Blockchain Adoption for Combating Deceptive Counterfeits

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In this paper, we study combating deceptive counterfeiting using blockchain technology while taking into account customer privacy concerns. We consider a market with a manufacturer and a deceptive counterfeiter. The manufacturer can either use blockchain technology or signal authenticity through pricing. While blockchain enables perfect authentication it increases a customer's privacy concern. We identify the optimal blockchain adoption strategy for the manufacturer. We find that blockchain should be used only when the counterfeit quality is intermediate or when customers have intermediate distrust about products in the market. When customers have serious distrust about products, differential pricing strategy is more effective than blockchain adoption. In cases where a government is interested in providing incentives for blockchain adoption through a subsidy to the manufacturer to optimize social welfare, we find that blockchain can be more effective than differential pricing strategy in eliminating post-purchase regret. We show that government subsidy can benefit both customers and the society and could be a better approach than government enforcement efforts.

Keywords: blockchain, customer privacy, deceptive counterfeit, government subsidy

1. Introduction

Counterfeiting is a severe problem in many sectors; the loss due to counterfeiting is estimated to be 98 billion USD annually in the clothing, cosmetics, handbags and watches industry (Business and Finance Market Research Reports 2018). The economic impact from counterfeit electronic parts is estimated to be approximately $169 billion (Havoscope Global Black Market Information 2016). Counterfeiting can occur in two different settings – (1) when a customer can distinguish yet decides to buy the counterfeit product (non-deceptive counterfeit); (2) when a customer cannot distinguish between a real and a counterfeit product *ex ante* (deceptive counterfeit). A customer typically buys a non-deceptive counterfeit due to economic reasons. For example, in the pharmaceutical industry, there are less expensive fake drugs that mimic the authentic drug, but are less effective. Nevertheless, some patients may buy the fake drug because they cannot afford the real one, and taking the fake drug may be better than not taking any medicine. For example, patients with Hepatitis C disease require a medicine called Sovaldi, which costs $84,000 for a 12-week cycle of treatment. Due to the high price, some patients buy counterfeit drugs from unregulated online pharmacies at costs that are less than $900 a cycle (Ford 2016). In the context of luxury products, buying a fake product knowingly (e.g., a fake Rolex) may allow a customer to signal status at a much lower cost. The second type of counterfeiting (deceptive counterfeit) occurs when the counterfeit product looks identical to the original product and the customer is unable to differentiate *ex ante* between the two product types. For example, there are many deceptive fake drugs that are sold in developing countries. According to a WHO study, approximately 10% of drugs sold in developing countries may be counterfeits (WHO 2017). Similarly, deceptive counterfeits cost the U.S. semiconductor industry an estimated $7.5 billion per year (Semiconductor Industry Association 2013). Even in the luxury industry where manufacturers have adopted technology such as RFID and hologram tags, there are cases of deceptive counterfeits. Recently, Chanel accused retailer RealReal of selling counterfeit handbags (Turk 2018). Both types of counterfeits (non-deceptive and deceptive) have negative impact on a manufacturer's profits and brand. Our focus in this paper is on the latter setting that relates to deceptive counterfeits.

Technologies such as RFID (Radio Frequency Identification) or holograms have been tried for addressing the counterfeiting issue (Toyoda et al. 2017). However, those solutions have had two major problems. First, it is possible for counterfeiters to copy the genuine product's tag. Second, once a product has been purchased, product authenticity can no longer be guaranteed since tags are removed at the sale counter. Therefore, second hand purchases cannot be easily verified. Blockchain technology has been identified as a promising solution for many operations and supply chain challenges including control of counterfeiting and providing supply chain transparency (Babich and Hilary 2018; Simchi-Levi 2018). Several blockchain based applications have been rolled out in the recent past. For example, BlockPharma is a blockchain solution provider for drug traceability that allows patients to check the authenticity of the medicine using their mobile app (Siyal et al. 2019). Chronicled is another firm that uses blockchain to maintain the integrity of the drug supply chain (Chakraborty 2018). Luxury industry giant LVMH and diamond company Chow Tai Fook allow their customers to access product information in the supply chain with blockchain based applications. In such cases, typically data about the source as well as history of transactions are captured and stored in a cloud based blockchain solution, which can then be retrieved by a customer usually through a mobile application by scanning a QR code on the product. Use of a private key from a customer along with a public key and complex cryptography, allows a blockchain to provide an immutable, irreversible and permanent record of transactions associated with a product. In this way a blockchain based solution provides better transparency and authenticity than other previous technologies. Further, with blockchain adoption a customer can easily and perfectly verify the authenticity of the product. However, there are costs associated with blockchain implementation and it is important to understand when such adoption is beneficial to a manufacturer as opposed to traditional price signalling strategies.

While blockchain can help deal with counterfeiting, customer privacy concern is a major roadblock to its implementation (Babich and Hilary 2018). Without blockchain, customers do not necessary have to leave any personal information if they purchase the product from a physical store with cash. With blockchain, customers must leave their information by downloading a mobile app or registering themselves as an owner in order to obtain a private key. In BlockPharma's case, such data contains patients' medical history which may have privacy implications for doctors and their patients (Siyal et al. 2019). Similarly, luxury industry giant LVMH and diamond company Chow Tai Fook both require their customers to download their mobile apps in order to access the blockchain information. This enables the firm to provide personalized promotion to customers. In the light of Facebook–Cambridge Analytica data scandal, customers may find such marketing activities as invasive to their privacy. While there is an increasing amount of concern about how firms may use data related to their customers even in normal settings, these concerns are more profound for blockchain implementation. Indeed, there is active discussion in the governance circles including the European Parliament around the role of blockchain and their implications for customer privacy (Kritikos 2018).

Governments in general have not been favorable to blockchain innovation as it relates to crypto-economic system (e.g., bitcoins and token/ICO). However, many governments are promoting permissioned blockchains that keep records digital and correct since they facilitate efficient governance and fraud prevention (Knowledge@Wharton 2018). Governments in general have been fostering the development of blockchain technology through various approaches including grants, competitions and collaborative efforts. For example, UK government has pledged £19 million through Innovate UK to fund blockchain companies. One of the firms that received funding is Provenance, which provides supply chain transparency (Proctor 2016). BlockCypher received funding from US's Department of Homeland Security for their work on online fraud detection in loyalty programs (Higgins 2017). The European Union Intellectual Property Office (EUIPO) organized an anti-counterfeiting blockchain competition in 2018. US's Food and Drug Administration has signed a two-year joint-development agreement with IBM Watson Health to use blockchain to enable secure patient data sharing (Macaulay 2019). The closest example to a government directly subsidizing blockchain development comes from the Indian government's Niti Aayog (National Institute for Transforming India) that is partnering with Strides Pharma Sciences (world's largest manufacturer of soft gel capsules), and Apollo Hospitals (largest private healthcare provider in India) in using Oracle blockchain solution to guard the integrity of the pharmaceutical supply chain from counterfeit drugs (Bhushan 2018). One of the main reasons for the

Indian government to take this initiative is because it is estimated that roughly 20% of drugs sold in India are fake. By subsidizing such implementation, the government expects to remove counterfeit drugs as well as improve social welfare in the economy as indicated by this statement by Mr. Kant, CEO of NITI, *"The problem of fake and counterfeit drugs is a major issue and the partnership will help drug manufacturers and healthcare experts eliminate fake drugs."* (The Hindu Business Line 2018). The Mongolian government has signed a one year contract with UK blockchain company FarmaTrust to address similar challenge of counterfeit drugs (Bitcoin News 2018). Our work provides insights on when it makes sense for a government to provide subsidies for blockchain implementation, particularly for products that might impact the society.

In this paper, we examine how blockchain technology can be used by firms and government to combat counterfeiting. In particular, we want to understand – (1) what type of market conditions encourages a manufacturer to adopt blockchain technology to combat counterfeiting? (2) how does customers' post purchase regret and concern about leaving a digital footprint affect blockchain implementation? (3) how well does government subsidy motivate the manufacturer to adopt blockchain to combat counterfeiting and how does it compare to a government enforcement strategy? To the best of our knowledge, this is the first paper that studies blockchain adoption to combat counterfeiting and provides prescriptive insights and conditions under which blockchain technology should be adopted and government subsidies provided.

We consider a setting where a manufacturer sells products to customers in the presence of a deceptive counterfeiter. The counterfeit product looks identical to the manufacturer's product, so customers cannot differentiate *ex ante*. The manufacturer can use blockchain technology to prove the genuineness of its product, but implementation is costly and customers have privacy concerns around blockchain usage. Alternatively, the manufacturer could use differential pricing strategy to signal product authenticity or use neither approach. The government has the option of providing subsidy to the manufacturer to incentivize the implementation of blockchain so that social welfare is maximized.

Without government intervention, our results show that in markets where the counterfeit quality is intermediate, or where customers have intermediate level of distrust about the products in the market such that the uncertainty about the authenticity is the highest, the manufacturer should implement blockchain. In markets where there is a greater proportion of counterfeit products, customers may have serious distrust about product (e.g., developing countries). In this case, differential pricing strategy is more effective in signaling product authenticity. Finally, in cases where the quality of the counterfeit product is comparable, a pooling strategy is the best option for the manufacturer. Further, blockchain is preferred in markets where customers of different types have intermediate difference in privacy cost. In a privacy-insensitive (privacy-hypersensitive) market, customers with various privacy concerns have similar (very different) privacy cost. In both these cases, the expected privacy cost over all customers is larger which makes adopting blockchain less attractive.

We show that when government subsidy is offered in the market, differential pricing strategy should never be used. It is optimal for the manufacturer to adopt blockchain technology in most of the cases unless the quality of the counterfeiter is very high. Not only is the manufacturer better off from government's involvement because subsidy lowers the implementation cost, customers are also better off because the manufacturer passes some of the subsidy to them in the form of lower retail price, and because customers are less likely to be deceived into purchasing counterfeits. Therefore, our result advocates that government should participate in the manufacturer's blockchain adoption decision. We also analyze government enforcement in which case the government exerts effort to catch the counterfeiter and if caught, penalizes them for tricking customers. We show that subsidy for blockchain implementation is better for overall social welfare in comparison to an enforcement strategy in majority of cases. Finally, we check the robustness of our results under endogenous quality, cost differences and alternative social welfare definitions.

The rest of the paper is organized as follows. We first discuss the relevant literature in the next section. Then, we introduce the mathematical model in Section 3, and the optimal solutions and insights in Section 4. In Section 5, we consider the setting where the government in an involved party. We demonstrate the robustness of our results in Section 6, and Section 7 contains concluding remarks and managerial insights.

2. Literature Review

Research in the area of counterfeiting is in its early stages. Mani et al. (2018) provide the first empirical investigation on the drivers of deceptive counterfeit parts on a supply chain using dataset of field programmable gate arrays. Below, we present several recent analytical papers that examine the impact of non-deceptive (Gao et al. 2016; Pun and Deyong 2017) and deceptive counterfeits (Qian 2014; Cho et. al 2015 and Qian et. al. 2015).

Both Gao et al. (2016) and Pun and DeYong (2017) consider competition between a branded manufacturer and a non-deceptive copycat. Gao et al. (2016) examine the impacts of different attributes and of consumer utilities to the copycat for luxury branded product. They use product quality as a deterrence strategy, and demonstrate that a higher branded product quality can deter copycat's entrance and can also increase consumer surplus. Under a two-period game with strategic customers, Pun and DeYong (2017) suggest that product quality, legal enforcement and customers' forward looking behavior can be used as deterrence strategies. Our paper differs from this stream of literature by considering a deceptive counterfeit.

Qian (2014) considers an industry with a manufacturer and a deceptive counterfeiter, and they study three deterrence strategies. First, they examine the impact of different quality levels of the authentic product. Second, they investigate the impact of enforcement effort such that the counterfeiter has some chance of being fined by the government. Third, they study a manufacturer that can use two types of non-price signals. The manufacturer can set up a company store (incurring a lump-sum cost) or the manufacturer can tag a hologram to each product (incurring a per-unit cost). They assume that the counterfeiter also has the capability of producing identical hologram. Hence, if it makes financial sense, the counterfeiter may tag the costly hologram to their products to fool customers. Our work is close to the second non-price signal approach in that we also consider a per unit cost for blockchain adoption. However, our model and analysis differ significantly because blockchain does not allow the counterfeiter to mimic the manufacturer's strategy.

Cho et al. (2015) examine a scenario where the manufacturer and the deceptive counterfeiter both sell through a common distributor, and this distributor decides the fraction of counterfeits among all products sold. They study two deterrence strategies. First, they consider the strategy where the manufacturer decides price and quality of its product. Second, they investigate how marketing, enforcement and technology can be used to combat counterfeit. Qian et al. (2015) examine an industry with a manufacturer and multiple symmetric deceptive counterfeiters. They introduce two dimensions of quality: searchable quality and experiential quality. The manufacturer chooses the quality level in these two dimensions and the price of its product and then the counterfeiters decide whether or not to enter and the quality/price of its product. They consider government enforcement as an anti-counterfeiting strategy that decreases the number of counterfeiters. In contrast to these two papers, our model setting is different. We do not consider a third-party distributor (Cho et al. 2015) nor multiple counterfeiters (Qian et al. 2015), so our benchmark (no blockchain) differs from these two papers. Moreover, in these two papers, customers may still be deceived into purchasing counterfeits even in the presence of these deterrence strategies which is not the case with blockchain implementation.

In summary, under all strategies studied in the existing literature, if it makes financial sense (e.g., considering the penalty from enforcement or the extra cost in manipulating the fake to look like the real), the counterfeiter can deceive customers. However, blockchain is fundamentally different from these anti-counterfeiting strategies. Products are tagged with a unique identifier, and this identifier cannot be duplicated because of blockchain's immutable, irreversible and permanent characteristics. All customers can identify whether a product is authentic or fake if the manufacturer deploys blockchain. As a result, when blockchain is adopted, the analysis moves from a deceptive counterfeit setting to a non-deceptive counterfeit setting, so the problem where customers buy a fake unknowingly is completely eliminated. More importantly, our paper also adds two new features to the literature. First, we consider the government as a decision maker who can provide subsidy to the manufacturer and second, we consider customers' privacy concern from blockchain adoption. Our paper is the first one to provide insights as to when blockchain technology should be adopted to combat counterfeiting.

3. The Model

Consider a setting where a manufacturer (firm M) sells products to customers, and there is a counterfeiter (firm C) in the market. Denote $i \in \{M, C\}$ as the firm. The customers' valuation for a product v is uniformly distributed between zero and one. The true quality of firm i's product is q_i, where $q_M = 1$ and $q_C = q < 1$. (We first assume that product qualities are exogenously given; in Subsection 6.1 we consider counterfeiter quality to be endogenous.) The counterfeit is deceptive, so the customers cannot differentiate between the two product types using product appearance. However, firm M can implement blockchain to prove the genuineness of its product; it can also use differential pricing strategy to signal product authenticity.

3.1. Blockchain Use

When the manufacturer uses blockchain to combat counterfeiting, customers know the authenticity of the product. However, they worry about leaving their digital footprint. Customers are heterogeneous in their concern. Specifically, customer type on privacy concern is θ, which is distributed between $\underline{\theta}$ and $\overline{\theta}$ based on the probability distribution function (pdf) $g(\theta)$. Customers with a higher θ have a higher privacy concern. The privacy cost of a type-θ customer is $f(\theta)$, where $f'(\theta) > 0$ (customers with a higher privacy concern would incur a higher privacy cost). We assume that customers' valuation v and their privacy concern θ are independent to each other. Then the customer has an utility $U_M^B(\theta) = v - p_M - f(\theta)$ when buying firm M's product at price p_M (we use superscript B to denote the "Blockchain" strategy).

We assume that the counterfeit product does not have blockchain technology. Hence, the utility of a type-θ customer when purchasing firm C's product at price p_C is $U_C^B(\theta) = qv - p_C$.

A customer gets zero utility if not buying any product. Then the demand for firm i from type-θ customers is $D_M^B(\theta) = \int_{v:U_M^B(\theta)>\max[U_C^B(\theta),0]} dv$ and $D_C^B(\theta) = \int_{v:U_C^B(\theta)>\max[U_M^B(\theta),0]} dv$. For simplicity, we normalize all production cost to zero. In Subsection 6.2 we consider the case where the two firms have asymmetric production cost.

Most manufacturers do not have the expertise and resource to build a blockchain platform from scratch. More likely, they will use blockchain-as-a-service platform. Therefore, a manufacturer's blockchain adoption cost depends on how much the blockchain provider is charging. In most cases, this cost is proportion to the sales volume. For example, IBM does not have a minimum fee; instead, it charges for the number of CPU-hour, which is directly proportion to the manufacturer's customer base and sales volume (IBM 2019). Therefore, we assume a linear cost of blockchain implementation c_B per-unit that includes the cost of cloud based implementation as well as the cost for the tag.[1] Firm M's profit is $\pi_M^B = (p_M - c_B)\int_{\underline{\theta}}^{\overline{\theta}} D_M^B(\theta)g(\theta)d\theta$. Note that when there is a fixed cost in adopting blockchain, the region where blockchain should be adopted shrinks, but all structural results presented in this paper hold. The counterfeiter does not have blockchain technology on its package, so its profit is $\pi_C^B = p_C\int_{\underline{\theta}}^{\overline{\theta}} D_C^B(\theta)g(\theta)d\theta$.

3.2. No Blockchain Use

When the manufacturer does not use blockchain, then the firm may use differential pricing strategy to signal product authenticity. Let $\varphi \in \{M, C\}$ be the product type, which can either be firm M's genuine product or firm C's counterfeit product. Furthermore, denote $u_i(\varphi)$ be the customers' *a priori* probability belief that product φ is from firm i, which initially has a value ϕ_i (i.e., $u_i(\varphi) = \phi_i$), where $\phi_M + \phi_C = 1$. The parameter ϕ_i can also be interpreted as the proportion of product from firm i. In regions where legislation against counterfeit is very strict, ϕ_M would be close to one; on the other hand, ϕ_M would be small in markets with more relaxed regulations such that customers have serious distrust about products sold in the market (e.g., developing countries). Our model and analysis for the case without blockchain is similar to Qian (2014) and Qian et al. (2015) who also consider competition with counterfeiters. We present these results only as a benchmark.

When the two firms charge a different price (i.e., $p_M \neq p_C$), customers update their belief about product φ after observing prices, such that $u_M(\varphi | p_\varphi > p_{-\varphi}) = 1$ and $u_C(\varphi | p_\varphi < p_{-\varphi}) = 1$, where $-\varphi$

[1] All results hold as long as $c_B + \int_{\underline{\theta}}^{\overline{\theta}} f(\theta)g(\theta)d\theta \geq 0$. When $c_B + \int_{\underline{\theta}}^{\overline{\theta}} f(\theta)g(\theta)d\theta < 0$, then firm M no longer chooses separating equilibrium in the optimal solution. The details are available from the authors upon request.

represents the other product type (i.e., $-\varphi \in \{M, C\}\backslash\varphi$). This captures the *separating equilibrium* scenario in which customers know for sure that the expensive product belongs to firm M and the cheap one is from firm C. Therefore, customers that buy a product with price p_i would know for sure that the product belongs to firm i, so the utility is (we use superscript N to denote the "No blockchain" strategy) $U_M^N = v - p_M$ and $U_C^N = qv - p_C$, and the demand for firm i is $D_M^N = \int_{v:U_M^N > \max[U_C^N, 0]} dv$ and $D_C^N = \int_{v:U_C^N > \max[U_M^N, 0]} dv$.

When the two firms have the same price (i.e., $p_M = p_C$), knowing this price does not provide further information. This is a *pooling equilibrium*, in which customers cannot differentiate between the two product types. They keep their *a priori* belief that product φ is from firm i has probability ϕ_i, i.e., $u_i(\varphi | p_\varphi = p_{-\varphi}) = \phi_i$. As a result, customers' perceived quality of product φ is $\phi_M q_M + \phi_C q_C$, so the utility of buying a product is:

$$U_i^N = (\phi_M q_M + \phi_C q_C)v - p_i \tag{1}$$

Recall that ϕ_i captures the proportion of product from firm i. Therefore, the demand for firm i is $D_i^N = \int_{v:U_i^N > 0} \phi_i dv$, and the profit is $\pi_i^N = p_i D_i^N$.

Similar to Qian et al. (2015), we use the Divinity Criteria (D1) proposed by Banks and Sobel (1987) to refine the Perfect Bayesian Equilibria. Specifically, given firm M's optimal price, in deviation, firm C is more likely to obtain a higher profit under a set of best responses associated with the possible out-of-equilibrium belief than firm M. D1 then requires that customers do not believe that the deviation type is firm M.

4. Results

In our analysis we assume the following game sequence. Firm M decides whether or not to implement blockchain. Since firm M is the market leader, it first sets price p_M, followed by firm C that sets price p_C. Lastly, customers make purchase decision. In this section, we present the optimal solutions and insights to the research questions. Proofs are presented in the appendix.

4.1. Equilibrium Analysis

4.1.1 No Blockchain Use

Similar to earlier research (see Qian 2014), we obtain the optimal solutions by backward induction. Consider the scenario where firm M does not implement blockchain. Given firm M's price, it can be shown that firm C would have profit $\pi_C^N(p_M;SE) = \frac{p_M^2 q}{4(1-q)}$ if it sets a different price (separating equilibrium) and would have profit $\pi_C^N(p_M;PE) = \frac{p_M\phi_C(\phi_M+\phi_C q-p_M)}{\phi_M+\phi_C q}$ if it sets the same price (pooling equilibrium). Therefore, the necessary and sufficient condition for separating equilibrium is:

$$\pi_C^N(p_M;SE) \geq \pi_C^N(p_M;PE) \Leftrightarrow p_M \geq \overline{p} \equiv \frac{4(1-q)(\phi_M+\phi_C q)}{(2-q)^2+q\phi_M/\phi_C} \tag{2}$$

Firm M's optimization problem when it does not use blockchain is such that it chooses the price p_M to maximize its profit, subject to (s.t.) firm C's pricing decision:

$$\max \begin{cases} \max\limits_{p_M} \pi_M^N(p_M;SE) \quad \text{s.t.}\, p_M \geq \overline{p} \quad \text{(3a)} \\ \max\limits_{p_M} \pi_M^N(p_M;PE) \quad \text{s.t.}\, p_M \leq \overline{p} \quad \text{(3b)} \end{cases} \tag{3}$$

Firm M sets p_M as the price leader, in anticipation of firm C's best response. This price decision indirectly determines the equilibrium to be separating or pooling. Specifically, the equilibrium would be separating if firm M sets a price that is larger than the threshold $\overline{p}$, because firm C would choose a different (low) price to have a larger profit (cf. Equation 2). On the other hand, if firm M sets a price that is smaller than $\overline{p}$, firm C would follow firm M's price (pooling equilibrium) to have a larger profit.

4.1.2 Blockchain Use

Define $\mathbb{S} \in \{B, PE, SE\}$, where B denotes that blockchain is implemented, and PE (SE) denotes that blockchain is not used and pooling (separating) equilibrium is chosen. Throughout the analysis, we focus on the interesting region where (1) customers believe that there is a reasonable chance of buying a counterfeit ($\phi_M < \overline{\phi}$) or else separating equilibrium will not be chosen for the subgame without blockchain, and (2) the counterfeit product quality is not very high ($q < \overline{q}$) and blockchain adoption is not very costly ($c_B < \overline{c}$) or else the manufacturer would have negative demand under the blockchain subgame and blockchain would not be profitable. The definitions of these three thresholds are given in the proof of Proposition 1 (Equations A1, A4 and A6 in the appendix). Then Proposition 1 presents the optimal strategy $\mathbb{S}^*$. All thresholds are presented in Table A1 in the appendix, where $\omega \equiv c_B + E_g[f(\theta)]$, and

$E_g[f(\theta)]$ is the expected value of $f(\theta)$ over pdf $g(\theta)$, i.e., $E_g[f(\theta)] = \int_{\underline{\theta}}^{\overline{\theta}} f(\theta)g(\theta)d\theta$. Figure 1 numerically illustrates the optimal strategy when blockchain implementation cost is $c_B = 0.03$, customers' privacy concern type $g(\theta)$ is uniformly distributed between $\underline{\theta} = 0$ and $\overline{\theta} = 0.06$, and type-θ has privacy cost $f(\theta) = \theta$.

Proposition 1: *Define $\hat{\lambda}_1(\omega, q)$ and $\hat{\lambda}_2(\omega, q)$ in Equations A2 and A3. Then the optimal strategy $\mathbb{S}^*$ is as follows:*

- *Choose separating equilibrium ($\mathbb{S}^* = SE$) if and only if $\phi_M \le \hat{\lambda}_1(\omega, q)$.*
- *Deploy blockchain ($\mathbb{S}^* = B$) if and only if $\hat{\lambda}_1(\omega, q) < \phi_M \le \hat{\lambda}_2(\omega, q)$.*
- *Choose pooling equilibrium ($\mathbb{S}^* = PE$) if and only if $\phi_M > \hat{\lambda}_2(\omega, q)$.*

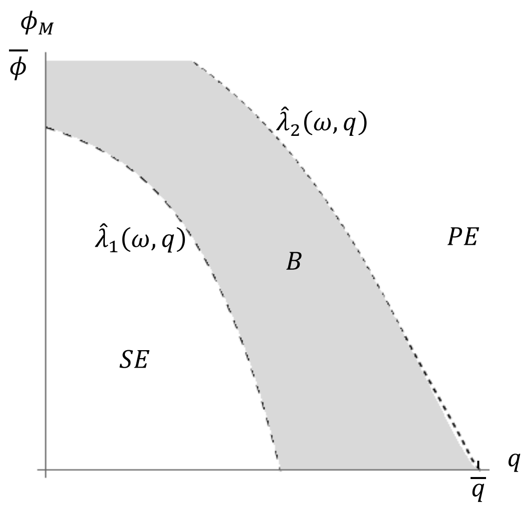

Figure 1: Optimal strategy $\mathbb{S}^*$

The optimal strategy for firm M depends on how much value it can derive from customers' ability to differentiate between the two product types. Firm M can disseminate product type information to customers either via pricing or via blockchain, but there is a rent to each approach. Specifically, having the constraint $p_M = \bar{p}$ in Equation 3a means that firm M may set a different price than the unconstrained problem (interior solution) to obtain separating equilibrium, so the rent for using the differential pricing strategy to disseminate product type information is the difference between firm M's profit under the unconstrained problem and under the constrained problem (Equation 3a). The rent for implementing blockchain strategy is ω, which depends on the implementation cost (c_B) and the expected value of the privacy cost $f(\theta)$ over all customers types $g(\theta)$, i.e., $c_B + E_g[f(\theta)]$. Then the optimal strategy is derived by comparing firm M's profit under blockchain adoption, separating equilibrium and pooling equilibrium.

At one extreme, when q is small, the two products are highly differentiated. The value that firm M can derive from customers' ability to differentiate the product is high. There is always a positive rent in implementing blockchain. Moreover, the price threshold $\bar{p}$ is small when q is small because firm C can likely get a higher profit under separating equilibrium than under pooling equilibrium for any given p_M (cf. Equation 2), so there is no rent to deploying the differential pricing strategy. Therefore, firm M would choose $\mathbb{S}^* = SE$ to signal product authenticity.

At the other extreme, when q is large, firm M would derive a negative value from customers differentiating the products. This is because both product types are very similar; so the two firms would engage in a severe price war when the customers can differentiate between the real from the fake. However, if firm C charges the same price as firm M, firm C pretends to be authentic and fools the customer. The customer cannot differentiate between the two product types, so the two firms avoid the differential price competition. While firm M splits the profit with firm C, it still has a larger profit than engaging in a price war with firm C. Consequently, firm M would take advantage of the customers' asymmetric information on product type and would choose a pooling equilibrium ($\mathbb{S}^* = PE$). This information value can be sufficiently negative such that firm M may not use blockchain even when the expectation of privacy cost is zero and when implementation is free (i.e., $E_g[f(\theta)] = c_B = 0$).

Firm M should use blockchain when the customers' *a priori* probability belief that the market has real product is intermediate ($\hat{\lambda}_1(\omega, q) < \phi_M < \hat{\lambda}_2(\omega, q)$). This is because customers are most uncertain about product types under this parameter setting, so firm M can derive a positive value from customers differentiating the products. However, firm M would need to price its product at $\bar{p}$ to incentivize firm C not to follow its price because the expected quality when customers cannot differentiate the two product types ($\phi_M + \phi_C q$) is sufficiently large (cf. Equation 2). The rent for choosing separating equilibrium becomes larger than the rent for implementing blockchain; this is because a larger price threshold $\bar{p}$ implies that the constraint $p_M \geq \bar{p}$ is more likely to be binding, so the constrained problem in Equation 3a is more likely to deviate from the unconstrained problem (interior solution). Consequently, firm M chooses blockchain to demonstrate the genuineness of its product ($\mathbb{S}^* = B$).

4.2 Effect of Counterfeit Quality and Market Structure

We define consumer surplus CS following the standard definition in the literature, and social welfare SW as the sum of firm M's profit, consumer surplus, and firm C's profit when customers can distinguish counterfeits from the authentic (i.e., separating equilibrium and blockchain). Note that all results hold whether or not social welfare includes firm C's profit for all three strategies (see Subsection 6.3). Moreover, to quantify the damage when customers purchase counterfeit products *unknowingly*, we define the post-purchase regret as the difference in utility between what the customers expect *ex ante* and what they receive *ex post*. The post-purchase regret is zero when the customers can distinguish between the genuine and the fake products (via blockchain $\mathbb{S}^* = B$, or via price signal $\mathbb{S}^* = SE$). However, when neither signal is presented ($\mathbb{S}^* = PE$), a fraction ϕ_C of customers would expect an *ex ante* utility given in Equation 1, but receive an *ex post* utility $qv - p_i$ from the fake product. (Customers do not have any post-purchase regret when they receive a real product.) Then the post-purchase regret over all customers is as follows:

$$R = \begin{cases} \int_{v:U_i^N>0} \phi_C[(\phi_M + \phi_C q)v - qv]dv & \text{if } \mathbb{S}^* = PE \\ 0 & \text{otherwise} \end{cases}$$

Proposition 2 presents the effect of counterfeit quality on the optimal solution. The analytical conditions for Proposition 2 are given in Lemma A1 in the Appendix.

Proposition 2:

a. *Firm M's profit may increase in the quality of the counterfeit product while consumer surplus and social welfare can both decrease if blockchain is not an option.*
b. *Firm M's profit may decrease in the quality of the counterfeit product while consumer surplus and social welfare can both increase if blockchain is an option.*

We observe the above phenomenon in the region where firm M chooses pooling equilibrium when the counterfeit product quality is intermediate when blockchain option is not available (cf. Proposition 1). We find that firm M's profit may increase in the quality of firm C's product when pooling equilibrium is used ($\mathbb{S}^* = PE$) in this intermediate region. This is because the customers' expected quality of a product is higher as the counterfeiter's quality increases. The price under pooling equilibrium increases, so firm M is better off. However, customers and social welfare can decrease when firm M switches from using differential pricing strategy as a signal ($\mathbb{S}^* = SE$) to simply playing along and using a pooling strategy ($\mathbb{S}^* = PE$). This is because customers no longer have any product information, so they may suffer from buying counterfeit product unknowingly.

With the possibility of blockchain, firm M would adopt blockchain when q is intermediate. Since customers can distinguish between the two product types, consumer surplus and social welfare would always increase when the counterfeit quality increases in this intermediate region. However, the manufacturer can no longer take advantage of a higher counterfeit quality because of more intense competition between the two product types. Further, the post-purchase regret from customers not knowing the two product types can be the largest in this intermediate region. The option of blockchain would eliminate this area, so this leads to the following insight:

Corollary 1: *Blockchain can be more effective than differential pricing strategy in eliminating customers' post-purchase regret.*

Next, Proposition 3 presents the impact of market structure on the optimal solution, and the analytical condition is given in Lemma A2 in the Appendix.

Proposition 3:

a. Consumer surplus and social welfare may increase in the customers' a priori probability belief that a product is genuine if blockchain is not an option.
b. Consumer surplus and social welfare may decrease in the customers' a priori probability belief that a product is genuine if blockchain is an option.

We see this behavior around the region where pooling equilibrium is optimal in the absence of blockchain. Since the customer's expected quality increases in the *a priori* probability belief, customers have higher utility from consuming the product when the counterfeit quality increases. Hence, consumer surplus and social welfare may increase. On the other hand, with the option of implementing blockchain, firm M would signal the genuineness of its product using blockchain when customers' *a priori* probability belief is small and would choose pooling equilibrium when *a priori* probability belief is large. Thus, when firm M switches from using blockchain strategy to not signaling product authenticity, consumer welfare would drop because customers can no longer differentiate between the two product types, and social welfare may also decrease.

4.3 Effect of Privacy Concerns

Recall that customer privacy concern of type-θ customer is $g(\theta)$, and the privacy cost of a type-θ customer is $f(\theta)$. For notational convenience, we use A to denote the region where $\mathbb{S}^* = B$, and the subscripts in Proposition 4a to denote the probability distribution function (pdf) that we take expectation on. For example, A_{g_1} denotes the region where blockchain should be adopted when the expected privacy cost over all customers is taken over the probability distribution function g_1.

Proposition 4:

a. *Assuming $g_1(\theta) \geq_{FSD} g_2(\theta)$. For any $f(\theta)$, blockchain is less likely to be adopted, firm M's profit, consumer surplus and social welfare are smaller under $g_1(\theta)$ than under $g_2(\theta)$, i.e., $A_{g_1} < A_{g_2}$, $(\pi_M^*)_{g_1} \leq (\pi_M^*)_{g_2}$, $CS_{g_1}^* \leq CS_{g_2}^*$ and $SW_{g_1}^* \leq SW_{g_2}^*$.*

b. *Assuming $g(\theta) \sim U[0,2a]$ and $f_k(\theta) = \frac{\theta^k}{a^{k-1}}$, where $k > 0$. Then the area where blockchain is implemented, firm M's profit, consumer surplus and social welfare is the largest when k is intermediate, i.e., $\frac{\partial A}{\partial k} < 0 \Leftrightarrow \frac{\partial \pi_M^*}{\partial k} < 0 \Leftrightarrow \frac{\partial CS^*}{\partial k} < 0 \Leftrightarrow \frac{\partial SW^*}{\partial k} < 0 \Leftrightarrow k > \frac{\ln e/2}{\ln 2}$.*

Proposition 4a presents the sensitivity analysis on $g(\theta)$. In particular, the pdf g_1 is first-order stochastic dominance (FSD) to the pdf g_2, implying that more customers have higher privacy concern under g_1 than under g_2. Therefore, for any given privacy cost function $f(\theta)$, the expected privacy cost across all customers is larger (i.e., $E_{g_1}[f(\theta)] > E_{g_2}[f(\theta)]$), so the rent for disseminating product type information through blockchain technology is higher. As a result, the region where firm M uses blockchain as an anti-counterfeiting strategy ($\mathbb{S}^* = B$) is smaller. Firm M, the customers and the society are (weakly) worse off because of a higher blockchain rent.

A type-θ customer has privacy cost $f(\theta)$, and Proposition 4b is the sensitivity analysis on the privacy cost. In particular, we assume that customers' privacy concern is uniformly distributed between 0 and $2a$, so the customer with type $\theta = a$ has an average privacy concern. The coefficient $\frac{1}{a^{k-1}}$ in $f_k(\theta)$ is for guaranteeing that this average-concern customer always has the same privacy cost for all k so that we can have a meaningful comparison across different k. As k increases, $f_k(\theta)$ becomes more asymmetrical around $\theta = a$. This implies that customers with a high (low) privacy concern has a higher (lower) privacy cost, so they are more (less) averse to buy a product with blockchain. Therefore, the sensitivity analysis on k can be interpreted as the analysis on the variance of the customers' averseness to privacy concern. We show in Proposition 4b that blockchain is used most often, and profit/welfare are the highest, when the parameter k is intermediate. In particular, in a privacy-insensitive market (k is extremely small), customers with various privacy concern have similar privacy cost when purchasing a product with blockchain. As k increases, the privacy cost of the low-concern customers decreases a lot more in comparison to the increase for the high concern customers, so the expected privacy cost over all customers, $E_g[f_k(\theta)]$, decreases. This makes adopting blockchain more attractive, and firm M, the customers and the society have higher profit/welfare. However, as k becomes larger, customers of different type have bigger differences in privacy cost. In this case, while increasing k does not have any significant impact to the privacy cost of low concern customers (or their acceptance of blockchain), the high privacy concern customers have a much higher cost, and increasing k significantly increases the privacy cost of these customers. Therefore, the expected privacy cost over all customers increases. This makes blockchain adoption less attractive; firm M, the customers and the society have lower profit/welfare. Blockchain is used most often, and profit/welfare are the highest, only when customers of different types have intermediate difference in privacy cost (k is intermediate).

5. Government Involvement

The manufacturer's decision on whether or not to implement blockchain may not always be beneficial from customers' standpoint. In situations where counterfeiting can do major harm to society (like the case of counterfeit drugs discussed in the introduction), the government may have a role to play as well.

5.1 Subsidy for Blockchain

In this section, we investigate how the government can incentivize the manufacturer to use blockchain technology. In particular, in the beginning of the game, the government decides whether or not to offer subsidy to firm M if it deploys blockchain; if the government decides to offer subsidy, it also decides on the level of per-unit subsidy, s. As a result of this subsidy, firm M's blockchain implementation cost reduces from c_B to $c_B - s$. Firm M's profit, as a function of the per-unit subsidy, becomes $\pi_M^B(s) = \left(p_M - (c_B - s)\right)\int_{\underline{\theta}}^{\overline{\theta}} D_M^B(\theta)g(\theta)d\theta$. The government's objective function is the social welfare, so the government's optimization problem is as follows:

$$\max\begin{cases}\max\limits_{s} \pi_M^B(s) + \pi_C^B(s) + CS^B(s) - s\int_{\underline{\theta}}^{\overline{\theta}} D_M^B(\theta)g(\theta)d\theta \quad \text{s.t.}\, \pi_M^B(s) \geq \pi_M^N & \text{(4a)} \\ SW^N & \text{(4b)}\end{cases} \tag{4}$$

The constraint of Equation 4a is the incentive compatibility constraint for firm M to use blockchain, and Equation 4b is the government's outside option (blockchain is not used and no subsidy is offered). The remaining of the game sequence is the same as that of the main math model (cf. Subsection 3.3). Proposition 5 presents the optimal strategy $\mathbb{S}^G$ to Equation 4. Figure 2 numerically illustrates the optimal strategy using the same parameter setting as Figure 1, i.e., $c_B = 0.03$, $g(\theta)\sim U[0,0.06]$ and $f(\theta) = \theta$. We use the superscript G to denote that this is the optimal solution when the government has the option of providing subsidy.

Proposition 5: *Define $\hat{\lambda}^G(\omega, q)$ in Equation A7 (in the appendix)*

a. *The optimal strategy for the manufacturer when government may provide subsidy $\mathbb{S}^G$ is as follows:*
   - *Never choose a separating equilibrium.*
   - *Deploy blockchain ($\mathbb{S}^G = B$) if and only if $\phi_M \leq \hat{\lambda}^G(\omega, q)$.*
   - *Choose pooling equilibrium ($\mathbb{S}^G = PE$) if and only if $\phi_M > \hat{\lambda}^G(\omega, q)$.*

b. $\frac{\partial s^{in}}{\partial q} < 0, \frac{\partial s^{in}}{\partial E[\theta]} < 0, \frac{\partial s^{bi}}{\partial E[\theta]} > 0$, *and* $\frac{\partial s^{bi}}{\partial q} < 0 \iff q > \tilde{q}_0(\omega, \phi_M)$

   The optimal subsidy level may decrease in q, $E_g[f(\theta)]$ and c_B.

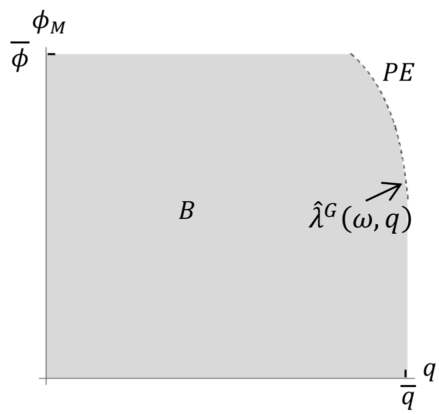

Figure 2: Optimal strategy $\mathbb{S}^G$ when the government may provide subsidy

To obtain the optimal solution in this model, the government decides whether or not to offer subsidy to firm M. This subsidy would only be offered if the manufacturer deploys blockchain and can alleviate the situation where a manufacturer's decision on whether or not to implement blockchain differs from the government's interest. Specifically, recall from Proposition 1 that without any government intervention, firm M would use differential pricing strategy to signal product authenticity when q and ϕ_M are small. However, with government subsidy, firm M would use blockchain to disseminate product type information to customers. This is because firm M incurs a rent in choosing a separating equilibrium, and this rent is the difference between firm M's profit under the unconstrained problem and under the constrained solution (cf. discussion in Proposition 1). While there is a cost for implementing blockchain and customers have privacy concern, the government subsidy is sufficiently lucrative to make the blockchain rent to be smaller than the pricing rent. Therefore, for those parameter settings where firm M chooses separating equilibrium when subsidy is not available, firm M now implements blockchain to combat counterfeits.

When firm C can produce a higher quality product, the government will lower the subsidy because customers' utility loss of buying a counterfeit product is lower. We also find that the government may provide a lower subsidy when blockchain implementation is costly or when customers have more concern about leaving their digital footprint. This is because the cost of finding out the product type increases with $E_g[f(\theta)]$ and c_B, so the government would less likely to support products with such characteristic.

Below, we derive further results about how government subsidy changes the manufacturer's blockchain adoption for combating deceptive counterfeits.

Proposition 6:

a. *In the presence of subsidy, blockchain is more likely to be implemented.*
b. *The manufacturer, the customers and the society are always better off from the government offering subsidy.*
c. *Without subsidy, the manufacturer's profit and the social welfare are non-monotonic in the quality of the counterfeit product. However, in the presence of government subsidy, the manufacturer is always worse off, while the society is always better off, when the counterfeit product is of higher quality.*
d. *With government subsidy, the manufacturer's profit, consumer surplus and social welfare are less dependent on customers' a priori belief on the market structure.*

With government subsidy, firm M should not choose separating equilibrium but instead should adopt blockchain (cf. Proposition 5a). Recall that the reason for firm M to choose a pooling equilibrium is to avoid engaging in a price war with firm C from customers knowing the real from the fake. Government subsidy reduces the attractiveness of this option. Hence, blockchain is always more likely to be implemented when government is involved. With that being said, blockchain should not always be used. In particular, when firm C can produce a product with very similar quality as firm M, firm M should still choose a pooling equilibrium to avoid the differential price competition with firm C. This is the region in Figure 2 in the top right corner.

Firm M is always better off from government intervention because subsidy lowers the blockchain implementation cost. Therefore, firm M is more willing to deploy blockchain. Customers are also better off because firm M passes some of the subsidy to customers in the form of lower retail price, and more customers would buy the product because of a lower price. Furthermore, since firm M is more likely to implement blockchain, another reason for customers to be better off is because they are less likely to be deceived into purchasing counterfeits. Finally, the social welfare increases from government's involvement because the optimal subsidy for aligning firm M's blockchain adoption incentive is sufficiently small when compared to the gain in firm M's profit and in customer surplus. The implication of Proposition 6b is that government should actively participate in firm M's blockchain adoption decision for combating deceptive counterfeits.

In the absence of government subsidy, firm M's profit and social welfare are non-monotonic in the quality of the counterfeit (cf. proof of Proposition 2b). This is because customers have a higher expected utility from a higher counterfeit quality under pooling equilibrium (cf. Equation 1). However, there is an upper limit to the price that firm M can charge in order to incentivize firm C to follow its price (the constraint in Equation 3b), and this constraint is not binding when the counterfeit product quality is small. This implies that firm M's profit would increase in the counterfeit quality in the non-binding region because customers are willing to pay a higher price for a higher expected utility and firm M does not need to deviate its price from the interior solution. Moreover, social welfare decreases when the optimal strategy shifts from deploying blockchain to choosing a non-binding pooling equilibrium; otherwise, the social welfare would always increase in the counterfeit quality. With government intervention, the subsidy is sufficiently lucrative such that the region for non-binding pooling equilibrium is replaced by blockchain adoption. Therefore, firm M is always worse off, while the society is always better off, when the counterfeit product has higher quality.

Customers use the *a priori* belief about market structure to derive their expected utility in buying a product when they cannot distinguish between the two product types. When government subsidy is available, unless the counterfeit product has a very similar quality as the authentic product, firm M would implement blockchain to signal product authenticity. Since the size of the region where pooling equilibrium is chosen becomes smaller, customers are less likely to use their *a priori* belief to evaluate the product's expected utility. Consequently, firm M's profit, consumer surplus and social welfare are less dependent on customers' *a priori* belief on market structure.

5.2 Government Enforcement

Instead of providing a subsidy, a government may decide to focus on enforcement. In this case, the government would carry out enforcement effort and penalize the counterfeiter for tricking customers to buy a fake product. Under the enforcement scheme, with probability $w(e)$, which is an increasing function of enforcement effort e, the counterfeiter will be caught and fined an amount π_C^{PE}, and the manufacturer will be the monopoly in this situation with profit $p_M(1-p_M)$. Thus, the firms' expected profit under pooling equilibrium becomes $E[\pi_M^{PE}] = (1-w)\pi_M^{PE} + wp_M(1-p_M)$ and $E[\pi_C^{PE}] =$

$(1-w)\pi_C^{PE} - w\pi_C^{PE}$. The two terms in these two functions represent the situation where firm C is caught or not. The necessary and sufficient condition for separating equilibrium (Equation 2) becomes the following: $\pi_C^N(p_M; SE) \geq E[\pi_C^N(p_M; PE)] \Leftrightarrow p_M \geq \bar{p}^E \equiv \frac{4(1-q)(1-2w)(q(1-\phi_M)+\phi_M)(1-\phi_M)}{q(8w(1-\phi_M)+5\phi_M-4)+q^2(1-\phi_M)+4(1-2w)(1-\phi_M)}$. It can easily be shown that $\bar{p}^E < \bar{p}$, so the condition for separating equilibrium becomes looser. In another word, the counterfeiter has less incentive to pool in the presence of enforcement.

Figure 3 plots the optimal strategy. The solid line is the benchmark case where there is no enforcement (cf. Figure 1), and the dotted line represents the solution to the enforcement model. We find that the region where blockchain is optimal ($\mathbb{S}^* = B$) is smaller under the enforcement scheme while the other two regions ($\mathbb{S}^* = SE$ and $\mathbb{S}^* = PE$) are larger. This is because with enforcement, firm C is penalized for deceiving customers (pooling equilibrium), implying that firm M can earn more from pooling equilibrium. In addition, the binding price ($\bar{p}^E$) is smaller, so the price constraint for separating equilibrium becomes less restrictive. Hence, firm M can also earn more under separating equilibrium when price is binding (Region SE_B). On the other hand, enforcement has no impact to the manufacturer's profit in region SE_{NB} and when blockchain is deployed (Region B).

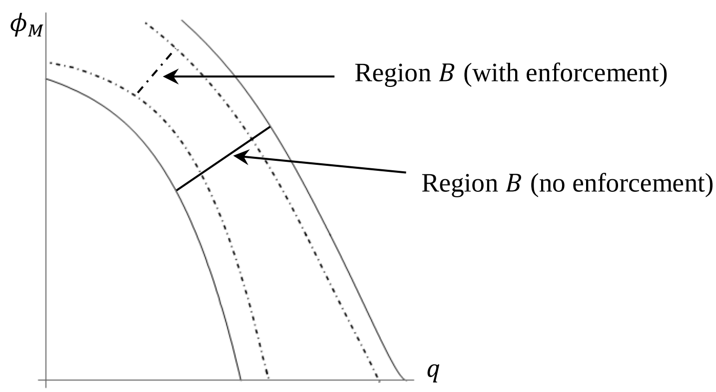

Figure 3: Optimal strategy comparison when there is enforcement

Next, we plot the post-purchase regret in Figure 4. We observe that the post-purchase regret is larger under the enforcement scheme. First, this is because post-purchase regret arises when customers are not able to distinguish between the genuine and the fake products, and the region for which pooling equilibrium is optimal ($\mathbb{S}^* = PE$) is larger under the enforcement scheme. Second, when pooling equilibrium is chosen under both schemes, retail price is lower under the enforcement scheme. As a result, there will be more demand as well.

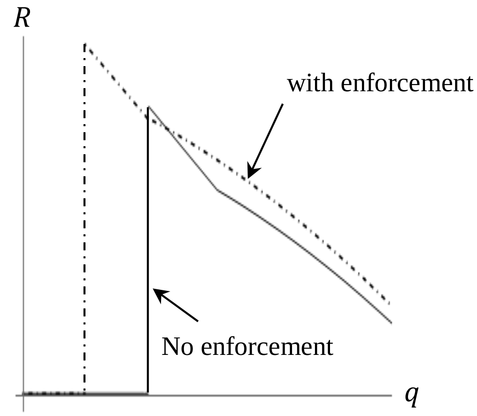

Figure 4: Post-purchase regret under enforcement scheme

5.3 Comparison between Subsidy and Enforcement

The government can either provide subsidy to assist blockchain adoption (Subsection 5.1) or exert enforcement efforts to eliminate the counterfeiter (Subsection 5.2). Figure 5 shows the dominant strategy in terms of maximizing social welfare when comparing blockchain subsidy with enforcement. We observe that in most cases, governments should use the subsidy scheme because it leads to a higher social welfare. This is because the region for PE is larger under the enforcement scheme and because the post-purchase regret is generally smaller under the subsidy scheme. Hence, subsidy is a more effective tool than enforcement in eliminates the situation where customers purchase counterfeit products unknowingly. However, when the quality of the counterfeiter is sufficiently high or when customers have low level of distrust about the products in the market, the government should use the enforcement scheme. This is because, as we have discussed in Proposition 5, at those parameter settings, the manufacturer and the counterfeiter would engage in a severe price war when the customers can differentiate between the real from the fake. Therefore, under the subsidy scheme, firm M would take advantage of the customers' asymmetric information on product type and would choose a pooling equilibrium. Hence, the social welfare would not be large under these settings because customers may be cheated to buy a deceptive counterfeit. However, under the enforcement scheme, there is a possibility that the counterfeiter is eliminated. Therefore, the manufacturer does not have to compete with a counterfeit; this also avoids the situation where customers buy the fake product unknowingly. As a result, social welfare is larger under the enforcement scheme.

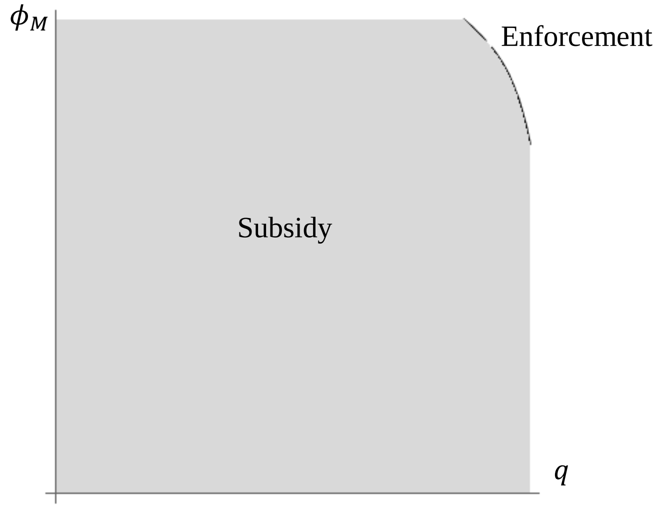

Figure 5: Comparison of social welfare between subsidy and enforcement schemes

6. Extensions

In this section, we examine the robustness of our results with respect to several possible extensions.

6.1. Endogenous Product Quality

In the main model, we have assumed that the product qualities are exogenous. In this subsection, we examine an extension where counterfeit product's quality is endogenously determined. In particular, we assume that firm C sets its product quality q_C, and the cost of quality is $q_C^2/2$ (e.g., Cho et al. 2015). Firm C's profits when blockchain is and is not implemented are $\pi_C^B = p_C \int_{\underline{\theta}}^{\overline{\theta}} D_C^B(\theta) g(\theta) d\theta - q_C^2/2$ and $\pi_C^N = p_C D_C^N - q_C^2/2$, respectively. Other variables and profit functions are the same as those given in the main model. Under this extension, firm M first decides whether or not to implement blockchain and then sets its price (p_M). Next, firm C decides the price (p_C) and quality (q_C) of its product. Lastly, each customer makes the purchase decision. This model is not analytically tractable, so we check the robustness of this model numerically using a wide array of parameter settings.

We plot the optimal strategy when firm C's quality is exogenous for a given q in Figure 6a, and when firm C's quality is endogenously determined in Figure 6b. When product qualities are exogenously given, we show in Proposition 1 that the manufacturer chooses pooling equilibrium when the *a priori* belief (ϕ_M) is sufficiently large (cf. Figure 6a). However, when firm C's product quality is endogenously determined, the additional profit that firm C can derive from customers not knowing the product type does not justify the quality investment cost. Firm C is better off producing a lower quality product and the manufacturer would no longer choose a pooling equilibrium. Instead, firm M would signal the authenticity of its product through blockchain ($\mathbb{S}^* = B$) when the implementation cost is low or through differential pricing ($\mathbb{S}^* = SE$) otherwise.

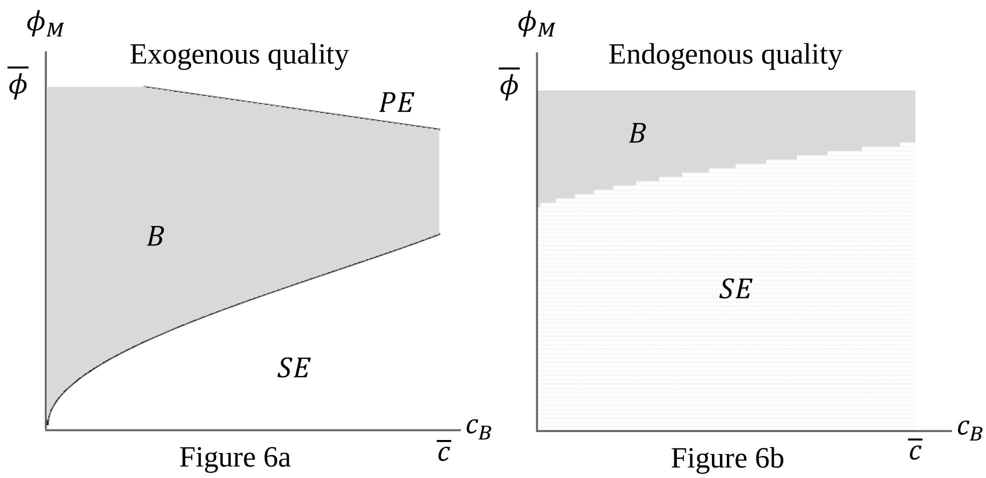

Figure 6: Optimal strategy $\mathbb{S}^*$ when there is no government subsidy

To obtain further insight, we plot the optimal firm C's quality (q^*) and its dependence on *a priori* belief ϕ_M (Figure 7a) and on blockchain implementation cost c_B (Figure 7b) when there is no government subsidy. We find that the optimal quality is independent of the *a priori* belief ϕ_M when blockchain is implemented (Region B in Figure 7a) or when separating equilibrium is chosen and the constraint in Equation 3a is non-binding (Region SE_{NB}). When ϕ_M is intermediate (Region SE_B), the constraint in Equation 3a is binding, so the manufacturer would adjust the retail price upward to incentivize firm C to choose a different price (cf. Equation 2). This gives more room for firm C to charge a higher price, so firm C would set higher quality for its product to justify the higher price. The optimal quality is independent of the blockchain implementation cost when separating equilibrium is chosen, whether Equation 3a is binding or not (Region SE in Figure 7b). Moreover, the optimal quality is increasing in c_B when blockchain is deployed (Region B). This is because firm M would charge a higher price as c_B increases, and firm C can also charge a higher price for its product, so firm C may set a higher quality to justify a higher retail price.

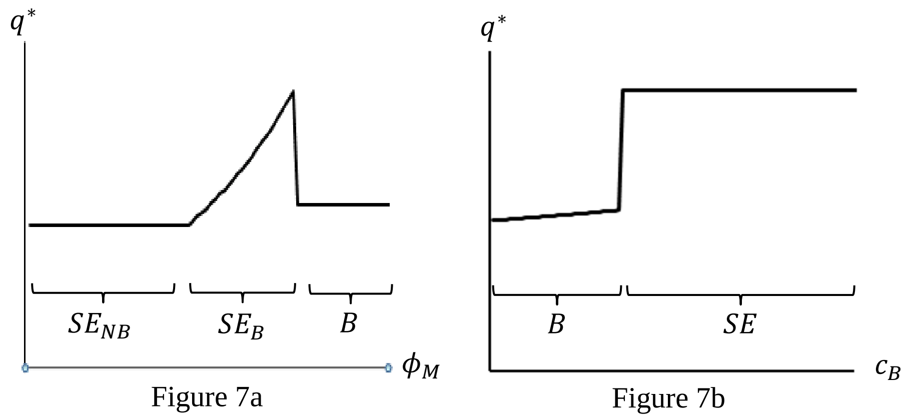

Figure 7: Optimal firm C's quality q^* when there is no government subsidy

Lastly, we examine the case when the government might provide subsidy to incentivize the manufacturer to deploy blockchain. When product qualities are exogenously given, we showed in Proposition 5a that the manufacturer would never choose a separating equilibrium. This result is robust even when firm C's product quality is endogenously determined. Moreover, as we have demonstrated in Figure 6b, firm M would not choose a pooling equilibrium because firm C would not produce a product that is of comparable quality with firm M. This implies that blockchain will always be implemented for endogenous quality in the presence of government subsidy. The optimal quality is independent of the *a priori* belief (ϕ_M) because blockchain is always used. Moreover, as we have discussed in Figure 7b, firm C's quality increases as blockchain implementation cost (c_B) increases.

6.2. Asymmetric Production Cost

To keep the mathematical model tractable, we assume that both firms have the same production cost. In this subsection, we demonstrate the robustness of our results numerically when the two firms have different production cost ($\Delta c = c_M - c_C$).

Figure 8 plots the optimal strategy (cf. Proposition 1 and Figure 1) when the counterfeiter has a higher production cost ($\Delta c = -0.02$), when the two firms have the same production cost ($\Delta c = 0$), and when the manufacturer has a higher production cost ($\Delta c = 0.02$). We use the same parameter setting as Figure 1 (i.e., $c_B = 0.03$, $g(\theta) \sim U[0,0.06]$ and $f(\theta) = \theta$), so Figure 8 generalizes Figure 1.

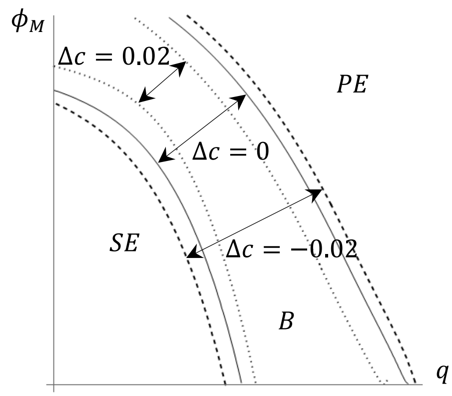

Figure 8: Optimal strategy $\mathbb{S}^*$ without government subsidy

The threshold that divides separating and pooling equilibrium increases as the difference in production cost Δc increases. This implies that pooling equilibrium is more likely to be chosen when the counterfeiter has a higher production cost, and separating equilibrium is more likely to be chosen when the manufacturer has a higher production cost. Moreover, as the difference in production cost Δc increases, the thresholds for blockchain adoption $\hat{\lambda}_1(\omega, q)$ increases but $\hat{\lambda}_2(\omega, q)$ decreases. This implies that blockchain is more likely to be used when the counterfeiter has a higher production cost, and is less likely to be implemented when the manufacturer has a higher production cost.

Next, Figure 9 plots the optimal strategy when the government may provide subsidy (cf. Proposition 5 and Figure 2). The top right corner is where pooling equilibrium is used and the rest of the area is where blockchain should be implemented. The result shows that the threshold for blockchain adoption decreases as the difference in production cost Δc increases. Similar to our observation in Figure 8 (without government subsidy), in the presence of government intervention, blockchain is more likely be adopted when the counterfeiter has a higher production cost.

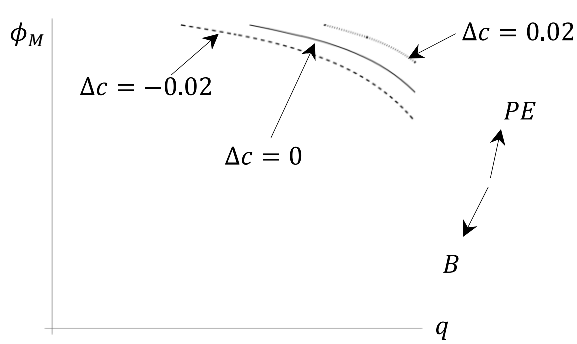

Figure 9: Optimal strategy $\mathbb{S}^G$ when the government may provide subsidy

6.3. Alternative Way to Model Social Welfare

In the main model, we have assumed that social welfare includes firm C's profit only if the customers can distinguish counterfeits from the real product (separating equilibrium and blockchain). However, there are two other possibilities to model social welfare. First, one can argue that social welfare should not include firm C's profit because firm C is illegitimate, i.e., $SW = \pi_M + CS - s\int_{\underline{\theta}}^{\overline{\theta}} D_M^B(\theta)g(\theta)d\theta$. Second, following a common definition in the literature (e.g., Qian et al. 2015; Gao et al. 2016), social welfare should include the profit of all parties, i.e., $SW = \pi_M + \pi_C + CS - s\int_{\underline{\theta}}^{\overline{\theta}} D_M^B(\theta)g(\theta)d\theta$ for all three strategies (separating equilibrium, pooling equilibrium, blockchain). All of our results (Propositions 1 – 6) hold whether or not social welfare includes firm C's profit for all three strategies.

7. Conclusions

Counterfeiting is an important problem; in this paper, we examine how blockchain as an emerging technology can be used to address this challenge when customers have concern about leaving their digital footprint. In particular, we consider a setting where the market consists of a manufacturer and a deceptive counterfeit. The manufacturer can use blockchain technology or differential pricing strategy to signal product authenticity. The government has the option of providing subsidy to incentivize the manufacturer to implement blockchain.

We find that blockchain adoption may not be beneficial to the manufacturer, even in the presence of government subsidy or when the implementation is costless. Specifically, when the counterfeit product has a comparable quality as the manufacturer's product, the manufacturer should take advantage of the customers' asymmetric information on product type and choose a pooling strategy to avoid engaging in a differential price competition. Without government subsidy, we find that blockchain technology should be used only when customers have intermediate distrust about the products in the market or when the counterfeit quality is intermediate. This is the parameter setting where counterfeit products would cause the most damage to customers. Blockchain, therefore, is effective in eliminating the post-purchase regret, which would arise when customers purchase counterfeit products unknowingly. In the presence of government intervention, the manufacturer should not use a price signal to validate product authenticity. Instead, the subsidy lowers the blockchain implementation cost, so blockchain is more likely to be adopted. The manufacturer is better off from government's involvement because of a lower implementation cost; the customers are also better off because of a lower retail price and because they are less likely to be deceived to buy a counterfeit product. Consequently, our result advocates for government's involvement in the manufacturer's blockchain decision because the manufacturer, the customers and the society can all benefit from government intervention. Further, we show that subsidy for blockchain adoption may lead to better social outcome than an enforcement strategy to combat counterfeiting.

Counterfeiting is difficult to be completely eliminated, but with the advancement in technology, blockchain has the potential of being an effective tool in combating counterfeit. Our paper is one of the first to study how the government can incentivize the manufacturer to implement blockchain technology to address to the counterfeit challenge. There are various paths to continue the research on how blockchain can be used to fight counterfeiting. First, one significant ability of the blockchain technology is full traceability. In our model, we consider a single-tier manufacturer; to fully explore the traceability feature, it would be interesting to consider a multi-tier supply chain, and how a manufacturer can motivate its suppliers to participate in the blockchain program. Second, most of the counterfeit products are sold through a C2C online platform (e.g., Alibaba's Taobao), and too many counterfeiters selling fake products would discourage customers from participating in the platform. Therefore, another fruitful avenue of future research would be to study how the online platform should motivate its sellers to deploy blockchain. Lastly it would also be interesting to investigate what firms can do to minimize customers' privacy concern about leaving their digital footprint.

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Appendix – Proof of Propositions

Proof of Proposition 1:

Blockchain subgame: We use the standard backward induction technique. In particular, $U_M^B(\theta) > U_C^B(\theta) \Leftrightarrow v > \frac{p_M - p_C + f(\theta)}{1-q}$ and $U_C^B(\theta) > 0 \Leftrightarrow v > \frac{p_C}{q}$, so $D_M^B(\theta) = 1 - \frac{p_M - p_C + f(\theta)}{1-q}$ and $D_C^B(\theta) = \frac{p_M - p_C + f(\theta)}{1-q} - \frac{p_C}{q}$. Then $\pi_M^B = (p_M - c_B)\int_{\underline{\theta}}^{\overline{\theta}}\left(1 - \frac{p_M - p_C + f(\theta)}{1-q}\right) g(\theta)d\theta = (p_M - c_B)\left(1 - \frac{p_M - p_C + E_g[f(\theta)]}{1-q}\right)$ and $\pi_C^B = p_C \int_{\underline{\theta}}^{\overline{\theta}}\left(\frac{p_M - p_C + f(\theta)}{1-q} - \frac{p_C}{q}\right) g(\theta)d\theta = p_C\left(\frac{p_M - p_C + E_g[f(\theta)]}{1-q} - \frac{p_C}{q}\right)$. π_C^B is concave in p_C; solving the first order condition (FOC) gives $p_C = \frac{1}{2}q(p_M + E_g[f(\theta)])$. π_M^B is concave in p_M, and solving the FOC gives $p_M^{B*} = \frac{2(1-q) + (c - E_g[f(\theta)])(2-q)}{2(2-q)}$.

No blockchain subgame: We follow the approach of Qian (2014) for solving a price-signaling game. Specifically, we use the Karush–Kuhn–Tucker (KKT) method to solve Equation 3; the subgame strategy is given in Figure A1. $\phi_1(q)$, $\phi_2(q)$ and $\phi_3(q)$ exist and are unique and are defined in Table A1. The subgame profit in each region are $\pi_M^{SE_{NB}} = \frac{1-q}{2(2-q)}$, $\pi_M^{SE_B} = \frac{4(1-q)(1-\phi_M)(q+\phi_M - q\phi_M)(q^2(3-2\phi_M)(1-\phi_M) + 4(1-\phi_M)^2 - q(8 - 15\phi_M + 6\phi_M^2))}{((2-q)^2 - (1-q)(4-q)\phi_M)^2}$, $\pi_M^{PE_{NB}} = \frac{\phi_M(q + \phi_M - q\phi_M)}{4}$ and $\pi_M^{PE_B} = \frac{4(1-q)q(1-\phi_M)\phi_M(q+\phi_M - q\phi_M)^2}{((2-q)^2 - (1-q)(4-q)\phi_M)^2}$. For notation convenience, define π_M^{N*} as firm M's corresponding subgame profit in the four regions. Note that $\bar{q}$ is defined in Table A1. Moreover

$$\pi_M^{SE_B}(q=0) > \pi_M^{PE_{NB}}(q=0) \Leftrightarrow \phi_M < \overline{\phi} \equiv \frac{4}{5} \tag{A1}$$

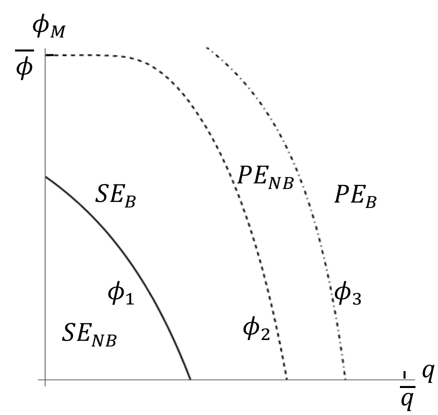

Figure A1: Subgame strategy when no blockchain

To eliminate any out-of-equilibrium, we apply the D1 criterion to refine the set of perfect Bayesian equilibria. Denote $\Omega(\varphi, p') \equiv \bigcup_{u_i(\varphi): u_i(\varphi|p_\varphi = p')=1}\{r \in MBR(u_i(\varphi), p') | \pi_i^*(\varphi) < \pi_i(\varphi, p', r)\}$ to be the set of mixed best responses (MBR), where r is customers' response to price p' and $u_i(\varphi | p_\varphi = p') = 1$ represents that after observing a product φ with price p', customers believe that this product comes only from firm i. Assuming that firm i is better off by setting p' than the optimal price p_i^*. Then we specify the off-the-equilibrium beliefs as follows:

Given a uniform price which is equal to p_M^*, customers have *a priori* belief that a product is from firm i with probability ϕ_i; otherwise, if they observe any other price, they perceive that all the products are from firm C. Besides, when customers observe two prices in the market, they believe that the expensive price which is equal to p_M^* is from firm M, and the other one is from firm C. However, if they observe any other two prices, they believe that the product is from firm C.

Separating equilibrium: When firm M deviates to $p' \neq p_M^*$, then its profit will be $\pi_M(p') = p'\left(1 - \frac{p'-p_C}{1-q}\right) < \max_{p_M} \pi_M(p_M) = \pi_M(p_M^*)$. Therefore $\Omega(M, p') \subset \Omega(C, p')$, where $\Omega(M, p')$ is an empty set, and hence $\varphi(p') = C$. It also means that firm C is more likely to deviate. Next, consider firm C's behavior. Given any price $p' \neq p_M^*$, customers believe the product is from firm C, i.e., $\varphi(p') = C$. When firm C deviates to $p' \neq p_C^*$, then its profit will be $\pi_C(p') = p'\left(\frac{p_M - p'}{1-q} - \frac{p'}{q}\right) < \max_{p_C} \pi_C(p_C) = \pi_C(p_C^*)$, so it also doesn't have incentive to deviate.

Pooling equilibrium: Assuming that firm M deviates to $p' \neq p_M^*$. Then its profit will be $\pi_M(p') = \phi_M p'\left(1 - \frac{p'}{q}\right) < \max_{p_M} \pi_M(p_M) = \pi_M(p_M^*)$. Therefore $\Omega(M,\ p') \subset \Omega(C,\ p')$, where $\Omega(M, p')$ is an empty set, and hence $\varphi(p') = C$. It also means that firm C is more likely to deviate. Similarly, given any price $p' \neq p_M^*$, customers believe the product is from firm C, i.e., $\varphi(p') = C$. Under pooling equilibrium, the profit that firm C could achieve by setting price p' is $\pi_C(p') = \phi_C p'\left(1 - \frac{p'}{q}\right) < \max_{p_C} \pi_C(p_C) = \pi_C(p_C^*)$, so it doesn't have incentive to deviate.

Thus, we can conclude that the off-the-equilibrium belief as discussed above is stable under the D1 criterion, implying that the equilibrium solution without blockchain is unique and survives D1 criterion.

<u>Optimal strategy</u>: We compare π_M^{B*} with π_M^{N*} in the four regions (cf. Figure A1) by using algebraic manipulation to demonstrate the unique and existence of the thresholds $\hat{\lambda}_1(\omega, q)$ and $\hat{\lambda}_2(\omega, q)$.

Region SE_{NB}: $\pi_M^{SE_{NB}} \geq \pi_M^{B*}$ because of $\pi_M^{SE_{NB}} = \pi_M^{B*}(\omega = 0)$, $\frac{\partial \pi_M^{SE_{NB}}}{\partial \omega} = 0$ and $\frac{\partial \pi_M^{B*}}{\partial \omega} < 0$. Hence, $\mathbb{S}^* = B$ is never true.

Region SE_B: $\pi_M^{B*} = \pi_M^{SE_B}$ can be expressed as a four-degree polynomial w.r.t. ϕ_M; while it can be shown that if $c_B < \bar{c} = \omega_0 - E_g[f(\theta)]$, the other three roots are outside of region SE_B. Note that $\bar{c}$ and ω_0 are defined in Equations A5 and A6 at a later stage, and are given in Table A1. Define $\hat{\lambda}_1(\omega, q)$ as the unique root in the feasible region. Hence, $\mathbb{S}^* = B \Leftrightarrow \phi_M > \hat{\lambda}_1(\omega, q)$, where

$$\hat{\lambda}_1(\omega, q) \text{ is the unique root of } \pi_M^{B*} = \pi_M^{SE_B} \text{ in region } SE_B \tag{A2}$$

Region PE_{NB}: $\pi_M^{B*} = \pi_M^{PE_{NB}}$ can be expressed as a quadratic polynomial w.r.t. ϕ_M; there are two roots and it can be shown that one of the roots is always outside of region PE_{NB}. Therefore, there is at most one root, $\hat{\lambda}_2'(\omega, q)$, that solves $\pi_M^{B*} = \pi_M^{PE_{NB}}$. The maximum value of π_M^{B*} is when $\omega = 0$ and it can be shown that the solution to $\pi_M^{B*}(\omega = 0) = \pi_M^{PE_{NB}}$, which is $\hat{\lambda}_2'(0, q)$, is inside region PE_{NB}. The minimum value of π_M^{B*} is when $\omega = \omega_0$ and the solution to $\pi_M^{B*}(\omega = \omega_0) = \pi_M^{PE_{NB}}$, which is $\hat{\lambda}_2'(\omega_0, q)$, is also inside region PE_{NB}. Since the maximum value of π_M^{B*} is not always greater than $\pi_M^{PE_{NB}}$, and the minimum value of π_M^{B*} is not always smaller than $\pi_M^{PE_{NB}}$, any intermediate value of π_M^{B*} (when $0 < \omega < \omega_0$) would have an intersection with $\pi_M^{PE_{NB}}$. Hence, there always exists a unique $\hat{\lambda}_2'(\omega, q)$ such that $\mathbb{S}^* = B \Leftrightarrow \phi_M < \hat{\lambda}_2'(\omega, q)$.

Region PE_B: We use a similar method as region PE_{NB} to show the unique and existence of the threshold. Specifically, $\pi_M^{B*} = \pi_M^{PE_B}$ can be expressed as a four-degree polynomial w.r.t. ϕ_M; it can be shown that three of the roots are always outside of region PE_B. Hence, there is at most one root, $\hat{\lambda}_2''(\omega, q)$, in region PE_B that solves $\pi_M^{B*} = \pi_M^{PE_B}$. It can be shown that the root to $\pi_M^{B*}(\omega = 0) = \pi_M^{PE_B}$, which is $\hat{\lambda}_2''(0, q)$, and the root to $\pi_M^{B*}(\omega = \omega_0) = \pi_M^{PE_B}$, which is $\hat{\lambda}_2''(\omega_0, q)$, are inside region PE_B. Hence, any intermediate value of π_M^{B*} would have an intersection with $\pi_M^{PE_B}$, so there always exists a unique $\hat{\lambda}_2''(\omega, q)$ such that $\mathbb{S}^* = B \Leftrightarrow \phi_M < \hat{\lambda}_2''(\omega, q)$.

Then define the following:

$$\hat{\lambda}_2(\omega, q) \equiv \begin{cases} \hat{\lambda}_2'(\omega, q) & \text{in region } PE_{NB} \\ \hat{\lambda}_2''(\omega, q) & \text{in region } PE_B \end{cases} \tag{A3}$$

Where $\hat{\lambda}_2'(\omega, q)$ is the unique root of $\pi_M^{B*} = \pi_M^{PE_{NB}}$ in region PE_{NB}, and $\hat{\lambda}_2''(\omega, q)$ is the unique root of $\pi_M^{B*} = \pi_M^{PE_B}$ in region PE_B.

Note that $\hat{\lambda}_2'(\omega, q) = \hat{\lambda}_2''(\omega, q)$ at ϕ_3 (the boundary between region PE_{NB} and PE_B). □

Method A1. This method is used to analyze whether the equilibrium solution is continuous or has a jump at the strategy shift line. For example, consider two bivariate polynomials, $\Gamma_1\ (q, \phi_M)$ and $\Gamma_2(q, \phi_M)$, with parameters q and ϕ_M. We want to compare the value of these two functions (whether $\Gamma_1\ (q, \phi_M)$ is larger or smaller than $\Gamma_2(q, \phi_M)$) at a line $\phi^{sh}(q)$. Note that for the purpose of the proof below, this line is the strategy shift line $\phi_M = \phi^{sh}(q)$. First, solving $\Gamma_1\ (q, \phi_M) = \Gamma_2(q, \phi_M)$ as a function of ϕ_M gives $\phi_M = \phi^{root}(q)$. Note that throughout the proof in our paper, there is always at most one root. If we have one $\phi^{root}(q)$ in the feasible region, then we have the following:

Case 1: $\phi^{root}(q) = \phi^{sh}(q)$

$\Gamma_1\ (q, \phi_M)$ and $\Gamma_2(q, \phi_M)$ are continuous at $\phi^{sh}(q)$, so $\Gamma_1\left(q, \phi^{sh}(q)\right) = \Gamma_2\left(q, \phi^{sh}(q)\right)$.

Case 2: $\phi^{root}(q) \neq \phi^{sh}(q)$

It can easily be shown that whether or not $\phi^{root}(q)$ and $\phi^{sh}(q)$ have any intersection point. Then there are two subcases:

Case 2a: $\phi^{root}(q)$ and $\phi^{sh}(q)$ do not intersect:

$\Gamma_1\left(q, \phi^{sh}(q)\right) > \Gamma_2\left(q, \phi^{sh}(q)\right)$ for all q if and only if $\Gamma_1\left(\hat{q}, \phi^{sh}(\hat{q})\right) > \Gamma_2(\hat{q}, \phi^{sh}(\hat{q}))$ for an arbitrary $q = \hat{q}$, and vice versa.

Case 2b: $\phi^{root}(q)$ and $\phi^{sh}(q)$ intersect:

We separate the strategy shift line $\phi^{sh}(q)$ into several segments, depends on the number of intersection point. Then for each segment, we use the method in Case 2a to compare between $\Gamma_1\left(q, \phi^{sh}(q)\right)$ and $\Gamma_2\left(q, \phi^{sh}(q)\right)$. For example, consider Figure A2. There are two intersection points, so there are three segments. Then we pick an arbitrary $q = \hat{q}_1$ inside segment 1 and compare between these two functions in this segment. We repeat this step for each segment.

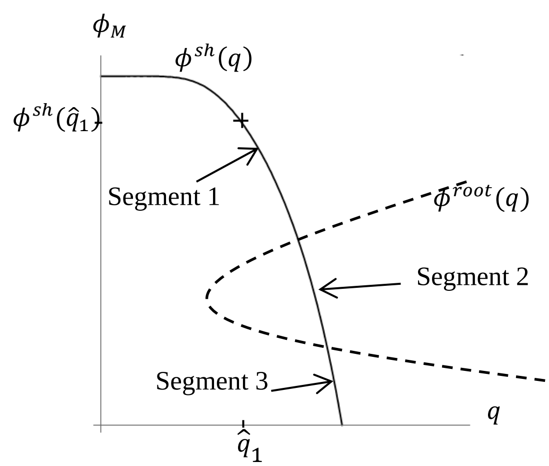

Figure A2: The illustration for case 2b

If there is no root inside the feasible region, then we can easily compare between $\Gamma_1\left(q, \phi^{sh}(q)\right)$ and $\Gamma_2\left(q, \phi^{sh}(q)\right)$ using an arbitrary $q = \hat{q}$ inside the feasible region.

Lemma A1: *Define $q_1(\phi_M)$ and $q_2(\phi_M)$ as the inverse function of ϕ_1 and ϕ_2. Moreover, define $\hat{q}_1(\phi_M, \omega)$ and $\hat{q}_2(\phi_M, \omega)$ as the inverse function of $\hat{\lambda}_1$ and $\hat{\lambda}_2$. Lastly, define ϵ as a small positive number*.

(a) *If blockchain is not an option*

(i) $\frac{\partial \pi_M^{N*}}{\partial q} > 0$ *if* $q_2(\phi_M) < q < \tilde{q}_1(\phi_M)$, *and* $\frac{\partial \pi_M^{N*}}{\partial q} < 0$ *otherwise.*

(ii) $\frac{\partial CS^{N*}}{\partial q} < 0$ *if* $q_1(\phi_M) < q < \tilde{q}_2(\phi_M)$, $CS^{N*}(q = q_2(\phi_M) - \epsilon) > CS^{N*}(q = q_2(\phi_M))$ *if* $\phi_M < \bar{\phi}_1$, $CS^{N*}(q = q_2(\phi_M) - \epsilon) < CS^{N*}(q = q_2(\phi_M))$ *if* $\phi_M > \bar{\phi}_1$, *and* $\frac{\partial CS^{N*}}{\partial q} > 0$ *otherwise.*

(iii) $\frac{\partial SW^{N*}}{\partial q} < 0$ *if* $q_1(\phi_M) < q < \tilde{q}_3(\phi_M)$, $SW^{N*}(q = q_2(\phi_M) - \epsilon) > SW^{N*}(q = q_2(\phi_M))$ *if* $\phi_M <$

$\bar{\phi}_2$, $SW^{N*}(q = q_2(\phi_M) - \epsilon) < SW^{N*}(q = q_2(\phi_M))$ *if* $\phi_M > \bar{\phi}_2$, *and* $\frac{\partial SW^{N*}}{\partial q} > 0$ *otherwise.*

(b) *If blockchain is an option*

(i) $\frac{\partial \pi_M^{B*}}{\partial q} > 0$ *if* $\hat{q}_2(\phi_M, \omega) < q < \tilde{q}_1(\phi_M)$, *and* $\frac{\partial \pi_M^{B*}}{\partial q} < 0$ *otherwise.*

(ii) $\frac{\partial CS^{B*}}{\partial q} < 0$ if $q_1(\phi_M) < q < \min\{\tilde{q}_2(\phi_M), \hat{q}_1(\phi_M, \omega)\}$, $CS^{B*}(q = \hat{q}_1(\phi_M, \omega) - \epsilon) < CS^{B*}(q = \hat{q}_1(\phi_M, \omega))$, $CS^{B*}(q = \hat{q}_2(\phi_M, \omega) - \epsilon) > CS^{B*}(q = \hat{q}_2(\phi_M, \omega))$, *and* $\frac{\partial CS^{B*}}{\partial q} > 0$ *otherwise.*

(iii) $\frac{\partial SW^{B*}}{\partial q} < 0$ if $q_1(\phi_M) < q < \min\{\tilde{q}_2(\phi_M), \hat{q}_1(\phi_M, \omega)\}$, $SW^{B*}(q = \hat{q}_1(\phi_M, \omega) - \epsilon) < SW^{B*}(q = \hat{q}_1(\phi_M, \omega))$, $SW^{B*}(q = \hat{q}_2(\phi_M, \omega) - \epsilon) > SW^{B*}(q = \hat{q}_2(\phi_M, \omega))$, *and* $\frac{\partial SW^{B*}}{\partial q} > 0$ *otherwise.*

Proof of Lemma A1:

Part a: (i) $\frac{\partial \pi_M^{SE_{NB}}}{\partial q} = \frac{-1}{2(2-q)^2} < 0$, $\frac{\partial \pi_M^{SE_B}}{\partial q} < 0$, and $\frac{\partial \pi_M^{PE_{NB}}}{\partial q} = \frac{(1-\phi_M)\phi_M}{4} > 0$. Moreover, $\frac{\partial \pi_M^{PE_B}}{\partial q} = 0$ can be expressed as a bivariate polynomial $\Gamma_1(q, \phi_M) = 0$. There is a unique root to $\Gamma_1(q, \phi_M) = 0$ in region PE_B, which we denote as $\tilde{q}_1(\phi_M)$. Hence, $\frac{\partial \pi_M^{PE_B}}{\partial q} > 0 \Leftrightarrow q < \tilde{q}_1(\phi_M)$ in region PE_B. Thus, $\frac{\partial \pi_M^{N*}}{\partial q} > 0$ can happen in regions PE_{NB} and PE_B, i.e. $\frac{\partial \pi_M^{N*}}{\partial q} > 0 \Leftrightarrow q_2(\phi_M) < q < \tilde{q}_1(\phi_M)$.

(ii) $\frac{\partial CS^{SE_{NB}}}{\partial q} = \frac{10-3q}{8(2-q)^3} > 0$, $\frac{\partial CS^{PE_{NB}}}{\partial q} = \frac{1-\phi_M}{8} > 0$, and $\frac{\partial CS^{PE_B}}{\partial q} > 0$. Moreover, $\frac{\partial CS^{SE_B}}{\partial q} = 0$ can be expressed as a bivariate polynomial $\Gamma_2(q, \phi_M) = 0$. There is a unique root to $\Gamma_2(q, \phi_M) = 0$ in region SE_B, which we denote as $\tilde{q}_2(\phi_M)$. Hence, $\frac{\partial CS^{SE_B}}{\partial q} < 0 \Leftrightarrow q < \tilde{q}_2(\phi_M)$ in region SE_B. Thus, $\frac{\partial CS^{N*}}{\partial q} < 0$ can happen in region SE_B, i.e. $\frac{\partial CS^{N*}}{\partial q} < 0 \Leftrightarrow q_1(\phi_M) < q < \tilde{q}_2(\phi_M)$. Furthermore, using Method A1, we can verify that the function CS^{N*} is continuous at $q = q_1(\phi_M)$ and $q = q_3(\phi_M)$. Moreover, solving $CS^{SE_B}(q, \phi_M) = CS^{PE_{NB}}(q, \phi_M)$ gives $\bar{q}(\phi_M)$, it can be shown that $\bar{q}(\phi_M)$ has one intersection point with $q_2(\phi_M)$. Define $\bar{\phi}_1$ as the unique solution of $\bar{q}(\phi_M) = q_2(\phi_M)$, then $CS^{N*}(q = q_2(\phi_M) - \epsilon) > CS^{N*}(q = q_2(\phi_M))$ if $\phi_M < \bar{\phi}_1$, and otherwise, $CS^{N*}(q = q_2(\phi_M) - \epsilon) < CS^{N*}(q = q_2(\phi_M))$.

(iii) $\frac{\partial SW^{SE_{NB}}}{\partial q} = \frac{6-5q}{8(2-q)^3} > 0$, $\frac{\partial SW^{PE_{NB}}}{\partial q} = \frac{1+\phi_M-2\phi_M^2}{8} > 0$, and $\frac{\partial SW^{PE_B}}{\partial q} > 0$. Moreover, $\frac{\partial SW^{SE_B}}{\partial q} = 0$ can be expressed as a bivariate polynomial $\Gamma_3(q, \phi_M) = 0$. There is a unique root to $\Gamma_3(q, \phi_M) = 0$ in region SE_B, which we denote as $\tilde{q}_3(\phi_M)$. Hence, $\frac{\partial SW^{SE_B}}{\partial q} < 0 \Leftrightarrow q < \tilde{q}_3(\phi_M)$ in region SE_B. Thus, $\frac{\partial SW^{N*}}{\partial q} < 0$ can happen in region SE_B, i.e. $\frac{\partial SW^{N*}}{\partial q} < 0 \Leftrightarrow q_1(\phi_M) < q < \tilde{q}_3(\phi_M)$. Furthermore, using Method A1, we can verify that the function SW^{N*} is continuous at $q = q_1(\phi_M)$ and $q = q_3(\phi_M)$. Moreover, SW^{N*} has an upward jump at $q = q_2(\phi_M)$ if $\phi_M > \bar{\phi}_2$ and has a downward jump if $\phi_M < \bar{\phi}_2$.

Part b: (i) $\frac{\partial \pi_M^{B*}}{\partial q} = -\frac{(2(1-\omega)-q(2-\omega))(2(1-q)+\omega(2-q))}{8(2-q)^2(1-q)^2}$, which is negative when $q \le \overline{q}(\omega)$. Recall Part a(i) and Method A1, then there is $\frac{\partial \pi_M^{B*}}{\partial q} > 0$ if $\hat{q}_2(\phi_M, \omega) < q < \tilde{q}_1(\phi_M)$, and $\frac{\partial \pi_M^{B*}}{\partial q} < 0$ otherwise.

(ii) $\frac{\partial CS^{B*}}{\partial q} = \frac{q^2(64-32\omega+6\omega^2)-q^3(12-8\omega+\omega^2)+8(5-2\omega+\omega^2)-4q(23-10\omega+3\omega^2)}{32(2-q)^3(1-q)^2} > 0$. Recall Part a (ii) and Method A1, then there is $\frac{\partial CS^{B*}}{\partial q} < 0$ if $q_1(\phi_M) < q < \min\{\tilde{q}_2(\phi_M), \hat{q}_1(\phi_M, \omega)\}$, and $\frac{\partial CS^{B*}}{\partial q} > 0$ otherwise. Furthermore, it can be shown that CS^{B*} has an upward jump at $q = \hat{q}_1(\phi_M, \omega)$ and has a downward jump at $\hat{q}_2(\phi_M, \omega)$.

(iii) $\frac{\partial SW^{B*}}{\partial q} = \frac{8(3+2\omega+7\omega^2)-q^3(20+8\omega+7\omega^2)+2q^2(32+16\omega+21\omega^2)-q(68+40\omega+84\omega^2)}{32(2-q)^3(1-q)^2} > 0$. Recall Part a (iii) and Method A1, then there is $\frac{\partial SW^{B*}}{\partial q} < 0$ if $q_1(\phi_M) < q < \min\{\tilde{q}_2(\phi_M), \hat{q}_1(\phi_M, \omega)\}$, and $\frac{\partial SW^{B*}}{\partial q} > 0$ otherwise. Furthermore, it can be shown that SW^{B*} has an upward jump at $q = \hat{q}_1(\phi_M, \omega)$ and has a downward jump at $\hat{q}_2(\phi_M, \omega)$.□

Proof of Proposition 2: The results can be obtained from Lemma A1. □

Proof of Corollary 1: It can easily be shown that R can decrease at around $q = \hat{\lambda}_2(\omega, q)$, so if blockchain is not available, R will be larger at $q = \hat{\lambda}_2^-(\omega, q)$ than at $q = \hat{\lambda}_2^+(\omega, q)$. □

Lemma A2:

(a) *If blockchain is not an option*

(i) $\frac{\partial \pi_M^{N*}}{\partial \phi_M} > 0$ *if* $\phi_2(q) < \phi_M < \tilde{\phi}_1(q)$, *and* $\frac{\partial \pi_M^{N*}}{\partial \phi_M} \le 0$ *otherwise.*

(ii) $\frac{\partial CS^{N*}}{\partial \phi_M} < 0$ *if* $\phi_1(q) < \phi_M < \tilde{\phi}_2(q)$, $CS^{N*}(\phi_M = \phi_2(q) - \epsilon) > CS^{N*}(\phi_M = \phi_2(q))$ *if* $\phi_M < \bar{\phi}_1$, $CS^{N*}(\phi_M = \phi_2(q) - \epsilon) < CS^{N*}(\phi_M = \phi_2(q))$ *if* $\phi_M > \bar{\phi}_1$, *and* $\frac{\partial CS^{N*}}{\partial \phi_M} \ge 0$ *otherwise.*

(iii) $\frac{\partial SW^{N*}}{\partial \phi_M} < 0$ *if* $\phi_1(q) < \phi_M < \tilde{\phi}_2(q)$, $SW^{N*}(\phi_M = \phi_2(q) - \epsilon) > SW^{N*}(\phi_M = \phi_2(q))$ *if* $\phi_M < \bar{\phi}_2$, $SW^{N*}(\phi_M = \phi_2(q) - \epsilon) < SW^{N*}(\phi_M = \phi_2(q))$ *if* $\phi_M > \bar{\phi}_2$, *and* $\frac{\partial SW^{N*}}{\partial \phi_M} > 0$ *otherwise.*

(b) *If blockchain is an option*

(i) $\frac{\partial \pi_M^{B*}}{\partial \phi_M} > 0$ *if* $\hat{\lambda}_2 < \phi_M < \tilde{\phi}_1(q)$, *and* $\frac{\partial \pi_M^{B*}}{\partial \phi_M} \le 0$ *otherwise.*

(i) $\frac{\partial CS^{B*}}{\partial \phi_M} < 0$ if $\phi_1(q) < \phi_M < \hat{\lambda}_1$, $CS^{B*}(\lambda = \hat{\lambda}_1 - \epsilon) < CS^{B*}(\lambda = \hat{\lambda}_1)$, $CS^{B*}(\lambda = \hat{\lambda}_2 - \epsilon) > CS^{B*}(\lambda = \hat{\lambda}_2)$.

(ii) $\frac{\partial SW^{B*}}{\partial \phi_M} < 0$ if $\phi_1(q) < \phi_M < \hat{\lambda}_1$, $SW^{B*}(\lambda = \hat{\lambda}_1 - \epsilon) < SW^{B*}(\lambda = \hat{\lambda}_1)$, $SW^{B*}(\lambda = \hat{\lambda}_2 - \epsilon) > SW^{B*}(\lambda = \hat{\lambda}_2)$.

Proof of Lemma A2:

Part a: (i) $\frac{\partial \pi_M^{SE_{NB}}}{\partial \phi_M} = 0$, $\frac{\partial \pi_M^{SE_B}}{\partial \phi_M} < 0$, $\frac{\partial \pi_M^{PE_{NB}}}{\partial \phi_M} = \frac{q+2(1-q)\phi_M}{4} > 0$. Moreover, $\frac{\partial \pi_M^{PE_B}}{\partial \phi_M}$ can be expressed as a bivariate polynomial $\Gamma_4(q, \phi_M) = 0$. There is a unique root to $\Gamma_4(q, \phi_M) = 0$ in region PE_B, which we denote as $\tilde{\phi}_1(q)$. Hence, $\frac{\partial \pi_M^{PE_B}}{\partial \phi_M} < 0 \Leftrightarrow \phi_M > \tilde{\phi}_1(q)$ in region PE_B. Thus, $\frac{\partial \pi_M^{N*}}{\partial \phi_M} > 0$ can happen in region PE_{NB} and PE_B, i.e. $\frac{\partial \pi_M^{N*}}{\partial \phi_M} > 0 \Leftrightarrow \phi_2(q) < \phi_M < \tilde{\phi}_1(q)$.

(ii) $\frac{\partial CS^{SE_{NB}}}{\partial \phi_M} = 0$, $\frac{\partial CS^{PE_{NB}}}{\partial \phi_M} = \frac{1-q}{8} > 0$,and $\frac{\partial CS^{PE_B}}{\partial \phi_M} > 0$. Moreover, $\frac{\partial CS^{SE_B}}{\partial \phi_M} = 0$ can be expressed as a bivariate polynomial $\Gamma_5(q, \phi_M) = 0$. There is a unique root to $\Gamma_5(q, \phi_M) = 0$ in region SE_B, which we denote as $\tilde{\phi}_2(q)$. Hence, $\frac{\partial CS^{SE_B}}{\partial \phi_M} < 0 \Leftrightarrow \phi_M < \tilde{\phi}_1(q)$ in region SE_B. Thus, $\frac{\partial CS^{N*}}{\partial \phi_M} < 0$ can happen in region SE_B, i.e. $\frac{\partial CS^{N*}}{\partial \phi_M} < 0 \Leftrightarrow \phi_1(q) < \phi_M < \tilde{\phi}_2(q)$. Furthermore, using Method A1, we can verify that the function CS^{N*} is continuous at $\phi_M = \phi_1$ and $\phi_M = \phi_3$. Moreover, CS^{N*} has an upward jump at $\phi_M = \phi_2$ if $\phi_M > \bar{\phi}_1$ and has a downward jump if $\phi_M < \bar{\phi}_1$.

(iii) $\frac{\partial SW^{SE_{NB}}}{\partial \phi_M} = 0$, $\frac{\partial SW^{PE_{NB}}}{\partial \phi_M} = \frac{1+q+4(1-q)\phi_M}{8} > 0$ and $\frac{\partial SW^{PE_B}}{\partial \phi_M} > 0$. Moreover, $\frac{\partial SW^{SE_B}}{\partial \phi_M} = 0$ can be expressed as a bivariate polynomial $\Gamma_6(q, \phi_M) = 0$. There is a unique root to $\Gamma_6(q, \phi_M) = 0$ in region SE_B, which we denote as $\tilde{\phi}_2(q)$ (the same with (ii)). Hence, $\frac{\partial SW^{SE_B}}{\partial \phi_M} < 0 \Leftrightarrow \phi_M < \tilde{\phi}_2(q)$ in region SE_B. Thus, $\frac{\partial SW^{N*}}{\partial \phi_M} < 0$ can happen in region SE_B, i.e. $\frac{\partial SW^{N*}}{\partial \phi_M} < 0 \Leftrightarrow \phi_1(q) < \phi_M < \tilde{\phi}_2(q)$. Furthermore, using Method A1, we can verify that the function SW^{N*} is continuous at $\phi_M = \phi_1$ and $\phi_M = \phi_3$. Moreover, SW^{N*} has an upward jump at $\phi_M = \phi_2$ if $\phi_M > \bar{\phi}_2$ and has a downward jump if $\phi_M < \bar{\phi}_2$.

Part b: (i) Because $\phi_2(q) < \hat{\lambda}_2 < \tilde{\phi}_1(q)$ and recall part a(i), then $\frac{\partial \pi_M^{B*}}{\partial \phi_M} > 0$ if $\hat{\lambda}_2 < \phi_M < \tilde{\phi}_1(q)$, and $\frac{\partial \pi_M^{B*}}{\partial \phi_M} \leq 0$ otherwise.

(ii) Because $\phi_1(q) < \hat{\lambda}_1 < \tilde{\phi}_2(q)$ and recall part a(ii), then $\frac{\partial CS^{B*}}{\partial \phi_M} < 0$ if $\phi_1(q) < \phi_M < \hat{\lambda}_1$. Furthermore, using Method A1, we can verify that the function CS^{B*} are continuous at $\phi_M = \phi_1$ and $\phi_M = \phi_3$. Moreover, CS^{N*} has an upward jump at $\phi_M = \hat{\lambda}_1$ and has a downward jump at $\phi_M = \hat{\lambda}_2$.

(iii) Because $\phi_1(q) < \hat{\lambda}_1 < \tilde{\phi}_2(q)$ and recall part a(iii), then $\frac{\partial SW^{B*}}{\partial \phi_M} < 0$ if $\phi_1(q) < \phi_M < \hat{\lambda}_1$. Furthermore, using Method A1, we can verify that the function SW^{B*} are continuous at $\phi_M = \phi_1$ and $\phi_M = \phi_3$. Moreover, SW^{B*} has an upward jump at $\phi_M = \hat{\lambda}_1$ and has a downward jump at $\phi_M = \hat{\lambda}_2$. □

Proof of Proposition 3: The results can be obtained from Lemma A2. □

Proof of Proposition 4:

The impact is only on the subgame with blockchain. $\frac{\partial \pi_M^{B*}}{\partial \omega} = -\frac{2(1-\omega)-q(2-\omega)}{4(1-q)} < 0$, which is true when $q \leq \overline{q}(\omega)$. Next, $\frac{\partial CS^{B*}}{\partial \omega} = -\frac{2(4-5q)(1-\omega)+q^2(2-3\omega)}{16(2-q)(1-q)} < 0$, and $\frac{\partial SW^{B*}}{\partial \omega} < 0$, which are both true when $0 < \omega < \omega_0 \wedge q \leq \overline{q}(\omega)$. Since $\omega = c_B + E_g[f(\theta)]$, below, we show the impacts of parts a and b on $E_g[f(\theta)]$.

Part a: $E_{g_1}[f(\theta)] \geq E_{g_2}[f(\theta)] \Leftrightarrow \int_{\underline{\theta}}^{\overline{\theta}} f(\theta)\big(g_1(\theta) - g_2(\theta)\big)d\theta = f(\theta)[G_1(\theta) - G_2(\theta)]|_{\underline{\theta}}^{\overline{\theta}} - \int_{\underline{\theta}}^{\overline{\theta}} f'(\theta)\big(G_1(\theta) - G_2(\theta)\big)d\theta > 0$, which is true because $g_1(\theta) \geq_{FSD} g_2(\theta) \Leftrightarrow G_1(\theta) \leq G_2(\theta)$.

Part b: $\frac{\partial E_g[f_k(\theta)]}{\partial k} = \int_0^{2a} \frac{\partial f_k(\theta)}{\partial k} g(\theta)d\theta = \frac{1}{2}\int_0^{2a} \frac{\partial}{\partial k}\left(\frac{\theta}{a}\right)^k d\theta = \frac{1}{2a^k}\int_0^{2a} \theta^k \ln\left(\frac{\theta}{a}\right)d\theta = \frac{2^k a}{k+1}\left(\ln 2 - \frac{1}{k+1}\right) > 0 \Leftrightarrow k > \frac{\ln e/2}{\ln 2}$. □

Proof of Proposition 5: We first show that $\mathbb{S}^G = B$ is always true in regions SE_{NB}, SE_B and PE_{NB}. Since π_M and SW are continuous at ϕ_3 (boundary between region PE_{NB} and PE_B), $\mathbb{S}^G = B$ can be true in region PE_B. Then we show that $\mathbb{S}^G = PE$ can also be true in region PE_B. Lastly, we solve for the threshold $\hat{\lambda}^G(\omega, q)$ that divides between $\mathbb{S}^G = B$ and $\mathbb{S}^G = PE$.

<u>Interior solution</u>: $\pi_C^B(s)$ is concave in p_C, and solving the FOC gives $p_C = \frac{1}{2}q(p_\mathrm{M} + E_g[f(\theta)])$. Next, $\pi_M^B(s)$ is concave in p_M, and solving the FOC gives $p_M = \frac{2(1-q)+(\omega-s)(2-q)}{2(2-q)}$. $SW^B(s)$ is concave in s, and solving the FOC gives $s^{in} \equiv \frac{2(4-3q)(1-q)-\omega(2-q)(4-q)}{(4-3q)(2-q)}$. $\pi_M^B(s^{in}) = \frac{2(q(7-4\omega)-q^2(3-\omega)-4(1-\omega))^2}{(4-3q)^2(2-q)(1-q)}$ and

$SW^B(s^{in}) = \frac{4(1-\omega)^2 - q(7-14\omega+4\omega^2)+q^2(3-6\omega+\omega^2)}{2(1-q)(4-3q)}$. Note that all non-negativity constraints are satisfied when

$$D_M^B(s^{in}) \geq 0 \Leftrightarrow q \leq \overline{q}(\omega) = \frac{7-4\omega-\sqrt{1+8\omega}}{2(3-\omega)} \tag{A4}$$

Regions SE_{NB}, SE_B and PE_{NB}: $\pi_M^B(s^{in}) - \pi_M^{SE_{NB}}$ can be expressed as a convex quadratic polynomial w.r.t. ω. Define $\widehat{\omega}_1(q) \equiv \frac{(4-3q)(1-q)}{2(2-q)^2}$ as the smaller root; the other root is outside of region SE_{NB}. It can be shown that $\omega_0 < \widehat{\omega}_1(q)$, so $\pi_M^B(s^{in}) > \pi_M^{SE_{NB}}$. Using the same methodology, it can be shown that $\pi_M^B(s^{in}) > \pi_M^{SE_B}$ in region SE_B and $\pi_M^B(s^{in}) > \pi_M^{PE_{NB}}$ in region PE_{NB}. Next, the social welfare in these three regions are $SW^{SE_{NB}} = \frac{12-9q+q^2}{8(2-q)^2}$, $SW^{SE_B} = \frac{16-q(32-q(8+(20-11q)q))-(1-q)\phi_M(32-2q(4+q(36-23q))+\phi_M(q((159-71q)q-84)-4(1-q)(4-3q)\phi_M(2-4q-(1-q)\phi_M)))}{2((2-q)^2-(4-q)(1-q)\phi_M)^2}$, $SW^{PE_{NB}} = \frac{(1+2\phi_M)(q+\phi_M-q\phi_M)}{8}$. It can be shown that in regions SE_B and PE_{NB}, these social welfares are smaller than $SW^B(s^{in})$, while in region SE_{NB},

$$SW^B(s^{in}) \geq SW^{SE_{NB}} \Leftrightarrow \omega < \omega_0 \equiv 0.0879 \tag{A5}$$

$$\overline{c} = \omega_0 - E_g[f(\theta)] \tag{A6}$$

Hence, in these regions, $\mathbb{S}^G = B$ when $c < \overline{c}$.

Region PE_B: We use the KKT method to solve Equation 4a. Then $s^{bi} \equiv \omega - \frac{2(1-q)((2-q)^2-(1-q)(4-q)\phi_M-2(q+\phi_M-q\phi_M)\sqrt{2(2-q)q(\phi_M-\phi_M^2)})}{(2-q)((2-q)^2-(1-q)(4-q)\phi_M)}$

Existence of $\mathbb{S}^G = PE$: $\lim_{q\to\overline{q}(\omega),\phi_M\to\overline{\phi}} \pi_M^{PE_B} = \frac{4(2\omega+\sqrt{1+8\omega}-1)(7-4\omega-\sqrt{1+8\omega})(31-12\omega-\sqrt{1+8\omega})^2}{25(97+16\omega^2-7\sqrt{1+8\omega}-4\omega(18-\sqrt{1+8\omega}))^2}$, $\lim_{q\to\overline{q}(\omega),\phi_M\to\overline{\phi}} SW^{PE_B} = \frac{(7-4\omega-\sqrt{1+8\omega})(31-12\omega-\sqrt{1+8\omega})^2(449+56\omega^2+\sqrt{1+8\omega}-4\omega(69-2\sqrt{1+8\omega}))}{200(3-\omega)(97+16\omega^2-7\sqrt{1+8\omega}-4\omega(18-\sqrt{1+8\omega}))^2}$, $\lim_{q\to\overline{q}(\omega),\phi_M\to\overline{\phi}} \pi_M^B(s^{in}) = 0$, $\lim_{q\to\overline{q}(\omega),\phi_M\to\overline{\phi}} SW^B(s^{in}) = \frac{1-\omega}{2}$, $\lim_{q\to\overline{q}(\omega),\phi_M\to\overline{\phi}} \pi_M^B(s^{bi}) = \frac{8(2\omega+\sqrt{1+8\omega}-1)(31-12\omega-\sqrt{1+8\omega})^2(17+\sqrt{1+8\omega}-2\omega(7+\sqrt{1+8\omega}))}{25(5+\sqrt{1+8\omega})(97+16\omega^2-7\sqrt{1+8\omega}-4\omega(18-\sqrt{1+8\omega}))^2}$ and $\lim_{q\to\overline{q}(\omega),\phi_M\to\overline{\phi}} SW^B(s^{bi}) = \frac{\overline{q}(176+992\overline{q}+24\overline{q}^2-704\overline{q}^3+163\overline{q}^4-26\overline{q}^5)-20\sqrt{2(2-\overline{q})\overline{q}}(16+4\overline{q}+4\overline{q}^2+\overline{q}^3)(\overline{q}(7-4\omega)-\overline{q}^2(3-\omega)-4(1-\omega))}{50(2-\overline{q})^2(4+\overline{q}^2)^2}$. It can be shown that $\lim_{q\to\overline{q}(\omega),\phi_M\to\overline{\phi}} \pi_M^{PE_B} \geq \lim_{q\to\overline{q}(\omega),\phi_M\to\overline{\phi}} \pi_M^B(s^*)$ and $\lim_{q\to\overline{q}(\omega),\phi_M\to\overline{\phi}} SW^{PE_B} > \lim_{q\to\overline{q}(\omega),\phi_M\to\overline{\phi}} SW^B(s^*)$, so $\mathbb{S}^* = PE$ when q and ϕ_M are large.

The constraint in Equation 4a: $\pi_M^B(s^{in}) - \pi_M^{PE_B}$ can be expressed as a convex quadratic polynomial w.r.t. ω. The larger root is infeasible, and the smaller root, $\widehat{\omega}_2(q,\phi_M)$, is smaller than ω_0, so Equation 4a is non-binding if $\omega < \widehat{\omega}_2(q,\phi_M)$ and is binding otherwise.

Non-binding: $SW^B(s^{in}) - SW^{PE_B}$ can be expressed as a four-degree polynomial w.r.t. ϕ_M, it can be shown that three of the roots are outside of region PE_B. Hence, there is at most one root, $\hat{\lambda}^{G\prime}(\omega,q)$, that solves $SW^B(s^{in}) = SW^{PE_B}$. It can be shown that $\hat{\lambda}^{G\prime}(\omega,q)$ is inside region PE_B. Hence, there always exists a unique $\hat{\lambda}^{G\prime}(\omega,q)$ such that $SW^B(s^{in}) > SW^{PE_B} \Leftrightarrow \phi_M < \hat{\lambda}^{G\prime}(\omega,q)$.

Binding: $SW^B(s^{bi}) - SW^{PE_B}$ is decreasing in ω; denote $\widehat{\omega}_3(q,\phi_M)$ to be the unique root such that $SW^B(s^{bi}) > SW^{PE_B} \Leftrightarrow \omega < \widehat{\omega}_3(q,\phi_M)$.

We define $\hat{\lambda}^G(\omega,q)$ as follows:

$$\hat{\lambda}^G(\omega,q) = \begin{cases} \hat{\lambda}^{G\prime}(\omega,q) & \text{if } \omega < \widehat{\omega}_2(q,\phi_M) \\ \text{solves } \omega = \widehat{\omega}_3(q,\phi_M) \text{ w.r.t.} \phi_M & \text{otherwise} \end{cases} \tag{A7}$$

Part b: $\frac{\partial s^{in}}{\partial q} = -\frac{2(4-3q)^2 - 8(2-q)^2\omega}{(4-3q)^2(2-q)^2} < 0$, $\frac{\partial s^{in}}{\partial E[\theta]} = \frac{\partial s^{in}}{\partial c_B} = -\frac{4-q}{4-3q} < 0$ and $\frac{\partial s^{bi}}{\partial E[\theta]} = \frac{\partial s^{bi}}{\partial c_B} = 1 > 0$. It can be shown that there is at most one root that solves $\frac{\partial s^{bi}}{\partial q} = 0$ w.r.t. q. Denote $\tilde{q}_0(\omega,\phi_M)$ to be the root if it exists, and $\tilde{q}_0(\omega,\phi_M) = -\infty$ otherwise. Then $\frac{\partial s^{bi}}{\partial q} < 0 \Leftrightarrow q > \tilde{q}_0(\omega,\phi_M)$. □

Proof of Proposition 6:

Part a: Without subsidy, $\mathbb{S}^* = B$ is true in part of regions SE_B and PE_{NB}; with subsidy, $\mathbb{S}^G = B$ is true in regions SE_{NB}, SE_B and PE_{NB} (cf. Proof of Propositoin1 and 5).

Region PE_B: Without subsidy, it can be shown that $\mathbb{S}^* = B$ when $\omega < \widehat{\omega}_4(q,\phi_M)$. With subsidy, define $\widehat{\omega}_5(q,\phi_M)$ as the root of $SW^B(s^{in}) = SW^{PE_B}$, so $SW^B(s^{in}) > SW^{PE_B} \wedge \pi_M^B(s^{in}) > \pi_M^{PE_B}$ when $\omega < \min[\widehat{\omega}_2(q,\phi_M), \widehat{\omega}_5(q,\phi_M)]$. Moreover, $SW^B(s^{bi}) > SW^{PE_B} \wedge \pi_M^B(s^{bi}) = \pi_M^{PE_B}$ when $\widehat{\omega}_2(q,\phi_M) < \omega < \widehat{\omega}_3(q,\phi_M)$ (cf. Proof of Proposition 5). Hence, $\mathbb{S}^G = B$ when $\omega < \min[\widehat{\omega}_3(q,\phi_M), \widehat{\omega}_5(q,\phi_M)]$. It can be shown that $\widehat{\omega}_4(q,\phi_M) < \min[\widehat{\omega}_3(q,\phi_M), \widehat{\omega}_5(q,\phi_M)]$, so the region of $\mathbb{S}^* = B$ is a subset of the region of $\mathbb{S}^G = B$.

Part b: We separate the proof into three regions.

$\mathbb{S}^G = B$-nonbinding: By definition, $SW^B(s^{in}) > SW^{N*} \wedge \pi_M^B(s^{in}) > \pi_M^{N*}$ or else blockchain would not be chosen. Moreover, in regions SE_{NB}, SE_B and PE_{NB}, $CS^B(s^{in}) - CS^{N*}$.is decreasing in ω, and it can be shown that $CS^B(s^{in}) > CS^{N*}$ when $\omega = \omega_0$. In region PE_B, it can be shown that $CS^B(s^{in}) > CS^{PE_B}$ when $\phi_M < \hat{\lambda}^{G\prime}(\omega, q)$.

Next, $\pi_M^B(s^{in}) > \pi_M^{B*} \Leftrightarrow \frac{16(q(7-4\omega)-q^2(3-\omega)-4(1-\omega))^2-(4-3q)^2(2-q(2-\omega)-2\omega)^2}{8(4-3q)^2(2-q)(1-q)} > 0$, $CS^B(s^{in}) > CS^{B*} \Leftrightarrow \frac{192(1-\omega)^2+q^4(60-52\omega+7\omega^2)-32q(19-30\omega+11\omega^2)-4q^3(86-87\omega+17\omega^2)+4q^2(175-218\omega+59\omega^2)}{32(2-q)^2(1-q)(4-3q)} > 0$ and $SW^B(s^{in}) > SW^{B*} \Leftrightarrow \frac{(q(14-6\omega)-q^2(6-\omega)-8(1-\omega))^2}{32(2-q)^2(1-q)(4-3q)} > 0$ which are true.

$\mathbb{S}^G = B$-binding : $\mathbb{S}^G = B$-binding $\Leftrightarrow \omega > \widehat{\omega}_2(q,\phi_M)$, and $\mathbb{S}^* = PE \Leftrightarrow \omega > \widehat{\omega}_4(q,\phi_M)$. It can be shown that $\widehat{\omega}_4(q,\phi_M) < \widehat{\omega}_2(q,\phi_M)$, so $\mathbb{S}^G = B$-binding is a subset of the area for $\mathbb{S}^* = PE$. By definition of binding, $\pi_M^B(s^{bi}) = \pi_M^{PE_B}$. Moreover, it can be shown that $CS^B(s^{bi}) > CS^{PE_B}$ and $SW^B(s^{bi}) > SW^{PE_B}$ in this region.

$\mathbb{S}^G = PE$: Proof of part a implies that this is a subset of $\mathbb{S}^* = PE$, and $\pi_M^G = \pi_M^{B*}$, $CS^G = CS^*$ and $SW^G = SW^*$.

Part c: Without subsidy, π_M^* and SW^* are non-monotonic in q (cf. Proposition 2). With subsidy, $\frac{\partial \pi_M^B(s^{in})}{\partial q} = \frac{2(q(7-4\omega)-q^2(3-\omega)-4(1-\omega))(20q(2+3\omega)-16(1+2\omega)+q^3(9+7\omega)-3q^2(11+12\omega))}{(2-q)^2(1-q)^2(4-3q)^3} < 0$, $\frac{\partial SW^B(s^{in})}{\partial q} = \frac{(12-16q+5q^2)\omega^2}{2(4-3q)^2(1-q)^2} > 0$. In region PE_B, it can be shown that $\frac{\partial \pi_M^B(s^{bi})}{\partial q} < 0$ and $\frac{\partial SW^B(s^{bi})}{\partial q} > 0$ are true.

Part d: Without subsidy, the optimal solutions depend on ϕ_M in regions SE_B, PE_{NB} and PE_B. With subsidy, the optimal solutions depend on ϕ_M only when $\mathbb{S}^G = PE$ or when $\mathbb{S}^G = B$-binding, which can only be true in region PE_B. The proof of parts a and b implies that $\mathbb{S}^G = PE$ and $\mathbb{S}^G = B$-binding are a subset of the area for $\mathbb{S}^* = PE$.□

Proof of Subsection 6.3:

Part a: $SW^N = \pi_M^N + \pi_C^N + CS^N$, $SW^B = \pi_M^B + \pi_C^B + CS^B - sD_M^B$, then $SW^{SE_{NB}}$ and SW^{SE_B} are same as before while $SW^{PE_{NB}} = \frac{3}{8}(q + (1-q)\phi_M)$ and $SW^{PE_B} = \frac{q(q+(1-q)\phi_M)^2(8-8q+q^2-(8-9q+q^2)\phi_M)}{2((2-q)^2-(4-5q+q^2)\phi_M)^2}$.

Proposition 1: There is no change because we do not consider social welfare.

Proposition 2: It can be shown that $\frac{\partial SW^{PE_{NB}}}{\partial q} = \frac{3(1-\phi_M)}{8} > 0$, and $\frac{\partial SW^{PE_B}}{\partial q} > 0$, so all results hold.

Proposition 3: It can be shown that $\frac{\partial SW^{PE_{NB}}}{\partial \phi_M} = \frac{3(1-q)}{8} > 0$ and $\frac{\partial SW^{PE_B}}{\partial \phi_M} > 0$. Moreover, $\underline{SW}^{B*} > SW^{PE_{NB}}$, $\underline{SW}^{B*} > SW^{PE_B}$ at $\phi = \hat{\lambda}_2(\omega, q)$. So all results hold.

Proposition 4: We only analyze the situation when $\mathbb{S}^* = B$, so there is no change to the result.

Proposition 5: There is no change to the optimal solution. For the existence of $\mathbb{S}^G = PE$, $\lim_{q \to \overline{q}(\omega), \phi_M \to \underline{\phi}} SW^{PE_B} =$
$\frac{(7-3\omega-\sqrt{1+14\omega+\omega^2})(7\omega+\sqrt{1+14\omega+\omega^2}-31)^2(3\omega^2-\omega(12-\sqrt{1+14\omega+\omega^2})+5(17+\sqrt{1+14\omega+\omega^2}))}{20(6-\omega)(97+7\omega^2-7\sqrt{1+14\omega+\omega^2}+\omega(3\sqrt{1+14\omega+\omega^2}-38))^2}$, and it can be shown that $\lim_{q \to \overline{q}(\omega), \phi_M \to \underline{\phi}} SW^{PE_B} > \lim_{q \to \overline{q}(\omega), \phi_M \to \underline{\phi}} SW^B(s^*)$ is also true. Moreover, it is not difficult to show that $\hat{\lambda}^G(\omega, q)$ is smaller under the new social welfare definition. There are no change to other parts.

Proposition 6: All structural results hold because we use the same logic, while the only change is that $SW^{PE_{NB}}$ and SW^{PE_B} become larger.

Part b: $SW^N = \pi_M^N + CS^N$, $SW^B = \pi_M^B + CS^B - sD_M^B$, then $SW^{PE_{NB}}$ and SW^{SP_B} are same as part a, and $SW^{SE_{NB}} = \frac{12-11q+3q^2}{8(2-q)^2}$, $SW^{SE_B} = \frac{8(2-4q+q^2)+3q^3(4-q)-(4-q)(1-q)\phi_M(2(4-7q^2)-q(19-23q)\phi_M-8(1-q)(1-2q)\phi_M^2+4(1-q)^2\phi_M^3)}{2((2-q)^2-(4-q)(1-q)\phi_M)^2}$, $SW^{B*} = \frac{48(1-\omega)^2-q^3(4-28\omega+5\omega^2)+4q^2(10-33\omega+8\omega^2)-4q(21-50\omega+17\omega^2)}{32(2-q)^2(1-q)}$.

Proposition 1: There is no change because we do not consider social welfare.

Proposition 2: It can be shown that $\frac{\partial SW^{SE_{NB}}}{\partial q}=\frac{2+q}{8(2-q)^3}>0$. We use the same logic as the proof of Proposition 2 to show that $\frac{\partial SW^{SE_B}}{\partial q}<0 \Leftrightarrow q<\tilde{q}_3'(\phi_M)$. Moreover, $\frac{\partial SW^{B*}}{\partial q}=\frac{q^3(4+8\omega+5\omega^2)-2q^2(16-15\omega)\omega+8(1-2\omega+5\omega^2)-4q(3-10\omega+15\omega^2)}{32(2-q)^3(1-q)^2}>0$. So all results hold.

Proposition 3: It can be shown that $\frac{\partial SW^{SE_{NB}}}{\partial \phi_M}=0$. We use the same logic as proof of Proposition 3 to show that $\frac{\partial SW^{SE_B}}{\partial \phi_M}<0 \Leftrightarrow \phi_M<\tilde{\phi}_1'(q)$ in region SE_B. Lastly, $\underline{SW}^{B*}>SW^{PE_{NB}}$, $\underline{SW}^{B*}>SW^{PE_B}$ at $\phi=\hat{\lambda}_2(\omega,q)$. So all results hold.

Proposition 4: $\frac{\partial SW^{B*}}{\partial \omega}=-\frac{24(1-\omega)+q^2(10-7\omega)-2q(17-13\omega)}{16(2-q)(1-q)}<0$, which is true when $0<\omega<\omega_0 \wedge q\leq \overline{q}(\omega)$. So all results hold.

Proposition 5: Using backward induction, we have $s^{in}\equiv\frac{2(4-q)(1-q)-\omega(2-q)(4-3q)}{(4-q)(2-q)}$. There is no change to s^{bi}. Then using the same method, it is not difficult to verify that $\mathbb{S}^G=B$ in regions SE_{NB}, SE_B and PE_{NB}. In region PE_B, we use the same logic as proof of Proposition 5 to show that $\mathbb{S}^*=PE$ when q and ϕ_M are large. Lastly, $\frac{\partial s^{in}}{\partial q}=-\frac{2(4-q)^2-8(2-q)^2\omega}{(4-q)^2(2-q)^2}<0$, $\frac{\partial s^{in}}{\partial E[\theta]}=\frac{\partial s^{in}}{\partial c_B}=-\frac{4-3q}{4-q}<0$. So all results hold.

Proposition 6: $\pi_M^B(s^{in})>\pi_M^{B*} \Leftrightarrow 2(4-q)(1-q)-\omega(2-q)(4-3q)>0$, $CS^B(s^{in})>CS^{B*} \Leftrightarrow \frac{(12-5q)\left(8-10q+2q^2-8\omega+10q\omega-3q^2\omega\right)^2}{32(4-q)^2(2-q)^2(1-q)}>0$ and $SW^B(s^{in})>SW^{B*} \Leftrightarrow \frac{\left((8-10q)(1-\omega)+q^2(2-3\omega)\right)^2}{32(4-q)(2-q)^2(1-q)}>0$ which are true. Next, $\frac{\partial \pi_M^B(s^{in})}{\partial q}=-\frac{2\left((1-\omega)(2-q)^2-q\right)\left(16+q^2(9-12\omega)-12q(2-\omega)-q^3(1-3\omega)\right)}{(4-q)^3(2-q)^2(1-q)^2}<0$, $\frac{\partial SW^B(s^{in})}{\partial q}=\frac{(4-q^2)w^2}{2(4-q)^2(1-q)^2}>0$. In region PE_B, it can be shown that $\frac{\partial \pi_M^B(s^{bi})}{\partial q}<0$ and $\frac{\partial SW^B(s^{bi})}{\partial q}>0$ are true. Hence, all results hold.

$\phi_1(q)$	Solves $\frac{1-q}{2-q} = \overline{p}$ w.r.t. ϕ_M (cf. P1, no blockchain)
$\phi_2(q)$	Solves $\pi_M^{SE_B} = \pi_M^{PE_{NB}}$ w.r.t. ϕ_M (cf. P1, no blockchain)
$\phi_3(q)$	Solves $\frac{q+\phi_M-q\phi_M}{2} = \overline{p}$ w.r.t. ϕ_M (cf. P1, no blockchain)
$\overline{\phi}$	4/5 (cf. P1, no blockchain)
ω_0	Solves $SW^B\left(s^{in}\right) = SW^{SE_{NB}}$ w.r.t. ω (cf. A5)
$\bar{c}$	$\omega_0 - E_g[f(\theta)]$ (cf. A6)
$\hat{\lambda}_1(\omega, q)$	Solves $\pi_M^{B*} = \pi_M^{SE_B}$ w.r.t. ϕ_M in region SE_B (cf. P1, region SE_B)
$\hat{\lambda}_2(\omega, q)$	$\begin{cases} \hat{\lambda}_2'(\omega, q) & \text{in region } PE_{NB} \\ \hat{\lambda}_2''(\omega, q) & \text{in region } PE_B \end{cases}$ (cf. P1)
$\overline{q}(\omega)$	$\frac{7-4\omega-\sqrt{1+8\omega}}{2(3-\omega)}$ (cf. P5)
$\tilde{q}_1(\phi_M)$	Solves $\frac{\partial \pi_M^{PE_B}}{\partial q} = 0$ w.r.t. q (cf. P1, region PE_B)
$\tilde{q}_2(\phi_M)$	Solves $\frac{\partial CS^{SE_B}}{\partial q} = 0$ w.r.t. q (cf. P1, region SE_B)
$\tilde{q}_3(\phi_M)$	Solves $\frac{\partial SW^{SE_B}}{\partial q} = 0$ w.r.t. q (cf. P1, region SE_B)
$\tilde{\phi}_1(q)$	Solves $\frac{\partial \pi_M^{PE_B}}{\partial \phi_M} = 0$ w.r.t. ϕ_M (cf. P1, region PE_B)
$\tilde{\phi}_2(q)$	Solves $\frac{\partial CS^{SE_B}}{\partial \phi_M} = 0$ w.r.t. ϕ_M (cf. P1, region SE_B)
$\bar{\phi}_1$	Solves $CS^{SE_B}(q = q_2) = CS^{PE_{NB}}(q = q_2)$ w.r.t. ϕ_M
$\bar{\phi}_2$	Solves $SW^{SE_B}(q = q_2) = SW^{PE_{NB}}(q = q_2)$ w.r.t. ϕ_M
s^{in}	$\frac{2(4-3q)(1-q)-\omega(2-q)(4-q)}{(4-3q)(2-q)}$ (cf. P5)
s^{bi}	$\omega - \frac{2(1-q)((2-q)^2-(1-q)(4-q)\phi_M - 2(q+\phi_M-q\phi_M)\sqrt{2(2-q)q(\phi_M-\phi_M^2)})}{(2-q)((2-q)^2-(1-q)(4-q)\phi_M)}$ (cf. P5)
$\hat{\lambda}^G(\omega, q)$	$\begin{cases} \hat{\lambda}^{G\prime}(\omega, q) & \omega < \widehat{\omega}_2(q, \phi_M) \\ \text{solves } \omega = \widehat{\omega}_4(q, \phi_M) \text{ w.r.t.} \phi_M & \omega > \widehat{\omega}_2(q, \phi_M) \end{cases}$ (cf. P5)
$\tilde{q}_0(\omega, \phi_M)$	Solves $\frac{\partial s^{bi}}{\partial q} = 0$ w.r.t. q

Table A1: Table of thresholds