $\pi_{sSrQ}^* \geq \pi_{sQrQ}^*$ ,  $\pi_{sSrS}^* \geq \pi_{sQrS}^*$  and  $\pi_{sSrQ}^* \geq \pi_{sQrS}^*$  in all instances<sup>16</sup>; and

<sup>16</sup> The magnitude of the profit increase and the degree of strategic region expansion naturally depend on the  $(c, g, \kappa, \nu)$  parameters and it is possible that the firm may opt to include only one component in the program (e.g., it may offer status reward only if  $g$  is very high, but  $\nu$  is very low). Given our interest we focused on the cases where both components co-existed.

(R2) The region of  $(c, g)$  over which the firm benefits from strategic consumers expands under the spending-based status program as compared to the quantity-based status program even when a redeemable points component.

We also generate the following additional managerial insights (labeled by Is below):

(I1) Imagine a firm running a status program (either quantity- or spending-based). If it wants to add a redeemable points component (either quantity- or spending-based) without changing the status-component design, which program design should it implement? Our results indicate that the difference between using a quantity-based versus spending-based design for the redeemable points component is very small if the status component is managed with the quantity-based design; conversely, it can be quite large if it is managed with the spending-based design. That is, we numerically observe that  $\pi_{sQrQ}^* \approx \pi_{sQrS}^*$ , but  $\pi_{sSrQ}^* \leq \pi_{sSrS}^*$ . We note that under restricted conditions, this follows theoretically from Lemma 3c.

(I2) Combining the latter point with the robustness check (R1), imagine a firm running a sQrQ program that wants to improve its profits by making a design change in only one component (either status or redeemable points), but not both. Which component should the firm change? Our results indicate that the benefit from a switch in the status component is bigger; thus, it should change the status component because  $\pi_{sQrS}^* \leq \pi_{sSrQ}^*$ .

The aforementioned insights might explain what some firms did in practice: for example, major airlines (United, Delta) added spending requirement to status in 2014, but they changed the redeemable miles component later in 2016, i.e., they changed the redeemable points component to spending *after* they changed the status component to spending.

To conclude the first robustness check, we extended our model to capture the redeemable points component (that many loyalty programs in practice have in addition to status) and showed that it does not impact our key qualitative insights. The brief analyses presented above certainly do not fully address the management of the redeemable points component; we believe that is an interesting topic for future research.

## 7.2. Dynamics of Acquiring Status

In practice, consumers need to purchase enough in one period (qualification period) to achieve status and enjoy its benefits in the following period (benefit period). However, as mentioned in Section 3, the majority of customers *maintain* their status year-over-year, and thus the distinction between qualification and benefit periods is insignificant: Gold consumers receive the benefits they earned “last year” while qualifying for “next year’s” status.

In this section we relax this assumption and consider the dynamics of acquiring status, meaning that if a consumer without status meets the status requirement in one period, she will receive

status and its respective benefits in the subsequent period. In addition, we consider the popular practice of a “courtesy period”; while program policies state that the status obtained in one year is valid for the next year, several firms allow consumers to enjoy the benefit immediately after they qualify<sup>17</sup>. While such courtesy is usually relevant for only a small proportion of consumers, as the majority obtains status at the end of the qualification period (data from an industrial partner shows that less than 6% of consumers obtain status prior to the last month of the qualification period), it would still be of interest to examine the implications of such practice. Next, we examine how both the dynamics of acquiring status and this courtesy policy impact the profitability of the quantity- versus spending-based designs.

As before, the firm decides on static prices,  $p, p^+$ , and status qualification requirements,  $Q, S$ , to maximize the expected discounted profit, but consumers now decide how much to purchase in each period,  $n_{i,t}^*, n_{i,t}^{+*}$ , for  $t = 1, 2, \dots \infty$ , by solving a dynamic program to maximize the expected discounted surplus. Let  $G_{i,t} = 1$  if consumer- $i$  has Gold status at the beginning of period  $t$ , and  $G_{i,t} = 0$  otherwise. Further, let  $j_{i,t}(n_{i,t}, n_{i,t}^+, a_i, G_{i,t})$  denote the instantaneous surplus of consumer- $i$  in period  $t$ . Without a courtesy period,  $j_{i,t}(n_{i,t}, n_{i,t}^+, a_i, G_{i,t}) = (a_i + gG_{i,t})(n_{i,t} + n_{i,t}^+) - \frac{(n_{i,t} + n_{i,t}^+)^2}{2} - (n_{i,t}p + n_{i,t}^+p^+)$ , which reflects that a consumer receives status benefits depending on his status at the beginning of the period ( $G_{i,t}$ ). However, when the firm offers a courtesy status extension, consumer  $i$  without status in period  $t$  (i.e., with  $G_{i,t} = 0$ ) can enjoy status benefits immediately upon meeting the qualification requirement. For instance, under the Q-design for  $n_{i,t} > Q$ ,  $j_{i,t}(n_{i,t}, n_{i,t}^+, a_i, G_{i,t}) = (a_i - p)Q + (a_i + g - p)(n_{i,t} - Q) - n_{i,t}^2/2$ , the first  $Q$  units (meaning before the status qualification is met) are consumed without the status benefit, but the remaining  $n_{i,t} - Q$  are consumed with the status benefit. Under the S-design a similar logic applies. Finally, the system transitions from one period to the next; under the Q-design  $G_{i,t+1} = 1$  if  $n_{i,t} \geq Q$ , and under the S-design  $G_{i,t+1} = 1$  if  $pn_{i,t} + p^+n_{i,t}^+ \geq S$ ; otherwise,  $G_{i,t+1} = 0$ . We solve the resultant dynamic models (with and without courtesy periods) numerically<sup>18</sup> and make two main observations.

<sup>17</sup> For example, United extends the status benefit “from the date when [you] qualified through the end of the following Program year.” <https://www.united.com/web/en-US/content/mileageplus/premier/qualify.aspx>

<sup>18</sup> We examined nine  $(c, g)$  pairs:  $c \in \{0.05; 0.25; 0.5\}, g \in \{0.25, 0.5; 0.75; 0.95\}, g > c$ , three correlation structures: static, where  $a_{i,t} \equiv a_{i,1}$  (reflecting our main model), and correlated such that  $corr(a_{i,t}, a_{i,t+1}) = 0.5$  or  $0.8$ , three status updating approaches: steady-state, where  $G_{i,t}$  is calculated as per our main model, curtesy, and no curtesy as explained above, two program designs:  $Q$  and  $S$ , and two probabilities of being Gold in the first period, 5% and 25%. The consumer purchasing dynamic program was solved through value iteration by varying  $n, n^+$  in step 0.01; a 10% discount factor was applied to calculate the value-to-go. The firm’s problem was simulated for 1000 trials, 100 periods each, embedding the optimal consumer dynamic response, to obtain the expected profit for a given set of decisions,  $(p, Q), (p, p^+, S)$ , depending on the design. The optimal decisions were selected via a grid search with the step of 0.01 to maximize such expected values. In all instances we used  $a_{i,1} = \{0.01, 0.02, \dots 0.99\}$ . The simulation was coded in Julia 0.5.0 and ran on a multi-core desktop PC.

- (R3) The insights from the steady-state model are robust with respect to introducing the dynamics. The S-design is more profitable in all cases, and the region of parameters where the firm benefits from strategic consumers expands. Further, exploring the detailed solutions, we noticed that consumer behavior quickly (often as fast as in  $t = 2$ ) converges to maintaining status, much as our steady-state model assumes. In other words, our model is a good approximation of the dynamic setting.
- (I3) Providing a courtesy status extension is beneficial; in our simulation, doing so resulted in a  $\sim 15\%$  larger relative profit improvement, on average. Allowing consumers to experience status benefits immediately upon qualification makes strategic behavior more valuable and, per Section 5, the firm, in many cases, benefits from strategic behavior. In other words, the practice of extending status benefits is not much of a courtesy at all: it is a profit-increasing action. This might provide support for the popularity of such a practice.

### 7.3. WTP Distributions

Lastly, to confirm the robustness of our results (derived assuming the uniform distribution), and to understand the impact of the different consumer type / WTP distributions on the firm's profit and consumer surplus, we numerically solve the Q- and S-problems, (6) and (9), under various distributions: skewed, centered, and bimodal. In each case, we measure the average, minimum and maximum profit increase from switching to a spending-based program over a unit quadrant of  $(c, g)$ . We also report the average change in consumer surplus. Table 2 summarizes the results.

Distribution	Average $\Delta\Pi$	Range $\Delta\Pi$	Average $\Delta CS$	Skewness	Kurtosis
$U[0, 1]$	4.8%	0-27.3%	9.5%	0.0	1.8
$Tr_{[0,1]}N(0.5, 0.5)$	7.1%	0-35.6%	9.8 %	0.0	1.9
$Tr_{[0,1]}N(0.5, 0.4)$	8.5%	0-45.7%	11.0%	0.0	2.0
$Tr_{[0,1]}N(0.5, 0.3)$	10.9%	0-53.2%	15.4%	0.0	2.2
$Tr_{[0,1]}N(0.5, 0.2)$	18.1%	0-93.5%	21.2%	0.0	2.6
$Tr_{[0,1]}N(0.5, 0.1)$	32.8%	0-246.5%	18.0%	0.0	3.0
75-25 Mix of $N(0.7, 0.1)$ & $N(0.3, 0.1)$	8.1%	0-72.9%	0.6%	-0.8	2.6
50-50 Mix of $N(0.7, 0.1)$ & $N(0.3, 0.1)$	8.6%	0-72.6%	20.5%	0	1.7
25-75 Mix of $N(0.7, 0.1)$ & $N(0.3, 0.1)$	28.3%	0-118.1%	26.0%	0.75	2.6
Beta(5,2)	4.0%	0-29.8%	3.0%	-0.6	2.9
Beta(4,2)	5.8%	0-39.3%	2.3%	-0.5	2.6
Beta(3,2)	8.2%	0-53.4%	14.4%	-0.3	2.4
Beta(2,3)	27.0%	0-126.59%	27.2%	0.3	2.4
Beta(2,4)	48.0%	0-197.9%	28.1%	0.5	2.6
Beta(2,5)	77.3%	0-319.9%	54.9%	0.6	2.9

**Table 2** Increase in profit and consumer surplus under different consumer type distributions.

We first observe that our main results are robust to the choice of distribution; both profit and the overall consumer surplus increase with a spending-based design for all WTP distributions. Beyond that, two additional insights emerge:

- (I4) For centered distributions, improvements increase as the standard deviation decreases; equivalently, as the kurtosis of the distribution increases.
- (I4) For skewed distributions, the improvements increase as the distribution becomes more right-skewed; equivalently as its skewness increases.

In other words, our simulations suggest that firms with a large mass of medium/low-WTP consumers stand to benefit more than those with more dispersed WTPs. This is connected to the discussion in Sections 4.2, 5.2 and Figure 4; the consumers who benefit from a spending-based design are those with lowest WTPs among the strategic Golds. In a right-skewed distribution, or one with a smaller standard deviation, the proportion of such consumers tends to be larger; hence, a firm facing such a distribution has more to gain from switching to a spending-based design.

## 8. Conclusions

This paper investigates the interaction between the firm’s pricing and the premium-status loyalty program and, specifically, the role that strategic, forward-looking, and status-seeking consumers play in this interaction. We consider a contemporaneous change that is occurring in practice, where firms across several industries are switching their loyalty programs from quantity-based designs to spending-based designs. This switch has been receiving significant attention in the media and it has been generally met with negative consumer reactions. The goal of our work is to rigorously study the impact of loyalty program design choices on the firm and its consumers.

A critical aspect of this problem is that consumers exhibit sophisticated strategic behavior; rather than viewing each purchase in isolation, they consider multiple purchase decisions to maximize the total surplus over a period of time. The prices they pay and the possible premium-status benefits in the loyalty program both play an important role in customers’ decision making. We first present a consumer behavior model that captures this strategic behavior and incorporates consumer response to the firm’s prices and loyalty program design decisions. Using this model, we endogenously derive consumer demand, characterize rich patterns of consumer behavior, and factor them into the firm’s pricing and loyalty program optimization problems. Then, we present a number of managerial insights on how the firm’s pricing and loyalty decisions impact, and are impacted by, strategic consumer behavior. We do so by comparing several program designs and considering various extensions and robustness checks. The key insights from our work are the following:

1. Strategic behaviors, resulting in “unnecessary” purchases (buying up) or paying high prices when low prices are available (paying up), are indeed utility maximizing for a segment of

- consumers. Switching to a spending-based program reduces buying up by providing incentives to pay more and buy less, but may not eliminate buying up completely.
2. In contrast with much of the literature, such strategic behavior could *benefit* firms with characteristics that fall within the “strategic regions” that we describe. Under spending-based designs such benefits increase and regions expand. That is, even if a firm is hurt by strategic behaviors under the quantity-based design, it may profitably offer a spending-based program.
  3. Contrary to the generally negative public and media reactions, a change to a spending-based program could be Pareto-improving, implying a win–win outcome for both consumers and the firm. As with strategic regions, we characterize which kinds of firms could achieve such a win–win outcome, and which cannot.
  4. Again, in contrast with a naive intuition, programs that impose strict requirements whereby consumers must meet quantity *and* spending thresholds to qualify for status are not necessarily better than those that impose softer rules of meeting either quantity *or* spending requirements. Both could result in the same profit which, nevertheless, is not higher than that of a simple spending program. The softer one, however, can achieve a status-quo Pareto-improving outcome, while the stricter one cannot. Adopting a quantity-or-spending design is therefore a natural recommendation for a firm that wants to gradually transition toward a spending-based program and minimize the negative consumer reactions that plagued those firms that changed to spending-based designs too abruptly.
  5. Finally, model extensions and numerical simulations confirm the robustness of these findings with respect to addition of redeemable points, dynamics of acquiring status, and WTP distributions, and also allow establishing additional insights, which agree with anecdotal evidence. Specifically, we observe that (i) managing redeemable points in a spending-based fashion is more profitable when status is also managed via spending; (ii) extending, by “courtesy,” the benefits period is profitable; and (iii) firms with more customers in the medium–low WTP range benefit most from switching to spending-based programs.

All of these insights speak directly to the contemporaneous changes in loyalty programs observed in practice, and to the media and public debate about their impacts on consumers and firms. We hope that these insights will be helpful for managers and the general public to better understand the changes they manage and observe; we further hope our paper will help ignite future research on the design and operations of loyalty programs.

As our paper is first to consider the role of strategic behavior in the design and management of loyalty programs, it is not without limitations. We mainly focus on a single aspect of the loyalty program design (in a stylized setting) where we believe strategic behavior is critical: premium

status. Future research can thus expand upon our study in several directions. For example, additional models can incorporate a richer pricing structure (for items and/or points), and/or address the problem of allocating capacity among differently priced products, which are pertinent in certain specific industries, e.g., airlines. One can also explicitly consider customer arrivals, as well as address other questions (beyond the premium status) regarding the design and control problem of loyalty programs. There could include studying not only the accumulation, but also the redemption of loyalty points/miles. Competition could add another layer to our analysis; for instance, could there be an industry equilibrium where some firms operate quantity-based programs while others use spending-based designs? Considering the impact of status exclusivity and the related externality (i.e., status is less valuable if more consumers have it), as well as studying the heterogeneity in valuing status, are plausible extensions of our work. An empirical investigation of strategic consumer behavior in response to pricing and loyalty program design is clearly of interest too. In the context of our model, this could mean validating the assumptions of strategic consumer behavior, as well as calibrating the utility formulations and distributions of consumers. Finally, the spending-based design that disproportionately rewards high-priced purchases can create the possibility of a moral hazard (e.g., a business traveller is incentivized to purchase a higher-priced product, even when a lower-price one is available), which poses interesting research questions as well.

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# Online Supplement

**Proof of Lemma 1.** Under a quantity-based status program consumer  $-i$  solves the following surplus maximization problem:

$$\max_n \left\{ (a_i + g \mathbf{1}_{n \geq Q} - p)n - \frac{n^2}{2} \right\}.$$

Consumer  $-i$  will “buy-up” to Gold, i.e., have  $n > Q$ , if:  $(a_i + g)Q - \frac{Q^2}{2} - pQ \geq a_i n_{i,R}^* - \frac{(n_{i,R}^*)^2}{2} - p n_{i,R}^* = \frac{(a-p)^2}{2}$ , which is equivalent to  $a_i \in [Q - \sqrt{2Qg} + p, Q + \sqrt{2Qg} + p]$ . However, consumers with  $n_{i,G}^* = a_i + g - p \geq Q$ , i.e., those with  $a_i \geq Q + p - g$ , are already qualifying for Gold without the need to buy-up. This leads to three thresholds:  $\bar{a}_Q^1 = p$ ,  $\bar{a}_Q^2 = Q - \sqrt{2Qg} + p$ ,  $\bar{a}_Q^3 = Q + p - g$ , and consequently up to four segments:  $A_Q^I = \{a_i, s.t. a_i \leq \bar{a}_Q^1\}$ ,  $A_Q^{II} = \{a_i, s.t. \bar{a}_Q^1 \leq a_i \leq \bar{a}_Q^2\}$ , etc., as is depicted on Figure 1.

The “up to” in the statement above refers to the fact that some of these segments may be empty. This depends on the parameters  $p, Q, g$ , and for any distribution of  $a_i$  with finite support also on  $a_{min} = \min \{a_i\}$  and  $a_{max} = \max \{a_i\}$ .

For ease of exposition, relabel the four potential segments as  $N$  (do Not buy),  $R$  (buy Regular as needed),  $B$  (Buy-up to Gold), and  $A$  (qualify for Gold Automatically). We first note that  $N$ -segment always exists since it is reasonable to assume  $a_{min} < p$ . Next, the meaning of “status” implies that there exists at least one regular and one Gold consumer. Regular exists when  $\bar{a}_Q^1 \leq \min[\bar{a}_Q^2, \bar{a}_Q^3]$ , which is equivalent to  $2g \leq Q$ . It is not difficult to verify that in this case  $\bar{a}_Q^2 < \bar{a}_Q^3$ , implying that Gold exists if  $\min[\bar{a}_Q^2, \bar{a}_Q^3] \leq a_{max}$  and further that the non-emptiness of A-segment guarantees the non-emptiness of B-segment. Summarizing, there are two feasible combinations of consumer behavior segments under the quantity-based program:  $NRB$ , which emerges if  $\bar{a}_Q^3 > a_{max}$ , and  $NRBA$  which emerges otherwise. Q.E.D.

**Proof of Corollary 1.** From the proof of Lemma 1, we have  $\bar{a}_Q^1 = p \leq \bar{a}_Q^2 = Q - \sqrt{2Qg} + p \leq \bar{a}_Q^3 = Q + p - g$  and  $2g \leq Q$ . That  $\bar{a}_Q^3$  decrease in  $g$  and increase in  $Q$  is obvious. Since  $\frac{\partial}{\partial Q} \bar{a}_Q^2 = 1 - \frac{g}{\sqrt{2gQ}} \leq \frac{\partial}{\partial Q} \bar{a}_Q^3 = 1$ , it proves part (i). For part (ii),  $\frac{\partial}{\partial g} \bar{a}_Q^2 = -\frac{Q}{\sqrt{2gQ}} \leq \frac{\partial}{\partial g} \bar{a}_Q^3 = -1$  given  $2g \leq Q$ . Q.E.D.

**Proof of Lemma 2.** Under a spending-based status program consumer  $-i$  solves the following surplus maximization problem:

$$\max_{n, n^+} \left\{ (a_i + g \mathbf{1}_{np + n^+ p^+ \geq S})(n + n^+) - \frac{(n + n^+)^2}{2} - (np + n^+ p^+) \right\}.$$

Consumer  $-i$  solves this problem in two steps. First, the consumer determines the optimal way to spend-up to Gold, i.e., to have  $np + n^+ p^+ \geq S$ . Second, the consumer decides whether the resultant surplus is higher than that of purchasing regular. We analyze these two steps in sequel.

To determine the best way to spend-up to Gold, a consumer decides on the number of “regular” purchases,  $n$ , and the number of “plus” purchases,  $n^+$  that would maximize:

$$(a_i + g)(n + n^+) - \frac{(n + n^+)^2}{2} - (pn + p^+ n^+), \text{ s.t. } pn + p^+ n^+ \geq S.$$

Let

$$n_{i,S}^* = \frac{-S + (a_i + g)p^+}{(p^+ - p)} \text{ and } n_{i,S}^{+*} = \frac{S - (a_i + g)p}{(p^+ - p)}. \quad (11)$$

where  $n^*$  is non-negative when  $a_i \geq S/p^+ - g$  and similarly  $n^{+*}$  is non-negative when  $a_i \leq S/p - g$ . Therefore:

- If  $a_i \leq S/p^+ - g$ , then the solution to the spend-up consumer problem is  $(0, S/p^+)$  and the resultant surplus equals  $Surplus1_i(0, S/p^+) = S \left( \frac{(2(a_i+g)p^+ - S)}{2(p^+)^2} - 1 \right)$ ,
- If  $a_i \in (S/p^+ - g, S/p - g)$ , then the solution to the spend-up consumer problem is  $(n_{i,S}^*, n_{i,S}^{+*})$ , as given in (11); the resultant surplus equals  $Surplus2_i(n_{i,S}^*, n_{i,S}^{+*}) = \frac{(a_i+g)^2 - 2S}{2}$ , and
- If  $a_i \geq S/p - g$  then the solution to the spend-up consumer problem is  $(S/p, 0)$  and the resultant surplus equals  $Surplus3_i(S/p, 0) = S \left( \frac{(2(a_i+g)p - S)}{2p^2} - 1 \right)$ .

The above solutions describe how a consumer would spend up to Gold if she decides to do so. The latter decision, however, will depend on whether the surplus from spending up to Gold is larger than the surplus of purchasing with regular status. The latter surplus as per Section 3 equals  $(a_i - p)^2/2$ . Comparing this surplus with the surpluses of spending up to Gold derived above, we obtain that:

- $Surplus1_i \geq (a_i - p)^2/2$  if  $a_i \in \left[ p + \frac{S}{p^+} - \sqrt{\frac{2S}{p^+}(g + p - p^+)}, p + \frac{S}{p^+} + \sqrt{\frac{2S}{p^+}(g + p - p^+)} \right]$ ,  $g + p - p^+ > 0$ ,
- $Surplus2_i \geq (a_i - p)^2/2$  if  $a_i \geq \frac{p-g}{2} + \frac{S}{p+g}$ ,
- $Surplus3_i \geq (a_i - p)^2/2$  if  $a_i \in \left[ p + \frac{S}{p} - \sqrt{\frac{2S}{p}g}, p + \frac{S}{p} + \sqrt{\frac{2S}{p}g} \right]$ .

Intersecting these ranges with the non-negativity ranges above we obtain a set of  $a_i$ s such that consumer  $-i$  will spend-up to Gold with thresholds  $\bar{a}_S^2 = p + \frac{S}{p^+} - \sqrt{\frac{2S}{p^+}(g + p - p^+)}$ ,  $\bar{a}_S^3 = S/p^+ - g$  and  $\bar{a}_S^4 = S/p - g$ . Similar to the quantity-based program, thresholds  $\bar{a}_S^1 = p$  and  $\bar{a}_S^5 = p + S/p - g$  define the boundaries of consumers who purchase regular or qualify for Gold automatically. That is, there exists five thresholds, and, consequently, up to six consumer segments:  $A_S^I = \{a_i, s.t. a_i \leq \bar{a}_S^1\}$ ,  $A_S^{II} = \{a_i, s.t. \bar{a}_S^1 \leq a_i \leq \bar{a}_S^2\}$ , etc., as is depicted on Figure 2.

As in the quantity-based program, the ‘‘up to’’ in the statement above refers to the fact that some of these segments may be empty. This depends on the parameters  $p, p^+, S, g$ , and for any distribution of  $a_i$  with finite support also on  $a_{min} = \min\{a_i\}$  and  $a_{max} = \max\{a_i\}$ . Relabel the potential segments as  $N$  (do Not buy),  $R$  (buy Regular as needed),  $S$  (Spend-up to Gold, a union of three segments  $A_S^{III}, A_S^{IV}, A_S^V$  depending on the conditions discussed above), and  $A$  (qualify for Gold Automatically). The  $N$ -segment always exists since it is reasonable to assume  $a_{min} < p$  and the meaning of ‘‘status’’ implies that there exists, at least one regular and one Gold consumer. Regular exists when  $\bar{a}_S^1 \leq \min[\bar{a}_S^2, \bar{a}_S^3, \bar{a}_S^4, \bar{a}_S^5]$ , which is equivalent to  $\{g \leq 2p^+ - p \wedge S > gp^+ + pp^+\} \vee \{g > 2p^+ - p \wedge S > 2gp^+ + 2pp^+ - 2(p^+)^2\}$ , under which  $\max[\bar{a}_Q^2, \bar{a}_Q^3, \bar{a}_Q^4] < \bar{a}_S^5$ , implying that non-emptiness of A-segment guarantees the non-emptiness of S-segment. Summarizing, similar to the quantity-based program, there are two feasible combinations of consumer behavior segments under the quantity-based program:  $NRS$ , which emerges if  $\bar{a}_S^5 > a_{max}$ , and  $NRSA$ , which emerges otherwise. However, within the S segment there could be up to seven potential permutations: only pay-up, i.e., non-empty  $A_S^{III}$  but empty  $A_S^{IV}, A_S^V$ , only mix, i.e., non-empty  $A_S^V$  but empty  $A_S^{III}, A_S^V$ , etc., defined by combining ranges presented earlier in the proof. Q.E.D.

**Proof of Corollary 2.** From the proof of Lemma 2, we have  $\bar{a}_S^1 = p$ ,  $\bar{a}_S^2 = p + \frac{S}{p^+} - \sqrt{\frac{2S}{p^+}(g + p - p^+)}$ ,  $\bar{a}_S^3 = \frac{S}{p^+} - g$ ,  $\bar{a}_S^4 = \frac{S}{p} - g$ ,  $\bar{a}_S^5 = p + \frac{S}{p} - g$ . That  $a_S^{3,4,5}$  decrease in  $g$  and increase in  $S$  is obvious. For part 1, because  $p^+ > p$ ,  $\frac{\partial}{\partial S} a_S^5 = 1/p = \frac{\partial}{\partial S} a_S^4 > 1/p^+ = \frac{\partial}{\partial S} a_S^3$ . Finally,  $\frac{\partial}{\partial S} a_S^2 = 1/p^+ - \sqrt{\frac{g+p-p^+}{2Sp^+}} < 1/p^+ = \frac{\partial}{\partial S} a_S^3$ . For part 2,  $\frac{\partial}{\partial g} a_S^2 = -\sqrt{\frac{S}{2p^+(g+p-p^+)}}$ , which is less than  $-1 = \frac{\partial}{\partial g} a_S^{3,4,5}$  under the condition  $S \geq 2p^+(g + p - p^+)$  (since  $a_S^1 \leq a_S^2$ ). Q.E.D.

**Proof of Proposition 1.** Expanding equation (10), the firm maximizes

$$\pi_M(p, ) = p_M \left( \int_p^{Q+p-g} (a_i - p) dF(a_i) + \int_{Q+p-g}^1 (a_i - p + g) dF(a_i) \right) - c \int_{Q+p-g}^1 (a_i - p + g) dF(a_i).$$

It is easy to verify that  $\frac{\partial \pi_M}{\partial Q} = cQ - gp$ , implying that  $\pi_M$  is convex in  $Q$  for any  $p$ . This, in turn, implies that there are two potentially optimal solutions: 1) the Gold threshold is so high that (almost) no consumer buys with Gold status, 2) the Gold threshold is so low that (almost) no consumer purchases without Gold status [a remark, “almost” refers to the fact that the marginal consumer, either one with  $a_i = p$  or with  $a_i = 1$  is, technically-speaking, indifferent]. We discuss these two cases in sequel:

*Case 1:* If (almost) no consumer purchases with Gold status, then  $Q = 1 - p + g$  and  $\pi_M = \int_p^1 p(a_i - p) da_i = \frac{1}{2}(p-1)^2 p$ , which is maximized at  $p_M^* = 1/3$ , implying  $Q_M^* = 2/3 + g$ , with the optimal value of  $\pi_{M,1}^* = 2/27$ ;

*Case 2:* If (almost) no consumer purchases without Gold status, then  $Q_M^* = g$  and  $\pi_M = (p - c) \int_p^1 (a_i + g - p) da_i$ , which is maximized at  $p_M^* = \frac{1}{3}(2 + c + 2g - \sqrt{c^2 - 2cg - 2c + 4g^2 + 2g + 1})$  with the optimal value of  $\pi_{M,2}^* = \frac{1}{54} \left( \sqrt{c^2 - 2c(g+1) + 4g^2 + 2g + 1} - c - 2g + 1 \right) \left( 2 - 2c + 2g - \sqrt{c^2 - 2c(g+1) + 4g^2 + 2g + 1} \right) \times \left( \sqrt{c^2 - 2c(g+1) + 4g^2 + 2g + 1} - c + 4g + 1 \right)$ .

Let  $\Delta = \pi_{M,2}^* - \pi_{M,1}^*$ , and let  $\hat{c}_M(g)$  be a root of  $\Delta = 0$  in  $c$ . We next establish that such a root necessarily exists and it is unique.

The root  $\hat{c}_M(g)$  exists on  $c \in [0, 1]$  if: (i)  $\Delta$  is continuous in  $c$  for any  $g$ , (ii) at  $c = 0$ ,  $\Delta \geq 0$  and, (iii) at  $c = 1$ ,  $\Delta = -2/27 \leq 0$  for all  $g \in [0, 1]$ . Here (i) and (iii) are obvious, and to see that (ii) is true note that:

$$\begin{aligned} \Delta|_{c=0} &= \frac{1}{54} \left( - \left( \sqrt{4g^2 + 2g + 1} - 2g - 2 \right) \left( \sqrt{4g^2 + 2g + 1} + 4g + 1 \right) \left( \sqrt{4g^2 + 2g + 1} - 2g + 1 \right) - 4 \right) \\ &= \frac{1}{54} \left( \sqrt{4g^2 + 2g + 1} (8g^2 + 4g + 2) - (16g^3 + 12g^2 - 6g + 2) \right). \end{aligned}$$

Observe that the expression is in the form of  $\frac{1}{54}(A - B)$ . Because  $A \geq 0$ , if  $A^2 > B^2$  then  $A > B$ . Squaring the two terms and rearranging, we obtain that  $A^2 - B^2 = 4g(108g^3 + 76g^2 + 3g + 12) \geq 0$ . This establishes that  $\Delta|_{c=0} \geq 0$  and the root therefore necessarily exists.

Finally, the root  $\hat{c}_M(g)$  is unique since  $\Delta$  is decreasing in  $c$  on  $c, g \in [0, 1]$ . Differentiating  $\Delta$  we obtain that:

$$\frac{\partial \Delta}{\partial c} = -\frac{1}{9} \left( (1 + g - c) \sqrt{3g^2 + (1 + g - c)^2} - (3g^2 - (1 + g - c)^2) \right).$$

Again, observe that the expression is in the form of  $-\frac{1}{9}(A - B)$  where  $A > 0$ . Thus, if  $A^2 > B^2$ , then  $A > B$  and  $\frac{\partial \Delta}{\partial c} < 0$ . By following the same rearranging and squaring steps as above,  $A^2 - B^2 = g^2(1 + 2g - c)(1 - c) \geq 0$  for all  $c, g \in [0, 1]$ . That is, the root  $\hat{c}_M(g)$  is unique which completes the proof of the Proposition. Q.E.D.

**Proof of Proposition 2.** From the proof of Lemma 1 there exists two potential customer segmentations: NRBA, when all segments exist (this happens when  $Q \leq 1 + g - p$ , and because in all  $Q$ -designs  $Q \geq 2g$ , this is equivalent to  $p \leq 1 - g$ ), and NRB, when segment  $A_Q^{IV}$  is empty, which happens when  $Q \geq 1 + g - p$ . We thus consider these two cases:

In NRBA:

$$\begin{aligned}\pi_{Q,1} &= p \int_p^{p+Q-\sqrt{2gQ}} (a_i - p) da_i + (p-c) \left( \int_{p+Q-\sqrt{2gQ}}^{p+Q-g} Q da_i + \int_{p+Q-g}^1 (a_i + g - p) da_i \right) \\ &= \frac{1}{2}(p-c)(1+g-p)^2 + \frac{1}{2}cQ \left( 2g+Q-2\sqrt{2gQ} \right).\end{aligned}$$

For  $p$ , it is easy to establish that  $p_Q^* = \frac{1}{3}(2c+g+1)$ , and from  $p \leq 1-g$ , we have this solution when  $g \leq 1/2 - c/2$ ; equivalently, when  $c \leq 1-2g$ . For  $Q$ , the derivative of the term in parenthesis equals  $1 - \frac{\sqrt{2g}}{\sqrt{gQ}}$  which is non-negative for  $Q \geq 2g$ . That is,  $\pi_{Q,1}$  increases in  $Q$  and therefore  $Q_Q^* = 1+g-p_Q^*$ . With these  $\pi_{Q,1}^* = \frac{1}{27}(-2)(-c+g+1) \left( 2c^2 + c \left( 6\sqrt{3}\sqrt{g(-c+g+1)} - 10g - 1 \right) - (g+1)^2 \right)$ ;

In NRB:

$$\begin{aligned}\pi_{Q,2} &= p \int_p^{p+Q-\sqrt{2gQ}} (a_i - p) da_i + (p-c) \int_{p+Q-\sqrt{2gQ}}^1 Q da_i \\ &= -p^2Q + \frac{1}{2}pQ(2c+2g-Q+2) + \frac{1}{2}Q \left( -2\sqrt{2}c\sqrt{gQ} + 2cQ - 2c \right).\end{aligned}$$

$\pi_{Q,2}$  is a concave quadratic function of  $p$ , which is maximized at  $p_Q^* = \frac{1}{4}(2+2c+2g-Q)$ . Substituting this expression into  $\pi_{Q,2}$  we show that  $\pi_{Q,2}$  is decreasing in  $Q$ . This is because  $\frac{\partial^2}{\partial c \partial g} \left[ \frac{\partial \pi_{Q,2}}{\partial Q} \right] = \frac{1}{2} \left( 1 - \frac{3Q}{\sqrt{2gQ}} \right) \leq 0$  because  $Q \geq 2g$ , i.e., the derivative  $\frac{\partial \pi_{Q,2}}{\partial Q}$  is submodular in  $(c, g)$ , meaning that the derivative in  $g$  is largest at  $c=0$ . But at  $c=0$ ,  $\frac{\partial^2 \pi_{Q,2}}{\partial Q \partial g} |_{c=0} = \frac{1}{2}(1+g-Q)$ , i.e.,  $\frac{\partial \pi_{Q,2}}{\partial Q} |_{c=0}$  is convex in  $g$ . Therefore,  $\frac{\partial \pi_{Q,2}}{\partial Q} |_{c=0}$  obtains its highest value either at  $g = \frac{1}{2}$  or at  $g = 1$ . Then,  $\frac{\partial \pi_{Q,2}}{\partial Q} |_{c=0, g=\frac{1}{2}} = \frac{1}{16}(3-3Q)(3-Q) \leq 0$  because from the boundaries of integration in the NRB case (evaluated at  $c=0, g = \frac{1}{2}$ ),  $Q \in [1, \frac{2}{9}(\frac{11}{2} + 2\sqrt{7}) \approx 2.4]$ , and  $\frac{\partial \pi_{Q,2}}{\partial Q} |_{c=0, g=1} = \frac{1}{16}(4-3Q)(4-Q) \leq 0$  because from the boundaries of integration in the NRB case (evaluated at  $c=0, g=1$ ),  $Q \in [2, \frac{32}{9} \approx 3.55]$ . Therefore,  $\pi_{Q,2}$  is decreasing in  $Q$ , and so  $Q_Q^* = 2g$  and, consequently,  $p_Q^* = \frac{1+c}{2}$ . From  $Q = 2g \geq 1+g-p$ , or equivalently,  $p = \frac{1+c}{2} \geq 1-g$ , we have this solution when  $g > 1/2 - c/2$ . Notice also that with  $Q = 2g$  NRB simplifies to NB (the R segment technically exists but consists only of the marginal consumer who is indifferent between purchasing regular or buying up), i.e.,  $\pi_{Q,2} = (p-c) \int_p^1 Q da_i$ . Substituting  $(Q_Q^*, p_Q^*)$  its optimal value equals  $\pi_{Q,2}^* = \frac{1}{2}(c-1)^2g$ .

We compare these solutions to the myopic solutions from Proposition 1 in order to establish the QSR regions. That is, we show that the following four relationships hold:

1.  $\pi_{Q,2}^* - \pi_{M,1}^* \geq 0$  for  $c \leq \hat{c}_{Q2}(g)$ ,  $g \geq 1/2 - c/2$ ;
2.  $\pi_{Q,1}^* - \pi_{M,1}^* \geq 0$  for  $c \leq \hat{c}_{Q1}(g)$ ,  $g \leq 1/2 - c/2$ ;
3.  $\pi_{Q,2}^* - \pi_{M,2}^* \geq 0$  for  $c \leq \hat{c}_M(g)$ ,  $g \geq 1/2 - c/2$ ;
4.  $\pi_{Q,1}^* - \pi_{M,2}^* \geq 0$  for  $c \leq \hat{c}_M(g)$ ,  $g \leq 1/2 - c/2$ .

For relationship #1 function  $\hat{c}_{Q2}(g)$  can be derived in closed form, because  $\pi_{Q,2}^* - \pi_{M,1}^* = \frac{1}{2}(c-1)^2g - \frac{2}{27}$  is a convex quadratic function with two roots,  $1 \pm \frac{2\sqrt{3}}{9\sqrt{g}}$ . Since  $c \leq 1$ , we therefore obtain that  $\pi_{Q,2}^* \geq \pi_{M,1}^*$  iff  $c \leq 1 - \frac{2\sqrt{3}}{9\sqrt{g}} \equiv \hat{c}_{Q2}(g)$ , where  $\hat{c}_{Q2}(1/3) = 1/3$ ,  $\hat{c}_{Q2}(1) = 1 - \frac{2\sqrt{3}}{9} \approx 0.61$ .

For relationship #2 we follow the process similar to that used in the proof of Proposition 1. Let  $\Delta = \pi_{Q,1}^* - \pi_{M,1}^*$  and let  $\hat{c}_{Q1}(g)$  be a root in  $c$  of  $\Delta = 0$ . For  $g \in [0, \frac{1}{3}]$  such a root exists because: (i)  $\Delta$  is continuous

in  $c$ , (ii)  $\Delta|_{c=0} = \frac{2}{27}((g+1)^3 - 1) \geq 0$ , and (iii)  $\Delta|_{c=1-2g} = 2g^3 - \frac{2}{27} \leq 0$ . For  $g \in [\frac{1}{3}, \frac{1}{2}]$  the root does not exist because on this interval  $2g^3 - \frac{2}{27} \geq 0$ . However, as we will show momentarily,  $\Delta$  is decreasing, which implies that  $\Delta > 0$  for  $g \in [\frac{1}{3}, \frac{1}{2}]$ . Combining the two cases, for  $g \in [0, \frac{1}{3}]$  the constraint  $g \leq 1/2 - c/2$  is not binding, and the  $QSR$  region is defined by  $\hat{c}_{Q1}(g)$ , but for  $g \in [\frac{1}{3}, \frac{1}{2}]$  the constraint  $g \leq 1/2 - c/2$  is binding, and the  $QSR$  region is defined by it. See Figure 3b for the illustration. Note also that  $\Delta|_{c=0, g=0} = 0$  implying that  $\hat{c}_{Q1}(0) = 0$  and  $\Delta|_{c=1/3, g=1/3} = 0$  implying that  $\hat{c}_{Q1}(1/3) = 1/3$ . Further, from the proof above, we have  $\hat{c}_{Q2}(1/3) = 1/3$ , the boundary of the  $QSR$  region is a continuous function (although, possibly, with a kink at  $c, g = 1/3$ ).

We next show that  $\Delta$  is monotone (decreasing) in  $c$  on  $c \in [0, 1 - 2g]$  for any  $g \in [0, 1/2]$ , i.e.,

$$\begin{aligned} \frac{\partial \Delta}{\partial c} &= \frac{2}{9} \left( 2c^2 + c \left( 5\sqrt{3}\sqrt{g(-c+g+1)} - 8g - 2 \right) - (g+1) \left( 2\sqrt{3}\sqrt{g(-c+g+1)} - 3g \right) \right) \\ &= \frac{2}{9} \left( (2c^2 - 8cg - 2c + 3g^2 + 3g) - \left( \sqrt{3}\sqrt{g(1-c+g)}(2 - 5c + 2g) \right) \right) \leq 0. \end{aligned}$$

Observe that the derivative is in the form of  $\frac{2}{9}(A - B)$ , which is negative if  $B > A$ . Note that (i)  $A|_{c=0} = 3g^2 + 3g \geq 0, B|_{c=0} = \sqrt{3g(1+g)}(2+2g) \geq 0$ , (ii)  $B = 0$  at  $c = \frac{2}{5}(1+g)$ , at which  $A|_{c=\frac{2}{5}(1+g)} = \frac{3}{25}(g^2 - 3g - 4) \leq 0$ , and (iii)  $A$  is a convex quadratic function in  $c$  which is minimized at  $c = \frac{1}{2}(4g+1) > \frac{2}{5}(1+g)$ . Combining these facts,  $A$  is decreasing on  $c \in [0, \frac{2}{5}(1+g)]$  and hence  $A \geq 0$  on  $c \in [0, \frac{2}{5}(1+g)]$  iff  $c \leq \frac{1}{2}(4g+1) - \frac{1}{2}\sqrt{10g^2+2g+1}$  (this expression is the first root of  $A = 0$ ); further,  $A$  crosses zero before  $B$  does. Therefore, we have three cases<sup>19</sup>:

*Case 1:*  $A \geq 0, B \geq 0$ , which happens for  $c \in [0, \frac{1}{2}(4g+1) - \frac{1}{2}\sqrt{10g^2+2g+1}]$ . For this case  $B > A$  if  $B^2 > A^2$ ;

*Case 2:*  $A < 0, B \geq 0$ , which happens for  $c \in [\frac{1}{2}(4g+1) - \frac{1}{2}\sqrt{10g^2+2g+1}, \frac{2}{5}(1+g)]$ . For this case there is nothing to prove since it is clear that  $B > A$ ;

*Case 3:*  $A < 0, B < 0$ , which happens for  $c \in [\frac{2}{5}(1+g), 1 - 2g]$ . For this case  $B > A$  if  $B^2 < A^2$ .

Now, we analyze  $B^2 - A^2 = (4 + g - 4c)(c^3 + c^2(11g - 1) - 12cg(g + 1) + 3g(g + 1)^2)$  for each case. The first term  $(4 + g - 4c)$  is clearly positive. Define the second term as  $\Delta_2 = (c^3 + c^2(11g - 1) - 12cg(g + 1) + 3g(g + 1)^2)$ ; for Case 1, we need to show that  $\Delta_2 \geq 0$  on  $c \in [0, \frac{1}{2}(4g + 1) - \frac{1}{2}\sqrt{10g^2 + 2g + 1}]$ , and for Case 3, we need to show that  $\Delta_2 \leq 0$  on  $c \in [\frac{2}{5}(1 + g), 1 - 2g]$ .

For Case 1: on  $c \in [0, \frac{1}{2}(4g + 1) - \frac{1}{2}\sqrt{10g^2 + 2g + 1}]$ , we show that: (C1-i)  $\Delta_2|_{c=0} \geq 0$ , (C1-ii)  $\Delta_2|_{c=\frac{1}{2}(4g+1)-\frac{1}{2}\sqrt{10g^2+2g+1}} \geq 0$ , and (C1-iii)  $\Delta_2$  is decreasing as follows:

- (C1-i) holds because  $\Delta_2|_{c=0} = 3g(g + 1)^2 \geq 0$ ;
- (C1-ii) holds because  $\Delta_2|_{c=\frac{1}{2}(4g+1)-\frac{1}{2}\sqrt{10g^2+2g+1}} = \frac{3}{4}g(98g^2 + 13g - (31g + 1)\sqrt{10g^2 + 2g + 1} + 5)$ , which, by following the same squaring-and-rearranging steps as above, is non-negative whenever  $\frac{27}{8}(4 - g)g^2(g + 1)^3 \geq 0$ , which is always true;
- (C1-iii) holds because  $\frac{\partial \Delta_2}{\partial c} = 3c^2 + c(22g - 2) - 12g(g + 1) \leq 0$ . To see that, define  $\Delta_3 = \frac{\partial \Delta_2}{\partial c}$  and apply the same steps as above again:

$$-\Delta_3|_{c=0} = -12g(1 + g) \leq 0;$$

<sup>19</sup> Depending on the value of  $g$  the corresponding intervals in the last two cases may be empty.

- $\Delta_3|_{c=\frac{1}{2}(4g+1)-\frac{1}{2}\sqrt{10g^2+2g+1}} = \frac{1}{2}(103g^2 + 5g - (34g + 1)\sqrt{10g^2 + 2g + 1} + 1)$ , which, after squaring and rearranging, is non-positive whenever  $-\frac{3}{4}g(g+1)^2(317g+20) \leq 0$ , which is always true;
- $\Delta_3$  is quadratic and convex in  $c$ .

For Case 3: on  $c \in [\frac{2}{5}(1+g), 1-2g]$ , we show that: (C3-i)  $\Delta_2|_{c=\frac{2}{5}(1+g)} = \frac{3}{125}(g-4)(1+g)^2 \leq 0$ , (C3-ii)  $\Delta_2|_{c=1-2g} = 9g^2(7g-2) \leq 0$  because non-emptiness of Case 3 implies  $\frac{2}{5}(1+g) < 1-2g$ , i.e.,  $g < \frac{1}{4}$ , and (C3-iii)  $\Delta_2$  is convex since  $\frac{\partial^2 \Delta_2}{\partial c^2} = 6c + 22g - 2 \geq 6\{\frac{2}{5}(1+g)\} + 22g - 2 = \frac{2}{5}(61g+1) \geq 0$ .

To summarize,  $\frac{\partial \Delta}{\partial c} = \frac{2}{9}(A-B) \leq 0$  on  $c \in [0, 1-2g]$  for all  $g \in [0, 1/2]$ ;  $\Delta$  is therefore decreasing, and the uniqueness of  $\hat{c}_{Q1}(g)$  follows.

Next, we prove relationship #3; in fact, we prove a slight generalization of it:  $\pi_{Q,2}^* - \pi_{M,2}^* \geq 0$  for  $g \geq 1/2 - c/2$  (i.e., that  $c \leq \hat{c}_M(g)$  is not necessary for  $\pi_{Q,2}^* - \pi_{M,2}^* \geq 0$ ). As before, define  $\Delta = \pi_{Q,2}^* - \pi_{M,2}^*$ . After simplifying and rearranging  $\Delta = \frac{1}{54} \left( \frac{1}{4}((c-1)^2(8c+75g-8) + g(-3c+8g+3)^2) - 2\sqrt{(3g^2 + (1+g-c)^2)^3} \right)$ . Observe that it is in the form of  $\frac{1}{54}(A - \sqrt{B})$ . Clearly  $B > 0$  and because  $g \geq 1/2 - c/2$  it is easy to see that  $A \geq 0$  as well, hence  $A - \sqrt{B} \geq 0$  if  $A^2 - B \geq 0$ . Squaring to open the root, and rearranging,  $A^2 - B = 27g(c-1)^2(g(2g-c+1)^2 + (c-1)^2(4c-4+10g)) \geq 0$  because for  $c \in [0, 1], g \geq 1/2 - c/2, (4c-4+10g) \geq 4c-4+5-5c = 1-c \geq 0$ .

We finally prove relationship #4; in fact, again, we prove a slight generalization of it:  $\pi_{Q,1}^* - \pi_{M,2}^* \geq 0$  for  $c \leq g \leq 1/2 - c/2$ . That is, we substitute  $c \leq \hat{c}_M(g)$  with a less restrictive condition  $c \leq g^{20}$ . Note also that  $g \geq c$  with  $g \leq 1/2 - c/2$  implies  $c \leq \frac{1}{3}$ .

As before, define  $\Delta = \pi_{Q,1}^* - \pi_{M,2}^*$ . After factoring and rearranging,  $\Delta = (1+g-c)(1+4c-5c^2+2g(1+11c+5g)) - 12c\sqrt{3g(1+g-c)^3} - \sqrt{(3g^2 + (1+g-c)^2)^3}$ . Observe that it is in the form of  $A - \sqrt{B} - \sqrt{C}$ . Clearly,  $B, C \geq 0$  and because  $c \in [0, 1], 1+4c-5c^2 \geq 0, A > 0$  as well. Thus,  $A - \sqrt{B} - \sqrt{C} \geq 0$  if  $A^2 - (\sqrt{B} + \sqrt{C})^2 \geq 0$ , which, in turn is non-negative if  $(A^2 - B - C)^2 - (2\sqrt{B}\sqrt{C})^2 = A^4 - 2A^2(B+C) + (B-C)^2 \geq 0$ . Define the latter as  $\tilde{\Delta}$  and observe that since  $A$  is a 4<sup>th</sup>-degree polynomial in  $g$ ,  $\tilde{\Delta}$  is a polynomial of degree 12 in  $g$ ; the expression is so large and un-insightful that we do not present it here.

To establish that  $\tilde{\Delta} \geq 0$ , we proceed through the series of 11 “steps” as in the proofs of case #2 above and in Proposition 1; throughout, we use  $X$  to represent the factorized positive constants for conciseness.

*Step 1:*  $\tilde{\Delta}_{g=0} = X(c-1)^8 c^2 (2c+1)^2 \geq 0$ ,  $\tilde{\Delta}|_{g=\frac{1}{2}-\frac{c}{2}} = X(c-1)^{10} (1307c^2 + 410c + 11) \geq 0$ , and  $\tilde{\Delta}$  is monotone (increasing), because:

*Step 2:* Define  $\tilde{\Delta}_2 = \frac{\partial}{\partial g} \tilde{\Delta}$ .  $\tilde{\Delta}_2|_{g=0} = X(c-1)^7 c^2 (17c^2 - 7c - 1) \geq 0$  on  $c \in [0, \frac{1}{3}]$ ,  $\tilde{\Delta}_2|_{g=\frac{1}{2}-\frac{c}{2}} = X(1-c)^9 (77 + 2414c + 6725c^2) \geq 0$ , and  $\tilde{\Delta}_2$  is monotone (increasing), because:

*Step 3:* Define  $\tilde{\Delta}_3 = \frac{\partial}{\partial g} \tilde{\Delta}_2$ .  $\tilde{\Delta}_3|_{g=0} = X(c-1)^6 c (-46c^3 + 21c^2 + 12c + 1) \geq 0$  on  $c \in [0, \frac{1}{3}]$  [showing this requires one similar nested step, which we omit for conciseness],  $\tilde{\Delta}_3|_{g=\frac{1}{2}-\frac{c}{2}} = X(c-1)^7 (90661c^3 - 59127c^2 - 40305c - 1597) \geq 0$  since  $c^3 \leq c^2 \leq c^1 \leq 1$  for  $c \in [0, 1]$ .  $\tilde{\Delta}_3$  is monotone (decreasing), because:

<sup>20</sup> This is because  $\hat{c}_M(g) < g$ , which follows from  $\Delta_{c=0} \geq 0, \Delta_{c=g} \leq 0$ . The first inequality is shown in the proof of Proposition 1. To see why the second holds, observe that  $\Delta_{c=g} = \frac{1}{27} \left( -(1+9c^2) + \sqrt{(3c^2+1)^3} \right)$ , which is in the form of  $-A + \sqrt{B}$ . Because  $A > 0, -A + \sqrt{B} < 0$  if  $A^2 > B$ . Squaring and rearranging  $A^2 - B = 9c^2(-3c^4 + 6c^2 + 1) \geq 0$  because  $c \in [0, 1]$ .

*Step 4:* Define  $\tilde{\Delta}_4 = \frac{\partial}{\partial g} \tilde{\Delta}_3$ .  $\tilde{\Delta}_4|_{g=0} = X(c-1)^5 c(97c^3 - 68c^2 + 25c + 6) \leq 0$  on  $c \in [0, 1]$  [showing this requires two similar nested steps, which we omit for conciseness],  $\tilde{\Delta}_4|_{g=\frac{1}{2}-\frac{\epsilon}{2}} = X(c-1)^6(56027c^3 - 37321c^2 - 34335c - 1651) \leq 0$  since  $c^3 \leq c^2 \leq c^1 \leq 1$  for  $c \in [0, 1]$ .  $\tilde{\Delta}_4$  is monotone (decreasing), because:

*Step 5:* Define  $\tilde{\Delta}_5 = \frac{\partial}{\partial g} \tilde{\Delta}_4$ .  $\tilde{\Delta}_5|_{g=c} = 25677c^8 - 33576c^7 - 748836c^6 - 918888c^5 - 373458c^4 - 94392c^3 - 13348c^2 - 696c - 3 \leq 0$  since  $c \in [0, 1]$ ,  $\tilde{\Delta}_5|_{g=\frac{1}{2}-\frac{\epsilon}{2}} = -X(-1+c)^4(27253 + 433484c - 139298c^2 - 833652c^3 + 519125c^4) \leq 0$  on  $c \in [0, \frac{1}{3}]$  [showing this requires two similar nested steps, which we omit for conciseness], and  $\tilde{\Delta}_5$  is monotone (decreasing), because:

*Step 6:* Define  $\tilde{\Delta}_6 = \frac{\partial}{\partial g} \tilde{\Delta}_5$ .  $\tilde{\Delta}_6|_{g=c} = 1189c^7 - 10947c^6 - 44955c^5 - 36843c^4 - 11865c^3 - 1969c^2 - 129c - 1 \leq 0$  since  $c \in [0, 1]$ ,  $\tilde{\Delta}_6|_{g=\frac{1}{2}-\frac{\epsilon}{2}} = X(c-1)^3(96911c^4 - 132804c^3 - 93054c^2 + 122572c + 9831) \leq 0$  on  $c \in [0, \frac{1}{3}]$  [showing this requires one similar nested steps, which we omit for conciseness], and  $\tilde{\Delta}_6$  is monotone (decreasing), because:

*Step 7:* Define  $\tilde{\Delta}_7 = \frac{\partial}{\partial g} \tilde{\Delta}_6$ .  $\tilde{\Delta}_7|_{g=c} = -X(621c^6 + 56976c^5 + 120933c^4 + 68088c^3 + 14663c^2 + 1176c + 15) \leq 0$ ,  $\tilde{\Delta}_7|_{g=\frac{1}{2}-\frac{\epsilon}{2}} = -X(c-1)^2(19585c^4 - 12940c^3 - 52370c^2 + 42372c + 4505) \leq 0$  on  $c \in [0, \frac{1}{3}]$  [showing this requires two similar nested steps, which we omit for conciseness], and  $\tilde{\Delta}_7$  is monotone (decreasing), because:

*Step 8:* Define  $\tilde{\Delta}_8 = \frac{\partial}{\partial g} \tilde{\Delta}_7$ .  $\tilde{\Delta}_8|_{g=c} = -X(741c^5 + 6663c^4 + 8487c^3 + 2995c^2 + 318c + 6) \leq 0$ ,  $\tilde{\Delta}_8|_{g=\frac{1}{2}-\frac{\epsilon}{2}} = X(-3327 - 18829c + 61786c^2 - 50442c^3 + 7093c^4 + 3719c^5) \leq 0$  on  $c \in [0, \frac{1}{2}]$  [showing this requires three similar nested steps, which we omit for conciseness], and  $\tilde{\Delta}_8$  is monotone (decreasing), because:

*Step 9:* Define  $\tilde{\Delta}_9 = \frac{\partial}{\partial g} \tilde{\Delta}_8$ .  $\tilde{\Delta}_9|_{g=c} = -X(2345c^4 + 9852c^3 + 7298c^2 + 1252c + 35) \leq 0$ ,  $\tilde{\Delta}_9|_{g=\frac{1}{2}-\frac{\epsilon}{2}} = X(6449c^4 - 57012c^3 + 98230c^2 - 38676c - 9247) \leq 0$  on  $c \in [0, \frac{1}{3}]$  [showing this requires two similar nested steps, which we omit for conciseness], and  $\tilde{\Delta}_9$  is monotone (decreasing), because:

*Step 10:* Define  $\tilde{\Delta}_{10} = \frac{\partial}{\partial g} \tilde{\Delta}_9$ .  $\tilde{\Delta}_{10}|_{g=c} = -X(3153c^3 + 6899c^2 + 2421c + 117) \leq 0$ ,  $\tilde{\Delta}_{10}|_{g=\frac{1}{2}-\frac{\epsilon}{2}} = -X(2559c^3 - 8373c^2 + 4813c + 1641) \leq 0$  [showing this requires two similar nested steps, which we omit for conciseness], and  $\tilde{\Delta}_{10}$  is monotone (decreasing), because:

*Step 11:* Define  $\tilde{\Delta}_{11} = \frac{\partial}{\partial g} \tilde{\Delta}_{10}$ .  $\tilde{\Delta}_{11}|_{g=c} = -X(998c^2 + 972c + 99) \leq 0$ ,  $\tilde{\Delta}_{11}|_{g=\frac{1}{2}-\frac{\epsilon}{2}} = X(2443c^2 - 2766c - 1485) \leq 0$  since  $c \in [0, 1]$ , and  $\frac{\partial}{\partial g} \tilde{\Delta}_{11} = -X(14c + 9g + 6)$ , i.e., it is monotone (decreasing).

That is,  $\tilde{\Delta} = A^4 - 2A^2(B+C) + (B-C)^2 \geq 0$  and hence  $A - \sqrt{B} - \sqrt{C} = \pi_{Q,1}^* - \pi_{M,2}^* \geq 0$ , which establishes relationship #4 and completes the proof of the Proposition. Q.E.D.

**Proof of Proposition 3.** Let  $p_Q^*, Q_Q^*$  and  $p_S^*, p_S^{*+}, S$  be the optimal solutions for the quantity-based and spending-based design, respectively. Part (a) is easy to verify: note that  $p_S = p_Q^*$  and  $S = p_Q^* Q_Q^*$  are feasible solutions of the spending-based loyalty program problem. Thus, consider a constrained version of the spending-based loyalty problem with  $p_S^* = p_Q^*$  and  $S = p_Q^* Q_Q^*$ , but allow  $p^+$  to change as to maximize the firm's profit. Then,  $\pi_S^* = \max_{p, p^+, S} \pi_S(p, p^+, S) \geq \max_{p^+} \pi_S(p_Q^*, p^+, \hat{S}) \geq \max_{p, Q} \pi_Q(p, Q) = \pi_Q^*$ .

For part (b), consider a spending-based design in which only segments  $A_S^I, A_S^{IV}$  are non-empty. From the proof of Lemma 2 this design emerges whenever (i)  $S/p - g > 1$  and (ii)  $S/p^+ - g = p$ , which clearly restricts

the feasible set of the variables, making the optimal solution to this design a lower-bound on  $\pi_S^*$ . As always, the marginal customer should be indifferent between purchasing regular, non-purchasing or being Gold, which, from Lemma 2, under this design holds when (iii)  $Surplus2_i(n_{i,S}^*, n_{i,S}^{+*}) = \frac{(a_i+g)^2-2S}{2} = (a_i-p)^2/2$  at  $a_i = p$ .

Under this design,  $\pi_S = \int_p^1 (S - c(a_i + g)) da_i = \frac{1}{2}(p-1)(c(2g+p+1) - 2S)$ . Note that  $\pi_S$  is independent of  $p^+$ : this is because in segment  $A_S^{IV}$  the revenue is  $S$  and the cost is  $c(n+n^+) = c(a_i+g)$ . The value of  $p^+$  is therefore determined from (ii):  $p^+ = S/(p+g)$ . Note further, that  $\pi_S$  is increasing in  $S$  and thus the conditionally optimal  $S$  for a given  $p$  is defined by the constraint (iii):  $S = \frac{1}{2}(g+p)^2$ . Substituting this function into  $\pi_S$  and optimizing over  $p$  we obtain that  $\pi_S$  is maximized at  $p = \frac{1}{3}(2c-g+2)$  with  $\tilde{\pi}^* = \frac{1}{54}(2c-g-1)(2c^2+7c(g+1)-4(g+1)^2)$  (here ‘‘tilde’’ denotes the optimal profit under this restricted design). We finally plug-in this solution and verify that constraint (i) is satisfied. Simple algebra shows that this happens when  $g \in [0.8, 1]$ ,  $c \in [0, \frac{g+1}{2} - \frac{3\sqrt{1-g^2}}{2}]$ ; the latter expression clearly increases in  $g$  and approaches 1 as  $g \uparrow 1$ . In other words, for high  $g$  the solution presented above holds effectively for all  $c$ .

From the proof of Proposition 2,  $\pi_{Q,2}^* < \pi_{M,1}^*$  if  $c > \frac{9\sqrt{g}-2\sqrt{3}}{9\sqrt{g}}$ . However, at  $c = \frac{9\sqrt{g}-2\sqrt{3}}{9\sqrt{g}} + \epsilon$  ( $\epsilon > 0$ ), i.e., outside  $QSR2$ ,  $\Delta = \tilde{\pi}_S^* - \pi_{M,1}^* = \frac{1}{54} \left( \frac{(36g^{3/2}+9\sqrt{g}(\epsilon+5)-2\sqrt{3})(9g^{3/2}-9\sqrt{g}(2\epsilon+1)+4\sqrt{3})^2}{729g^{3/2}} - 4 \right)$ . While this expression is not positive everywhere,  $\lim_{g \uparrow 1, \epsilon \downarrow 0} \Delta = \frac{2(81-8\sqrt{3})}{6561} > 0$ , which establishes that there exists  $(c, g)$  outside  $QSR2$  region at which the presented lower-bound spending-based program attains profit that exceeds the myopic profit, i.e., the region of  $(c, g)$  over which the firm benefits from strategic behavior expands under the spending-based design. Q.E.D.

**Proof of Proposition 4.** Consider spending-based programs in strategic regions QSR1 and QSR2.

QSR1: With the optimal quantity-based solutions,  $p_Q^* = \frac{1}{3}(2c+g+1)$ ,  $Q_Q^* = \frac{2}{3}(1+g-c)$  consider a status-quo Pareto-improving spending design with  $p_S = p_Q^*$ ,  $S_S = p_S^* Q_Q^*$  and check whether there exist consumers who purchase at  $p^+ > p_S$ . From Lemma 2 such consumers can be of two kinds: (1) those who buy some items at  $p^+$  with some items at  $p$  (mix) and (2) those who purchase all their items at  $p^+$  (payup). First consider the mix segment. From the proof of Lemma 2 the interval of  $a_i$  over which mixing is potentially optimal is no wider than  $(\sqrt{2S} - g, \frac{S}{p} - g)$ . Let  $\Delta_{S,1}$  be the width of this interval. It is easy to verify that  $\Delta_{S,1} = \frac{2}{3}(-c+g+1) - \frac{2}{3}\sqrt{(-c+g+1)(2c+g+1)} < 0$  for any  $g > 0$  and  $c > 0$ . That is, a mix segment is empty and there is no consumer who purchase both  $p^+$  and  $p$ . Now, consider the latter case, the payup segment and note that w.l.o.g. we can define  $p^+ = p + \epsilon$  for some  $\epsilon > 0$ . From the proof of Lemma 2 the interval of  $a_i$  over which buying all items at  $p^+$  is potentially optimal is no wider than  $(p + \frac{S}{p+\epsilon} - \sqrt{2\frac{S}{p+\epsilon}(g-\epsilon)}, \frac{S}{p+\epsilon} - g)$ . Again, let  $\Delta_{S,1}$  be the width of this interval. Then,

$$\Delta_{S,1} = \frac{1}{3} \left( 2\sqrt{3} \sqrt{-\frac{(c-g-1)(2c+g+1)(g-\epsilon)}{2c+g+3\epsilon+1}} - 2c - 4g - 1 \right).$$

Since

$$\frac{\partial \Delta_{S,1}}{\partial \epsilon} = -\frac{(2c+4g+1)\sqrt{\frac{(-2c^2+c(g+1)+(g+1)^2)(g-\epsilon)}{2c+g+3\epsilon+1}}}{\sqrt{3}(g-\epsilon)(2c+g+3\epsilon+1)} < 0,$$

the interval is widest when  $\epsilon \downarrow 0$ , in which case  $\Delta_{S,1} \uparrow \frac{1}{3} \left( 2\sqrt{3}\sqrt{g(-c+g+1)} - (2c+4g+1) \right) < 0$ . To see why this holds, observe that the expression is in the form of  $A - B$  where  $A > 0, B > 0$ . Thus, if  $A^2 < B^2$  then

$A < B$ . Squaring the two terms and rearranging, we obtain that  $A^2 - B^2 = -4c^2 - 4c(7g + 1) - (1 - 2g)^2 < 0$  for any  $g > 0$  and  $c > 0$ . Thus, there exists no consumer who purchases all items at  $p^+$ , either, and so there exists no status-quo Pareto-improving spending-based design under *QSR1*.

*QSR2*: With the optimal quantity-based solutions,  $p_Q^* = \frac{1+c}{2}$ ,  $Q_Q^* = 2g$  consider a status-quo Pareto-improving spending design  $p_S = p_Q^*$ ,  $S_S = p_S^* Q_Q^*$ . As before, check whether there exist consumers who purchase at  $p^+ > p_S$ . Consider the mix segment and let  $\Delta_{S,2}$  be the width of the mix-segment interval. Since the lower bound  $\sqrt{2S_S} - g = \sqrt{2}\sqrt{(c+1)g} - g < p_S$ , we obtain

$$\Delta_{S,2} = -\frac{c}{2} + g - \frac{1}{2}.$$

We can further verify that  $\Delta_{S,2} > 0$  if  $g > \frac{1}{2} + \frac{c}{2}$ . That is, for any  $g$  and  $c$  in *QSR2*, as long as  $g > \frac{1}{2} + \frac{c}{2}$ , there exists some consumers who will be better-off by purchasing some items at  $p^+$ . At such a solution,  $\pi_S > \pi_Q^*$  since these buy-up consumers become mix consumers and contribute the same revenue, but cost less. Therefore, in the part of *QSR2* with  $g > \frac{1}{2} + \frac{c}{2}$  there always exists a status-quo Pareto-improving spending design. Q.E.D.

**Proof of Proposition 5.** Part (a) contains three inequalities: (i)  $\pi_Q^* \leq \pi_{QaS}^*$ , (ii)  $\pi_S^* \leq \pi_{QaS}^*$  and (iii)  $\pi_{QaS}^* \leq \pi_S^*$ ; the latter two imply the equality stated in part (a). To see why (i) holds, note that the optimal quantity-based solution  $(p_Q^*, Q_Q^*)$  is a feasible solution to the *QaS* problem with  $\hat{p}^+ = p_Q^*$  and  $\hat{S} = p_Q^* Q_Q^*$ . Thus,  $\pi_Q^* = \max_{p,Q} \pi_Q(p, Q) = \max_{p,Q} \pi_{QaS}(p, \hat{p}^+, Q, \hat{S}) \leq \max_{p,p^+,Q,S} \pi_{QaS}(p, p^+, Q, S) = \pi_{QaS}^*$ . (ii) holds by the same argument as above since the optimal spending-based solution  $(p_S^*, p_S^{+*}, S_S^*)$  coupled with  $Q = 0$  is a feasible solution for the *QaS* problem.

To prove (iii), suppose that there exists a *QaS* design with  $\pi_{QaS}^* > \pi_S^*$ . Then consider a variation of the *QaS* design as follows: remove the quantity requirement and fix  $p = p_{QaS}^*$ ,  $p^+ = p_{QaS}^{+*}$ ,  $S = S_{QaS}^*$ . Refer to this design as  $\tilde{S}$ -design and note that: (i)  $\pi_{\tilde{S}} \leq \pi_S^*$  by the same argument above, and (ii) by construction, the optimal *QaS* design can be obtained from the  $\tilde{S}$ -design by adding a quantity-based constraint. In other words, that  $\pi_{QaS}^* > \pi_S^* \geq \pi_{\tilde{S}}$  implies that there exists an optimal value of  $Q$  that can be added to the  $\tilde{S}$ -design to increase profit.

1.  $Q \geq S/p$  cannot be optimal because the program would collapse to a  $Q$ -design, which would imply a contradiction by Proposition 3;
2.  $Q \leq S/p^+$  cannot be optimal either because the program would collapse to the  $\tilde{S}$ -design, which cannot be better than *QaS* by construction.

Therefore, the potentially optimal  $Q \in (\frac{S}{p^+}, \frac{S}{p})$ . On this interval, it is easy to verify that the quantity and the spending constraints are both binding at  $n^{crit} = \frac{Qp^+ - S}{p^+ - p}$ ,  $n^{+crit} = \frac{S - Qp}{p^+ - p}$  and for  $n > n^{crit}$  the quantity constraint is not binding, and otherwise the spending constraint is not binding.

From Lemma 2, under the  $\tilde{S}$ -design consumers make optimal decisions  $n_S^* = \max(0, \frac{(a_i + g)p^+ - S}{p^+ - p})$  and  $n_S^{+*} = \max(0, \frac{S - (a_i + g)p}{p^+ - p})$ . Because  $n_S^* > n^{crit}$  is equivalent to  $a_i > Q - g$ , adding  $Q \in (\frac{S}{p^+}, \frac{S}{p})$  to the  $\tilde{S}$ -design impacts the consumer behavior under the *QaS* design as follows:

- Consumers with  $a_i > Q - g$ , which include all consumers in segments  $A_S^V$ ,  $A_S^{VI}$  as well as the “upper- $a_i$ ” portion of  $A_S^{IV}$ , are not affected. The firms’ profit from these consumers under *QaS* is therefore the same as under  $\tilde{S}$ .

- Consumers with  $a_i \leq Q - g$ , which include consumers in the “lower- $a_i$ ” portion of  $A_S^{IV}$  and all consumers in  $A_S^{III}$  must decide on the optimal purchasing decisions  $n^*, n^{+*}$  and compare the resultant Gold surplus with the surplus of purchasing as Regular. When the quantity constraint is binding, i.e.,  $n^+ = Q - n$ , the gradient of the consumer surplus in  $n$  (see Lemma 2) equals:

$$\frac{\partial}{\partial n} \Big|_{n^+=Q-n} \left( (a_i + g)(n + n^+) - \frac{(n + n^+)^2}{2} - (np + n^+p^+) \right) = p^+ - p > 0, \forall a_i.$$

Therefore,  $n^* = n^{crit}$  and  $n^{+*} = n^{+crit}$ , meaning that strategic consumers with  $a_i \leq Q - g$  spend  $S$ , but purchase  $Q \geq a_i + g = n_s^* + n_s^{+*}$  leading to higher costs. The firms’ profit from these consumers under  $QaS$  is therefore no larger than under  $\tilde{S}$ . Also, some consumers with  $a_i \leq Q - g$  will find it more beneficial to become Regular if the surplus of purchasing as Regular is larger than the resultant Gold surplus at  $(n^{crit}, n^{+crit})$ . This, however, cannot be profitable for the firm, either; as per the discussion in Section 5 (see Figure 4), those consumers are the most profitable of all strategic consumers and hence if depriving them of status can be profitable, then so would be depriving other strategic Golds of their status. To see why, consider a strategic consumer  $X$  with  $a_X \in A_S^{III}$ . Under  $\tilde{S}$ -design the firm’s profit from  $X$  is  $\frac{S}{p^+}(p^+ - c)$ , and if  $X$  loses status, then the profit will become  $(a_X - p)p$ . Let  $\Delta_X = \frac{S}{p^+}(p^+ - c) - (a_X - p)p$  and note that it represents the incremental profit from  $X$  being strategic Gold (instead of being Regular). Now consider another consumer,  $Y$ , and assume  $a_Y \in A_S^V$ . Similarly define  $\Delta_Y = \frac{S}{p}(p - c) - (a_Y - p)p$ . Because  $p^+ \geq p$  and  $a_X < a_Y$ , clearly  $\Delta_X > \Delta_Y$ . Similarly, we can obtain  $\Delta_X > \Delta_Z$  for a consumer  $Z$  with  $a_Z \in A_S^{IV}$ . Combining these results, if depriving  $X$  of status can be profitable (meaning  $\Delta_X < 0$ ), then the firm should also deprive all other strategic Golds under  $\tilde{S}$ -design of their status. This requires setting  $Q > S/p$ , which is a contradiction.

Summarizing, there does not exist a value of  $Q$  that can be profitably added to the  $\tilde{S}$ -design. However, by construction the optimal  $QaS$  design, with the profit higher than  $\pi_{\tilde{S}}$ , is obtained by adding some  $Q$  to  $\tilde{S}$ -design. We therefore obtain a contradiction, and part (a) of the Proposition follows.

For part (b), a status-quo Pareto-improvement means that the firm operating with the optimal quantity-based prices and qualifications in strategic regions of QSR1 and QSR2 (i.e., with  $p_{QaS} = p_Q^*$ ,  $Q_{QaS} = Q_Q^*$ , and  $S_{QaS} = p_Q^*Q_Q^*$ ) can manage  $p^+$  to improve its profit. Because consumers must spend  $S_{QaS}$  to meet the qualification constraint, this can only be done through cost reduction from some consumers who purchase items at  $p^+$ . Per Lemma 2 this can be possibly better for consumers in  $A_S^{II}$  who can improve surplus by purchasing  $S_{QaS}/p^+ < Q_Q^*$ . However, under a QaS-design, consumers also have to purchase at least  $Q_Q^*$  items to qualify for status, and hence there is no incentive for them to purchase at  $p^+$ . Thus, when the firm switches from a quantity-based design to a QaS-design, a status-quo Pareto improvement is not possible. Q.E.D.

**Proof of Proposition 6.** For part(a) the proof generally follow the same structure as the proof of Proposition 5. The inequalities  $\pi_{QoS}^* \geq \pi_Q^*$  and  $\pi_{QoS}^* \geq \pi_S^*$  follow because quadruples of  $(p, p^+, Q, S)$  defined by  $(p_Q^*, p_Q^*, Q_Q^*, \infty)$  and  $(p_S^*, p_S^{+*}, \infty, S_S^*)$ , i.e., the optimal  $Q$ - and  $S$ -design solutions, are feasible solutions to the  $QoS$ -design. To establish  $\pi_{QoS}^* \leq \pi_S^*$  construct the  $\tilde{S}$ -design instance as in the proof of Proposition 5 and observe that:

1.  $Q \geq S/p$  cannot be optimal because all consumers would be better-off by satisfying the spending requirement and not quantity, and the program would therefore collapse to an  $S$ -design, which would imply a contradiction to  $\pi_{QoS}^* > \pi_{\tilde{S}}^*$ ;
2.  $Q \leq S/p^+$  cannot be optimal because all consumers would be better-off by satisfying the quantity requirement, and the program would therefore collapse to a  $Q$ -design, which would imply a contradiction by Proposition 3.

Therefore, the potentially optimal  $Q \in (\frac{S}{p^+}, \frac{S}{p})$ . On this interval, recall that there exist  $(n^{crit}, n^{+crit})$  at which both quantity and spending constraints are binding. Ceteris paribus, strategic consumers would prefer to obtain status by purchasing and spending less; therefore, the spending constraint is binding for  $n < n^{crit}$  and the quantity constraint is binding. Adding  $Q \in (\frac{S}{p^+}, \frac{S}{p})$  to the  $\tilde{S}$ -design impacts the consumer behavior under the  $QoS$  design as follows:

- Consumers with  $a_i < Q - g$ , which include all consumers in segments  $A_S^{II}, A_S^{III}$  as well as the “lower- $a_i$ ” portion of  $A_S^{IV}$ , are not affected. The firms’ profit from these consumers under  $QoS$  is therefore the same as under  $\tilde{S}$ .
- Consumers with  $a_i \geq Q - g$ , which include all consumers in  $A_S^V$ , and the “upper- $a_i$ ” portion of  $A_S^{IV}$ , must decide on the optimal purchasing decisions  $n^*, n^{+*}$ . For them, however, the problem is that of the  $Q$ -design described in Lemma 1:  $n^{+*} = 0$  and  $n^* = Q$  for those with  $a_i < p + Q - g$ , and otherwise  $n^* = a_i + g - p < S/p$  (the latter inequality holds because otherwise those consumers would be in segment  $A_S^{VI}$  and not in segments  $A_S^{IV}, A_S^V$ ). The firm’s profit from the affected consumers would be smaller than under the  $\tilde{S}$  because: (i) those who were in  $A_S^V$  would contribute  $Q(p - c)$  instead of  $S/p(p - c)$  where  $Q < S/p$ , and (ii) those who were in  $A_S^{IV}$  were more profitable than those in  $A_S^V$  as they were contributing the same revenue ( $S$ ), but were purchasing fewer units hence had a smaller cost.

Summarizing, there does not exist a  $Q$  that can be profitably added to the  $\tilde{S}$ -design. However, by construction the optimal  $QoS$  design, with the profit higher than  $\pi_{\tilde{S}}$ , is obtained by adding some  $Q$  to  $\tilde{S}$ -design. We therefore obtain a contradiction, and part (a) of the Proposition follows.

For part (b), consider a constrained version of the  $QoS$ -design under strategic regions of QSR1 and QSR2 with  $p_{QoS} = p_Q^*$  and  $S_{QoS} = p_Q^* Q_Q^*$ . Under this setting, the revenues remain the same, and thus the firm can improve the profit through cost reduction from consumers who purchase fewer items but at price  $p^+$  (by self-selecting to satisfy the spending threshold rather than quantity). From the proof of Proposition 4, there exists consumers who purchases items at  $p^+$  in QSR2, but not in QSR1. Therefore, when the firm switches from a quantity-based design to a  $QoS$ -design, the status-quo Pareto-improving is always possible in the part of QSR2 with  $g > 1/2 + c/2$  per Proposition 4. Q.E.D.

**Proof of Lemma 3.** From Table 1, for s\*rQ, we first derive  $n_{i,R}^* = a_i - p + \nu x$  and for s\*rS,  $n_{i,R}^* = a_i - p + \nu y$  (if  $\nu y \leq 1$ ) or  $n_{i,R}^* = a_i - p^+ + p^+ \nu y$  (if  $\nu y > 1$ ). Since the case of  $\nu y > 1$  where the value of points awarded for \$1 purchase exceeds \$1 is not reasonable (and certainly not realistic), we focus on the case where  $\nu y \leq 1$ . For both designs, we have  $n_{i,G}^* = n_{i,R}^* + g$ .

■ sQrQ: A consumer of type  $i$  will “buy-up” to Gold if

$$(a_i + g)Q - \frac{Q^2}{2} - pQ + \nu xQ \geq a_i n_{i,R}^* - \frac{(n_{i,R}^*)^2}{2} - p n_{i,R}^* + \nu x n_{i,R}^* = \frac{(a - p + \nu x)^2}{2},$$

which is equivalent to saying that consumers with  $a_i \in [Q - \sqrt{2Qg} + p - \nu x, Q + \sqrt{2Qg} + p - \nu x]$ . However, consumers with  $n_{i,Q}^* = a_i + g - p + \nu x \geq Q$ , i.e., those with  $a_i \geq Q + p - g - \nu x$ , are already qualifying for Gold without the need to buy-up. That is, consumers with  $a_i \in [Q - \sqrt{2Qg} + p - \nu x, Q + p - g - \nu x]$  are buying up to Gold status, and thus there exist up to four consumer segments: no-purchase, purchase as needed (regular), buy-up to Gold, and auto-Gold.

■ sQrS: A consumer of type  $i$  will “buy-up” to Gold if

$$(a_i + g)Q - \frac{Q^2}{2} - pQ + \nu ypQ \geq a_i n_{i,R}^* - \frac{(n_{i,R}^*)^2}{2} - pn_{i,R}^* + \nu ypn_{i,R}^* = \frac{(a - p + p\nu y)^2}{2}.$$

As before, consumers with  $a_i \geq Q + p - g - \nu yp$  qualify for Gold automatically. Thus, consumers with  $a_i \in [Q - \sqrt{2Qg} + p - \nu yp, Q + p - g - \nu yp]$  are buying up to Gold status, and there exist up to four consumer segments as in sQrQ.

■ sSrQ: A consumer who purchases as needed at price  $p$  will qualify for Gold if  $p(a_i + g - p + \nu x) \geq S$ , i.e., if  $a_i \geq \frac{S}{p} + p - g - \nu x$ . Otherwise, a consumer will consider spending-up to Gold and decides on the number of “regular” purchases,  $n$ , and the number of “plus” purchases,  $n^+$  that would maximize:

$$(a_i + g)(n + n^+) - \frac{(n + n^+)^2}{2} - (pn + p^+n^+) + \nu x(n + n^+), \text{ s.t. } pn + p^+n^+ \geq S.$$

Let  $n_{i,S}^* = \frac{-S + (a_i + g + \nu x)p^+}{(p^+ - p)}$  and  $n_{i,S}^{+*} = \frac{S - (a_i + g + \nu x)p}{(p^+ - p)}$ . Note that  $n^*$  is non-negative when  $a_i \geq S/p^+ - g - \nu x$  and similarly  $n^{+*}$  is non-negative when  $a_i \leq S/p - g - \nu x$ . Thus,

- If  $a_i \leq S/p^+ - g - \nu x$ , then the solution to this consumer problem is  $(0, S/p^+)$ . Let the resultant surplus of spending up to Gold for a customer of type  $i$  be *Surplus1<sub>i</sub>*.
- If  $a_i \in (S/p^+ - g - \nu x, S/p - g - \nu x)$ , then the solution to this consumer problem is  $(n_{i,S}^*, n_{i,S}^{+*})$ . Let the resultant surplus of spending up to Gold for a customer of type  $i$  be *Surplus2<sub>i</sub>*.
- If  $a_i \geq S/p - g - \nu x$  then the solution to this consumer problem is  $(S/p, 0)$ . Let the resultant surplus of spending up to Gold for a customer of type  $i$  be *Surplus3<sub>i</sub>*.

Comparing these surpluses to the surplus of purchasing as needed (regular),  $(a_i - p - \nu x)^2/2$ , we obtain that:

- *Surplus1<sub>i</sub>*  $\geq (a_i - p - \nu x)^2/2$  if  $a_i \in \left[ p + \frac{S}{p^+} - \sqrt{\frac{2S}{p^+}(g + p - p^+) - \nu x}, p + \frac{S}{p^+} + \sqrt{\frac{2S}{p^+}(g + p - p^+) - \nu x} \right]$ ,
- *Surplus2<sub>i</sub>*  $\geq (a_i - p - \nu x)^2/2$  if  $a_i \geq \frac{p-g}{2} + \frac{S}{p+g} - \nu x$ , and
- *Surplus3<sub>i</sub>*  $\geq (a_i - p - \nu x)^2/2$  if  $a_i \in \left[ p + \frac{S}{p} - \sqrt{\frac{2S}{p}g - \nu x}, p + \frac{S}{p} + \sqrt{\frac{2S}{p}g - \nu x} \right]$ .

Thus, there exist up to six consumer segments: no-purchase, purchase as needed (regular), pay-up, mix, and buy-up to Gold, and auto-Gold.

■ sSrS: Similar to sSrQ case, a consumer with  $a_i \geq \frac{S}{p} + p - g - p\nu y$  qualifies for Gold automatically. Otherwise, a consumer will consider spending-up to Gold and decides  $n$  and  $n^+$  that would maximize:

$$(a_i + g)(n + n^+) - \frac{(n + n^+)^2}{2} - (pn + p^+n^+) + \nu y(pn + p^+n^+), \text{ s.t. } pn + p^+n^+ \geq S.$$

Let  $n_{i,S}^* = \frac{-S + (a_i + g)p^+}{(p^+ - p)}$  and  $n_{i,S}^{+*} = \frac{S - (a_i + g)p}{(p^+ - p)}$ . Note that  $n^*$  is non-negative when  $a_i \geq S/p^+ - g$  and similarly  $n^{+*}$  is non-negative when  $a_i \leq S/p - g$ . Thus,

- If  $a_i \leq S/p^+ - g$ , then the solution to this consumer problem is  $(0, S/p^+)$ . Let the resultant surplus of spending up to Gold for a customer of type  $i$  be *Surplus1<sub>i</sub>*.

- If  $a_i \in (S/p^+ - g, S/p - g)$ , then the solution to this consumer problem is  $(n_{i,S}^*, n_{i,S}^{+*})$ . Let the resultant surplus of spending up to Gold for a customer of type  $i$  be  $Surplus2_i$ .
- If  $a_i \geq S/p - g$  then the solution to this consumer problem is  $(S/p, 0)$ . Let the resultant surplus of spending up to Gold for a customer of type  $i$  be  $Surplus3_i$ .

Comparing these surpluses to the surplus of purchasing as needed (regular),  $(a_i - p - \nu yp)^2/2$ , we obtain that:

- $Surplus1_i \geq (a_i - p - \nu yp)^2/2$  if  $a_i \in \left[ p + \frac{S}{p^+} - \sqrt{\frac{2S}{p^+}(g + p - p^+ - \nu y(p + p^+))} - \nu yp, p + \frac{S}{p^+} + \sqrt{\frac{2S}{p^+}(g + p - p^+ - \nu y(p + p^+))} - \nu yp \right]$ ,
- $Surplus2_i \geq (a_i - p - \nu yp)^2/2$  if  $a_i \geq \frac{(y-1)(p^2(y-1)-2S)-g^2}{2(g-py+p)}$ , and
- $Surplus3_i \geq (a_i - p - \nu yp)^2/2$  if  $a_i \in \left[ p + \frac{S}{p} - \sqrt{\frac{2S}{p}g} - \nu yp, p + \frac{S}{p} + \sqrt{\frac{2S}{p}g} - \nu yp \right]$ .

Thus, there exist up to six consumer segments: no-purchase, purchase as needed (regular), pay-up, mix, and buy-up to Gold, and auto-Gold. Q.E.D.