Conspicuous by Its Absence: Diagnostic Expert Testing under Uncertainty

Tinglong Dai Shubhranshu Singh*

February 3, 2019

Abstract

We study the problem a diagnostic expert (e.g., a physician) faces when offering a diagnosis to a client (e.g., a patient) that may be based only on her own diagnostic ability or supplemented by a diagnostic test revealing the client's true condition. The expert's diagnostic ability (or type) is her private information. The expert is impurely altruistic in that she cares about both the client's utility and her own reputational payoff that depends on the peer perception about her diagnostic ability. The decision of whether to perform the test, which is costly for the client, provides the expert with an opportunity to influence that perception. We show a unique separating equilibrium exists in which the high-type expert does not resort to diagnostic testing and offers a diagnosis based only on her own diagnostic ability, whereas the low-type expert performs the test. Furthermore, we establish that the high-type expert may skip necessary diagnostic tests to separate her from the low-type expert. Interestingly, the effect of reputational payoff on under-testing is non-monotonic, and the desire to appear of high type leads to under-testing only when the reputational payoff is intermediate. Our results also suggest a more altruistic expert may be more likely to engage in under-testing. Furthermore, efforts to encourage testing by providing financial incentives or by raising malpractice-lawsuit concerns may, surprisingly, help fuel under-testing in the equilibrium.

*Both authors contributed equally to this work and are listed alphabetically. Address: Carey Business School, Johns Hopkins University, Baltimore, MD 21202. Email: {dai, shubhranshu.singh}@jhu.edu. We thank Mehmet Ayvaci, Sarang Deo, Pedro M. Gardete, Liang Guo, Chao-Wei Hwang, Robert Shumsky, Bobby Zhou, seminar participants at Chinese University of Hong Kong, University of Chicago, and University of South Carolina, as well as session participants at INFORMS Annual Meeting, INFORMS Marketing Science Conference, INFORMS Healthcare Conference, INFORMS International Meeting, MSOM Conference, POMS Annual Meeting, POMS-HK International Conference, and University of Texas at Dallas Frank M. Bass FORMS Conference for their helpful feedback and discussions.

P.S.—If you don't receive this letter, write and let me know, and I'll send you another.

— Anonymous

1. Introduction

Few issues in the healthcare market are more salient than the provision of diagnostic tests: an estimated 30% of medical-testing decisions are deemed inappropriate (Brody 2010; Fisher et al. 2003), which may entail either over- or under-provision. All too frequently, the public attention has centered on over-provision. By comparison, under-provision of diagnostic testing has received little attention in the mass media, but frequently appears in the medical literature: a Harvard Medical School research team examining 15 years' worth of medical-testing literature—covering 1.6 million results from 46 of the 50 most commonly used diagnostic tests—finds under-testing may well be more prevalent than over-testing (Zhi et al. 2013). According to Dr. Ramy Arnaout, a member of the research team, "underutilization is at least as bad a problem as overutilization... This is a robust finding. This is for real" (O'Reilly 2014). Under-testing is the leading cause of what Landro (2013) refers to as "the biggest mistake doctors make"—diagnostic error, which is attributed to severe harm and death for approximately 160,000 patients per year (Newman-Toker et al. 2013; Singh et al. 2013). The economic impact of under-testing is equally striking, with some estimating it as high as 38% of total healthcare expenditure (Sollman 2015).

Motivated by the phenomenon of under-testing in the healthcare market, our paper develops an analytical lens for understanding its drivers. Specifically, we examine a physician's diagnostic decision under both information asymmetry and diagnostic uncertainty. Consider the physician-patient encounter in a physician's office. A patient consults with a physician about the nature of a medical problem, which may be either positive (sick) or negative (healthy). During a consultation, the physician collects and synthesizes patient history, which helps her figure out the likelihood or the base rate that the patient's condition is positive. The physician also acquires a private signal—a "hunch"—indicative of the nature of the problem.[1] A more competent physician is able to generate a more accurate signal of the patient's condition. Even the best physicians may not be able to perfectly infer the patient's health. Diagnostic testing (e.g., blood test, ultra-sound, and X-ray) is often called for. The physician does not always perform a diagnostic test, which imposes monetary and other burdens on the patient. Although a physician reputed for high diagnostic ability among

[1] Siddhartha Mukherjee (2017, p. 53), in a *New Yorker* article in which he details clinical encounters of Dr. Lindsey Bordone, a dermatologist from Columbia University, provides an example of this private signal: "The diagnostic moment [of an encounter] came to Bordone in a flash of recognition."

primary care physicians, triage nurses, OPD doctors, and other experts (collectively referred to as "generalist peers" or simply "peers") may enjoy more referrals, her competence level may not be immediately obvious to her peers. However, the physician can possibly manage perceptions about her diagnostic ability by making certain diagnostic-testing decisions. Anticipating such observational learning, the physician has an opportunity to choose a diagnostic pathway (i.e., the process to reach the eventual diagnosis, which may or may not involve diagnostic testing) to influence the perception of her diagnostic ability. We would like to emphasize that this paper focuses on the diagnostic aspect of the physician-patient encounter. We do not incorporate treatments in our analysis, because we do not intend the physician's incentives to profit from her informational advantage to contaminate her diagnostic decisions.[2]

Our model captures several features of the physician-patient encounter. In a healthcare setting, similar to many other regulated services, the service providers do not set the prices. Payers (e.g., Medicare) usually set them; the Affordable Care Act has been more directed at setting prices. In addition, the *opacity* in pricing in the healthcare market is a well-known phenomenon that separates this market from markets for most goods and services, as Uwe Reinhardt (2013, p. 1927) pointedly remarks in a *JAMA* article:

> [P]rices were kept as trade secrets. Rare are the physicians, hospitals, imaging centers, or other clinicians or health care centers who post on their websites the prices for frequently performed procedures. Furthermore, few health care practitioners or centers are willing to quote prices over the phone for even standard procedures, such as a normal vaginal delivery. As a consequence, the often advanced idea that American patients should have "more skin in the game" through higher cost sharing, inducing them to shop around for cost-effective health care, so far has been about as sensible as blindfolding shoppers entering a department store in the hope that inside they can and will then shop smartly for the merchandise they seek. So far the application of this idea in practice has been as silly as it has been cruel.

For decades, health economists such as Reinhardt (2013) have called in vain for price transparency. The bipartisan Congressional Budget Office (2008), on the other hand, by citing empirical evidence from other industries, contends that increasing transparency in the healthcare market can result in higher prices. Consistent with the industry practice we aim to capture, we assume experts (e.g., physicians) do not set prices for their diagnostic services.

[2] In our main analysis, we abstract away from the physician's financial incentives, which we investigate in an extension presented in Section 6.1.

In a healthcare setting, considerable uncertainty is associated with physicians' diagnostic accuracy. Furthermore, the difference in the skill levels across experts may not be transparent to patients (Gawande 2004; Makary 2013). Indeed, information asymmetry plays a pivotal role in the expert's decision-making process. The information asymmetry manifests itself in two different aspects: (1) the patient as well as the physician's peers, who refer the patient to the expert physician, do not know about the diagnostic ability of the physician ex ante, which is the physician's private information; and (2) the patient or the peers cannot observe the physician's private signal, which the physician cannot credibly communicate. In addition, the patient does not have the medical expertise to analyze her own medical history and determine the likelihood of having some medical condition.

A patient visits a physician, who may perform a diagnostic test, when referred to by an OPD doctor, primary care physician, triage nurse, or some other expert. A diagnostic test imposes a cost on the patient. This cost may include any monetary or inconvenience cost the patient incurs. The payer—not the physician—typically determines this cost, which has little, if anything, to do with the physician's diagnostic ability. We also assume the physician is "impurely altruistic" in the sense that she is concerned about both the patient's welfare and her own reputational payoff. The physician's reputational payoff captures her gain from peer referrals when she is believed to be of high ability as opposed to low ability. It may also include respect among peers, advantageous employment prospects, and job satisfaction, among other—potentially intrinsic—benefits.

We consider a client, whose state as assigned by nature is either positive or negative, visits a diagnostic expert to learn about her state.[3] The expert, who can be of either high or low type, learns the likelihood of the client's state being positive and receives an informative signal of the client's state. A high-type expert receives a more informative signal of the client's state than a low-type one does. The expert's type information and private signal are both unobservable to the client. The expert cares about both the client's utility and her own reputational payoff. The expert offers a diagnosis that may be either based only on her private signal or on a diagnostic test that perfectly reveals the client's state. Our model reflects the observation that diagnostic expertise and testing may substitute each other in a variety of settings (see, e.g., Johnson 1988; Doyle et al. 2010; Clark et al. 2012; Silver 2016; Rosenbaum 2017).[4] The belief about the expert's diagnostic ability

[3] In the model, we refer to the physician as a diagnostic expert, or simply an expert, and to the patient as a client.

[4] In particular, Doyle et al. (2010) examine a large sample of patients randomly assigned to two physician teams, and found that one team had significantly lower costs than the other team. The difference in costs stem from differences in diagnostic testing; that is, patients assigned to one team were more likely to experience diagnostic tests than patients assigned to the other team. Still, the two teams achieved comparable treatment outcomes by all measures, including mortality and readmissions.

or type is updated by her peers based on the diagnostic pathway the expert chooses. We model the expert's sequential decision-making process and characterize the perfect Bayesian equilibrium; peers have prior beliefs about the type of the expert (either high or low), which are updated using Bayes' rule after observing the expert's testing decisions. We look for separating equilibria in which the two types of experts have *externally* different pathways leading to the eventual diagnosis. Consider, for example, a candidate equilibrium in which the high-type expert performs the test regardless of whether her private signal is positive or negative, whereas the low-type expert performs the test if and only if her private signal is negative. In this case, the peers cannot separate the two types by merely observing the expert's chosen diagnostic pathway, because the two types of experts' diagnostic pathways are different only when each of them receives a positive private signal, yet the expert's private signal is unobservable to the peers. Furthermore, the physician does not have pricing power, and therefore prices cannot serve as a signaling device.

An important objective of the work is to investigate whether the expert's diagnostic pathway can act as a signal of her diagnostic ability. Our analysis approaches the question using a generic modeling framework. We show a unique separating equilibrium exists in which the high-type expert does not perform the test and offers a diagnosis that is consistent with her private signal, whereas the low-type expert tests. The intuition is as follows. A diagnostic test offers the benefit of precise learning of the client's state, but comes at a cost to the client. The cost of testing does not vary across expert types. However, the incremental benefit is larger when the low-type expert performs the test, because the low-type expert receives a less informative signal of the client's state. The high-type expert forgoes the small incremental benefit of performing the test for the reputational payoff she would receive as a result of the belief that she is a high-type expert. The low-type expert finds not performing the test is too costly, because of her less precise private signal.

We also show the separating equilibrium exists only when the reputational payoff is neither too high nor too low. In other words, the effect of the reputational payoff on the expert's signaling incentive is non-monotonic. If the reputational payoff is low, the high-type expert's incentive-compatibility constraints drive the equilibrium. An increase in the reputational payoff makes separation more attractive to the high-type expert, and the equilibrium exists in the wider range of the parameter space. On the other hand, if the reputational payoff is high, the low-type expert's incentive-compatibility constraints drive the equilibrium. An increase in the reputational payoff makes mimicking the high-type expert more attractive for the low-type expert, and therefore reduces the high-type expert's incentive to separate. As a result, the equilibrium exists over a smaller range of the parameter space.

Given the unique separating equilibrium in which the high-type expert does not perform the test, a natural question arises: Does the high-type expert under-test? We show the high-type expert chooses to skip testing for certain clients, although testing would have been deemed necessary for these clients in the full-information case. The information asymmetry about the expert type induces the high-type expert to perform too few tests in order to prevent her from being perceived as a low-type expert. The low-type expert, on the other hand, does not over-test and uses the same diagnostic strategy as in the full-information case. In the separating equilibrium, the high-type expert sacrifices client utility by under-testing for her own reputational gains. Would a more altruistic expert be more or less likely to engage in under-testing? We find a more altruistic expert may be more likely to engage in under-testing, because if the expert is more altruistic, the low-type expert is more likely to test and less likely to mimic the high-type expert, who forgoes testing for reputational payoffs.

Building on our baseline model, we consider an extension in which the expert receives a financial incentive for performing the diagnostic test. In the healthcare context, this type of financial incentive is referred to as "fee-for-service" and is frequently viewed as a major source of over-provision of medical tests (Epstein et al. 1986). Notwithstanding the potential misalignment under a fee-for-service environment, we show that in certain cases, a strong financial incentive for diagnostic tests may help mitigate under-testing and improve client welfare. In other cases, quite surprisingly, a strong financial incentive for diagnostic tests can facilitate the high-type expert's under-testing. Along similar lines, an effort to incentivize testing by raising lawsuit concerns among diagnostic experts can potentially backfire and result in more under-testing.

Complementing our main analysis, we also examine a case with a tamper-proof technology (see, e.g., Ichikawa et al. 2017) that allows the expert to reliably disclose her private evaluation. We show that in this case, a separating equilibrium in which the type-h expert signals her ability by disclosing her private evaluation and then performing the test does *not* exist. We also confirm that the separating equilibrium discussed in the baseline model continues to exist even when signal disclosure is possible. Interestingly, a pooling equilibrium with potential over-testing may exist in which the expert, regardless of her type, chooses to disclose her private evaluations before testing.

Our result that under-testing arises as a separating device echoes the widely held belief in the medical decision-making literature (e.g., Schroeder et al. 1974; Yeh 2014) that "high utilizers would tend to be less competent physicians who attempt to compensate for clinical deficiencies" (Schroeder et al. 1974, p. 710). It is also directionally aligned with a recent finding by, for example, Arkes et al. (2007), Probst (2008), Joshi and Wolf (2011), and Wolf (2014), that peers tend to "derogate the diagnostic ability of physicians" who rely on diagnostic tools in reaching their diagnosis (Wolf 2014, p. 288). The result has important implications for the US healthcare system, in which under-testing has emerged as an important source of misdiagnosis but has not received due attention from either the public or the healthcare research community. Our paper represents an initial attempt to model physician-patient encounters behind this phenomenon, and in doing so, establishes a formal linkage between diagnostic uncertainty and information asymmetry. In addition, we derive several surprising results including a non-monotonic effect of reputational payoff, that a more altruistic expert may engage in more under-testing, and that efforts to incentivize testing by offering financial incentives or by raising lawsuit concerns may backfire and result in more under-testing. The findings of the paper may be applicable to other settings where the diagnostic experts (e.g., auto mechanics, failure analysis engineers, lawyers, and management consultants) cannot use prices to signal their types, possibly because they are employees of a firm and receive largely fixed compensation that does not crucially depend on their utilization.

1.1 Literature

Much of the literature on expert services has focused on the joint provision of diagnosis and intervention. In this literature, the expert's incentives to maximize profits when selling products or services influence her diagnosis. In a seminal paper, Darby and Karni (1973) show that branding and customer relationships can serve as a monitoring mechanism to reduce fraud by experts selling diagnoses and services. Taylor (1995) considers an expert who determines if the consumer (or her product) is healthy or diseased, and performs treatment. He shows the information asymmetry may create demand for health insurance by risk-neutral consumers. Emons (1997) argues a non-fraudulent equilibrium can possibly exist because consumers may be able to infer an expert's incentives after observing market data. Ely and Välimäki (2003) consider an auto mechanic who cares about her reputation for being scrupulous. They show reputational concerns may induce the mechanic to perform minor repairs even when a major repair is in order. In the context of an expert providing a diagnosis and solving an uninformed customer's problems, Fong (2005) proposes that expert cheating may arise as a substitute for price discrimination: experts may target high-valuation and high-cost customers. Jiang, Ni, and Srinivasan (2014) consider experts who may be either purely altruistic or purely self-interested, and study experts' pricing strategy that serves to signal the expert's type.

Diagnostic experts do not always sell additional products or services. The diagnoses these experts provide may not be influenced by their incentive to perform a subsequent, costly, and often unnecessary service. Dublin and Iyer (2009) consider an expert who cares about her reputation for incorruptibility, and offers advice (or a diagnosis) to an uninformed decision-maker in the presence of a third party that tries to influence the expert's advice through unobservable bribes. The authors show bribes can help restore truthful communication that would otherwise not occur. Singh (2017) considers an agent who evaluates competing firms and recommends a firm for purchase in a procurement auction. He shows the buyer's increased monitoring effort of the agent can result in a higher likelihood of selection of an inferior firm. Gardete and Bart (2018) study how information environment and communication costs influence an expert's communication strategy and market outcomes. We contribute to the literature on diagnostic expert services by examining an expert's incentives to offer a diagnosis that is based solely on her imperfect private information or to perform a diagnostic test that perfectly reveals the client's state. Most of the literature assumes the expert acquires *perfect* client information that is unknown to the clients. In our model, by contrast, the expert receives an informative but *imperfect* signal of the client's state. Aligned with empirical evidence on diagnostic expertise in the labor economics literature (see, e.g., Currie and MacLeod 2017), we focus on the case in which experts may differ in their diagnostic accuracy, and highlight the consideration that even an expert with the best expertise and the best intentions may be unable to reach a perfect assessment of the client's status.

Our paper is related to the marketing literature on diagnostic services. Sarvary (2002) shows a market for second opinions may arise in the expert diagnosis service as a result of temporal differentiation. Arora and Fosfuri (2005) examine pricing of diagnostic information to the consumers who are privately informed about the value of the information. Jiang, Ni, and Srinivasan (2014) study experts' pricing strategy that serves to signal the expert's type. They assume the expert can perfectly and costlessly acquire client information, and focus on the case in which the expert can be either ethical or purely self-interested. By contrast, in our baseline model, the expert is not driven by financial interests but is nevertheless impurely altruistic in that the expert has a reputation consideration that may be tied to peers' observational learning of the expert's diagnostic pathway. Also, the expert in our setting is *imperfectly* informed about the client's state unless she performs a diagnostic test. We investigate the expert's financial incentives in an extension of our baseline model.

Our paper is also related to the work by Miklos-Thal and Zhang (2013) on a firm's "de-marketing" strategy that entails purposely reducing its salesforce efforts to create a perception that its product is of high quality. In our paper, a high-type expert may choose not to perform necessary tests to signal her type to clients. Note that in our model, not performing a diagnostic test does not reduce demand for expert services. Our paper also differs from Miklos-Thal and Zhang (2013) in that ours models a diagnostic expert whose principal focus is to choose a course of action to diagnose the client condition, which may be either positive or negative. By contrast, their paper models a firm that chooses the optimal level of sales effort to maximize its own profit.

Our paper, by examining the phenomenon of under-provision, contributes to the literature on the economics of service provision. This literature focuses on the phenomenon of *over*-provision by diagnostic experts and uncovers myriad drivers such as insurance structure (Dai, Akan, and Tayur 2017), unverifiable service requirements (Debo, Toktay, and van Wassenhove 2008), lawsuit concerns (DeKay and Asch 1998), and conflicts of interest (Alger and Salanie 2006; Paç and Veeraraghavan 2015). In contrast to this stream of literature, our paper is motivated by *under*-provision, which is an equally important aspect in many service industries (particularly healthcare) but has not received due attention. Our paper proposes and examines a novel strategic reason for the phenomenon of under-provision by diagnostic experts: high-ability experts may use under-testing as a way to signal their ability, because diagnostic testing and expertise can substitute for each other.

Our paper also contributes to the marketing literature on health-related topics. Amaldoss and He (2009) investigate seemingly wasteful direct-to-consumer advertising of prescription drugs. Dukes and Tyagi (2009) examine in vitro fertilization clinics' incentives to offer money-back guarantees, and the effect of these guarantees on couples' choices. Cui, Desai, and Wang (2016) characterize a medical insurance plan's decision to include drugs in the formulary to reduce costs, the bargaining process, and the copay amount. Bala, Bhardwaj, and Chintagunta (2017) study a drug manufacturer's allocation decision for the category-defense effort and direct-sales effort when facing likely recall of a competing drug. We contribute to this evolving literature by examining a diagnostic expert's (or physician's) incentives that may result in prescribing fewer diagnostic tests than optimal.

The rest of the paper is organized as follows. Section 2 describes our modeling environment and the full-information benchmark. Section 3 characterizes the equilibrium. Section 4 presents the analysis. Section 5 analyzes the case in which the expert can opt to disclose her private evaluations before testing. Section 6 presents three extensions of the baseline model. Section 7 concludes.

2. Model

Consider a client whose state (θ) is either positive ($\theta = 1$) or negative ($\theta = 0$). In a physician-patient encounter, for instance, a positive state means the patient suffers from a medical condition, whereas a negative state means the patient is healthy. At the beginning of the game, nature draws the state from a Bernoulli distribution and assigns it to the client. The client's state is positive ($\theta = 1$) with prior probability α, and is negative ($\theta = 0$) with the complementary probability. The prior-probability α is a client characteristic and represents the base rate with which she acquires a certain medical condition. The client does not observe the true state θ. If the state θ is correctly revealed to the client, the adverse effects associated with a positive state can be remedied.

The client visits a diagnostic expert to seek diagnosis. The expert's service starts with consultation, during which she learns client's α and receives a *private* signal of the client's state. The expert's private signal is captured by $s_e \in \{0, 1\}$, where $e \in \{l, h\}$ represents the type of the expert and determines whether the precision of the expert's signal is low (ρ_l) or high (ρ_h). The prior probability that the expert is of type h is $\gamma \in (0, 1)$. The expert's type e and her private signal s_e are unobservable to the client. The signal $s_e = 1$ indicates a positive state and $s_e = 0$ indicates a negative state. The expert cannot verifiably deliver her private signal to the client.[5] A type-e expert's private signal has a precision of ρ_e, that is,

$$\Pr(s_e = 0|\theta = 0) = \Pr(s_e = 1|\theta = 1) = \rho_e, \text{ for } e = h, l.$$

We assume $\rho_h > \rho_l$ to reflect that a type-h expert tends to make more accurate judgments than a type-l expert; both ρ_h and ρ_l are above 1/2, so the diagnostic accuracy of either type is better than tossing a coin. The expert can either reach a diagnosis that is based on the signal she has received about the client's state or perform a diagnostic test. Let $t \in \{0, 1\}$ denote the expert's testing decision and let $a \in \{0, 1\}$ denote the expert's diagnosis decision. After the consultation, the expert has three possible actions: (1) perform the diagnostic test (i.e., $t = 1$) and diagnose according to its outcome; (2) provide a positive diagnosis without performing the test (i.e., $t = 0, a = 1$); and (3) provide a negative diagnosis without performing the test (i.e., $t = 0, a = 0$).

We assume the diagnostic test, if performed, reveals the client's true condition to both the expert and the client. Performing the test comes at a cost of c, incurred to the client. Here, we provide two motivating examples for this setup. In the case of an emergency department (ED), when patients complain of dizziness/vertigo, an estimated 35% of underlying strokes are missed, despite the fact that a diagnostic test can "identify more than 99% of strokes" (Newman-Toker et al. 2013). As another example, in a catheterization laboratory (commonly known as "cath lab"), an interventional cardiologist often decides whether to conduct a percutaneous coronary intervention (PCI, also known as "stenting") by visually assessing a coronary angiogram ("eyeballing"), a process

[5]In Section 5, we provide a formal discussion of the case in which the expert attempts to signal her ability by first disclosing her private signal and then performing the test.

Table 1: Client payoff as a function of the expert's diagnosis outcome

	Expert's diagnosis	
Client's true state	$a = 1$ (positive diagnosis)	$a = 0$ (negative diagnosis)
$\theta = 1$ (positive)	B	$-D$
$\theta = 0$ (negative)	$-d$	0

with significant subjectivity and associated with a significant proportion of inappropriate PCI procedures (Desai et al. 2015). An advanced intracoronary test such as fractional flow reserve (FFR) provides a nearly objective measure of the appropriateness of a PCI procedure, and "almost seems too good to be true," according to Dr. William Fearon of Stanford University (Fornell 2013). The FFR test, however, is intrusive and may introduce additional health risk to patients (Topol 2008).

The value of the expert's service is B if the expert diagnosis correctly identifies a positive client state (either through consultation only or through both consultation and diagnostic testing), and is $-D$ if the expert provides a negative diagnosis when the true state is positive; both B and D are positive.[6] Consider, for example, a patient suffering from a medical condition. If the physician correctly diagnoses the condition, the patient may be put on a treatment and fully recover; that is, the correct diagnosis of the medical condition offers a benefit B to the patient. However, if the physician's diagnosis incorrectly suggests the patient is healthy, the patient's condition might deteriorate, requiring more extensive treatment or incomplete recovery; the parameter D captures this loss due to a misdiagnosis. Now suppose the patient is healthy but the physician diagnoses the patient's state as positive. In this case, the patient may be put on an unnecessary treatment or incur a psychological burden. We capture the disutility of the expert's positive diagnosis, when the client's true state is negative, by $-d$ ($d > 0$). We further assume $c < d$. We normalize the client's payoff from a true-negative diagnosis (i.e., when both the true state and the expert's diagnosis are negative) to zero. We summarize the client's payoff from the diagnostic service (without accounting for the cost of a potential diagnostic test) as a result of the expert's diagnosis outcome in Table 1 and illustrate the sequence of events in Figure 1.

We derive the type-e expert's beliefs $b_e(\alpha|s_e)$ about the probability of the client's state being $\theta = 1$ using Bayes' rule. The beliefs are given by

$$b_e(\alpha|s_e) \triangleq \begin{cases} \frac{\alpha\rho_e}{\alpha\rho_e+(1-\alpha)(1-\rho_e)} & \text{if } s_e = 1, \\ \frac{\alpha(1-\rho_e)}{\alpha(1-\rho_e)+(1-\alpha)\rho_e} & \text{if } s_e = 0. \end{cases} \tag{1}$$

[6]Throughout the paper, we define the client's payoff as the change in client utility as a result of the diagnostic service. Alternatively, one can define the client's payoff as the client's absolute utility. In that case, B can have a negative value, and we can replicate all our results.

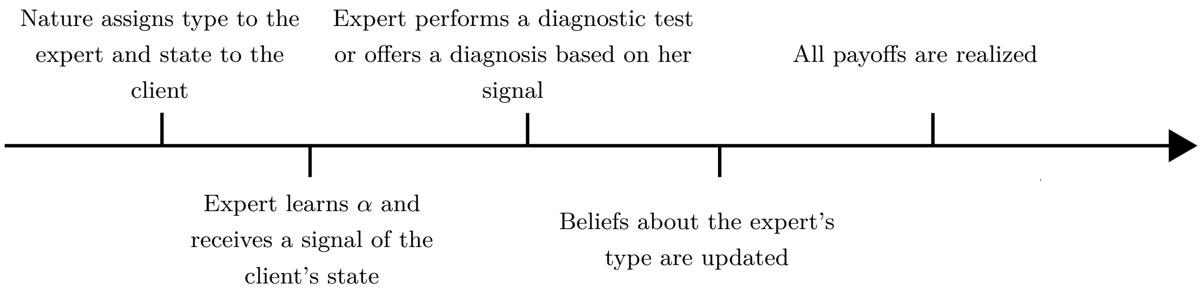

Figure 1: Timing of the Game

Now we describe the client's expected utility given the expert's signal s_e and her decisions t and a. If the expert tests ($t = 1$), the client incurs a cost c and the test reveals her state θ. The client learns the state is $\theta = 1$ with probability $b_e(\alpha|s_e)$ and receives a benefit B, whereas with probability $1 - b_e(\alpha|s_e)$, the client learns the state is $\theta = 0$ and receives a payoff of zero. Therefore, in this case, the client's expected utility is $b_e(\alpha|s_e) \cdot B - c$. If the expert chooses not to perform a diagnostic test, she would have to decide whether to follow her private signal in reaching her diagnosis. Suppose her diagnosis is consistent with her signal ($a = s_e$). The client's expected utility in this case is given by $b_e(\alpha|s_e) \cdot B + [1 - b_e(\alpha|s_e)] \cdot (-d)$ if $s_e = 1$, and $b_e(\alpha|s_e) \cdot (-D)$ if $s_e = 0$. However, if the expert's diagnosis is not consistent with her signal (i.e., $a = 1$ if $s_e = 0$ and $a = 0$ if $s_e = 1$), the client's expected utility is given by $b_e(\alpha|s_e) \cdot (-D)$ if $s_e = 1$, and $b_e(\alpha|s_e) \cdot B + [1 - b_e(\alpha|s_e)] \cdot (-d)$ if $s_e = 0$. A purely altruistic expert, who is solely concerned about the client's utility, would choose to perform the test if the client's expected utility is higher when the diagnosis is based on the test instead of her private signal.

The expert is impurely altruistic in that she is concerned about both the client's utility and her own reputation as a type-h professional among her peers. The expert cares about her reputation among peers because an expert who is believed to be of high ability receives more referrals from her peers. In the healthcare industry, studies have widely documented that patients are unable to observe the quality difference between experts (Gawande 2004; Makary 2013). Partly as a result of this knowledge gap, patients do not actively choose physicians on their own (Harris 2003; Victoor et al. 2012); rather, they largely rely on the referrals from experts' peers (e.g., primary care physicians, triage nurses, OPD doctors, and other experts) for advice on visiting specialized experts (Dealey 2005). On the physician's side, the literature has shown physicians rely on referrals from their generalist peers, and their patient volume increases as their reputation formed among those peers grows (Navathe and David 2009). Consistent with the aforementioned observation that diagnostic testing and expertise may substitute for each other, a high-ability expert receives a more informative signal of the client's state and is thus less likely than a low-ability expert to recommend a diagnostic test; in this sense, better expertise leads to higher value. For this reason, a diagnostic expert perceived by her peers as being of high ability might receive more referrals. We capture the reputational payoff by r, which can be interpreted as the gain in the present value of the expert's future payoff through higher referrals when her peers believe she is of type-h instead of type-l.

For example, imagine a two-period model in which in the first period, the expert signals her type using her diagnostic pathway, and in the second period, peers make referrals to a new set of patients to receive diagnosis from the expert. The expert has an incentive to signal her type to her peers in the hope of generating a large number of future referrals. Her peers make referrals decisions based on their updated beliefs about the expert type, because they not only prefer their patients to receive better diagnostic service (Choudhry et al. 2014), but also benefit from their association with a highly reputable colleague (Shortell and Anderson 1971). In this case, the reputational payoff r represents the present value of the expert's payoff gain in the second period due to higher referrals when the expert is believed to be a type-h expert instead of a type-l expert.

Peers know the client's prior probability α and update their beliefs $\beta(t, a)$ about the expert type after observing the diagnostic pathway the expert chose.[7] The expert's diagnostic pathway can become observable to her peers in many different ways. Specifically, because of the prevalence of electronic medical records, consolidation of hospitals and medical practices, and an emphasis of care coordination and communications between physicians, a client's medical records are increasingly shared among general practitioners and specialists providing care. In addition, hospitals routinely conduct peer-review programs through which an expert's peers evaluate her medical decisions for the purpose of in-house training or enhancing care quality (Landro 2017).

We normalize the reputational payoff of the expert known to be of type-l to zero. Therefore, the reputational payoff of the type-h expert is simply r. The expert's payoff is the weighted sum of (1) her reputational payoff $r \cdot \beta(t, a)$ and (2) the client's expected utility U from her diagnostic service. The expert's expected payoff is given by

$$u_e = \phi U + (1 - \phi) r \beta(t, a),$$

where $\phi \in [0, 1]$ is the weight the expert puts on the client's utility. It is convenient to define $\omega \triangleq (1 - \phi)/\phi$ as the relative weight the expert puts on her own reputational payoff compared to the client's utility. The parameter ω captures the extent of selfishness of the expert. A fully altruistic

[7]All the results presented in the paper continue to hold even if we assume the treatment outcomes are also observation to the expert's peers. The intuition is that the expert knows her diagnostic ability but is only imperfectly informed about the client's state. Therefore, the chosen diagnostic pathway becomes more informative about expert's ability than the random treatment outcomes given her diagnostic decision.

expert has $\omega = 0$, and a completely selfish one has $\omega = \infty$. To rule out the trivial case in which the expert never performs the test for any clients, we make the assumption that $B + D > cd/(d - c)$.

2.1 Full-Information Benchmark

As a benchmark, we first consider the full-information case in which the expert's type is common knowledge. The expert is only concerned about her clients' expected utility when deciding her diagnostic pathway. The following proposition provides the expert's optimal testing policy under the full-information case. For convenience of exposition, we define (derived in the appendix)

$$\underline{\alpha}_1^e \triangleq \frac{(1-\rho_e)c}{(1-\rho_e)c + \rho_e(B+D-c)} \text{ and } \bar{\alpha}_1^e \triangleq \frac{(1-\rho_e)(d-c)}{(1-\rho_e)(d-c) + \rho_e c}$$

as the decision thresholds for the case in which the type-e expert receives a positive private signal ($s_e = 1$). In addition, we define

$$\underline{\alpha}_0^e \triangleq \frac{\rho_e c}{\rho_e c + (1-\rho_e)(B+D-c)} \text{ and } \bar{\alpha}_0^e \triangleq \frac{\rho_e(d-c)}{\rho_e(d-c) + (1-\rho_e)c}$$

as the decision thresholds for the case in which the type-e expert receives a negative private signal ($s_e = 0$). All proofs are presented in the appendix.

Proposition 1. *Under symmetric information about expert type, for $s = 0, 1$ and $e = h, l$, a type-e expert*

(i) provides a positive diagnosis without performing the test if $\alpha > \bar{\alpha}_s^e$,

(ii) performs the test if $\underline{\alpha}_s^e < \alpha \leq \bar{\alpha}_s^e$,

(iii) provides a negative diagnosis without performing the test if $\alpha \leq \underline{\alpha}_s^e$.

The results presented in the above proposition are fairly intuitive. If the prior probability α that the client's state is positive is sufficiently high or sufficiently low, the test is not needed. If α is high enough ($\alpha > \bar{\alpha}_s^e$), the expert diagnoses the state as $\theta = 1$. If α is sufficiently low, she diagnoses the state as $\theta = 0$. A costly test is valuable only when the uncertainty about the client's state is sufficiently high (i.e., $\underline{\alpha}_s^e < \alpha \leq \bar{\alpha}_s^e$). A comparison of the testing thresholds reveals $\underline{\alpha}_1^e < \underline{\alpha}_0^e$ and $\bar{\alpha}_1^e < \bar{\alpha}_0^e$ for $e = h, l$. An expert, regardless of her type, is more likely to diagnose the client as positive (and less likely to diagnose the client as negative) if her private signal is positive, and is less likely to diagnose the client as positive (and more likely to diagnose the client as negative) if her private signal is negative. We also find $\bar{\alpha}_1^e - \underline{\alpha}_1^e$ and $\bar{\alpha}_0^e - \underline{\alpha}_0^e$ are both decreasing in ρ_e, indicating a high-ability expert performs the test over a smaller range of α compared to a low-ability expert. This finding is consistent with the empirical evidence that expertise may substitute for testing in the healthcare context (see, e.g., Doyle et al. 2010; Clark et al. 2012; Silver 2016; Rosenbaum 2017). To ensure the analysis captures all the possible combinations of expert decisions (e.g., both types do not perform test, only type-h performs test, only type-l performs test, and both types perform test), we assume $\underline{\alpha}_0^e < \bar{\alpha}_1^e$.

We now compare both types of expert's diagnostic pathways. A comparison of expert's diagnostic decisions in the different ranges of α reveals no α exists for which the two types of experts choose externally separating diagnostic pathways. In order words, an observer, who is not informed about the expert type or her signal, will be unable to identify the two types of experts based on their chosen strategies for any α. For example, no α exists for which one type of expert always performs the test and the other type does not. This result has the following implication: suppose the clients do not have knowledge about the expert's type information; then a costless separating equilibrium—in which both types of experts behave as in the full-information benchmark—does not exist, because any equilibrium identical to the full-information benchmark cannot be externally separating to clients.

3. Diagnostic Pathway and Expert Type

In this section, we consider the asymmetric-information case in which peers have no information about the expert's type ex ante. They have a prior belief γ about the probability that the expert is of a high type, and update the belief $\beta\,(t, a)$ based on their observation of the expert's diagnostic pathway. We will examine whether the expert is able to signal her type by strategically choosing the diagnostic pathway. To do so, we begin with defining the expert's strategy space as consisting of externally separating diagnostic pathways and enumerating all possible candidates for separating equilibria, which allows us to establish the uniqueness of the form of the separating equilibrium, in which a type-h expert does not perform the test and a type-l expert does. We then characterize the condition for such a separating equilibrium to exist.

3.1 Candidate Equilibria

Suppose a separating equilibrium exists in which the type-h expert credibly signals her type by choosing a particular diagnostic pathway different from that of the type-l expert. What would each type's strategy be in that equilibrium? The answer is not immediately obvious, due to the generic nature of our modeling environment. Thus, we start with defining the expert's strategy space and enumerating possible separating equilibria. In our setting, for an equilibrium to be separating, each type of expert must exhibit an *externally* different diagnostic pathway than the other type. In other words, in a separating equilibrium, if a type-h expert chooses one of the three possible actions (performing the test, providing a positive diagnosis without testing, providing a negative diagnosis without testing), a type-l expert would have to choose from the remaining set of actions. Using this criterion, we identify 18 possible candidates for separating equilibria, as listed in Table 2 in the appendix. For example, under candidate equilibrium 1, the type-h expert performs a diagnostic test regardless of the private signal she receives, whereas the type-l expert chooses not to perform the test and diagnoses based on her private signal.

A candidate separating equilibrium must survive a set of incentive-compatibility (IC) and individual-rationality (IR) constraints to qualify as an equilibrium. The underlying logic is that each type of expert must be internally consistent in its choice of diagnostic decisions; that is, neither type of expert would have the incentive to masquerade as the other type, or deviate from the diagnostic pathway specified in the equilibrium. In addition, the chosen diagnostic pathway must result in a non-negative payoff for the expert. In total, we need eight IC constraints and four IR constraints to specify each candidate separating equilibrium. We provide an illustrative example of the IC and IR constraints for a candidate equilibrium in the appendix.

By examining the IC and IR constraints for all 18 candidate separating equilibria, we generate two properties for the separating equilibrium to sustain, and present them in Lemmas 1 and 2.

Lemma 1. *No separating equilibrium exists in which the type-h expert performs the test.*

The proof is fairly technical, and here we provide some basic intuition. If the type-h expert performs the test, for the two types of experts to have externally separating diagnostic pathways, the type-l expert must not perform the test. In this case, the type-l expert has a strong incentive to mimic the type-h expert, which provides the dual benefits of (1) being perceived as a type-h expert and (2) delivering a more accurate diagnosis to the client. As a result, if the type-h expert attempts to separate from the type-l expert by performing the test, the type-l expert would find mimicking the type-h expert's strategy to be more lucrative. A consequence of Lemma 1 is that if a separating equilibrium exists, it necessarily involves the type-h expert's not performing the test.

Next, we present another lemma that helps us screen the candidate separating equilibria.

Lemma 2. *In any separating equilibrium, if the type-h expert does not perform the test, her diagnosis must be consistent with her private signal.*

Here, we provide the guiding intuition behind Lemma 2. One expects the type-h expert to offer a diagnosis that is inconsistent with her private signal in two cases: (1) if her private signal $s_h = 1$, and the prior probability that the client's state is positive (α) is very low; and (2) if her private signal $s_h = 0$, and α is very high. However, when α is very high or very low, the type-l expert may not need to perform the test either. Thus, separating herself from the type-l expert is challenging for the type-h expert in these cases.

Remarkably, by jointly applying Lemmas 1 and 2, we can rule out 17 of the 18 candidates for separating equilibria (see Table 2 in the appendix): Candidates 1–4 and Candidates 9–12 violate Lemma 1, whereas Candidates 6–8 and Candidates 13–18 violate Lemma 2. Candidate 5—whereby the type-h expert does not perform the test and offers a diagnosis consistent with her private signal, and the type-l expert performs the test regardless of her private signal—emerges as the only surviving candidate separating equilibrium. Thus, we have the following proposition:

Proposition 2. *There is only one type of candidate separating equilibrium, in which*

(i) the type-h expert chooses $t = 0$ and a diagnosis $a = s_e$; that is, the type-h expert does not perform the test and offers a diagnosis consistent with her signal; and

(ii) the type-l expert chooses $t = 1$; that is, she performs the test regardless of her private signal.

Proposition 2 states that if a separating equilibrium exists at all, it must be the case that the type-h expert does not perform the test whereas the type-l does. This uniqueness property is a significant result, especially because we start with a fairly generic setting.

3.2 Can the Diagnostic Pathway Signal Expert Type?

In the previous section, we established that the only remaining candidate separating equilibrium is one in which the type-h expert does not perform the test and provides a diagnosis consistent with her private signal, whereas the type-l expert performs the test regardless of her private signal. Understanding the expert's trade-offs in her testing decision is useful before examining the conditions that must hold for the separating equilibrium to exist. The expert is concerned about the implications of her actions for a client's utility. As a result of performing the test, the client may receive a benefit B. She may also avoid the cost due to a false negative diagnosis (D) and the cost due to a false positive diagnosis (d). In the expert's mental accounting system, two costs are associated with the decision to perform the test: (1) the client would incur a cost (c), and (2) the expert would not be able to earn the reputational payoff (r). The expert's decision to test is also driven by her own signal precision (i.e., ρ_e), and she is more likely to perform the test under a lower precision, because a low signal precision increases her likelihood of reaching an incorrect diagnosis. We capture these trade-offs in the expert's testing decisions below.

To be able to separate herself from the type-l expert, according to Proposition 2, the type-h expert does not perform the test. If, however, the benefit to the client from a true positive diagnosis (B) is sufficiently large, the expert would find it desirable to perform the test. We define an upper bound $\overline{B}$ (derived in the appendix), beyond which the separating equilibrium would not exist, because the type-h expert's incentives to perform the test would be too strong, as

$$\overline{B} \triangleq \left(\frac{\rho_h}{1-\rho_h}\right)^2 \cdot \frac{(r\omega + c)^2}{d - r\omega - c} + r\omega + c - D. \tag{2}$$

In addition, we expect the type-l expert to perform the test in the equilibrium. Correspondingly, a lower bound $\underline{B}$ exists such that if $B < \underline{B}$, the type-l expert's expected payoff from performing the test would be so low that she would not have the incentive to perform the test. We define $\underline{B}$ as

$$\underline{B} \triangleq \left(\frac{\rho_l}{1-\rho_l}\right)^2 \cdot \frac{(r\omega + c)^2}{d - r\omega - c} + r\omega + c - D. \tag{3}$$

The separating equilibrium exists only if $B \in [\underline{B}, \overline{B}]$. In the separating equilibrium, each type of expert must find that following her own equilibrium strategy dominates the other type's strategy, and that her expected payoff is non-negative. An examination of these conditions provides the necessary and sufficient conditions for the existence of the only surviving candidate separating equilibrium characterized in Proposition 2. We present these conditions in the following proposition. For simplicity of exposition, we define a threshold

$$\hat{B} \triangleq \frac{\rho_h}{1-\rho_h} \cdot \frac{\rho_l}{1-\rho_l} \cdot \frac{(r\omega + c)^2}{d - r\omega - c} + r\omega + c - D.$$

Proposition 3. *A unique separating equilibrium—in which the type-h expert does not perform the test and offers a diagnosis consistent with her private signal, and the type-l expert performs the test regardless of her private signal—exists if and only if $\alpha \in [\underline{\alpha}, \bar{\alpha}]$, where*

$$(\underline{\alpha}, \bar{\alpha}) = \begin{cases} \left(\frac{\rho_l(r\omega+c)}{\rho_l(r\omega+c)+(1-\rho_l)(B+D-r\omega-c)}, \frac{(1-\rho_l)(d-r\omega-c)}{\rho_l(r\omega+c)+(1-\rho_l)(d-r\omega-c)}\right) & \text{if } \underline{B} \le B \le \hat{B}, \\ \left(\frac{(1-\rho_h)(d-r\omega-c)}{\rho_h(r\omega+c)+(1-\rho_h)(d-r\omega-c)}, \frac{\rho_h(r\omega+c)}{\rho_h(r\omega+c)+(1-\rho_h)(B+D-r\omega-c)}\right) & \text{if } \hat{B} < B \le \overline{B}. \end{cases}$$

The range $[\underline{B}, \overline{B}]$—in which the separating equilibrium exists—depends on the precision of the expert's signal. As the difference in the two types widens, the separating equilibrium exists in a larger range of B. The intuition is as follows. As the type-h expert's signal becomes more precise (i.e., as ρ_h increases), the expected utility loss from the expert's not performing the test becomes smaller. The type-h expert becomes increasingly willing to reach her diagnosis based on her private signal without performing the test. Similarly, as the type-l expert's signal becomes less precise, she becomes likely to perform the test. Therefore, the more dissimilar the two types are in their signal precision, the larger the range of α in which the separating equilibrium exists.

The magnitude of the client's benefit from a true positive diagnosis (B) drives the existence of the equilibrium in the following way: the equilibrium is dictated by the type-l expert's IC constraints under a small B ($B \leq \hat{B}$), and by the type-h expert's under a large B ($B > \hat{B}$). The basic intuition is that all else being equal, the type-l expert is more willing to perform the test than the type-h expert, due to the noisier signal she receives. Nevertheless, if B is small, the type-l expert may be tempted to skip testing despite her relatively imprecise private signal. For the separating equilibrium to hold, the overarching constraint is that the type-l expert has no incentive to mimic the type-h expert. In other words, the type-l expert has to be able to generate at least an equal amount of surplus from performing the diagnostic test as from following her private signal. If B is large, however, the type-h expert may be tempted to perform the test because it offers attractive benefits to the client if a positive condition is correctly revealed, which may be compromised if the type-h expert follows her private signal in reaching the diagnosis. Thus, for the equilibrium to sustain, the overarching constraint is that the type-h expert has no incentive to mimic the type-l expert.

4. Analysis

In this section, we generate managerial insights based on the unique separating equilibrium characterized in the previous section. We compare the asymmetric-information case (Section 3) with the full-information benchmark (Section 2.1) to generate implications for the provision of diagnostic testing. Then we weigh the role of reputational payoff. We conclude this section with a brief note of client utility from different types of experts.

First, we draw implications on the expert's provision of diagnostic testing. In the equilibrium, the type-h expert does not perform the test and provides a diagnosis consistent with her private signal, whereas the type-l expert performs the test regardless of her private signal. Two questions arise: (1) Does the type-h expert under-test? and (2) Does the type-l expert over-test? Note the type-h expert's equilibrium strategy of not performing the test cannot necessarily be interpreted as under-testing. Likewise, the type-l expert's equilibrium strategy of always performing the test cannot necessarily be interpreted as over-testing. To draw meaningful conclusions about over- or under-testing, we would need to compare the expert's strategies in the asymmetric-information equilibrium against those in the full-information benchmark (Section 2.1). We say the expert *under-tests* if she does not perform the test in the asymmetric-information equilibrium but does perform the test in the full-information benchmark. On the flip side, we say the expert *over-tests* if the expert performs the test in the asymmetric-information equilibrium but does not perform the test in the full-information benchmark. We have the following proposition:

Proposition 4. *If a separating equilibrium exists, a continuum of α in which the type-h expert under-tests must also exist. Furthermore, the type-l expert does not over-test in the equilibrium.*

In the separating equilibrium, the type-h expert is less likely to perform the test than in the full-information benchmark, to separate herself from the type-l expert. Because the precision of the type-l expert's signal is lower than that of type-h expert's, the type-l expert finds mimicking the type-h expert's strategy of not performing the test to be too costly. Although the type-l expert always performs the test in the equilibrium, she does not over-test, because her testing strategy is the same as in the full-information case and not influenced by the information asymmetry about the expert type.

Under-testing that originates from the information asymmetry about the expert type may be among the factors accounting for the prevalence of under-testing in the US healthcare market reported in the literature (see, e.g., Zhi et al. 2013). The policy implication of this finding is echoed by the healthcare community's call for better transparency in the quality of care (see, e.g., Makary 2013)—policymakers should aim to eliminate or reduce the information asymmetry about the expert type. For example, experts may be required to make their academic credentials public. Disclosing experts' success and failure stories may also help. Reducing the information asymmetry about the expert type may lead to fewer instances of missed diagnoses. Because our focus in this paper is on offering an explanation for under-testing in the healthcare context, we restrict our attention to the separating equilibrium (in which under-testing arises due to the high-ability expert's reputational concern). Pooling equilibria also exist in our model setting. For example, a pooling equilibrium, in which both types of experts diagnose the client's condition as positive (negative) regardless of their private evaluation, exists if the prior probability α is sufficiently high (low).

Next, we analyze the impact of reputational payoff. The type-h expert takes the costly action of not performing the test to separate herself from the type-l expert, because an expert who is believed to be of type-h receives a reputational payoff r. We examine the effect of an increase in the reputational payoff r on the type-h expert's incentives to separate from the type-l expert. The range of α, in which the separating equilibrium exists, captures the expert's incentives to separate. The following proposition describes the effect of the reputational payoff on the range of α in which the separating equilibrium exists. (The thresholds $\underline{r}$, $\hat{r}$, and $\bar{r}$ are defined in the appendix.)

Proposition 5. *The type-h expert fails to separate from the type-l expert if r is too low (i.e., $r < \underline{r}$) or too high (i.e, $r > \bar{r}$). In the intermediate range (i.e., $\underline{r} \leq r \leq \bar{r}$), as the reputational payoff r increases, the range of α for which the separating equilibrium exists first increases (if $r \leq \hat{r}$) and then decreases (if $r > \hat{r}$).*

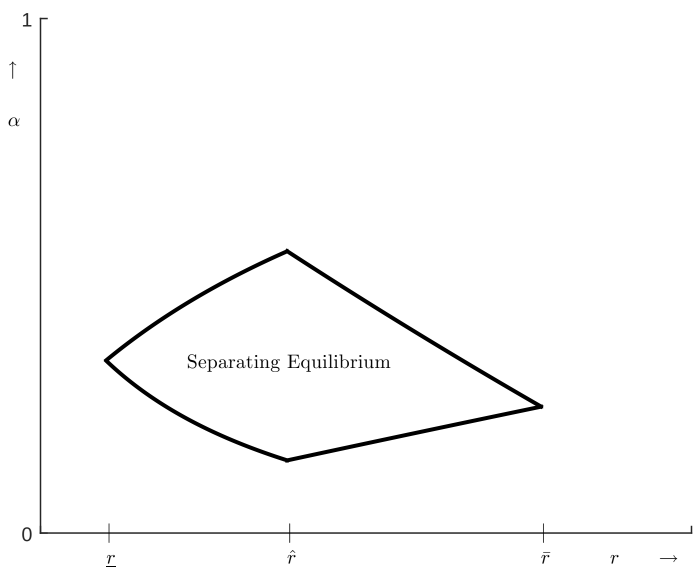

FIGURE 2: Effect of reputational payoff r

The results above are graphically presented in Figure 2. Clearly, the reputational payoff r plays an instrumental role in the characterization of the equilibrium. We discover an interesting non-monotonic effect of r on the range of α in which the equilibrium exists. If reputational gains are too small ($r < \underline{r}$), the type-h expert does not find it worthwhile to sacrifice client utility for reputational gain. Therefore, if r is too small, no α exists for which the separating equilibrium exists. If r is further increased ($\underline{r} \leq r \leq \hat{r}$), the equilibrium is driven by the type-h expert's incentive not to perform the test. The type-h expert finds reputational incentives strong enough to justify her decision not to perform the test. The type-l expert, on the other hand, continues to find performing the test worthwhile, because her private signal is less precise. An increase in r decreases the type-h expert's incentive to perform the test. As a result, the range of α in which the equilibrium exists expands. If $\hat{r} < r \leq \bar{r}$, the equilibrium is now primarily driven by the type-l expert's incentives. The reputational incentives become strong enough such that the type-l experts may also be tempted not to perform the test. Because an increase in r makes the type-l expert more likely not to perform the test, the range of α—in which the separating equilibrium arises—becomes narrower with an increase in r. Finally, if r is sufficiently high ($r > \bar{r}$), suppose the type-h expert chooses not to perform the test; the type-l expert would then be tempted to mimic the type-h expert by not performing the test either—in the expert's mental accounting system, the loss in patient utility would be compensated for by the gain from peer perception. The separating equilibrium collapses.

Next, we discuss the effect of the expert's selfishness ω on the type-h expert's incentive to separate herself from the type-l expert. The separating equilibrium exists only if the expert is neither too selfish nor too altruistic ($\underaccent{\bar}{\omega} \leq \omega \leq \bar{\omega}$). (The thresholds $\underaccent{\bar}{\omega}$, $\hat{\omega}$, and $\bar{\omega}$ are defined in the appendix.) If the expert is too altruistic ($\omega < \underaccent{\bar}{\omega}$), she would not want to sacrifice client utility for her own reputational gain. Also, if the expert is too selfish ($\omega > \bar{\omega}$), she would always want to sacrifice client utility for her own reputational gain. The parameter space in which the separating equilibrium exists first expands (if $\underaccent{\bar}{\omega} \leq \omega \leq \hat{\omega}$) and then contracts (if $\hat{\omega} \leq \omega \leq \bar{\omega}$) with an increase in the selfishness ω of the expert. If $\underaccent{\bar}{\omega} \leq \omega \leq \hat{\omega}$, the type-$h$ expert's incentive to not perform the test drives the equilibrium. An increase in the selfishness ω makes reputational gain more rewarding for the type-h expert. The type-h expert becomes less willing to perform the test, and the range of α in which the separating equilibrium exists expands. If $\hat{\omega} \leq \omega \leq \bar{\omega}$, the type-$l$ expert's incentive to perform the test drives the equilibrium. An increase in the selfishness ω makes the type-l expert less willing to perform the test. As a result, the parameter space in which the separating equilibrium exists contracts.

Finally, we discuss the client's expected utility in the separating equilibrium. We often hold the belief that more competent experts offer better services—a type-h expert receives a more precise signal of the client's state and should therefore be able to offer a better diagnosis. This scenario is indeed the case in the full-information benchmark. Under asymmetric information, however, the expert has a desire to manage her own reputation and may thus distort her decision to induce favorable beliefs. In the separating equilibrium characterized in Section 3, the type-h expert chooses not to perform the test in cases in which she should perform the test, which undermines the client's expected utility from visiting the type-h expert. Notwithstanding the fact that the client has no information about the expert type ex ante, one natural question arises: Could a client receive an even lower expected utility from visiting a type-h expert as opposed to a type-l one? We answer this question by comparing the client's expected utility from visiting each type of expert in the separating equilibrium. We find that in the separating equilibrium, the client has a lower expected utility if she happens to be diagnosed by a type-h expert than if she is diagnosed by a type-l expert if $c < (1-\rho_h)\left[\alpha\left(B+D\right)+(1-\alpha)\,d\right]$, and vice versa. This result is driven by the type-h expert's under-testing behavior—skipping the test when doing so is in the client's best interest—as a signaling device. A low cost of testing means, all else being equal, the client is more likely to receive the test in the full-information benchmark. When the cost of diagnostic testing is sufficiently low, the client's expected utility from visiting a type-l expert becomes higher than that from visiting a type-h expert, because the type-l expert performs the test but the type-h does not (even if, or particularly because, the diagnostic test comes at a very low cost).

5. Disclosing Private Evaluations before Testing

In the baseline model presented in Section 2, we assumed the expert's private evaluation (i.e., private signal of her client's state) cannot be used to signal her ability because she cannot verifiably communicate her private evaluation to her peers. Now suppose a tamper-proof technology (e.g., Ichikawa et al. 2017) is in place such that the expert cannot modify her disclosure after conducting a test. In this section, we examine the expert's expanded strategy space in which she may disclose her private evaluation (e.g., by making a note in the patient record that her peers can access) before offering a diagnosis to the client that may be based only on her private signal or supplemented by testing. All the other assumptions are the same as in the baseline model. In this setting, peers may use the observed difference between the disclosure and the outcome of the diagnosis to update their beliefs about the expert's type. We explore the possibilities of a separating equilibrium and a pooling equilibrium, respectively.

5.1 (Non-)Existence of Separating Equilibrium

We start with investigating the existence of a separating equilibrium. We represent the expert's private-evaluation-disclosure decision by $b \in \{0,1\}$, where $b=1$ indicates the private signal is disclosed and $b=0$ indicates otherwise. We look for a separating equilibrium in which the type-h expert signals her expertise by disclosing the private evaluation before subsequently performing the test. The type-l expert must play one of the following three strategies in the separating equilibrium: (1) not disclosing private evaluation ($b=0$), not performing the test ($t=0$), and providing a diagnosis that is consistent with the private evaluation; (2) not disclosing private evaluation ($b=0$), not performing the test ($t=0$), and providing a diagnosis that is inconsistent with the private evaluation; and (3) not disclosing private evaluation ($b=0$), and performing the test ($t=1$). If the type-l expert discloses her private evaluation but does not perform the test, the signal recipient (peers) would ignore the disclosed information. In other words, the private-evaluation disclosure is not credible unless it is accompanied by testing. A formal analysis (see the appendix for a sketch) reveals the type-l expert has no incentive to follow any of the aforementioned three strategies for the following reason. Along the equilibrium path, updated beliefs are not dependent on whether test results actually confirm the private evaluation; they are simply a function of chosen strategies. A type-l expert, who mimics the type-h expert's equilibrium strategy, is believed to be of type-h regardless of the outcome of the test. Therefore, disclosing private evaluations before testing does not function as a signal of high expert ability. In addition, we confirm that the separating equilibrium from the baseline model continues to exist even with the disclosure option.

5.2 Existence of Pooling Equilibrium

Next, we investigate the existence of a pooling equilibrium. We specify the belief that the expert is of type h for out-of-equilibrium diagnostic pathways as zero. In other words, whenever the expert chooses to deviate from the pooling equilibrium, peers will form the belief that the probability that the expert is of type h is zero. According to this specification, whenever the expert chooses to conduct a test, she is better off disclosing her private evaluation beforehand. Conversely, if an expert deviates from the diagnostic pathway specified in the pooling equilibrium, she will not perform a test and will reach her diagnosis based on her own diagnostic ability.

Upon observing the expert's disclosure and the result of the subsequent test, peers use Bayes' rule to form a belief about the expert's type, depending on whether her disclosure is confirmed by the result of the subsequent test: if her disclosure is confirmed by the test, peers form a belief that she is of type h with probability $\bar{\gamma} = \frac{\gamma\rho_h}{\gamma\rho_h+(1-\gamma)\rho_l}$; if her disclosure is contradicted by the test, peers form a belief that she is of type h with probability $\underline{\gamma} = \frac{\gamma(1-\rho_h)}{\gamma(1-\rho_h)+(1-\gamma)(1-\rho_l)}$. Clearly, $0 < \underline{\gamma} < \gamma < \bar{\gamma} < 1$. In other words, when peers observe that the expert discloses a private evaluation supported by the test, they would believe her probability of being of type h is above the prior (γ), and vice versa. For ease of exposition, we define $r_1 \triangleq \bar{\gamma}\omega r$ and $r_2 \triangleq \underline{\gamma}\omega r$. In the following proposition, we characterize the condition under which a pooling equilibrium sustains for the range of $\alpha \in [1-\rho_l, \rho_l]$.

Proposition 6. *If $c \leq \min\{r_1, \frac{r_1+r_2+d}{2}\}$, for $\alpha \in [1-\rho_l, \rho_l]$, regardless of whether the private evaluation is positive or negative, neither type of expert has an incentive to deviate from the pooling equilibrium in which she discloses her private evaluation before subsequently conducting a test.*

We now jointly examine the pooling equilibrium characterized above and the full-information benchmark analyzed in Section 2.1. Recall from Proposition 1 that, in the full-information benchmark, the type-h expert conducts a test only if $\alpha \in [\underset{\sim}{\alpha}_1^h, \bar{\alpha}_0^h]$, where $\underset{\sim}{\alpha}_1^h = \frac{(1-\rho_h)c}{(1-\rho_h)c+\rho_h(B+D-c)}$ and $\bar{\alpha}_0^h = \frac{\rho_h(d-c)}{\rho_h(d-c)+(1-\rho_h)c}$. By comparing this range of α with that in Proposition 6, we have the following corollary.

Corollary 1. *If $c < \min\{r_1, \frac{d}{2}\}$, in the pooling equilibrium in which she discloses her private evaluation before subsequently conducting a test, scenarios exist in which the type-h expert overtests.*

Corollary 1 shows the existence of the over-testing in the pooling equilibrium in which the expert discloses her private evaluation before subsequently conducting a test. Note that this pooling equilibrium is possible thanks to the availability of the aforementioned tamper-proof technology. This result, along with our analysis in sections 4 and 6 of the paper, has implications for ways to address the phenomenon of under-testing.

6. Extensions

In this section, we present three extensions of our baseline model. Section 6.1 extends the model by incorporating payment-related issues. In Section 6.2, we discuss the impact of malpractice concerns on diagnostic testing decisions. Section 6.3 provides an analytical foundation generating testable predictions that can help distinguish between our reputation-based theory and an alternative theory of overconfidence.

6.1 Financial Incentives

So far, in our analysis, we have assumed the expert is not influenced by possible payments from performing a diagnostic test and that the client bears the full cost of the diagnostic test (if any). In the real world, the expert may receive a payment that is contingent on performing the test, and the client, due to insurance coverage, is only partially responsible for the cost of diagnostic testing. In this section, we first relax the assumption about no monetary motive on the expert's side. Recall that in the baseline model, the client incurs a cost c if the expert performs the test. We incorporate the financial-incentive consideration by assuming that if the expert performs the test, she receives as a fee a proportion δ of the cost c incurred to the client. In addition, toward the end of this section, we briefly discuss the impact of insurance coverage along the same line of reasoning.

As a benchmark, we consider the symmetric-information setup in which the client is informed of the expert's type. The expert's incentive to perform the test intensifies as a result of the fee. As one would expect, we find the expert performs the test under a larger parameter space regardless of her signal: as δ increases, both $\bar{\alpha}_0^e$ and $\bar{\alpha}_1^e$ become larger, and both $\underline{\alpha}_0^e$ and $\underline{\alpha}_1^e$ become smaller.

Next, we examine the asymmetric-information setup in which the expert type is unknown to the client. Similar to the baseline model, a unique separating equilibrium exists for $\alpha \in [\underline{\alpha}^f, \bar{\alpha}^f]$ in which the type-h expert does not perform the test and the type-l expert does. The diagnosis of the type-h expert is consistent with her signal. The effect of the financial incentive (δ) on the parameter space ($\triangle\alpha^f \triangleq \bar{\alpha}^f - \underline{\alpha}^f$) in which the separating equilibrium exists depends on the magnitude of the client's benefit from a true positive diagnosis (B). We describe this result in the following proposition.

Proposition 7. *There exist some threshold $\hat{B}_f$ such that a higher-power financial incentive (i.e., a larger δ)*

(i) reduces the range of α for which the separating equilibrium exists ($\Delta\alpha^f$) if the client's payoff from a true positive diagnosis is large enough (i.e., $B > \hat{B}_f$), and

(ii) increases the range of α for which the separating equilibrium exists ($\Delta\alpha^f$) if the client's payoff from a true positive diagnosis is large enough (i.e., $B \leq \hat{B}_f$).

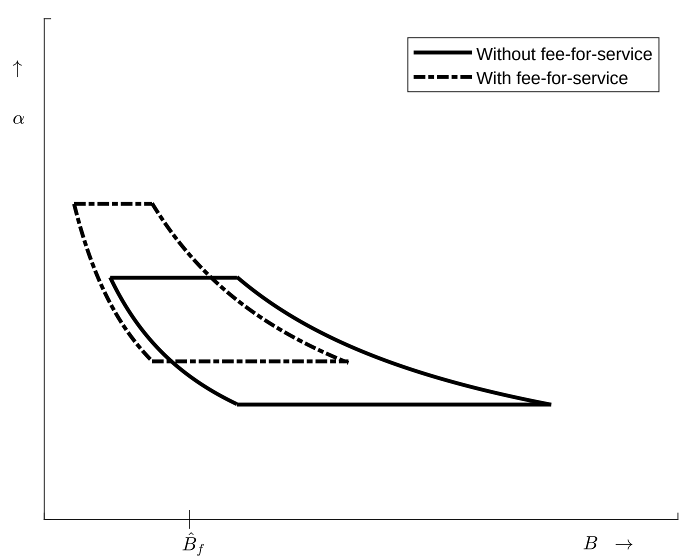

FIGURE 3: Effect of financial incentive on the existence of separating equilibrium

The above results are illustrated in Figure 3. Intuitively and as expected, if $B > \hat{B}_f$, a higher-powered financial incentive results in the existence of a separating equilibrium (under-testing by type-h expert) over a smaller parameter space. Surprisingly, however, if $B \leq \hat{B}_f$, a higher-powered financial incentive can lead to under-testing over a *wider* parameter space. The intuition is the following. The presence of (or expansion in) the fee-for-service environment increases the expert's incentives to perform the test. As in the baseline model, if the benefit from correctly identifying the positive state (B) is large, the type-h expert may experience a large disutility as a result of not performing the test. Providing a financial incentive for testing only intensifies this effect. As a result, the range of α for which the separating equilibrium exists becomes smaller. If B is small, the type-l expert does not incur a large cost when she does not perform the test. The presence of (or increase in) a financial incentive increases the cost of not performing the test for the type-l expert. Mimicking the type-h expert becomes increasingly difficult for the type-l expert. As a result, the separating equilibrium exists over a larger set of α.

As we have seen from Section 4, under-testing by diagnostic experts may result in a lower surplus for the clients. Accordingly, policymakers may weigh initiatives with the potential to induce the high-type expert to deviate from resorting to under-testing as a signaling device. One may expect that providing a financial incentive for performing the test would result in more tests by both types of experts. The above result about the effect of δ has interesting and unexpected implications in this aspect and reveals the incentive effect hinges on the client's payoff from a true positive diagnosis (B). On the one hand, if B is high (i.e., $B > \hat{B}_f$), a higher-power financial incentive will discourage under-testing. On the other hand, if B is small (i.e., $B \leq \hat{B}_f$), a higher-power financial incentive, quite unexpectedly, will result in *more* under-testing. Thus, a sensible policy initiative may entail either strengthening or curbing the financial incentive, depending on the magnitude of the client's payoff from a true positive diagnosis.

Impact of Insurance Coverage. Before we conclude this section, we briefly discuss the impact of the client's insurance coverage (e.g., health insurance in a healthcare context) on the expert's under-testing behavior. Insurance coverage reduces a client's out-of-pocket expense. As a result, one might expect under-testing to be less salient because the expert is less concerned about the cost of the diagnostic test. Our modeling framework, on the other hand, implies insurance coverage may lead to either more or less salient under-testing behavior; see the Appendix for a proof sketch. The intuition behind this result is as follows: when B is small (i.e., $B \leq \hat{B}$), the type-l expert does not suffer from a large loss (from client utility) when she does not perform the test. Insurance coverage increases a client's net benefit from diagnostic testing, which, equivalently, increases the cost of not performing the test for the type-l expert. Therefore, mimicking the type-h expert becomes more costly for the type-l expert. For this reason, due to insurance coverage, the separating equilibrium exists over a larger set of α.

6.2 Malpractice Concerns

We now consider an extension in which the expert has concerns about possible malpractice lawsuits. One way to model malpractice concerns is to introduce a "misdiagnosis cost" $m \cdot \mathbf{1}(t=0)$, where $m > 0$ is the cost the expert incurs when she opts not to order a diagnostic test. The expert's payoff function is now $u_e = \phi U + (1-\phi)\, r\beta\,(t, a) - m \cdot \mathbf{1}(t=0)$. In the context of healthcare, this misdiagnosis cost reflects a cost to the physician because of concerns about potential lawsuits in the future. In anticipation of the legal future, the physician views imaging tests not only as diagnostic tools, but also as evidence that can be presented to the court when needed. The misdiagnosis cost is a real cost incurred to the physician, and essentially captures the non-financial aspect of the physician's expected costs due to potential malpractice lawsuits—in practice, the physician can present tests in court as evidence of providing adequate medical care in the case of a malpractice lawsuit. Thus, it is a "burden of proof" that decreases in the intensity of testing (i.e., increases in the chosen service rate). For example, the assumption that misdiagnosis costs decrease with additional testing may simply reflect the fact that physicians attach psychological costs to skipping certain tests. This assumption is robust in healthcare environments in which no evidence exists that the objective probability of malpractice suits (or the premiums for malpractice insurance) increases in the frequency of diagnostic testing. Nevertheless, a physician's subjective expectation of malpractice suits may decrease in the frequency of diagnostic testing, which would translate itself into lower psychological costs.[8]

With the help of some algebra, we can show the additional term $m \cdot \mathbf{1}(t=0)$ in the expert's payoff function above can be incorporated into client utility U by redefining the cost of diagnostic testing as $\hat{c} = c - m/\phi$. In other words, in terms of modeling the impact on physician decision-making, incorporating malpractice concerns is equivalent to a reduction in the net cost of diagnostic testing. We can proceed to show under-testing exists. Indeed, due to this additional consideration, under-testing may be *more* salient; see the Appendix for a sketch of the proof. To understand the

[8] Echoing our above formulation, Kessler and McClellan (2002) highlight the notion of "malpractice pressure," and contend such pressure can be both financial and non-financial. The financial part does not play a significant role because "malpractice insurance is community rated" and the premium rarely depends on malpractice claims. However, "no insurance is possible against the unpleasant experiences and considerable time commitment over months or years. For example, in discovery, a physician may be required to answer both written and oral questions about her competence and judgment and to respond to questions and other requests from lawyers for the patient, for the malpractice insurer, and for the hospital and its malpractice lawyer." In a similar spirit, Currie and MacLeod (2008) state, "Doctors' premiums are not experience-rated, but are set at the specialty-area level. Hence, short of moving from a high-premium area to another area, or leaving her specialty entirely, there is little a doctor can do to affect her premiums." Thus, Currie and MacLeod contend, "doctors generally face little financial risk from malpractice claims." Yet doctors "apparently care so deeply about the problem of legal liability" and their concerns constitute a real cost because of noninsurable costs that include the psychological and time burden in response to malpractice lawsuits.

intuition behind this result, note that when B is small (i.e., $B \leq \hat{B}$), the cost of the diagnostic test becomes an important concern in the expert's mental accounting problem. The existence of malpractice concerns—or, mathematically, a change from c to $\hat{c}$—increases the type-l expert's cost of not performing the test, and thus makes mimicking the type-h expert more costly for the type-l. As a result, the separating equilibrium exists over a larger set of α.

6.3 Overconfidence

One alternative explanation for under-testing is expert overconfidence. In this paper, to highlight the role of reputational payoff, we have abstracted away from considering overconfidence in our baseline model. Notwithstanding that overconfidence may lead to under-testing, in this section, we show that under-testing due to reputation concerns is likely to arise when the prior of a positive condition (α) is neither very high nor very low, whereas under-testing due to overconfidence tends to arise when the prior of a positive condition (α) is very high or very low.

Below, we present our analysis to elucidate the idea. Because under-testing is specific to type-h experts in our model, we will only present our analysis for the case in which the expert is type-h. Consider a type-h expert who suffers from overconfidence.[9] To be specific, whereas her actual diagnostic precision is ρ_h, the expert's self-perceived diagnostic precision is $\hat{\rho}_h > \rho_h$. For ease of exposition, we define the following functions:

$$\underline{\alpha}_1(\rho) \triangleq \frac{(1-\rho)c}{(1-\rho)c + \rho(B+D-c)}, \bar{\alpha}_1(\rho) \triangleq \frac{(1-\rho)(d-c)}{(1-\rho)(d-c)+\rho c};$$
$$\underline{\alpha}_0(\rho) \triangleq \frac{\rho c}{\rho c + (1-\rho)(B+D-c)}, \bar{\alpha}_0(\rho) \triangleq \frac{\rho(d-c)}{\rho(d-c)+(1-\rho)c}.$$

In the Appendix, we characterize the diagnostic-testing policies of both an overconfident type-h expert and a self-aware expert. By comparing the diagnostic-testing policies across the overconfident expert and the self-aware expert, we find the overconfident expert misses tests for clients with α satisfying either (a) $\bar{\alpha}_1(\hat{\rho}_h) \leq \alpha < \bar{\alpha}_1(\rho_h)$ if $s_h = 1$, or (b) $\underline{\alpha}_0(\rho_h) < \alpha \leq \underline{\alpha}_0(\hat{\rho}_h)$ if $s_h = 0$. Drawing from the above two cases, we find an overconfident expert under-tests when the prior of the problem "agrees with" what the expert's private signal suggests, that is, when α is sufficiently high (i.e., $\alpha \geq \bar{\alpha}_1(\hat{\rho}_h)$) and the expert's private signal is positive, or when α is sufficiently low (i.e., $\alpha \leq \underline{\alpha}_0(\hat{\rho}_h)$) and the expert's private signal is negative. For a third-party observer's perspective, the overconfident expert misses tests when α is sufficiently high or sufficiently low.

[9] An overconfident type-l expert acts in a similar fashion and the expressions for her decision thresholds can be obtained by replacing $\hat{\rho}_h$ with $\hat{\rho}_l$.

We now consider the case in which an expert is perfectly self-aware but has reputation concerns. Note that our paper has established this type of under-testing arises only when the expert is of type h. As Proposition 3 alludes, the type-h expert would choose to skip necessary tests only when α is neither very large nor very small. Specifically, Proposition 4 suggests that in the presence of reputation concerns, the expert under-tests only when $\underline{\alpha} \leq \alpha \leq \bar{\alpha}$, and this range is the same regardless of the expert's private signal.

By comparing the analyses of under-testing due to overconfidence and reputation concerns, respectively, we generate the following testable prediction that can help distinguish between a reputational theory and a theory of overconfidence: whereas under-testing due to overconfidence tends to arise when the prior of a positive condition (α) is very high or very low, under-testing due to reputation concerns is likely to arise when the prior of a positive condition (α) is neither very high nor very low.

7. Concluding Remarks

In many professional services, diagnostic experts may not be able to immediately reach correct diagnoses for their clients' conditions, and often resort to diagnostic testing with cost implications (e.g., money, time, privacy, discomfort, or side effect) to the clients. For example, in the diagnosis of dementia among elderly persons, history-taking and mental examination during consultations are essential, but laboratory testing is often required for a more definite assessment of patient conditions. Under-testing has been well documented in this situation and presents health hazards (NIH 1987).

One may expect experts' diagnostic testing decisions to reflect both uncertainty underlying clients' situations and the experts' diagnostic accuracy. When experts' diagnosis accuracy is their private information and they desire to be perceived as high-ability professionals among peers, they have an opportunity to choose a diagnostic pathway (i.e., the process to reach the eventual diagnosis, which may or may not involve diagnostic testing) to influence perception of their ability.

In this paper, we formulate a diagnostic expert's pathway-selection problem when the peers observe these decisions and form beliefs about the expert's skill level accordingly. We have shown how a high-type diagnostic expert may use her diagnostic pathway to credibly inform peers of her skill level. We find that, due to information asymmetry, the high-type expert's optimal diagnostic pathway may entail not performing the test even when the test generates a positive surplus to the clients. Furthermore, we show this type of under-testing pattern is the unique pattern allowing the high-type expert to credibly signal her type.

We have established the existence of the separating equilibrium depends on the magnitude of reputational payoff in a non-monotonic fashion: for separation between different types of experts to occur, the reputational payoff can be neither too low nor too high. The desire to be viewed as being of high ability leads to under-testing only when the expert's reputational payoff is intermediate. Furthermore, we show that under some conditions, a more altruistic expert may be more likely than a less altruistic expert to engage in harmful under-testing.

We generally think of monetary incentives (e.g., fee-for-service in the healthcare setting) as presenting a source of misalignment between an expert's and her client's interests. Our model provides a more balanced view: receiving additional payments for performing the diagnostic test may induce a behavior-modification effect in experts. Specifically, although a low-type expert may be more likely to perform unnecessary diagnostic tests, a high-type expert, because of this financial incentive, may be more likely to act in the best interests of her clients, with a lower tendency to under-test. On the other hand, we also show that in some cases, providing a stronger financial incentive to perform the test may lead to more salient under-testing by the high-type expert.

Our paper represents an initial attempt to formalize the linkage between information asymmetry about expert type and the diagnostic pathway. When the diagnostic pathway shapes expert-client communication, the decision of whether to perform the diagnostic test not only affects the quality of the diagnostic service, but may also serve as a signaling device of expert type. Our model broadly reflects and has implications for various professional service settings. For example, in the US healthcare market, whereas what dominates the contemporary discourse has been over-testing, recent medical research (e.g., Zhi et al. 2013) has revealed the prevalence of under-testing, with crucial impacts on the quality of medical care. We expect some of our findings that are relevant to physicians' under-testing behavior, especially driven by monetary considerations (e.g., a stronger financial incentive may lead to more salient under-testing), may be tested in the laboratory or in the field. We also generate testable hypotheses about comparison of overconfidence-driven under-testing and reputation-driven under-testing.

Relevant to the phenomenon of under-provision of diagnostic testing, multiple alternative theories exist, several of which hinge on uncertainty in decision-making (see, e.g., Davis et al. 2000; Epstein, Begg, and McNeil 1984). Our contribution in this paper is that in addition to capturing the uncertain nature of diagnostic expert decision-making, we theoretically explore a novel and little-explored aspect of the complicated piece of puzzle. Among multiple, concurrent factors behind the phenomenon, our research uncovers a compelling driving force that may guide policymakers as they navigate through strategies to elicit appropriate provision of healthcare resources.

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Appendix

Proof of Proposition 1.

Let us say the type-e expert receives a signal $s_e = 0$. The expert compares the client's utility from three possible decisions, that is, (1) $t = 0$, $a = 1$, (2) $t = 1$, and (3) $t = 0$, $a = 0$, in arriving at her decisions. We write the client's utility for the three decisions as follows:

$$U(\alpha|s_e = 0) = \begin{cases} b\left(\alpha|s_e\right) \cdot B + \left[1 - b_e\left(\alpha|s_e\right)\right] \cdot (-d) & \text{if } t = 0, a = 1 \\ b_e\left(\alpha|s_e\right) \cdot B - c & \text{if } t = 1 \\ b_e\left(\alpha|s_e\right) \cdot (-D) & \text{if } t = 0, a = 0 \end{cases},$$

where $b_e\left(\alpha|s_e\right)$ is the type-e expert's beliefs that the client's state is $\theta = 1$ and is described in (1). A comparison of the client's expected utility corresponding to the three possible decisions reveals the expert (1) does not perform the test and diagnoses the client as positive ($t = 0$ and $a = 1$) if $\alpha > \bar{\alpha}_0^e$, where $\bar{\alpha}_0^e \triangleq \frac{\rho_e(d-c)}{\rho_e(d-c)+(1-\rho_e)c}$, (2) does not perform the test and diagnoses the client as negative ($t = 0$ and $a = 0$) if $\alpha \leq \underline{\alpha}_0^e$, where $\underline{\alpha}_0^e \triangleq \frac{\rho_e c}{\rho_e c+(1-\rho_e)(B+D-c)}$, and (3) performs the test ($t = 1$), otherwise.
The proof for the case in which the expert receives signal $s_e = 1$ proceeds in the same manner. The corresponding thresholds ($\bar{\alpha}_1^e$ and $\underline{\alpha}_1^e$) are $\frac{(1-\rho_e)(d-c)}{(1-\rho_e)(d-c)+\rho_e c}$ and $\frac{(1-\rho_e)c}{(1-\rho_e)c+\rho_e(B+D-c)}$, respectively. *Q.E.D.*

List of Candidate Separating Equilibria

In Table 2, we list the 18 possible separating equilibria. In each candidate equilibrium, different types of experts exhibit *externally* different diagnostic pathways. In other words, in each candidate separating equilibrium, if a type-h expert chooses one of the three possible actions (performing the test, providing a positive diagnosis without testing, providing a negative diagnosis without testing), a type-l expert has to choose from the remaining set of actions.

An Illustrative Example of IC and IR Constraints for a Candidate Equilibrium

Below, as an illustrative example, we describe all the IC and IR constraints for the candidate equilibrium 5 specified in Table 2 (the type-h expert does not test and diagnose following the private signal; the type-l expert performs a diagnostic test). The constraints for all other candidate equilibria can be written in a similar manner. We represent the type-e expert's expected utility when she chooses not to perform the test (i.e., $t = 0$) and reaches a diagnosis of $a_0 \in \{0, 1\}$ given a private signal of $s_0 \in \{0, 1\}$ by $u_e(t = 0, a = a_0|s_e = s_0)$. Also, we represent the expert's expected utility when she performs the test by $u_e(t = 1|s_e = s_0)$. We have the following IC constraints:

Table 2: List of candidate separating equilibria

Number	Private signal s_e	Type-h expert's action	Type-l expert's action
1	1	$t = 1$	$t = 0, a = 1$
	0	$t = 1$	$t = 0, a = 0$
2	1	$t = 1$	$t = 0, a = 0$
	0	$t = 1$	$t = 0, a = 1$
3	1	$t = 1$	$t = 0, a = 0$
	0	$t = 1$	$t = 0, a = 0$
4	1	$t = 1$	$t = 0, a = 1$
	0	$t = 1$	$t = 0, a = 1$
5	1	$t = 0, a = 1$	$t = 1$
	0	$t = 0, a = 0$	$t = 1$
6	1	$t = 0, a = 0$	$t = 1$
	0	$t = 0, a = 1$	$t = 1$
7	1	$t = 0, a = 0$	$t = 1$
	0	$t = 0, a = 0$	$t = 1$
8	1	$t = 0, a = 1$	$t = 1$
	0	$t = 0, a = 1$	$t = 1$
9	1	$t = 1$	$t = 0, a = 1$
	0	$t = 0, a = 0$	$t = 0, a = 1$
10	1	$t = 1$	$t = 0, a = 0$
	0	$t = 0, a = 1$	$t = 0, a = 0$
11	1	$t = 0, a = 0$	$t = 0, a = 1$
	0	$t = 1$	$t = 0, a = 1$
12	1	$t = 0, a = 1$	$t = 0, a = 0$
	0	$t = 1$	$t = 0, a = 0$
13	1	$t = 0, a = 1$	$t = 1$
	0	$t = 0, a = 1$	$t = 0, a = 0$
14	1	$t = 0, a = 0$	$t = 1$
	0	$t = 0, a = 0$	$t = 0, a = 1$
15	1	$t = 0, a = 1$	$t = 0, a = 0$
	0	$t = 0, a = 1$	$t = 1$
16	1	$t = 0, a = 0$	$t = 0, a = 1$
	0	$t = 0, a = 0$	$t = 1$
17	1	$t = 0, a = 1$	$t = 0, a = 0$
	0	$t = 0, a = 1$	$t = 0, a = 0$
18	1	$t = 0, a = 0$	$t = 0, a = 1$
	0	$t = 0, a = 0$	$t = 0, a = 1$

1. The type-l expert performs the test along the equilibrium path, but the type-h expert does not. The type-h expert should prefer to play her own equilibrium strategy (of not performing the test, and providing a diagnosis that is consistent with her private signal) instead of mimicking the type-l expert by performing the test, that is,

$$u(t=0, a=0|s_h=0) \geq u_h(t=1|s_h=0), \tag{IC-1}$$

$$u(t=0, a=1|s_h=1) \geq u_h(t=1|s_h=1). \tag{IC-2}$$

2. The type-l expert should prefer to perform the test and not mimic the type-h expert by not performing the test, and providing a diagnosis according to her private signal, that is,

$$u_l(t=1|s_l=0) \geq u_l(t=0, a=0|s_l=0), \tag{IC-3}$$

$$u(t=1|s_l=1) \geq u_l(t=0, a=1|s_l=1). \tag{IC-4}$$

3. In the equilibrium, we expect the type-h expert to provide a diagnosis that is consistent with her private signal. The type-h expert should be unwilling to provide a diagnosis that is not consistent with her private signal. The following two constraints ensure the type-h expert has no incentive to deviate from the equilibrium by choosing not to perform the test, and providing a diagnosis inconsistent with her private signal, that is,

$$u(t=0, a=0|s_h=0) \geq u_h(t=0, a=1|s_h=0), \tag{IC-5}$$

$$u_h(t=0, a=1|s_h=1) \geq u_h(t=0, a=0|s_h=1). \tag{IC-6}$$

4. The constraints (IC-3) and (IC-4) above ensure the type-l expert does not deviate by not performing the test, and providing a diagnosis that is consistent with her private signal. We must also ensure the type-l expert has no incentive to deviate from the equilibrium by choosing not to perform the test, and providing a diagnosis inconsistent with her private signal, that is,

$$u_l(t=1|s_l=0) \geq u_l(t=0, a=1|s_l=0), \tag{IC-7}$$

$$u_l(t=1|s_l=1) \geq u_l(t=0, a=0|s_l=1). \tag{IC-8}$$

In addition, four IR constraints ensure each type of expert has a non-negative utility along the equilibrium path under any private signal:

$$u(t=0, a=1|s_h=1) \geq 0, \tag{IR-1}$$

$$u_h(t=0, a=0|s_h=0) \geq 0, \tag{IR-2}$$

$$u_l(t=1|s_l=1) \geq 0, \tag{IR-3}$$

$$u(t = 1|s_l = 0) \geq 0. \tag{IR-4}$$

Proof of Lemma 1.

We prove this lemma by contradiction, and consider three possible cases, all of which entail the type-h expert performing the test.

(i) Suppose there is a separating equilibrium in which the type-h expert performs the test regardless of her signal. Because the type-h expert performs the test, for the two types of experts to exhibit externally separating diagnostic pathways, the type-l expert must not perform the test. The type-l expert does not recommend $a = 1$ when she receives $s_l = 1$, because $u_l\,(t = 0, a = 1|s_l = 1) < u_l\,(t = 1|s_l = 1)$ if the type-h expert performs the test on $s_h = 1$ in the equilibrium. Similarly, the type-l expert does not recommend $a = 0$ when she receives $s_l = 0$, because $u_l\,(t = 0, a = 0|s_l = 0) < u_l\,(t = 1|s_l = 0)$ if the type-h expert performs the test on $s_h = 0$ in the equilibrium. Therefore, the type-l expert's recommendation cannot be consistent with her signal. We must have $u_l\,(t = 0, a = 1|s_l = 0) \geq u_l\,(t = 0, a = 0|s_l = 0)$ and $u_l\,(t = 0, a = 0|s_l = 1) \geq u_l\,(t = 0, a = 1|s_l = 1)$. However, both conditions cannot be simultaneously satisfied for $\rho_l > \frac{1}{2}$. This is a contradiction. Therefore, a separating equilibrium does not exist in which the type-h expert performs the test regardless of her signal.

(ii) Suppose a separating equilibrium exists in which the type-h expert tests only when she receives $s_h = 1$. Since type-h expert performs the test only when she receives $s_h = 1$, she has to choose a diagnosis of either $a = 0$ or $a = 1$ when she receives $s_h = 0$. Suppose she chooses $a = 0$. The type-l expert must recommend $a = 1$. However, as shown in Part (i), in a separating equilibrium, if type-h expert performs the test on receiving $s_h = 1$, the low type finds mimicking the type-h expert to be more profitable than recommending $a = 1$. Therefore, the type-h expert does not recommend $a = 0$ when she receives $s_h = 0$. Now suppose she recommends $a = 1$. The type-l expert, to exhibit an externally separating diagnostic pathway, has to choose $a = 0$. However, in an equilibrium, if the type-h expert recommends $a = 1$ on receiving $s_h = 0$, $u_l\,(t = 0, a = 1|s_l = 0) > u_l\,(t = 0, a = 0|s_l = 0)$. This is a contradiction. Therefore, no separating equilibrium exists in which the type-h expert tests only when she receives $s_h = 1$.

(iii) Suppose a separating equilibrium exists in which the type-h expert tests only when she receives $s_h = 0$. The proof is by contradiction, and proceeds in a similar fashion as in that of part (ii), and is therefore not presented here.

By jointly examining all the above cases, we have enumerated all pairs of externally separating diagnostic pathways in which the type-h expert performs the test. Therefore, a separating equilibrium does not exist in which the type-h expert performs the test. *Q.E.D.*

Proof of Lemma 2.

The type-h expert's recommendation may be inconsistent with her signal in three different ways that we will eliminate one at a time below.

(i) The type-h expert recommends $a = 0$ regardless of her signal, making her recommendation inconsistent with her signal if $s_h = 1$.

Because the type-h expert recommends $a = 0$, for the two types of experts to have externally separating diagnostic pathways, the type-l expert has to either perform the test or recommend $a = 1$. However, in an equilibrium, if the type-h expert recommends $a = 1$ on receiving $s_h = 1$, the type-l expert prefers to mimic the type-h expert instead of performing the test or recommending $a = 1$. Therefore, a separating equilibrium does not exist in which the type-h expert recommends $a = 0$ regardless of her signal.

(ii) The type-h expert recommends $a = 1$ regardless of her signal, making her recommendation inconsistent with her signal if $s_h = 0$.

In this case, the type-l expert must either perform the test or recommends $a = 0$. However, given that the type-h expert recommends $a = 1$ on receiving $s_h = 0$ the type-l expert would find it more lucrative to mimic the type-h expert instead of performing the test or recommending $a = 0$ when she receives $s_l = 0$. Therefore a separating equilibrium in which the type-h expert recommends $a = 1$ regardless of her signal is not possible.

(iii) The type-h expert recommends $a = 0$ if $s_h = 1$ and $a = 1$ if $s_h = 0$.

Here, the type-l expert must perform the test but she finds deviation to not testing and recommending $a = 0$ more attractive when she receives $s_l = 1$. As a result, this scenario is also ruled out.

Therefore, the type-h expert's recommendation must be consistent with her signal in any separating equilibrium. The type-l expert must perform the test regardless of her signal. *Q.E.D.*

Proof of Proposition 2.

Proposition 2 follows directly from Lemma 1 and Lemma 2. According to Lemma 1, a candidate separating equilibrium that involves the type-h expert performing the test cannot survive. Candidates 1–4 and Candidates 9–12 (in described in Table 2) involve the type-h expert performing the test and therefore violate Lemma 1. Lemma 2 requires the type-h expert's diagnosis to be consistent with her signal. Candidates 6–8 and Candidates 13–18 involve the type-h expert offering a diagnosis that is inconsistent with her private signal and therefore violate Lemma 2. The only candidate equilibrium that survives both Lemmas 1 and 2 is the candidate equilibrium 5. In the candidate equilibrium 5, the type-h expert does not perform the test and her diagnosis is consistent with her signal, whereas the type-l expert performs the test. *Q.E.D.*

Proof of Proposition 3.

Consider the following separating equilibrium: The type-h expert never performs the test, and always provides a diagnosis that is consistent with the her signal; the type-l expert always performs the test. For this outcome to constitute an equilibrium, we need to check both IC and IR constraints.

IC Constraints: Eight IC constraints exist as described above in Section 3.1. We start by simplifying (IC-1) (having received a private signal of $s_h = 0$, the type-h prefers to play her equilibrium strategy instead of mimicking type-l) . The expert's relevant expected utilities can be written as $u_h(t=0, a=0|s_h=0) = \frac{\phi(1-\rho_h)\alpha(-D)}{(1-\rho_h)\alpha+\rho_h(1-\alpha)} + (1-\phi)$ and $u_h(t=1|s_h=0) = \phi\left(\frac{(1-\rho_h)\alpha B}{(1-\rho_h)\alpha+\rho_h(1-\alpha)} - c\right)$. Substituting utilities in the expression for (IC-1) gives

$$r\omega + c \geq \frac{\alpha(1-\rho_h)(B+D)}{\alpha(1-\rho_h)+(1-\alpha)\rho_h}. \tag{IC-1}$$

Similarly, (IC-2), which means the type-h prefers to play her equilibrium strategy instead of mimicking type-l in the case of $s_h = 1$, reduces to

$$r\omega + c \geq \frac{(1-\alpha)(1-\rho_h)d}{(1-\alpha)(1-\rho_h)+\alpha\rho_h}. \tag{IC-2}$$

The constraints (IC-3) and (IC-4) ensure the type-l expert has no incentive to mimic the type-h expert by not performing the test and providing a diagnosis according to the signal observed during the consultation. Simplifying these constraints yields

$$r\omega + c \leq \frac{\alpha(1-\rho_l)(B+D)}{\alpha(1-\rho_l)+(1-\alpha)\rho_l}, \tag{IC-3}$$

$$r\omega + c \leq \frac{(1-\alpha)(1-\rho_l)d}{(1-\alpha)(1-\rho_l)+\alpha\rho_l}. \tag{IC-4}$$

The constraints (IC-5) and (IC-6) ensure the type-h expert has no incentive to deviate from the equilibrium by choosing not to perform the test and providing a diagnosis that is not completely consistent with the observed signal. They reduce to:

$$\alpha \leq \frac{\rho_h d}{\rho_h d + (1-\rho_h)(B+D)}, \tag{IC-5}$$

$$\alpha \geq \frac{(1-\rho_h)d}{\rho_h(B+D)+(1-\rho_h)d}. \tag{IC-6}$$

The constraints (IC-7) and (IC-8) ensure the type-l expert has no incentive to deviate from the equilibrium by choosing not to perform the test and providing a diagnosis that is not completely consistent with the observed signal. These constraints reduce to:

$$r\omega + c \leq \frac{(1-\alpha)\rho_l d}{(1-\alpha)\rho_l + \alpha(1-\rho_l)}, \tag{IC-7}$$

$$r\omega + c \leq \frac{\alpha\rho_l(B+D)}{\alpha\rho_l+(1-\alpha)(1-\rho_l)}. \tag{IC-8}$$

IR Constraints. Four IR constraints (as described in Section 3.1) exists. Substituting appropriate utility expressions, we have

$$u_h(t=0, a=1|s_h=1) = \phi\left(\frac{\alpha\rho_h B - (1-\alpha)(1-\rho_h)d}{\alpha\rho_h + (1-\alpha)(1-\rho_h)}\right) + (1-\phi)\,r \geq 0, \tag{IR-1}$$

$$u_h(t=0, a=0|s_h=0) = \phi\left(\frac{\alpha(1-\rho_h)(-D)}{\alpha(1-\rho_h)+(1-\alpha)\rho_h}\right) + (1-\phi)\,r \geq 0, \tag{IR-2}$$

$$u_l(t=1|s_l=1) = \phi\left(\frac{\alpha\rho_l B}{\alpha\rho_l + (1-\alpha)(1-\rho_l)} - c\right) \geq 0, \tag{IR-3}$$
$$u(t=1|s_l=0) = \phi\left(\frac{\alpha(1-\rho_l)B}{\alpha(1-\rho_l) + (1-\alpha)\rho_l} - c\right) \geq 0. \tag{IR-4}$$

It is straightforward to show (IC-7) and (IC-8) are redundant given (IC-4) and (IC-3), respectively. The IC constraints (IC-1)–(IC–6) are equivalent to

$$\alpha \geq \max\left\{\frac{(1-\rho_h)(d-r\omega-c)}{\rho_h(r\omega+c)+(1-\rho_h)(d-r\omega-c)}, \frac{\rho_l(r\omega+c)}{\rho_l(r\omega+c)+(1-\rho_l)(B+D-r\omega-c)}, \frac{(1-\rho_h)d}{\rho_h(B+D)+(1-\rho_h)d}\right\},$$

and

$$\alpha \leq \min\left\{\frac{\rho_h(r\omega+c)}{\rho_h(r\omega+c)+(1-\rho_h)(B+D-r\omega-c)}, \frac{(1-\rho_l)(d-r\omega-c)}{\rho_l(r\omega+c)+(1-\rho_l)(d-r\omega-c)}, \frac{\rho_h d}{(1-\rho_h)(B+D)+\rho_h d}\right\}.$$

Claim 1: *All the four IR constraints are satisfied given $r\omega(B+D+d) \geq dD$, and $r\omega B \geq cD$.*

First, (IR-3) follows from (IR-4). We only need to consider (IR-1), (IR-2), and (IR-4).

Second, we show that given $r\omega(B+D+d) \geq dD$, Constraint (IR-1) is redundant, because $\alpha \geq \frac{(1-\rho_h)d}{\rho_h(B+D)+(1-\rho_h)d}$ gives

$$\begin{aligned}
&\alpha\rho_h B - (1-\alpha)(1-\rho_h)d + r\omega[\alpha\rho_h + (1-\alpha)(1-\rho_h)] \\
&\geq \frac{1}{\rho_h\,(B+D) + (1-\rho_h)\,d} \cdot \rho_h(1-\rho_h)\left[-dD + r\omega(B+D+d)\right] \geq 0,
\end{aligned}$$

which yields

$$\frac{\alpha\rho_h B - (1-\alpha)(1-\rho_h)d}{\alpha\rho_h + (1-\alpha)(1-\rho_h)} + r\omega \geq 0$$

given that $r\omega(B+D+d) \geq dD$.

Third, we show (IR-2) is redundant under the condition that $r\omega B \geq cD$. This is because (IC-1) is equivalent to

$$\alpha \leq \frac{\rho_h\,(r\omega+c)}{\rho_h(r\omega+c)+(1-\rho_h)(B+D-r\omega-c)}, \tag{4}$$

whereas (IR-2) is equivalent to

$$\alpha \leq \frac{\rho_h r\omega}{\rho_h r\omega + (1-\rho_h)(D-r\omega)}. \tag{5}$$

We can verify that (5) follows from (4) given $r\omega B \geq cD$.

Last, note that (IR-4) is also redundant under the condition that $r\omega B \geq cD$. This is because (IC-7) is equivalent to

$$\alpha \geq \frac{\rho_l(r\omega+c)}{\rho_l(r\omega+c)+(1-\rho_l)(B+D-r\omega-c)}, \tag{6}$$

whereas (IR-4) is equivalent to

$$\alpha \geq \frac{\rho_l c}{\rho_l c + (1-\rho_l)(B-c)}. \tag{7}$$

We can verify that (7) follows from (6) given $r\omega B \geq cD$.

Claim 2: *If* $\frac{1}{B+D} + \frac{1}{d} > \frac{1}{r\omega+c}$*, the thresholds for* α *can be simplified to*

$$\underset{\sim}{\alpha} = \frac{\rho_l(r\omega+c)}{\rho_l(r\omega+c)+(1-\rho_l)(B+D-r\omega-c)} \quad \textit{and} \quad \bar{\alpha} = \frac{(1-\rho_l)(d-r\omega-c)}{\rho_l(r\omega+c)+(1-\rho_l)(d-r\omega-c)}.$$

(Note, in Claims 3 and 4 we show the separating equilibrium does not exist in this parameter space.)

To show this, note the condition $\frac{1}{B+D} + \frac{1}{d} > \frac{1}{r\omega+c}$ yields $\frac{r\omega+c}{d-r\omega-c} > \frac{B+D-r\omega-c}{r\omega+c}$, $\frac{B+D}{d-r\omega-c} > \frac{B+D-r\omega-c}{r\omega+c}$, $\frac{B+D-r\omega-c}{r\omega+c} < \frac{r\omega+c}{d-r\omega-c}$, and $\frac{B+D}{d} < \frac{r\omega+c}{d-r\omega-c}$. In addition, we have from $\rho_h > \rho_l > 1/2$ that $\frac{\rho_h}{1-\rho_h} > \frac{1-\rho_l}{\rho_l}$, and $\frac{1-\rho_h}{\rho_h} < \frac{\rho_l}{1-\rho_l}$. Therefore, we have

$$\begin{aligned}
\underset{\sim}{\alpha} &= \max\left\{\frac{(1-\rho_h)(d-r\omega-c)}{\rho_h(r\omega+c)+(1-\rho_h)(d-r\omega-c)}, \frac{\rho_l(r\omega+c)}{\rho_l(r\omega+c)+(1-\rho_l)(B+D-r\omega-c)}, \frac{(1-\rho_h)d}{\rho_h(B+D)+(1-\rho_h)d}\right\}, \\
&= \max\left\{\frac{1}{\frac{\rho_h}{1-\rho_h}\frac{r\omega+c}{d-r\omega-c}+1}, \frac{1}{1+\frac{1-\rho_l}{\rho_l}\frac{B+D-r\omega-c}{r\omega+c}}, \frac{1}{1+\frac{\rho_h}{1-\rho_h}\frac{B+D}{d}}\right\} \\
&= \frac{1}{1+\frac{1-\rho_l}{\rho_l}\frac{B+D-r\omega-c}{r\omega+c}} = \frac{\rho_l(r\omega+c)}{\rho_l(r\omega+c)+(1-\rho_l)(B+D-r\omega-c)},
\end{aligned}$$

and

$$\begin{aligned}
\bar{\alpha} &= \min\left\{\frac{\rho_h(r\omega+c)}{\rho_h(r\omega+c)+(1-\rho_h)(B+D-r\omega-c)}, \frac{(1-\rho_l)(d-r\omega-c)}{\rho_l(r\omega+c)+(1-\rho_l)(d-r\omega-c)}, \frac{\rho_h d}{\rho_h d+(1-\rho_h)(B+D)}\right\} \\
&= \min\left\{\frac{1}{1+\frac{1-\rho_h}{\rho_h}\frac{B+D-r\omega-c}{r\omega+c}}, \frac{1}{1+\frac{\rho_l}{1-\rho_l}\frac{r\omega+c}{d-r\omega-c}}, \frac{1}{1+\frac{1-\rho_h}{\rho_h}\frac{B+D}{d}}\right\} \\
&= \frac{1}{1+\frac{\rho_l}{1-\rho_l}\frac{r\omega+c}{d-r\omega-c}} = \frac{(1-\rho_l)(d-r\omega-c)}{\rho_l(r\omega+c)+(1-\rho_l)(d-r\omega-c)}.
\end{aligned}$$

Claim 3: *A necessary condition for a separating equilibrium to sustain is* $\frac{1}{B+D} + \frac{1}{d} \leq \frac{1}{r\omega+c}$. (Note, in Claim 4 we show this condition is redundant.)

We prove the above claim by contradiction. Suppose the condition is not satisfied. In that case, following Claim 2, we can show that $\underset{\sim}{\alpha} > \bar{\alpha}$. This is because $\underset{\sim}{\alpha} = \frac{\rho_l(r\omega+c)}{\rho_l(r\omega+c)+(1-\rho_l)(B+D-r\omega-c)} = \frac{1}{1+\frac{1-\rho_l}{\rho_l}\frac{B+D-r\omega-c}{r\omega+c}} > \frac{1}{1+\frac{\rho_l}{1-\rho_l}\frac{r\omega+c}{d-r\omega-c}}$. Therefore, the condition $\frac{1}{B+D} + \frac{1}{d} \leq \frac{1}{r\omega+c}$ must be satisfied in the separating equilibrium.

Claim 4: *The separating equilibrium exists in the interval*

$$(\underset{\sim}{\alpha}, \bar{\alpha}) = \begin{cases} \left(\frac{\rho_l(r\omega+c)}{\rho_l(r\omega+c)+(1-\rho_l)(B+D-r\omega-c)}, \frac{(1-\rho_l)(d-r\omega-c)}{\rho_l(r\omega+c)+(1-\rho_l)(d-r\omega-c)}\right) & \textit{if } \underline{B} \leq B \leq \hat{B} \\ \left(\frac{(1-\rho_h)(d-r\omega-c)}{\rho_h(r\omega+c)+(1-\rho_h)(d-r\omega-c)}, \frac{\rho_h(r\omega+c)}{\rho_h(r\omega+c)+(1-\rho_h)(B+D-r\omega-c)}\right) & \textit{if } \hat{B} < B \leq \overline{B} \end{cases},$$

where $\hat{B} = r\omega + c - D + \frac{\rho_h}{1-\rho_h}\frac{\rho_l}{1-\rho_l}\frac{(r\omega+c)^2}{(d-r\omega-c)}$.
The condition $\frac{1}{B+D} + \frac{1}{d} \leq \frac{1}{r\omega+c}$, implies

$$\frac{r\omega + c}{d - r\omega - c} \leq \frac{B + D}{d} \leq \frac{B + D - r\omega - c}{r\omega + c}.$$

The implication is that (IC-5) and (IC-6) are redundant. We may proceed to verify that

$$\frac{(1-\rho_h)(d - r\omega - c)}{\rho_h(r\omega + c) + (1-\rho_h)(d - r\omega - c)} > \frac{\rho_l(r\omega + c)}{\rho_l(r\omega + c) + (1-\rho_l)(B + D - r\omega - c)},$$

if and only if $B > r\omega + c - D + \frac{\rho_h}{1-\rho_h}\frac{\rho_l}{1-\rho_l}\frac{(r\omega+c)^2}{(d-r\omega-c)}$. In addition,

$$\frac{\rho_h(r\omega + c)}{\rho_h(r\omega + c) + (1-\rho_h)(B + D - r\omega - c)} < \frac{(1-\rho_l)(d - r\omega - c)}{\rho_l(r\omega + c) + (1-\rho_l)(d - r\omega - c)}.$$

if and only if $B < r\omega + c - D + \frac{1-\rho_h}{\rho_h}\frac{1-\rho_l}{\rho_l}\frac{(r\omega+c)^2}{(d-r\omega-c)}$.
Next, we show the condition

$$r\omega + c - D + \frac{\rho_l^2}{(1-\rho_l)^2}\frac{(r\omega + c)^2}{(d - r\omega - c)} \leq B \leq r\omega + c - D + \frac{\rho_h^2}{(1-\rho_h)^2}\frac{(r\omega + c)^2}{(d - r\omega - c)}. \tag{8}$$

is the sufficient and necessary condition for the separating equilibrium to exist, because (8) is equivalent to

$$\frac{(1-\rho_h)(d - r\omega - c)}{\rho_h(r\omega + c) + (1-\rho_h)(d - r\omega - c)} \leq \frac{\rho_h(r\omega + c)}{\rho_h(r\omega + c) + (1-\rho_h)(B + D - r\omega - c)} \text{ and}$$
$$\frac{\rho_l(r\omega + c)}{\rho_l(r\omega + c) + (1-\rho_l)(B + D - r\omega - c)} \leq \frac{(1-\rho_l)(d - r\omega - c)}{\rho_l(r\omega + c) + (1-\rho_l)(d - r\omega - c)},$$

and is thus equivalent to $\underline{\alpha} \leq \bar{\alpha}$.
Also note the condition $B \leq r\omega + c - D + \frac{\rho_h^2}{(1-\rho_h)^2}\frac{(r\omega+c)^2}{(d-r\omega-c)}$ implies $\frac{1}{B+D} + \frac{1}{d} \leq \frac{1}{r\omega+c}$. Therefore, the condition $\frac{1}{B+D} + \frac{1}{d} \leq \frac{1}{r\omega+c}$ is redundant. *Q.E.D.*

Proof of Proposition 4.

(i). Proposition 1 implies that in the full-information benchmark, the type-h physician performs the test for $\alpha \in [\underline{\alpha}_1^h, \bar{\alpha}_0^h]$. Thus, we need to show the intersection of the two sets $[\underline{\alpha}, \bar{\alpha}]$ and $[\underline{\alpha}_1^h, \bar{\alpha}_0^h]$ is not an empty set. We can establish this result by examining the following two cases:

(a) If $\frac{(r\omega+c)^2}{(d-r\omega-c)(B+D-r\omega-c)} < \frac{1-\rho_h}{\rho_h}\frac{1-\rho_l}{\rho_l}$, we have from Proposition 3 that

$$\bar{\alpha} = \frac{\rho_h(r\omega + c)}{\rho_h(r\omega + c) + (1-\rho_h)(B + D - r\omega - c)} \text{ and } \underline{\alpha} = \frac{(1-\rho_h)(d - r\omega - c)}{\rho_h(r\omega + c) + (1-\rho_h)(d - r\omega - c)}.$$

Thus,

$$\underline{\alpha}_1^h = \frac{(1-\rho_h)c}{(1-\rho_h)c+\rho_h(B+D-c)} < \frac{\rho_h c}{\rho_h c+(1-\rho_h)(B+D-c)} < \frac{\rho_h(r\omega+c)}{\rho_h(r\omega+c)+(1-\rho_h)(B+D-r\omega-c)} = \bar{\alpha},$$

and

$$\bar{\alpha}_0^h = \frac{\rho_h(d-c)}{\rho_h(d-c)+(1-\rho_h)c} > \frac{(1-\rho_h)(d-r\omega-c)}{\rho_h(r\omega+c)+(1-\rho_h)(d-r\omega-c)} = \underline{\alpha}.$$

Because $\underline{\alpha}_1^h < \bar{\alpha}$ and $\underline{\alpha} < \bar{\alpha}_0^h$, the intersection of the two sets $[\underline{\alpha}, \bar{\alpha}]$ and $[\underline{\alpha}_1^h, \bar{\alpha}_0^h]$ is non-empty.

(b) If $\frac{(r\omega+c)^2}{(d-r\omega-c)(B+D-r\omega-c)} \geq \frac{1-\rho_h}{\rho_h}\frac{1-\rho_l}{\rho_l}$, we have from Proposition 3 that

$$\bar{\alpha} = \frac{(1-\rho_l)(d-r\omega-c)}{\rho_l(r\omega+c)+(1-\rho_l)(d-r\omega-c)} \text{ and } \underline{\alpha} = \frac{\rho_l(r\omega+c)}{\rho_l(r\omega+c)+(1-\rho_l)(B+D-r\omega-c)}.$$

Thus,

$$\underline{\alpha}_1^h = \frac{(1-\rho_h)c}{(1-\rho_h)c+\rho_h(B+D-c)} < \frac{\rho_l c}{\rho_l c+(1-\rho_l)(B+D-c)} < \frac{\rho_l(r\omega+c)}{\rho_l(r\omega+c)+(1-\rho_l)(B+D-r\omega-c)} = \underline{\alpha},$$

and

$$\bar{\alpha}_0^h = \frac{\rho_h(d-c)}{\rho_h(d-c)+(1-\rho_h)c} > \frac{(1-\rho_l)(d-r\omega-c)}{\rho_l(r\omega+c)+(1-\rho_l)(d-r\omega-c)} = \bar{\alpha}.$$

Therefore, we have $[\underline{\alpha}, \bar{\alpha}] \subset [\underline{\alpha}_1^h, \bar{\alpha}_0^h]$.

(ii). When $\frac{(r\omega+c)^2}{(d-r\omega-c)(B+D-r\omega-c)} < \frac{1-\rho_h}{\rho_h}\frac{1-\rho_l}{\rho_l}$, we have from Proposition 3 that

$$\bar{\alpha} = \frac{\rho_h(r\omega+c)}{\rho_h(r\omega+c)+(1-\rho_h)(B+D-r\omega-c)} \text{ and } \underline{\alpha} = \frac{(1-\rho_h)(d-r\omega-c)}{\rho_h(r\omega+c)+(1-\rho_h)(d-r\omega-c)}.$$

In addition, $\frac{(r\omega+c)^2}{(d-r\omega-c)(B+D-r\omega-c)} < \frac{(1-\rho_h)(1-\rho_l)}{\rho_h\rho_l}$ ensures that

$$\frac{\rho_h\left(r\omega+c\right)}{\rho_h\left(r\omega+c\right)+\left(1-\rho_h\right)\left(B+D-r\omega-c\right)} < \frac{\left(1-\rho_l\right)\left(d-r\omega-c\right)}{\rho_l\left(r\omega+c\right)+\left(1-\rho_l\right)\left(d-r\omega-c\right)},$$

and

$$\frac{\left(1-\rho_h\right)\left(d-r\omega-c\right)}{\rho_h\left(r\omega+c\right)+\left(1-\rho_h\right)\left(d-r\omega-c\right)} > \frac{\rho_l\left(r\omega+c\right)}{\rho_l\left(r\omega+c\right)+\left(1-\rho_l\right)\left(B+D-r\omega-c\right)}.$$

Therefore, to show the type-l expert does not over-test in the entire range of α in which the separating equilibrium exists, it suffices to show that

$$\underline{\alpha}_0^l < \frac{\rho_l\left(r\omega+c\right)}{\rho_l\left(r\omega+c\right)+\left(1-\rho_l\right)\left(B+D-r\omega-c\right)}, \text{ and}$$

$$\bar{\alpha}_1^l > \frac{(1-\rho_l)(d-r\omega-c)}{\rho_l(r\omega+c)+(1-\rho_l)(d-r\omega-c)}.$$

It is straightforward to show that both the above conditions hold for all $r > 0$. Therefore, the type-l expert does not over-test in the separating equilibrium. *Q.E.D.*

Proof of Proposition 5.

First, we define thresholds $\underline{r}$, $\hat{r}$, and $\bar{r}$ such that (1) the type-h expert prefers not to perform the test if $r < \underline{r}$, (2) the type-l expert considers not performing the test if $r > \hat{r}$, and (3) the type-l expert does not performs the test if $r > \bar{r}$. The expressions for $\underline{r}$, $\bar{r}$, and $\hat{r}$ are positive solutions to the following equations respectively:

$$\underline{r}\omega + c - B - D + \left(\frac{\rho_h}{1-\rho_h}\right)^2 \frac{(\underline{r}\omega+c)}{(d-\underline{r}\omega-c)} = 0,$$
$$\bar{r}\omega + c - B - D + \left(\frac{\rho_l}{1-\rho_l}\right)^2 \frac{(\bar{r}\omega+c)}{(d-\bar{r}\omega-c)} = 0,$$
$$\hat{r}\omega + c - B - D + \left(\frac{\rho_h}{1-\rho_h}\right)\left(\frac{\rho_l}{1-\rho_l}\right) \frac{(\hat{r}\omega+c)}{(d-\hat{r}\omega-c)} = 0.$$

We also have $\underline{r} < \hat{r} < \bar{r}$. The range of α for which the separating equilibrium exists, as described in Proposition 3, can be written as

$$(\underline{\alpha}, \bar{\alpha}) = \begin{cases} \left(\frac{(1-\rho_h)(d-r\omega-c)}{\rho_h(r\omega+c)+(1-\rho_h)(d-r\omega-c)}, \frac{\rho_h(r\omega+c)}{\rho_h(r\omega+c)+(1-\rho_h)(B+D-r\omega-c)}\right), & \text{if } \underline{r} \le r \le \hat{r} \\ \frac{\rho_l(r\omega+c)}{\rho_l(r\omega+c)+(1-\rho_l)(B+D-r\omega-c)}, \frac{(1-\rho_l)(d-r\omega-c)}{\rho_l(r\omega+c)+(1-\rho_l)(d-r\omega-c)}, & \text{if } \hat{r} \le r \le \bar{r}. \end{cases}$$

The parameter space $[\underline{\alpha}, \bar{\alpha}]$ is empty, if $r < \underline{r}$ or $r > \bar{r}$. In addition,

$$\frac{\partial}{\partial r}(\bar{\alpha} - \underline{\alpha}) \begin{cases} > 0 & \text{if } \underline{r} < r < \hat{r} \\ < 0 & \text{if } \hat{r} < r < \bar{r}. \end{cases}$$

This completes the proof. *Q.E.D.*

The Effect of Altruism in Section 4.

The proof is similar to the proof of Proposition 5. Here we provide only the main results. We define thresholds $\underline{\omega}$, $\hat{\omega}$, and $\bar{\omega}$ such that (1) the type-h expert prefers not to perform the test if $\omega < \underline{\omega}$, (2) the type-$l$ expert considers not performing the test if $\omega > \hat{\omega}$, and (3) the type-l expert does not performs the test if $\omega > \bar{\omega}$. The expressions for $\underline{\omega}$, $\hat{\omega}$, and $\bar{\omega}$ are given by the positive solutions to the following equations respectively:

$$r\underline{\omega} + c - B - D + \left(\frac{\rho_h}{1-\rho_h}\right)^2 \frac{(r\underline{\omega}+c)}{(d-r\underline{\omega}-c)} = 0,$$

$$r\hat{\omega}+c-B-D+\left(\frac{\rho_h}{1-\rho_h}\right)\left(\frac{\rho_l}{1-\rho_l}\right)\frac{(r\hat{\omega}+c)}{(d-r\hat{\omega}-c)}=0,$$
$$r\bar{\omega}+c-B-D+\left(\frac{\rho_l}{1-\rho_l}\right)^2\frac{(r\bar{\omega}+c)}{(d-r\bar{\omega}-c)}=0.$$

It follows that $\underline{\omega}<\hat{\omega}<\bar{\omega}$. A unique separating equilibrium exists for the range of α given by

$$(\underline{\alpha},\bar{\alpha})=\begin{cases}\left(\frac{(1-\rho_h)(d-r\omega-c)}{\rho_h(r\omega+c)+(1-\rho_h)(d-r\omega-c)},\frac{\rho_h(r\omega+c)}{\rho_h(r\omega+c)+(1-\rho_h)(B+D-r\omega-c)}\right), & \text{if } \underline{\omega}\leq\omega\leq\hat{\omega}\\ \frac{\rho_l(r\omega+c)}{\rho_l(r\omega+c)+(1-\rho_l)(B+D-r\omega-c)},\frac{(1-\rho_l)(d-r\omega-c)}{\rho_l(r\omega+c)+(1-\rho_l)(d-r\omega-c)}, & \text{if } \hat{\omega}\leq\omega\leq\bar{\omega}.\end{cases}$$

No separating equilibrium exists if $\omega<\underline{\omega}$ or $\omega>\bar{\omega}$. The parameter space $[\underline{\alpha},\bar{\alpha}]$ is empty if $\omega<\underline{\omega}$ or $\omega>\bar{\omega}$. In addition,

$$\frac{\partial}{\partial\omega}\left(\bar{\alpha}-\underline{\alpha}\right)\begin{cases}>0 & \text{if } \underline{\omega}<\omega<\hat{\omega}\\ <0 & \text{if } \hat{\omega}<\omega<\bar{\omega}.\end{cases}$$

Q.E.D.

Analysis of Client Utility in Section 4.

Throughout the proof, we consider the separating equilibrium characterized in Propositions 2–3. Consider a client (who does not know the expert type) who is diagnosed by a type-l expert. If the client's true state is 1 (with probability of α), the client's payoff is $B-c$. However, if the true state is 0 (with probability of $(1-\alpha)$), the client's payoff is $-c$. Thus, the client's expected payoff from visiting a type-l expert is given by $U_l=\alpha B-c$. Now suppose the client, unaware about the expert type, visits a type-h expert. The type-h expert does not perform the test and her diagnosis is consistent with her signal. Therefore, if the client's true state is 1 (with probability of α), the client's expected payoff is $\rho_h B+(1-\rho_h)(-D)$, and if her true state is 0 (with probability of $(1-\alpha)$), the client's expected payoff is $(1-\rho_h)(-d)$. The client's expected payoff from visiting a type-h expert can be written as $U_h=\alpha[\rho_h B-(1-\rho_h)D]-(1-\alpha)(1-\rho_h)d$. The client's expected utility from visiting a type-h expert is higher than her expected utility from visiting a type-l expert if $u_h>u_l$, or, equivalently, $c>(1-\rho_h)\left[\alpha(B+D)+(1-\alpha)d\right]$. The client's expected utility from visiting a type-h expert is lower than her expected utility from visiting a type-l expert, otherwise. *Q.E.D.*

Proof of Results in Section 5.1

Because none of the three possible candidate equilibria exists, we only provide a subset of conditions that establish that the particular candidate equilibrium does not exist.

(1) First, consider the candidate equilibrium in which the type-h expert discloses her private signal ($b=1$) and performs the test ($t=1$), whereas the type-l player does not disclose her private signal ($b=0$), does

not perform the test ($t = 0$), and provides a diagnosis that is consistent with her private signal. The type-h expert must prefer to disclose her private signal ($b = 1$) and perform the test ($t = 1$) compared to mimicking the type-l expert if her private signal $s_h = 0$. This condition can be written as $u_h(b = 1, t = 1|s_h = 0) \geq u_h(b = 0, t = 0, a = 0|s_h = 0)$, or, equivalently, $\phi\left(\frac{(1-\rho_h)\alpha B}{(1-\rho_h)\alpha+\rho_h(1-\alpha)} - c\right) + (1-\phi)r \geq \phi\left(\frac{(1-\rho_h)\alpha(-D)}{(1-\rho_h)\alpha+\rho_h(1-\alpha)}\right)$, which can be reorganized as

$$\omega r - c \geq \frac{-(1-\rho_h)\alpha(B+D)}{(1-\rho_h)\alpha + \rho_h(1-\alpha)}.$$

Similarly, the type-l expert should prefer to not disclose her signal ($b = 0$), to not perform the test ($t = 0$), and to offer a diagnosis ($a = 0$) that is consistent with her private evaluation ($s_l = 0$) instead of mimicking the type-h expert. The condition can be written as$u_l(b = 0, t = 0, a = 0|s_l = 0) \geq u_l(b = 1, t = 1|s_l = 0)$, which can be reorganized as

$$\omega r - c \leq \frac{-(1-\rho_l)\alpha(B+D)}{(1-\rho_l)\alpha + \rho_l(1-\alpha)}.$$

Because the above two conditions cannot be simultaneously satisfied, the candidate separating equilibrium described above does not exist.

(2) Next, consider the candidate separating equilibrium in which the type-h expert discloses her private signal ($b = 1$) and then performs the test ($t = 1$), whereas the type-l expert does not disclose her private signal ($b = 0$), does not perform the test ($t = 0$), and provides a diagnosis that is inconsistent with the private signal. The type-l expert must prefer to offer a diagnosis that is inconsistent with her private signal to one that is consistent with her private signal. Equivalently,

$$\begin{aligned} u_l(b = 0, t = 0, a = 1|s_l = 0) &\geq u_l(b = 0, t = 0, a = 0|s_l = 0), \\ u_l(b = 0, t = 0, a = 0|s_l = 1) &\geq u_l(b = 0, t = 0, a = 1|s_l = 1). \end{aligned}$$

Similar to part (1) above, it is straightforward to show the above two conditions cannot be satisfied simultaneously. Therefore, this candidate separating equilibrium does not exist.

(3) Now consider the candidate separating equilibrium in which the type-h expert discloses her private signal ($b = 1$) and then performs the test ($t = 1$), whereas the type-l expert does not disclose private signal ($b = 0$) but does perform the test ($t = 1$). If the type-l expert deviates and discloses her private signal, she would be believed to be a type-h expert regardless of whether the test confirms the private signal. Because disclosing the private signal has no associated costs, disclosing her own private signal is a profitable deviation for the type-l expert. Therefore, this candidate separating equilibrium does not exist. *Q.E.D.*

Proof of Proposition 6.

To establish the proposition, we introduce and prove three lemmas, namely, Lemmas 3–5.

First, we rule out the case in which either type of expert has an incentive to falsely disclose her private evaluation, which leads to a condition stated in the following lemma:

Lemma 3. *In the pooling equilibrium in which the expert discloses her private evaluation before subsequently testing, for her to truthly disclose the private evaluation, a necessary condition is:*

$$\rho_l \leq \alpha \leq 1 - \rho_l. \tag{9}$$

Proof. In the case of $s_e = 1$, we achieve so by requiring that the expert's reputational payoff from disclosing a positive evaluation is (weakly) higher than from disclosing a negative evaluation; that is,

$$b_e(\alpha|1)(B + r_1) + [1 - b_e(\alpha|1)]r_2 - c \geq b_e(\alpha|1)r_2 + [1 - b_e(\alpha|1)](B + r_1) - c,$$

which gives

$$b_e(\alpha|1) \geq \frac{1}{2}, \tag{10}$$

or, equivalently,

$$\alpha \geq 1 - \rho_e. \tag{11}$$

In a similar fashion, we rule out the case in which the expert has an incentive to falsely disclose her private evaluation by requiring that in the case of $s_e = 0$, the expert's reputational payoff from disclosing a negative evaluation is (weakly) higher than otherwise; that is,

$$b_e(\alpha|0)(B + r_2) + [1 - b_e(\alpha|0)]r_1 - c \geq b_e(\alpha|0)(B + r_1) + [1 - b_e(\alpha|0)]r_2 - c,$$

which gives

$$b_e(\alpha|0) \leq \frac{1}{2}, \tag{12}$$

or, equivalently,

$$\alpha \leq \rho_e. \tag{13}$$

For (11) and (13) to hold for both expert types, because $\rho_h > \rho_l$, we need

$$\alpha \in \mathcal{A} = [1 - \rho_l, \rho_l], \tag{14}$$

which we will impose as a necessary condition for the pooling equilibrium to sustain. The proof of Lemma 3 is complete.

Next, we consider the conditions for the pooling equilibrium to sustain for the cases in which the private evaluation is positive (i.e., $s_e = 1$) and negative (i.e., $s_e = 0$), leading to Lemmas 4 and 5, respectively. We first examine the case in which the expert's private evaluation is positive ($s_e = 1$). By disclosing her private evaluation before subsequently conducting a test, she gains a utility of

$$\phi \left\{ b_e(\alpha|1)(B + r_1) + [1 - b_e(\alpha|1)]r_2 - c \right\}. \tag{15}$$

We represent the expert's utility by deviating from the diagnostic pathway specified in the pooling equilibrium. In this case, because of our specification for out-of-equilibrium diagnostic pathways, the expert gains a reputational payoff of zero. Thus, the expert's utility is

$$\begin{aligned} &\phi \max\{b_e(\alpha|1) \cdot B + [1 - b_e(\alpha|1)] \cdot (-d), b_e(\alpha|1) \cdot (-D)\} \\ &= \begin{cases} \phi \left\{ b_e(\alpha|1) \cdot B + [1 - b_e(\alpha|1)] \cdot (-d) \right\}, & \text{if } b_e(\alpha|1) \geq \frac{d}{B+D+d}, \\ \phi \left[b_e(\alpha|1) \cdot (-D) \right] & \text{otherwise.} \end{cases} \end{aligned} \tag{16}$$

Comparing (15) with (16), we have the following two conditions, one of which has to hold for the pooling equilibrium to sustain:

$$b_e(\alpha|1) \geq \frac{d}{B + D + d} \text{ and } (r_1 - r_2 - d)b_e(\alpha|1) \geq c - r_2 - d, \tag{17}$$

$$b_e(\alpha|1) < \frac{d}{B + D + d} \text{ and } b_e(\alpha|1) \geq \frac{c - r_2}{B + D + r_1 - r_2}. \tag{18}$$

We have the following lemma from conditions (9), (17), and (18):

Lemma 4. *If $c \leq \min\{r_1, \frac{r_1+r_2+d}{2}\}$, for $\alpha \in [1 - \rho_l, \rho_l]$, in the presence of a positive private evaluation, neither type of expert has an incentive to deviate from the pooling equilibrium in which she discloses her private evaluation before subsequently conducting a test.*

Proof. Clearly, condition (18) cannot be satisfied, because $b_e(\alpha|1) < \frac{d}{B+D+d}$ contradicts (10). So we only need to analyze (17).

(a) First, we examine the case in which $r_1 - r_2 > d$. In this case, we simplify (17) as $b_e(\alpha|1) \in \mathcal{B}_{1a}$, in which

$$\mathcal{B}_{1a} = \begin{cases} [\frac{d}{B+D+d}, 1] & \text{if } c \leq \frac{(B+D)(r_2+d)+dr_1}{B+D+d} \\ [\frac{c-r_2-d}{r_1-r_2-d}, 1] & \text{if } \frac{(B+D)(r_2+d)+dr_1}{B+D+d} < c \leq r_1 \\ \emptyset & \text{if } c > r_1 \end{cases},$$

which is equivalent to $\alpha \in \mathcal{A}^e_{1a}$, where

$$\mathcal{A}^e_{1a} = \begin{cases} [\frac{d(1-\rho_e)}{d(1-\rho_e)+(B+D)\rho_e}, 1] & \text{if } c \leq \frac{(B+D)(r_2+d)+dr_1}{B+D+d} \\ [\frac{(c-r_2-d)(1-\rho_e)}{(c-r_2-d)(1-\rho_e)+(c-r_1)\rho_e}, 1] & \text{if } \frac{(B+D)(r_2+d)+dr_1}{B+D+d} < c \leq r_1 \\ \emptyset & \text{if } c > r_1 \end{cases}.$$

By incorporating condition (14), we have the following range of α in which the pooling equilibrium sustains:

$$\alpha \in \mathcal{A} \cap \mathcal{A}^h_{1a} \cap \mathcal{A}^l_{1a} = \begin{cases} [1-\rho_l, \rho_l] & \text{if } c \leq \frac{r_1+r_2+d}{2} \\ [\frac{(c-r_2-d)(1-\rho_l)}{(c-r_2-d)(1-\rho_l)+(c-r_1)\rho_l}, \rho_l] & \text{if } \frac{r_1+r_2+d}{2} < c \leq r_1 \\ \emptyset & \text{if } c > r_1 \end{cases}. \tag{19}$$

(b) Next, we examine the case in which $r_1 - r_2 = d$. In this case, $(r_1 - r_2 - d)b_e(\alpha|1) \geq c - r_2 - d$ is equivalent to $c \leq r_2 + d = r_1$. Thus, (17) is equivalent to $b_e(\alpha|1) \in \mathcal{B}_{1b}$, in which

$$\mathcal{B}_{1b} = \begin{cases} [\frac{d}{B+D+d}, 1] & \text{if } c \leq r_1 \\ \emptyset & \text{if } c > r_1 \end{cases},$$

which is equivalent to $\alpha \in \mathcal{A}^e_{1b}$, where

$$\mathcal{A}^e_{1b} = \begin{cases} [\frac{d(1-\rho_e)}{d(1-\rho_e)+(B+D)\rho_e}, 1] & \text{if } c \leq r_2 + d \\ \emptyset & \text{if } c > r_1 \end{cases}.$$

By incorporating condition (14), we have the following range of α in which the pooling equilibrium sustains:

$$\alpha \in \mathcal{A} \cap \mathcal{A}^h_{1b} \cap \mathcal{A}^l_{1b} = \begin{cases} [1-\rho_l, \rho_l] & \text{if } c \leq r_1 \\ \emptyset & \text{if } c > r_1 \end{cases}. \tag{20}$$

(c) Finally, we examine the case in which $r_1 - r_2 < d$. In this case, $(r_1 - r_2 - d)b_e(\alpha|1) \geq c - r_2 - d$ is equivalent to $b_e(\alpha|1) \leq \frac{r_2+d-c}{d-r_1+r_2}$. Thus, (17) is equivalent to $b_e(\alpha|1) \in \mathcal{B}_{1c}$, in which

$$\mathcal{B}_{1c} = \begin{cases} [\frac{d}{B+D+d}, 1] & \text{if } c \leq r_1 \\ [\frac{d}{B+D+d}, \frac{r_2+d-c}{d-r_1+r_2}] & \text{if } r_1 < c \leq \frac{(B+D)(r_2+d)+dr_1}{B+D+d} \\ \emptyset & \text{if } c > \frac{(B+D)(r_2+d)+dr_1}{B+D+d} \end{cases},$$

which is equivalent to $\alpha \in \mathcal{A}_{1c}^{e}$, where

$$\mathcal{A}_{1c}^{e} = \begin{cases} [\frac{d(1-\rho_e)}{d(1-\rho_e)+(B+D)\rho_e}, 1] & \text{if } c \leq r_1 \\ [\frac{d(1-\rho_e)}{d(1-\rho_e)+(B+D)\rho_e}, \frac{(c-r_2-d)(1-\rho_e)}{(c-r_2-d)(1-\rho_e)+(c-r_1)\rho_e}] & \text{if } r_1 < c \leq \frac{(B+D)(r_2+d)+dr_1}{B+D+d} \\ \emptyset & \text{if } c > \frac{(B+D)(r_2+d)+dr_1}{B+D+d}. \end{cases}$$

By incorporating condition (14), we have the following range of α in which the pooling equilibrium sustains:

$$\alpha \in \mathcal{A} \cap \mathcal{A}_{1b}^{h} \cap \mathcal{A}_{1b}^{l} = \begin{cases} [1-\rho_l, \rho_l] & \text{if } c \leq r_1 \\ [1-\rho_l, \frac{(c-r_2-d)(1-\rho_h)}{(c-r_2-d)(1-\rho_h)+(c-r_1)\rho_h}] & \text{if } r_1 < c \leq \frac{(B+D)(r_2+d)+dr_1}{B+D+d} \\ \emptyset & \text{if } c > \frac{(B+D)(r_2+d)+dr_1}{B+D+d} \end{cases} \tag{21}$$

We complete the proof of Lemma 4 using (19)–(21).

We now move on to the case in which the expert's private evaluation is negative ($s_e = 0$). By disclosing her private evaluation before subsequently conducting a test, she gains a utility of

$$\phi \left\{ b_e(\alpha|0)(B + r_2) + [1 - b_e(\alpha|0)]r_1 - c \right\}. \tag{22}$$

We represent the expert's utility by deviating from the diagnostic pathway specified in the pooling equilibrium as

$$\begin{aligned} &\phi \max\{b_e(\alpha|0) \cdot B + [1 - b_e(\alpha|0)] \cdot (-d), b_e(\alpha|0) \cdot (-D)\} \\ &= \begin{cases} \phi \left\{ b_e(\alpha|0) \cdot B + [1 - b_e(\alpha|0)] \cdot (-d) \right\}, & \text{if } b_e(\alpha|0) \geq \frac{d}{B+D+d}, \\ \phi \left[b_e(\alpha|0) \cdot (-D) \right] & \text{otherwise.} \end{cases} \end{aligned} \tag{23}$$

Comparing (22) with (23), we have the following two conditions, one of which has to hold for the pooling equilibrium to sustain:

$$b_e(\alpha|0) \geq \frac{d}{B+D+d} \text{ and } (r_2 - r_1 - d)b_e(\alpha|0) \geq c - r_1 - d, \tag{24}$$

and

$$\frac{c - r_1}{B + D + r_2 - r_1} \leq b_e(\alpha|0) \leq \frac{d}{B + D + d}. \tag{25}$$

The above conditions, along with condition (9), give the lemma that follows:

Lemma 5. *If $c \leq r_1$, for $\alpha \in [1 - \rho_l, \rho_l]$, in the presence of a negative private evaluation, neither type of expert has an incentive to deviate from the pooling equilibrium in which she discloses her private evaluation* *before subsequently conducting a test.*

Proof. In (24), clearly, $r_2 - r_1 - d < 0$. Thus, (24) is equivalent to

$$\frac{d}{B+D+d} \leq b_e(\alpha|0) \leq \frac{r_1+d-c}{d-r_1+r_2}. \tag{26}$$

For the pooling equilibrium to sustain, either (25) or (26) has to be satisfied, which is equivalent to $b_e(\alpha|0) \in \mathcal{B}_0$, where

$$\mathcal{B}_0 = \begin{cases} [\frac{c-r_1}{B+D+r_2-r_1}, 1] & \text{if } 0 < c \leq r_2 \\ [\frac{c-r_1}{B+D+r_2-r_1}, \frac{d+r_1-c}{d+r_1-r_2}] & \text{if } r_2 < c \leq \frac{(B+D)(r_1+d)+dr_2}{B+D+d} \\ \emptyset & \text{if } c > \frac{(B+D)(r_1+d)+dr_2}{B+D+d} \end{cases},$$

which is equivalent to $\alpha \in \mathcal{A}_0^e$, where

$$\mathcal{A}_0^e = \begin{cases} [\frac{(c-r_1)\rho_e}{(c-r_1)\rho_e+(B+D+r_2-c)(1-\rho_e)}, 1] & \text{if } c \leq r_1 \\ [\frac{(c-r_1)\rho_e}{(c-r_1)\rho_e+(B+D+r_2-c)(1-\rho_e)}, \frac{(d+r_1-c)\rho_e}{(d+r_1-c)\rho_e+(d-r_2+c)(1-\rho_e)}] & \text{if } r_1 < c \leq \frac{(B+D)(r_2+d)+dr_1}{B+D+d} \\ \emptyset & \text{if } c > \frac{(B+D)(r_2+d)+dr_1}{B+D+d} \end{cases}.$$

By incorporating condition (14), we have the following range of α in which the pooling equilibrium sustains:

$$\alpha \in \mathcal{A} \cap \mathcal{A}_0^h \cap \mathcal{A}_0^l = \begin{cases} [1-\rho_l, \rho_l] & \text{if } c \leq r_1 \\ [1-\rho_l, \frac{(d+r_1-c)\rho_l}{(d+r_1-c)\rho_l+(d-r_2+c)(1-\rho_l)}] & \text{if } r_1 < c \leq \frac{(B+D)(r_2+d)+dr_1}{B+D+d} \\ \emptyset & \text{if } c > \frac{(B+D)(r_2+d)+dr_1}{B+D+d}. \end{cases} \tag{27}$$

The above condition completes the proof of Lemma 5.

Finally, Proposition 6 is immediate from Lemmas 4 and 5. *Q.E.D.*

Proof of Corollary 1.

Note that $c < \min\{r_1, \frac{d}{2}\}$ means $w \leq \min\{r_1, \frac{r_1+r_2+d}{2}\}$, meaning the condition in Proposition 6 is satisfied. So the pooling equilibrium characterized therein sustains. In addition, $c < \min\{r_1, \frac{d}{2}\}$ means $c < \frac{d}{2} < \frac{B+D}{2}$, which, along with $\rho_h > \rho_l$, gives $[\underline{\alpha}_1^h, \bar{\alpha}_0^h] \subset [1-\rho_l, \rho_l]$. In other words, there exists some $\alpha \in [1-\rho_l, \rho_l]$ such that the type-h physician does not conduct a test in the full-information equilibrium but does in the pooling equilibrium, meaning over-testing exists as a result of pooling. *Q.E.D.*

Proof of Proposition 7.

Because the proof of results presented in this section proceed in the manner as those from the baseline model, we only list the main results here.

First, consider the full information case. Suppose the type e expert receives signal $s_e = 0$. The expert does not perform the diagnosis test ($t = 0$) and provides a positive diagnosis ($a = 1$) if $\alpha > \bar{\alpha}_0^e$; she performs no diagnosis test ($t = 0$) and provides a negative diagnosis ($a = 0$) if $\alpha < \underline{\alpha}_0^e$; and she performs the test ($t = 1$) otherwise. The cutoffs $\underline{\alpha}_0^e$ and $\bar{\alpha}_0^e$ are given by

$$\underline{\alpha}_0^e = \frac{\rho_t c(1-\delta)}{(1-\rho_t)(B+D)+(2\rho_t-1)c(1-\delta)}, \text{ and } \bar{\alpha}_0^e = \frac{\rho_t[d-c(1-\delta)]}{\rho_t d-c(1-\delta)(2\rho_t-1)}.$$

Similarly, if the expert receives a signal $s_e = 1$, the cutoffs $\underline{\alpha}_1^e$ and $\bar{\alpha}_1^e$ are given by

$$\underline{\alpha}_1^e = \frac{c(1-\delta)(1-\rho_t)}{\rho_t(B+D)-c(1-\delta)(2\rho_t-1)}, \text{ and } \bar{\alpha}_1^e = \frac{[d-c(1-\delta)](1-\rho_t)}{(1-\rho_t)d+c(1-\delta)(2\rho_t-1)}.$$

Next, consider the asymmetric information setup in which the client is not informed about the expert type. A separating equilibrium exists if and only if

$$\frac{1-\rho_h}{\rho_h} \leq \frac{r\omega + c(1-\delta)}{\sqrt{[B+D-r\omega-c(1-\delta)][d-r\omega-c(1-\delta)]}} \leq \frac{1-\rho_l}{\rho_l}.$$

The interval $[\underline{\alpha}^f, \bar{\alpha}^f]$ for which the separating equilibrium exists is described by

$$\{\underline{\alpha}^f, \bar{\alpha}^f\} = \begin{cases} \left\{ \frac{(1-\rho_h)[d-r\omega-c(1-\delta)]}{\rho_h[r\omega+c(1-\delta)]+(1-\rho_h)[d-r\omega-c(1-\delta)]}, \frac{\rho_h[r\omega+c(1-\delta)]}{\rho_h[r\omega+c(1-\delta)]+(1-\rho_h)[B+D-r\omega-c(1-\delta)]} \right\}, & if\, B > \tilde{B}_f \\ \left\{ \frac{\rho_l[r\omega+c(1-\delta)]}{\rho_l[r\omega+c(1-\delta)]+(1-\rho_l)[B+D-r\omega-c(1-\delta)]}, \frac{(1-\rho_l)[d-r\omega-c(1-\delta)]}{\rho_l[r\omega+c(1-\delta)]+(1-\rho_l)[d-r\omega-c(1-\delta)]} \right\}, & otherwise \end{cases}$$

where $\tilde{B}_f \triangleq r\omega + c(1-\delta) - D + \frac{\rho_h\rho_l[r\omega+c(1-\delta)]^2}{(1-\rho_h)(1-\rho_l)[d-r\omega-c(1-\delta)]}$. It is straightforward to show that $\frac{\partial}{\partial\delta}\left(\bar{\alpha}^f - \underline{\alpha}^f\right) < 0$ if $B > \tilde{B}_f$, and $\frac{\partial}{\partial\delta}\left(\bar{\alpha}^f - \underline{\alpha}^f\right) > 0$ if $B \leq \tilde{B}_f$. Also, note that $\frac{\partial}{\partial\delta}\tilde{B}_f < 0$. Therefore, if we increase δ from some $\delta_l \geq 0$ to $\delta_h = \delta_l + \epsilon$, there exists some $\hat{B}_f \in \left(\tilde{B}_f|_{\delta=\delta_l}, \tilde{B}_f|_{\delta=\delta_h}\right)$ such that $[\underline{\alpha}^f|_{\delta=\delta_h}, \bar{\alpha}^f|_{\delta=\delta_h}] \subset [\underline{\alpha}^f|_{\delta=\delta_l}, \bar{\alpha}^f|_{\delta=\delta_l}]$ if $B > \hat{B}_f$ and $[\underline{\alpha}^f|_{\delta=\delta_h}, \bar{\alpha}^f|_{\delta=\delta_h}] \supset [\underline{\alpha}^f|_{\delta=\delta_l}, \bar{\alpha}^f|_{\delta=\delta_l}]$ if $B \leq \hat{B}_f$. The range $\triangle\alpha^f$ for which the separating equilibrium exists shrinks if $B > \hat{B}_f$ and expands if $B \leq \hat{B}_f$. *Q.E.D.*

***Proof of the Result about Insurance Coverage in Section* 6.1.**

Recall from Proposition 3 that if $\underline{B} \leq B \leq \hat{B}$, the signaling range is $[\underline{\alpha}, \bar{\alpha}]$, where $\underline{\alpha} = \frac{1}{1+\frac{1-\rho_l}{\rho_l}\cdot\frac{B+D-r\omega-c}{r\omega+c}}$ increases in c, and $\bar{\alpha} = \frac{1}{\frac{\rho_l}{1-\rho_l}\cdot\frac{r\omega+c}{d-r\omega-c}+1}$ decreases in c. Therefore, a reduction in c can lead to a wide signaling range and allude to more salient under-testing behavior. *Q.E.D.*

***Proof of the Result about Malpractice Concerns in Section* 6.2.**

We can draw from Proposition 3 that if $\underline{B} \leq B \leq \hat{B}$, the type-$h$ expert's signaling range is $[\underline{\alpha}, \bar{\alpha}]$, where $\underline{\alpha} = \frac{1}{1+\frac{1-\rho_l}{\rho_l}\cdot\frac{B+D-r\omega-\hat{c}}{r\omega+\hat{c}}} < \frac{1}{1+\frac{1-\rho_l}{\rho_l}\cdot\frac{B+D-r\omega-c}{r\omega+c}}$ and $\bar{\alpha} = \frac{1}{\frac{\rho_l}{1-\rho_l}\cdot\frac{r\omega+\hat{c}}{d-r\omega-\hat{c}}+1} > \frac{1}{\frac{\rho_l}{1-\rho_l}\cdot\frac{r\omega+c}{d-r\omega-c}+1}$. Therefore, a reduction in the cost of diagnostic testing from c to $\hat{c}$ can lead to a wide signaling range. *Q.E.D.*

***Proof of Results in Section* 6.3.**

Using a procedure similar to the proof of Proposition 1, we can show that an overconfident type-h expert chooses the following diagnostic-testing policy: if the expert receives a signal of $s_h = 0, 1$, she provides a positive diagnosis without performing the test if $\alpha > \bar{\alpha}_{s_h}(\hat{\rho}_h)$, performs the test if $\underline{\alpha}_{s_h}(\hat{\rho}_h) < \alpha \leq \bar{\alpha}_{s_h}(\hat{\rho}_h)$, and provides a negative diagnosis without performing the test if $\alpha \leq \underline{\alpha}_{s_h}(\hat{\rho}_h)$.

A self-aware type-h expert, by comparison, chooses a similar diagnostic-testing policy but with different decision thresholds: if the expert receives a signal of $s_h = 0, 1$, the expert provides a positive diagnosis without performing the test if $\alpha > \bar{\alpha}_{s_h}(\rho_h)$, performs the test if $\underline{\alpha}_{s_h}(\rho_h) < \alpha \leq \bar{\alpha}_{s_h}(\rho_h)$, and provides a negative diagnosis without performing the test if $\alpha \leq \underline{\alpha}_{s_h}(\rho_h)$.

We do not consider the trivial case in which the expert suffers from an extreme level of overconfidence such that the expert misses all the necessary tests. By comparing the diagnostic-testing policies across the overconfident expert and the self-aware expert, we identify the following two scenarios in which the overconfident expert misses necessary diagnostic tests: (i) in the case in which the private signal is positive (i.e., $s_h = 1$), because $\bar{\alpha}_1(\rho)$ is a decreasing function, we have $\bar{\alpha}_1(\hat{\rho}_h) < \bar{\alpha}_1(\rho_h)$, indicating the overconfident expert misses tests for clients with α satisfying $\bar{\alpha}_1(\hat{\rho}_h) \leq \alpha < \bar{\alpha}_1(\rho_h)$; (ii) in the case in which the private signal is negative (i.e., $s_h = 0$), because $\underline{\alpha}_0(\rho)$ is an increasing function, we have $\underline{\alpha}_0(\hat{\rho}_h) > \underline{\alpha}_0(\rho_h)$, indicating the overconfident expert misses tests for clients with α satisfying $\underline{\alpha}_0(\rho_h) < \alpha \leq \underline{\alpha}_0(\hat{\rho}_h)$. *Q.E.D.*