## Nash Bargaining with Asymmetric Bargaining Power: Bargaining over Profit in a Simple Supply Chain Opher Baron • Oded Berman

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In this note we apply a Nash bargaining model with non-symmetric bargaining power to negotiation over profit division within a simple supply chain. We show that this bargaining leads to a profit sharing in proportion to the bargaining power within the chain. We also discuss a few implications of this bargaining model.

Keywords: Nash bargaining; Supply chain coordination.

### 1 Introduction

Supply chain coordination has attracted a lot of attention in the last two decades from both practitioners and researchers. In both communities it is widely accepted that coordinating a supply chain is a promising approach to attain a sustainable competitive advantage. Moreover, to coordinate a supply chain it is necessary to align the incentives of the firms in the chain, e.g., Lee, (2004) [14]. If a coordinated supply chain maximizes its profit and so do all the firms in the chain, a direct way to coordinate a supply chain might be to share the profit within the chain. Then, firms would try to increase their share of the chain's profit by bargaining over the profit division within the chain. The results of this bargaining process depend, of course, on the bargaining power of the different firms.

Note that in monopolistic markets with a single chain a Vertically Integrated (VI) supply chain is coordinated, e.g., Bernstein and Federgruen (2005) [2] and Cachon (2003) [5]. Consequently, much research has been devoted to supply chain contracts that induce retailers and suppliers to act as if they are vertically integrated, e.g. two part tariff, Bonnano and Vickers (1988) [4], buy back, Pasternack (1985) [17], quantity flexibility, Tsay (1999) [18] and Weng (1995) [19], and revenue sharing Cachon and Larivierre (2005) [6], see also Larivierre and Porteus (2001) [13]. For a survey of this literature see Cachon (2003) [5]. A salient attribute of these coordination contracts is that they support a wide array of profit divisions between the manufacturer and the retailer. However, these contracts do not dictate how to share the profit within the chain.

Thus, Cachon (2003) [5] (in pages 8-9) states that the division of the profit within a coordinated chain is a well worth research topic that has not been properly addressed in the literature. Exceptions are Ertogral and Wu (2001) [9] and Wu (2004) [20] who present a model of bargaining on the profit within a coordinated chain. Similar to Dukes et al. (2006) [8], they consider the outside options as the factor for asymmetry between firms. Thus, their model basically supports only a single profit division. In this paper, in contrast, we model asymmetric bargaining power within the chain explicitly and independently of the differences in the outside options. Our approach is in line with the use of the Nash Bargaining Product as explained in Binmore et al. [3]. (A similar Nash bargaining framework to investigate supply chain coordination in the presence of competition is proposed in Baron et. al. (2007) [1].) Moreover, due to the different bargaining power structures our results support a continuum division of profit.

As we mentioned above we investigate bargaining on the profit sharing using an asymmetric Nash bargaining framework. We believe that capturing asymmetry in the bargaining procedure within a supply chain is essential, because in many cases firms in the supply chain have different bargaining power (an example is WalMart). Our main contribution is applying the Nash bargaining concept for bargaining on the profit division within a supply chain. This helps to fill an existing gap in the literature on the division of profit within the supply chain. We show that our approach leads to a profit division that is proportional to the bargaining power of the firms within the chain.

The outline of the paper is as follows. In the next section we revisit the Nash bargaining model within the supply chain in accordance with Binmore et al. [3]. Section 3 presents a few models for bargaining on the profit within a supply chain. We summarize the paper and suggest research extensions in Section 4.

## 2 The Nash Bargaining Model

Here we review Nash's (1950) [16] model for bargaining between two players, based on Binmore et al. (1986) [3]. To apply this model for bargaining within the supply chain, we assume that the supply chain is composed of a manufacturer serving a single retailer. We denote the utilities of the retailer, the manufacturer, and the supply chain by  $\pi^R$ ,  $\pi^M$ , and  $\pi^{SC}$ , respectively.

The basic strategic bargaining model between two parties starts with the first party suggesting an offer that the second party can choose to accept or reject. If the second party accept the offer the bargaining stops and the outcome is the offer of the first party. However, if the second firm rejects the offer, it will come back with a counter offer after some time  $\Delta$ (where  $\Delta$  might approach 0). Now the first party can choose to accept or reject the offer. Again, if the offer is accepted the bargaining stops with that outcome, otherwise the first party comes back with a counter offer after time  $\Delta$ . The bargaining game continues with this procedure until either an agreement is achieved or until it is evident that an agreement can never be reached.

Binmore et al. (1986) [3] reviewed two strategic bargaining models:

The strategic bargaining model with time preferences: This model's main additional assumption is that the two bargaining parties prefer any reasonable result earlier rather than later.<sup>1</sup> We note that when the parties are bargaining over money, this model coincides with the use of a discount factor by both parties.

The strategic bargaining model with exogenous risk of breakdown: This Model's main assumption is that between any two offers there is a possibility that the parties will not come back to the negotiation table.<sup>2</sup> An example is when both parties bargain over gains from some business opportunity. Then as time passes along there is a chance that a third party would grasp this opportunity, leaving nothing to negotiate about.

Binmore et al. (1986) [3] gives a condition for both strategic bargaining models to have a unique perfect equilibrium. Furthermore, they establish that when  $\Delta$ , the time between offers, approaches 0, the outcome of this unique equilibrium is identical to a properly defined Nash bargaining model.

The Nash bargaining model, presented by Nash (1950) [16] and discussed by Kalai and Smordinsky, 1975 [12], can be formulated as a Nash Bargaining Product, e.g., Binmore et al. [3]. To apply this model to bargaining within a supply chain we assume that both the manufacturer and the retailer are profit maximizers and that they bargain on a value of  $\theta$ that influences their profits. For concreteness in the supply chain context one can think of  $\theta$  as the profit. We let the bargaining power of the manufacturer, denoted by  $\alpha \in [0, 1]$ , be

<sup>&</sup>lt;sup>1</sup>By Assumption 5 of Binmore et al. (1986) [3] reasonable result means that it is a result that if accepted at time 0 its payoff is higher than the option of never to agree.

<sup>&</sup>lt;sup>2</sup>By Assumption 9 of Binmore et al. (1986) [3] it is also required that the parties are risk averse. This still holds when assuming that firms are risk neutral, as Assumption 9 does not require strict risk aversion.

exogenously given; and thus  $1 - \alpha$  is the bargaining power of the retailer. Then, the cases  $\alpha = 0$  and  $\alpha = 1$  allow the retailer and manufacturer, respectively, to dictate the choice of  $\theta$ , and  $\alpha = 0.5$  reflects a balance power between the manufacturer and the retailer.

With this notation the Nash Bargaining Product model can be written as:

$$Max_{\theta}\left\{\widetilde{\Phi}\left(\theta\right)\right\} = Max_{\theta}\left\{\left(\pi^{M} - \widetilde{f}^{M}\right)^{\alpha}\left(\pi^{R} - \widetilde{f}^{R}\right)^{1-\alpha}\right\},\$$

where  $\tilde{\Phi}(\theta)$  is the Nash bargaining product,  $\tilde{f}^M$ ,  $\tilde{f}^R$  are the manufacturer's and retailer's threat points, respectively, and we suppress the explicit dependency of  $\pi^M$  and  $\pi^R$  in  $\theta$ . Additional requirement of the Nash bargaining approach is to have individual rationality, i.e., that the optimal profit for each firm is at least as high as when they do not bargain, e.g., Iyer and Villas-Boas (2003) [11] and Assumption 4 of Kalai and Smordinsky (1975) [12].

Thus, to properly define a Nash bargaining model we need to define both parties' utilities, threat points, and bargaining power. We consider the firms utilities as their profits, as is common in the modeling of risk neutral firms. As for the choice of the threat points, Binmore et al. (1986) [3] state that they should reflect the status quo between the negotiating parties rather than outside options. For the strategic bargaining model with time preferences this status quo is the utility gained by the parties during their negotiation. For the strategic bargaining model with exogenous risk of breakdown this status quo is the utility gained by the parties if the bargaining breaks down. We will explicitly discuss the values for the threat points when applying the Nash Bargaining model. Binmore et al. (1986) [3] suggest that asymmetry in bargaining power, which is often defined imprecisely, can be attributed to four main different factors: the asymmetry in preferences, threat points, beliefs on the environment, and the bargaining procedure. They note that the first two asymmetries are captured by the model. However, the last two lead to asymmetry that should be explicitly captured through differences in the bargaining power of the firms,  $\alpha$ .

While common sense might suggest that in practice firms have different bargaining power, it is not clear what factors actually contribute to different bargaining power structures. Below we discuss a few such factors, which were suggested in the literature, and their potential effect on the bargaining power in practice.

Binmore et al. (1986) [3] demonstrate that a different generation time for offers and counter offers causes an asymmetry in the bargaining procedure and therefore in the bargaining power. Specifically, if firm 1's time to generate offers is  $\Delta_1$  and firm 2's time to generate offers is  $\Delta_2$ , then  $\alpha$  the bargaining power of firm 1 is  $\Delta_2/(\Delta_1 + \Delta_2)$ . Thus, if one firm is slower to react the other firm's power increases. In practice a different time to return with a counter offer might be attributed to different operational efficiencies and decision making processes in the firms. For the time preference model Binmore et al. (1986) [3] highlight different discount factors as leading to asymmetry. In practice, different discount factors might be attributed to different sizes of firms (e.g., due to its sheer size WalMart discount factor is likely lower than the one faced by its suppliers). For the exogenous breakdown risk model, Binmore et al. (1986) [3] demonstrate that different believes on the probability of breakdown lead to asymmetric bargaining power. Recently, Hananay and Gal (2007) [10] show how asymmetries in information between bargaining parties result in an asymmetric bargaining power. They consider one party as a "surprised" one, i.e., only this party believes that some events are possible. Then, they suggested a simple bargaining procedure that implements the resulting asymmetric Nash bargaining solution.

The remaining of this note applies the asymmetric Nash bargaining framework to bargaining on the profit division in a simple supply chain, without further investigating the causes to the asymmetry in the bargaining power.

# 3 Nash Bargaining on Profit's Division in a Supply Chain

Here we present a few different bargaining procedures over profit division in a non coordinated supply chain or on the division of the extra profit resulting from coordinating the chain assuming a given bargaining power structure. When a supply chain coordinates its action to improve profitability its firms could bargain on the division of the extra profit. We denote this additional earnings by K and therefore the profit of the coordinated chain is  $\pi^{SC} + K = \pi^M + \pi^R + K$ . We next present additive and multiplicative models for bargaining over this profit. The equilibrium in these models suggest to divide the profit proportionally to the bargaining power.

### 3.1 Additive Bargaining Procedures

We discuss bargaining on the division of the extra profit, resulting from coordination of the chain. Within this model we discuss a different choice of threat points and show that this choice is equivalent to bargaining over the entire profit resulting in profit sharing within the chain.

#### 3.1.1 Bargaining on the Extra Profit

We assume that the firms negotiate on a proportion  $\gamma \in [0, 1]$  of the extra profit, K, to be given to the manufacturer. Mathematically, they solve

$$\max_{\gamma \in [0,1]} \left( Z\left(\gamma\right) \right) = \max_{\gamma \in [0,1]} \left( \pi^{M} + \gamma K - f^{M} \right)^{\alpha} \left( \pi^{R} + (1-\gamma) K - f^{R} \right)^{1-\alpha}.$$
 (1)

We consider the status quo as no coordinating takes place; thus, we set the threat points  $f^M = \pi^M$  and  $f^R = \pi^R$ , to solve

$$\max_{\gamma \in [0,1]} \left( Z\left(\gamma\right) \right) = \max_{\gamma \in [0,1]} K \gamma^{\alpha} \left( 1 - \gamma \right)^{1-\alpha}.$$

It is easy to verify that  $\gamma^* = \alpha$  is optimal. This leads to an extra profit for the manufacturer of  $K\alpha$  and the rest to the retailer. Thus, the extra profit are divided proportionally to the bargaining power.

This model can be considered an extension of the bargaining game for supply chain contracting proposed by Ertogral and Wu (2001) [9] and reviewed in Wu (2004) [20]. Using the notation of the latter, we let  $\psi$  denote the exogenous probability of breakdown in the bargaining process. Then after some algebra it can be seen that in their model the first firm to make an offer receives a portion  $1 - \frac{\psi^2}{2(2-\psi)}$  of the extra profit. This implies that the bargaining power of that firm is  $1 - \frac{\psi^2}{2(2-\psi)}$ . Thus, they consider the bargaining process as the sole factor determining the power of the different firms in the bargaining. However, as discussed above, we feel that in some supply chains their may be additional sources for the asymmetry in the bargaining power. Then, our model can be more appropriate.

#### 3.1.2 Profit Sharing

The Nash bargaining model in (1) becomes a model of profit sharing when we choose the threat points in a different manner. For example, if the Nash bargaining approximates the bargaining model with time preference the threat points,  $f^M$  and  $f^R$ , should reflect the status quo during negotiation. If during this time the firms do not operate their income is zero, the threat points are  $f^M = f^R = 0$ . This threat point choice implies the following Nash bargaining product

$$\max_{\gamma \in [0,1]} \left( Z\left(\gamma\right) \right) = \max_{\gamma \in [0,1]} \left( \pi^M + \gamma K \right)^{\alpha} \left( \pi^R + (1-\gamma) K \right)^{1-\alpha}$$

It is easy to verify that  $\gamma^* = \frac{\alpha(\pi^{SC}+K)-\pi^M}{K}$  is optimal. This leads to a manufacturer's profit of  $\alpha(\pi^{SC}+K)$  and the rest goes to the retailer. Thus, if this choice for  $\gamma^*$  satisfies

the individual rationality constraints, the total profit of the coordinated chain is divided proportional to the bargaining power, hence the name profit sharing. Verifying that this solution satisfies the individual rationality should be done given the explicit profits of the firms in the non coordinated chain.

### 3.2 Multiplicative Bargaining Procedures

Here we consider a multiplicative bargaining model and show its equivalence to the additive bargaining model above. In this case the profit of the manufacturer is multiplied by a factor  $\gamma_M \geq 1$  and the profit of the retailer are multiplied by  $\gamma_R \geq 1$ :

$$\max_{\gamma_{M},\gamma_{R}} \left( Z\left(\gamma_{M},\gamma_{R}\right) \right) = \max_{\substack{\gamma_{M},\gamma_{R} \\ s.t.}} \left( \pi^{M} \gamma_{M} - f^{M} \right)^{\alpha} \left( \pi^{R} \gamma_{R} - f^{R} \right)^{1-\alpha}$$

$$s.t.$$

$$\gamma_{M} \geq 1$$

$$\gamma_{R} \geq 1$$

$$\pi_{M} \left( \gamma_{M} - 1 \right) + \pi_{R} \left( \gamma_{R} - 1 \right) = K.$$

Using the third constraint,  $\gamma_R = \frac{\pi^M(1-\gamma_M) + \pi^R + K}{\pi^R}$  and then the constraint  $\gamma_R \ge 1$  can be written as

$$\gamma_M \le \frac{\pi^M + K}{\pi^M}.\tag{2}$$

We now choose  $\gamma_M = 1 + K/\pi^M$  and observe that this choice satisfies (2) with an equality and satisfies  $\gamma_M \ge 1$  if and only if  $\gamma \in [0, 1]$  in the additive model. After some algebra, it is seen that with this choice the multiplicative bargaining on extra profit becomes equivalent to the additive bargaining one, as given in (1). Therefore it is clear that it leads to the same profit division as for the additive bargaining model (for the same choice of threat points).

### 4 Discussion

There are a few implications of our results. First, our models show that the profit or extra profit, will be shared in proportion to the bargaining power of the participants in the supply chain. This might be used to estimate the relative bargaining power of a party (retailer or manufacturer) within a supply chain as its proportion of the supply chain's profit.

The empirical observations from Draganska et al. (2010) [7] that consider the German ground coffee industry agree with this estimation. Indeed, looking at their Tables 6 and 7

we observe that the relative bargaining power of the manufacturers and retailers reported are nearly identical to the relative manufacturer's and retailer's margin out of the of the total supply chain's margins.

A second implication is the use of profit sharing as a supply chain coordination mechanism. With such a contract firms would take their decisions (e.g., price, quantity, and effort level) based on a proportion of the chain's profit. Thus, they imitate a VI chain. Still, we believe that a more in depth investigation of profit sharing contracts is required before recommending to apply them.

Finally, if a VI chain is coordinated, our models support the use of coordination mechanism that assume that all profit goes to one of the parties and would later be shared (e.g. two part tariff as in Bonanno and Vickers (1988) [4]). This is true because our models suggest that even if the bargaining on the profit is done ex post, it would cause the parties to act as if the chain is vertically integrated. Note that the converse is also true. Our models do not support the use of such contracts when a VI is not coordinating, as in the presence of competition, e.g., the seminal work of McGuire and Staelin (1983) [15].

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