Expanding the donor pool: Incentivizing the use of marginal organs for transplantation

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Despite efforts to increase the supply of donated organs for transplantation, organ shortage keeps increasing as the demand outpaces supply. We study the use of marginal organs for transplantation in a queueing-theoretic framework. We establish that self-interested individuals who optimize their own well-being while competing with other candidates set their utilization levels more conservatively in equilibrium than the socially efficient level. To help reduce the resulting gap, we offer policy incentives through candidates returning to the waiting list for re-transplantation. We show that a lenient policy that compensates returning candidates, who have accepted marginal organs, for giving up their position on the waiting list increases the equilibrium utilization of organs while also improving social welfare. We also show that the degree of improvement increases monotonically with the level of compensation provided by the policy.

In practice, such a policy can be implemented by preserving some fraction of the waiting time previously accumulated by the returning candidates. Detailed numerical study for the U.S. renal transplant system suggests that introduction of such a policy helps significantly reduce kidney discard rate (baseline: 17.4%). Depending on the strength of response given by the population to the policy, discard rate can be as low as 5.4% (strong response), 9.5% (moderate response), or 15.7% (weak response), which translates to 1746, 1148, or 241 more transplants per year, respectively. Moreover, KDPI of transplanted kidneys increases by 0.8-3.7 depending on the strength of response, but the resulting graft survival 1-year post-transplant remains stable around 94.7%-94.9% versus 95.0% for baseline.

1. Introduction

Transplantation is the preferred mode of treatment for thousands of patients with organ failure. In the United States, the number of organ transplants since the year 2000 has exceeded half a million, increasing at an annual rate of 3% (OPTN 2018a). In this study, we focus on deceased donor organs, which compose 82% of all organs harvested for transplantation (OPTN 2018a).[1] Allocation of deceased donor organs in the U.S. are managed through nationwide waiting lists. Every 10 minutes a new candidate is added to the transplant waiting lists, while demand-supply imbalance results in 15 patient deaths every day while waiting for a transplant. There are close to 115,000 candidates currently waiting for an organ in the U.S. transplant waiting lists.

Despite the growing need for donor organs, those harvested for transplantation are frequently declined by patients/physicians and discarded in large volumes. In 2016, more than 14% of all organs recovered for transplantation are discarded, with highest rates for kidney (20%) and pancreas (24%). Reasons for high discard rates include inefficiencies in the organ allocation systems that expend the window of viability of organs before finding willing recipients (Massie et al. 2009) and behavioral attitudes of patients/physicians (e.g., concerns over using relatively higher risk organs) (Schold et al. 2009). In 2016, almost 40% of discarded kidneys were due to not locating any recipient after exhausting the entire waiting list (Israni et al. 2018).

Increasing pressures towards meeting the demand of the growing waiting lists recently promoted the idea of using, instead of discarding, organs that are less than optimal for transplantation, also known as *marginal organs*. Although there is no consensus on the definition, the lower part of the organ quality spectrum is regarded as marginal organs. In particular, an organ is generally considered marginal if there is a risk of initial poor function, primary non-function, or if it carries any factor (e.g., elderly donor or a donor with known diseases) that may cause late graft failure. Marginal organs, albeit having worse outcomes than standard organs,[2] are argued as viable alternatives for patients dying while waiting for a transplant (Busuttil and Tanaka 2003). An increasing number of retrospective observational studies in the recent medical literature document the benefits of using marginals organs, signaling a room for improvement in utilizing these organs. Massie et al. (2014) report that older patients and patients in centers with high median time to transplant benefit the most from accepting a marginal kidney, instead of exposing themselves to the risk of death while waiting for a better kidney. Ojo et al. (2001) further report that recipients of marginal kidneys have a substantial reduction in mortality and improvement in life expectancy compared

[1] While living donors offer an important additional source, transplants in the U.S. are predominantly due to organs recovered from deceased donors; living donors compose 31.8% of all transplanted kidneys, 4.1% of transplanted livers, and they are practically nonexistent for all other solid organ transplants (Israni et al. 2018, OPTN 2018a).

[2] For instance, average kidney graft half-life is 8.8 years for standard deceased-donor transplants, while 6.4 years for higher risk transplants (Lamb et al. 2011).

to dialysis patients in the waiting list. These, however, are small, single-center reports and do not provide a full picture of the effect of utilizing marginal organs on the overall welfare.

A direct approach to increase the utilization of available organs is to penalize or prevent rejection of offered organs. However, such penalties are prohibited by the U.S. laws that establish that the final decision to accept/reject an offered organ is the prerogative of the patient and/or the transplant surgeon/physician responsible for the care of the patient. Another approach would be to restrict marginal organ offers to those candidates, who declare their willingness to accept such offers. Some form of this approach is readily implemented in the U.S. by allowing wait-listed candidates to declare certain criteria before organs are offered to them (e.g., they can declare maximum acceptable donor age and cold ischemic time as well as their willingness to accept non-heart-beating donors, or donors with Hepatitis B, C, a history of diabetes or hypertension). Although the effectiveness of this program has not been formally evaluated, the current state of the U.S. system indicates a need for additional approaches to help reduce discarded organs.

Our goal in this paper is to investigate the effect of implementable nudge mechanisms, such as those that promote (but not enforce) acceptance of certain types of organ offers, within the confines of the current system, instead of mechanisms that redesign the system. Our main contribution emanates from the following simple observation. When accepting an organ offer, a patient loses her position in the waiting list, foregoing the opportunity of transplanting a higher quality organ in the future. Can an incentive mechanism that compensates the potential losses of patients accepting organs of certain types induce the desired behavior of higher utilization of available organs? Assuming the desired behavior is achieved, would it also result in increased social welfare?

We settle both questions affirmatively through identifying such a mechanism that exploits a rather unexplored component of the transplant system: return events. Most transplant recipients live long, healthy lives after transplantation and do not need a repeat transplant during the rest of their lifetime. However, some recipients outlive the life of the grafts they have received and, upon experiencing graft failure, they return back to the waiting list for a repeat transplant opportunity. OPTN data indicates that this group amounts to more than 11% overall, with highest rates for kidney (12.7%) and pancreas (17.5%), for which the discard rates are also highest (OPTN 2018a). Transplants using marginal organs are typically associated with higher rates of graft failure (Lamb et al. 2011) and concomitantly with higher rates of returning candidates to the waiting list. Therefore, increasing the use of marginal organs, on the one hand, intensifies the competition for the limited set of organs, while at the same time it softens the competition by increasing the pool of available organs. The net effect to the equilibrium behavior of patients is less than clear.

Recognizing that the fear of being in need of a repeat transplant may drive more conservative patient behavior towards marginal organs, we introduce an incentive mechanism that compensates the returning recipients of marginal organs for exposing themselves to higher risks of graft failure. This mechanism simply classifies candidates as eligible or ineligible based on their willingness to accept a set of organs, defined a priori, and provides probabilistic priority to the eligible returning candidates. While this mechanism is rich theoretically, its implementation in practice, for example, can correspond to preserving some fraction of the previously accumulated waiting times for eligible returning candidates, hence it provides the opportunity to reclaim the previous (or even better) waiting list positions for such recipients.

We theoretically analyze this mechanism via developing a multiclass queueing model with reneging and delayed feedback. Our model allows heterogeneity in the types of candidates in need of an organ transplant and in the quality of organs. We use quality-adjusted life expectancy (QALE) to measure candidates' utility from transplantation and define the social welfare function as the difference between the total QALE obtained from transplantation and the social costs associated with transplantation (caused by the heterogeneity in organ qualities). We identify the socially efficient utilization of organs that maximize social welfare and the equilibrium behavior of self-interested individuals who optimize their own well-being while competing with other candidates. Our results suggest that candidates set their utilization level more conservatively in equilibrium than the socially efficient level. This finding offers an explanation to the low utilization of marginal organs observed in current practice. We next establish that the equilibrium outcome of the proposed mechanism strictly increases the utilization of organs, helping reduce the observed gap, as well as the social welfare. We also show that the degree of improvement increases monotonically with the degree of incentive (i.e., level of compensation) offered by the mechanism.

We complement our theoretical analysis with detailed simulation results for the U.S. renal transplant system. Our simulation results suggest that introduction of the proposed mechanism helps significantly reduce kidney discard rate (baseline: 17.4%). Depending on the strength of response given by the population to the policy, discard rate can be as low as 5.4% (strong response), 9.5% (moderate response), or 15.7% (weak response), which translates to 1746, 1148, or 241 more transplants per year, respectively. Moreover, KDPI of transplanted kidneys deteriorates by 0.8-3.7 depending on the strength of response, but the resulting graft survival 1-year post-transplant remains stable around 94.7%-94.9% versus 95.0% for baseline.

Rest of the paper is organized as follows. Current U.S. organ allocation rules and relevant literature on organ transplantation is briefly summarized in Section 2. Section 3 presents the details of our baseline model formulation and its analysis, establishing the gap between socially efficient and equilibrium utilization of organs. Section 4 presents a revised model that incorporates returning candidates to the baseline model and derives structural results for the revised model. In particular, it corrects the equilibrium characterization obtained in Section 3 in the absence of returning candidates, and establishes that equilibrium is, in fact, less conservative but a positive gap still exists. Section 5 introduces the policy incentive and its analysis concluding with the proposed incentive as a possible remedy that helps achieve the desired effects. Our simulation results in Section 6 demonstrate the magnitude of improvements and we conclude in Section 7. Proofs of all theoretical results are provided in Appendix A.

2. Background

2.1. An Overview of the U.S. Transplantation System

With the enactment of National Organ Transplant Act (NOTA) in 1984, US Congress established the Organ Procurement and Transplantation Network (OPTN) to regulate the management and allocation of donated organs (NOTA 1984). Later in 1998, the U.S. Department of Health and Human Services (DHHS) published the "Final Rule" as a regulatory binding document to "achieve equitable allocation of organs among patients" (DHHS 1998). The Final Rule established the fundamental principles of an allocation policy including: placing emphasis on medical urgency, seeking to achieve effective use of organs, preventing organ wastage while preserving patients' ability of declining an offer, and reducing disparities in waiting times (Gibbons et al. 2000). In accordance with the Final Rule, separate allocation policies are developed for each organ, which have been reviewed periodically and revised as appropriate to this date (See Stegall et al. (2017), Schilsky and Moini (2016), Meyer et al. (2015), Pullen (2018) for brief history of allocation policies).

Each allocation policy employs a sophisticated algorithm to determine the offer sequence among the active candidates in the waiting list for each harvested organ. We provide an overview of the current allocation rules for adult candidates in the US for the three largest transplant volume organ types and refer interested readers to the OPTN policy document (OPTN 2018b) for other details. Geography plays an important role in all allocation rules. The US is divided into 11 mutually exclusive collectively exhaustive geographic regions, which are further subdivided into 58 local donation service areas (DSA). Each DSA is served by one organ procurement organization (OPO) that coordinates the harvesting and matching of donated organs to candidates, and by one or more transplant hospitals, donor hospitals, and histocompatibility laboratories.

The current kidney allocation system (KAS) classifies adult candidates through Estimated Post Transplant Survival (EPTS) scoring based on their (*i*) time on dialysis, (*ii*) diagnosis of diabetes, (*iii*) prior organ transplant, and (*iv*) age to identify candidates with longer expected post-transplant survival time. KAS also classifies deceased donors kidneys through Kidney Donor Profile Index (KDPI) that represents the cumulative risk of graft failure compared to a reference population based on several donor characteristics including age, ethnicity, creatinine level, history of hypertension, diabetes and Hepatitis C (medical complications), cause of death, height and weight. Accordingly, KAS categorizes kidneys based on KDPI scores, and assigns allocation rules for each category, where it prioritizes candidates with top %20 EPTS when offering an organ with %20 KDPI. Within each classification, candidates are sorted first according to kidney allocation points, which is calculated based on waiting time, whether the candidate is a prior living organ donor, candidate's likelihood of being incompatible with an average donor (estimated through Calculated Panel Reactive Antibody (CPRA) score) or the intended donor, and whether the candidate is under 18 years of age, and then according to their date of registration. Conforming to these priorities, kidneys from deceased donors are matched to candidates with permissible blood types searching through the DSA and region of the harvested kidney as well as through the nation.

The current liver allocation system classifies and prioritizes adult candidates based on their geography and medical urgency. For each harvested liver, candidates are geographically re-labeled as local, regional, or national depending on the relative geographies of the candidate and the harvested organ. Candidates with a life expectancy of less than 7 days without a transplant are labeled as Status 1, while all other candidates are labeled with one of 35 MELD (Model for End-Stage Liver Disease) categories, which reflects the candidate's probability of death within the next 3 months. Within each pair of (geography, medical urgency) classification, candidates are prioritized according to their blood type compatibility and waiting time.

The current heart allocation system classifies and prioritizes adult candidates through (i) a heart status, namely Status 1A, 1B and 2, that reflects a candidate's medical urgency for transplant, (ii) a geographical label, based on whether their transplant hospital is in the same DSA as the hospital harvesting the heart, and if not, then the distance between the two hospitals, (iii) blood type matching using a primary or secondary match. Within each classification, candidates are sorted according to their respective waiting times.

2.2. Related Literature

This paper contributes to the growing operations management literature on organ transplantation. One stream in the literature focuses on individual's accept/reject decisions while waiting for a transplant as an optimal stopping time problem (David and Yechiali 1985, Ahn and Hornberger 1996, Howard 2002, Alagoz et al. 2007, Sandıkçı et al. 2013). Another stream studies the design of optimal organ allocation policies considering the trade-off between efficiency and equity (David and Yechiali 1995, Zenios et al. 2000, Su and Zenios 2005, 2006, Akan et al. 2012, Bertsimas et al. 2013). Several papers also develop simulation models to analyze the performance of alternative allocation policies (Zenios et al. 1999, Shechter et al. 2005, Hasankhani and Khademi 2017). Our paper consolidates these three streams, and focuses on self-interested patient's accept/reject decisions concomitantly considering the effect of these decisions on the overall welfare of the society, proposes policy incentives to direct decentralized behavior of individual candidates to socially desirable levels, and illustrates the impact of the proposed policy using a clinically detailed simulation model.

Our work is relevant to several studies examining the queueing models for organ allocation. Su and Zenios (2006) utilize mechanism design to propose efficient incentive compatible kidney allocation policies, where patients declare their own type information to choose the queue they would like to join. They identify the optimal assignment of kidneys to patients in terms of maximizing the efficiency or the equity of the allocation. Our work focuses on accept/reject decisions of patients, and aim to promote better utilization of available organs under the existing allocation policy. Dai et al. (2017) uses a queueing theoretic approach to study whether incentivizing donation through promising priority to individuals who sign up as donors if they ever need an organ transplant in the future can increase the pool of donors. They adapt an approach provided by Kessler and Roth (2012, 2014) to define the social cost of being a donor, and analyze the impact of donor prioritization on organ donation decisions in equilibrium. Our work studies the problem of expanding the pool of organs from a different perspective; we assume that donor organs are scarce and limited, and focus on the utilization of available organs.

Alleviating the growing concern for organ wastage, due to high discard rates of "marginal" organs, is at the core of this paper. Su and Zenios (2004) study the organ wastage problem by conducting welfare analysis for different queueing disciplines while allowing individual patients to defer organ offers. They exploit an idea proposed in Hassin et al. (1985), and conclude that, when allocating organs, switching from a first-come-first-serve queue to a last-come-first-serve queue, although being practically infeasible, increases organ utilization and social welfare. They assume that patients are homogeneous, and welfare does not include any social costs. We model heterogeneous set of patients, who faces discrepancies in accessing to organs, and consider social cost of transplantation. Bandi et al. (2018) study the problem of estimating the waiting time of a customer with incomplete system information under a multiclass queueing system using a mixed-integer programming formulation. They illustrate an application of their model in the kidney allocation system with the motivation of inducing better informed offer acceptance decisions through reliable waiting time estimates. We assume that patients possess accurate estimations about the system dynamics, including waiting times, and concentrate on accept/reject decisions of candidates. Finally, Agarwal et al. (2017) perform an empirical analysis to study the organ wastage problem by predicting the accept/reject decisions from real data, and comparing the outcome metrics of a set of alternative mechanisms. Our work distinguishes itself from the literature by incorporating candidates returning to the waiting list for a repeat transplant opportunity and offering a viable incentive mechanism through these candidates to mitigate the burden of organ wastages.

3. Model Formulation

We formulate a multiclass queueing model with reneging to study the organ transplant waiting list. Consider K different types of candidates in need of an organ transplant. Candidate types can be formed, for example, around age groups, geographies (e.g., those candidates that join the waiting list from the same DSA, the same region, or the same zone), medical conditions (e.g., MELD scores in liver, listing status in heart, time on dialysis in kidney), or comorbidities (e.g., those candidates that have a history of diabetes or hypertension). Each candidate type arrives to the waiting list according to a Poisson process independent of everything else. The total arrival rate of candidates to the waiting list is given by $\lambda = \sum_{k=1}^{K} \lambda_k$, where λ_k denotes the arrival rate of type-k candidates.

Candidates depart from the waiting list either when they die while waiting for an organ or when they receive a transplant, whichever happens first. The time until death for type-k candidates is assumed to follow an exponential distribution with mean $1/d_k$. The transplant event depends on the availability of organs as well as the accept/reject decisions of candidates on the waiting list. Organs arrive to the system according to an independent Poisson process with a rate normalized to $\mu = 1$. We consider the heterogeneity among the quality of organs for an accurate representation of the diversity of post-transplant benefits. Accordingly, we define a quality score q with $q \in [\underline{q}, \overline{q}]$ corresponding to each organ. Although the definition of the term "marginal donor organ" still requires clarification, it usually refers to suboptimal quality organs whose survival benefits are inferior compared with those of standard organs. We categorize organs into two groups based on their quality scores using a threshold $q^m \in [\underline{q}, \overline{q}]$. Organs with quality score $q \geq q^m$ are called standard donor organs, and those with quality score $q < q^m$ are called marginal organs. Organ quality scores are iid uniform on $[\underline{q}, \overline{q}]$. A general cumulative distribution function can also be used; however, the clinical data shows evidence for uniformity over donated organ quality. As an example, Figure 1 provides the distribution of donor organs recovered between 2012 and 2017 in terms of KDPI scores and shows that donor KDPI score is almost uniform over [0,100].

Current organ allocation policies result in discrepancies in patients' access to organs, which is best observed in transplant statistics stratified by patient type and/or organ quality. For example, the median waiting time until transplantation varies greatly by medical status of the candidates (e.g., 87 days for Status 1A recipients versus 726 days for Status 2 recipients in heart transplantation (Colvin et al. 2018)), by their blood type (e.g., 127 days for blood type AB recipients versus 1,662 days for type O recipients in liver transplantation (Kim et al. 2018)), and by the geographic location of the candidates (e.g., between 285 and 1,872 days depending on the recipient's listing DSA in kidney transplantation (Hart et al. 2018)). Furthermore, patients' access to organs can also vary by the quality of harvested organs (e.g., since 2014, patients 55 years of age or older received about the same number ($\sim 21,000$) of kidney transplants as those younger than 55; but only 10.5% of the

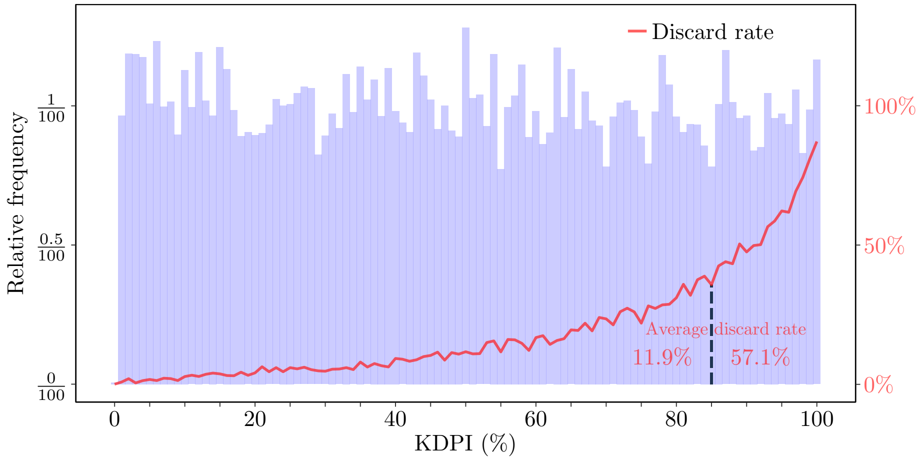


Figure 1 The distribution of donor kidneys in terms of KDPI scores between 2012 and 2017.

kidneys received by the older group was high quality (KDPI $\leq 20\%$) while this fraction triples to 30.6% for the younger group). We model such discrepancies with a static randomized policy through the parameter $p_k(q)$, which denotes the probability that an arriving organ of quality q is offered to type-k candidates. The offer probability $p_k(q)$ imposes the level of access of type-k candidates to organs of quality q, and also determines the stationary waiting time until transplantation for type-k candidates. In Appendix B, we provide a closed form representation of stationary waiting time until transplantation as a function of $p_k(q)$.

Organ offers in practice do not always translate to transplantation since patients have the prerogative to reject offers. We assume risk-neutral patients decide on offered organs jointly with their physicians. An offer is rejected typically when a better quality organ is anticipated for the patient under consideration. (Detailed discussions of the accept/reject problem in organ transplantation can be found in, for example, Sandıkçı et al. 2013, 2008, Alagoz et al. 2007, Howard 2002). We capture this phenomenon through focusing on equilibrium behavior of patients, in which a type-k candidate sets a threshold q_k that represents the organ quality for which the patient is indifferent between accepting or rejecting, accepts organs with quality higher than q_k and rejects those with quality lower than q_k. The scarcity of available organs raises competition between candidates and the offer probability $p_k(q)$ regulates this competition for a particular organ of quality q. In particular, given thresholds q_k for $k = 1, \cdots, K$,

(i) $p_k(q) = 0$ for $q \in \left[\underline{q}, q_k\right)$ for all k; that is, we model type-k candidates' rejection of organs that do not meet their quality threshold q_k through their offer probabilities, and

(ii) $\sum_{k\,:\,q_k \leq q} p_k(q) = 1$ for all q; that is, we model the competition among candidate types who are willing to accept an organ of quality q through their offer probabilities.

Without loss of generality, we associate lower indices with candidate types that have higher access to the top quality organs $\overline{q}$, and assume that this ordering is preserved for any other organ q among patient types who are interested in q. That is, $p_k(q) > p_\ell(q)$ for $k < \ell$ and $q \in [q_k, \overline{q}]$. As a result, at the extreme, candidates who have the lowest access (i.e., type-K candidates) face a notable disadvantage when competing for highly demanded organs. If, however, these candidates settle for organs that are not of interest to other candidates, they can significantly increase their share from the organ pool.

Given threshold q_k and offer probabilities $p_k(\cdot)$, the arrival rate $\mu_k(\cdot)$ of organs to type-k candidates can be obtained by adjusting the organ pool that is of interest to this group with the offer probabilities to these patients. In particular,

$$\mu_k(q_k) = \frac{\overline{q} - q_k}{\overline{q} - \underline{q}} \cdot \int_{q_k}^{\overline{q}} p_k(q) \frac{1}{\overline{q} - q_k} \, dq = \frac{\int_{q_k}^{\overline{q}} p_k(q) \, dq}{\overline{q} - \underline{q}} \tag{1}$$

For any patient type k, we assume that candidate and organ arrival rates, λ_k and μ_k, are considerably larger than the death rate, d_k. This assumption enables the use of asymptotic results to characterize the stationary behavior of our multiclass queueing model .

We use quality-adjusted life expectancy (QALE) to measure candidates' utility from transplantation. The life expectancy of a candidate is composed of pre- and post-transplant periods, which are functions of the organ quality threshold set by the candidate. In particular, the pre-transplant life expectancy Φ_k of type-k candidates is defined as the duration between the time of arrival to the waiting list and the times of death or receiving a transplant, whichever occurs first, and can be calculated as

$$\begin{aligned}\Phi_k(q_k) = &\text{ Expected time until transplant given transplant occurs before death} \cdot \pi_k(q_k) \\ &+ \text{Expected time until death given death occurs before transplant} \cdot [1 - \pi_k(q_k)],\end{aligned}$$

where $\pi_k(q_k)$ denotes the probability that a type-k candidate having the organ threshold q_k will be offered an organ. In steady-state, this probability can be approximated by the fraction of organs that are offered to type-k candidates. Therefore, we have

$$\pi_k(q_k) \approx \min\left\{1, \frac{\mu_k(q_k)}{\lambda_k}\right\}. \tag{2}$$

Furthermore, given that transplant occurs before death, expected time until transplant asymptotically approaches zero, and therefore, pre-transplant life expectancy can be approximated as

$$\Phi_k(q_k) \approx \frac{1}{d_k} \max\left\{0, \frac{\lambda_k - \mu_k(q_k)}{\lambda_k}\right\}. \tag{3}$$

Relying on a fluid limit approximation, Zenios (1999) provides a formal proof of the closed-form expressions given in equations (2) and (3).

The post-transplant life expectancy of type-k candidates is defined as the life years the candidate is expected to enjoy following the transplant and is given by

$$\mathbb{E}_k[\Psi_k(q) \mid q \geq q_k] = \int_{q_k}^{\overline{q}} \Psi_k(q) \frac{p_k(q)}{\int_{q_k}^{\overline{q}} p_k(q)\, dq}\, dq, \tag{4}$$

where $\mathbb{E}_k[\cdot]$ is the expectation taken over q with respect to $p_k(\cdot)$, $\Psi_k(q)$ denotes the average life span of a type-k candidate after transplanting an organ of quality q. We assume that $\Psi_k(q)$ is strictly increasing in q for all k to represent that higher quality organs provide longer life benefits.

A quality-of-life score between 0 (denoting death) and 1 (denoting perfect health state) is typically used in the medical literature to adjust for the quality of life under different medical conditions compared to having perfect health (Guyatt et al. 1993). Following this convention, we denote the quality-of-life score for the pre- and post-transplant periods as α and β, respectively, and assume that the quality of life improves after transplantation (i.e., $\beta > \alpha$).

We can, therefore, write the expected QALE for type-k candidates having threshold q_k as

$$L_k(q_k) = \alpha \Phi_k(q_k) + \beta \pi_k(q_k) \mathbb{E}_k[\Psi_k(q) \mid q \geq q_k]. \tag{5}$$

In the remainder of the paper, to represent organ scarcity, we assume that $\mu_k(\underline{q}) \leq \lambda_k$ for all k. The analysis of the case where $\mu_k(\underline{q}) > \lambda_k$ for a type-k candidate is not of practical interest because of the gap present between the demand and supply for organs in the United States. As a result, using equations (1) to (5), the expected QALE for type-k candidates reduces to

$$L_k(q_k) = \alpha \frac{1}{d_k} \left(1 - \frac{\int_{q_k}^{\overline{q}} p_k(q)\, dq}{\lambda_k(\overline{q} - \underline{q})} \right) + \beta \frac{1}{\lambda_k(\overline{q} - \underline{q})} \int_{q_k}^{\overline{q}} \Psi_k(q) p_k(q)\, dq. \tag{6}$$

Proposition 1 provides the QALE maximizing thresholds of each type-k candidate to optimize their expected QALE.

PROPOSITION 1. *QALE-maximizing threshold q_k^* of a type-k candidate is unique over the interval $[\underline{q}, \overline{q}]$ and given by*

$$q_k^* = \min\left\{ \overline{q}, \max\left\{ \underline{q}, \Psi_k^{-1}\left(\frac{\alpha}{\beta d_k} \right) \right\} \right\}, \quad \textit{for } k = 1, \dots, K.$$

The boundary conditions $q_k^* = \overline{q}$ and $q_k^* = \underline{q}$ in Proposition 1 are provided for mathematical completeness, but they are unlikely to be observed in practice. The solution $q_k^* = \overline{q}$ happens only if the life expectancy from transplanting the highest quality organ is lower than that from staying in the waiting list until death, implying a system where transplantation is a non-value adding activity. The solution $q_k^* = \underline{q}$ represents the other extreme case and happens only if the life expectancy from transplanting the lowest quality organ is higher than that from staying in the waiting list until death, implying a system where staying in the waiting list is non-value adding. Staying on the waiting list may be more valuable than transplanting certain types of organs, as evidenced by frequent rejection of organs in practice, as well as a lack of interest declared in advance by some transplant centers for particular types of organ offers (Israni et al. 2018, Jay and Schold 2017). Therefore, neither of these extremes represent a scenario observable in practice. As a result, expected QALE of type-k candidates is maximized when they set their quality threshold to $q_k^* = \Psi_k^{-1}\left(\frac{\alpha}{\beta d_k}\right)$, implying that type-$k$ candidates should accept all organs that have a longer life expectancy than staying in the waiting list to maximize their QALE.

The expected QALE in equation (6) considers only life benefits, whereas one should also acknowledge the social costs associated with the heterogeneity in organ qualities (Kessler and Roth 2012). Compared to a typical organ that is generally accepted by transplant centers as standard, transplanting a lower quality organ induces a cost. This cost can manifest itself in the form of real costs such as lower life expectancy and medical complications, or psychological costs such as feeling underprivileged, or a combination of both. Transplanting a higher quality organ, on the other hand, may be perceived as an additional reward due to, for example, higher life expectancy, anticipation of fewer medical complications, or feeling privileged. Similar to Kessler and Roth (2012), we define $c(q)$ as the cost of transplanting an organ with quality q and assume that it is (i) decreasing in q, and (ii) additively separable from expected QALE. The case when $c(q) > 0$ is interpreted as the cost, while the case when $c(q) < 0$ is interpreted as the reward of transplanting an organ with quality q. The expected cost paid by a type-k candidate having threshold q_k is given by

$$
\begin{aligned}
C_k(q_k) &= \pi_k(q_k) \cdot \mathbb{E}_k[c(q) \mid q \geq q_k] && \text{(7a)} \\
&= \frac{\int_{q_k}^{\overline{q}} p_k(q)\, dq}{\lambda_k(\overline{q} - \underline{q})} \int_{\underline{q}_k}^{\overline{q}} c(q) \frac{p_k(q)}{\int_{q_k}^{\overline{q}} p_k(q)\, dq}\, dq && \text{(7b)} \\
&= \frac{1}{\lambda_k(\overline{q} - \underline{q})} \int_{q_k}^{\overline{q}} c(q) p_k(q)\, dq. && \text{(7c)}
\end{aligned}
$$

Thus, the social welfare function can be written as

$$
S(q_1, \ldots, q_K) = \sum_{k=1}^{K} \frac{\lambda_k}{\lambda} [L_k(q_k) - C_k(q_k)]. \tag{8}
$$

Proposition 2 specifies socially efficient thresholds for all patient types that maximize the social welfare function.

PROPOSITION 2. *Socially efficient threshold q_k^s of a type-k candidate is unique over the interval $[\underline{q}, \overline{q}]$ and given by*

$$
q_k^s = \min\{\overline{q}, \max\{\underline{q}, q_k'\}\}, \quad \textit{for } k = 1, \ldots, K,
$$

where q'_k is the unique solution of the following equation:

$$\beta\Psi_k(q') - c(q') = \frac{\alpha}{d_k}.$$

Similar to Proposition 1, the boundary conditions $q_k^s = \overline{q}$ and $q_k^s = \underline{q}$ in Proposition 2 are again provided for mathematical completeness. In the remainder of the paper, we assume that these two solutions are not attainable for notational convenience. We establish an ordering of the QALE-maximizing and socially efficient quality thresholds in Corollary 1.

Corollary 1. *For any candidate type k,*

$$q_k^* < q_k^s \qquad \text{if } c(q_k^*) > 0,$$
$$q_k^* \geq q_k^s \qquad \text{if } c(q_k^*) \leq 0.$$

The case $c(q_k^*) \leq 0$ in Corollary 1 corresponds to a situation where there is a social reward associated with transplanting organs of quality q_k^*—those organs that maximize the expected QALE of type-k candidates. Therefore, the social planner who recognizes this reward decides to set the socially efficient threshold q_k^s lower than the QALE-maximizing threshold q_k^*. The case $c(q_k^*) > 0$ corresponds to the alternative where there is a social cost associated with transplanting quality-q_k^* organs. Therefore, the social planner who wants to avoid this cost acts more conservatively and sets the socially efficient threshold q_k^s higher than the QALE-maximizing threshold q_k^*. In both cases, the social planner sets the socially efficient threshold at the expense of expected QALE, but compensates for this loss by maximizing the overall welfare to the society that also includes the social costs/rewards of transplantation.

Corollary 1 offers some insight for the idea that gains increasing popularity in the medical literature—the idea of using marginal organs for individuals that have a significant disadvantage in accessing standard organs (e.g., the proposal to use organs from elderly donors for elderly candidates). Marginal organs are perceived as 'marginal' in the medical community because of their sub-standard performance when used for transplantation. Accordingly, marginal organs in our modeling framework would correspond to those that are associated with positive costs (i.e., $c(q) > 0$). When this is the case, Corollary 1 implies that QALE-maximizing planner, who ignores the social costs of transplanting, would utilize more marginal organs than the socially efficient level. However, marginal organs have relatively higher cost among all utilized organs, and therefore, the social planner uses such organs more conservatively despite their QALE benefits.

Self-interested individuals, on the other hand, optimize their own well-being while competing with other candidates, and their decisions may not coincide with those of a social planner or a

QALE-maximizing planner. We next examine the equilibrium decisions of self-interested individuals. When an organ is offered to a candidate, she faces a trade-off between the costs/benefits of accepting and rejecting. Accepting the organ provides the QALE benefit as well as the cost of transplanting that particular organ, increases the pool of organs utilized by her type, but forgoes the possibility of a higher-quality organ. Rejecting, on the other hand, allows for the possibility of obtaining a higher benefit of transplantation through a higher-quality organ, but forgoes the benefit of transplanting the current organ, exposes the patient to the risk of death without receiving a transplant, and decreases the pool of organs utilized by her type. In equilibrium, type-k candidates are indifferent between accepting or rejecting the organs of quality q_k^e. All type-k candidates accept organs that are at least as good in quality as the equilibrium threshold q_k^e, and reject those that do not meet the threshold q_k^e. We characterize the equilibrium behavior of candidates in Proposition 3.

PROPOSITION 3. *The equilibrium threshold q_k^e of type-k candidates exists uniquely and is given by the solution of the following equation*

$$\beta\Psi_k(q') - c(q') = \frac{\alpha}{d_k} \cdot [1 - \pi_k(q')] + \mathbb{E}_k[\beta\Psi_k(q) - c(q) \mid q \geq q'] \cdot \pi_k(q'). \tag{9}$$

Comparing the equilibrium threshold q_k^e with the socially efficient threshold q_k^s, we establish in Corollary 2 that candidates set their quality thresholds more conservatively in equilibrium than the socially efficient level.

COROLLARY 2. *For any candidate type k, $q_k^s < q_k^e$.*

Corollary 2 offers a potential explanation to the low utilization of marginal organs observed in practice. The gap between the socially efficient and the equilibrium thresholds results from the lack of appropriate incentive mechanisms for candidates to accept lower quality organs for the common good. Instead of transplanting these organs having relatively lower QALE benefits and higher costs, candidates would prefer staying in the waiting list enjoying, in expectation, the average benefits that anyone with their access level can get. Lowering the equilibrium threshold q_k^e, for any candidate type k, until the socially efficient threshold q_k^s creates a positive externality to the system. However, individuals' failing to internalize this externality results in the gap observed in Corollary 2.

One approach to close this gap would be through increasing the left-hand side of equation (9); since its right-hand side is decreasing beyond q_k^s (see the proof of Proposition 3), any increase in its left-hand side would shift the equilibrium threshold q_k^e towards q_k^s. A popular means of achieving such an increase is to raise awareness of the social benefits, which corresponds to decreasing the social costs, of transplanting marginal organs that are wastefully discarded in the current system. Promoting and publicizing these benefits is a major motivation for the burgeoning medical literature on the utilization of marginal organs (Reese et al. 2016, Massie et al. 2014, Heuer et al. 2010, Lee et al. 2005, Busuttil and Tanaka 2003, Alexander and Zola 1996). The intended increase in the left-hand side of equation (9) can also be achieved by offering, for example, monetary compensation for accepting marginal organs. Using financial incentives to alleviate the burden of organ shortage have indeed been proposed by economists (e.g., see Becker and Elias (2007)). However, financial incentives related to any phase of organ donation and transplantation are generally perceived as unethical (Arnold et al. 2002) and currently unlawful in the US (NOTA 1984). Therefore, offering monetary compensations of any form to promote marginal organs can be assumed impractical.

Alternatively, the gap between q_k^s and q_k^e, for any candidate type k, can be reduced by decreasing the right-hand side of equation (9); since its left-hand side is strictly increasing (see the proof of Proposition 3), any decrease in its right-hand side would shift the equilibrium threshold q_k^e towards q_k^s. Such a decrease may be achieved by mechanisms that reduce the access of type-k candidates to relatively higher quality organs in exchange for higher access to lower quality organs. This exchange among access levels to different organ types would correspond to decreasing the expectation term in equation (9). If, however, the mechanism also lowers the probability of receiving any offer $\pi_k(\cdot)$, it would create further injustice and render itself unimplementable. Therefore, any successful implementation that lowers q_k^e should offer some increase in $\pi_k(\cdot)$, while appropriately altering the access levels, for a net decrease in the right-hand side of equation (9). The Eurotransplant Senior Program introduced in Section 1 constitutes a nice example in this direction.

To help reduce the gap between the socially efficient and the equilibrium thresholds, we offer yet another alternative through considering the candidates returning to the waiting list for re-transplantation—a significant aspect of the organ transplant systems that has not received much attention in the operations literature. In the next section, we expand our queueing model to include returning candidates and analyze its impact on the system outcomes, which leads us to the alternative mechanisms that help reduce the gap without compromising social welfare.

4. Returning Candidates Due to Failed Organs

A transplant recipient in practice can outlive the life of the graft she has received. Such cases arise when the recipient experiences irreversible graft failure but she is not necessarily dead and, in fact, can live longer if an alternative is found to replace the functions of the failed graft (e.g., dialysis for kidney recipients or retransplantation). Such transplant recipients typically[3] return back to the waiting list for a new transplant opportunity. Data summarized in Table 1 shows that 10.6% of all additions to the waiting list and 9.5% of all transplant recipients were repeat transplant candidates. Table 1 also displays variation in the rate of repeat transplantation across organs, with kidney and pancreas facing the highest rates. In this section, due to their significance in transplant systems, we incorporate returning candidates into our model.

[3] Graft-failed transplant recipients do not always return back to the waiting list if they cannot clear the eligibility protocol of the transplant center for wait-listing.

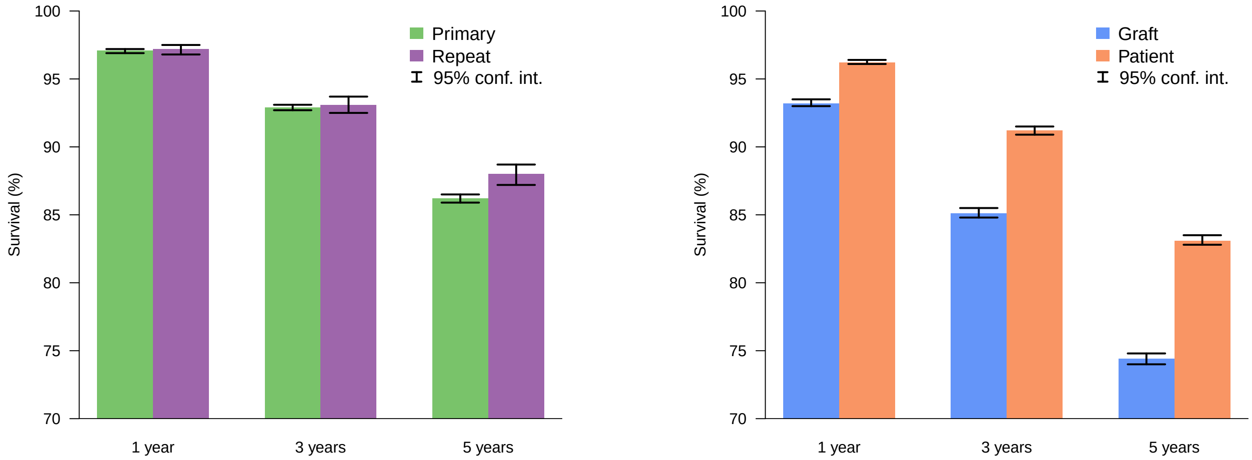
Table 1 Previous transplant status by organ between January 1, 1995 and March 31, 2018

	Waiting list additions		Transplant recipients	
	Primary	Repeat	Primary	Repeat
Kidney	604,968 (87.1 %)	89,678 (12.9 %)	320,086 (88.3 %)	42,596 (11.7 %)
Liver	234,029 (93.0 %)	17,586 (7.0 %)	128,077 (92.7 %)	10,111 (7.3 %)
Pancreas	13,978 (82.6 %)	2,946 (17.4 %)	7,059 (87.9 %)	972 (12.1 %)
Heart	80,839 (95.5 %)	3,821 (4.5 %)	53,392 (96.6 %)	1,897 (3.4 %)
Lung	48,548 (95.5 %)	2,294 (4.5 %)	32,498 (96.2 %)	1,268 (3.8 %)
Intestine	4,136 (89.2 %)	500 (10.8 %)	2,546 (89.5 %)	299 (10.5 %)
All Organs	986,498 (89.4 %)	116,825 (10.6 %)	543,658 (90.5 %)	57,143 (9.5 %)

When compared to Section 3, the competition for organs becomes stiffer with the involvement of returning candidates, since the supply of organs is not affected by these returns. Furthermore, this involvement becomes even more prominent when patients increasingly utilize marginal organs, as such organs are, by definition, associated with lower graft survival. As a result, while increasing the use of marginal organs may soften the competition through increasing the pool of available organs, it can also intensify the competition for the limited set of donated organs.

We analyze the resulting tradeoff and its impact on equilibrium outcomes by incorporating returning candidates as delayed feedback in our queueing model. Let $r_k(q)$ denote the probability that a type-k candidate who has received an organ transplant of quality q outlives her graft and returns to the waiting list. Assume that $r_k(q)$ is nonincreasing in q for all patient types k, which is consistent with the definition of quality q. We consider any general probability function for $r_k(\cdot)$ in our analysis. But, for practical applications, it can take simple forms such as a step function that takes value 0 for high quality organs q above some level $\hat{q}$ (i.e., recipients are outlived by the graft) and the value 1 for low quality organs $q \leq \hat{q}$ (i.e., recipients outlive the graft).

For analytical tractability of the model, we assume that returning patients inherit their original types. An important implication of this assumption is that repeat transplant recipients enjoy the same post-transplant life distribution as that of primary transplant recipients. For kidney recipients, this assumption is justified by Figure 2a, which displays Kaplan-Meier patient survival estimates for primary and repeat transplants. This figure demonstrates that survival rates are not significantly different between primary and repeat transplant recipients, except repeat transplant recipients seems to have a slightly higher 5-year survival probabilitiy, which is explained by selection bias in practice (repeat transplant recipients are younger than primary transplant recipients). In agreement with this figure, analyzing long-term survival trends, Andre et al. (2014) and El-Husseini et al. (2017) also report that kidney allograft outcomes and patient survival are comparable in primary and repeat kidney transplant recipients.



(a) Patient survival for primary vs. repeat transplant (b) Graft versus patient survival

Figure 2 Kaplan-Meier survival estimates for kidney transplants performed during 2008-2015 (based on OPTN data as of April 6, 2018)

4.1. Preliminaries

The delayed feedback to the queue, in general, can potentially distort the memoryless property of the effective arrival process. However, if the arrivals of the first-time candidates crowds out the arrivals of the returning candidates, then the dependency between the two arrival processes becomes negligible and the effective arrival process converges to a Poisson process. The crowding out naturally happens when the returns are rare relative to first-time arrivals, which seems to hold in the US transplant system, where, on average, only 10 repeat arrivals occur for every 90 first-time arrivals (see Table 1). Peköz and Joglekar (2002) formalizes this observation by scaling up the delay time distribution and proves the convergence of the effective arrival process to a Poisson process. Therefore, the effective arrival rate equals the first-time arrival rate inflated by the expected number of returns. We allow candidates to return back to the waiting list at most once, since multiple returns are not widely observed in organ transplantation.[4] As a result, the effective arrival rate of type-k candidates having quality threshold q_k is given by

$$\tilde{\lambda}_k(q_k) := \lambda_k + \mu_k(q_k)\mathbb{E}_k[r_k(q) \mid q \geq q_k], \tag{10}$$

which is equivalent to $\tilde{\lambda}_k(q_k) = \lambda_k(1+\pi_k(q_k)\mathbb{E}_k[r_k(q) \mid q \geq q_k])$. Observe that the return event is only defined for transplanted organs and, therefore, the mean number of returns includes the probability $\pi_k(q_k)$, since only those candidates that receive a transplant may return.

Allowing patients to return back to the waiting list impacts several quantities of interest including expected QALE and equilibrium decisions of each candidate type as well as social welfare. With

[4] Based on OPTN data as of September 30, 2017, the fraction of patients who received two or more repeat transplants (of the same organ type) has peaked at 0.8% (for kidney recipients) and can be as low as 0.1% (for lung recipients).

candidates returning back to the waiting list for a re-transplant, the probability of type-k candidate having a quality threshold q_k being offered a transplant is revised as

$$\tilde{\pi}_k(q_k) = \frac{\mu_k(q_k)}{\tilde{\lambda}_k(q_k)}, \tag{11}$$

and the pre-transplant life expectancy is revised as

$$\tilde{\Phi}_k(q_k) = \frac{1}{d_k}\left[1 - \frac{\mu_k(q_k)}{\tilde{\lambda}_k(q_k)}\right]. \tag{12}$$

With the introduction of the possibility of returning to the waiting list for a re-transplant opportunity, it becomes critical to separate graft survival from patient survival. Graft survival is concerned with the time until irreversible graft failure, signified by a need for retransplantation or patient death, whereas patient survival is only concerned with the time until patient death. Kaplan-Meier survival estimates for kidney transplant recipients in Figure 2b show significant differences between graft and patient survivals, emphasizing the importance of distinguishing between these two concepts. For this purpose, let $\tilde{\Psi}_k(q)$ denote the average life of an organ of quality q transplanted to a type-k candidate. We should note that, when patients outliving their graft are not allowed to return to the waiting list as in Section 3, the distinction between graft and patient survivals is less critical since the quality-adjusted average life span $\beta\Psi_k(q)$ used in Section 3 can be viewed as

$$\beta\Psi_k(q) = \beta\tilde{\Psi}_k(q) + \alpha r_k(q)\frac{1}{d_k}. \tag{13}$$

The right-hand side of equation (13) is composed of the life expectancy from the graft and the life expectancy without a transplant after a failed graft adjusted by the probability of patient outliving the graft, in which case patient was assumed to not return for a retransplant opportunity.

Using the updated quantities in equations (10) to (13), the expected QALE for type-k candidates having threshold q_k is revised as

$$\tilde{L}_k(q_k) = \left(\alpha\tilde{\Phi}_k(q_k) + \beta\tilde{\pi}_k(q_k)\mathbb{E}_k\left[\tilde{\Psi}_k(q) \;\middle|\; q \geq q_k\right]\right) \cdot \frac{\tilde{\lambda}_k(q_k)}{\lambda_k}, \tag{14}$$

where the term inside the parantheses corresponds to expected QALE per arrival, and therefore, it is adjusted by the factor $\frac{\tilde{\lambda}_k(q_k)}{\lambda_k}$ to obtain expected QALE per candidate. Similarly, the expected cost incurred by a type-k candidate is revised as

$$\tilde{C}_k(q_k) = \tilde{\pi}_k(q_k) \cdot \mathbb{E}_k[c(q) \mid q \geq q_k] \cdot \frac{\tilde{\lambda}_k(q_k)}{\lambda_k} = \pi_k(q_k) \cdot \mathbb{E}_k[c(q) \mid q \geq q_k] = C_k(q_k). \tag{15}$$

Observe that the revised expected cost $\tilde{C}_k(\cdot)$ is the same as the original expected cost $C_k(\cdot)$, since the social cost $c(\cdot)$ of transplanting an organ depends only on the quality of the organ (and, therefore, it is independent of whether the organ is used in a primary or a repeat transplant). Equations (14) and (15) yield the revised social welfare function:

$$\tilde{S}(q_1,\dots,q_K)=\sum_{k=1}^{K}\frac{\lambda_k}{\lambda}\Big(\tilde{L}_k(q_k)-\tilde{C}_k(q_k)\Big). \tag{16}$$

The difference $\tilde{L}_k(q_k)-\tilde{C}_k(q_k))$ in equation (16) is measured in welfare generated per candidate, and therefore, it is weighted by $\frac{\lambda_k}{\lambda}$, the fraction of type-$k$ candidates.

4.2. Structural Results

Proposition 4 characterizes QALE-maximizing and socially efficient thresholds for each type-k candidate when candidates are allowed to return to the waiting list. Compared to Propositions 1 and 2, Proposition 4 captures the possibility of outliving the graft when maximizing expected QALE or social welfare, and it reveals that the potential benefit from returning to the waiting list should also be considered when setting QALE-maximizing and socially efficient thresholds.

PROPOSITION 4. *When returns are allowed, for any type-k candidate,*

(a) *QALE-maximizing threshold $\tilde{q}_k^*$ is unique the unique solution of the following equation:*

$$\beta\tilde{\Psi}_k(\tilde{q})=\frac{\alpha}{d_k}(1-r_k(\tilde{q})), \tag{17}$$

(b) *socially efficient threshold $\tilde{q}_k^s$ is the unique solution of the following equation:*

$$\beta\tilde{\Psi}_k(\tilde{q})-c(\tilde{q})=\frac{\alpha}{d_k}(1-r_k(\tilde{q})). \tag{18}$$

Corollary 3 follows immediately from Propositions 1, 2 and 4, offering a rather surprising result. Accordingly, introduction of returning candidates to the waiting list due to failed organs has no effect on the QALE-maximizing or the socially efficient utilization levels of organs, as measured by organ quality thresholds, which can be explained by two main factors acting together: (i) a QALE-maximizing planner or a social planner has no preference between allocating an organ to primary or repeat transplant candidates, and (ii) organ arrivals are not affected by returning candidates.

COROLLARY 3. *For any candidate type k, $\tilde{q}_k^*=q_k^*$ and $\tilde{q}_k^s=q_k^s$.*

We next characterize, in Proposition 5, the equilibrium threshold of type-k candidates when returns are allowed. Compared to Proposition 3, Proposition 5 identifies an equilibrium through exploiting the possibility of patient outliving the graft and getting re-listed, in addition to the immediate benefits of transplanting.

Proposition 5. *When returns are allowed, for any type-k candidate, equilibrium threshold $\tilde{q}_k^e$ exists and is given by a solution to the following equation*

$$\beta\tilde{\Psi}_k(\tilde{q}) - c(\tilde{q}) = \left(\frac{\alpha}{d_k}[1-\tilde{\pi}_k(\tilde{q})] + \mathbb{E}_k\left[\beta\tilde{\Psi}_k(q) - c(q) \,\middle|\, q \geq \tilde{q}\right]\tilde{\pi}_k(\tilde{q})\right)\left(\frac{\tilde{\lambda}_k(\tilde{q})}{\lambda_k} - r_k(\tilde{q})\right). \tag{19}$$

Furthermore, the equilibrium is unique if $\frac{r_k(q)}{\tilde{\lambda}_k(q)}$ is nondecreasing in q.

The condition for the uniqueness of the equilibrium can be interpreted as follows. Relaxing organ acceptance threshold for type-k candidates from q to $\hat{q}$ increases their worst-case return probability as well as their effective arrival rate. The condition stipulates that the relative change $\frac{r_k(\hat{q})}{r_k(q)}$ in the worst-case return probability should be no more than the relative change $\frac{\tilde{\lambda}_k(\hat{q})}{\tilde{\lambda}_k(q)}$ in the effective arrival rate. Violating this condition would imply that the increase in the return probability by accepting a lower quality organ $\hat{q}$ may not be matched with an increase in the effective arrival rate, and thus, candidates who used to be in equilibrium at q can exploit this mismatch and benefit from accepting the inferior quality $\hat{q}$, potentially resulting in multiple equilibria.

Although QALE-maximizing and socially efficient thresholds remain unchanged with the introduction of the possibility of returning to the waiting list, we find in Corollary 4 below that allowing returns strictly decreases equilibrium thresholds. That is, considering the potential benefits from retransplantation allows self-interested individuals to accept lower quality organs than those when they overlook the possibility of retransplantation. Corollary 4 further establishes that the equilibrium thresholds with and without considering returns coincide if and only if it is impossible for the patient to outlive her graft, which happens only if her organs of interest are supreme quality.

Corollary 4. *For any candidate type k, $\tilde{q}_k^s < \tilde{q}_k^e \leq q_k^e$. Furthermore, $\tilde{q}_k^e = q_k^e$ iff $r_k(\tilde{q}_k^e) = 0$.*

Although considering returns helped move the equilibrium thresholds closer to the socially efficient level, we find in Corollary 4 that it falls short of completely closing the gap between the two. The existence of this positive gap explains the flurry of activity in the medical literature focusing on documenting the realized benefits of transplanting marginal organs (Ojo et al. 2001, Busuttil and Tanaka 2003, Massie et al. 2014). However, to the best of our knowledge, the correlation between transplanting marginal organs and returning back to the waiting list has not been spotted as an opportunity to incentivize the use of marginal organs. We next exploit the return events to offer novel incentive mechanisms that further increase the equilibrium utilization of organs towards the socially efficient level.

5. A Remedy: Preserve Waiting Times

This remedy is motivated from marketing strategies (i) to promote a substandard product with low reliability (Li et al. 2013) and (ii) to encourage consumers with valuation uncertainty, who face risk bearing decisions (Su 2009). Lenient return policies and warranty incentives are also shown to play an important role in customer decisions (Davis 2001). Manufacturers may compensate risk-bearing decisions of customers through insurance mechanisms to promote their products (Xie and Shugan 2001). A transplant candidate may reject a marginal organ offer because she may not have any intrinsic motivation to accept this higher risk organ with substandard survival expectancy, which would also require giving up her current position in the waiting list. In light of this observation, a policy that compensates the recipient of a marginal organ for freeing up her spot in the waiting list in exchange for a substandard organ may be very appealing for some candidates to be receptive of such organs. Since the organ does not necessarily always last shorter, which would have forced its recipient into a re-transplant situation, we may consider offering the compensation only if the graft fails and the recipient returns back seeking re-transplantation. This may as well be deemed more fair, since no candidate is forced, but instead those who suffer are compensated. In practice, this compensation may correspond to preserving some fraction of the previously accumulated waiting times of the candidates returning to the waiting list for a re-transplant.

We formalize the above discussion by introducing the $(\delta, \mathring{q})$ policy to our queueing framework. Under this policy, candidates who transplant organs with quality at most $\mathring{q}$ are considered *eligible* for priority. If an eligible type-k candidate returns back to the waiting list for a repeat transplant opportunity, she is given priority in her queue (of type-k candidates) with probability $\delta \in [0,1]$. At one extreme, setting $\delta = 0$ corresponds to the baseline studied in Section 4, in which accumulated waiting time is lost at the time of transplantation. At the other extreme, setting $\delta = 1$ corresponds to inflating the waiting time by a large value (e.g., ∞), and therefore, giving absolute priority to eligible returning candidates. In between these two extremes, one may consider partial-priority policies that preserve fractions (or, if desired, multiples) of the accumulated waiting time until first transplantation. The full spectrum of partial-priority policies correspond to allowing any value between 0 and 1 for δ.

Eligibility parameter $\mathring{q}$ in a $(\delta, \mathring{q})$ policy can take any value in $[\underline{q}, \overline{q}]$; setting $\mathring{q} = q^m$ corresponds to giving eligibility to marginal organ recipients only, $\mathring{q} = \overline{q}$ makes every recipient eligible while $\mathring{q} = \underline{q}$ makes no recipient eligible.[5] Social planner strategically sets $\mathring{q}$ as a means to influence the equilibrium behavior of candidates and achieve socially efficient utilization levels of organs. In doing so, setting the eligibility parameter too conservatively (i.e., $\mathring{q} \ll \tilde{q}^e_k$ for any patient type k) would weaken the strength of incentive for type-k candidates, and the $(\delta, \mathring{q})$ policy may not at all achieve the desired effect over such patients. On the other hand, setting $\mathring{q}$ too generously (i.e., $\mathring{q} \gg \tilde{q}^e_k$) would correspond to promoting a subset of organs that type-k candidates are willing to accept

[5] Observe that, in a $(\delta, \mathring{q})$ policy, $\mathring{q}$ is irrelevant when $\delta = 0$, and similarly, δ is irrelevant when $\mathring{q} = \underline{q}$.

in the absence of any promotion, and thus, it may not achieve the full potential of an optimally designed $(\delta, \mathring{q})$ policy. In Proposition 6, we characterize the equilibrium behavior of candidates after introducing the $(\delta, \mathring{q})$ policy for any $\mathring{q}$. Compared to Proposition 5 that considers the expected benefits of re-listing without any priority, Proposition 6 identifies an equilibrium through exploiting the additional possibility of receiving priority when being re-listed.

We start with preliminaries that are used in Proposition 6. Let $\overset{\star}{\lambda}_k(\cdot)$ and $\mathring{\lambda}_k(\cdot)$ denote the arrival rates of eligible and ineligible returning type-k candidates, respectively. Observe that $\tilde{\lambda}_k(q_k) = \lambda_k + \overset{\star}{\lambda}_k(q_k) + \mathring{\lambda}_k(q_k)$ and, for any $(\delta, \mathring{q})$ policy, $\overset{\star}{\lambda}_k(q_k)$ and $\mathring{\lambda}_k(q_k)$ satisfy

$$\overset{\star}{\lambda}_k(q_k) = \begin{cases} 0 & \text{for } q_k > \mathring{q} \\ \overset{\star}{\mu}_k(q_k)\mathbb{E}_k[r_k(q) \mid q_k \le q \le \mathring{q}] & \text{for } q_k \le \mathring{q} \end{cases} \tag{20}$$

and

$$\mathring{\lambda}_k(q_k) = \begin{cases} \mu_k(q_k)\mathbb{E}_k[r_k(q) \mid q_k \le q] & \text{for } q_k > \mathring{q} \\ [\mu_k(q_k) - \overset{\star}{\mu}_k(q_k)]\mathbb{E}_k[r_k(q) \mid \mathring{q} \le q] & \text{for } q_k \le \mathring{q} \end{cases} \tag{21}$$

where $\overset{\star}{\mu}_k(q_k)$ denotes the pool of organs that makes their recipients eligible and is given by

$$\overset{\star}{\mu}_k(q_k) = \frac{\mathring{q} - q_k}{\overline{q} - \underline{q}} \cdot \int_{q_k}^{\mathring{q}} p_k(q) \frac{1}{\mathring{q} - q_k} \, \mathrm{d}q = \frac{\int_{q_k}^{\mathring{q}} p_k(q) \, \mathrm{d}q}{\overline{q} - \underline{q}}. \tag{22}$$

Observe that $\overset{\star}{\lambda}_k(q_k) \le \overset{\star}{\mu}_k(q_k) \le \mu_k(q_k)$ for any q_k, and therefore, under any $(\delta, \mathring{q})$ policy, a δ fraction of $\overset{\star}{\lambda}_k(q_k)$ receive an organ with probability that asymptotically approaches 1, and their expected time until transplant converges to 0 (Zenios 1999). The probability of receiving an organ offer for the remaining set of candidates (arriving with rate $\tilde{\lambda}_k(q_k) - \delta\overset{\star}{\lambda}_k(q_k)$), which includes eligible candidates that return to the waiting list but do not receive priority as well as returning ineligible candidates and candidates arriving for the first time, is given by

$$\tilde{\pi}_k^{\delta}(q_k) = \frac{\mu_k(q_k) - \delta\overset{\star}{\lambda}_k(q_k)}{\tilde{\lambda}_k(q_k) - \delta\overset{\star}{\lambda}_k(q_k)}.$$

The priority probability δ can take any value in $[0, 1]$. However, from the perspective of a social planner, some values of δ may not be easily implementable due to fairness considerations, or they may even be undesirable. In particular, the degree of incentive brought in by some values of δ may encourage lowering organ acceptance thresholds below the socially efficient level. Such an outcome, however, would be undesirable by the social planner since it implies a loss in realized social welfare compared to what can be achieved at the socially efficient level. That is, the social planner would only implement $(\delta, \mathring{q})$ policies, called admissible policies, that avoids such outcomes. We next provide technical definition of admissible policies from the perspective of a social planner, and defer intuitive explanation of the terms that define such a policy until after Proposition 6.

Definition 1. An *admissible* $(\delta, \mathring{q})$ *policy* is any $(\delta, \mathring{q})$ policy such that $\delta \in [0, \delta^s]$, where

$$\delta^s := \min\Big\{1,\ \min_k \{\delta_k^s\}\Big\},$$

and, for any candidate type-k, δ_k^s is the solution[6] to the following equation:

$$\delta_k^s + (1 - \delta_k^s) \cdot \tilde{\pi}_k^{\delta}(\tilde{q}_k^s) = \Gamma_k, \tag{23}$$

where
$$\Gamma_k = \frac{\frac{\tilde{\lambda}_k(\tilde{q})}{\lambda_k} \cdot \tilde{\pi}_k(\tilde{q}_k^s) \cdot \mathbb{E}_k\left[\beta \tilde{\Psi}_k(q) - c(q) - \frac{\alpha}{d_k} + \frac{\alpha}{d_k} r_k(q) \;\middle|\; q \geq \tilde{q}_k^s\right]}{r_k(\tilde{q}_k^s) \cdot \mathbb{E}_k\left[\beta \tilde{\Psi}_k(q) - c(q) - \frac{\alpha}{d_k} \;\middle|\; q \geq \tilde{q}_k^s\right]}.$$

In Proposition 6, we characterize the equilibrium behavior of candidates under any admissible $(\delta, \mathring{q})$ policy and emphasize the importance of setting the eligibility parameter $\mathring{q}$ in this characterization. In particular, we establish in part (b) that the $(\delta, \mathring{q})$ policy would not achieve any desired effect for type-k candidates, if the social planner chooses a $\mathring{q}$ below $\tilde{q}_k^s$. Therefore, the planner should only consider setting $\mathring{q}$ above $\tilde{q}_k^s$. However, setting $\mathring{q}$ in the interval $[\tilde{q}_k^s, \tilde{q}_k^e)$ risks the possibility of not achieving any effect for type-k candidates, as $\tilde{q}_k^e$ is preserved as an equilibrium when $\mathring{q} \in [\tilde{q}_k^s, \tilde{q}_k^e)$ (see part (a-2)). On the other hand, setting $\mathring{q}$ in the interval $[\tilde{q}_k^e, \overline{q}]$ eliminates $\tilde{q}_k^e$ from being preserved as an equilibrium, which we formally prove in Corollary 5 below, and therefore, guarantees achieving the desired effect (see part (a-1)). Furthermore, part (a-1) also establishes a sufficient condition for the uniqueness of the equilibrium when $\mathring{q} \in [\tilde{q}_k^e, \overline{q}]$. Observe that the term $\frac{r_k(q)\tilde{\pi}_k^{\delta}(q)}{\mu_k(q)}$ in this sufficient condition is a simple generalization of the similar term $\frac{r_k(q)}{\tilde{\lambda}_k(q)} = \frac{r_k(q)\tilde{\pi}_k(q)}{\mu_k(q)}$ given in Proposition 5. In the rest of this section, for clarity of presentation, we assume that the uniqueness condition specified in Proposition 5 holds.[7]

Proposition 6. *Under any admissible* $(\delta, \mathring{q})$ *policy, for any type-*k *candidate, any equilibrium threshold* $\tilde{q}_k^{e,(\delta,\mathring{q})} \in [\underline{q}, \mathring{q}]$ *solves the following equation:*

$$\begin{aligned}
&\overbrace{\beta\tilde{\Psi}_k(\tilde{q}) - c(\tilde{q})}^{\substack{\text{immediate benefit from}\\ \text{transplanting } \tilde{q}}} + \overbrace{r_k(\tilde{q})}^{\substack{\text{prob. of}\\ \text{return}}} \left(\overbrace{\delta\, \mathbb{E}_k\left[\beta\tilde{\Psi}_k(q) - c(q) \;\middle|\; q \geq \tilde{q}\right]}^{\substack{\text{expected benefit from transplan-}\\ \text{tation given priority}}} + (1-\delta) \left(\overbrace{\frac{\alpha}{d_k}\left(1 - \tilde{\pi}_k^{\delta}(\tilde{q})\right) + \mathbb{E}_k\left[\beta\tilde{\Psi}_k(q) - c(q) \;\middle|\; q \geq \tilde{q}\right] \tilde{\pi}_k^{\delta}(\tilde{q})}^{\substack{\text{expected benefit from wait-listing}\\ \text{given no priority}}} \right) \right) \\
&= \Bigg(\underbrace{\frac{\alpha}{d_k}\left(1 - \tilde{\pi}_k^{\delta}(\tilde{q})\right) + \mathbb{E}_k\left[\beta\tilde{\Psi}_k(q) - c(q) \;\middle|\; q \geq \tilde{q}\right] \tilde{\pi}_k^{\delta}(\tilde{q})}_{\substack{\text{expected benefit from wait-listing}\\ \text{given no priority}}} \Bigg) \Bigg(1 + \underbrace{\frac{(1-\delta)\overset{\star}{\lambda}_k(\tilde{q}) + \mathring{\lambda}_k(\tilde{q})}{\lambda_k}}_{\substack{\text{fraction of}\\ \text{non-prioritized}\\ \text{returns}}} \Bigg) + \underbrace{\frac{\delta\overset{\star}{\lambda}_k(\tilde{q})}{\lambda_k}}_{\substack{\text{fraction of}\\ \text{prioritized}\\ \text{returns}}} \underbrace{\mathbb{E}_k\left[\beta\tilde{\Psi}_k(q) - c(q) \;\middle|\; q \geq \tilde{q}\right]}_{\substack{\text{expected benefit from}\\ \text{transplantation given priority}}},
\end{aligned} \tag{24}$$

and any equilibrium threshold $\tilde{q}_k^{e,(\delta,\mathring{q})} \in [\mathring{q}, \overline{q}]$ *solves equation* (24) *with* $\delta = 0$.

(a) *If* $\mathring{q} \geq \tilde{q}_k^s$, *and*

[6] Observe that the solution of equation (23) is given by $\delta_k^s = \left[\Gamma_k - \tilde{\pi}_k(\tilde{q}_k^s)\right] \cdot \left[1 + (\Gamma_k - 1)\frac{\overset{\star}{\lambda}_k(\tilde{q}_k^s)}{\tilde{\lambda}_k(\tilde{q}_k^s)} - \tilde{\pi}_k(\tilde{q}_k^s)\right]^{-1}$.

[7] When this assumption is relaxed and multiple equilibria exists, $\tilde{q}_k^e$ is taken as the maximum of those equilibria.

(a-1) *If* $\mathring{q} \geq \tilde{q}^e_k$, *then* $\tilde{q}^{e,(\delta,\mathring{q})}_k$ *exists only in the interval* $[\underline{q}, \mathring{q}]$ *and it is unique when* $\frac{r_k(q)\tilde{\pi}^\delta_k(q)}{\mu_k(q)}$ *is nondecreasing in* q.

(a-2) *If* $\mathring{q} < \tilde{q}^e_k$, *then* $\tilde{q}^{e,(\delta,\mathring{q})}_k$ *exists uniquely in the interval* $(\mathring{q}, \overline{q}]$ *and equals* $\tilde{q}^e_k$. *Furthermore, any solution to equation* (24) *in the interval* $[\underline{q}, \mathring{q}]$ *leads to multiple equilibria.*

(b) *If* $\mathring{q} < \tilde{q}^s_k$, *then* $\tilde{q}^{e,(\delta,\mathring{q})}_k$ *exists uniquely in the interval* $[\underline{q}, \overline{q}]$ *and equals* $\tilde{q}^e_k$.

Moreover, no equilibrium exists in the interval $[\underline{q}, \tilde{q}^s_k)$.

Equation (24) plays a key role in the equilibrium characterization of organs that are of interest to type-k candidates under any $(\delta, \mathring{q})$ policy. Its left hand-side is interpreted as the expected payoff from accepting the organ of quality $\tilde{q}^{e,(\delta,\mathring{q})}_k$, which is composed of the life benefits from transplanting the organ in question and, if the candidate outlives this organ, a continuation payoff associated with returning to the waiting list for a repeat transplant opportunity. This continuation payoff includes the possibility of receiving priority when the candidate returns to the waiting list. The right hand-side of equation (24), on the other hand, is interpreted as the expected payoff from rejecting $\tilde{q}^{e,(\delta,\mathring{q})}_k$ in hopes of a better organ. When the candidate has no priority —this happens for all first-time candidates as well as for all ineligible returning candidates and $(1-\delta)$-fraction of eligible returning candidates—, she is exposed to the risk of death without a transplant while waiting for the better organ. When the candidate has priority —this happens only for δ-fraction of eligible returning candidates—, with probability asymptotically approaching 1, she receives an organ that is better, on average, than the current organ in question.

In light of Proposition 6, we can intuitively interpret the terms in Definition 1. Observe, after some algebra, that equation (23) is equivalent to equation (24) evaluated at $\tilde{q} = \tilde{q}^s_k$. Therefore, δ^s_k denotes the degree of incentive δ required by type-k candidates to set their equilibrium threshold equal to the socially efficient level. A solution $\delta^s_k > 1$ of equation (23) implies that the degree of incentive under any $(\delta, \mathring{q})$ policy would not be strong enough (since, by definition, $\delta \in [0,1]$) to achieve the socially efficient utilization of organs for type-k candidates. On the other hand, a solution $\delta^s_k \leq 1$ of equation (23) implies that the socially efficient utilization of organs is achievable by type-k candidates and, therefore, any $\delta \in (\delta^s_k, 1]$ is undesirable by the social planner. Observe that $\delta^s_k > 1$ if and only if $\Gamma_k > 1$. To interpret Γ_k, note that at $q = \tilde{q}^s_k$, Proposition 4(b) implies

$$\beta\tilde{\Psi}_k(\tilde{q}^s_k) - c(\tilde{q}^s_k) + \frac{\alpha}{d_k} r_k(\tilde{q}^s_k) = \frac{\alpha}{d_k}. \tag{25}$$

Adding the numerator of Γ_k to the right hand-side of equation (25) yields the expected benefit from rejecting the organ of quality $\tilde{q}^s_k$, which equals the right hand-side of equation (24). Therefore, the numerator of Γ_k corresponds to the *incremental* expected benefit from a prospective transplant compared to dying without a transplant, after rejecting $\tilde{q}^s_k$. Similarly, adding the denominator of

Γ_k to the left hand-side of equation (25) yields the expected benefit from accepting $\tilde{q}_k^s$ assuming absolute priority if re-listed, which equals the left hand-side of equation (24) when $\delta = 1$. Therefore, the denominator of Γ_k corresponds to the *incremental* expected benefit from a prospective *re*-transplant compared to dying without a transplant when re-listed, after accepting $\tilde{q}_k^s$. As a result, for any type-k candidate, $\Gamma_k > 1$ corresponds to the case when the expected benefit from rejecting $\tilde{q}_k^s$ dominates that from accepting $\tilde{q}_k^s$ even under absolute priority, implying that socially efficient utilization of organs is not achievable by type-k candidates under any degree of incentive $\delta \leq 1$.

The equilibrium characterization in Proposition 6 implies that $(\delta, \mathring{q})$ policy may result in multiple equilibria. By establishing, in Corollary 5(a-1), that no equilibrium emerges outside the interval $[\tilde{q}_k^s, \tilde{q}_k^e]$ for any type-k candidate, we conclude that all admissible $(\delta, \mathring{q})$ policies are innocuous to the system. We also find in Corollary 5(a-2) that any $\delta > 0$ strictly improves the utilization of organs with an appropriate selection of $\mathring{q}$, and that maximum possible organ utilization of type-k candidates can only be achieved by setting $\delta = \delta_k^s$. If $\delta_k^s > 1$, however, socially efficient utilization of organs by type-k candidates is not achievable under any admissible $(\delta, \mathring{q})$ policy (Corollary 5(a-3)). For any admissible $(\delta, \mathring{q})$ policy, to distinguish the social planner's preference among multiple equilibria, we define the *Pareto efficient equilibrium* for type-k candidates as the one that maximally utilizes organs.[8] In Corollary 5(b), we establish that the utilization of organs achieved at the Pareto efficient equilibrium strictly increases with increasing degree of incentive δ.

COROLLARY 5. *Under any admissible $(\delta, \mathring{q})$ policy, for candidate type k:*

(a) *Any equilibrium $\tilde{q}_k^{e,(\delta,\mathring{q})}$ satisfies the following:*

(a-1) $\tilde{q}_k^{e,(\delta,\mathring{q})} \in [\tilde{q}_k^s, \tilde{q}_k^e]$.

(a-2) *If $\mathring{q} \geq \tilde{q}_k^e$, then $\tilde{q}_k^{e,(\delta,\mathring{q})} \in (\tilde{q}_k^s, \tilde{q}_k^e)$ for $\delta \notin \{0, \delta_k^s\}$.*

(a-3) *If $\delta_k^s > 1$, then $\tilde{q}_k^{e,(\delta,\mathring{q})} \in (\tilde{q}_k^s, \tilde{q}_k^e]$.*

(b) *Among Pareto efficient equilibria, $\tilde{q}_k^{e,(\delta,\mathring{q})} < \tilde{q}_k^{e,(\delta',\mathring{q})}$ for $\delta' < \delta$.*

The performance of an admissible $(\delta, \mathring{q})$ policy is clearly affected by the social planner's choice of the eligibility parameter $\mathring{q}$. We characterize the effect of choosing $\mathring{q}$ on the equilibrium utilization of organs in Corollary 6, where it is assumed, for expositional clarity, that the uniqueness condition identified in Proposition 6 holds.[9]

COROLLARY 6. *For any admissible $(\delta, \mathring{q})$ policy with $\delta \neq 0$, with respect to utilization of organs by type-k candidates,*

(a) *if $\mathring{q} \in (\tilde{q}_k^e, \overline{q}]$, then $(\delta, \mathring{q})$ policy is strictly dominated by the $(\delta, \tilde{q}_k^e)$ policy,*

[8] Note that if $\mathring{q} \geq \tilde{q}_k^e$ and the assumption in Proposition 6(a-1) holds, then the Pareto efficient equilibrium coincides with the unique equilibrium.

[9] When the uniqueness assumption is relaxed, Corollary 6 remains valid for Pareto efficient equilibria.

(b) *if* $\mathring{q} \in [\underline{q}, \tilde{q}_k^e)$*, and the realized equilibrium is*

(b-1) $\tilde{q}_k^{e,(\delta,\mathring{q})} = \tilde{q}_k^e$*, then* $(\delta,\mathring{q})$ *policy is strictly dominated by any* (δ, q) *policy with* $q \geq \tilde{q}_k^e$*,*

(b-2) $\tilde{q}_k^{e,(\delta,\mathring{q})} \leq \mathring{q}$*, then* $(\delta,\mathring{q})$ *policy strictly dominates any* (δ, q) *policy with* $q > \mathring{q}$*.*

Corollary 6(a) establishes that setting $\mathring{q} > \tilde{q}_k^e$ would be a poor design choice. The suboptimality of such a choice is intuitively explained through observing that it wastes the potential benefits of the $(\delta,\mathring{q})$ policy by encouraging the use of organs that are already being demanded by type-k candidates without any incentive. An immediate consequence of Corollary 6(a) is that setting $\mathring{q}$ above the equilibrium threshold of the most conservative candidate type would be an ineffective choice for the overall utilization of organs, and thus, for the resulting social welfare.

On the other hand, for any admissible $(\delta,\mathring{q})$ policy with $\mathring{q} < \tilde{q}_k^e$, Proposition 6(a-2) implies that the baseline equilibrium $\tilde{q}_k^e$ is always preserved, while new equilibria $\tilde{q}_k^{e,(\delta,\mathring{q})} \leq \mathring{q}$ may also emerge, for type-k candidates. The question of whether the selected $\mathring{q}$ is a good design choice (i.e., achieves the desired effect) or not depends on which equilibrium is realized. If the baseline equilibrium $\tilde{q}_k^e$ is realized, then Corollary 6(b-1) implies that setting $\mathring{q} < \tilde{q}_k^e$ would be a poor choice. If, however, the equilibrium $\tilde{q}_k^{e,(\delta,\mathring{q})} < \mathring{q}$ is realized, then Corollary 6(b-2) implies that setting $\mathring{q} < \tilde{q}_k^e$ would be a good choice. Although any of these equilibria may be realized, one of them, known as the *focal equilibrium*, may distinguish itself as being more relevant and/or natural than others (Schelling 1980, Myerson 2007). For example, the focal equilibrium may correspond to the baseline equilibrium $\tilde{q}_k^e$ as it represents the status quo, rendering the $(\delta,\mathring{q})$ policy ineffective in achieving its desired effect. Moreover, by definition, the choice of $\mathring{q} < \tilde{q}_k^e$ provides no incentive for any organ of quality $q \in (\mathring{q}, \tilde{q}_k^e)$, and thus, when candidates are forced to accept these non-incentivized organs to gain access to the set of incentivized organs, it may be perceived as a burden to any equilibrium $\tilde{q}_k^{e,(\delta,\mathring{q})} < \mathring{q}$. It can be argued that such a perception of burden would also push the focal equilibrium to $\tilde{q}_k^e$. When $\mathring{q}$ is set more conservatively, the burden of non-incentivized organs increases, resulting in increased attractiveness of $\tilde{q}_k^e$. At the extreme, if this burden exceeds beyond a threshold, then no equilibrium $\tilde{q}_k^{e,(\delta,\mathring{q})} < \mathring{q}$ emerges, making $\tilde{q}_k^e$ the unique equilibrium.

Recognizing the burden of non-incentivized organs, however, candidates may no longer behave as imposed by the model, in which a single acceptance threshold is assumed. In particular, for any patient type k, setting $\mathring{q} < \tilde{q}_k^e$, may result in anomalous behavior involving disjoint acceptance regions, in which the candidate is *not* interested in any organs of quality $q \in (\mathring{q}, \tilde{q}_k^e)$ to avoid the burden, but she is interested in all organs of quality $q \in [\tilde{q}_k^{e,(\delta,\mathring{q})}, \mathring{q}] \cup [\tilde{q}_k^e, \overline{q}]$, where $\tilde{q}_k^{e,(\delta,\mathring{q})}$ solves equation (24) after adjusting $p_k(q)$ such that $\int_{\tilde{q}_k^e}^{\mathring{q}} p_k(q)\,\mathrm{d}q = 0$. Removing the burden through disjoint acceptance regions increases the attractiveness of $\tilde{q}_k^{e,(\delta,\mathring{q})} < \mathring{q}$, resulting in increased likelihood of achieving the desired effect. When disjoint acceptance regions are allowed, since our derivations do not impose any assumptions on $p_k(q)$, all of the results in this section continue to hold (except Corollary 6(b-2), in which case the domination guarantees are dropped).

Having characterized the candidates' equilibrium behavior under a $(\delta, \mathring{q})$ policy, we are now ready to study the resulting social welfare. In Proposition 7, we establish that social welfare improves with the introduction of a $(\delta, \mathring{q})$ policy.

PROPOSITION 7. *Introduction of any admissible* $(\delta, \mathring{q})$ *policy with* $\delta \neq 0$ *increases social welfare.*

We conclude this section with an intuitive explanation for the underlying causes of why an appropriately designed $(\delta, \mathring{q})$ policy achieves the desired effect of increased marginal organ utilization while also improving social welfare. Any $(\delta, \mathring{q})$ policy constitutes an incentive mechanism for candidates (particularly, those having relatively lower access to organs) to internalize the benefits of accepting a target set of organs. Social planner can specify the target set as he wishes, but it makes sense to define it as the set of organs that he struggles allocating (e.g., marginal organs that are currently being rejected in the absence of an incentive). By accepting such an organ, a candidate clearly unloads the waiting list. If the recipient of such an organ returns back to the waiting list due to graft failure, then she re-loads the waiting list, resulting in no effective change on the size of the waiting list. The real concern in this case, however, is the candidates' fear of losing her position on the waiting list by accepting the organ in question. Such concerns can be alleviated by promising the candidate her previous position, which is exactly what a $(\delta, \mathring{q})$ policy is designed for. (Note that, the $(\delta, \mathring{q})$ policy gives the planner the flexibility to assign a returning candidate to any position on the prioritized waiting list through the parameter δ.) If, on the other hand, the recipient of such an organ does not experience graft failure, and thus, not return to the waiting list, then the candidate is served by the transplant system as best as feasibly possible, while others on the waiting list enjoy the reduced load. Therefore, the candidate as well as the rest of the society are better off with such a transplantation.

6. Numerical Study

Our theoretical results are very strong and encouraging, but they only inform about the direction of change with the proposed mechanism. To quantify the magnitude of impact associated with the proposed mechanism, we resort to simulation. Although the proposed $(\delta, \mathring{q})$ policy is not limited to any specific organ, we demonstrate its application focusing on the U.S. kidney transplantation system. In our implementation, we take the eligibility parameter $\mathring{q}$ as the KDPI cutoff and the priority parameter δ as the waiting time adjustment factor set by the system designer. In particular, setting KDPI cutoff $\mathring{q} \in [0\%, 100\%]$ makes a recipient eligible for priority if she has accepted a kidney with a KDPI score of $\mathring{q}$ or worse (i.e., KDPI score $\geq \mathring{q}$). Setting waiting time adjustment factor $\delta \in [0, \infty]$ grants eligible candidates, if they return back to the waiting list, their previously accumulated waiting times multiplied by δ.

The U.S. transplant policy makers use a software called kidney-pancreas simulated allocation model (KPSAM) to evaluate proposals regarding the kidney transplant system. When kidney allocation was extensively revised in 2014, KPSAM was used to demonstrate the projected changes in the demographics of transplant recipients and post-transplant outcomes under their proposed revisions (Gustafson et al. 2016). KPSAM is also made available to the public as an executable file. However, its lack of source code offers very little flexibility to its users in evaluating different policies. In fact, KPSAM could not be used in its current form to evaluate our proposed mechanism.

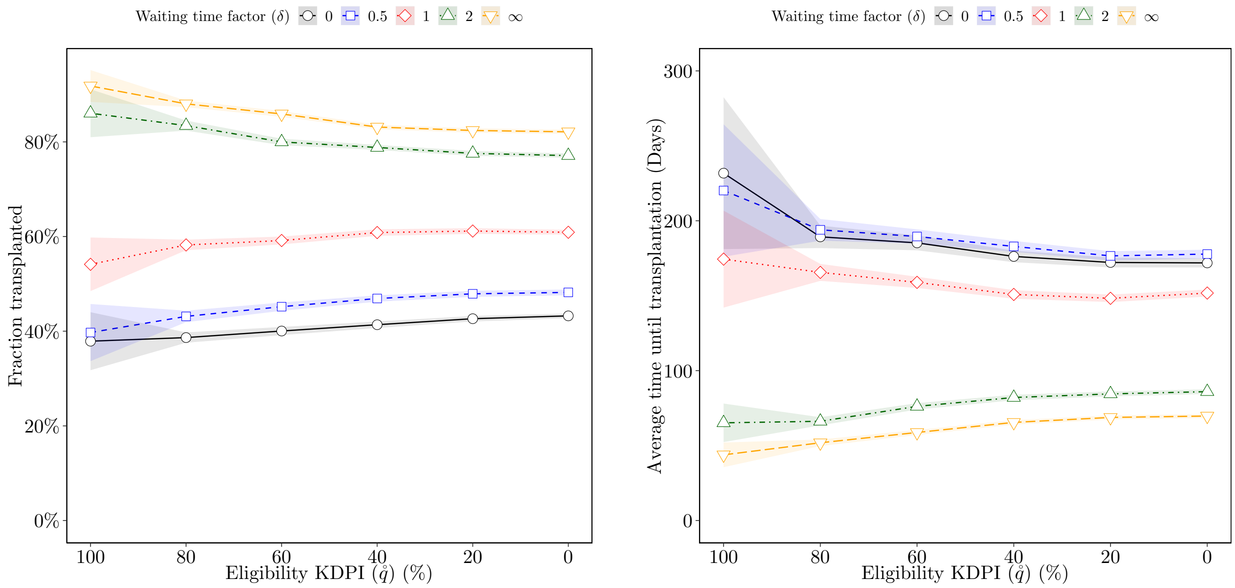
We have developed a clinically detailed simulation model for the entire U.S. kidney transplant system as an alternative to KPSAM. Similar to KPSAM, key events in the simulation, such as arrivals of organs and patients, as well as status updates for candidates (including, among many other things, active/inactive status and removal from the waiting list due to death or other reasons), are linked to an actual transplant database obtained from OPTN/SRTR. This key database includes all transplant candidates that were listed in the US transplant waiting lists between 1987 and 2018, history of updates for all wait-listed candidates, follow-up records for transplant recipients, as well as all deceased donor organs that were harvested during the same time frame. The database contains de-identified records with demographic as well as clinical variables, and collectively amounts to more than 30 million records and close to 1,000 variables. This simulation model implements the most recent UNOS deceased-donor kidney allocation policy (OPTN 2018b) and is demonstrated to closely match the real outcomes indicated from the database.

In §6.1-6.2, we present results quantifying the impact on several outcomes of the $(\delta, \mathring{q})$ policy for various choices of $\delta \in [0, \infty]$ and $\mathring{q} \in [0\%, 100\%]$. For this purpose, given a pair of $(\delta, \mathring{q})$ values, we simulate the U.S. kidney waiting list for a 3-year period from January 2015 to January 2018. We report statistics for each $(\delta, \mathring{q})$ after 100 independent replications of the simulation.

6.1. How does an individual candidate benefit from the $(\delta, \mathring{q})$ policy?

As we assert at the beginning of Section 5, the proposed mechanism, when compared to the current system without the mechanism, would yield more favorable results in access to transplantation for eligible re-listed candidates. We now demonstrate the validity of this assertion under the assumption that candidates maintain their offer acceptance behavior as in the current system without the $(\delta, \mathring{q})$ policy. We choose this simplified setting for clarity of exposition, but note that our results remain structurally identical when this assumption is relaxed as in Section 6.2.2.

Figures 3a and 3b present the fraction transplanted and time until transplantation, respectively, for re-listed candidates during the simulation period under various $(\delta, \mathring{q})$ pairs. Observe that setting



(a) Fraction transplanted (b) Time until transplantation

Figure 3 Access to transplantation for re-listed candidates during the simulation period under the $(\delta, \mathring{q})$ policy. Colored shadings around each line denote 99% confidence interval around the point estimates.

$\delta = 0$ corresponds to the current system, in which accumulated waiting time is lost at the time of transplantation. It can also be viewed as a delusive implementation of the proposed mechanism, as it publicizes a set of 'incentivized kidneys' without offering any compensation. As δ increases, for any fixed $\mathring{q}$, not only a larger fraction of eligible re-listed candidates receive a transplant, but they also access to transplantation in significantly shorter durations, validating the aforementioned assertion. For instance, when $\mathring{q} = 85\%$, as we increase δ from 0 to ∞, the fraction transplanted for eligible re-listed candidates monotonically increases from 38.31% to 89.28%, while, at the same time, their average time until transplantation decreases from 193.3 days to 48 days. On the other hand, as expected, varying δ for any $\mathring{q}$ does not affect the access to transplantation for ineligible re-listed candidates (results not included in Figure 3 to avoid overcrowding).

Figure 3 offers additional insights about the proposed mechanism. First, in contrast to theory (see Section 5), the fraction transplanted among eligible re-listed candidates remains strictly less than 100% and their time until transplantation remains strictly above 0, despite $\delta \to \infty$. This is mainly caused by the fact that real-life queueing dynamics is affected by several factors other than waiting time (e.g., blood type compatibility, HLA typing, sensitization). Second, setting $\delta = 2$ achieves comparable access levels as setting $\delta = \infty$, implying that one does not require impractical (and potentially inadmissible) levels of δ to attain an impactful implementation. Third, the twofold benefit of the proposed mechanism may not always materialize, particularly for small values of δ. For instance, setting $\delta = 0.5$, when compared to $\delta = 0$, leads to a larger fraction of transplantation among eligible relistings, but their time to transplantation do not differ significantly.

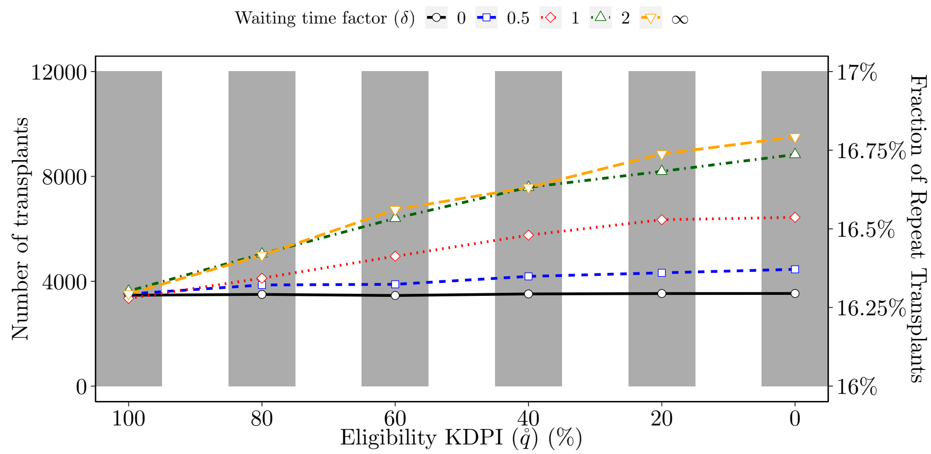


Figure 4 Simulation results under baseline offer acceptance.

Last, but not least, the impact with respect to $\mathring{q}$ is influenced by δ. Figure 3 displays that access to transplantation improves (worsens) as the set of incentivized kidneys expands when δ is low (high, respectively). This rather nontrivial finding can be explained by two competing forces. On the one hand, the definition of $\mathring{q}$ introduces a selection bias. As $\mathring{q}$ increases, the set of eligible candidates narrows down to those facing increasing difficulty in receiving kidney offers due to exogenous factors (e.g., hard to match HLA typing, blood type, geography). Therefore, access to transplantation is expected to worsen with increasing $\mathring{q}$. On the other hand, a larger value for $\mathring{q}$ results in weaker competition among eligible relistings, who, consequently enjoy more from the priority offered by the proposed mechanism. Therefore, access to transplantation is now expected to improve with increasing $\mathring{q}$. We find in Figure 3 that the former force prevails when δ is low, while the latter prevails when δ is high. The resulting funnel shapes in Figure 3 implies that the marginal impact of δ on an eligible re-listed candidate's access to transplantation can be increased by selecting $\mathring{q}$ more conservatively.

6.2. Societal impact of the $(\delta, \mathring{q})$ policy

Introducing a change to the organ allocation policy may naturally result in changes in patients' offer acceptance behavior. In our baseline simulation model (representing the current system without the $(\delta, \mathring{q})$ policy), we have implemented the kidney offer acceptance model developed by SRTR (SRTR 2018) to determine each candidate's likelihood of accepting a kidney offer. This model uses a detailed logistic regression, which includes a generalized linear model with a logit link and a semi-parametric baseline hazard function, to estimate odds ratios of acceptance from kidney match run data. The model is stratified across donor quality and adult/pediatric status, and is adjusted for donor and candidate characteristics including donor-candidate interactions.

Figure 4 presents simulation results under this baseline acceptance behavior and reveals a potential concern associated with the introduction of the $(\delta, \mathring{q})$ policy. While this policy does not affect the annual number of transplants overall (about 12,000 per year), it alters the distribution of transplants between primary versus repeat candidates, where repeat candidates receive increasingly higher share with increasing δ or decreasing $\mathring{q}$. We are interested in the impact of this alteration to overall social welfare.

Section 5 (Proposition 7) established that introduction of $(\delta, \mathring{q})$ policies does no harm to social welfare. To elaborate on this result, we use the given baseline acceptance behavior in Section 6.2.1 and estimate the worst-case societal impact of the proposed mechanism. We next modify this baseline acceptance behavior in Section 6.2.2 to capture candidates' responses to the introduction of the $(\delta, \mathring{q})$ policy and obtain a more realistic estimate of the societal impact.

6.2.1. Worst-Case Analysis: Candidates are Oblivious to the Proposed Mechanism The worst-case scenario associated with the proposed mechanism arises when it is introduced at full strength (i.e., $\mathring{q} = 0$ and $\delta = \infty$), but it is not reciprocated in candidates' acceptance behavior. In such a case, the social planner not only offers eligibility to every recipient but also grants absolute priority to every returning candidate, distorting the system dynamics at the policy's maximum potential. Nevertheless, candidates remain oblivious to such a generous offering and thereby nullifying the efforts of the planner.

Several simulated outcomes under this setting are summarized in Table 2 along with baseline results. We find that there is no significant difference in the numbers of kidneys discarded (or, equivalently, kidneys transplanted), candidates died while waiting, or the size of the waiting list at the end of the simulation period. Furthermore, post-transplant outcomes (namely, the number of graft failures as well as 1-year graft survival rate) also remain comparable. The only statistically significant difference is observed in the time until transplantation, which is about a week shorter under the worst-case scenario and a result of shorter waiting times experienced by prioritized relistings (see Figure 3b). These results demonstrate that even an aggressive implementation of the $(\delta, \mathring{q})$ policy coupled with no response in candidates' acceptance behavior does not hurt the social welfare, which is a function of the pre- and post-transplant outcomes attained by the population.

6.2.2. Candidates Respond to the Proposed Mechanism In this section, we model candidates' response to the introduction of the $(\delta, \mathring{q})$ policy by adjusting SRTR's offer acceptance model. This acceptance model estimates an individual's offer acceptance probability using the inverse logit function $\frac{e^{\boldsymbol{\beta}\mathbf{x}}}{1+e^{\boldsymbol{\beta}\mathbf{x}}}$, where $e^{\boldsymbol{\beta}\mathbf{x}}$ denotes the *odds* of acceptance and $\mathbf{x}$ is a vector encapsulating donor and candidate attributes. We model candidates' response through adjusting their baseline odds of acceptance, so that the odds of accepting an organ of quality q after the introduction of the $(\delta, \mathring{q})$ policy is multiplied by a factor, which we call the *relative odds*(q).

Table 2 Worst case performance analysis results for the $(\delta, \mathring{q})$ policy

	Baseline			Worst-case ($\mathring{q}=0,\ \delta=\infty$)		
Outcome	Median	Mean	99% CI	Median	Mean	99% CI
Number of discarded kidneys†	2 528	2 529	± 7.80	2 529	2 532	± 7.73
Number died while waiting†	4 304	4 303	± 2.34	4 303	4 303	± 2.41
Size of the waiting list*	92 867	92 873	± 24.23	92 854	92 854	± 20.92
Time until transplantation (days)	797	1 011	± 1.25	789	1 005	± 1.25
Number of graft failures†	729	732	± 4.24	729	730	± 3.88
1-year graft survival (%)	95.00	94.99	± 0.03	94.99	94.99	± 0.03

† Average per year from January 1, 2015 to January 1, 2018.
* Number of candidates as of January 1, 2018.

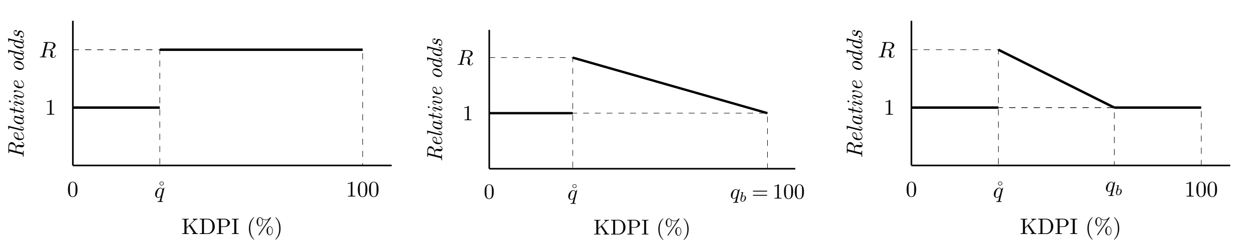
We model relative odds using a simple yet expressive linear form. It is natural to expect that candidates respond only to a subset of incentivized kidneys, which we take as those with a KDPI in the interval $[\mathring{q}, q_b]$ with parameter $q_b > \mathring{q}$ denoting the point where baseline response is restored. Assuming that peak response, denoted R, is attained at $\mathring{q}$ (i.e., the best in the set of incentivized kidneys), we obtain:

$$\text{relative odds}(q) = 1 + \phi(q) \cdot (R - 1), \tag{26}$$

where

$$\phi(q) = \begin{cases} \frac{q_b - q}{q_b - \mathring{q}} & \text{if } q \in [\mathring{q}, q_b], \\ 0 & \text{otherwise.} \end{cases} \tag{27}$$

Observe that $\phi(q) = 0$ corresponds to the baseline response, where relative odds is 1; and $\phi(\mathring{q}) = 1$ (i.e., relative odds attains its peak R at $\mathring{q}$). We take $q_b = \infty$ to capture the scenario that relative odds remains constant at R for all $q \geq \mathring{q}$. Choosing $R < 1$ implies that introduction of the proposed incentive mechanism discourages candidates and makes them even more selective than their baseline acceptance levels. However, such a choice is impractical, as candidates should maintain their baseline acceptance behavior in the worst-case had they perceived the mechanism is not in their best interest. Figure 5 illustrates various forms of interest that the relative odds function can take.



(a) Equal response (b) Gradual response (c) Local response

Figure 5 Modeling candidates' response to the introduction of the $(\delta, \mathring{q})$ policy.

Figure 5a illustrates equal response to all incentivized kidneys $q \geq \mathring{q}$. Note that this setup does not imply equal likelihood of acceptance for all $q \geq \mathring{q}$; it rather increases the baseline odds of acceptance by the same factor for all $q \geq \mathring{q}$, hence retaining the ratio of odds unchanged for any $q_1, q_2 \geq \mathring{q}$. On the other hand, given a particular implementation of the $(\delta, \mathring{q})$ policy, patients may not respond equally to all incentivized organs as in Figure 5a. In fact, their response may worsen as the quality of the organ decreases, since all incentivized organs are compensated at the same level δ but higher quality organs are expected to provide higher post-transplant benefit. Figures 5b and 5c illustrate this phenomenon: the former demonstrates gradually decreasing response as the quality of kidneys decrease and the latter demonstrates a confined response to a local neighborhood of $\mathring{q}$.

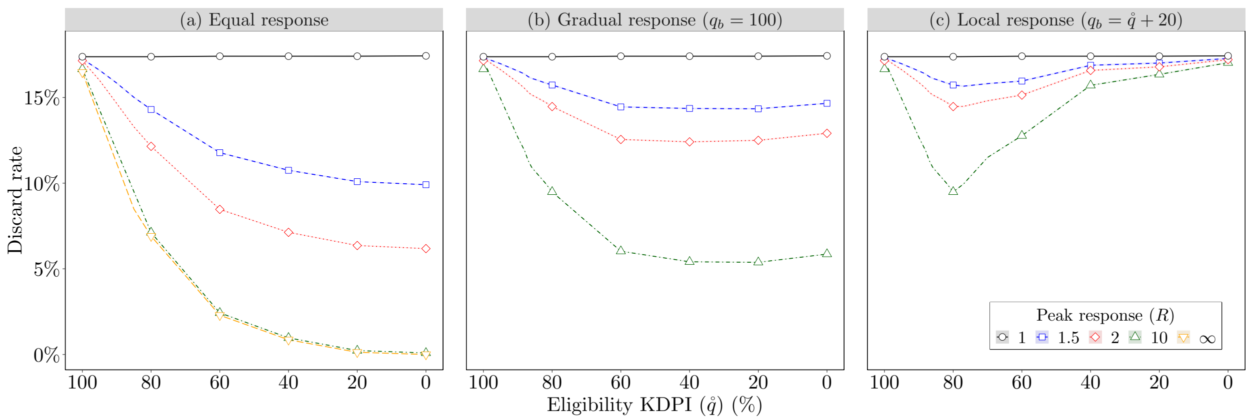


Figure 6 **Overall discard rate of deceased-donor kidneys under the $(\delta, \mathring{q})$ policy ($\delta = 1$).**

Using the response adjustment model (26)-(27), we are now able to provide more realistic estimates of the societal impact of the $(\delta, \mathring{q})$ policy. Figure 6 presents discard rates of donated kidneys for various choices of $\mathring{q}$, q_b, and R. We find the overall discard rate of kidneys to be 17.4% in the baseline. Results in Figure 6a demonstrate that, given an acceptance level measured by the peak response parameter R, discard rate decreases monotonically with $\mathring{q}$ and attains its minimum at $\mathring{q} = 0$ (i.e., when the policy makes all recipients eligible). Under such a policy, if odds of acceptance doubles from its baseline level, discard rate decreases by 11.2 percentage points to 6.2%. Moreover, discards can be entirely eliminated if the policy stimulates very strong response (denoted by $R = \infty$). Observing that there is no distinguishable difference in discard rates between $R = \infty$ and $R = 10$ in Figure 6a, we drop $R = \infty$ for clarity of exposition in the rest of our results.

Figures 6b and 6c present discard rates when candidates' relative odds of acceptance does not remain constant across incentivized kidneys. Observe that the savings in kidneys discarded decrease as we move from Figure 6a (stronger response) to 6c (weaker response), reflecting the reductions in the strength of population's response to the policy. As response weakens, discard is minimized with a more conservative setting of $\mathring{q}$. If candidates' response decreases gradually as in Figure 5b, optimal $\mathring{q}$ is found around 40% (see Figure 6b). If, under such a setting, peak response R doubles

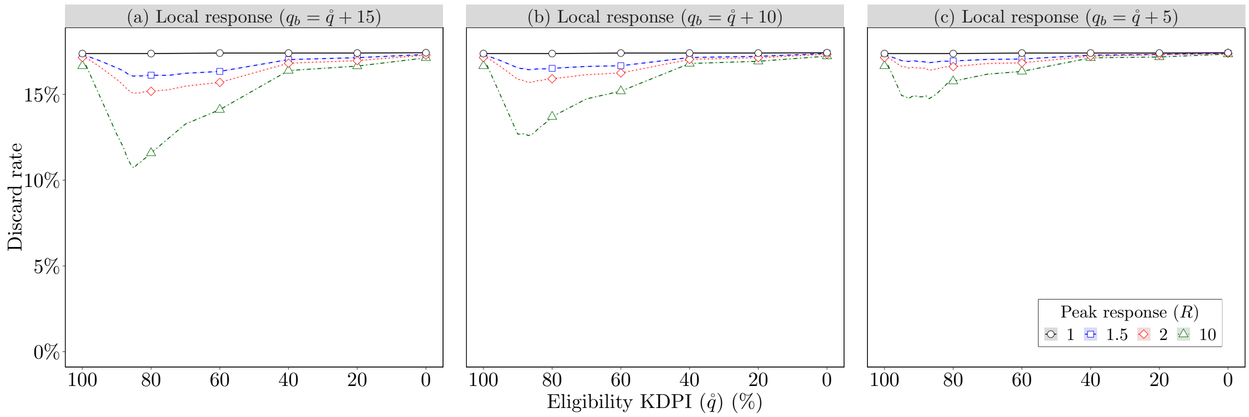


Figure 7 **Overall discard rate with weakening candidate response to the $(\delta, \mathring{q})$ policy ($\delta = 1$).**

from its baseline level, discard rate decreases by 5 percentage points to 12.4%. On the other hand, if candidates' response remains local as in Figure 5c, selection of $\mathring{q}$ becomes more critical in minimizing the discard rate (see Figure 6c). In particular, while setting $\mathring{q}$ around 80% minimizes discard rate, we also find that deviating from this level in either direction can waste the entire potential of the proposed mechanism. The definition of locality (i.e., $q_b - \mathring{q}$) clearly influences the discard rates. As $q_b - \mathring{q}$ decreases (i.e., response weakens), we find that the savings in kidneys discarded decreases and optimal $\mathring{q}$ to around 85% (see Figure 7).

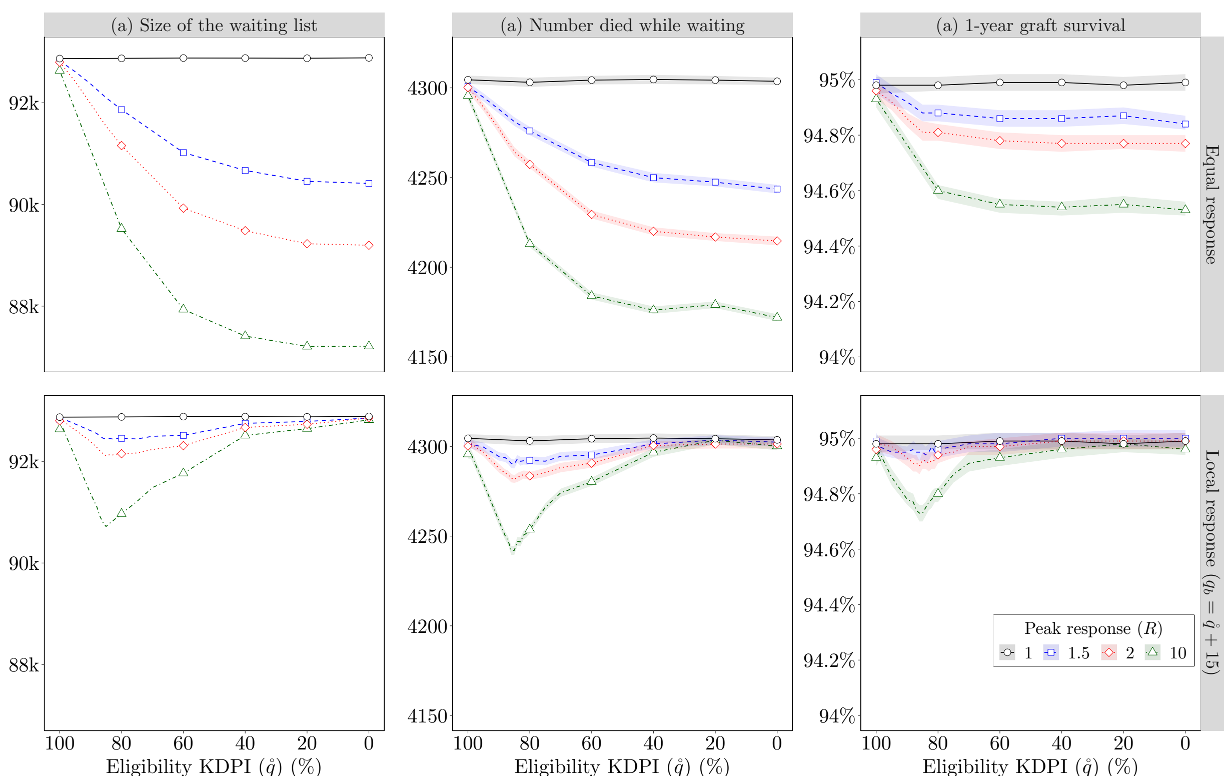


Figure 8 **Other simulated outcomes under the $(\delta, \mathring{q})$ policy.**

Benefits of the $(\delta, \mathring{q})$ policy is not limited to alleviating the burden of discarded kidneys, as demonstrated by the results summarized in Figure 8. As a direct consequence of the kidneys saved from getting discarded, we find significant reductions in the size of the waiting list (Figure 8a) and the number of patient deaths while waiting for a transplant (Figure 8b). It is also important to note that, despite using relatively inferior kidneys compared to the baseline, the $(\delta, \mathring{q})$ policy maintains comparable 1-year graft survival post-transplantation (Figure 8c).

7. Concluding remarks

Organ transplantation is life-saving. The demand for this life-saving treatment continues to increase, adding up to more than 100,000 patients currently waiting in the United States. Availability of donated organs forms the main bottleneck in offering this life-saving treatment to the thousands dying while waiting. Despite the widely documented scarcity, more than 14% of all organs recovered for transplantation are discarded, because organs arrive in many shapes and forms, but most candidates wish to only use 'better' organs. This traditional approach is recently being challenged in the transplantation community. Less than ideal organs, that are traditionally discarded, are now argued as viable alternatives for the many who would die without a transplant (Reese et al. 2016, Massie et al. 2014, Lee et al. 2005). Yet, there is still a wide gap between what is proposed in the medical community and how individual decisions are made.

We have studied the problem of how to increase the utilization of available organs within the confines of the current U.S. allocation systems. Our initial theoretical analysis establishes that a gap between the socially efficient and equilibrium utilization of self-interested individuals is inevitable. To mitigate this observed gap, we have proposed an incentive mechanism that compensates the recipient of marginal organs by promising them priority in case they return back to the waiting list for a re-transplant. Our theoretical results suggest that such a mechanism indeed shifts the equilibrium behavior of self-interested individuals towards socially desirable levels and does so without harming social welfare. We have also analyzed the optimal design of the proposed mechanism.

An incentive mechanism, similar to ours in the spirit of not enforcing, called the Eurotransplant Senior Program (ESP), has been successfully implemented in Europe (Branger and Samuel 2016). ESP was first established in 1999 for kidney transplantation as an old-to-old allocation system, where kidneys from deceased donors aged 65 years or older are exclusively allocated to ESP-enrolled candidates of the same age group. Since the implementation of ESP, not only the fraction of kidneys retrieved from donors over 65 years of age increased from 10% to 14.2% (Cohen et al. 2004), but also the ratio of kidney recipients older than 65 years also increased from 3.6% to 19.7% (De Fijter 2009). Overall, ESP has been associated with a notable increase in marginal organ transplantation (Bahde et al. 2014, Fabrizii et al. 2005).

A program similar to ESP, however, currently does not exist in the U.S. transplant system. Our results in this paper provide theoretical as well as numerical evidence into the achievability of lower organ discards through establishing appropriately defined incentive mechanisms. In particular, numerical simulations for the U.S. renal transplant system suggests that an optimal implementation of the proposed mechanism can lower the kidney discard rate from 17.4% (baseline) down to 5.4% (when the mechanism faces strong response from the candidates), 9.5% (moderate response), or 15.7% (weak response). Compared to the baseline, these reductions translate to 1746, 1148, or 241 more transplants per year, respectively. Moreover, despite utilizing marginally lower quality kidneys (as measured by the KDPI score) under the proposed mechanism, the resulting graft survival 1-year post-transplant remains stable around 94.8% (versus 95.0% for baseline). A potential concern associated with the mechanism is that it distorts the distribution of transplants across primary versus repeat cases in favor of repeat cases. However, our worst-case analysis concludes that such a distortion, when compared to the baseline, does not cause inferior transplant outcomes. In fact, candidates who choose to accept incentivized kidneys benefit from significantly shorter waiting times and higher likelihood of transplantation.

Considering the optimization of the δ and $\mathring{q}$ parameters that define the proposed mechanism, we find that one does not need impractical values for δ to achieve a notable impact. For instance, $\delta = 1$, which grants eligible returning candidates their previous waiting time without any inflation, serves the purpose. Considering $\mathring{q}$, we find that the optimal KDPI cutoff is around 85%, which coincides with the generally accepted definition of marginal organs in the medical community.

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Appendix A: Proofs of Results

Proposition 1. *QALE-maximizing threshold q_k^* of a type-k candidate is unique over the interval $[\underline{q}, \overline{q}]$ and given by*

$$q_k^* = \min\left\{\overline{q}, \max\left\{\underline{q}, \Psi_k^{-1}\left(\frac{\alpha}{\beta d_k}\right)\right\}\right\}, \quad \text{for } k = 1, \ldots, K.$$

Proof of Proposition 1. From equation (6), we have

$$\frac{\partial}{\partial q_k} L_k(q_k) = \frac{p_k(q_k)}{\lambda_k(\overline{q} - \underline{q})}\left[\frac{\alpha}{d_k} - \beta \Psi_k(q_k)\right], \tag{28}$$

which gives $\frac{\partial}{\partial q_k} L_k(q_k') = 0$, when q_k' satisfies,

$$\Psi_k(q_k') = \frac{\alpha}{\beta d_k}. \tag{29}$$

Since $\Psi_k(q)$ is strictly increasing in $q \in [\underline{q}, \overline{q}]$, we find (i) q_k' satisfying equation (29) is unique, (ii) $\frac{\partial}{\partial q_k} L_k(q_k) > 0$ for $q_k < q_k'$, and (iii) $\frac{\partial}{\partial q_k} L_k(q_k) < 0$ for $q_k > q_k'$. Hence $L_k(q_k)$ is strictly increasing in q_k for $q_k < q_k'$ and strictly decreasing in q_k for $q_k > q_k'$. Then q_k^* satisfying $\Psi_k(q_k^*) = \min\{\Psi_k(\overline{q}), \max\{\Psi_k(\underline{q}), \frac{\alpha}{\beta d_k}\}\}$, or equivalently, $q_k^* = \min\{\overline{q}, \max\{\underline{q}, \Psi_k^{-1}(\frac{\alpha}{\beta d_k})\}\}$ is the unique maximizer of $L_k(q)$ over $q \in [\underline{q}, \overline{q}]$. □

Proposition 2. *Socially efficient threshold q_k^s of a type-k candidate is unique over the interval $[\underline{q}, \overline{q}]$ and given by*

$$q_k^s = \min\{\overline{q}, \max\{\underline{q}, q_k'\}\}, \quad \text{for } k = 1, \ldots, K,$$

where q_k' is the unique solution of the following equation:

$$\beta\Psi_k(q') - c(q') = \frac{\alpha}{d_k}.$$

Proof of Proposition 2. Consider any candidate type $k = 1, \ldots, K$. Using equation (8), we have

$$\frac{\partial}{\partial q_k} S(\cdot) = \frac{\partial}{\partial q_k} \frac{\lambda_k}{\lambda} [L_k(q_k) - C_k(q_k)] = \frac{p_k(q_k)}{\lambda(\overline{q} - \underline{q})} \left[\frac{\alpha}{d_k} - \beta\Psi_k(q_k) + c(q_k)\right], \tag{30}$$

which gives $\frac{\partial}{\partial q_\cdot} S(\cdot) = 0$ when q_k' satisfies

$$\beta\Psi_k(q_k') - c(q_k') = \frac{\alpha}{d_k}, \tag{31}$$

which has a unique solution because $\beta\Psi_k(q_k) - c(q_k)$ is increasing in q_k since $\frac{\partial}{\partial q}\Psi_k(q) > 0$ and $\frac{\partial}{\partial q}c(q) \leq 0$ for $q \in [\underline{q}, \overline{q}]$. Similar to the proof of Proposition 1, we find that $S(\cdot)$ is strictly increasing in q_k for $q_k < q_k'$ and strictly decreasing in q_k for $q_k > q_k'$, where q_k' is the unique solution of equation (31). Therefore, the socially efficient threshold of type-k candidates, q_k^s, maximizing $S(\cdot)$ is given by $q_k^s = \min\{\overline{q}, \max\{\underline{q}, q_k'\}\}$. $\square$

Corollary 1. *For any candidate type k,*

$$\begin{aligned} q_k^* &< q_k^s \qquad \text{if } c(q_k^*) > 0, \\ q_k^* &\geq q_k^s \qquad \text{if } c(q_k^*) \leq 0. \end{aligned}$$

Proof of Corollary 1. We prove the results by contradiction. Consider any patient type k and the case $c(q_k^*) > 0$. Assume, to the contrary, that $q_k^* \geq q_k^s$. Then it must be that $\Psi_k(q_k^*) \geq \Psi_k(q_k^s)$ because $\Psi(q)$ is strictly increasing in q. Furthermore, $0 < c(q_k^*) \leq c(q_k^s)$, where the last inequality follows since $c(q)$ is decreasing in q. However, from Propositions 1 and 2, we have

$$\beta\Psi_k(q_k^s) = \frac{\alpha}{d_k} + c(q_k^s) = \beta\Psi_k(q_k^*) + c(q_k^s) > \beta\Psi_k(q_k^*),$$

which contradicts with the condition $\Psi_k(q_k^*) \geq \Psi_k(q_k^s)$. It must be that the initial assumption is incorrect, and therefore, $q_k^* < q_k^s$ must hold.

The proof for the case when $c(q_k^*) \leq 0$ follows similarly, and is omitted. $\square$

Proposition 3. *The equilibrium threshold q_k^e of type-k candidates exists uniquely and is given by the solution of the following equation*

$$\beta\Psi_k(q') - c(q') = \frac{\alpha}{d_k} \cdot [1 - \pi_k(q')] + \mathbb{E}_k[\beta\Psi_k(q) - c(q) \mid q \geq q'] \cdot \pi_k(q'). \tag{9}$$

Proof of Proposition 3. The equilibrium threshold q_k^e of a type-k candidate is the point where she is indifferent between accepting or rejecting the offered organ. For an organ with quality q_k^e, accepting the organ provides the benefit of transplantation, whereas rejecting this organ gives the patient an opportunity to transplant a higher quality organ at the risk of dying without a transplant. That is,

$$\beta\Psi_k(q_k) - c(q_k) = \frac{\alpha}{d_k}\cdot[1-\pi_k(q_k)] + \mathbb{E}_k[\beta\Psi_k(q) - c(q) \mid q \geq q_k]\cdot\pi_k(q_k) \tag{32a}$$

$$= \frac{\alpha}{d_k}[1-\pi_k(q_k)] + \pi_k(q_k)\cdot\int_{q_k}^{\overline{q}}[\beta\Psi_k(q) - c(q)]\frac{p_k(q)}{\int_{q_k}^{\overline{q}} p_k(q)\,dq}\,dq \tag{32b}$$

$$= \frac{\alpha}{d_k}\left(1 - \frac{\int_{q_k}^{\overline{q}} p_k(q)\,dq}{\lambda_k\left(\overline{q} - \underline{q}\right)}\right) + \beta\frac{1}{\lambda_k\left(\overline{q} - \underline{q}\right)}\int_{q_k}^{\overline{q}}\Psi_k(q)p_k(q)\,dq - \frac{1}{\lambda_k(\overline{q} - \underline{q})}\int_{q_k}^{\overline{q}} c(q)p_k(q)\,dq \tag{32c}$$

For notational convenience in the rest of the proof, let $\mathcal{L}(q_k)$ and $\mathcal{R}(q_k)$ denote the left- and right-hand side of equation (32c), respectively. Since $\Psi_k(q)$ is strictly increasing and $c(q)$ is decreasing in $q \in [\underline{q}, \overline{q}]$, $\mathcal{L}(q_k)$ is a strictly increasing function of q_k. Furthermore, note that $\mathcal{R}(q_k) = L_k(q_k) - C_k(q_k)$ (see equations (6) and (7c)). We have found in the proof of Proposition 2 that $L_k(q_k) - C_k(q_k)$ is decreasing when $q_k \geq q'_k$, where q'_k is the unique solution of the following equation:

$$\mathcal{L}(q'_k) := \beta\Psi_k(q'_k) - c(q'_k) = \frac{\alpha}{d_k}. \tag{33}$$

Observe that

$$\mathcal{R}(q'_k) = \frac{\alpha}{d_k} + \frac{1}{\lambda_k\left(\overline{q} - \underline{q}\right)}\left[\int_{q'_k}^{\overline{q}}\left(\beta\Psi_k(q) - c(q) - \frac{\alpha}{d_k}\right)p_k(q)\,dq\right] \tag{34a}$$

$$> \frac{\alpha}{d_k} + \frac{1}{\lambda_k\left(\overline{q} - \underline{q}\right)}\left[\int_{q'_k}^{\overline{q}}\left(\beta\Psi_k(q'_k) - c(q'_k) - \frac{\alpha}{d_k}\right)p_k(q)\,dq\right] \tag{34b}$$

$$= \frac{\alpha}{d_k} \tag{34c}$$

where inequality (34b) follows since $\beta\Psi_k(q) - c(q) = \mathcal{L}(q)$ is strictly increasing in q, which, when combined with equation (33), yields equation (34c). As a result, we find $\mathcal{L}(q'_k) < \mathcal{R}(q'_k)$. Furthermore,

$$\begin{aligned}\mathcal{R}^{)}(\overline{q}) &= \frac{\alpha}{d_k}\cdot[1-\pi_k(\overline{q})] + \mathbb{E}_k[\beta\Psi_k(q) - c(q) \mid q \geq \overline{q}]\cdot\pi_k(\overline{q}) \\ &\leq [\beta\Psi_k(\overline{q}) - c(\overline{q})]\cdot[1-\pi_k(\overline{q})] + [\beta\Psi_k(\overline{q}) - c(\overline{q})]\cdot\pi_k(\overline{q}) \\ &= \mathcal{L}(\overline{q}),\end{aligned}$$

where the inequality follows from equation (33) along with that $\beta\Psi_k(q) - c(q)$ is strictly increasing in q, and $\pi_k(\overline{q}) \in [0,1]$. This concludes, by the intermediate value theorem, that the equilibrium threshold q_k^e exists, and it is unique, in $(q'_k, \overline{q}]$.

On the other hand, when $q_k < q'_k$, we have (from equation (32b))

$$\begin{aligned}\mathcal{R}^{)}(q_k) &= \frac{\alpha}{d_k}[1-\pi_k(q_k)] + \pi_k(q_k)\cdot\int_{q_k}^{\overline{q}}[\beta\Psi_k(q) - c(q)]\frac{p_k(q)}{\int_{q_k}^{\overline{q}} p_k(q)\,dq}\,dq \\ &> \mathcal{L}(q_k)\cdot[1-\pi_k(q_k)] + \pi_k(q_k)\cdot[\beta\Psi_k(q_k) - c(q_k)] \\ &= \mathcal{L}(q_k),\end{aligned} \tag{35}$$

where inequality (35) follows since $\beta\Psi_k(q) - c(q) = \mathcal{L}(q)$ is strictly increasing in q and $\mathcal{L}(q_k) < \mathcal{L}(q'_k) = \frac{\alpha}{d_k}$. Therefore, equation (32) can not have any solution when $q_k < q'_k$.

This concludes that the equilibrium defined in equation (32) uniquely exists over $\left[\underline{q}, \overline{q}\right]$. $\square$

Corollary 2. *For any candidate type k, $q_k^s < q_k^e$.*

Proof of Corollary 2. For any candidate type k, consider the socially efficient threshold q_k^s and the equilibrium threshold q_k^e. Proposition 2 requires that q_k^s satisfies $\beta\Psi_k(q_k^s) - c(q_k^s) = \frac{\alpha}{d_k}$. We have also found in the proof of Proposition 3 that $q_k^e > q_k'$, where q_k' is the unique solution of equation (33) and, therefore, it coincides with q_k^s. This concludes that $q_k^e > q_k^s$. □

Proposition 4. *When returns are allowed, for any type-k candidate,*

(a) *QALE-maximizing threshold $\tilde{q}_k^*$ is unique the unique solution of the following equation:*

$$\beta\tilde{\Psi}_k(\tilde{q}) = \frac{\alpha}{d_k}(1 - r_k(\tilde{q})), \tag{17}$$

(b) *socially efficient threshold $\tilde{q}_k^s$ is the unique solution of the following equation:*

$$\beta\tilde{\Psi}_k(\tilde{q}) - c(\tilde{q}) = \frac{\alpha}{d_k}(1 - r_k(\tilde{q})). \tag{18}$$

Proof of Proposition 4. Consider any candidate type $k = 1, \ldots, K$.

(a) From equation (14), we have,

$$\begin{aligned}\tilde{L}_k(q_k) &= \frac{\alpha}{d_k}\left(1 + \frac{\mu_k(q_k)}{\lambda_k}\mathbb{E}_k[r_k(q) \mid q \geq q_k] - \frac{\mu_k(q_k)}{\lambda_k}\right) + \beta\frac{\mu_k(q_k)}{\lambda_k}\mathbb{E}_k\left[\tilde{\Psi}_k(q) \,\middle|\, q \geq q_k\right] \\ &= \frac{\alpha}{d_k}\left(1 - \frac{\int_{q_k}^{\overline{q}} p_k(q)\,dq}{\lambda_k(\overline{q} - \underline{q})}\right) + \frac{1}{\lambda_k(\overline{q} - \underline{q})}\int_{q_k}^{\overline{q}}\left[\frac{\alpha}{d_k}r_k(q)p_k(q) + \beta\tilde{\Psi}_k(q)p_k(q)\right]dq,\end{aligned}$$

and

$$\frac{\partial}{\partial q_k}\tilde{L}_k(q_k) = \frac{p_k(q_k)}{\lambda_k(\overline{q} - \underline{q})}\left[\frac{\alpha}{d_k} - \frac{\alpha}{d_k}r_k(q_k) - \beta\tilde{\Psi}_k(q_k)\right]. \tag{36}$$

Equation (13), combined with the assumption that $\Psi_k(q)$ is strictly increasing in q, implies that $\beta\tilde{\Psi}_k(q_k) + \frac{\alpha}{d_k}r_k(qk)$ is strictly increasing in q_k. Therefore, $\frac{\partial}{\partial q_k}\tilde{L}_k(q_k) \geq 0$ for $q_k \leq q'$ and $\frac{\partial}{\partial q_k}\tilde{L}_k(q_k) < 0$ for $q_k > q'$, where q' is the unique solution of

$$\frac{\alpha}{d_k} - \frac{\alpha}{d_k}r_k(q_k) - \beta\tilde{\Psi}_k(q_k) = 0. \tag{37}$$

Therefore, $\tilde{q}_k^*$ satisfying equation (17) is the unique maximizer of $\tilde{L}_k(q)$ over $q \in [\underline{q}, \overline{q}]$.

(b) From equation (16), we have

$$\begin{aligned}\frac{\partial}{\partial q_k}\tilde{S}(\cdot) &= \frac{\partial}{\partial q_k}\frac{\lambda_k}{\lambda}\left(\tilde{L}_k(q_k) - C_k(q_k)\right) \\ &= \frac{p_k(q_k)}{\lambda(\overline{q} - \underline{q})}\left[\frac{\alpha}{d_k} - \beta\tilde{\Psi}_k(q_k) - \frac{\alpha}{d_k}r_k(q_k) + c(q_k)\right],\end{aligned}$$

which yields $\frac{\partial}{\partial q_k}\tilde{S}(\cdot) = 0$ when q_k satisfies

$$\frac{\alpha}{d_k} - \beta\tilde{\Psi}_k(q_k) - \frac{\alpha}{d_k}r_k(q_k) + c(q_k) = 0, \tag{38}$$

which has a unique solution since $c(q_k)$ is decreasing, and $\beta\tilde{\Psi}_k(q_k) + \frac{\alpha}{d_k}r_k(q_k)$ is strictly increasing in q_k, and accordingly $\beta\tilde{\Psi}_k(q_k) + \frac{\alpha}{d_k}r_k(q_k) - c(q_k)$ is strictly increasing in q_k. As a result, $\tilde{S}(\cdot)$ is strictly increasing for $q_k \leq q'$, and strictly decreasing for $q_k > q'$, where q' is the unique solution of equation (38). Therefore, the socially efficient threshold $\tilde{q}_k^s$ of type-k candidates satisfying equation (18) is the unique maximizer of $\tilde{S}(\cdot)$ over $q \in [\underline{q}, \overline{q}]$. □

COROLLARY 3. *For any candidate type k, $\tilde{q}_k^* = q_k^*$ and $\tilde{q}_k^s = q_k^s$.*

Proof of Corollary 3. Consider any candidate type $k = 1, \dots, K$. We show that $\tilde{q}_k^s = \tilde{q}_k^s$, and omit the proof of $\tilde{q}_k^* = q_k^*$ as it follows similarly. Proposition 4(b) establishes that, when returns are allowed, the socially efficient threshold $\tilde{q}_k^s$ is obtained at the unique solution to the equation

$$\beta \tilde{\Psi}_k(q) + \alpha r_k(q) \frac{1}{d_k} - c(q) = \frac{\alpha}{d_k}. \tag{39}$$

Equations (13) and (39) imply that

$$\beta \Psi_k(q) - c(q) = \frac{\alpha}{d_k}. \tag{40}$$

Furthermore, Proposition 2 establishes that, when returns are not allowed, the socially efficient threshold q_k^s is the unique solution of equation (40). Therefore, the unique solutions $\tilde{q}_k^s$ and q_k^s must coincide. $\square$

PROPOSITION 5. *When returns are allowed, for any type-k candidate, equilibrium threshold $\tilde{q}_k^e$ exists and is given by a solution to the following equation*

$$\beta \tilde{\Psi}_k(\tilde{q}) - c(\tilde{q}) = \left(\frac{\alpha}{d_k} [1 - \tilde{\pi}_k(\tilde{q})] + \mathbb{E}_k \left[\beta \tilde{\Psi}_k(q) - c(q) \,\middle|\, q \geq \tilde{q} \right] \tilde{\pi}_k(\tilde{q}) \right) \left(\frac{\tilde{\lambda}_k(\tilde{q})}{\lambda_k} - r_k(\tilde{q}) \right). \tag{19}$$

Furthermore, the equilibrium is unique if $\frac{r_k(q)}{\tilde{\lambda}_k(q)}$ is nondecreasing in q.

Proof of Proposition 5. The equilibrium threshold $\tilde{q}_k^e$ of a type-k candidate is the point where she is indifferent between accepting or rejecting the offered organ. Accepting an organ with quality $\tilde{q}_k^e$ provides the benefit of transplanting $\tilde{q}_k^e$ and, if the candidate outlives this organ, a continuation payoff associated with returning to the waiting list for a repeat transplant opportunity. Rejecting $\tilde{q}_k^e$, on the other hand, exposes the patient to the risk of dying without a transplant but it also offers a potential benefit from transplanting a higher quality organ and, if the candidate receives an organ and outlives it, a continuation payoff associated with returning to the waiting list for a repeat transplant opportunity. That is, $\tilde{q}_k^e$ is a solution to

$$\begin{aligned}
&\beta \tilde{\Psi}_k(q_k) - c(q_k) + r_k(q_k) \cdot \left(\frac{\alpha}{d_k} \cdot [1 - \tilde{\pi}_k(q_k)] + \mathbb{E}_k \left[\beta \tilde{\Psi}_k(q) - c(q) \,\middle|\, q \geq q_k \right] \cdot \tilde{\pi}_k(q_k) \right) \\
&= \left(\frac{\alpha}{d_k} \cdot [1 - \tilde{\pi}_k(q_k)] + \mathbb{E}_k \left[\beta \tilde{\Psi}_k(q) - c(q) \,\middle|\, q \geq q_k \right] \cdot \tilde{\pi}_k(q_k) \right) \cdot \left(1 + \frac{\mu_k(q_k)}{\lambda_k} \mathbb{E}_k [r_k(q) \mid q \geq q_k] \right),
\end{aligned} \tag{41}$$

which, after substituting for $\tilde{\pi}_k(q_k)$ and $\mu_k(q_k)$, and rearranging terms, can be equivalently written as

$$\begin{aligned}
&\beta \tilde{\Psi}_k(q_k) - c(q_k) + \frac{\alpha}{d_k} r_k(q_k) \\
&= - \frac{r_k(q_k)}{\tilde{\lambda}_k(q_k)(\overline{q} - \underline{q})} \int_{q_k}^{\overline{q}} \left(\beta \tilde{\Psi}_k(q) - c(q) - \frac{\alpha}{d_k} \right) p_k(q) \, \mathrm{d}q \\
&\quad + \frac{\alpha}{d_k} + \frac{1}{\lambda_k(\overline{q} - \underline{q})} \int_{q_k}^{\overline{q}} \frac{\alpha}{d_k} r_k(q) p_k(q) \, \mathrm{d}q + \frac{1}{\lambda_k(\overline{q} - \underline{q})} \int_{q_k}^{\overline{q}} \left(\beta \tilde{\Psi}_k(q) - c(q) - \frac{\alpha}{d_k} \right) p_k(q) \, \mathrm{d}q \\
&= \frac{\alpha}{d_k} + \frac{1}{\lambda_k(\overline{q} - \underline{q})} \int_{q_k}^{\overline{q}} \frac{\alpha}{d_k} r_k(q) p_k(q) \, \mathrm{d}q + \left[\frac{1}{\lambda_k(\overline{q} - \underline{q})} - \frac{r_k(q_k)}{\tilde{\lambda}_k(q_k)(\overline{q} - \underline{q})} \right] \int_{q_k}^{\overline{q}} \left(\beta \tilde{\Psi}_k(q) - c(q) - \frac{\alpha}{d_k} \right) p_k(q) \, \mathrm{d}q \\
&= \frac{\alpha}{d_k} + \frac{r_k(q_k)}{\tilde{\lambda}_k(q_k)(\overline{q} - \underline{q})} \int_{q_k}^{\overline{q}} \frac{\alpha}{d_k} r_k(q) p_k(q) \, \mathrm{d}q \\
&\quad + \left[\frac{1}{\lambda_k(\overline{q} - \underline{q})} - \frac{r_k(q_k)}{\tilde{\lambda}_k(q_k)(\overline{q} - \underline{q})} \right] \int_{q_k}^{\overline{q}} \left(\beta \tilde{\Psi}_k(q) - c(q) - \frac{\alpha}{d_k} + \frac{\alpha}{d_k} r_k(q) \right) p_k(q) \, \mathrm{d}q
\end{aligned} \tag{42}$$

For notational convenience in the rest of the proof, let $\mathcal{L}(q_k)$ and $\mathcal{R}(q_k)$ denote the left and right hand-side of equation (42), respectively. Observe that, since $c(q)$ is decreasing in q, equation (13) implies that $\mathcal{L}(q_k)$ is strictly increasing in q_k. Furthermore, for $q_k \geq q'$, where q' satisfies

$$\beta\tilde{\Psi}_k(q') - c(q') - \frac{\alpha}{d_k} + \frac{\alpha}{d_k} r_k(q') = 0, \tag{43}$$

we have

$$\begin{aligned}
\frac{\partial}{\partial q_k}\mathcal{R}(q_k) = & \\
& \frac{\alpha}{d_k}\left[\frac{\partial r_k(q_k)}{\partial q_k}\left(1 + \frac{\lambda_k(\overline{q}-\underline{q})}{\int_{q_k}^{\overline{q}} r_k(q)p_k(q)\,\mathrm{d}q}\right) - \frac{\lambda_k(\overline{q}-\underline{q})r_k(q_k)p_k(q_k)}{\left(\int_{q_k}^{\overline{q}} r_k(q)p_k(q)\,\mathrm{d}q\right)^2} r_k(q_k)\right]\left(1 + \frac{\lambda_k(\overline{q}-\underline{q})}{\int_{q_k}^{\overline{q}} r_k(q)p_k(q)\,\mathrm{d}q}\right)^{-2} && \text{(44a)}\\
& - \left[\frac{1}{\lambda_k(\overline{q}-\underline{q})} - \frac{r_k(q_k)}{\tilde{\lambda}_k(q_k)(\overline{q}-\underline{q})}\right]\cdot\left(\beta\tilde{\Psi}_k(q_k) - c(q_k) - \frac{\alpha}{d_k} + \frac{\alpha}{d_k}r_k(q_k)\right)p_k(q_k) && \text{(44b)}\\
& - \frac{1}{(\overline{q}-\underline{q})}\left(\frac{\partial}{\partial q_k}\frac{r_k(q_k)}{\tilde{\lambda}_k(q_k)}\right)\cdot\int_{q_k}^{\overline{q}}\left(\beta\tilde{\Psi}_k(q) - c(q) - \frac{\alpha}{d_k} + \frac{\alpha}{d_k}r_k(q)\right)p_k(q)\,\mathrm{d}q && \text{(44c)}\\
& \leq 0, && \text{(44d)}
\end{aligned}$$

where the last inequality follows from observing (i) $\frac{\partial r_k(q_k)}{\partial q_k} \leq 0$ since $r_k(q_k)$ is assumed to be decreasing, and therefore, 44a is non-positive; (ii) $r_k(q_k) \leq 1$ and $\tilde{\lambda}_k(q_k) \geq \lambda_k$ imply that the term in square brackets in 44b is non-negative, and $\beta\tilde{\Psi}_k(q) - c(q) - \frac{\alpha}{d_k} + \frac{\alpha}{d_k}r_k(q)$ increasing in q and equals zero at $q = q'$, where q' is the solution of equation (43), imply that the term in parantheses in 44b is non-negative for $q_k \geq q'$, and therefore 44b is non-positive; and (iii) $\int_{q_k}^{\overline{q}}\left(\beta\tilde{\Psi}_k(q) + \frac{\alpha}{d_k}r_k(q) - c(q) - \frac{\alpha}{d_k}\right)p_k(q)\,\mathrm{d}q$ is non-negative for $q_k \geq q'$ due to the observation in (ii) and $\frac{\partial}{\partial q_k}\frac{r_k(q_k)}{\tilde{\lambda}_k(q_k)} \geq 0$ since $\frac{r_k(q)}{\tilde{\lambda}_k(q)}$ is assumed to be increasing in q, and therefore 44c is non-positive. It follows from inequality 44d that $\mathcal{R}(q_k)$ is decreasing in q_k for $q_k \geq q'$, where q' is the unique solution of equation (43). Therefore, if an equilibrium exists to the right of q', it must be unique. We proceed to prove that such an equilibrium exists by showing that $\mathcal{L}(q)$ crosses $\mathcal{R}(q)$ at some $q \in (q', \overline{q}]$. Observe that

$$\begin{aligned}
\mathcal{R}(q') &= \frac{\alpha}{d_k} + \frac{r_k(q')}{\tilde{\lambda}_k(q')(\overline{q}-\underline{q})}\cdot\int_{q'}^{\overline{q}}\frac{\alpha}{d_k}r_k(q)p_k(q)\,\mathrm{d}q \\
&\quad + \left[\frac{1}{\lambda_k(\overline{q}-\underline{q})} - \frac{r_k(q')}{\tilde{\lambda}_k(q')(\overline{q}-\underline{q})}\right]\cdot\int_{q'}^{\overline{q}}\left(\beta\tilde{\Psi}_k(q) - c(q) - \frac{\alpha}{d_k} + \frac{\alpha}{d_k}r_k(q)\right)p_k(q)\,\mathrm{d}q \\
&> \frac{\alpha}{d_k} + \left[\frac{1}{\lambda_k(\overline{q}-\underline{q})} - \frac{r_k(q')}{\tilde{\lambda}_k(q')(\overline{q}-\underline{q})}\right]\cdot\int_{q'}^{\overline{q}}\left(\beta\tilde{\Psi}_k(q) - c(q) - \frac{\alpha}{d_k} + \frac{\alpha}{d_k}r_k(q)\right)p_k(q)\,\mathrm{d}q \\
&\geq \frac{\alpha}{d_k} \\
&= \mathcal{L}(q'),
\end{aligned}$$

and

$$\begin{aligned}
\mathcal{R}(\overline{q}) &= \left(\frac{\alpha}{d_k}[1 - \tilde{\pi}_k(\overline{q})] + \mathbb{E}_k\left[\beta\tilde{\Psi}_k(q) - c(q)\,\middle|\, q \geq \overline{q}\right]\tilde{\pi}_k(\overline{q})\right)\cdot\left(\frac{\tilde{\lambda}_k(\overline{q})}{\lambda_k} - r_k(\overline{q})\right) + \frac{\alpha}{d_k}r_k(\overline{q}) \\
&\leq \left[[\beta\tilde{\Psi}_k(\overline{q}) - c(\overline{q})]\cdot[1 - \tilde{\pi}_k(\overline{q})] + [\beta\tilde{\Psi}_k(\overline{q}) - c(\overline{q})]\cdot\tilde{\pi}_k(\overline{q})\right]\cdot\left(\frac{\tilde{\lambda}_k(\overline{q})}{\lambda_k} - r_k(\overline{q})\right) + \frac{\alpha}{d_k}r_k(\overline{q})
\end{aligned}$$

$$
\begin{aligned}
&\leq \beta\tilde{\Psi}_k(\overline{q}) - c(\overline{q}) + \frac{\alpha}{d_k} r_k(\overline{q}) \qquad (45)\\
&= \mathcal{L}(\overline{q}),
\end{aligned}
$$

where inequality (45) follows from $\frac{\tilde{\lambda}_k(\overline{q})}{\lambda_k} - r_k(\overline{q}) = (1 + \pi_k(q_k)\mathbb{E}_k[r_k(q) \mid q \geq q_k]) - r_k(\overline{q}) \leq 1$. As a result, the equilibrium threshold $\tilde{q}_k^e$ that solves equation (41) exists uniquely in $(q', \overline{q}]$.

We next show that equation (41) cannot have any solution in $[\underline{q}, q')$. For any $q_k < q'$, we have

$$
\begin{aligned}
\mathcal{R}(q_k) = &\frac{\alpha}{d_k} + \left[\frac{1}{\lambda_k(\overline{q} - \underline{q})} - \frac{r_k(q_k)}{\tilde{\lambda}_k(q_k)\big(\overline{q} - \underline{q}\big)}\right] \cdot \int_{q_k}^{\overline{q}} \left(\beta\tilde{\Psi}_k(q) - c(q) - \frac{\alpha}{d_k} + \frac{\alpha}{d_k} r_k(q)\right) p_k(q)\,\mathrm{d}q\\
&+ \frac{r_k(q_k)}{\tilde{\lambda}_k(q_k)\big(\overline{q} - \underline{q}\big)} \cdot \int_{q_k}^{\overline{q}} \frac{\alpha}{d_k} r_k(q) p_k(q)\,\mathrm{d}q \qquad (46a)\\
> &\frac{\alpha}{d_k} \cdot \left[1 - \left(\frac{1}{\lambda_k(\overline{q} - \underline{q})} - \frac{r_k(q_k)}{\tilde{\lambda}_k(q_k)\big(\overline{q} - \underline{q}\big)}\right) \int_{q_k}^{\overline{q}} p_k(q)\,\mathrm{d}q\right]\\
&+ \left[\frac{1}{\lambda_k(\overline{q} - \underline{q})} - \frac{r_k(q_k)}{\tilde{\lambda}_k(q_k)\big(\overline{q} - \underline{q}\big)}\right] \cdot \left(\beta\tilde{\Psi}_k(q_k) - c(q_k) + \frac{\alpha}{d_k} r_k(q_k)\right) \int_{q_k}^{\overline{q}} p_k(q)\,\mathrm{d}q \qquad (46b)\\
\geq &\mathcal{L}(q_k) \cdot \left[1 - \left(\frac{1}{\lambda_k(\overline{q} - \underline{q})} - \frac{r_k(q_k)}{\tilde{\lambda}_k(q_k)\big(\overline{q} - \underline{q}\big)}\right) \int_{q_k}^{\overline{q}} p_k(q)\,\mathrm{d}q\right]\\
&+ \mathcal{L}(q_k) \cdot \left(\frac{1}{\lambda_k(\overline{q} - \underline{q})} - \frac{r_k(q_k)}{\tilde{\lambda}_k(q_k)\big(\overline{q} - \underline{q}\big)}\right) \int_{q_k}^{\overline{q}} p_k(q)\,\mathrm{d}q \qquad (46c)\\
= &\mathcal{L}(q_k),
\end{aligned}
$$

where inequality 46b follows from rearranging 46a after dropping its non-negative last term and that $\beta\tilde{\Psi}_k(q) - c(q) + \frac{\alpha}{d_k} r_k(q) = \mathcal{L}(q)$ is strictly increasing in q; and inequality 46c follows after observing that $\mathcal{L}(q_k) \leq \frac{\alpha}{d_k}$ for $q_k \leq q'$, and $\left(\frac{1}{\lambda_k(\overline{q} - \underline{q})} - \frac{r_k(q_k)}{\tilde{\lambda}_k(q_k)\big(\overline{q} - \underline{q}\big)}\right) \int_{q_k}^{\overline{q}} p_k(q)\,\mathrm{d}q \leq \frac{1}{\lambda_k(\overline{q} - \underline{q})} \int_{q_k}^{\overline{q}} p_k(q)\,\mathrm{d}q = \pi_k(q_k) \leq 1$. This shows that equation (41) cannot have any solution in $[\underline{q}, q')$, and concludes the proof. $\square$

Corollary 4. *For any candidate type k, $\tilde{q}_k^s < \tilde{q}_k^e \leq q_k^e$. Furthermore, $\tilde{q}_k^e = q_k^e$ iff $r_k(\tilde{q}_k^e) = 0$.*

Proof of Corollary 4. We have established in the proof of Proposition 5 that $\tilde{q}_k^e > q'$, where q' is the unique solution of equation (43), which coincides with $\tilde{q}_k^s$ (see Proposition 4(b)). This establishes $\tilde{q}_k^s < \tilde{q}_k^e$.

Furthermore, we have

$$
\begin{aligned}
\mathcal{L}(\tilde{q}_k^e) :=&\beta\Psi_k(\tilde{q}_k^e) - c(\tilde{q}_k^e)\\
=&\beta\tilde{\Psi}_k(\tilde{q}_k^e) - c(\tilde{q}_k^e) + \frac{\alpha}{d_k} r_k(\tilde{q}_k^e) \qquad (47a)\\
=&\frac{\alpha}{d_k} + \frac{1}{\lambda_k(\overline{q} - \underline{q})} \cdot \int_{\tilde{q}_k^e}^{\overline{q}} \left(\beta\tilde{\Psi}_k(q) + \frac{\alpha}{d_k} r_k(q) - c(q) - \frac{\alpha}{d_k}\right) p_k(q)\,\mathrm{d}q\\
&- \frac{r_k(\tilde{q}_k^e)}{\big(\overline{q} - \underline{q}\big)\tilde{\lambda}_k(\tilde{q}_k^e)} \cdot \int_{\tilde{q}_k^e}^{\overline{q}} \left(\beta\tilde{\Psi}_k(q) - c(q) - \frac{\alpha}{d_k}\right) p_k(q)\,\mathrm{d}q \qquad (47b)\\
\leq&\frac{\alpha}{d_k} + \frac{1}{\lambda_k(\overline{q} - \underline{q})} \cdot \int_{\tilde{q}_k^e}^{\overline{q}} \left(\beta\tilde{\Psi}_k(q) + \frac{\alpha}{d_k} r_k(q) - c(q) - \frac{\alpha}{d_k}\right) p_k(q)\,\mathrm{d}q \qquad (47c)\\
=&\frac{\alpha}{d_k} \cdot [1 - \pi_k(\tilde{q}_k^e)] + \mathbb{E}_k[\beta\Psi_k(q) - c(q) \mid q \geq \tilde{q}_k^e] \cdot \pi_k(\tilde{q}_k^e)\\
=&: \mathcal{R}(\tilde{q}_k^e),
\end{aligned}
$$

where equations (47a) and (47b) follow from equation (13) and Proposition 5, respectively, and inequality 47c follows from the assumption that transplantation, on average, is associated with higher life benefits than that from waiting until death without a transplant (i.e., $\mathbb{E}_k\left[\beta\tilde{\Psi}_k(q) - c(q) \,\middle|\, q \geq \tilde{q}_k^e\right] > \frac{\alpha}{d_k}$).

We have established in the proof of Proposition 3 that $\mathcal{L}(q)$ is strictly increasing in q, $\mathcal{R}(q)$ is strictly decreasing in q for $q \geq q_k^s = \tilde{q}_k^s$ (last equality is due to Corollary 3), and $\mathcal{L}(q_k^e) = \mathcal{R}(q_k^e)$. This combined with $\mathcal{L}(\tilde{q}_k^e) \leq \mathcal{R}(\tilde{q}_k^e)$ implies $\tilde{q}_k^e \leq q_k^e$, with strict inequalities holding if and only if $r_k(\tilde{q}_k^e)$ is nonzero. $\square$

Proposition 6. *Under any admissible $(\delta, \mathring{q})$ policy, for any type-k candidate, any equilibrium threshold $\tilde{q}_k^{e,(\delta,\mathring{q})} \in [\underline{q}, \mathring{q}]$ solves the following equation:*

$$\begin{aligned}&\underbrace{\beta\tilde{\Psi}_k(\tilde{q}) - c(\tilde{q})}^{\substack{\text{immediate benefit from}\\ \text{transplanting } \tilde{q}}} + \overbrace{r_k(\tilde{q})}^{\substack{\text{prob. of}\\ \text{return}}} \Bigg(\delta\, \overbrace{\mathbb{E}_k\left[\beta\tilde{\Psi}_k(q) - c(q) \,\middle|\, q \geq \tilde{q}\right]}^{\substack{\text{expected benefit from transplan-}\\ \text{tation given priority}}} + (1-\delta)\Bigg(\overbrace{\frac{\alpha}{d_k}\left(1 - \tilde{\pi}_k^\delta(\tilde{q})\right) + \mathbb{E}_k\left[\beta\tilde{\Psi}_k(q) - c(q) \,\middle|\, q \geq \tilde{q}\right]\tilde{\pi}_k^\delta(\tilde{q})}^{\substack{\text{expected benefit from wait-listing}\\ \text{given no priority}}}\Bigg)\Bigg)\\ &= \Bigg(\underbrace{\frac{\alpha}{d_k}\left(1 - \tilde{\pi}_k^\delta(\tilde{q})\right) + \mathbb{E}_k\left[\beta\tilde{\Psi}_k(q) - c(q) \,\middle|\, q \geq \tilde{q}\right]\tilde{\pi}_k^\delta(\tilde{q})}_{\substack{\text{expected benefit from wait-listing}\\ \text{given no priority}}}\Bigg)\Bigg(1 + \underbrace{\frac{(1-\delta)\overset{\star}{\lambda}_k(\tilde{q}) + \mathring{\lambda}_k(\tilde{q})}{\lambda_k}}_{\substack{\text{fraction of}\\ \text{non-prioritized}\\ \text{returns}}}\Bigg) + \underbrace{\frac{\delta\overset{\star}{\lambda}_k(\tilde{q})}{\lambda_k}}_{\substack{\text{fraction of}\\ \text{prioritized}\\ \text{returns}}}\ \underbrace{\mathbb{E}_k\left[\beta\tilde{\Psi}_k(q) - c(q) \,\middle|\, q \geq \tilde{q}\right]}_{\substack{\text{expected benefit from}\\ \text{transplantation given priority}}},\end{aligned} \tag{24}$$

and any equilibrium threshold $\tilde{q}_k^{e,(\delta,\mathring{q})} \in [\mathring{q}, \overline{q}]$ solves equation (24) with $\delta = 0$.

(a) *If $\mathring{q} \geq \tilde{q}_k^s$, and*

(a-1) *If $\mathring{q} \geq \tilde{q}_k^e$, then $\tilde{q}_k^{e,(\delta,\mathring{q})}$ exists only in the interval $[\underline{q}, \mathring{q}]$ and it is unique when $\frac{r_k(q)\tilde{\pi}_k^\delta(q)}{\mu_k(q)}$ is nondecreasing in q.*

(a-2) *If $\mathring{q} < \tilde{q}_k^e$, then $\tilde{q}_k^{e,(\delta,\mathring{q})}$ exists uniquely in the interval $(\mathring{q}, \overline{q}]$ and equals $\tilde{q}_k^e$. Furthermore, any solution to equation (24) in the interval $[\underline{q}, \mathring{q}]$ leads to multiple equilibria.*

(b) *If $\mathring{q} < \tilde{q}_k^s$, then $\tilde{q}_k^{e,(\delta,\mathring{q})}$ exists uniquely in the interval $[\underline{q}, \overline{q}]$ and equals $\tilde{q}_k^e$.*

Moreover, no equilibrium exists in the interval $[\underline{q}, \tilde{q}_k^s)$.

Proof of Proposition 6. Consider any candidate type $k = 1, \ldots, K$. Under any admissible $(\delta, \mathring{q})$ policy, the equilibrium threshold $\tilde{q}_k^{e,(\delta,\mathring{q})}$ of a type-k candidate is the point where she is indifferent between accepting or rejecting the offered organ. Candidates utilizing $\tilde{q}_k^{e,(\delta,\mathring{q})} \in [\underline{q}, \mathring{q}]$ as their acceptance threshold are *potentially* eligible for priority. Therefore, any equilibrium $\tilde{q}_k^{e,(\delta,\mathring{q})} \in [\underline{q}, \mathring{q}]$ should solve equation (24). On the other hand, candidates utilizing $\tilde{q}_k^{e,(\delta,\mathring{q})} \in (\mathring{q}, \overline{q}]$ as their acceptance threshold are *not* eligible for priority. Therefore, the search for any equilibrium in $(\mathring{q}, \overline{q}]$ should be performed under the baseline setting studied in Section 4, in which re-listed candidates are not prioritized. Accordingly, $\tilde{q}_k^{e,(\delta,\mathring{q})} \in (\mathring{q}, \overline{q}]$ should solve equation (24) after setting $\delta = 0$, which is equivalent to equation (19).

The left hand-side of equation (24) can be re-organized as

$$\begin{aligned}&\beta\tilde{\Psi}_k(\tilde{q}) - c(\tilde{q}) + r_k(\tilde{q})\left(\delta\mathbb{E}_k\left[\beta\tilde{\Psi}_k(q) - c(q) \,\middle|\, q \geq \tilde{q}\right] + (1-\delta)\left[\frac{\alpha}{d_k}\left(1 - \tilde{\pi}_k^\delta(\tilde{q})\right) + \mathbb{E}_k\left[\beta\tilde{\Psi}_k(q) - c(q) \,\middle|\, q \geq \tilde{q}\right]\tilde{\pi}_k^\delta(\tilde{q})\right]\right)\\ &= \beta\tilde{\Psi}_k(\tilde{q}) - c(\tilde{q}) + r_k(\tilde{q})\left(\delta\left[\frac{\alpha}{d_k} + \mathbb{E}_k\left[\beta\tilde{\Psi}_k(q) - c(q) - \frac{\alpha}{d_k} \,\middle|\, q \geq \tilde{q}\right]\right] + (1-\delta)\left[\frac{\alpha}{d_k} + \mathbb{E}_k\left[\beta\tilde{\Psi}_k(q) - c(q) - \frac{\alpha}{d_k} \,\middle|\, q \geq \tilde{q}\right]\tilde{\pi}_k^\delta(\tilde{q})\right]\right)\\ &= \beta\tilde{\Psi}_k(\tilde{q}) - c(\tilde{q}) + \frac{\alpha}{d_k}r_k(\tilde{q}) + r_k(\tilde{q}) \cdot \left[\delta + (1-\delta)\tilde{\pi}_k^\delta(\tilde{q})\right] \cdot \mathbb{E}_k\left[\beta\tilde{\Psi}_k(q) - c(q) - \frac{\alpha}{d_k} \,\middle|\, q \geq \tilde{q}\right].\end{aligned} \tag{48}$$

Similarly, the right-hand side of equation (24) can be re-organized as

$$
\begin{aligned}
&\left(\frac{\alpha}{d_k}\left(1-\tilde{\pi}_k^\delta(\tilde{q})\right)+\mathbb{E}_k\left[\beta\tilde{\Psi}_k(q)-c(q)\,\middle|\,q\geq\tilde{q}\right]\tilde{\pi}_k^\delta(\tilde{q})\right)\left(1+\frac{(1-\delta)\overset{\star}{\lambda}_k(\tilde{q})+\mathring{\lambda}_k(\tilde{q})}{\lambda_k}\right)+\frac{\delta\overset{\star}{\lambda}_k(\tilde{q})}{\lambda_k}\mathbb{E}_k\left[\beta\tilde{\Psi}_k(q)-c(q)\,\middle|\,q\geq\tilde{q}\right]\\
&=\left(\frac{\alpha}{d_k}\left(1-\tilde{\pi}_k^\delta(\tilde{q})\right)+\mathbb{E}_k\left[\beta\tilde{\Psi}_k(q)-c(q)\,\middle|\,q\geq\tilde{q}\right]\tilde{\pi}_k^\delta(\tilde{q})\right)\left(\frac{\tilde{\lambda}_k(\tilde{q})-\delta\overset{\star}{\lambda}_k(\tilde{q})}{\lambda_k}\right)+\frac{\delta\overset{\star}{\lambda}_k(\tilde{q})}{\lambda_k}\mathbb{E}_k\left[\beta\tilde{\Psi}_k(q)-c(q)\,\middle|\,q\geq\tilde{q}\right]\\
&=\frac{\alpha}{d_k}\left(\frac{\tilde{\lambda}_k(\tilde{q})-\delta\overset{\star}{\lambda}_k(\tilde{q})}{\lambda_k}-\frac{\mu_k(\tilde{q})-\delta\overset{\star}{\lambda}_k(\tilde{q})}{\lambda_k}\right)+\mathbb{E}_k\left[\beta\tilde{\Psi}_k(q)-c(q)\,\middle|\,q\geq\tilde{q}\right]\left(\frac{\mu_k(\tilde{q})-\delta\overset{\star}{\lambda}_k(\tilde{q})}{\lambda_k}+\frac{\delta\overset{\star}{\lambda}_k(\tilde{q})}{\lambda_k}\right)\\
&=\frac{\alpha}{d_k}\left(\frac{\tilde{\lambda}_k(\tilde{q})}{\lambda_k}-\frac{\mu_k(\tilde{q})}{\lambda_k}\right)+\mathbb{E}_k\left[\beta\tilde{\Psi}_k(q)-c(q)\,\middle|\,q\geq\tilde{q}\right]\frac{\mu_k(\tilde{q})}{\lambda_k} \qquad (49)\\
&=\left(\frac{\alpha}{d_k}[1-\tilde{\pi}_k(\tilde{q})]+\mathbb{E}_k\left[\beta\tilde{\Psi}_k(q)-c(q)\,\middle|\,q\geq\tilde{q}\right]\tilde{\pi}_k(\tilde{q})\right)\cdot\frac{\tilde{\lambda}_k(\tilde{q})}{\lambda_k}.\\
&=\tilde{L}_k(q_k)-\tilde{C}_k(q_k).
\end{aligned}
$$

Combining equations (48) and (49), equation (24) can be equivalently written as

$$
\begin{aligned}
&\beta\tilde{\Psi}_k(\tilde{q})-c(\tilde{q})+\frac{\alpha}{d_k}r_k(\tilde{q})\\
=&\frac{\alpha}{d_k}\left(\frac{\tilde{\lambda}_k(\tilde{q})}{\lambda_k}-\frac{\mu_k(\tilde{q})}{\lambda_k}\right)+\mathbb{E}_k\left[\beta\tilde{\Psi}_k(q)-c(q)\,\middle|\,q\geq\tilde{q}\right]\frac{\mu_k(\tilde{q})}{\lambda_k}-r_k(\tilde{q})\cdot\left[\delta+(1-\delta)\tilde{\pi}_k^\delta(\tilde{q})\right]\cdot\mathbb{E}_k\left[\beta\tilde{\Psi}_k(q)-c(q)-\frac{\alpha}{d_k}\,\middle|\,q\geq\tilde{q}\right]\\
=&\frac{\alpha}{d_k}+\frac{\mu_k(\tilde{q})}{\lambda_k}\cdot\mathbb{E}_k\left[\frac{\alpha}{d_k}r_k(q)\,\middle|\,q\geq\tilde{q}\right]+\frac{\mu_k(\tilde{q})}{\lambda_k}\cdot\mathbb{E}_k\left[\beta\tilde{\Psi}_k(q)-c(q)-\frac{\alpha}{d_k}\,\middle|\,q\geq\tilde{q}\right]\\
&-r_k(\tilde{q})\cdot\left[\delta+(1-\delta)\tilde{\pi}_k^\delta(\tilde{q})\right]\cdot\mathbb{E}_k\left[\beta\tilde{\Psi}_k(q)-c(q)-\frac{\alpha}{d_k}\,\middle|\,q\geq\tilde{q}\right] \qquad (50)
\end{aligned}
$$

For notational convenience, let $\mathcal{L}(\tilde{q})$ and $\mathcal{R}^{(\delta,\mathring{q})}(\tilde{q})$ denote the left and right hand-side of equation (50), respectively, under the $(\delta,\mathring{q})$ policy. We next show that $\mathcal{R}^{(\delta,\mathring{q})}(\cdot)$ is decreasing in δ.

$$
\begin{aligned}
\frac{\partial}{\partial\delta}\mathcal{R}^{(\delta,\mathring{q})}(\tilde{q})=&-\frac{\partial}{\partial\delta}\left\{r_k(\tilde{q})\cdot\left[\delta+(1-\delta)\tilde{\pi}_k^\delta(\tilde{q})\right]\cdot\mathbb{E}_k\left[\beta\tilde{\Psi}_k(q)-c(q)-\frac{\alpha}{d_k}\,\middle|\,q\geq\tilde{q}\right]\right\} \qquad (51)\\
=&-r_k(\tilde{q})\mathbb{E}_k\left[\beta\tilde{\Psi}_k(q)-c(q)-\frac{\alpha}{d_k}\,\middle|\,q\geq\tilde{q}\right]\\
&\cdot\frac{\left(\tilde{\lambda}_k(\tilde{q})-\mu_k(\tilde{q})-\overset{\star}{\lambda}_k(\tilde{q})\right)\left(\tilde{\lambda}_k(\tilde{q})-\delta\overset{\star}{\lambda}_k(\tilde{q})\right)+\overset{\star}{\lambda}_k(\tilde{q})\left(\delta\tilde{\lambda}_k(\tilde{q})+(1-\delta)\mu_k(\tilde{q})-\delta\overset{\star}{\lambda}_k(\tilde{q})\right)}{\left(\tilde{\lambda}_k(\tilde{q})-\delta\overset{\star}{\lambda}_k(\tilde{q})\right)^2}\\
=&-r_k(\tilde{q})\mathbb{E}_k\left[\beta\tilde{\Psi}_k(q)-c(q)-\frac{\alpha}{d_k}\,\middle|\,q\geq\tilde{q}\right]\cdot\frac{\left(\tilde{\lambda}_k(\tilde{q})-\mu_k(\tilde{q})\right)\left(\tilde{\lambda}_k(\tilde{q})-\overset{\star}{\lambda}_k(\tilde{q})\right)}{\left(\tilde{\lambda}_k(\tilde{q})-\delta\overset{\star}{\lambda}_k(\tilde{q})\right)^2}\\
<&0, \qquad (52)
\end{aligned}
$$

where inequality (52) follows since, by assumption, the expectation term is positive (transplantation, on average, provides higher life benefits than time until death without a transplant) and $\tilde{\lambda}_k(\tilde{q})\geq\mu_k(\tilde{q})$.

(a) Assume $\mathring{q}\geq\tilde{q}_k^s$.

(a-1) Given $\mathring{q}\geq\tilde{q}_k^e$. We first show that no equilibrium exists in the interval $(\mathring{q},\overline{q}]$. Assume, to the contrary, that there exists an equilibrium $\tilde{q}_k^{e,(\delta,\mathring{q})}\in(\mathring{q},\overline{q}]$. As noted above, $\tilde{q}_k^{e,(\delta,\mathring{q})}$ should solve equation (19). Proposition 5 establishes that the unique solution of equation (19) is $\tilde{q}_k^e$, but $\tilde{q}_k^{e,(\delta,\mathring{q})}=\tilde{q}_k^e\leq\mathring{q}$ contradicts with the assumption that $\tilde{q}_k^{e,(\delta,\mathring{q})}\in(\mathring{q},\overline{q}]$.

We next show that an equilibrium exists in the interval $[\underline{q}, \mathring{q}]$. For $\tilde{q} = \overline{q}$, since $\mathcal{R}^{(\delta,\mathring{q})}(\tilde{q})$ is decreasing in δ, we have $\mathcal{R}^{(\delta,\mathring{q})}(\overline{q}) \le \mathcal{R}^{(0,\mathring{q})}(\overline{q}) \le \mathcal{L}(\overline{q})$, where the last inequality follows from inequality (45). For $\tilde{q} = \tilde{q}_k^s$, where $\tilde{q}_k^s$ is the socially efficient threshold satisfying equation (18), we have $\mathcal{L}(\tilde{q}_k^s) = \frac{\alpha}{d_k}$ and $\mathcal{R}^{(\delta,\mathring{q})}(\tilde{q}_k^s) \ge \mathcal{R}^{(\delta_k^s,\mathring{q})}(\tilde{q}_k^s)$ since $\delta \in [0, \delta_k^s]$ for any admissible $(\delta, \mathring{q})$ policy, where δ_k^s is defined in Definition 1. Substituting equation (23) from Definition 1 into equation (50), we find $\mathcal{R}^{(\delta_k^s,\mathring{q})}(\tilde{q}_k^s) = \frac{\alpha}{d_k}$, implying that $\mathcal{R}^{(\delta,\mathring{q})}(\tilde{q}_k^s) \ge \mathcal{L}(\tilde{q}_k^s)$ for any admissible $(\delta, \mathring{q})$ policy. By intermediate value theorem, we conclude that there exists an equilibrium $\tilde{q}_k^{e,(\delta,\mathring{q})}$ that solves equation (24) in $[\tilde{q}_k^s, \mathring{q}]$.

Finally, to show the uniqueness of the equilibrium, we start with showing that equation (50) cannot have any solution in $\tilde{q} \in [\underline{q}, \tilde{q}_k^s)$. For any admissible $(\delta, \mathring{q})$ policy and $\tilde{q} \in [\underline{q}, \tilde{q}_k^s)$, we have

$$\begin{aligned}
&\mathcal{R}^{(\delta,\mathring{q})}(\tilde{q}) \ge \mathcal{R}^{(\delta_k^s,\mathring{q})}(\tilde{q}) \\
&\quad = \frac{\alpha}{d_k} + \left(\frac{\mu_k(q')}{\lambda_k} - r_k(\tilde{q}) \cdot \left[\delta_k^s + (1-\delta_k^s) \tilde{\pi}_k^{\delta_k^s}(\tilde{q}) \right] \right) \mathbb{E}_k \left[\beta \tilde{\Psi}_k(q) - c(q) - \frac{\alpha}{d_k} + \frac{\alpha}{d_k} r_k(q) \,\middle|\, q \ge \tilde{q} \right] \\
&\qquad\qquad + r_k(\tilde{q}) \cdot \left[\delta_k^s + (1-\delta_k^s) \tilde{\pi}_k^{\delta_k^s}(\tilde{q}) \right] \cdot \mathbb{E}_k \left[\frac{\alpha}{d_k} r_k(q) \,\middle|\, q \ge \tilde{q} \right] && \text{(53a)} \\
&\quad \ge \frac{\alpha}{d_k} \left[1 - \left(\frac{\mu_k(q')}{\lambda_k} - r_k(\tilde{q}) \cdot \left[\delta_k^s + (1-\delta_k^s) \tilde{\pi}_k^{\delta_k^s}(\tilde{q}) \right] \right) \right] \\
&\qquad\qquad + \left(\frac{\mu_k(q')}{\lambda_k} - r_k(\tilde{q}) \cdot \left[\delta_k^s + (1-\delta_k^s) \tilde{\pi}_k^{\delta_k^s}(\tilde{q}) \right] \right) \left(\beta \tilde{\Psi}_k(\tilde{q}) - c(\tilde{q}) + \frac{\alpha}{d_k} r_k(\tilde{q}) \right) && \text{(53b)} \\
&\quad > \mathcal{L}(\tilde{q}), && \text{(53c)}
\end{aligned}$$

where inequality 53b follows from rearranging 53a after dropping its non-negative last term and that $\beta \tilde{\Psi}_k(q) - c(q) + \frac{\alpha}{d_k} r_k(q) = \mathcal{L}(q)$ is strictly increasing in q; and inequality 53c follows after observing that $\mathcal{L}(\tilde{q}) < \frac{\alpha}{d_k}$ for $\tilde{q} < \tilde{q}_k^s$. Inequality 53c implies that equation (50) cannot have a solution in $[\underline{q}, \tilde{q}_k^s)$.

To complete the proof for the uniqueness of the equilibrium, observe that equation (50) can be equivalently written as

$$\begin{aligned}
&\beta \tilde{\Psi}_k(\tilde{q}) - c(\tilde{q}) + r_k(\tilde{q}) \left(\delta \mathbb{E}_k \left[\beta \tilde{\Psi}_k(q) - c(q) \,\middle|\, q \ge \tilde{q} \right] + (1-\delta) \frac{\alpha}{d_k} \right) \\
&\quad = \frac{\alpha}{d_k} + \frac{\mu_k(\tilde{q})}{\lambda_k} \cdot \mathbb{E}_k \left[\frac{\alpha}{d_k} r_k(q) \,\middle|\, q \ge \tilde{q} \right] + \left(\frac{\mu_k(\tilde{q})}{\lambda_k} - (1-\delta) r_k(\tilde{q}) \tilde{\pi}_k^{\delta}(\tilde{q}) \right) \mathbb{E}_k \left[\beta \tilde{\Psi}_k(q) - c(q) - \frac{\alpha}{d_k} \,\middle|\, q \ge \tilde{q} \right] \\
&\quad = \frac{\alpha}{d_k} + \frac{\mu_k(\tilde{q})}{\lambda_k} \mathbb{E}_k \left[\frac{\alpha}{d_k} r_k(q) \,\middle|\, q \ge \tilde{q} \right] + \left(\frac{1}{\lambda_k} - (1-\delta) \frac{r_k(\tilde{q}) \tilde{\pi}_k^{\delta}(\tilde{q})}{\mu_k(\tilde{q})} \right) \mu_k(\tilde{q}) \mathbb{E}_k \left[\beta \tilde{\Psi}_k(q) - c(q) - \frac{\alpha}{d_k} \,\middle|\, q \ge \tilde{q} \right]. && \text{(54)}
\end{aligned}$$

The left-hand side of equation (54) is strictly increasing in $\tilde{q}$ since

$$\begin{aligned}
&\frac{\partial}{\partial \tilde{q}} \left(\beta \tilde{\Psi}_k(\tilde{q}) - c(\tilde{q}) + r_k(\tilde{q}) \left(\delta \mathbb{E}_k \left[\beta \tilde{\Psi}_k(q) - c(q) \,\middle|\, q \ge \tilde{q} \right] + (1-\delta) \frac{\alpha}{d_k} \right) \right) \\
&\quad = \frac{\partial}{\partial \tilde{q}} \left(\beta \tilde{\Psi}_k(\tilde{q}) - c(\tilde{q}) + (1-\delta) \frac{\alpha}{d_k} r_k(\tilde{q}) \right) + \delta \frac{\partial}{\partial \tilde{q}} \left(r_k(\tilde{q}) \mathbb{E}_k \left[\beta \tilde{\Psi}_k(q) - c(q) \,\middle|\, q \ge \tilde{q} \right] \right) \\
&\quad \ge \frac{\partial}{\partial \tilde{q}} \left(\beta \tilde{\Psi}_k(\tilde{q}) - c(\tilde{q}) + (1-\delta) \frac{\alpha}{d_k} r_k(\tilde{q}) \right) + \delta \mathbb{E}_k \left[\beta \tilde{\Psi}_k(q) - c(q) \,\middle|\, q \ge \tilde{q} \right] \frac{\partial}{\partial \tilde{q}} r_k(\tilde{q}) && \text{(55a)} \\
&\quad > \frac{\partial}{\partial \tilde{q}} \left(\beta \tilde{\Psi}_k(\tilde{q}) - c(\tilde{q}) + r_k(\tilde{q}) \left[\beta \tilde{\Psi}_k(\overline{q}) - c(\overline{q}) \right] \right) && \text{(55b)} \\
&\quad \ge 0, && \text{(55c)}
\end{aligned}$$

where inequality 55a follows from that $\frac{\partial}{\partial \tilde{q}}\mathbb{E}_k\left[\beta\tilde{\Psi}_k(q) - c(q) \,\middle|\, q \geq \tilde{q}\right] > 0$ since $\beta\tilde{\Psi}_k(q) - c(q)$ is strictly increasing in $\tilde{q}$; inequality 55b follows since $\frac{\partial}{\partial \tilde{q}} r_k(\tilde{q}) \leq 0$ and $\beta\tilde{\Psi}_k(q) - c(q)$ is strictly increasing in $\tilde{q}$; and equation (55c) follows since $\frac{\partial}{\partial \tilde{q}} c(\tilde{q}) \leq 0$ and equation (13) implies $\beta\tilde{\Psi}_k(\tilde{q}) + r_k(\tilde{q})\beta\tilde{\Psi}_k(\overline{q})$ is nondecreasing in $\tilde{q}$. Furthermore, $\frac{r_k(\tilde{q})\tilde{\pi}_k^{\delta}(\tilde{q})}{\mu_k(\tilde{q})}$ is nondecreasing in $\tilde{q}$ by assumption, and we have

$$\frac{1}{\lambda_k} - (1-\delta)\frac{r_k(\tilde{q})\tilde{\pi}_k^{\delta}(\tilde{q})}{\mu_k(\tilde{q})} \geq \frac{1}{\lambda_k} - (1-\delta)\frac{r_k(\tilde{q})}{\tilde{\lambda}_k(\tilde{q}) - \delta\tilde{\lambda}_k(\tilde{q})} = \frac{1}{\lambda_k} - \frac{r_k(\tilde{q})}{\tilde{\lambda}_k(\tilde{q})} \geq 0. \tag{56}$$

Therefore, each of the three terms in the right hand-side of equation (54) is positive decreasing in $\tilde{q}$, which imply that the right hand-side of equation (54) is decreasing in $\tilde{q}$. This concludes that the equilibrium threshold is unique.

(a-2) Given $\mathring{q} < \tilde{q}_k^e$. We first show that $\tilde{q}_k^{e,(\delta,\mathring{q})}$ exists uniquely in the interval $(\mathring{q}, \overline{q}]$ and equals $\tilde{q}_k^e$. As noted above, $\tilde{q}_k^{e,(\delta,\mathring{q})}$ should solve equation (19). Observing that $\tilde{q}_k^e$ is the unique solution of equation (19) (see Proposition 5), and $\mathring{q} < \tilde{q}_k^e$, we conclude that $\tilde{q}_k^{e,(\delta,\mathring{q})} = \tilde{q}_k^e$ is the unique equilibrium in the interval $(\mathring{q}, \overline{q}]$. Furthermore, if equation (24) also has a solution $\tilde{q}_k^{e,(\delta,\mathring{q})}$ in the interval $[\underline{q}, \mathring{q}]$, then multiple equilibria exist.

(b) Assume $\mathring{q} < \tilde{q}_k^s$. We have shown in the proof of part (a-1) that equation (24) has no solution in the interval $[\underline{q}, \tilde{q}_k^s)$, implying that no equilibrium can exist in the interval $[\underline{q}, \mathring{q}]$ since, by assumption, $\mathring{q} < \tilde{q}_k^s$. Therefore, any equilibrium, if exists, must be in the interval $(\mathring{q}, \overline{q}]$. As noted above, an equilibrium in $(\mathring{q}, \overline{q}]$ should solve equation (19), which has a unique solution $\tilde{q}_k^e$ (see Proposition 5). Since $\mathring{q} < \tilde{q}_k^s < \tilde{q}_k^e$ (see Corollary 4), $\tilde{q}_k^e$ emerges as the only equilibrium in the interval $(\mathring{q}, \overline{q}]$, which concludes the proof.

Moreover, no equilibrium can exists in the interval $[\underline{q}, \tilde{q}_k^s)$, since equation (24) has no solution in $[\underline{q}, \tilde{q}_k^s)$ (see the proof of part (a-1)) and equation (19) only has solution in $(\tilde{q}_k^s, \overline{q}]$ (see the proof of part (b)). □

Corollary 5. *Under any admissible $(\delta, \mathring{q})$ policy, for candidate type k:*

(a) *Any equilibrium $\tilde{q}_k^{e,(\delta,\mathring{q})}$ satisfies the following:*

(a-1) $\tilde{q}_k^{e,(\delta,\mathring{q})} \in [\tilde{q}_k^s, \tilde{q}_k^e]$.

(a-2) *If $\mathring{q} \geq \tilde{q}_k^e$, then $\tilde{q}_k^{e,(\delta,\mathring{q})} \in (\tilde{q}_k^s, \tilde{q}_k^e)$ for $\delta \notin \{0, \delta_k^s\}$.*

(a-3) *If $\delta_k^s > 1$, then $\tilde{q}_k^{e,(\delta,\mathring{q})} \in (\tilde{q}_k^s, \tilde{q}_k^e]$.*

(b) *Among Pareto efficient equilibria, $\tilde{q}_k^{e,(\delta,\mathring{q})} < \tilde{q}_k^{e,(\delta',\mathring{q})}$ for $\delta' < \delta$.*

Proof of Corollary 5. Proposition 6 states that for any candidate type k, any equilibrium of an admissible $(\delta, \mathring{q})$ policy with $\mathring{q} \in (\tilde{q}_k^e, \overline{q}]$ should solve equation (50). Following the notation used in the proof of Proposition 6, let $\mathcal{L}(\tilde{q})$ and $\mathcal{R}^{(\delta,\mathring{q})}(\tilde{q})$ denote the left and right hand-side of equation (50), respectively. Note that $\mathcal{R}^{(\delta,\mathring{q})}(\cdot)$ is decreasing in δ (see the proof of Proposition 6).

(a) Consider any admissible $(\delta, \mathring{q})$ policy and any type-k candidate.

(a-1) $\tilde{q}_k^{e,(\delta,\mathring{q})} \geq \tilde{q}_k^s$ follows trivially from Proposition 6, which establishes that no equilibrium exists in the interval $[\underline{q}, \tilde{q}_k^s)$. We next prove that $\tilde{q}_k^{e,(\delta,\mathring{q})} \leq \tilde{q}_k^e$. If $\mathring{q} < \tilde{q}_k^e$, then $\tilde{q}_k^{e,(\delta,\mathring{q})} = \tilde{q}_k^e$ is the unique equilibrium in the interval $(\mathring{q}, \overline{q}]$ (see Proposition 6(a-2)), implying $\tilde{q}_k^{e,(\delta,\mathring{q})} \leq \tilde{q}_k^e$. To prove the result when $\mathring{q} \geq \tilde{q}_k^e$, assume, to the contrary, that there exists an equilibrium $\tilde{q}_k^{e,(\delta,\mathring{q})} > \tilde{q}_k^e$ for some δ. Proposition 6 states that, if $\mathring{q} \geq \tilde{q}_k^e$, any equilibrium $\tilde{q}_k^{e,(\delta,\mathring{q})}$ solves $\mathcal{R}^{(\delta,\mathring{q})}(\tilde{q}_k^{e,(\delta,\mathring{q})}) = \mathcal{L}(\tilde{q}_k^{e,(\delta,\mathring{q})})$, which implies

$\mathcal{R}^{(0,\mathring{q})}(\tilde{q}_k^{e,(\delta,\mathring{q})}) \geq \mathcal{R}^{(\delta,\mathring{q})}(\tilde{q}_k^{e,(\delta,\mathring{q})}) = \mathcal{L}(\tilde{q}_k^{e,(\delta,\mathring{q})})$ since $\mathcal{R}^{(\delta,\mathring{q})}(\cdot)$ is decreasing in δ. Note that $\tilde{q}_k^{e,(0,\mathring{q})} = \tilde{q}_k^e$, and therefore, Proposition 5 implies that $\tilde{q}_k^{e,(0,\mathring{q})}$ is unique. We also have $\mathcal{R}^{(0,\mathring{q})}(\overline{q}) \leq \mathcal{L}(\overline{q})$ (see proof of Proposition 6(a-1)), implying by the intermediate value theorem that there exists an equilibrium $\tilde{q}_k^{e,(0,\mathring{q})} \geq \tilde{q}_k^{e,(\delta,\mathring{q})}$, which contradicts with the assumption that $\tilde{q}_k^{e,(\delta,\mathring{q})} > \tilde{q}_k^e = \tilde{q}_k^{e,(0,\mathring{q})}$.

(a-2) Consider any $\delta \notin \{0, \delta_k^s\}$. The proof for $\tilde{q}_k^{e,(\delta,\mathring{q})} > \tilde{q}_k^s$ follows similarly to the proof in part (a-1). To show that $\tilde{q}_k^{e,(\delta,\mathring{q})} < \tilde{q}_k^e$, assume, to the contrary, that $\tilde{q}_k^{e,(\delta,\mathring{q})} \geq \tilde{q}_k^e$. This implies that $\tilde{q}_k^{e,(\delta,\mathring{q})} = \tilde{q}_k^e$, since part (a-1) established $\tilde{q}_k^{e,(\delta,\mathring{q})} \leq \tilde{q}_k^e$. We, therefore, find $\mathcal{L}(\tilde{q}_k^e) = \mathcal{L}(\tilde{q}_k^{e,(\delta,\mathring{q})}) = \mathcal{R}^{(\delta,\mathring{q})}(\tilde{q}_k^{e,(\delta,\mathring{q})}) = \mathcal{R}^{(\delta,\mathring{q})}(\tilde{q}_k^e) < \mathcal{R}^{(0,\mathring{q})}(\tilde{q}_k^e) = \mathcal{L}(\tilde{q}_k^e)$, where the equalities between $\mathcal{L}(\cdot) = \mathcal{R}^{(\cdot,\mathring{q})}(\cdot)$ follows from the equilibrium conditions and the strict inequality follows since $\mathcal{R}^{(\delta,\mathring{q})}(\cdot)$ is decreasing in δ and $\delta \neq 0$.

(a-3) Follows similarly to the proof in part (a-1).

(b) Assume, to the contrary, that $\tilde{q}_k^{e,(\delta,\mathring{q})} \geq \tilde{q}_k^{e,(\delta',\mathring{q})}$ for some $\delta' < \delta$. Since $\mathcal{R}^{(\delta,\mathring{q})}(\cdot)$ is decreasing in δ, we have $\mathcal{R}^{(\delta,\mathring{q})}(\tilde{q}_k^{e,(\delta',\mathring{q})}) < \mathcal{R}^{(\delta',\mathring{q})}(\tilde{q}_k^{e,(\delta',\mathring{q})}) = \mathcal{L}(\tilde{q}_k^{e,(\delta',\mathring{q})})$. We also have $\mathcal{R}^{(\delta,\mathring{q})}(\tilde{q}_k^s) \geq \mathcal{L}(\tilde{q}_k^s)$ (see proof of Proposition 6(a-1)), implying by the intermediate value theorem that there exists an equilibrium in the interval $[\tilde{q}_k^s, \tilde{q}_k^{e,(\delta',\mathring{q})})$ associated with δ, which contradicts with the assumption that $\tilde{q}_k^{e,(\delta,\mathring{q})}$ is the Pareto efficient equilibrium for δ. $\square$

Corollary 6. *For any admissible $(\delta,\mathring{q})$ policy with $\delta \neq 0$, with respect to utilization of organs by type-k candidates,*

(a) *if $\mathring{q} \in (\tilde{q}_k^e, \overline{q}]$, then $(\delta,\mathring{q})$ policy is strictly dominated by the $(\delta,\tilde{q}_k^e)$ policy,*

(b) *if $\mathring{q} \in [\underline{q}, \tilde{q}_k^e)$, and the realized equilibrium is*

(b-1) *$\tilde{q}_k^{e,(\delta,\mathring{q})} = \tilde{q}_k^e$, then $(\delta,\mathring{q})$ policy is strictly dominated by any (δ,q) policy with $q \geq \tilde{q}_k^e$,*

(b-2) *$\tilde{q}_k^{e,(\delta,\mathring{q})} \leq \mathring{q}$, then $(\delta,\mathring{q})$ policy strictly dominates any (δ,q) policy with $q > \mathring{q}$.*

Proof of Corollary 6. Consider any admissible $(\delta,\mathring{q})$ policy with $\delta \neq 0$.

(a) For any $\mathring{q} \in (\tilde{q}_k^e, \overline{q}]$, following the notation used in the proof of Proposition 6, let $\mathcal{L}(\tilde{q})$ and $\mathcal{R}^{(\delta,\mathring{q})}(\tilde{q})$ denote the left and right hand-side of equation (50), respectively. We have

$$\begin{aligned}
\frac{\partial}{\partial \mathring{q}} \mathcal{R}^{(\delta,\mathring{q})}(\tilde{q}) &= r_k(\tilde{q}) \mathbb{E}_k\left[\beta \tilde{\Psi}_k(q) - c(q) - \frac{\alpha}{d_k} \,\middle|\, q \geq \tilde{q}\right] \\
&\quad \cdot \frac{\delta r_k(\mathring{q}) p_k(\mathring{q}) \left(\tilde{\lambda}_k(\tilde{q}) - \delta \overset{\star}{\lambda}_k(\tilde{q})\right) - \delta r_k(\mathring{q}) p_k(\mathring{q}) \left(\delta \tilde{\lambda}_k(\tilde{q}) + (1-\delta)\mu_k(\tilde{q}) - \delta \overset{\star}{\lambda}_k(\tilde{q})\right)}{(\overline{q} - \underline{q}) \left(\tilde{\lambda}_k(\tilde{q}) - \delta \overset{\star}{\lambda}_k(\tilde{q})\right)^2} \\
&= r_k(\tilde{q}) \mathbb{E}_k\left[\beta \tilde{\Psi}_k(q) - c(q) - \frac{\alpha}{d_k} \,\middle|\, q \geq \tilde{q}\right] \cdot \frac{\delta(1-\delta) r_k(\mathring{q}) p_k(\mathring{q}) \left(\tilde{\lambda}_k(\tilde{q}) - \mu_k(\tilde{q})\right)}{(\overline{q} - \underline{q}) \left(\tilde{\lambda}_k(\tilde{q}) - \delta \overset{\star}{\lambda}_k(\tilde{q})\right)^2} \\
&> 0,
\end{aligned} \tag{57}$$

where inequality (57) follows since the expectation term is positive (by assumption, transplantation, on average, provides higher life benefits than time until death without a transplant), and $\tilde{\lambda}_k(\tilde{q}) \geq \mu_k(\tilde{q})$. Let $\tilde{q}_k^{e,(\delta,\mathring{q})}$ denote the equilibrium threshold of a type-k candidate under $(\delta,\mathring{q})$ policy. We show that there exists an equilibrium threshold under the $(\delta,\tilde{q}_k^e)$ policy that is strictly less than any equilibria that emerges under any admissible $(\delta,\mathring{q})$ policy with $\mathring{q} \in (\tilde{q}_k^e, \overline{q}]$. Assume, to the contrary, that there exists a $(\delta, \mathring{q})$ policy with $\mathring{q} \in (\tilde{q}_k^e, \overline{q}]$ such that $\tilde{q}_k^{e,(\delta,\mathring{q})}$ is less than any equilibrium threshold $\tilde{q}_k^{e,(\delta,\tilde{q}_k^e)}$. Inequality (57) implies $\mathcal{R}^{(\delta,\tilde{q}_k^e)}(\tilde{q}_k^{e,(\delta,\mathring{q})}) < \mathcal{R}^{(\delta,\mathring{q})}(\tilde{q}_k^{e,(\delta,\mathring{q})}) = \mathcal{L}(\tilde{q}_k^{e,(\delta,\mathring{q})})$.

We also have $\mathcal{R}^{(\delta,\tilde{q}_k^e)}(\tilde{q}_k^s) \geq \mathcal{L}(\tilde{q}_k^s)$ (see proof of Proposition 6(a-1)). These two inequalities imply, by the intermediate value theorem, that there exists an equilibrium $\tilde{q}_k^{e,(\delta,\tilde{q}_k^e)}$ to the left of $\tilde{q}_k^{e,(\delta,\mathring{q})}$, which contradicts with the assumption that $\tilde{q}_k^{e,(\delta,\mathring{q})}$ is less than any equilibrium threshold $\tilde{q}_k^{e,(\delta,\tilde{q}_k^e)}$.

(b) For any $\mathring{q} \in [\underline{q}, \tilde{q}_k^e)$, and if the realized equilibrium is

(b-1) $\tilde{q}_k^{e,(\delta,\mathring{q})} = \tilde{q}_k^e$, then the result follows immediately from Corollary 5(a-2).

(b-2) $\tilde{q}_k^{e,(\delta,\mathring{q})} \leq \mathring{q}$, then the proof follows similarly to the proof in part (a). □

Proposition 7. *Introduction of any admissible $(\delta, \mathring{q})$ policy with $\delta \neq 0$ increases social welfare.*

Proof of Proposition 7. In the proof of Proposition 6, we find that the right hand-side of equation (24) is equivalent to $\tilde{L}_k(q_k) - \tilde{C}_k(q_k)$. As a consequence, the social welfare function remains unchanged with the introduction of the $(\delta, \mathring{q})$ policy, which implies that the socially efficient threshold $\tilde{q}_k^s$ for any patient type k as well as the social welfare targeted by the social planner are unaffected by the $(\delta, \mathring{q})$ policy.

In the proof of Proposition 4(b), we find that, for any given k, the social welfare function $\tilde{S}(q_1, \ldots, q_K)$ is strictly decreasing in q_k for $q_k > \tilde{q}_k^s$. This implies that

$$\tilde{S}(\tilde{q}_1^{e,(\delta,\mathring{q})}, \ldots, \tilde{q}_K^{e,(\delta,\mathring{q})}) > \tilde{S}(\tilde{q}_1^e, \ldots, \tilde{q}_K^e),$$

since $\tilde{q}_k^{e,(\delta,\mathring{q})} \in [\tilde{q}_k^s, \tilde{q}_k^e]$ for all k and $\tilde{q}_k^{e,(\delta,\mathring{q})} \in [\tilde{q}_k^s, \tilde{q}_k^e)$ for some k (Corollary 5(a-1) and (a-2), respectively). □

Appendix B: Estimating waiting time

Let W_k denote the stationary waiting time until transplantation for type-k candidates. In steady state, the fraction of type-k candidates that receive an organ offer is $\frac{\mu_k}{\lambda_k}$, where μ_k denotes the arrival rate of organs that are offered to type-k candidates and is given in equation (1). This fraction corresponds to the probability that a type-k candidate receives an organ offer, which is characterized by the event that waiting time until transplantation is no more than the time until death (i.e., $W_k \leq D_k$). Therefore, we have

$$\frac{\mu_k}{\lambda_k} = P(\text{type-}k \text{ candidate receives a transplant offer}) = P(W_k \leq D_k) \tag{58}$$

$$= e^{-W_k/d_k}, \tag{59}$$

where the last equality follows since the time until death is exponentially distributed. Taking natural logarithms of both sides of equation (59) results in the following approximation for W_k,

$$W_k = d_k \ln(\frac{\lambda_k}{\mu_k}) = d_k \ln\left(\frac{\lambda_k \left(\overline{q} - \underline{q}\right)}{\int_{q_k}^{\overline{q}} p_k(q)\, dq,}\right) \tag{60}$$

where the last equality is obtained after substituting for μ_k using equation (1). Zenios (1999) also proves equation (60) using a fluid limit approximation.