A Game Between a Terrorist and a Passive Defender

Opher Baron[*], Oded Berman, Arieh Gavious[†]

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Abstract

In the last two decades, terrorism has become a major issue around the world. We analyze a conflict between a terrorist (Terrorist) and a passive defender (Defender) using a simple game theoretical model. Defender is passive as her actions can only influence the costs (damages) when Terrorist attacks. We consider single and multi period games. In each period, Terrorist may attack Defender who may try to prevent damage. The games take into account the available technologies for Terrorist and his potential learning. Based on the equilibrium in these games, we make several conjectures related to political events that may change the level of violence, the technology used by Terrorist, and possible cease-fire agreements. We use three sources of data on the conflict between Israel, as Defender, and terrorist groups from the Gaza strip, as Terrorist. Based on these data, we estimate parameters for the models and present numerical examples. We show that for this conflict's political situation our data do not reject our conjectures. In addition, our conjectures provide insights into long term conflicts in general.

Keywords: Game Theory, Anti-Terrorism.

1 Introduction

In the last two decades, terrorism has become a major threat in many countries. Terrorist attacks have a devastating effect, damaging property and causing human suffering including decrease moral, injuries, and deaths. We distinguish between two types of terrorism. In one type, terrorists focus their efforts on major sporadic attacks, such as the attack on the USA on September 11, 2001, that killed nearly 3000 people and injured thousands more, or the bombing of four trains in Madrid on March 11, 2004, when 191 people were killed and nearly 1800 were injured. In the other type of terrorism, terrorists focus their efforts on "continuous" (e.g., daily) attacks, such as the first Chechen war where Russia fought Chechen guerillas during 1994–1996, or the mortar shell and missile attacks on Israel since 2001.

In this paper, we consider the latter — continuous conflict. When countries face a continuous conflict, they can fight back by making active attacks on the terrorists, their infrastructure, and resources, or by passive means, such as providing the population with tools to defend themselves (such as shelters) or trying to prevent specific attacks. In practice, countries often use both approaches. Keohane and Zeckhauser (2003) use the terminology "stocks" and "flows" to refer to passive and active approaches, respectively. Rubin (2011) uses similar terminology: "active defence" for actions aiming to destroy the enemy's force before it is applied and "passive defence" for

[*]Rotman School of Management, University of Toronto, 105 St. George Street, Toronto, Ontario, Canada M5S 3E6.

[†]Department of Industrial Engineering and Management, Faculty of Engineering Sciences, Ben-Gurion University, P.O. Box 653, Beer-Sheva 84105, Israel.

the provision of shelters and alarm systems or evacuation of population centers. We develop stylized and simple models for a defender that uses preventive actions, such as the deployment of the Iron Dome in Israel. To this end we consider several games pitting terrorists (hereafter called Terrorist) against defenders (hereafter called Defender), present the possible resulting Nash equilibriums, and discuss the implication of these equilibriums. We assume that a successful preventive action by Defender may prevent damage caused by Terrorist's attack and may even damage Terrorist, but Defender is passive because her actions will have no effect if Terrorist does not attack. We also discuss extensions to include active Defender.

One objective of this paper is to investigate whether simple, stylized models can improve our understanding of very complex, continuous conflicts, such as the one between Israel and the Palestinians in the Gaza strip or between Russia and Chechen guerillas. Our models have obvious shortcomings, such as ignoring exogenous, external, and internal factors (e.g., internal politics and public opinion). While it is clear that such generations old conflicts cannot be resolved and or completely understood using easily tractable models, using data from the Israeli Palestinian conflict, we show that such models shed some light on several aspects of continuous conflicts.

As the basis for our analysis, the first and simplest game presented is a single period game where Terrorist may attack a Defender using a single technology (e.g., firing mortar shells in the Israeli-Palestinian conflict). In this case Terrorist has only two possible actions: Attack or Not Attack. Defender also has only two possible actions: Respond (e.g., try to intercept a missile) or Not Respond. (We are considering simultaneous games but prefer to use the term Respond to imply Defender is taking a preventive action and Not Respond to imply she is taking no action.)

In continuous conflicts occurring over years, Terrorist and Defender technologies may improve. Therefore, to understand such conflicts it is important to consider the effect of different technologies on the games' results. Therefore, the second game is identical to the first except Terrorist can use two technologies to attack (e.g., Mortar shells and Qassam missiles in the Israeli-Palestinian conflict). As we do not wish to focus on the arms race, we look at a single Defender's technology, but allow its efficacy to vary as discussed below.

Of course, as we aim at understanding continuous conflicts, we also need to consider multi-period games. While upgrading technologies, as in the second game, captures an important aspect of long term conflicts, other changes may also occur. In our example, in the short term, i.e., between each two periods, Terrorist attack cost may change as a result of learning from past experience. Therefore, the third game has a finite number of repetitions of the first game where Terrorist attack cost in a period changes based on the realizations in the previous period. More specifically, this cost decreases as a result of learning after Terrorist's attacks, or increases as a result of learning following a successful response by Defender. Finally, because many continuous conflicts appear to have no end in sight, it is important to consider infinite periods games. Accordingly, the last game presents an infinite repetition of the first game (possibly with several technologies).

To demonstrate the games we use the conflict between Israel as Defender and various terrorist groups from the Gaza strip as Terrorist. Starting in 2001 the terrorists in the Gaza strip have used mortar shells to attack Israel. In 2002, they added Qassam missiles (over several generations), and in 2006, they started launching Grad missiles. During the years covered by IICC (2007) there were almost 5000 attacks recorded, on average, about 2.5 attacks per day. In 2007 alone, they were 1423 attacks representing about 3.9 attacks per day. Clearly attacks by terrorists and Israel's responses to these attacks have significantly affected the life of thousands of people on both sides.

We use three data sets to improve our understanding of the conflict and to calibrate our model's parameters.

The first data set is based on IICC (2007), the second includes data we collected for the November 5, 2008, to January 17, 2009, and the third comes from the Israeli security agency, Shabak (2010). We discuss these data sets in Section 6.2. Depending on the games' parameters, several possible equilibriums can be derived using pure or mixed strategies. As discussed below, the relevant equilibriums in this study are often mixed, and in our theoretical analysis we discuss possible effects of such equilibrium. Based on this analysis we make several conjectures about the nature of a conflict:

- The level of violence in different periods: The level of violence is likely to change with a change in the leadership of Terrorist and Defender regimes because a change of leadership may affect the players' payoffs. This conjecture is supported by data; while this finding is not surprising, it provides some evidence that our models do not lead to counter intuitive or unreasonable results.

- The sustainability of cease-fire agreements: When the political situation is unstable, there is no single authority to ensure a cease-fire; thus, agreements during such periods are not practical. Our theory suggests that even when the political atmosphere is stable, cease-fire agreements are not likely to hold for long. This conjecture ignores payoffs that are exogenous to the model. The logic behind this conjecture is that Terrorist is not likely to maintain a cease-fire in the absence of exogenous payoffs. In other words, short term payoffs to Terrorist will still not result in a sustainable cease fire. Therefore, we recommend cease-fire discussions in long term conflicts focus on long lasting payoffs.

- The number of technologies used by Terrorist in different periods: When the political atmospheres is stable, several technologies could be used by Terrorist, but only a single technology will be used when the political atmospheres is unstable (even if several technologies are available to Terrorist).

We also consider situations where Defender's response can increase Terrorist's attack cost to a level where further attacks do not benefit him. In a multi-period game with such limitations, there is a potential for a quiet equilibrium without violence. Based on our numerical results, we see that the possibility of reaching a quiet period decreases as Terrorist's technology becomes more effective (i.e., causes more damage).

In view of the political situation in the Gaza strip and Israel, we investigate whether our data support these conjectures and find insufficient evidence to reject them, thus supporting our contention that our models shed some light on several aspects of continuous conflicts. We believe the second conjecture on the sustainability of the cease fire agreements is the most important while the conjecture on the number of technologies is the most surprising. Models similar to the ones presented here could guide political decisions, such as peace negotiations. We, of course, do not claim these models should be the sole factor guiding such decisions.

The paper continues as follows. In Section 2 we review the relevant literature. In Section 3 we present and analyze a single period single technology model. In Section 4, we consider a single period model with two technologies. We extend the model used in Section 3 by considering a multi period game with and without Terrorist learning in Section 5. We discuss our conjectures and their correspondence with the data in view of numerical results in Section 6. We summarize the paper and suggest extensions in Section 7. A detailed discussion of the parameters' estimation is provided in Appendix A, and all proofs appear in Appendix B.

2 Literature Review

In recent years, the amount of literature addressing the fight against terrorism has burgeoned. One stream addresses the logistic problem of resource allocation faced by governments defending themselves against terrorism while another concentrates on the strategic aspects of the fight against terrorism. Sandler and Siqueira (2009) review some of the research applying game theory published in this field and classify the literature into seven categories. Enders and Sandler (2012) offer a broad discussion of theoretical aspects of terrorism models and empirical data. Below, we review only those articles especially relevant to our study.

The common assumption in research considering the problem of resource allocation is that Defender (the government), who is passive, allocates resources before an attack takes place and does not take any further actions to stop Terrorist (the terrorist group) before the attack. When initiating an attack, Terrorist may be informed about Defender's allocation of resources or he may be unaware. There are two cases studies with such assumptions.

The first case, when Terrorist is aware of Defender's decision, results in a two-stage leader-follower game (Stackelberg game). This two-stage game studied by e.g., Berman and Gavious (2007) is solved by backward induction and yields a sub-game perfect Nash equilibrium. Brown et al. (2006) consider two- and three-level actions where in the three-level setting, Defender invests in protecting some of his resources against attack; then, Terrorist is informed about Defender's actions and chooses which of the resources to attack; in the last stage, Defender decides how to use the remaining resources optimally. Brown et al. (2006) demonstrate their model on the Louisiana petroleum infrastructure. Powell (2007), Zhuamg and Bier (2007) and Farrow (2007) consider a leader-follower setting with preventive action taken by Defender.

In the second case, when Terrorist is unaware of the Defender's decision, the setting is a simultaneous game usually solved for a mixed strategies Nash equilibrium, such as in a recent study by Berman, Gavious, and Huang (2011). Early studies consider a simultaneous move game as presented by Dresher (1956) and generalized by Karlin (1959) and Cohen (1966). In these studies, the authors investigated a simultaneous zero-sum game where both Defender and Terrorist simultaneously allocate a variety of resources in different locations. Note that the typical solution of simultaneous games between Terrorist and Defender is a mixed strategies equilibrium since Defender conceals her decision by applying a mixed strategy, forcing Terrorist to randomize as well. If the decision made by Defender is not to randomize, Terrorist will react optimally against Defender's decision (in the sense of best response strategy) and will attack Defender's weakest spot. Dresher (1956) shows that the equilibrium features mixed strategies and demonstrates the "No Soft-Spot principle" whereby in equilibrium, Terrorist will concentrate his resources on a single target selected randomly while Defender will split her resources among all targets considered by Terrorist. However, in the Dresher (1956) setting, in equilibrium, only Terrorist uses mixed strategies while Defender has a pure strategy. In our paper, we consider a passive Defender whose actions may prevent all damage from an attack. The solution is usually a mixed strategy as in the simultaneous resource allocation setting, Defender responds to Terrorist's attacks as they occur, and we ignore the prior stage of resource allocation.

Another branch of research presents models with an active Defender who invests resources to prevent an attack or reduce its damage. Hausken (2008) suggests a two period simultaneous model. In the first period Defender attacks Terrorists' resources aiming to reduce them and Terrorists protects his resources. In the second period, Terrorist attacks Defender who protects her infrastructure. Both players make efforts simultaneously, generating pure strategies equilibrium. In contrast, we consider single and multi period models where the decisions are binary for both players: attack-don't attack for Terrorist and respond–don't respond for Defender. This yields mixed strategies equilibrium even in the single period model. Poveda and Tauman (2011) and a generalization by Bandyopadhyay and Sandler (2011) offer a model similar to Hausken (2008). Poveda and Tauman (2011) consider many governments (Defenders) all investing resources to fight a Terrorist. The multinational setting allows them to study international conflict between nations. In the first stage, some nations invest resources to reduce Terrorist's resources. In the second stage, all nations invest resources to defend themselves. In the last stage, Terrorist attacks by allocating his surviving resources among nations. We, in contrast, consider a conflict that includes a single Terrorist (terrorist group) and a single Defender (nation). Kaplan et al. (2010) use Lanchester models of imperfect intelligence when considering how Defender's action aimed at Terrorist strongholds can harm civil population on both sides of the conflict .

The next stream of the literature addresses the balance between Terrorists' attack and Defenders' counter measures where the dynamics of the process are the main interest. An early study by Brophy-Baermann and Conybeare (1994) looks at these in a non-game theoretical framework known as rational expectations. This approach assumes that since Terrorist is rational, his expectation about Defender's action is taken into account when he chooses his action. As a result, Terrorist already discounts any future retaliation actions. We, in contrast, use game theory, providing insights into why players act as they do.

Jacobson and Kaplan (2007) study multi period interactions between Defender who considers targeted killings and Terrorist who considers a suicide bombings. Both players have a continuous set of strategies where, at every period, Terrorist first decides on the number of suicide bombings; Defender is then informed about Terrorist's decision and decides how many targeted killings to launch. The outcome of every period affects Terrorist's resources in the next period, and the resulting equilibrium is one of pure strategies. We, in contrast, focus on a passive Defender response that may only affect Terrorist when he attacks.

The literature relevant for the theory of repeated games is broad and we note several known results in Section 5.2.2. In a repeated game, there are many possible equilibriums. Thus, we prefer to concentrate on specific equilibriums that have an appealing structure that leads to some insight. This is similar to De Mesquita (2005) who considers an infinitely repeated game with two Terrorist (two terrorist groups) and one Defender. In that model, the decision made by Defender at every period inclues a concession to Terrorists, along with the amount of resources invested in the fight against terrorism. Each terrorist (group) decides whether to accept the concession (and stop terrorism forever) or refuse the concession and invest in terrorism. If either Terrorist decides to accept the concession, he then decides whether to help Defender in her fight against the other Terrorist.

The literature on anti terrorism involving asymmetric information traditionally deals with signaling as an action by Terrorist to signal about his strength or resources (see, for example, Lapan and Sandler 1993 and Arce and Sandler 2007). Our model does not consider intelligence.

3 The Single Period Single Technology Model and its Analysis

Consider a Terrorist who can attack a Defender using a specific technology. Terrorist's actions are "Attack" and "Not Attack" denoted by $A^T \in \{A, NA\}$. The probability that an attack will hit the target (without interference from Defender) is P. Defender's actions are "Respond" or "Not Respond" denoted by $A^D \in \{R, NR\}$, respectively. Recall that we use the former term respond to imply that Defender takes a preventive action. (We prefer "R"

Terrorist's action	Defender's action	Outcome	Probability	Terrorist's payoff	Defender's payoff (cost)
NA	R	Terrorist does not shoot	1	0	$-c^D$
NA	NR	Terrorist does not shoot	1	0	0
A	NR	Terrorist shoots and hits	P	$-c^T + K^T$	$-K^D$
A	NR	Terrorist shoots and misses	$1 - P$	$-c^T$	0
A	R	Both shoot Terrorist hits	$P(1 - Q)$	$-c^T + K^T$	$-c^D - K^D$
A	R	Both shoot both miss	$(1 - Q)(1 - P)$	$-c^T$	$-c^D$
A	R	Both shoot Defender hits	Q	$-c^T - K^S$	$-c^D$

Table 1: Single period single technology game: outcomes, probabilities and payoffs given different actions.

$T \backslash D$	NR	R
NA	$0, 0$	$0, -c^D$
A	$-c^T + K^T P, -K^D P$	$-c^T + K^T P(1 - Q) - K^S Q, -c^D - K^D P(1 - Q)$

Table 2: Game table for the single person single technology model.

for respond rather than "P" for prevent to avoid confusion with the standard usage of "P" for probability.) We consider a passive Defender whose response is to try to intercept an attack. The probability that Defender will intercept an attack (if she responds) is Q. We assume a successful interception is not correlated with the (potential) success of the attack. The game proceeds as follows. Terrorist decides whether to attack a Defender's target and Defender decides whether to respond. We assume both actions are performed simultaneously, and immediately after decisions are made, the success and failure of the actions are determind resulting in some payoffs (possibly negative ones) for both players. Here and in the sequel, we denote quantities related to Terrorist and Defender with a superscript, T and D, respectively. As is common in the literature, we assume Terrorist and Defender are both rational and risk neutral.

Let the cost of action A for Terrorist be $c^T > 0$ and the cost of action R for Defender be $c^D > 0$. Let $K^T > 0$ be the benefit for Terrorist if he hits the target, and $K^D > 0$ be the cost for Defender when the target is hit. We also let $K^S \geq 0$ denote the cost for Terrorist when Defenders makes a successful interception.

Note that while c^D and c^T are easily related to dollar amounts the other costs K^D, K^T, and K^S may be related to casualties. We discuss translation of casualties to dollars in Appendix A.

3.1 Solution of the Single Period Game

With the description above we can find for each pair of actions the probability of different outcomes of the game and the resulting expected payoffs. The possible outcomes and payoffs are summarized in Table 1. These possible outcomes lead to the single period table game depicted in Table 2, where we use R^T and R^D to denote Terrorist's and Defender's expected payoffs, respectively. The solution of this game is given in Proposition 1 below.

Proposition 1 *Part a – pure equilibrium strategies:*

1. If $K^T P - c^T \leq 0$, then (NA, NR) is an NE leading to payoffs $\left(R^T, R^D\right) = (0, 0)$.

2. If $K^T P - c^T \geq 0$ and $K^D PQ - c^D \leq 0$, then (A, NR) is an NE with payoffs $\left(R^T, R^D\right) = (K^T P - c^T, -K^D P)$.

3. If $K^D PQ - c^D \geq 0$ and $K^T P(1 - Q) - K^S Q - c^T \geq 0$, then (A, R) is an NE with payoffs $(R^T, R^D) = (K^T P(1 - Q) - c^T - K^S Q, -K^D P(1 - Q) - c^D)$.

Part b – mixed equilibrium strategies:

If $0 < K^T P - c^T < Q\left(K^T P + K^S\right)$ and $K^D PQ - c^D > 0$, the mixed strategies $\left((q_0^T, 1 - q_0^T), (q_0^D, 1 - q_0^D)\right)$ given by

$$\left(\left(\frac{PK^D Q - c^D}{PK^D Q}, \frac{c^D}{PK^D Q}\right), \left(\frac{c^T - K^T P + Q\left(K^T P + K^S\right)}{Q\left(K^T P + K^S\right)}, \frac{K^T P - c^T}{Q\left(K^T P + K^S\right)}\right)\right) \tag{1}$$

consist a unique NE, leading to expected payoffs $\left(R^T, R^D\right) = \left(0, \frac{-c^D}{Q}\right)$.

Recalling that Terrorist and Defender are both rational and risk neutral, the different pure equilibrium strategies in part a) seem uninteresting. Specifically, in a1) when $K^T P - c^T \leq 0$, Terrorist's attack cost is higher than Terrorist's expected gain from such an attack, giving Terrorist no incentive to attack; in a2) when $K^D PQ - c^D \leq 0$, Defender response cost is higher than the expected cost savings from a successful response, giving Defender no incentive to respond. Such a model is reasonable at the start of terror campaigns when Terrorist has the "first strike" advantage or when he introduces a new technology, and in a3) when $-c^T + K^T P(1 - Q) - K^S Q \geq 0$, Terrorist's expected payoff from an attack is positive even if Defender responds, so Terrorist will always attack. While some of the above models may be reasonable in practice, from a theoretical point of view, we learn little from these models in these settings. Accordingly, the mixed strategy equilibrium is the sensible equilibrium in our model. (We discuss the uniqueness of the different equilibriums from Proposition 1 after its proof in Appendix B.) Therefore, in the rest of the paper we assume the following.

Assumptions:

$$\begin{array}{rcl} K^D PQ & > & c^D, \text{ and} \\ K^T P - c^T - Q\left(K^T P + K^S\right) & < 0 < & K^T P - c^T. \end{array} \tag{A.1}$$

With the above assumptions, only the mixed strategy NE is used and the expected payoff of Terrorist is always 0. This 0 represents the value Terrorist has if he selects the option of not acting. This payoff follows whenever Defender is passive, because Terrorist can always choose NA and guarantee a 0 payoff. Furthermore, the expected payoff of Defender is always nonpositive and only depends on c^D and Q. The expected payoff of both parties is independent of Terrorist payoff characteristics, such as K^T, c^T, and P. An important implication is that the payoffs of both parties are independent of Terrorist's technology.

It turns out that the probability of Defender to respond in the mixed NE is important in our analysis below. Thus, to simplify the exposition, we define

$$G := \frac{K^T P - c^T}{Q\left(K^T P + K^S\right)} = \Pr\left(Defender\ respond\right), \tag{2}$$

G is the ratio between Terrorist's payoff when he attacks and Defender does not respond to the Terrorist expected payoff reduction (from $K^T P - c^T$) when Defender does respond. Note that G depends on several parameters, and we will emphasize this dependency as necessary; e.g., when discussing changes in Terrorist's attack cost we will use $G\left(c^T\right)$ and when discussing technology i, we will use G^i.

$T\backslash D$	NR	R
NA	$0, 0$	$0, -c^D$
$A1$	$-c^{T1} + K^{T1}P^1, -K^{D1}P^1$	$-c^{T1} + K^{T1}P^1\left(1 - Q^1\right) - K^{S1}Q^1, -c^D - K^{D1}P^1\left(1 - Q^1\right)$
$A2$	$-c^{T2} + K^{T2}P^2, -K^{D2}P^2$	$-c^{T2} + K^{T2}P^2\left(1 - Q^2\right) - K^{S2}Q^2, -c^D - K^{D2}P^2\left(1 - Q^2\right)$

Table 3: Game table for the single period 2 technologies model.

4 The Single Period Two Technologies Model and its Analysis

Let A_i, $i = 1, 2$, denote Terrorist's decision to attack using technology i. In our model, Terrorist's strategy is captured by the values of c_i^T, P^i, Q^i, $K^{\mathrm{D}i}$, K^{Ti}, and K^{Si}. This leads to the table game in Table 3. In the context of the Israeli-Palestinian conflict, a new technology may be a rocket with a longer range and (i) is more expensive to launch, as is captured by $c^{T2} > c^{T1}$, (ii) has different precision, as captured via P^i, (a χ^2 test for the accuracy of different technologies, based on the data reported in Section 6.2, suggests that these differ) (iii) may have a different chance of being captured by Defender's response, as captured by Q^i, (iv) can cause more damage because it can reach more densely populated areas or has more explosive material, as captured by $K^{D2} > K^{D1}$ and $K^{T2} > K^{T1}$, and (v) may differ in damage as a result of Defender's successful response as captured by $K^{S1} \neq K^{S2}$.

Note that c^D is most likely independent of the technology used by Terrorist, because preventing damage may require the same amount of resources independently of the attacking technology.

We assume each technology satisfies Assumption (A.1), e.g., that $K^{Di}P^iQ^i > c^D$ for $i = 1, 2$. Then, the solution of the game in Table 3 is

Proposition 2 *(a) If*

$$G^2 < G^1, \tag{3}$$

the mixed strategies $\left((q_0^T, q_1^T, q_2^T), (q_0^D, 1 - q_0^D)\right)$ *given by*

$$\left(\left(\frac{P^1K^{D1}Q^1 - c^D}{P^1K^{D1}Q^1}, \frac{c^D}{P^1K^{D1}Q^1}, 0\right), \left(1 - G^1, G^1\right)\right)$$

consist a unique NE, leading to expected payoffs $\left(R^T, R^D\right) = \left(0, \frac{-c^D}{Q^1}\right)$*; if the inequality in (3) is in the other direction, the unique NE is a similar mixed strategy where Terrorist mixes NA and $A2$.*
(b) In the case when the two ratios in (3) are equal there are infinitely many NEs where Terrorist mixes all 3 strategies.

Remark 1 *The use of a single technology even when several technologies are available is consistent with an arms race in a continuous conflict. In such a race, once Defender's technology is effective in limiting the damage caused by a particular technology, Terrorist tries to develop or use a more effective one. See the discussion in the preface of (NRC 2007).*

Remark 2 *The surprising result of Proposition 2 is that even when Terrorist has several non-dominating technologies, he will use only the one (i.e., Terrorist does not mix attacks with both technologies) that causes Defender to respond with a higher probability. Therefore, Terrorist's action increases the level of violence used by Defender.*

We discuss the implication of this result on the equilibrium in a real conflict in Section 8.

Example: changes in c_{n+1}^T				
Terrorist's action	Defender's action	Result	c_{n+1}^T	c_{n+1}^T
NA	R/NR	Terrorist did not shot	c_n^T	c_n^T
A	NR	Terrorist shot (hits or misses)	$f_{-1}(\cdot)$	θc_n^T
A	R	Both shot, Defender hits (with probability Q)	$f_1(\cdot)$	γc_n^T
A	R	Both shot, Defender misses (with probability $1-Q$)	$f_{-1}(\cdot)$	θc_n^T

Table 4: Terrorist's attack cost changes from learning.

5 Multi period Models with Terrorist Learning

5.1 Single Technology Game: Model and Analysis

We denote the time of the first period by $n = 1$ and consider a game over several periods $n = 1, 2, ..., N$, where $N < \infty$. We ignore a discount factor but similar results hold if such a factor exists. Here we consider a case where the table game depicted in Table 2 is played at each period and the costs c_n^T are changed as explained next. We denote the expected payoff for an n period game for Terrorist and Defender by R_n^T and R_n^D respectively.

We assume that if Terrorist does not attack in period n, his next period attacking cost is given by $c_{n+1}^T = c_n^T$. Similarly, with every successful attack, without interference from Defender, Terrorist advances on the learning curve and reduces his attacking cost in the next period; that is, with every attack, $c_{n+1}^T = f_{-1}\left(c_n^T\right)$, where $f_{-1}\left(c_n^T\right) \leq c_n^T$ represents the cost reduction due to the experience gained by Terrorist. To simplify the exposition, we assume the same cost reduction in cases when the attack succeeds despite Defender's response. Finally, upon a successful interception by Defender, we assume $c_{n+1}^T = f_1\left(c_n^T\right)$, where $f_1\left(c_n^T\right) \geq c_n^T$. This models an increase in Terrorist's attack cost due to loss of experience (e.g., due to loss of trained people). To summarize, we assume,

$$f_{-1}(c) \leq c \leq f_1(c) \quad \forall c \in (0, \infty). \tag{4}$$

For example, the $f_{-1}\left(c^T\right) \leq c^T \leq f_1\left(c^T\right)$ functions may represent multiplicative learning. If Terrorist does not attack, his next period attacking cost remains the same; i.e., $c_{n+1}^T = c_n^T$; if an attack by Terrorist fails due to a successful response by Defender, we let $c_{n+1}^T = f_1\left(c_n^T\right) = \gamma c_n^T$, where $\gamma > 1$; in all other cases, we let $c_{n+1}^T = f_{-1}\left(c_n^T\right) = \theta c_n^T$, with $\theta < 1$. The multiplicative learning example with $\gamma = 1/\theta$ occurs when an experience period is lost upon Defender's successful interception. The $f_i\left(c^T\right)$ functions for the example are given in Table 4.

In the n^{th} period, we let q_{0n}^T and q_{0n}^D denote the probability that Terrorist and Defender do not attack or respond. The different NE solutions for the two periods game are given in Proposition 3, as follows:

Proposition 3 *The two periods game with Terrorist attacking cost updated as with $f_i(\cdot)$, $i = -1, 1$ satisfying (4), has the following Nash equilibriums:*
(i) If $K^T P \geq f_1\left(c_1^T\right)$ the players will employ the mixed strategy from (1) in both periods. This leads to expected payoffs $\left(R_2^T, R_2^D\right) = \left(0, -2c^D/Q\right)$.

(ii) If $f_1\left(c_1^T\right) > K^T P \geq c_1^T$: in period 1, the players use the mixed strategy $\left((q_1^T, 1-q_1^T), (q_1^D, 1-q_1^D)\right)$ given by

$$\left(\left(\frac{PK^DQ}{PK^DQ+c^D}, \frac{c^D}{PK^DQ+c^D}\right), \left(1-G\left(c_1^T\right), G\left(c_1^T\right)\right)\right) \tag{5}$$

and in period 2 they employ the strategies given in Proposition 1 when c^T is replaced by the resulting c_1^T. This leads to expected payoffs $\left(R_2^T, R_2^D\right) = \left(0, -\frac{c^D\left(2PK^DQ+c^D\right)}{(c^D+PK^DQ)Q}\right)$.
(iii) If $c_1^T > K^T P$ both players follow the (NA, NR) strategy in both periods leading to payoffs $\left(R_2^T, R_2^D\right) = (0,0)$.

Note that the expected payoffs for Defender in case (ii) is given by $-c^D/Q*\left(1+q_1^T\right) = -c^D/Q*\left(2-\left(1-q_1^T\right)\right)$. The right hand side of this expression means that as the probability of Terrorist's attack in period $n=1$ increases, Defender's expected cost decreases. The only way for Defender not to pay in (expectation) $-c^D/Q$ twice is if in the second period, the NE results in the outcome $(0,0)$. In case (ii) this can only happen if Terrorist attacks in the first period. Thus, the expected payoff for Defender is decreasing in q_1^T.

We can make several interesting corollaries based on the solution for the two periods game for a general N period game. For this we define recursively for $k = 1, 2, ...$the k^{th} changes caused by the $f_i(c)$ function $f_i^k(c) = f_i\left(f_i^{k-1}(c)\right)$, where $f_i^1(c) = f_i(c)$. Now consider the N period game where the payoff in the n^{th} period is given with c_n^T replacing c^T in Table 2 and c_n^T Terrorist's attacking cost are updated with $f_i(\cdot)$, $i = -1, 1$ satisfying (4) (i.e., there are no external changes for c^T or other parameters). This yields the following:

Corollary 1 *For N period game with learning we have*
(a) If for some $n \in [1, ..., N]$, we have $K^T P \geq f_1^{N-n}\left(c_n^T\right)$, the solution for any period $n, ..., N$ is the mixed NE strategy given in part (b) of Proposition 1. Then, Defender's expected payoff is $R_n^D = -(N-n+1)\,c^D/Q$. Specifically, if $K^T P \geq f_1^{N-1}\left(c_1^T\right)$ the players will use this mixed strategy during all N periods, resulting in $R_N^D = -Nc^D/Q$.
(b) If for some $n \in [1, ..., N]$, we have $K^T P - c_n^T < 0$, the solution for periods $n, ..., N$ is the pure $(0,0)$ NE strategy in each period and $\left(R_n^T, R_n^D\right) = (0,0)$. Specifically, if $K^T P - c_1^T < 0$, there is an NE for any N period game in which Terrorist never attacks and Defender never responds. This is equivalent to stable N quiet periods.
(c) If for some $n \in [1, ..., N]$, we have $c_n^T \leq K^T P < f_1^{N-n}\left(c_n^T\right)$, the solution up to this period (i.e., for periods $n+1, .., N$) includes both mixed and pure strategies.
(d) For any $n \in [1, ..., N]$, $R_n^T = 0$ this is the lowest payoff Defender can enforce on Terrorist. This is independent of the changes in c^T.
(e) If $K^T P - c_n^T \geq 0$, the mixed strategy of Defender in the n^{th} period is $\left(1-G\left(c_n^T\right), G\left(c_n^T\right)\right)$. That is, in each period Defender's actions are identical to her actions in a single period game with $c^T = c_n^T$. Specifically, Defender's actions in the first of an N period game are identical to her actions in a single period game with the same Terrorist attack cost.

From part (e) we see that the probability of Defender to respond is decreasing with c^T. Thus, if, due to the actions in the n^{th} period, $c_{n+1}^T < c_n^T$, Defender will have an increased likelihood of response in the $n+1^{st}$ period (in comparison to this probability in the n^{th} period). Based on the results of Corollary 1, we develop an algorithm solving the N period game (results are reported in Section 6.3).

We next investigate, in Proposition 4, how Terrorist's attack costs change with time. Once Defender's response is efficient, there may be a threshold $c^{T*} \in \left(0, K^T P\right)$ representing a stable value for c_n^T, that is, if $c_{n-1}^T < c^{T*}$, c_n^T is expected to increase towards c^{T*} and if $c_{n-1}^T > c^{T*}$, c_n^T is expected to decrease towards c^{T*}. Thus, we call c^{T*} the equilibrium Terrorist attack cost. Specifically,

Proposition 4 *In the mixed strategies equilibrium, i.e., when $c_{n-1}^T < K^T P$, we have*

1. *If $Qf_1\left(c^T\right) \leq c^T$, then $E\left(c_n^T | c_{n-1}^T < K^T P\right) < c_{n-1}^T$; that is, Terrorist's expected attack cost is decreasing*
2. *If $Qf_1\left(c^T\right) > c^T$ (and assuming $Qf_1(c) \neq f_{-1}(c)$), then $c_{n-1}^T >, (=, <)\ c^{T*}$, where*

$$c^{T*} \quad : \quad = \arg\left\{K^T P - Q\left(K^T P + K^S\right)\frac{c^{T*} - f_{-1}\left(c^{T*}\right)}{Qf_1\left(c^{T*}\right) - f_{-1}\left(c^{T*}\right)} = c^{T*}\right\} \tag{6}$$

$$G\left(c^{T*}\right) \quad = \quad \frac{c^{T*} - f_{-1}\left(c^{T*}\right)}{Qf_1\left(c^{T*}\right) - f_{-1}\left(c^{T*}\right)}. \tag{7}$$

results in $E\left(c_n^T | c_{n-1}^T < K^T P\right) <, (=, >)\ c_{n-1}^T$, respectively; that is, Terrorist's expected attack cost moves towards c^{T*}.

The ratio $Qf_1\left(c^T\right)/c^T$ from Proposition 4 represents the relative efficiency of Defender's technology to Terrorist loss of experience (measured by the probability, Q, times the increase in Terrorist's attacking cost from a hit, relative to the change in the attacking cost of Terrorist's loss of experience). We call this the *Defenders Efficiency Ratio*. This ratio is only influenced by the event "both shoot, Defender hits, Terrorist misses." Proposition 4 shows that in the case where this ratio is small (i.e., less than 1 or even somewhat larger than 1, in case 2), Defender's response is not efficient enough and Terrorist's expected attacking costs decrease with time. In the multiplicative learning example from Table 4, $Qf_1\left(c^T\right)/c^T \leq 1$, implies $Q\gamma \leq 1$; that is, c_n^T will decrease with n if the probability of a successful Defender's response, Q, is lower than Terrorist's relative cost increase after Defender's hit, $1/\gamma$. This case suggests the ability of Defender to push the multi period game into its quiet NE is limited. Moreover, since, $G\left(c^T\right)$, Defender's probability of responding, is decreasing with c^T, an inefficient response by Defender suggests an escalation in the conflict.

In fact, even if this relative efficiency is higher than 1, c^{T*} as defined in (6) is negative, Terrorist's attacking cost are still expected to decrease. In contrast, if $c^{T*} \geq K^T P$, whenever the mixed strategy is used, Terrorist's attacking cost are expected to increase. Thus, a quiet period is expected.

5.2 Multi period Games with Several Technologies

5.2.1 Finitely Repeated Game

Now consider the multi period game with learning and several technologies. Rather than providing a detailed solution to this model, we discuss the equilibrium actions under the earlier assumptions. An important observation is that if there is a unique equilibrium in the single period game, the unique equilibrium in the repeated game is to play this unique NE at each period. Thus, given our analysis of the 2-technologies game, at each period, only a single technology (the one with the higher G^i, Defender's response probability) may be used by Terrorist. Without parameter changes from one period to another, due to either exogenous or endogenous factors such as learning, there will only be a single technology used by Terrorist in any finite period game.

However, once we allow changes to the parameters from one period to the next, the situation may differ. To see this, let c_n^{Ti} denote Terrorist's attack costs in period n when using technology i and consider the case where $G^2\left(c_n^{T2}\right) < G^1\left(c_n^{T1}\right)$. Given our discussion and results above, at this period, Terrorist attacks with technology 1. Now, if $G^1\left(c_n^{T1}\right) > G^1\left(c^{T1*}\right)$, the period may end up with $c_n^{T1} < c_{n-1}^{T1}$ and $G^1\left(c_n^{T1}\right) > G^1\left(c_{n-1}^{T1}\right)$. If $G^1\left(c^{T*}\right) < G^2\left(c_n^{T2}\right)$ and n is large enough, there will be some period j when $G^1\left(c_j^T\right) < G^2\left(c_j^T\right) = G^2\left(c_n^T\right)$ (as long as technology 2 is not used, we assume the corresponding attack cost does not change). Therefore, after several periods where Terrorist uses technology 1, he will switch to technology 2. Extending this line of thought, we see that if $c^{T1*} \approx c^{T2*}$ (and both $c^{T1*}, c^{T2*} > 0$), a different technology may be used at any period when n is large enough.

Remark 3 *Note that the RHS in (7) depends only on the learning evolution, which should be similar across technologies, and Defender's probability of interception, Q^i. Therefore, technologies with similar Q^is may be used in different periods of a finite horizon game, whereas technologies with substantially different Q^is may not.*

5.2.2 Infinitely Repeated Game

Consider a long term conflict in which the single period model is infinitely repeated. A vast literature addresses the problem of repeated games, yielding so-called *folk theorems* that identify possible equilibriums.

An important insight from folk theorem (e.g., Friedman 1971) is that both players benefit from the infinitely repeated game, in the sense of a higher average payoff per period (as opposed to benefit in absolute terms). In our settings, Terrorist's only way to increase his expected payoff from 0 is if *there is a higher proportion of outcomes where he attacks and Defender does not respond*. Thus, a long term conflict may result in more violence against citizens in Defender's territory with no response by Defender (i.e., Terrorist attacks and Defender does not respond).

The idea behind equilibriums in a long term conflict is that players select those actions yielding better payoffs. Any deviation will be punished by switching to the single period equilibrium strategy either forever or for a cycle of periods of punishments. Such strategies are called trigger strategies.[1] With cyclic polices, in cases of deviation, the players switch to the single period equilibrium for a finite number of repetitions until any gain generated by the deviation is lost; the players then return to the equilibrium actions. Such cyclic policies enable Terrorist and Defender to "forgive" mistakes or "punish" deviations or even successful hits; thus, the resulting payoff for a particular period may be substantially different than the expected payoff for a period.

Remark 4 *In contrast to the equilibrium in the single period games the infinitely repeated game allows an equilibrium where Terrorist uses several technologies. As an example, any equilibrium where Defender plays NR at every period and Terrorist mixes NA, $A1$ and $A2$ (or more technologies if available) as long as the total damage to Defender per period is below this damage in the single period game equilibrium. In contrast to the finite repeated game, such equilibriums are feasible in a long term conflict even without making changes to the game's parameters from one period to the other. Therefore, in a long term conflict. technologies with substantially different Q^is can still be used in different periods.*

5.3 Multi period Games with Exogenous Changes to Model Parameters

The equilibriums considered above depend on the model's parameters. For example, in the single period game, Terrorist's attack probability is $c^D/K^D PQ$. The analyses of these models ignore exogenous changes to the parameters, including changes in the environment, to the ruling regimes, in public opinion, as well as increasing pressure from other parties not directly involved in the conflict (such as the international community). Note that

[1]It is possible to consider other equilibriums where players switch to other subgame perfect equilibrium once a deviation occurs. In this case, the punisher threatens to play the equilibrium generating the worst payoff for the player who deviates (see Abreu 1988). For simplicity we avoid such solutions.

the parameters K^D, K^T, and K^S are all subjective measures of damage, including the cost attributed to injury or death. Because these parameters are hard to quantify, we give them less attention in the following analysis, except for a brief discussion of their possible effect on the equilibrium.

6 Conjectures and Their Correspondence with Data

In this section we first make several conjectures based on our models. We then describe the data, discuss how we use them to estimate the different parameters and present a numerical example. We then review the history of the political situation in Israel and the Gaza strip from January 2001 until November 2007. Given this political situation, we make conjectures on this conflict and determine whether our data agree with these conjectures. Note that while we found quite a substantial agreement between these conjectures and our data, we do not claim our model is the only viable explanation. To emphasize this, we say "the data do not contradict the conjecture" rather than the more common phrase "the data support the conjecture."

6.1 Conjectures

Our conjectures (denoted by "*C*") rely on the political atmosphere on both sides. When it is stable, i.e., the governing bodies are in power for a reasonably long horizon, the conflict can be modeled as long term and equilibriums that are supported by the infinite period model should occur. In contrast, when the governing body of either party is unstable, the equilibrium should be similar to the one derived by a finite period model.

We first consider the sustainability of cease-fire agreements when the political situation is stable. Following the discussion of the infinite period model in Section 5.2.2, any equilibrium that can be supported in an infinite period game should lead to a higher (than 0) payoff for Terrorist per period. As discussed, all such equilibriums have a positive proportion of Terrorist attacks without Defender's response. Thus, a cease-fire agreement — where Terrorist does not attack — cannot be an equilibrium. Our infinite period model suggests that in the absence of a substantial external payoff related to the long term benefits of a quiet period, such a quiet period is not sustainable:

C1: When both governing bodies are stable, cease-fire agreements are not likely to hold for long.

In the context of our models, Terrorist can always guarantee at least a 0 payoff by being passive. In a long term conflict, this remains the case, even if Defender always responds. Therefore, *C1* remains valid even when Defender is not passive.

Conjecture *C1* ignores payoffs exogenous to the model. Specifically, exogenous payoffs made to Terrorist may lead to a sustainable cease-fire agreement. As exogenous payoffs are often provided by the international community, *C1* suggests that without a permanent payoff resulting from external involvement, searching for a cease-fire agreement may be a lost cause even when both regimes are stable. Moreover, if the exogenous payoff is not permanent and could be used to improve Terrorist's technology, a short term cease-fire agreement may result in an increase in the violence level.

We wish to clarify the difference between a cease-fire and a peace agreement. The latter relies on substantive changes to the environment providing both parties with positive payoffs that are sustainable for the long term. In contrast, cease-fires do not change the environment and are only expected to provide a short term solution, e.g., several months. Therefore, *C1* is not surprising; it suggests that in a conflict with Terrorist, with a stable leadership, cease-fire agreements may only hold for a very short term, e.g., weeks rather than months.

We also wish to clarify the difference between a cease-fire between two countries and that between Terrorist and Defender. When two countries are involved in a conflict, both can maintain a 0 payoff if they stick to the cease-fire and may face a negative payoff as a result of the other country's attacks (see Defender's payoff in our model). This is also true in an infinite game, so both countries may be better off maintaining the cease-fire, preventing the negative payoff resulting from an active conflict.

We next consider the relationship between the political situation and the number of technologies used by Terrorist. Remember that the single period model predicts that only a single technology will be used, and the finite period model suggests that several Terrorist technologies can be used, if the *Defenders' Efficiency Ratio* with respect to these technologies is similar; in contrast, as discussed in Remark 4, in the infinite period model, several technologies can be used.

We therefore conjecture:

C2: When both governing bodies are stable, several technologies may be used by Terrorist.

C3: When either one of the governing bodies is unstable, only Terrorist technologies with similar Defender's Efficiency Ratio will be used by Terrorist.

Recall that changes in the ruling regimes may affect the level of violence. In the case where only Defender is stable, we expect the single period equilibrium. In a single period game, Terrorist's attack probability is $c^D / \left(K^D PQ\right)$. The ratio c^D/K^D measures the relative cost of Defender's response to damage caused by Terrorist. This ratio depends on the subjective value, K^D, attributed to different injuries or deaths. As mentioned in Section 5.3, given this subjective evaluation, a change in Defender's leadership may change the ratio c^D/K^D, leading to a different equilibrium in the single period model. Similarly, even when both parties in the conflict are stable, a change in leadership may change the subjective relative value of the parameters dictating the equilibrium.

C4: When the leadership of a stable body changes, the level of violence is likely to change.

Given our discussion in Section 5.3 on the effect of exogenous changes on the parameters in the model on the equilibrium, this conjecture is far from surprising and is actually quite intuitive. Still it is interesting to see if changes appear in the data, as such substantiation will validate our models.

6.2 Data and Its Analysis

We use three data sets to calibrate the parameters of our model. Data set 1 is based on IICC (2007); data set 2 comes from major Israeli newspapers from November 5, 2008, to January 17, 2009; data set 3 is based upon periodical reports published by the Israeli Security Agency, Shabak (2010). An additional description of the development of the conflict between Israel and the Gaza strip terrorists including technical details for their rockets can be found in Rubin (2011).

6.2.1 Description and Summary of Data

For data set 1, we included the number of mortar shells and missiles (Qassam or Grad) launched from the Gaza strip and the resulting number of injuries and deaths, based on IICC (2007). These are summarized in Table 5. The IICC data on the number of attacks, collected from different Israeli security forces, may be lower by 20% from the actual number (because some hits may be hard to identify). Note that the data in IICC (2007) do not differentiate between Qassam and Grad missiles and both may be used.

From Table 5 it seems that the total number of attacks and the amount of damage were increasing during

Year		Attacks	Injuries	Deaths
2001	Mortar Shell	245	24	1
	Qassam	4	0	0
2002	Mortar Shell	257	9	0
	Qassam	35	3	0
2003	Mortar Shell	265	19	0
	Qassam	155	37	0
2004	Mortar Shell	876	68	3
	Qassam	281	148	4
2005	Mortar Shell	238	23	4
	Qassam	179	61	2
2006	Mortar Shell	22	1	2
	Qassam+Grad	946	86	2
2007	Mortar Shell	640	7	0
	Qassam+Grad	783	98	2
Total	Mortar Shell	2543	151	10
	Qassam+Grad	2383	433	10

Table 5: Number of attacks, injuries, and deaths during 2001-2007, based on IICC (2007).

these years. The exception is 2005, when the unilateral disengagement of Israel from the Gaza strip and Northern Samaria was announced and carried out. During the years covered in IICC data, there were almost 5000 attacks recorded from the Gaza strip, on average about 2.5 per day when 2005 is left out, this average is closer to 3 attacks per day). In 2007 alone, the 1423 attacks recorded in the report represent about 3.9 attacks per day. From these data, it is clear that the attacks by terrorists from the Gaza strip and Israel's responses significantly affect the life of thousands of people on both sides.

Figure 1 illustrates the number of rockets and mortar shell attacks from the Gaza strip by month during January 2001–November 2007 based on IICC (2007). We later refer to this figure when investigating conjectures specific to the Israeli-Palestinian conflict in the Gaza strip.

The second data set includes data on mortar shells and missiles launched from the Gaza strip into Israel for the period November 5, 2008, to January 17, 2009. At the end of 2008, a Twitter account called QassamCount was set up. This account features updates on missiles launched into Israel. Every update is accompanied by a link to Haaretz and NRG (Maariv) (two leading news providers in Israel). Every item includes information on the (approximate) location where the missile hit the ground and the number of deaths, wounded, and property damage. The owner of the Twitter account published the same information on two web sites (http://qassamcount.com/ and at http://twitter.com/#!/QassamCount). After verifying the data with the original publication, we use these data to estimate the parameters for our model.

The second data set includes 525 attacks of which 420 can be traced to one of the three technologies, mortar shell, Qassam, or Grad missile. We use a superscript $i = 1$ to denote a mortar shell, $i = 2$ to denote a Qassam (of either generation) missile, and $i = 3$ to denote a Grad missile. (Note that from November 5, 2008, to January 17, 2009, the average number of attacks recorded per day exceeded 7.) In estimating our models' parameters we ignore the 105 attacks that cannot be attributed to a specific technology. These include 41 on December 20, 2008, that injured two people and caused heavy property damage. We define attack incidents as those consisting of possibly several attacks. We identify 420 attacks and 205 attack incidents in this data set.

Data set 2 is summarized in Table 6. For each technology, i, we present 5 levels of hit accuracy (denoted

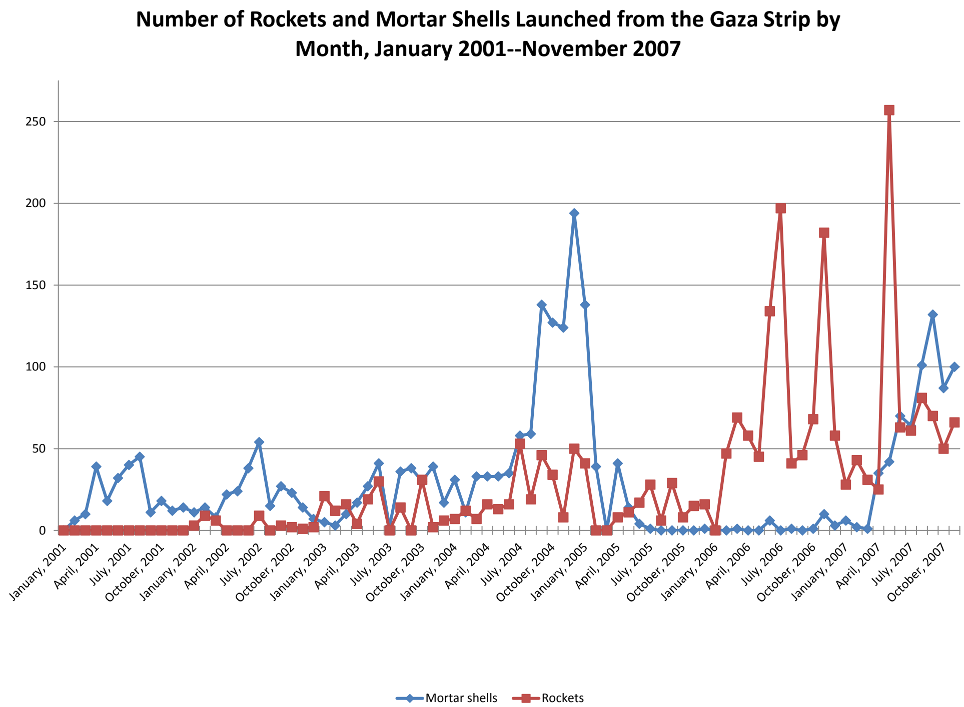

Figure 1: Number of Terrorist attacks by Months from IICC(2007).

by H_j^i or $\tilde{H}_j^i$; as discussed below, the superscript is omitted) and 5 levels of damage (denoted by D and $\tilde{D}$, as discussed below).

For each known attack incident and each technology, i, we record 5 levels of hits: H_0^i -hit in the Palestinian territory, H_1^i- hit in an unpopulated open space in Israel, H_2^i-hit in a populated area open space in Israel, H_3^i-hit causing only property damage, and H_4^i-hit causing injuries or deaths. Note that in some cases, data are aggregated as attack incidents; that is, in cases where several simultaneous attacks using a specific technology caused a number of injuries, we cannot relate the injuries to a specific attack. In such cases, we record these as attacks causing injuries and our estimate of H_4 may be higher than its actual value. We also estimate $\tilde{H}_j^i$ $i = 1, 2, 3$, and $j = 0, ..., 4$ counting only the number of attack incidents (i.e., assuming that only a single missile/ mortar shell is launched at each attack incident with a hit). These estimates give us a lower bound on the hit

	H_0	H_1	H_2	H_3	H_4	$\tilde{H}_0$	$\tilde{H}_1$	$\tilde{H}_2$	$\tilde{H}_3$	$\tilde{H}_4$	Total H^i	Total $\tilde{H}^i$
Mortar Shell	0	21	28	3	0	0	4	9	2	0	52	15
Qassam	2	33	140	61	27	2	17	73	23	8	263	123
Grad	0	19	52	28	6	0	16	32	14	5	105	67
Total	2	73	220	92	33	2	37	114	39	13	420	205

	D_0	D_1	D_2	D_3	D_4	$\tilde{D}_0$	$\tilde{D}_1$	$\tilde{D}_2$	$\tilde{D}_3$	$\tilde{D}_4$	Total D	Total $\tilde{D}$
Mortar Shell	49	1	2	0	0	13	1	1	0	0	52	15
Qassam	165	59	12	15	1	87	22	6	15	1	252	131
Grad	63	16	18	20	1	44	8	9	20	1	118	82
Total	277	76	32	35	2	144	31	16	35	2	417	223

Table 6: Hit accuracy and damage for the three technologies for November 5, 2008, to January 17, 2009, based on Israeli newspapers.

Year		Attacks	Attack incidents	Deaths	Combined Injuries
2001	Mortar Shell		510	1	25
	Qassam				
2002	Mortar Shell		455	0	10
	Qassam		17		
2003	Mortar Shell		514	0	44
	Qassam		123		
2004	Mortar Shell		882	4	99
	Qassam		276	4	
2005	Mortar Shell	854	574	1	68
	Qassam	401	286	5	
2006	Mortar Shell	55	28	0	163
	Qassam+Grad	1722	1247	2	
2007	Mortar Shell	1531	663	0	343
	Qassam+Grad	1276	938	2	
2008	Mortar Shell	1668	912	3	464
	Qassam+Grad	2048	1270	5	
2009	Mortar Shell	289	197	5	180
	Qassam+Grad	569	404	0	
Total	Mortar Shell	4397	4735	14	1396
	Qassam+Grad	6016	4561	18	

Table 7: Number of attacks, events,deaths, and injuries (reported jointly for all technologies) during 2001-2009, based on Shabak (2010).

probability.

For each attack incident (and technology) that hit Israel, we record: D_0^i — number of attacks causing no damage. D_1^i — number of attacks causing property damage only, D_2^i — number of attacks causing shock (and possibly some property damage but no injuries or deaths), D_3^i — number of injuries and D_4^i — number of deaths. That is, an attack causing both injuries and deaths adds to both D_3^i and D_4^i, but does not add to the number of attacks causing shock. We find the number of injuries and deaths is more relevant than the number of attacks causing them. We also record $\tilde{D}_j^i$ for $i = 1, 2, 3,$ and $j = 0, ..., 4$ when counting the number of attack incidents rather than attacks (similar to the definitions of the $\tilde{H}_j^i$ above).

The third data set, taken from the Shabak report, includes yearly data on the number of attacks, attacks incidents, and deaths, caused by mortar shells and missiles for the years 2001-2009. The data also report the combined injuries from both mortar shells and missiles per year. The data are summarized in Table 7.

As can be seen when comparing the data in Table 7 to those in Tables 5 and 6, the number of attacks reported in Tables 5 and 6 is lower than the total number of attacks recorded by the Shabak.

We further note that data sets one (IICC) and three (Shabak) both differentiate between mortar shell and missile attack, but do not differentiate between the Grad and Qassam. This is likely because the range of mortar shells (typically up to 2 or 3 km) is much lower than the range of missiles ($6-14$ km for Qassam and 20 km for Grad; in 2012 this range increased to about 40 km).

6.2.2 Parameter Estimation

We use the data from Tables 5, 6 and 7 to estimate the parameters for our models. During the process we make several rough estimations. The estimates are all denoted with a "ˆ" and are given by (note: technologies 1, 2, and 3, are respectively, mortar shell, Qassam and Grad):

$$\begin{aligned}
\hat{P}_1 &= 0.34, \quad \hat{P}_2 = 0.75, \quad \hat{P}_3 = 0.73, \\
\hat{K}^{D1} &= \hat{K}^{S1} = \hat{K}^{T1} = 0.015, \ \hat{K}^{D2} = \hat{K}^{S2} = \hat{K}^{T2} = 0.017, \text{ and } \hat{K}^{D3} = \hat{K}^{S3} = \hat{K}^{T3} = 0.049, \\
\hat{c}^{T1} &= 0.001, \quad \hat{c}^{T2} = 0.003, \text{ and } \hat{c}^{T3} = 0.008, \\
\hat{Q} &\in \{1, 0.8, 0.6\} \\
\text{and } \hat{c}^{D} &\in \{0.0005, 0.001, 0.002\}\,.
\end{aligned} \tag{8}$$

A detailed discussion of the parameter estimation is given in Appendix A. Note that Zucker and Kaplan (2014) recently discussed the level of damage caused by rocket attacks on Sderot, a small Israeli city suffering about 5,000 rocket attacks during the 2001-2010 period. They observed that the actual damage caused is three to nine times smaller than the potential damage that could be expected from such attacks. They reason that Israel has invested extensively in passive defence mechanisms such as fortified rooms, shelters, rockets alarm, and the Iron Dome anti-rocket system. The predictions in Zucker and Kaplan (2014) imply that the expected damage of Terrorist attacks may have changed during the period we investigate, as a result of the introduction of such passive defence mechanisms. For simplicity, our estimation ignores such possible changes.

6.3 Numerical Examples

We use the estimated values discussed above to numerically investigate the resulting equilibriums in a single period and an eight period game starting with the same parameters (detailed below). We expect to see a different equilibrium in the first period of a multi period game only if there is a chance for a quiet period equilibrium. For the eight period game with the parameter estimation in (8), part a of Corollary 1 implies there is a chance for a quiet period if $\gamma^7 \geq K^{Ti}P_i/c^{Ti}$. That is, for Mortar shell, Qassam, and Grad, there is a chance for a quiet period in an eight period game if $\gamma \geq 1.26$, 1.23, 1.24, respectively. Given hundreds of attacks, a reasonable increase in Terrorist attack cost should be in the order of several percentage points at most; thus, we suspect increases of more than 20% are quite unlikely in practice. For any γ value smaller than these, i.e., any reasonable γ values, the equilibrium attack and response probabilities in the first period of an eight period game would be identical to these probabilities in a single period game (the parameter θ is less significant in affecting the results in the first period).

We let Terrorist learning parameter $\theta = .9$, i.e., Terrorist learning is slow, and $\gamma = 1.4$, i.e., every Terrorist action successfully intercepted by Defender increases Terrorist's attack cost by 40%. This unrealistic choice is made to create a positive probability for a quiet period within eight periods. With these values, with $Q = 0.6$, we have $Qf_1\left(c^T\right) \leq c^T$, so that Defender's technology is not efficient enough and Terrorist's attack cost is expected to decrease. In contrast, when $Q = 0.8$ or $Q = 1$, we have $Qf_1\left(c^T\right) > c^T$ and the values of c^{T*} from Proposition 4 are 0.0051, 0.0127, and 0.0355 for Mortar shell, Qassam, and Grad, respectively. Therefore, Terrorist's attack costs are expected to increase in these examples.

Remember that the single and finite multi period games with multiple technologies analyzed in Section 4 typically result in an equilibrium where only a single technology is used. Therefore, we do not present results for two technology games.

The numerical results are summarized in Table 8, parts a, b, and c for the Mortar shell, Qassam, and Grad, respectively. For each of these technologies, we used $\hat{P}_i$, $\hat{K}^{Di}$, $\hat{K}^{Si}$, $\hat{K}^{Ti}$, and $\hat{c}^{Ti}$ from (8). The top 3 lines in each table have $c^D = 0.0005$, letting Q decrease from 1 to 0.8, and to 0.6. The second and third groups of three lines are similar but with $c^D = 0.001$, and $c^D = 0.002$, respectively. The values q_0^D, q_1^D, q_0^T, q_1^T, and R^D are these probabilities and Defender's expected payoff for the first period in the eight period game; these are calculated using backward induction. The respective quantities for the single period game are given in closed form in Proposition 1 and are thus not reported. We let q_1^{T8} denote Terrorist's attack probability in the first period of the eight period game. We report, $\%\Delta q_1^T$, where $\%\Delta q_1^T = 100\left(q_1^{T8} - q_1^T\right)/q_1^T$; this column presents the percentage increase in Terrorist's attack probability between the first period of the eight period game to these probabilities in a single period game with the same c^{Ti}. Remember that Defender's expected damage over the eight period with no quiet periods is 8$*$ Defender's expected damage in a single period game (as expected from Section 5, Terrorist's expected payoff remains 0). We report $\%\Delta R^D$, where $\%\Delta R^D = 100\left(R_8^D - 8R_1^D\right)/\left(8R_1^D\right)$; this column presents the percentage increase in Defender's expected damage for an eight period game comparing to the expected damage from eight repetitions of a single period game (note that Defender's expected damage is always negative).

We examine the equilibrium in a single period game and observe the following:

1. As expected, when Q decreases, both players become more active (i.e., q_1^D and q_1^T are increasing) and the expected damage to Defender is larger.

2. As expected from Proposition 1, in the single period game, for each c^D value, Defender's expected damages are independent of the technology, because the ratio c^D/Q is fixed. As expected from part e of Corollary 1, q_0^D and q_1^D in the first of the eight period game are identical to q_D^0 and q_D^1 of the single period game.

3. The differences in the equilibrium results for different technologies are not trivial. Comparing mortar shell to either Qassam or Grad, we observe that Defender's response probability increases, whereas Terrorist's attack probability decreases (while maintaining the same damage to Defender). These changes are attributed to the differences in the hit probabilities of these technologies. When comparing Qassam to Grad, we see that Defender's response probability with Grad is a bit lower than with Qassam. These relations hold even though the Qassam hit probability (0.75) is higher than Grad (0.73). These differences are attributed to the difference in K^S between these technologies.

4. Defender's response probability is independent of c^D (as long as Assumption (A.1) is satisfied). Terrorist, however, becomes more active as c^D increases, causing extra damage to Defender.

5. Terrorist attack probabilities are decreasing as Terrorist's technology improves (from mortar shell to Qassam and then to Grad). In fact, only when Terrorist's technology is not efficient (i.e., mortar shells) and $c^D \geq c^{T1}$ Terrorist is more active than Defender. In all our other numerical results, Defender is more active than Terrorist. So once Terrorist technology's efficiency passes some threshold, a passive Defender is forced to be "on the watch" and quite active in her responses.

We next examine the results for the eight period game. Remember that by design, in all our cases, a quiet period can be reached within eight periods. We observe:

a. Mortar Shell

c^D	Q	q_0^D	q_1^D	q_0^T	q_1^T	R^D	$\%\Delta q_1^T$	$\%\Delta R^D$
0.0005	1	0.7960	0.2040	0.9020	0.0980	-0.0040	-0.013	-0.002
	0.8	0.7450	0.2550	0.8775	0.1225	-0.0050	-0.039	-0.007
	0.6	0.6600	0.3400	0.8368	0.1632	-0.0067	-0.143	-0.024
0.001	1	0.7960	0.2040	0.8045	0.1955	-0.0080	-0.316	-0.053
	0.8	0.7450	0.2550	0.7569	0.2431	-0.0100	-0.799	-0.134
	0.6	0.6600	0.3400	0.6811	0.3189	-0.0133	-2.414	-0.406
0.002	1	0.7960	0.2040	0.6256	0.3744	-0.0159	-4.528	-0.767
	0.8	0.7450	0.2550	0.5533	0.4467	-0.0197	-8.878	-1.527
	0.6	0.6600	0.3400	0.3400	0.5384	-0.0258	-17.618	-3.154

b. Qassam

c^D	Q	q_0^D	q_1^D	q_0^T	q_1^T	R^D	$\%\Delta q_1^T$	$\%\Delta R^D$
0.0005	1	0.6723	0.3277	0.9608	0.0392	-0.0040	0.000	0.000
	0.8	0.5903	0.4097	0.9510	0.0490	-0.0050	-0.002	0.000
	0.6	0.4538	0.5462	0.9346	0.0654	-0.0067	-0.002	0.000
0.001	1	0.6723	0.3277	0.9216	0.0784	-0.0080	-0.004	-0.001
	0.8	0.5903	0.4097	0.9020	0.0980	-0.0100	-0.013	-0.002
	0.6	0.4538	0.5462	0.8693	0.1307	-0.0133	-0.050	-0.009
0.002	1	0.6723	0.3277	0.8433	0.1567	-0.0160	-0.114	-0.002
	0.8	0.5903	0.4097	0.8045	0.1955	-0.0200	-0.298	-0.054
	0.6	0.4538	0.5462	0.7411	0.2589	-0.0266	-0.957	-0.179

c. Grad

c^D	Q	q_0^D	q_1^D	q_0^T	q_1^T	R^D	$\%\Delta q_1^T$	$\%\Delta R^D$
0.0005	1	0.6724	0.3276	0.9860	0.0140	-0.0040	0.000	0.000
	0.8	0.5905	0.4095	0.9825	0.0175	-0.0050	0.000	0.000
	0.6	0.4540	0.5460	0.9767	0.0233	-0.0067	0.000	0.000
0.001	1	0.6724	0.3276	0.9720	0.0280	-0.0080	0.000	0.000
	0.8	0.5905	0.4095	0.9651	0.0349	-0.0100	0.000	0.000
	0.6	0.4540	0.5460	0.9534	0.0466	-0.0133	0.000	-0.002
0.002	1	0.6724	0.3276	0.9441	0.0559	-0.0160	-0.002	0.000
	0.8	0.5905	0.4095	0.9301	0.0699	-0.0200	-0.003	0.000
	0.6	0.4540	0.5460	0.9068	0.0932	-0.0267	-0.011	0.000

Table 8: Results for eight-period games for different technologies

1. $\Delta q_1^T \leq 0$, i.e., Terrorist's attack probability in the first of the eight period game is lower than in the single period game. Moreover, Δq_1^T seems to decrease with c^T and increase with Q for each of the technologies. This observation implies the possibility of a quiet period, resulting from an increase in Terrorist's attack cost, decreases the level of violence in earlier periods as well.

2. ΔR^D is decreasing (in some cases the decrease is not significant). Moreover, as Q increases, the decreases in ΔR^D are less significant, which can be attributed to the lower q_1^T in these cases. A lower q_1^T reduces the chance of learning to affect future costs and, thus, decreases the probability of reaching a quiet period. Similarly, for the more effective technologies, the reduction in ΔR^D is less significant. That is, Defender's chances of reaching a quiet period are lower as Terrorist's technology advances.

6.4 Correspondence between Conjectures and Data

Before investigating the conjectures in Subsection 6.1, we add a conjecture on the level of usage of different technologies based upon the results of our numerical examples. These results show that the equilibrium probability of Terrorist's attack using mortar shells is higher than the corresponding probability of using Missiles. Assuming the results would hold in the infinite period model, we conjecture:

C5: Whenever both mortar shells and missiles are used (i.e., in periods corresponding to an infinite period game) the number of attacks using mortar shells will be higher than the number using missiles.

To investigate our conjectures, we use the data collected from the conflict between Israel and the Gaza strip from January 2001 to November 2007, the period covered in IICC (2007). We first highlight some important political changes in Israel and the Gaza strip during this period.

Israel is a democratic state and its response is guided by its prime minister. The prime ministers from March 2001 to March 2009, Ariel Sharon (to January 2006) and Ehud Ulmert (to March 2009) represented the same political party. (Both moved from the Likud to the new Kadima party in November 2005.) Israel elections were held on March 28, 2006, and Ehud Ulmert, who replaced Ariel Sharon following Sharon's stroke, led the government. The main act related to our work was Israel's unilateral disengagement from the Gaza strip, scheduled to start on July 2005; it actually started in August 2005 and was completed in September 2005.

The political situation in the Palestinian National Authority (PNA) during the period covered in IICC (2007) was quite involved. The first president of the PNA, Yasser Arafat, was widely accepted by Palestinians (including support from both Hamas and Fatah). President Arafat became terminally ill on October 2004 and died in November 2004; he was succeeded by Rawhi Fattuh as the interim president until January 2005, when Mahmoud Abbas was elected as the second president of the PNA. While, at first, President Abbas was widely accepted by Palestinians, as the Israeli planned disengagement approached, he lost support in the Gaza strip. This (and other factors) led President Abbas to announce upcoming elections to the Palestinian Legislative Council on August 9, 2005. This announcement was followed by increasing internal conflicts, many between Hamas and Fatah. In March 17, 2007, a unity government led by Ismail Haniyeh from Hamas was established. But this government was dissolved in June 2007 by President Abbas; then, Hamas violently took control of the Gaza strip. (To date, Hamas remains the de facto governing body.)

As for the attack technologies available in the Gaza strip, the first technology employed, starting in March 2001, was mortar shells. These attacked Israeli settlements within and outside the Gaza strip. Qassam rockets were first launched over Israel in January 2002. In March 2006, the Grad rocket technology was launched for the

Period	Average # of Attacks/month	Min. # of Attacks/month	Max # of Attacks/month
August 2005-January 2006	12.5	0, January 06	29, September 05
February 2006-March 2007	77	32, March 07	197, July 06

Table 9: Average, minimum, and maximum number of attacks in the two periods.

first time.

To summarize the main factors influencing the political nature of the conflict during the period under investigation: until July 2005 and starting again in April 2007, a single governing body controlled the Gaza strip, but between these periods, various organizations fought for control. Israel was always led by a single governing body that changed in January 2006. Therefore, in the periods before July 2005 and after April 2007, the political leadership was stable; these should correspond to an infinite period game model. Other periods should correspond to a finite period game model.

In view of this discussion and *C1* we expect:

C1.1: During the periods before July 2005 and after April 2007 (when an infinite period model is adequate), cease-fire agreements are not likely to hold for long.

The data on the conflict do not contradict *C1.1.* There were two main attempts to achieve a cease-fire, one during each of the above periods. The first cease-fire on June 26, 2003, followed the Aqaba summit. This agreement only lasted until July 2003. The second cease-fire started in November 2006, following the *Autumn Clouds* operation carried out by the Israel Defense Forces. This agreement did not last long either; the number of attacks in December 2006 was 61, only slightly lower than the average of 77 attacks per month for the period of February 2006 to March 2007 (see Table 9).

In game theory terminology, we can see peace discussions as a cooperative game rather than a non-cooperative one. Presumably, in such cooperative models, equilibriums with better payoffs for both players can be supported. However, when Terrorist's payoff is similar to the one in our model, Terrorist has no incentive to stick to the Not Attack action. In the Israeli-Palestinian conflict, it appears that Terrorist is more concerned with the possible immediate payoffs of acting when Defender does not respond (as implied by a cease-fire agreement), than with the potential long term benefits of a cease-fire. This conjecture suggests that short term payoffs to Terrorist will not result in a sustainable cease-fire. Therefore, we recommend that cease-fire discussions in long term conflicts focus on long lasting payoff changes.

Note that if we were to observe cease-fire agreements holding for a longer period, say several months, we would likely reject *C1.1.* After several months, exogenous events not captured in our model may lead to the end of the cease-fire. We further note that during the cease-fire discussions with the PNO, in July 2003, several Palestinian groups (including Hamas) were trying to gain support in Gaza, and there were claims that these groups attacked Israel as a way of improving their internal political power in Gaza. This suggests the infinite horizon model may be less than perfect for the period before July 2005 and provides another explanation of the failure of this cease-fire agreement.

We next consider *C2* and *C3*. In view of *C2*, and the political situation in the Gaza strip, the periods in C1.1 should be analyzed using the infinite period model, and we conjecture:

C2.1: Until July 2005 and from April 2007 (when a finite period model is adequate), several technologies could be used by Terrorist organizations in Gaza.

The other periods should be analyzed using the finite period games model. From the discussion of the finite period model in Subsection 5.2.1, we expect a single technology will be used if this technology results in a higher probability of Defender's response, G, and technologies with similar G may be used in parallel. With the estimated values in (8) we can calculate G for each of the technologies. We note that G as given in (2) is decreasing in Q. So for mortar shells, the highest G^1 is achieved with $Q = 0.6$, then $G^1 = 0.34$; in contrast, for Qassam, the lowest G^2 is with $Q = 1$, then $G^2 = 0.37$; similarly, for Grad, the lowest G^3 is $G^3 = 0.33$.

Because based on our estimations, the lowest G^2 is higher than the highest G^1 and given the availabilities of different technologies for terrorists, we conjecture:

C3.1: From August 2005 until March 2006 (when Grad was introduced) only Qassam would be used.

We further observe that with similar Qs, G^2 and G^3 are quite similar. In fact, due to missiles long range, their time in the air is longer than the time required for mortar shells. Such differences also affect Q^i, the probability of successful response by Defender. A successful response to a mortar shell attack is less likely than for missiles. Moreover, following our discussion of learning, given the significant differences between missiles and mortar shell technologies, we may assume the learning related to these different technologies is also quite different. For example, if the attackers are hurt and new attackers must be trained, the cost to train squads to fire missiles is higher than the cost required to train attackers using mortar shells. With these differences (between missiles and mortar shells) and similarities (between Qassam and Grad, i.e., both missiles), it is reasonable to assume the Defenders Efficiency Ratio, $Qf_1\left(c^T\right)/c^T$, would be similar for the different missile type and differ for mortar shells and missiles. Together with *C3*, this discussion suggests that either mortar shells or missiles would be used when both governing bodies are stable.

C3.2: From April 2006 until March 2007 (when Grad, Qassam, and mortar shell are available for Terrorist and a finite period model is adequate) Qassam is more likely to be used but Grad may be used as well; in contrast mortar shells should not be used .

The data in IICC (2007) do not contradict *C2.1, C3.1* and *C3.2*. In the period August 2005 to March 2007, mortar shells were hardly used; rather, Terrorist used almost solely Qassam and starting on March 2006 Grad rockets. Table 10 summarizes the average number of mortar shells and missiles (Qassam or Grad) used during each of the relevant periods. (The data on mortar shells used during July 2005 to April 2007 includes the 6 mortar shells used as diversion on June 2006 when Gilad Shalilt was abducted; without these, the average mortar shells for this period would be 1.3.)

An alternative explanation for the decrease in the number of mortar shell attacks in July 2005 is that after the disengagement, mortar shells were not as effective; due to their limited range, such attacks could not aim at Israel forces. In addition, after the disengagement, it was easier for Terrorist to prepare for launching missiles in the absence of Israeli forces. (But mortar shells were used after the disengagement starting in April 2007.) Other factors influencing the mix of technologies are cost and availability. The cost of mortar shells is significantly lower and missiles may be unavailable to some Terrorist organizations. Similarly, different level of activity among organization, with different resources, may cause differences in the technologies used. Note: it is assumed that for several months starting in April 2007 Hamas focused on attacks using mortar shells whereas Fatah focused on attacks using missiles. But the joint use of both technologies has continued past this period, even after the Fatah lost most of its influence and its activity levels went down. Therefore, neither alternative clearly explains why Terrorist switched from using only mortar shells or only missiles to attacking with both technologies.

We find Conjectures C2.1, C3.1 and C3.2 yield insight into the level of violence and the mix of Terrorist's

Period	Average # of Mortar Shell/month	Average # of Missiles/month	Average # of Attacks/month
January 2001-July 2005	34.2	10.5	44.7
August 2005-March 2007	1.6	56.1	57.7
April 2007-November 2007	78.9	84.1	163

Table 10: Average number of mortar shell and missile attacks during the three periods

technologies used in similar conflicts. We note that data that would contradict these conjectures would show similar usage patterns of the different technologies in the periods studied. For example, during January 2001 to July 2005 about 23% of attacks were with Qassam missiles (this proportion is 30% if focusing on the period after the availability of the first Qassam, starting in January 2002). If a similar proportion of mortar shells were recorded during the period of August 2005 to March 2007, the data would contradict these conjectures.

It can be seen from Figure 2 that April 2007 marks the point of change from the second period to the third. Indeed, in March 2007, a single mortar shell and 31 missile attacks were recorded, whereas in August there were 35 mortar shell and 25 missile attacks. The change appears to occur in the middle of June 2005. (The number of mortar shells and missiles attacks recorded are: in May: 14 and 11; in June: 4 and 17; and in July: 1 and 28 respectively.) It appears Terrorist organizations started to act in accordance with the finite period equilibrium when Israeli disengagement approached and the uprising against President Abbas began.

Also, from these data and conjectures, it appears the Defenders Efficiency Ratio is similar among missiles and different for mortar shells. It seems both Qassam and Grad missiles are used by Terrorist.

In view of *C4* and as Israel single governing body was changed in January 2006, we conjecture:

C4.1: The level of violence may change in January 2006.

The data in IICC (2007) do not contradict *C4.1.* There seems to be a sharp increase in the level of violence in February 2006. Table 9 summarizes the average minimum, and maximum of number attacks during August 2005 to January 2006 and February 2006 to March 2007 (when the finite period model is adequate).

It can be seen from Figure 2 that February 2006 marks the point of change in the level of violence. In January 2006 no attacks were recorded, and in February 47 attacks were recorded. We note that in September 2005, 19 Palestinians were killed during a rally in Jabalyia refugee camp. Hamas blamed Israel for this and launched a barrage of 15 rockets. Thus, the 29 attacks in September 2005 represent an outlier without which the results above are even sharper.

Similarly, in view of *C4* and the changes in the Presidency in the Palestinian authority in October 2004 and January 2005 we conjecture:

C4.2: The level of violence may change in October 2004 and in January 2005.

The data in IICC (2007) do not contradict *C4.2.* Table 11 summarizes the average, minimum, and maximum number of attacks during each of the periods: February 2001, when the first attacks are recorded in IICC (2007), to September 2004, just before Arafat became terminally ill, October 2004 to January 2005, during the interim presidency of Rawhi Fattuh, and from February 2005 to July 2005, when President Abbas was still well supported in the Gaza strip. (The data in Figure 2 indicate a change in the violence level in September 2004, rather than October 2004 as *C4.2* predicts. This change may be attributed to the approval of the Israeli unilateral disengagement in October 2004, by the Israeli Parliament, the Kneset. The discussions of the disengagement started much earlier; in September 2004, the Israeli cabinet approved a plan to compensate Israelis who lived in

Period	Average # of Attacks/month	Min. # of Attacks/month	Max # of Attacks/month
February 2001-September 2004	35.8	2, July 03	184, September 04
October 2004- January 2005	179	132, November 04	244, December 04
February 2005-July 2005	27.2	0, March 05	49, April 05

Table 11: Average, minimum and maximum number of attacks in the three periods.

the Gaza strip.) As seen in Table 11, the level of violence changed in February 2005 once President Abbas came into power thus completely agreeing with *C4.2*.

Again, the findings for C4.1 and C4.2 are not surprising given our discussion in Section 5.3; however, it is worth noting that these conjectures based on our models appear to agree with the data.

In view of C5 and the political situation in the Gaza strip, we conjecture (similar to C2.1):

C5.1: Until July 2005 and after April 2007, i.e., when several technologies could be used by Terrorist organizations in Gaza, the number of attacks using mortar shells will be higher than using missiles.

We first note that an additional explanation for a higher number of attacks using mortar shells may be that mortar shells are cheaper than missiles and some Terrorist organizations in the Gaza strip may lack the money to purchase more missiles. The data we have may reject *C5.1*. Figure 2 shows that when both missiles and mortar shells are used, the number of mortar shells is typically higher than the number of rocket launches. Similar support for the conjecture can be obtained from the data in Table 6 for the period up to 2005 (and maybe in 2007). However, in both 2008 and 2009, there are more attacks using missiles than using mortar shells. This lack of support hints the insights from a finite period model do not carry through to the infinite settings. (This is not surprising; e.g., in a finite period game only a single technology will be used).

7 Summary

In this paper, we consider a continuous conflict type, where Terrorist routinely attacks and a passive Defender uses preventive actions. We discuss several games of Terrorist against Defender, their possible Nash equilibriums, and the implication of the equilibriums.

The first game is a single period game where Terrorist has a single technology and only two possible actions Attack or Not Attack. Defender also has two possible actions: Respond or Not Respond. In the second game, Terrorist can use two technologies. The third game is a finite multi period game where Terrorist's attack cost changes in accordance with the results of the previous period. The last game is an infinite repetition of the first game (possibly with several technologies).

We consider the conflict between Israel as Defender and different terrorist groups from the Gaza Strip as Terrorist. Starting in 2001, Terrorist has used mortar shells and different types of missiles to attack Israel. We use three data sets on these attacks to estimate the models' parameters.

Based on our theoretical development, we make several conjectures about the nature of the conflict. For example, our theory suggests that even when the political atmosphere is stable, cease-fire agreements are not likely to hold for long. Specifically, without permanent exogenous payoffs to Terrorist, cease-fire agreements are not sustainable. Moreover, if the exogenous payoffs are not permanent and could be used to improve Terrorist's technology, the cease-fire agreement may end up increasing the level of violence.

$T \backslash D$	NR	R
NA	$0, 0$	$-K^S Q, -c^D$
A	$-c^T + K^T P, -K^D P$	$-c^T + K^T P(1-Q) - K^S Q, -c^D - K^D P(1-Q)$

Table 12: Game table for the single period game with an active Defender.

Other conjectures are linked to the number of technologies used by Terrorist in different periods, and the level of violence in these periods. In view of the political situation in the Gaza strip and Israel we investigate whether our data support these conjectures and find they do not contradict our conjectures.

Note that our stylized models ignore several important issues such as public opinion and international support or pressure. Public opinion is often reflected in and influenced by the media. For example, an equilibrium when Defender is not responding might have followed the Israeli disengagement in 2005. Then, the number of Terrorist launches was relatively low. However, the public opinion in Israel often criticized the government for not reacting more harshly to these attacks. Recalling the subjective evaluation of the costs attributed to injuries or deaths motivating our conjecture 4, the level of violence may change not only with a change in a regime but also with public opinion. However, shifts that occur due to public opinion are typically slower than those related to a change in leadership. Therefore, shifts due to changes in public opinion are harder to investigate with our data.

Finally, we note that our qualitative results and conjectures are quite robust. For example, they hold even if Defender is active, as seen in the analysis of the model with active Defender below.

7.1 Active Defender

Consider the single period game where Defender is not passive, i.e., upon response she causes a damage of K^S to Terrorist with probability Q even if Terrorist does not attack, i.e., the game table for a single period is shown in Table 12.

Then, similar to Proposition 1 (proof is omitted) we get:

Proposition 5 *The mixed strategies* $\left((q_0^T, 1-q_0^T), (q_0^D, 1-q_0^D)\right)$ *given by*

$$\left(\left(\frac{PK^DQ - c^D}{PK^DQ}, \frac{c^D}{PK^DQ}\right), \left(\frac{c^T - K^TP + QK^TP}{QK^TP}, \frac{K^TP - c^T}{QK^TP}\right)\right)$$

consist a unique NE, leading to expected payoffs $\left(R^T, R^D\right) = \left(-\frac{K^TP - c^T}{K^TP}K^S, \frac{-c^D}{Q}\right)$.

When comparing the results of this game with an active Defender to that of the passive Defender game given in Proposition 1 we see that:

When the mixed NE is played, the expected payoff of Terrorist is negative, which is lower than in the passive Defender case. That is, an active Defender decreases Terrorist's expected payoff. This decrease is linearly increasing with K^S, the efficiency of Defender's attack. Moreover, Terrorist's attacking probability and therefore Defender's expected payoff are unchanged. An active Defender reduces Terrorist's payoff but does not change the violence level experienced by Defender's population. Finally, if Terrorist has several technologies, results similar to those in Sections 4-6 still hold. For example, in a single period game Terrorist would use a single technology and Remark 4 still holds: several technologies may be used in a long term conflict. Similar results hold in the presence of learning.

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A Detailed Discussion of Parameters Estimation

We next discuss the parameters' estimation. We note that (i) only data set 2, reported in Table 6, includes individual data on each of the three technologies, (ii) the data on accuracy and damage caused by Qassam rockets may be biased toward the 3^{rd} generation that was more heavily used (unfortunately, granularity with respect to the different generations of Qassam was not reported in either of our data sets), and (iii) our data capture hits possibly after Defender's response, causing us to underestimate the hit probability.

Based on the data from Table 6, we estimate $\bar{P}^i$ the probability of a successful hit by technology $i = 1, 2, 3$ as

$$\bar{P}^i = \frac{\sum_{j=2}^4 H_j^i}{\sum_{j=0}^4 H_j^i},$$

giving

$$\bar{P}^1 = 0.6, \quad \bar{P}^2 = 0.87, \quad and \quad \bar{P}^3 = 0.82.$$

Similarly, we estimate $\tilde{P}^i$ the probability of a successful hit by technology $i = 1, 2, 3$ as

$$\tilde{P}^i = \frac{\sum_{j=2}^4 \tilde{H}_j^i}{\sum_{j=0}^4 \tilde{H}_j^i},$$

giving

$$\tilde{P}^1 = 0.34, \quad \tilde{P}^2 = 0.75, \quad and \quad \tilde{P}^3 = 0.73.$$

χ^2 tests, not reported here in detail, suggest the accuracy of the three technologies is different, but the accuracy of technologies 2 and 3 may be identical. Finally, in the estimation of P^i in (8) denoted by $\hat{P}^i$ we prefer the latter, lower, estimations and let $\hat{P}^i = \tilde{P}^i$.

For estimating the average cost to Defender from an attack, $\hat{K}^{D_i}$, we need to aggregate the average cost of damage to property and life and then to relate these to each technology. The relative costs of injuries and deaths are subjective. Nevertheless, Miller et al. (1989) estimate the cost of different severities of injury types relative to death as follows: minor injury, $\alpha_1 = 0.002$, moderate injury, $\alpha_2 = 0.0155$, serious injury, $\alpha_3 = 0.0575$, severe injury, $\alpha_4 = 0.1875$, critical injury, $\alpha_5 = 0.7625$, and fatal injury $\alpha_6 = 1$.

Because data set 3 provides no details on the specific technology causing the injuries and data set 1 does not differentiate between missiles, we need to find an appropriate method to relate the total injuries to the different technologies. This is discussed next.

We first present the estimations based on data set 2, the most detailed one. Because this data set includes only 4 levels of damage, we estimate the expected damage from a successful hit caused by technology $i = 1, 2, 3$ as

$$\bar{K}^{Di} = \frac{\alpha_1 D_1^i + \alpha_2 D_2^i + \alpha_4 D_3^i + \alpha_6 D_4^i}{\sum\limits_{j=2}^{4} H_j^i},$$

leading to

$$\bar{K}^{D1} = 0.001, \quad \bar{K}^{D2} = 0.018, \quad \textit{and} \quad \bar{K}^{D3} = 0.059.$$

We note that the effect of D_1^i and D_2^i on the estimation of $\bar{K}^{Di}$ only influences the third digit (i.e., in the order of 0.1%); that is, the injuries and deaths due to attacks cause most of the damage. Therefore, we can also use data sets 1 and 3, which only include data on deaths and injuries, to estimate K^{D_i}.

Next, we estimate these damages based on data set 1. With regards to estimating the damage from mortar shells we note these data report 2543 mortar shells leading to 151 injuries and 10 deaths; thus, we estimate:

$$\dot{K}^{D1} = \frac{151\alpha_4 + 10\alpha_6}{2543} = 0.015.$$

Remember that the first usage of Grad was on March 2006, so the data in data set 1 on missiles hits during 2001-2005 pertain only to Qassam. We thus estimate based on Table 5 ($654 = 4 + 35 + 155 + 281 + 179, 249 = 0 + 3 + 37 + 148 + 61, 6 = 0 + 0 + 0 + 0 + 4 + 2$):

$$\dot{K}^{D2} = \frac{249\alpha_4 + 6\alpha_6}{654} = 0.081.$$

To estimate the average damage caused by a Grad missile, we divide the missile damage for 2006-2007 between Qassam and Grad in accordance with this division in data set 2, as reported in Table 6. The total numbers of rockets in data set 2 (reported under total H^i in Table 6) is 368, of which 263 or 71.5% are Qassam and 105 or 28.5%, are Grad rockets. Further, the total number of injuries from rockets in data set 2 (reported under D_3) is 35, of which 15 or 42.86% are injuries due to Qassam and 20 or 57.14% are due to Grad rockets. We follow the same procedure to estimate the percentage of deaths, starting in 2006 (after the first use of Grad) 50% of the deaths are due to Qassam and 50% are due to Grad. (We note that a more elaborate allocation of damage to the different technologies can be used, but the results are not significantly different.) Based on the data from 2006-2007 in data set 2 we estimate:

$$\begin{aligned}
\tilde{K}^{D2} &= \frac{184\alpha_4 * 0.4286 + 4\alpha_6 * 0.5}{1729 * 0.715} = 0.014 \\
\tilde{K}^{D3} &= \frac{184\alpha_4 * 0.5714 + 4\alpha_6 * 0.5}{1729 * 0.285} = 0.046
\end{aligned}$$

(where from Table 5, in 2006-2007, $1729 = 946 + 783$, $184 = 86 + 98$ and $4 = 2 + 2$).

To estimate the damages, $\overset{+}{K}^{Di}$, based on the third data set from the Shabak, we use a similar normalization. First, we normalize the number of injuries between mortar shells and missiles in accordance with the ratio of these injuries from data set 1 by IICC (2007), as reported in Table 5. From Table 7, in 2001, all 25 injuries are due to mortar shells; in 2002-2005, injuries are due to either mortar shells or Qassam, so the injuries are allocated to the technology according to the proportion of injuries due to this technology in this year in Table 5 (e.g., in 2002 $9/(9+3)$ of the 10 injuries are allocated to mortar shells); in 2006-2009, injuries may be due to any of the three technologies, so the injuries are allocated first between mortar shells and missiles (as for years 2002-2005 where, 42.86% of the injuries due to missiles are allocated to Qassam). In 2008-2009, where there are no detailed data from Table 5, the allocation between mortar shell and missiles is based on the aggregated numbers reported in this table. Specifically, the total number of injuries reported in Table 5 is 584, of which 151 or 25.86% are injuries due to mortar shells and 433 or 74.14%, are due to missiles (e.g., 25.86% of the 464 injuries in 2008 reported in Table 6 are allocated to mortar shells). With this procedure, we estimate the number of injuries due to mortar shells as 397.05, to missiles as 724, of which 375.5 are due to Qassam and 348.5 to Grad missiles. This leads to the following damage estimations:

$$\begin{aligned}
\overset{+}{K}^{D1} &= \frac{\alpha_4 * 288.5 + \alpha_6 * 14}{4397} = 0.015 \\
\overset{+}{K}^{D2} &= \frac{\alpha_4 * 375.5 + \alpha_6 * (9 + 9/2)}{6016 * .715} = 0.02 \\
\overset{+}{K}^{D3} &= \frac{\alpha_4 * 348.5 + \alpha_6 * (9/2)}{6016 * .285} = 0.041.
\end{aligned}$$

To summarize, we have the following different estimators

$$\begin{aligned}
\bar{K}^{D1} &= 0.001, \quad \bar{K}^{D2} = 0.018, \quad \textit{and} \quad \bar{K}^{D3} = 0.059. \\
\dot{K}^{D1} &= 0.015, \quad \dot{K}^{D2} = 0.081 \\
\tilde{K}^{D2} &= 0.014 \quad \tilde{K}^{D3} = 0.046 \\
\overset{+}{K}^{D1} &= 0.015 \quad \overset{+}{K}^{D2} = 0.02 \quad \overset{+}{K}^{D3} = 0.041.
\end{aligned}$$

Among these numbers there are two outliers. The first is $\bar{K}^{D1} = 0.001$, which is much smaller than the other estimations for the damage caused by mortar shells. This difference may be attributed to the relatively small sample of mortar shell attacks in the data shown in Table 6 (for example, if there was a single death caused by these attacks, the estimate for $\bar{K}^{D1}$ would be 0.093) so we ignore the estimates $\bar{K}^{D1}$. The second outlier is $\dot{K}^{D2} = 0.081$, which is much bigger than the other estimates for the damage caused by Qassam missiles. This difference may be attributed to the fact that this estimate is based on data before the disengagement, when Qassam missiles were often aimed at Israeli citizens in close range, i.e., within the Gaza strip. Thus, we ignore this estimate as well. Giving identical weight to the other estimates, we get our final estimates:

$$\begin{aligned}
\hat{K}^{D1} &= \frac{\dot{K}^{D1} + \overset{+}{K}^{D1}}{2} = 0.015, \\
\hat{K}^{D2} &= \frac{\bar{K}^{D2} + \tilde{K}^{D2} + \overset{+}{K}^{D2}}{3} = 0.017, \\
\hat{K}^{D3} &= \frac{\bar{K}^{D3} + \tilde{K}^{D3} + \overset{+}{K}^{D3}}{3} = 0.049.
\end{aligned}$$

As we have no data to estimate the value Terrorist gains from a successful hit, we normalize the expected benefit to Terrorist from a successful hit by technology i, $\hat{K}^{Ti}$, to equal the expected damage to Defender, K^{Di}. This normalization scales the utilities so that death is a single unit of utility for Terrorist and disutility for Defender. We normalize the expected cost to Terrorist from a successful interception, $\hat{K}^{\mathrm{Si}}$ in a similar fashion[2] leading to the estimates $\hat{K}^{Si}$ and $\hat{K}^{Ti}$ in (8).

Because data on the accuracy of interception trials, Q^i are confidential, we have no detailed data. We will use three values for accuracy. For simplicity, we assume these values are independent of the technology. Because the accuracy of the missiles in our data is between 0.6 and 0.9 and the accuracy of the response should be in the same range and because the Iron Dome missile interception unit employed by Israel in November 2012, during the "Pillar of Defense" operation, is reported to have a success probability of 80% (see Jerusalem Post 2013) we choose: $Q \in \{1, 0.8, 0.6\}$.

We will estimate $\hat{c}^{T_i}$ and $\hat{c}^D$ in a similar fashion. Recall from Proposition 1 that when $c^{Ti} < K^{Ti}P^i$ the single period game results in non violence. Similarly, Assumption (A.1) requires $c^D < K^D PQ$ (recall we assumed Defender's response costs are independent of Terrorist's technology). Therefore, because we assume $K^{Di} = K^{Ti}$, the costs c^{T_i} and c^D are of the same magnitude. This gives us the possibility of investigating three relations between these costs: when the cost c^D of responding relative to the damage caused to Defender from a hit is twice as large, equal, or half the cost c^T of attacking relative to the benefit to Terrorist from a hit. We thus estimate $\hat{c}^{Ti} = \lambda \hat{K}^{Ti} \hat{P}^i$. Given our explanation above, any choice of $\lambda \in [0, 0.5)$ would work; we use $\lambda = 0.25$ to get:

$$\hat{c}^{T1} = 0.001, \quad \hat{c}^{T2} = 0.003, \quad \mathit{and} \quad \hat{c}^{T3} = 0.008.$$

We assume the cost of responding is independent of the technology and choose $\min_i \left\{ \hat{c}^{Ti} \right\} = 0.001$, and

$$\hat{c}^D \in \left\{ .5\hat{c}^{T1}, \hat{c}^{T1}, 2\hat{c}^{T1} \right\} = \{0.0005, 0.001, 0.002\}.$$

Note that with this choice, even for $Q = 0.6$, our estimates satisfy Assumption (A.1) ($0.002 < \min_i \left\{ K^{Di} P^i Q \right\} = 0.015 * 0.34 * 0.6 = 0.003$).

B Proofs

Proof. of Proposition 1: The different equilibriums in Part a are easy to verify directly using iterative (weak) dominance: in a-1 NA is a (weak) dominant strategy for Terrorist. In a-2 NR is a (weak) dominant strategy for Defender and in a-3 A is a (weak) dominant strategy for Terrorist.

For Part b, we look for a mixed strategies Nash equilibrium $\left((q_0^T, 1 - q_0^T), (q_0^D, 1 - q_0^D) \right)$. For such an equilibrium, we need E(*payoff if Defender plays* NR) $=$ E(*payoff if Defender plays* R) and E(*payoff if Terorist plays* NA) $=$ E(*payoff if Terorist plays* A) . Thus

$$-\left(1 - q_0^T\right) K^D P = -q_0^T c^D - \left(1 - q_0^T\right) \left(c^D + PK^D (1 - Q)\right)$$

[2] We normalize K^{Si} in this fashion because it is consistent with the normalization of the damages from a hit by each technology. That a successful interception of a technology resulting in higher expected damage causes higher cost to Terrorists can be supported by the following: (i) the latter may be positively correlated with the amount of explosive in the technology — so that an interception causes more damage to the surrounding, and (ii) Terrorist operating a more advanced technology may be better trained and therefore more valuable.

and

$$0 = q_0^D\left(-c^T + PK^T\right) + \left(1 - q_0^D\right)\left(-c^T + PK^T - Q\left(PK^T + K^S\right)\right) \tag{4}$$

and the result follows. ■

Remark 5 *If we eliminate weakly dominated strategies in the proof of Proposition 1 we avoid multiplicity of equilibriums. Specifically, we get a unique NE if (i)* $K^TP - c^T < 0$ *or if* $K^TP - c^T = 0$ *and* $K^DPQ - c^D > 0$*, then* (NA, NR) *is a unique NE; (ii)* $K^TP - c^T > 0$ *and* $K^DPQ - c^D < 0$ *or if* $K^DPQ - c^D = 0$ *and* $K^TP(1-Q) - c^T - K^SQ < 0$*, then* (A, NR) *is a unique NE, or if (iii)* $K^TP(1-Q) - c^T - K^SQ \geq 0$ *and* $0 < K^DPQ - c^D$*, then* (A, R) *is a unique NE. If we do not eliminate the weakly dominant strategies, we get an infinite number of equilibriums if (i)* $K^DPQ - c^D = 0$ *and* $K^TP(1-Q) - K^SQ - c^T \geq 0$*, then there are multiple equilibriums* $(A, \cdot)$ *where Defender is indifferent between his strategies given that Terrorist is playing A and players' payoffs are* $(h_1, -K^DP)$ *where* $K^TP(1-Q) - K^SQ - c^T \leq h_1 \leq K^TP - c^T$*, or if (ii)* $K^TP - c^T = 0$ *and* $K^DPQ - c^D \leq 0$*, then there are multiple equilibriums* $(\cdot, NR)$ *where Terrorist is indifferent between his strategies given that Defender is playing* NR *and players' payoffs are* $(0, h_2)$ *where* $0 \leq h_2 \leq -K^DP$*.*

Proof. of Proposition 2: (a) If $K^{Tj}P^j - c^{Tj} < 0$ and $K^{Ti}P^i - c^{Ti} > 0$ $i = 1, 2,$ $j \neq i$ or $0 < -c^{Tj} + K^{Tj}P^j < -c^{Ti} + K^{Ti}P^i$ and $-c^{Tj} + K^{Tj}P^j\left(1 - Q^j\right) < -c^{Ti} + K^{Ti}P^i\left(1 - Q^i\right) < K^{\mathrm{S}i}$ $i = 1, 2,$ $j \neq i$, then strategy Ai dominates Aj and the players play the single technology game with the solution as above. We next check the mixed strategies solutions when neither of Terrorist's strategies immediately dominates the others.

First consider the case where Terrorist mixes strategies $A1$ and $A2$. In this case, Defender's dominant strategy would be to respond, causing Terrorist to prefer not to attack. Thus, this case does not result in an equilibrium.

Next consider the case when Terrorist mixes between NA and $A1$. This case is identical to the single technology game. Thus:

$$q_0^D = \frac{c^{T1} - P^1K^{T1} + Q^1P^1K^{T1} + Q^1K^{S1}}{Q^1\left(P^1K^{T1} + K^{S1}\right)}. \tag{5}$$

This leads to the payoffs: $\left(R^T, R^D\right) = \left(0, \frac{-c^D}{Q^i}\right)$. Terrorist mixing between NA and $A1$ is an equilibrium only if Terrorist payoff from deviating and playing the pure strategy $A2$ is negative, i.e., if:

$$\begin{aligned}
-c^{T2} + P^2K^{T2} - q_1^DQ^2\left(P^2K^{T2} + K^{S2}\right) &< 0 \\
\frac{-c^{T2} + P^2K^{T2}}{Q^2\left(P^2K^{T2} + K^{S2}\right)} &< q_1^D \\
\frac{-c^{T2} + P^2K^{T2}}{Q^2\left(P^2K^{T2} + K^{S2}\right)} &< \frac{-c^{T1} + P^1K^{T1}}{Q^1\left(P^1K^{T1} + K^{S1}\right)}.
\end{aligned}$$

This is the unique NE whenever (3) holds, because no pure strategy is an NE in this setting. Clearly, when the inequality sign in (3) changes direction. a similar argument results in the other NE in the proposition. Finally, part (b) follows because mixing $A1$ and $A2$ leads to the same expected payoff for both players. Standard derivation and some algebra leads to

$$\begin{aligned}
\left(q_0^D, 1 - q_0^D\right) &= \left(\frac{c^{T1} - K^{T1}P^1 + Q^1\left(K^{T1}P^1 + K^{S1}\right)}{Q^1\left(K^{T1}P^1 + K^{S1}\right)}, \frac{K^{T1}P^1 - c^{T1}}{Q^1\left(K^{T1}P^1 + K^{S1}\right)}\right) \\
&= \left(\frac{c^{T1} - K^{T1}P^1 + Q^1\left(K^{T1}P^1 + K^{S1}\right)}{Q^1\left(K^{T1}P^1 + K^{S1}\right)}, \frac{K^{T1}P^1 - c^{T1}}{Q^1\left(K^{T1}P^1 + K^{S1}\right)}\right)
\end{aligned}$$

and for and $q_0^T \in (0,1)$ q_1^T and q_2^T are the solution of the following two linear equations with two unknowns:

$$\begin{aligned} q_1^T + q_2^T &= 1 - q_0^T \\ q_1^T K^{D1} P^1 Q^1 + q_2^T K^{D2} P^2 Q^2 &= c^D, \end{aligned}$$

leading to expected payoffs $\left(R^T, R^D\right) = \left(0, \frac{-c^D}{Q^1} + q_2^T K^{D2} P^2 \left(\frac{Q^2}{Q^1} - 1\right)\right)$. ■

Proof. Of Proposition 3 First, we note that Terrorist's payoff in the last period ($n = 2$) is 0 independent of the results in period $n = 1$. Thus, Terrorist's expected payoff in the $N = 2$ game is the same as his payoff in a single period game. Therefore, the solution for Terrorist is identical to those given in Proposition 1.

Proof of part (i): If $K^T P \geq f_1\left(c_1^T\right)$, then independently of the results of the first period ($n = 1$) game, the NE in the second period ($n = 2$) game will be the one given in (b) of Proposition 1. This results in an expected payoff $\left(R_1^T, R_1^D\right) = \left(0, -c^D/Q\right)$; then the expected payoff in the first period's game is similar to the one given in table 2 but adding $-c^D/Q$ for the payoffs of Defender in all cells. Note however, that this addition does not change the optimal strategy of Defender or Terrorist, because it is a linear transformation of their utility. Thus, the solution given in (1) is optimal for this period as well.

Proof of part (ii): if $f_1\left(c_1^T\right) > K^T P \geq c_1^T$, then, if period 1 ended with "Terrorist Attack, Defender Response, Defender hits and Terrorist misses" the next period's attacking cost will be $c_2^T = f_1\left(c_1^T\right) > K^T P$. Thus, in period 2 the players would play the unique pure strategy NE given in (a1) of Proposition 1. However, for any other result in period 1, we would have $c_2^T \leq f_1\left(c_1^T\right) \leq K^T P$ so the next period's game would result in the mixed strategy NE given in (b) of Proposition 1. This results in an expected payoff $-c^D/Q$. Indeed, the expected payoff of period 2 in these cases (the result of period 1 game is not "Terrorist Attack, Defender Response, Defender hits and Terrorist misses") is independent of c_1^T. Thus, for the $N = 2$ periods game with $f_1\left(c_1^T\right) > K^T P \geq c_1^T$, the expected payoff in period 1 is given by:

$T \backslash D$	NR	R
NA	$0, -c^D/Q$	$0, -c^D\left(1 + 1/Q\right)$
A	$-c_1^T + K^T P, -K^D P - c^D/Q$	$-c_1^T + K^T P\left(1 - Q\right) - K^S Q,$ $-c^D - K^D P\left(1 - Q\right) - \left(1 - Q\right) c^D/Q$

that after rearranging terms and adding $-c^D/Q$ to all payoffs of Defender (this can be done without loss of generality because a linear transformation keeps the payoffs ordered as before) is equivalent to

$T \backslash D$	NR	R
NA	$0, 0$	$0, -c^D$
A	$-c_1^T + K^T P, -K^D P$	$-c_1^T + K^T P\left(1 - Q\right) - K^S Q, -K^D P\left(1 - Q\right)$

(Note the only difference between this table and the single period game is in Defender's payoff for the case (A, R); the payoff is higher in c^D.) It is easy to verify because $-c_1^T + K^T P > 0$ the last table game has no pure strategy NE. We now look for a mixed strategy Nash equilibrium $\left((q_{01}^T, 1 - q_{01}^T), (q_{01}^D, 1 - q_{01}^D)\right)$. In equilibrium, it should satisfy $E\left(payoff\ if\ Defender\ plays\ \ 0\right) = E\left(payoff\ if\ Defender\ plays\ \ 1\right)$; thus,

$$\begin{aligned} -\left(1 - q_1^T\right) K^D P &= -q_{01}^T c^D - \left(1 - q_1^T\right)\left(P K^D \left(1 - Q\right)\right) \\ q_1^T &= \frac{P K^D Q}{P K^D Q + c^D}, \end{aligned}$$

which (after adding the outcome of the first period $-c^D/Q$ to the payoff) leads to the expected payoff given in the Proposition.

Proof of part (iii) follows that of case (a) in Proposition 1. ■

Proof. Of Corollary 1 Assertions (a-d) of the Corollary follow by induction. The steps and logic behind the induction are similar to the analysis in Proposition 3. Assertion (e) follows, since by assertion (d) the expected payment faced by Terrorist in period n is the same as in the single period game, with c_n^T replacing c^T. Thus, the mixed strategy of Defender solves (4)–with c_n^T replacing c^T. ■

Proof. Of Proposition 4 If in period $n-1$ the mixed strategy is played (i.e., $c_{n-1}^T < K^T P$) we have

$$
\begin{aligned}
E\left(c_n^T | c_{n-1}^T < K^T P\right) &= \Pr\left(Terrorist\ Attacks\right) * \Pr\left(Defender\ \ \mathrm{Re}\,sponds\right) * Qf_1\left(c_{n-1}^T\right) \\
&\quad + \Pr\left(Terrorist\ Attacks\right) * \Pr\left(Defender\ \ doesn't\ \ \mathrm{Re}\,spond\right) f_{-1}\left(c_{n-1}^T\right) \\
&\quad + \Pr\left(Terrorist\ Doesn't\ Attack\right) c_{n-1}^T.
\end{aligned}
$$

Mathematically, we have

$$
\begin{aligned}
E\left(c_n^T | c_{n-1}^T < K^T P\right) &= \frac{c^D}{PK^D Q} \frac{\left(K^T P - c_{n-1}^T\right) Qf_1\left(c_{n-1}^T\right) + \left(QK^T P + QK^S - K^T P + c_{n-1}^T\right) f_{-1}\left(c_{n-1}^T\right)}{Q\left(K^T P + K^S\right)} \\
&\quad + \left(\frac{PK^D Q - c^D}{PK^D Q}\right) c_{n-1}^T \\
&= \frac{c^D}{PK^D Q} \frac{\left(K^T P - c_{n-1}^T\right)\left(Qf_1\left(c_{n-1}^T\right) - f_{-1}\left(c_{n-1}^T\right)\right)}{Q\left(K^T P + K^S\right)} \\
&\quad + \frac{c^D}{PK^D Q}\left(f_{-1}\left(c_{n-1}^T\right) - c_{n-1}^T\right) + c_{n-1}^T
\end{aligned}
$$

Then, because $\frac{c^D}{PK^D Q * Q\left(K^T P + K^S\right)} > 0$, we have $E\left(c_n^T | c_{n-1}^T < K^T P\right) > c_{n-1}^T$ if:

$$
\begin{aligned}
\left(K^T P - c_{n-1}^T\right)\left(Qf_1\left(c_{n-1}^T\right) - f_{-1}\left(c_{n-1}^T\right)\right) + Q\left(K^T P + K^S\right)\left(f_{-1}\left(c_{n-1}^T\right) - c_{n-1}^T\right) &> 0 \\
G\left(c_{n-1}^T\right) \frac{Qf_1\left(c_{n-1}^T\right) - f_{-1}\left(c_{n-1}^T\right)}{c_{n-1}^T - f_{-1}\left(c_{n-1}^T\right)} &> 1 \qquad (6)
\end{aligned}
$$

First, we observe that if $Qf_1\left(c_{n-1}^T\right)/c_{n-1}^T \leq 1$, the LHS above is always smaller than 1, implying part (i). In contrast, once $Qf_1\left(c_{n-1}^T\right)/c_{n-1}^T > 1$ and $c_{n-1}^T = c^{T*}$ as defined above, the LHS of (6) will equal 1 and we have $E\left(c_n^T | c_{n-1}^T < K^T P\right) = c_{n-1}^T$. If $c_{n-1}^T > c^{T*}$, i.e., $G\left(c_{n-1}^T\right) < G\left(c^{T*}\right)$ (a lower $P\left(Defender\ responses\right)$, then at $c_{n-1}^T = c^{T*}$), we have $E\left(c_n^T | c_{n-1}^T < K^T P\right) < c_{n-1}^T$, so that $G\left(c_{n-1}^T\right) < G\left(c_n^T\right)$; similarly $c^{T*} > c_{n-1}^T$ maintains the above inequality and results in $E\left(c_n^T | c_{n-1}^T < K^T P\right) > c_{n-1}^T$. This establishes part (ii). ■