# Consumer Value-Maximizing Sweepstakes \& Contests 

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#### Abstract

Sweepstakes and contests are one of the most frequently used promotional tools. Consumers participating in sweepstakes or contests have an opportunity to win prizes through a random draw. The authors examine how commonly used sweepstakes formats that vary in the number of winners and the allocation of the total reward money between the winners impact consumer valuations of the promotion. Given a fixed amount of reward money, the paper examines alternative reward formats based on the promotional objectives, consumer risk aversion and degree of subaddivity. The authors test the analytical results using an experimental approach.


Key words: Sweepstakes, Sales promotion, Cumulative prospect theory

Sweepstakes and contests promotions are commonly employed with the Cox Direct $20^{\text {th }}$ Annual Survey of Promotional Practises reporting that $73 \%$ of the firms surveyed in 1997 used sweepstakes. According to the 2007 Promotional Marketing Association's Annual Promotions Industry Trend report, ${ }^{1}$ firms spent $\$ 1.83$ billion on sweepstakes and contests with expenditures forecasted to grow to $\$ 1.85$ billion in 2009. In the same survey, $10.5 \%$ of the marketers cited sweepstakes as their biggest expense in 2006. Firms spend considerable amounts of their communication budgets on the rewards as well as advertising these promotions. For example, in 2005, McDonalds' Monopoly Sweeps advertised \$200 million in prize money. There is considerable anecdotal evidence suggesting that managers consider sweepstakes a very effective tool in generating sales (e.g. Marketing News, 2000). For example, the "text-to-win" sweepstakes by Alltel in 2005 featuring a $\$ 1$ million grand prize received 71 million entries. ${ }^{2}$

Consider the following examples of sweepstakes promotions: In 2005, Sirius Satellite Radio conducted a sweepstake where one winner received a $\$ 2$ million prize the odds of winning depended on the number of entries received. Also in 2005, Quilted Northern Bath Tissue conducted a sweepstake where the odds of winning the $\$ 1$ million grand prize were 1 in $11,000,000$. In addition, there were 100,100 smaller prizes, where the odds of winning were 100100 divided by the number of entries received. Starbucks conducted a sweepstake in 2008 where they gave away 20 Costa Rica vacations worth $\$ 3,500$ each. Finally, in 2005, Allstate conducted a sweepstake rewarding each of the five winners a Cadillac worth \$50,000 with odds of winning were $1: 283,800$. Further, 1,000 consumers each won an Olympic sports bag worth $\$ 50$ with the odds of winning being 1:1419.

These examples illustrate the wide variation in how the rewards in sweepstakes and contests are allocated. The number of winners ranges from one in the Sirius Satellite Radio example to 100,101 in the Quilted Northern Bath Tissue case. The number of prize levels
awarded also varies from one level for Starbucks to several levels in the case of Quilted Northern Bath sweepstakes. Further, the differences in amounts between the levels of prizes are also considerably different. These examples raise the issue of how reward structure impacts consumer valuation of the promotion. Existing literature on sales promotions (e.g. Schultz, Robinson, and Petrison 1998), while emphasizing the critical importance of prize structure in designing sweepstakes and contests, offers no specific guidance. The prevailing industry wisdom is both sparse and lack of consensus. For example, Promotion Magazine suggests that the sweepstakes should offer at least 1:3 chance of winning. ${ }^{3}$ According to the research firm Envoy, ${ }^{4}$ a sweepstakes is effective only if "the prize is above $\$ 1000$ and a fair number of secondary prizes are included". The objective of this paper is to investigate how consumers' valuations of sweepstakes vary based on the sweepstakes design and consumer characteristics. To the best of our knowledge, this paper provides the first formal analysis on the design of a sweepstakes prize structure.

Literature investigating people's incentive to participate in lotteries offers insights to understand why sweepstakes are potentially effective. Friedman and Savage (1948) argue that since people's utility function is concave up to a point but later becomes convex, winning accords a possibility of reaching a state of high income that provides disproportionate benefits. Kwang (1965) presents an "indivisibility of expenditures" explanation according to which high ticket items like cars or boats cannot be divided infinitely. Since limited resources do not allow consumers to purchase fractions of products, rational consumers with limited income wishing to purchase both products will participate in lotteries. Another argument is the availability bias. Usually, a firm's communication messages emphasize only the winners, which increase the availability of the positive consequences of participation in the consumers' minds (Tversky and Kahneman, 1974). Finally, consumers may derive utility by just participating in some contests and sweepstakes (Chandon, Wansink and Laurent 2000).

Sweepstakes and contests differ in that while sweepstakes are promotions where winning is based on chance, contests also require some level of effort or skill. Very often, the skill required in most contests is minimal and does not differentiate between consumers. For example, in 2008, Degree deodorant used an online campaign to promote a contest where consumers entered into a drawing if they correctly answered trivia questions related to a TV show. Degree provided the answers by posting the content adjacent to the questions. We use the terms contests and sweepstakes interchangeably but our results are applicable only to those contests where the skill or effort level does not impact the outcome.

Designing a sweepstake or contest involves several interrelated decisions such as determining the total reward money, the number of winners and allocating the reward between the winners. Other decisions include deciding the theme of contest, type of prizes (cash or products), the duration and frequency of the sweepstake, whether the rewards are immediate or delayed and the amount of effort the consumer has to expend to participate. In some cases, contests are designed so that the odds of winning are based on the number of entries received. Therefore, sweepstake contests can either have risky prospects (actual winning odds are announced) or have uncertain prospects (actual winning odds are not known and depend on the number of entries received). In some of the motivational examples used earlier, the firm has provided the odds of winning on the packages.

The focus in this paper is only on the decisions pertaining to the reward structure. We examine some commonly used sweepstakes formats and provide insights on how consumer valuations depend on the number of winners, the number of levels of prizes, and the difference in the awards between the levels (reward spread). We first apply Cumulative Prospect Theory to investigate how consumers value different sweepstakes. We analyse the optimal sweepstakes design that maximizes consumers' anticipated value of participation.

We then present experimental studies that examine consumer preferences for different types of sweepstakes. The studies offer empirical validation for the key theoretical results.

## THEORETICAL MODEL

Our objective is to identify the sweepstakes formats that maximize consumers' valuation. We characterize the target consumers with two factors. The first is consumers' brand preferences. Brand preferences have been identified as a useful variable to design promotions. For example, brand switchers tend to use coupons quicker than brand loyals (Neslin 1990) implying that coupons should have shorter expiration dates (Krishna and Zhang 1999). Raju, Dhar and Morrison (1994) demonstrate that instantly redeemable coupons are more effective for targeting brand switchers while in-pack coupons are better for holding loyal consumers. The second factor we examine is based on how consumers evaluate risky prospects. Since a sweepstakes promotion has an uncertain outcome, we expect a consumer's valuation for such a promotion to depend on the consumers' risk attitudes. As our focus is limited to the design of the sweepstakes structure, we assume the price of the product as well as consumer's brand valuation to be fixed.

We use Cumulative Prospect Theory (CPT) to model consumers' valuation of the sweepstakes has two key elements: a value function with loss aversion and a weighting function that overweighs small probabilities. Later advances in the decision weighting functions develop a cumulative function, which transforms cumulative rather than individual probabilities. CPT incorporates both the original prospect theory and advances in cumulative weighting function (Luce and Fishburn 1991, 1995, Tversky and Kahneman 1992). Though widely accepted, CPT is not without controversy (see Birnbaum 2008).

## Sweepstakes Reward Structure

As we limit our focus to the design of the sweepstakes structure, we assume that the firm has allocated a fixed budget of $R$ to be distributed as prize money to the winners. We let $S$ denote the prize structure of a sweepstakes promotion where

$$
\begin{equation*}
S=\left\{r_{1}, m_{1} ; r_{2}, m_{2} ; \ldots r_{n}, m_{n}\right\} . \tag{1}
\end{equation*}
$$

In equation (1), the number of prize levels is denoted by $n ; r_{j}$ and $m_{j}$ denote the size of prize and number of winners for $j^{\text {th }}$ prize, respectively, both $r_{j}$ and $m_{j}$ are positive. Thus, the sweepstake defined by (1) offers $m_{l}$ number of first prize at the amount of $r_{1}, m_{2}$ number of second prize at the amount of $r_{2}$ and so on. Without any loss of generality, we assume that the prizes offered are in the form of cash. We acknowledge but do not explicitly model heterogeneity in valuation of non-cash prizes. Given the promotion budget constraint ( $R$ ), a feasible prize structure for the sweepstakes should satisfy the following budget conditions:

$$
\sum_{j=1}^{n} m_{j} r_{j}=R \text { and } r_{i}>r_{j}>0 \text { for any } 1 \leq i<j \leq n .
$$

## Transaction Utility

Consumers face a decision on whether or not to purchase the brand conducting the sweepstakes. For expositional simplicity, we consider purchase of multiple units as independent purchases. We use the term 'participate' in the contest to reflect that the consumer has decided to both purchase the brand and to avail of the opportunity to win a prize. We do not consider consumers who enter the sweepstakes without purchasing the brand as these rates are small. A consumer (i)'s value depends on whether the consumer participates in the sweepstakes and the amount of reward received at the end. We let $v_{i}^{0}$ represent the value that consumer $i$ would receive without sweepstakes. Then $v_{i}^{0}$ serves as the reference value that consumer $i$ should employ in judging the gain and loss from participating.

In evaluating a sweepstakes, a consumer's anticipated gain and loss depends on the consumers' preference for the brands as well as the sweepstakes outcome. To capture consumers' heterogeneous brand preferences, for simplicity, we divide the market into two segments: the first segment is defined as the high-brand valuation segment (denoted as $i=H$ ), and the second segment as low-brand valuation (denoted as $i=L$ ). The high-brand valuation segment consists of consumers who have a relatively higher preference for the brand relative to the competitors. Firms may target high-brand valuation consumers to increase consumption through purchase of additional units or encouraging purchase acceleration (Blattberg and Neslin 1990). Another reason is to reward current consumers (Kotler 1997) so as to increase their brand preferences in future periods and prevent defection (Schultz, Robinson and Petrison 1998). For example, several airlines place sweepstakes forms in their in-flight magazines that are available only to flyers who are current customers. Low-brand valuation customers prefer competitors' brands at the time the sweepstake is conducted. The primary objective of the sweepstakes or contests targeting these low-brand valuation consumers is to encourage brand switching or brand trial.

These two segments of consumers anticipate different levels of post-purchase utility. Since consumers of high-brand valuation segment prefer the brand on the particular purchase occasion with or without the sweepstakes promotion, their reference value $\left(v_{H}^{0}\right)$ is same as the value that they would receive without winning any prizes. These consumers will experience a gain from any prize that they might win. In contrast, the low-brand valuation consumers will select another brand if the sweepstake is not offered as they value the competing brand more on the particular purchase occasion. Therefore, the low-brand valuation segment's reference value is higher than the value that they would receive without winning any prize. The sweepstakes itself does not alter brand valuation. Rather, it alters their overall utility of purchasing the brand. Therefore, a low-brand valuation consumer will gain
from purchasing the firm's brand only if she wins a prize. If the consumer does not win any prize, then she experiences a loss which is the opportunity cost. $\tau$ denotes the opportunity (switching) cost of purchasing the less preferred brand. For simplicity, we do not consider important elements like the utility of participation and the disutility of effort involved in a sweepstake. When a consumer switches to the less-preferred brand with sweepstakes promotion, the true switching cost $\tau$ may include the cost of participation, e.g. time spent and anxiety (or fun) experienced.

For a sweepstake $S=\left\{r_{1}, m_{1} ; r_{2}, m_{2} ; \ldots ; r_{\mathrm{n}}, m_{\mathrm{n}}\right\}$, we allow for the possibility that some prizes could be smaller than the switching cost $\tau$. Suppose only the top $J$ prizes are larger than the switching cost. That is, $r_{j}>\tau$ for $l \leq j \leq J$ and $r_{j} \leq \tau$ for any $J<j \leq n$. Then the prizes $r_{j}(J<j \leq n)$ are not large enough to cover the opportunity cost that the low-brand valuation consumers incur. We now define the gain function as $g($.$) and the loss function as l($.$) . Both$ the gain and loss function depend on the size of prize that consumers receive. We recognize but do not explicitly model that consumers may not always claim the prizes that they win particularly when the prize is small and the transaction cost to receive the prize is high. Suppose that a consumer receives a prize $r$. If the consumer belongs to the high-brand valuation segment, then she experiences a gain of $g(r)$. If the consumer belongs to the lowbrand valuation segment, then she will experience a gain of $g(r-\tau)$ if the reward is greater than the opportunity cost $(r \geq \tau)$ and will experience a loss of $l(r-\tau)$ if the reward is lower than the opportunity cost $(r<\tau)$. Both the gain function $g($.$) and the loss function l($.$) are positive,$ monotonically increasing, and concave:

$$
\begin{equation*}
g(x)>0, g^{\prime}(x)>0, g^{\prime \prime}(x) \leq 0, l(x)>0, l^{\prime}(x)>0 \text {, and } l^{\prime \prime}(x) \leq 0 . \tag{2}
\end{equation*}
$$

A loss is more significant than a gain of the same amount due to loss-aversion behaviour; that is, $g(x)<l(x)$ for any positive value of $x$. Loss-aversion behaviour was originally modelled by

Tversky and Kahneman (1991) and has been demonstrated in the marketing domain (for examples see Loewenstein and Prelec 1993 and Hardie, Johnson, and Fader 1993).

Following Cumulative Prospect Theory (Luce and Fishburn 1991, 1995, Tversky and Kahneman 1992), consumers form an anticipated value from a sweepstake $(S)$ given in equation (1) by assigning decision weights $\omega_{\mathrm{j}}$ to the valuation of the prize $r_{j}$ (either a gain or a loss). The anticipated value for the sweepstakes $S$, denoted as $\mathrm{V}_{\mathrm{i}}(\mathrm{S})$ for segment $i$, can be formulated as follows for the high- and low-brand valuation segments respectively:

$$
\begin{align*}
& V_{H}(S)=\sum_{j=1}^{n} \omega_{j} g\left(r_{j}\right)+\omega_{0} g(0),  \tag{3}\\
& V_{L}(S)=\sum_{j=1}^{J} \omega_{j} g\left(r_{j}-\tau\right)-\sum_{j=J+1}^{n} \omega_{j} l\left(\tau-r_{j}\right)-\omega_{0} l(\tau) . \tag{4}
\end{align*}
$$

Naturally, decision weights $\omega_{j}$ depend on the odds of winning the prize $r_{j}$. Both the utility formulation (3) and (4) and above assumptions are standard in Cumulative Prospect Theory. Next, we discuss the properties of the weighting functions in more depth.

## The Weighting Function

In CPT, the value of each outcome is multiplied by a decision weight. The decision weight measures the perceived likelihood of an event. CPT adopts rank-dependent cumulative decision weighting functions (Quiggin 1982, Tversky and Kahneman 1992, Luce and Fishburn 1991, 1995, and Tversky and Wakker 1995). Consider the case of risky prospect sweepstakes where the consumers know the odds of winning. For the sweepstakes defined in (1), we denote the total number of product units as $N$. When consumers do not know the actual number of entries, sweepstakes become uncertain prospects and may introduce ambiguity. In our analysis $N$ is replaced by consumers' expected number of entries formed according to their beliefs on a probabilistic distribution of $N$. A different $N$ may change the quantitative results, e.g. the optimal number of small prizes; however, it will not
change the qualitative results stated in the propositions. We focus on the case with known odds.

The participant's actual chance to win the first prize is $m_{l} / N$, the chance to win the second prize is $m_{2} / N$, and so on. The cumulative probability for the event of winning one of top $j$ prizes is $\sum_{k=1}^{j} m_{k} / N$. The corresponding cumulative decision weight for the event of winning one of top $j$ prizes is $\omega\left(\sum_{1}^{j} m_{k} / N\right)$ where $\omega$ is the cumulative decision weighting function. The decision weight $\omega_{\mathrm{j}}$, associated with winning the $j^{\text {th }}$ prize $(j=1,2 \ldots . n)$ is the difference between the cumulative weights of the event of winning one of top $j$ prizes and winning one of top ( $j$-1) prizes; that is, $\omega_{j}=\omega\left(\sum_{k=1}^{j} m_{k} / N\right)-\omega\left(\sum_{k=1}^{i-1} m_{k} / N\right)$. We use the term rank of the prize, where rank of 1 implies that it is the first prize corresponding to the highest value. For any rank $j=1,2 \ldots . . n$, we obtain the decision weight $\omega_{\mathrm{j}}$ associated with winning the $j^{\text {th }}$ prize as follows.

$$
\begin{align*}
& \omega_{1}=\omega\left(\frac{m_{1}}{N}\right), \omega_{2}=\omega\left(\frac{m_{1}+m_{2}}{N}\right)-\omega\left(\frac{m_{1}}{N}\right), \ldots \omega_{j}=\omega\left(\sum_{k=1}^{j} m_{k} / N\right)-\omega\left(\sum_{k=1}^{j-1} m_{k} / N\right), \ldots \\
& \omega_{n}=\omega\left(\sum_{k=1}^{n} m_{k} / N\right)-\omega\left(\sum_{k=1}^{n-1} m_{k} / N\right), \text { and } \omega_{0}=1-\omega\left(\sum_{k=1}^{n} m_{k} / N\right) \tag{5}
\end{align*}
$$

where $\omega_{0}$ denotes the decision weight for the event of "not winning any prize". The cumulative decision weighting function $\omega($.$) is a s$-shaped function, $\omega(0)=0$ and $\omega(1)=1$. Such a weighting function over-weighs small probabilities and under-weighs moderate and high probabilities. The $s$-shaped weighting function has been empirically verified in many studies, e.g., Kahneman and Tversky (1979), Tversky and Kahneman (1992), Camerer and Ho (1994), and Wu and Gonzalez (1996), etc. Specific $s$-shaped functions have also been
suggested by Tversky and Kahneman (1992) and Prelec (1998) and further estimated using experimental data by Camerer and Ho (1994) and Wu and Gonzalez (1996).

Due to the rank-dependent nature of the weighting function, weights of winning large prizes are evaluated earlier and hence inflated more (relative to actual probabilities). Thus, the ratio of decision weight and actual probability of winning a particular prize, which measures the degree of overweighting the probability of winning, is larger at higher ranks:

$$
\begin{equation*}
\frac{\omega_{j}}{m_{j}}>\frac{\omega_{k}}{m_{k}} \text { for any rank } j<k \tag{6}
\end{equation*}
$$

Consumers' risk attitude depends on both the value functions and the decision weights. First, following the standard approach in CPT, we conceptualise risk aversion only through the value function where a consumer's risk aversion with gain (loss) increases with the concavity of gain (loss) function. Second, in order to compare alternative decision weight functions, we adopt the measure of subadditivity proposed by Tversky and Wakker (1995). Higher subadditivity is interpreted as an ordering by departure from the actual probability of the corresponding outcome. This measure is taken independent of the value functions.

Several design elements of a sweepstakes can manipulate the level of subadditivity. For example, increasing the effort level required to participate (e.g. completing a form or collecting game pieces) or selecting own lottery numbers (Gonzalez and Wu, 1999) could create an illusion of control (Langer 1975) that may increase subadditivity.

## Value-Maximizing Sweepstakes

Our objective is to investigate the design of sweepstakes that maximizes consumers' valuation. We use value rather than profit maximizing as we do not explicitly model the costs of implementing alternative formats. Clearly, increasing the amount of prize money will increase consumer valuations. We only compare prize structures that meet the same budget
condition $\sum_{j=1}^{n} m_{j} r_{j}=R$. We use $\mathrm{V}_{\mathrm{H}}(\mathrm{S})$ and $\mathrm{V}_{\mathrm{L}}(\mathrm{S})$ to denote consumers' objective function for sweepstakes targeting the high-brand and low-brand valuation segment respectively.

Utilizing the value functions in (3) and (4) and decision weights in (5), the consumers' anticipated utility of sweepstakes participation can be characterized as follows:

$$
\begin{align*}
V_{H}(S)= & \omega\left(\frac{m_{1}}{N}\right) g\left(r_{1}\right)+\sum_{j=2}^{n}\left(\omega\left(\sum_{k=1}^{j} m_{k} / N\right)-\omega\left(\sum_{k=1}^{j-1} m_{k} / N\right)\right) g\left(r_{j}\right)+\left(1-\omega\left(\sum_{k=1}^{n} m_{k} / N\right)\right) g(0)(7) \\
V_{L}(S)= & \omega\left(\frac{m_{1}}{N}\right) g\left(r_{1}-\tau\right)+\sum_{j=2}^{J}\left(\omega\left(\sum_{k=1}^{j} m_{k} / N\right)-\omega\left(\sum_{k=1}^{j-1} m_{k} / N\right)\right) g\left(r_{j}-\tau\right)- \\
& \sum_{j=J+1}^{n}\left(\omega\left(\sum_{k=1}^{j} m_{k} / N\right)-\omega\left(\sum_{k=1}^{j-1} m_{k} / N\right)\right) l\left(\tau-r_{j}\right)-\left(1-\omega\left(\sum_{k=1}^{n} m_{k} / N\right)\right) l(\tau) \tag{8}
\end{align*}
$$

In equation (7), a high-brand valuation consumer anticipates a gain with any winning reward from the sweepstakes, and no gain otherwise. The weights allocated to rewards are rankdependent and cumulative in the sense that the decision weight for winning one of the top $j$ prizes is $\omega\left(\sum_{k=1}^{j} m_{k} / N\right)$. As a result, the weight allocated to $j t h$ reward is the net difference between cumulative weights for top $j$ and $j-1$ prizes, i.e. $\omega_{j}=\omega\left(\sum_{k=1}^{j} m_{k} / N\right)-\omega\left(\sum_{k=1}^{j-1} m_{k} / N\right)$. For example, the weight for the highest reward is $\omega_{1}=\omega\left(m_{1} / N\right)$, weight for the second reward is $\omega_{2}=\omega\left(m_{1}+m_{2} / N\right)-\omega\left(m_{1} / N\right)$, and so on. In equation (8), a low-brand valuation consumer anticipates a gain if she expects to win one of the top $J$ prizes, but a loss otherwise. The size of gain or loss is determined by the absolute value of difference between switching $\operatorname{cost} \tau$ and reward $r$. The decision weights in equation (8) are rank-dependent and cumulative, too, as in equation (7).

## MODEL ANALYSIS

As suggested in marketing texts (e.g. Kotler 1997, Schultz, Robinson, and Petrison 1998), the strategic use of sweepstakes and contests should start with a clear promotion objective: either to retain loyal customers (high-brand valuation segment) or to acquire switchers (low-brand valuation segment). We first analyse the case where the sweepstakes promotion targets the high-brand valuation segment and follow with targeting of the low-brand valuation segment case. The prize structure of the sweepstakes most valued by each target segment is examined and conclusions drawn for general scenarios as well as for specific type of consumers' risk attitudes. Finally, the formats for both segments are compared.

## High-brand valuation Segment

Problem (P1) solves the value-maximizing sweepstakes design for consumers of the highbrand valuation segment.

$$
\begin{equation*}
\operatorname{Max}_{s} V_{H}(S)=\omega\left(\frac{m_{1}}{N}\right) g\left(r_{1}\right)+\sum_{j=2}^{n}\left(\omega\left(\frac{\sum_{k=1}^{j} m_{k}}{N}\right)-\omega\left(\frac{\sum_{k=1}^{j-1} m_{k}}{N}\right)\right) g\left(r_{j}\right)+\left(1-\omega\left(\frac{\sum_{j=1}^{n} m_{j}}{N}\right)\right) g(0) \tag{P1}
\end{equation*}
$$

$$
\text { s.t. } \sum_{j=1}^{n} m_{j} r_{j}=R
$$

We solve problem (P1) in web appendix 1 and obtain the following optimality condition:

$$
\begin{equation*}
\frac{\omega_{j}}{m_{j}} g^{\prime}\left(r_{j}^{*}\right)=M_{H}^{*}(j=1,2, . . n) \tag{9}
\end{equation*}
$$

where $M_{H}^{*}$ is the marginal anticipated value from an increase in prize, which is identical across all positive prizes. Equation (9) shows that $M_{H}^{*}$ increases with $\omega_{j} / m_{j}$ (the degree of actual winning probability being overweighed) and $g^{\prime}\left(r_{j}^{*}\right)$ (marginal utility from gain).

Next, we discuss the implications of optimality condition (9) on the design of the sweepstakes based on the consumer's risk aversion characteristics. We first analyse the
value-maximizing format of sweepstakes targeting high-brand valuation consumers who are risk neutral in gain. We will later study the implications of risk aversion.

Sweepstakes for High-brand valuation Consumers Risk Neutral in Gain
When high-brand valuation consumers are risk neutral in gain, the marginal value of gain $\mathrm{g}^{\prime}\left(\mathrm{r}_{\mathrm{j}}\right)$
becomes constant. Since $\frac{\omega_{j}}{m_{j}}>\frac{\omega_{k}}{m_{k}}$ for any $j<k$ (equation (6)), optimality condition (9)
implies only one prize level $(n=1)$. In addition, among the sweepstakes that have only one level of prize, the value-maximizing solution is to have only one winner because $\frac{\omega(1 / N)}{1 / N}>\frac{\omega\left(m_{1} / N\right)}{m_{1} / N}$ for any $m_{l}>1$. We summarize the above result in Proposition 1.

Proposition 1: When high-brand valuation consumers are risk neutral in gain, the valuemaximizing format of sweepstakes is to offer one winner-take-all grand prize only; that is, $S_{H}^{*}=\left\{r_{1}^{*}=R, m_{1}^{*}=1 ; r_{j}^{*}=0, j>1\right\}$.

The intuition behind the result of Proposition 1 lies in the rank-dependent $s$-shaped decision weighting function. Such a decision weighting function overweighs the chance of winning larger prizes (or higher ranks). Therefore, when high-brand valuation consumers are risk neutral in gain, allocating more of the prize money to larger prizes increases high-brand valuation consumers' anticipated value for the sweepstakes. Additionally, for a specific high rank, since a lower winning probability is overweighed more, it is value-maximizing to offer one grand prize only.

## Sweepstakes for High-brand valuation Consumers Risk Averse in Gain

When high-brand valuation consumers are risk averse, more specifically, when $\omega_{1} \mathrm{~g}^{\prime}(\mathrm{R})<\omega_{2} \mathrm{~g}^{\prime}(0)$ and $\mathrm{m}_{1}=\mathrm{m}_{2}=1$, they should prefer sweepstakes that offer multiple levels of prizes. This condition implies that when a grand prize is slightly reduced to create a small second prize, a consumer's sweepstakes valuation will increase. On one hand, the probabilities of winning higher ranks are still overweighed more $\left(\omega_{1}>\omega_{2}\right)$. On the other hand,
since the top prize is already sufficiently large, adding the remaining budget to the prize will increase the consumers' valuations for the sweepstakes by a very small amount. The consumers' valuations can be much higher if the remaining budget is used to create some smaller prizes $\left(\mathrm{g}^{\prime}(\mathrm{R})<\mathrm{g}^{\prime}(0)\right)$.

To design a sweepstake that offers multiple levels of prizes, the firm needs to decide the number of winners for each rank of prize $\left\{\mathrm{m}_{1}, \mathrm{~m}_{2}, \ldots, \mathrm{~m}_{\mathrm{n}}\right\}$ and the inter-rank spread (difference in size of prize between consecutive ranks). For example, if the first prize is $\$ 800,000$ and the second prize is $\$ 200,000$ then the spread is $\$ 600,000$. If the first prize is $\$ 700,000$ and the second prize is $\$ 300,000$, the spread is $\$ 400,000$. Surprisingly, we find that the value-maximizing number of winners for each rank of prize is one (See web appendix 1 for proof). Our proof shows that if a sweepstake has multiple winners for rank $j$, it is always possible to improve consumers' anticipated value of participation by dividing the reward money to multiple levels of rewards. We therefore conclude that the consumer valuemaximizing prize structure should offer only a single prize at every level. However, this theoretical guideline is rarely observed in practice. We speculate on two possible reasons for this departure from theory. First, the cost of implementing and communicating such a prize structure could be high. Second, when the prizes are products rather than cash, the firm may obtain quantity discounts which leads to prizes that are highly valued by consumers but are low cost to the firm. Thus, a trade-off needs to be made between using several levels of prizes to increasing the attractiveness of the sweepstake and the implementation costs of administering these levels of prizes.

When a sweepstakes consists of a single prize at every level, optimality condition (9) becomes

$$
\begin{equation*}
(\omega(j / N)-\omega(j-1 / N)) g^{\prime}\left(r_{j}^{*}\right)=M_{H}^{*} \tag{10}
\end{equation*}
$$

for $j=1,2, . . n$. Condition (10) implies that, given a decision weighting function ( $\omega$ ), the ratio of marginal gains from consecutive ranks $\left(g^{\prime}\left(r_{j}^{*}\right) / g^{\prime}\left(r_{j+1}^{*}\right)\right)$ is a constant. When consumers are increasingly risk averse, their marginal utility functions become steeper. For a given value of $r_{j+1}^{*}$, a smaller $r_{j}^{*}$ is required to satisfy condition (10) leading to a smaller inter-rank spread $d_{j}^{*}$. Therefore, when high-brand valuation consumers are more risk averse, they prefer a sweepstake with more winners and smaller spreads. We can also see from condition (10) that when the decision-weighting function is more sub-additive, the ratio $\left(g^{\prime}\left(r_{j}^{*}\right) / g^{\prime}\left(r_{j+1}^{*}\right)\right)$ is smaller. Following a similar line of reasoning as above, we can conclude that given a fixed level of risk aversion, high-brand valuation consumers prefer sweepstakes that have larger inter-rank spreads and fewer winners. We summarize these results in following proposition.

## Proposition 2:

(i) If high-brand valuation consumers are sufficiently risk averse $\left(\omega_{1} \mathrm{~g}^{\prime}(\mathrm{R})<\omega_{2} \mathrm{~g}^{\prime}(0)\right)$, the value-maximizing sweepstakes should consist of more than one prize.
(ii) The optimal number of prizes increases but the inter-rank spread decreases with (a) increasing consumer risk aversion and (b) decreasing subadditivity.

## Low-brand valuation Segment

A key distinction from the high valuation segment is that the low-brand valuation consumers will experience a loss when they do not win any prize - they incur an opportunity cost of not purchasing the preferred brand on the specific purchase occasion. Problem (P2) defines the value-maximizing sweepstakes for the low-brand valuation segment.

$$
\begin{align*}
\operatorname{Max}_{s}(S) & =\omega\left(\frac{m_{1}}{N}\right) g\left(r_{1}-\tau\right)+\sum_{j=2}^{J}\left(\omega\left(\sum_{k=1}^{i} m_{k} / N\right)-\omega\left(\sum_{k=1}^{j-1} m_{k} / N\right)\right) g\left(r_{j}-\tau\right)  \tag{P2}\\
- & \sum_{j=J+1}^{n}\left(\omega\left(\sum_{k=1}^{j} m_{k} / N\right)-\omega\left(\sum_{k=1}^{i-1} m_{k} / N\right)\right) l\left(\tau-r_{j}\right)-\left(1-\omega\left(\sum_{j=1}^{n} m_{j} / N\right)\right) l(\tau)
\end{align*}
$$

s.t. $\sum_{j=1}^{n} m_{j} r_{j}=R$

## Size of the Minimum Prize

Our first finding is that value-maximizing prize structure of the sweepstakes for lowbrand valuation segment should only consist of prizes at least as large as opportunity cost $\tau$. If a consumer does win a prize, she should anticipate either a gain or at least indifference (no gain and no loss). We state the result in Lemma 1 (See web appendix 2 for a formal proof).

Lemma 1: In the value-maximizing sweepstakes for the low-brand valuation segment, the lowest prize should be at least as large as the opportunity cost $\tau$.

Lemma 1 is driven by two assumptions. First, since the loss function is concave, the marginal utility becomes larger when the size of prize increases closer to the switching cost. Second, given the same budget, an increase in amount of the lowest prize leads to a decrease in the number of last prize winners. This reduces the probability of winning the last prize (while the chance to win all other prizes remain the same), and implies a greater marginal decision weight for the last prize. Combining these two effects, we conclude that a marginal increase in the amount of the last prize closer to opportunity $\operatorname{cost} \tau$ will always increase the low-brand valuation consumers' anticipated value for the sweepstakes. In the rest of analysis on problem (P2) we will apply the result of Lemma 1.

Similar to our analysis of the high-brand valuation segment, we now derive the optimality conditions for the low-brand valuation segment, $S_{L}^{*}=\left\{r_{1}^{*}, m_{1}^{*} ; r_{2}^{*}, m_{2}^{*} ; \ldots r_{n}^{*}=\tau\right.$, $\left.m_{n}^{*}\right\}$. (Please see web appendix 2 for the derivation.)

$$
\begin{align*}
& \frac{\omega_{j}}{m_{j}} g^{\prime}\left(r_{j}^{*}\right)=M_{L}^{*} \quad(j=1,2, . . n-1)  \tag{11}\\
& \frac{\omega_{n}}{m_{n}} l^{\prime}(0) \geq M_{L}^{*} \geq \frac{\omega_{n}}{m_{n}} g^{\prime}(0) \tag{12}
\end{align*}
$$

where $M_{L}^{*}$ is the marginal value that is identical across all the top $(n-1)$ levels of prizes.

Optimal sweepstakes contains a bottom prize equal to $\tau$ if (12) holds. Since $l^{l^{\prime}(0)} / g^{\prime}\left(r_{j}\right)$
measures the degree of loss aversion, conditions (11) and (12) indicate that the valuemaximizing sweepstakes for the low-brand valuation consumers should consist of a small prize when the low-brand valuation consumers are sufficiently loss averse. Since consumers are loss averse, they anticipate a much higher value of participation once the chance to experience the loss is reduced.

## Sweepstakes for Low-brand valuation Consumers Risk Neutral in Gain

When low-brand valuation consumers are risk neutral in gain, we find the valuemaximizing sweepstake consists of only two levels of prizes: one should be larger than opportunity $\operatorname{cost}(\tau)$ and the other should be equal to $\tau$. The reason for only one large prize follows the same logic as behind Proposition 1: since the probability of winning the top prize is overweighed most, only a grand prize can satisfy equation (11).

In addition to the 'grand' prize, the value-maximizing sweepstakes for the low-brand valuation segment also contains many small prizes equal to $\tau$. According to (12), the sufficient condition for such a prize structure to be value-maximizing is

$$
\begin{equation*}
l^{\prime}(0) \frac{\omega_{2}}{m_{2}} \geq g^{\prime}\left(r_{1}-\tau\right) \frac{w_{1}}{1} \tag{13}
\end{equation*}
$$

This condition holds when the low-brand valuation consumers are sufficiently loss averse.
Next, we analyse the optimal size of the first prize $\left(r_{1}\right)$ and number of small-prize winners $\left(m_{2}\right)$. In web appendix 2 we derive the optimality condition for the valuemaximizing number of small-prize winners:

$$
\begin{equation*}
\omega^{\prime}\left(1+m_{2}^{*} / N\right)=\frac{\omega(1 / N)}{1 / N} \frac{g^{\prime}\left(r_{1}^{*}-\tau\right)}{l(\tau) / \tau} \tag{14}
\end{equation*}
$$

Equation (14) indicates that, when consumers are more loss averse and/or consumers have lower switching costs, the ratio $\frac{g^{\prime}\left(r_{1}^{*}\right)}{l(\tau) / \tau}$ is smaller and thus implies a larger number of small-
prize winners $\left(m_{2}\right)$. Similarly, as the slope $\frac{\omega(1 / N)}{1 / N}$ increases with subadditivity of the decision weighting function, the number of small-prize winners $\left(m_{2}\right)$ should decrease with subadditivity of the decision weighting function. Since the size of first prize decreases with increasing number of small-prize winners, we can infer that the spread between the first prize and small prize should decrease with loss aversion but increase with subadditivity of the decision weighting function. The results of the above analysis on the value-maximizing structure of sweepstakes for low-brand valuation consumers lead to the following proposition.

## Proposition 3:

(i) If the low-brand valuation segment is risk neutral with respect to gain and sufficiently loss averse, then the value-maximizing prize structure for the sweepstakes should consist of a grand prize and many small prizes equal to $\tau$. That is, $s_{L}^{*}=\left\{r_{1}^{*}, m_{1}^{*}=1 ; r_{2}^{*}=\tau, m_{2}^{*}\right\}$.
(ii) In a value-maximizing sweepstakes, the number of small-prize winners ( $m_{2}^{*}$ ) determined by (14) increases with loss aversion and decrease with subadditivity of the decision weighting function.

## Sweepstakes for Low-brand valuation Consumers Risk Averse in Gain

Similar to the analysis for the high-brand valuation consumers who are risk averse, low-brand valuation consumers who are sufficiently risk averse that $\omega_{1} \mathrm{~g}^{\prime}(\mathrm{R})<\omega_{2} \mathrm{~g}^{\prime}(0)$ and $\mathrm{m}_{1}=\mathrm{m}_{2}=1$, also prefer sweepstakes that offer multiple levels of prizes above opportunity cost $\tau$. With risk aversion in gain, the necessary condition (11) can hold for multiple levels of rewards.

To design a sweepstake that offers multiple levels of prizes, decisions on the number of winners at each rank of prize $\left(m_{1}, m_{2}, \ldots, m_{n}\right)$ and the inter-rank spread need to be made. Similar to what we find for the high-brand valuation consumers, the value-maximizing number of winner for each rank of prize (above $\tau$ ) is one. The intuition behind this result is same as discussed for high-brand valuation segment. When a sweepstakes has only one winner at each level of prize, the optimality condition (11) becomes

$$
\begin{equation*}
\left(\omega\left(\frac{j}{N}\right)-\omega\left(\frac{j-1}{N}\right)\right) g^{\prime}\left(r_{j}^{*}-\tau\right)=M_{L}^{*}(j=1,2, . . n-l) \tag{15}
\end{equation*}
$$

First, Condition (15) implies that, given all else equal, when low-brand valuation consumers are more risk averse, they prefer a sweepstake with a smaller inter-rank spread $\left(\mathrm{r}_{\mathrm{j}}-\mathrm{r}_{\mathrm{j}+1}\right)$ and larger number of winners. Second, when decision-weighting function is more subadditive, a sweepstake that has larger inter-rank spreads for the prizes and a fewer number of winners provides maximum value.

The number of last-prize winners also changes with consumer risk aversion. We derive the optimality condition for value-maximizing number of winners in web appendix 2 as follows:

$$
\begin{equation*}
\omega^{\prime}\left(n-1+m_{n}^{*} / N\right)=\frac{\omega(j / N)-\omega(j-1 / N)}{1 / N} \frac{g^{\prime}\left(r_{j}-\tau\right)}{l(\tau) / \tau} \quad(j=1,2, \ldots n-1) \tag{16}
\end{equation*}
$$

We can infer from condition (16) that the number of last-prize winners increases with loss aversion but decreases with the size of opportunity cost. Also, the number of last-prize winners increases with the low-brand valuation consumers' risk aversion.

## Proposition 4:

(i) If low-brand valuation consumers are risk averse $\left(\omega_{1} \mathrm{~g}^{\prime}(\mathrm{R})<\omega_{2} \mathrm{~g}^{\prime}(0)\right)$, the value-maximizing sweepstakes should consist of multiple big prizes and many small prizes equal to $\tau$.
(ii) The number of prizes that are larger than opportunity cost $\tau$ should increase but the interrank spread decrease with degree of consumer risk aversion.
(iii) The number of winners of last prize, which is determined by (16), should increase with loss aversion and risk aversion, but decrease with the size of switching cost and subadditivity of the decision weighting function.

We summarize our results on the design of value-maximizing sweepstakes in Table 1.
The major distinction between the designs of value-maximizing sweepstakes for the highbrand valuation vs. low-brand valuation segment is that the reward structure should include more prizes of smaller amounts for the latter. A sweepstakes that offers many small prizes to the low-brand valuation consumers efficiently reduces the anticipated loss resulting from
switching from her preferred brand. This is unnecessary for a sweepstakes that targets the high-brand valuation consumers.

The implication of risk aversion is that consumers prefer sweepstakes that offer a number of large prizes rather than just one grand prize. This pattern should be the same for both the high-brand valuation segment and the low-brand valuation segment. However, for a sweepstakes that targets low-brand valuation consumers, when the level of loss aversion is high, the number of small prizes should be increased to maximize consumers' valuation. Thus, in addition to offering multiple large prizes rather than just one grand prize, increased number of small prizes also decreases the chance for low-brand valuation consumers to experience a loss from participating in sweepstakes.

Next we discuss three experiments conducted to test the propositions developed in above theoretical analysis.

## EXPERIMENTS

The propositions contain two sets of results: one set relate to the value-maximizing sweepstakes for risk-neutral consumers, the second set concern the directional effects of risk aversion and subadditivity on the design of value-maximizing sweepstakes.

Risk neutral consumers should value sweepstakes with a single grand prize if their brand valuation is high (Proposition 1) and should prefer a prize structure with a single grand prize and many small prizes if their brand value is low (Proposition 3). In reality, very few consumers or subjects are likely to be risk neutral. We therefore amend the propositions to include low risk averse consumers in the following hypothesis:

Hypothesis 1: Consumers whose risk aversion is relatively low (a) are more likely to most prefer Grand Prize Only sweepstakes if they are high brand-valuation consumers and (b) are more likely to most prefer Grand Prize \& Many Small Prizes sweepstakes if they are low brand-valuation consumers.

Propositions 2 and 4 collectively show that, consumers' value-maximizing sweepstakes depends on three factors: brand valuation, risk aversion, and subadditivity. The testable hypotheses from these propositions are summarized next:

Hypothesis 2: Given everything else equal (a) consumers who are lower in risk aversion are more likely to prefer Grand Prize only over Multiple Large Prizes (b) low brand-valuation consumers' preference for a sweepstakes increases when the sweepstakes has many smaller prizes (c) consumers who are higher in subadditivity are more likely to prefer Grand Prize only sweepstakes over Multiple Large Prizes;

The analysis suggests that consumers' anticipated value of participation can be improved by dividing the reward money to multiple levels of rewards leading to H 3 .

Hypothesis 3: Given everything else equal, consumers are more likely to prefer sweepstakes that has one prize at each level rather than sweepstakes with many prizes at the same level.

## Experiment 1

89 undergraduate business majors in a major eastern university participated to get extra course credit. The experiment was conducted as a part of a one hour session consisting of other experiments. The task given to subjects was to select between alternative sweepstakes. Subjects were asked to imagine that they were at a grocery store where they could select between two brands in each category. Based on pre-tests, two categories were selected where students' usage rates of the product categories were high. The first category was cookies and the two brands were M\&M Cookies and Fig Newtons. The second category used was mints with Altoids and Lifesavers as the two brands.

First, subjects were asked to sample the brands in both categories. Then their brand valuations were elicited for each brand using two seven point scales anchored between "Dislike Very Much" to "Like Very Much" and between "Very Bad" to "Very Good". Subjects were then told that they would have an opportunity to earn money in a subsequent experiment based on their decisions and performance. They were instructed that they could
use part of their earnings to buy products and that if they elected to purchase the products, the price would be deducted from their final earnings.

The first choice given to them was to purchase either Fig Newtons or M\&Ms in the cookies category each priced 50 cents. They were told that M\&M was running a sweepstake promotion where different packages had different prize structures with varying prizes and odds of winning a prize.

The subjects were asked to look at the six options available to them that included not making any purchase, buying Fig Newtons that had no sweepstake or one of four packages of M\&M cookies. Each M\&M pack had a different sweepstakes promotion but the total amount in prize money was always $\$ 1000$. The stimuli of the alternative sweepstakes formats are provided in Table 2. The first format in Pack 1 is the "Single Grand Prize (GP)" where there is a one winner of a $\$ 1000$ grand prize. The second format is "Grand Prize \& Small Prizes (GP\&Small)" where Pack 2 is a Grand Prize of $\$ 500$ and 250 prizes of $\$ 2$. The third format is "Multiple Large Prizes (MLP)" where the prize amount is allocated between a small set of winners. Pack 3 sweepstakes consists of multiple large prizes with 10 prizes of $\$ 50$ and 20 prizes of \$25. Finally, the fourth format consists of several "Multiple Prizes \& Small Prizes (MLP\&Small)". Thus, Pack 4 has one level of relatively large multiple prizes of $\$ 25$ with several small prizes of $\$ 2$.

Subjects were asked to rank order the six alternatives in order of preference. Then they were asked to make a choice in the mints category between Altoids and Lifesavers which were priced at $\$ 1.00$ each. The alternative sweepstakes (prizes and odds of winning) were described similarly as the first product category. The stimuli are provided in Table 2. After subjects completed their responses for the sweepstakes, they participated in an unrelated experiment which took approximately 30 minutes. Next, subjects were handed another booklet where their sweepstakes preferences were elicited again in a manner identical
to the first task except that the brand with the sweepstakes promotion was switched (i.e., Fig Newtons had the identical sweepstakes to that in first set whereas M\&Ms had no sweepstake). The questionnaires were counterbalanced to eliminate order effects. The order in which the sweepstakes were presented was also randomized. Subjects then again completed another unrelated experiment after which measures of risk aversion were obtained using ten certainty equivalence questions (e.g. Johnson \& Schkade 1989) which are provided in Table 3.

To ensure that the task was taken seriously, subjects were informed that they would participate in one of the category sweepstakes for real. At the end of the session, a random number was drawn to determine the category. Based on the preferences indicated in their responses, subjects participated in the sweepstakes where the prizes were the same as in the stimuli. Additionally, subjects also received the brand of their choice.

The pooled results for both sweepstakes are provided in Table 4. The percentages reflect the most preferred sweepstake selected by the subjects. Using the two brand preference scales, we construct a brand-valuation indicator variable using strict preference for one brand over the other. Risk aversion was determined using responses to the certainty equivalent questions. The top and bottom one-third of the subjects were categorized as the low and high-risk averse subjects.

## Results

Hypothesis 1 predicts the design of value-maximizing sweepstakes for low riskaversion consumers. From Table 4, we find that for the High-brand Valuation subjects with Low Risk Aversion, the most preferred sweepstakes format is the Single Grand Prize (55.00\%). For Low-brand Valuation subjects with Low Risk Aversion, the modal choice was the Grand Prize \& Small Prizes format (46.55\%). Both results are consistent with Hypothesis 1. Within the subjects who are categorized as Low Risk Aversion, the choices made by the High-brand Valuation segment are substantially different from those made by the Low-brand

Valuation segment. Specifically, the Low-brand Valuation group is much less likely to choose Single Grand Prize format (difference of 43\%), but much more likely to choose Grand Prize \& small Prizes format (difference of $40 \%$ ). The difference in choices between the Highbrand Valuation segment and the Low-brand Valuation segment is also statistically significantly ( $\chi_{1,3}^{2}=46.12, p<.001$ ).

Hypothesis 2(a) predicts the effect of risk aversion on sweepstakes preferences. First, Table 4 indicates that the High-brand Valuation and High Risk Aversion subjects were more likely to prefer sweepstakes with multiple prizes than the High-brand Valuation and Low Risk Aversion subjects ( $38.98 \%+25.42 \%=64.4 \%$ versus $33.33 \%+5 \%=38.33 \%$ ). The difference in choices between these segments of subjects is statistically significant ( $\chi_{1,3}^{2}=30.66, p<.001$ ). Similarly, among the Low-brand Valuation subjects, those of High Risk Aversion were more likely to choose sweepstakes with multiple prizes than those of Low Risk Aversion (48.08\%+28.85\%=76.93\% versus $15.52 \%+25.86 \%=41.38 \%)$. The effect of risk aversion in these Low-brand Valuation subjects is also statistically significant ( $\chi_{1,3}^{2}=18.85, p<.001$ ). These results are consistent with Hypothesis 2(a).

Hypothesis 2(b) concerns the effect of brand valuation and loss aversion on sweepstakes preferences for High Risk Averse consumers. In Table 4, for the Low-brand Valuation subjects with High Risk Aversion, the modal choice is the Multiple \& Small Prizes ( $48.08 \%$ ). The model choice for the High-brand Valuation and High Risk Averse subjects is Multiple Large prizes (38.98\%). For these subjects with High Risk Aversion, the choices made by the High-Brand Valuation segment differ significantly from the low-brand valuation segment ( $\chi_{1,3}^{2}=8.04, p<.05$ ). This result is consistent with Hypothesis 2(b).

For a better test of H2, measures of the individual's subadditivity are also required. We adopt the measurement procedure proposed by Prelec (2000), which requires
parameterization of a consumer's decision making. Specifically, we assume following gain and loss functions:

$$
v(x)= \begin{cases}x^{\sigma} & \text { for } x>0  \tag{17}\\ -\lambda(-x)^{\sigma} & \text { for } x<0\end{cases}
$$

where parameter $\sigma$ measures the degree of risk aversion, and $\lambda$ measures the degree of loss aversion. Loss aversion hypothesis implies that $\lambda>1$. A consumer's risk aversion decreases with value of $\sigma$. When $\sigma=1$, a consumer is risk neutral. The weighting function is as follows:

$$
\begin{equation*}
\omega^{+}(p)=\omega^{-}(p)=\exp \left[-(-\ln p)^{\alpha}\right] . \tag{18}
\end{equation*}
$$

When $0<\alpha<1$, the weighting function exhibits the type of inverse $S$-shape as demonstrated by several studies (e.g., Tversky and Kahneman 1992). A consumer's subadditivity decreases with the value of $\alpha$. When $\alpha=1$, the weighting function is linear, and the weight is equal to the actual probability.

Prelec (1998) has demonstrated that the specification given above by (17) and (18) are consistent with the regular preference axioms. The data required to estimate risk aversion ( $\sigma$ ) and subadditivity $(\alpha)$ consists of certainty equivalents for a series of one-prize prospects (gambles), $\left\{\left(\mathrm{x}_{\mathrm{i}} ; \mathrm{y}_{\mathrm{i}}, \mathrm{p}_{\mathrm{i}}\right), i=1,2 ., . . N.\right\}$ where $\mathrm{x}_{\mathrm{i}}$ is the certainty equivalent for a prospect that offers a chance of $p_{i}$ to win $y_{i}$ ( $y_{i}$ can be negative). The indifference (certainty equivalent) implies that

$$
\begin{equation*}
v\left(x_{i}\right)=v\left(y_{i}\right) \omega\left(p_{i}\right) \tag{19}
\end{equation*}
$$

Substituting (17) and (18) into (19), we obtain

$$
\begin{equation*}
\left(x_{i} / y_{i}\right)^{\sigma}=\exp \left[-\left(-\ln p_{i}\right)^{\alpha}\right] . \tag{20}
\end{equation*}
$$

The equation above can be transformed into the following.

$$
\begin{equation*}
\ln \left(-\ln x_{i} / y_{i}\right)=-\ln \sigma+\alpha \ln \left(-\ln p_{i}\right) . \tag{21}
\end{equation*}
$$

Equation (21) shows that estimating a linear equation is sufficient to obtain the parameters for risk aversion $(\sigma)$ and the shape of weighting function $(\alpha)$. After estimating equation (21), we obtain estimates for both risk aversion and subadditivity for each of the 89 subjects. The range of the risk parameter estimate $(\sigma)$ is 0.08 to 1.24 , with a mean equal to 0.71. For 11 subjects, the estimates of parameter $\sigma$ were above 1 , outside the assumed range. The estimate for subadditivity parameter $\alpha$ ranges from 0.23 to 2.70 , with a mean equal to 0.65. For 3 subjects, the estimates of parameter $\alpha$ were above 1 , also outside the assumed range. We focus on the 76 subjects with both estimates within the interval $[0,1]$. Note that we do not estimate loss aversion. Pretests indicated that the loss aversion did not explain preferences in this experimental scenario.

We finally run pair-wise logistic regressions to test the hypotheses derived from the propositions. (We include the directional results in Table 5, and the detailed estimation results in the web appendix.) The dependent variables comprise the relative preference of one sweepstakes type over the other. To illustrate, consider the 4 sweepstakes with purchase of M\&M cookies. We develop 6 pair comparisons, and hence 6 dummy variables according to relative ranking of the sweepstakes. For example, the first dependent variable in Table 5, MLP \& Small vs GP $=\left\{\begin{array}{l}1 \text { if a consumer ranks the MLP \& Small sweepstakes }(20, \$ 25 ; 250, \$ 2) \\ \quad \text { over the Grand Prize }(1, \$ 1,000) \\ 0 \text { otherwise }\end{array}\right.$

In each regression, we include three independent variables: Sigma $(\sigma)$ for risk neutrality (opposite of risk aversion), Alpha ( $\alpha$ ) for (negative) subadditivity, and an indicator variable I (LowValue) for low brand valuation. The first two variables $\sigma$ and $\alpha$, estimated earlier from regression equation (21), are subject-specific and are the same for all 24 logit regression. The Low-brand valuation indicator variable, I(LowValue) varies based on which brand is being promoted in the stimuli.

The hypothesized signs of estimates (relations) from H 2 are summarized in Table 5. The notation ( + ) and (-) indicates the hypothesized direction and question mark (?) denotes that there is no clear hypothesis. For example, H2(a) predicts when risk aversion is low (large sigma( $\sigma$ )) MLP\&Small is less likely to be chosen over GP. H2 (b) predicts that a lowbrand valuation consumer is more likely to choose MLP\&Small because it contains a large number of small prizes. Finally H 2 (c) states that when subadditivity is low (large $\alpha$ ) MLP\&Small will be preferred to GP. Table 5 reveals that $80 \%$ of estimates are statistically significant and consistent with the hypotheses. The confirmation rates are similar for risk aversion, subadditivity, and loss aversion. The remaining $20 \%$ of estimates are either not significant or without clear predictions emanating from Hypotheses 2. There is no single incidence of a significant parameter estimate inconsistent with Hypothesis 2.

Overall, the results confirm that consumers' preferences for sweepstakes format differ based on their brand valuations, level of risk aversion and subadditivity. The results provide evidence suggesting that marketers should match the reward structure of their sweepstakes promotion with the characteristics of target consumers. The key difference between the experimental set-up and real sweepstakes is that subjects selected between alternative sweepstake formats whereas most sweepstakes do not offer that choice. In Experiment 2, we test Hypothesis 2(b) using a between subjects design.

## Experiment 2

A limitation of Experiment 1 is that subjects were required to make judgments about alternative sweepstakes. We use this approach as several pretests indicated that in a between subjects design, when asked to make a choice between similarly priced brands, one with no sweepstakes and one with a sweepstakes, most subjects selected the brand conducting the
sweepstakes. In this experiment, using a between subjects design, ${ }^{5}$ we test the hypothesis that high brand valuation customers will prefer large prize sweepstakes.

122 subjects participated for extra course credit. Subjects were told that they would be given $\$ 1.00$ and that they could use the money to purchase a one litre bottle of Diet Coke. Subjects were told that two types of Diet Coke bottle were available. One bottle was priced at 50 cents. The price of the second bottle was $\$ 1.00$ but they could also participate in a sweepstake if they purchased the more expensive bottle. The design was a single factor between-subject design (sweepstake prize structures) with two levels (MLP vs MLP \& Small). There were two prizes in the MLP condition: First Prize of $\$ 300$ (Odds of Winning 1:600); Second prize of $\$ 200$ (Odds of Winning 1:400). In the MLP \& Small condition, there were 10 first prizes of $\$ 25$ (Odds of Winning 1:50) and 125 prizes of $\$ 2$ (Odds of Winning 1:4). Brand preference was measured using the same two scales used in Experiment 1.

Results
In the MLP condition, $70.49 \%$ of the subjects elected to participate in the sweepstakes while $62.30 \%$ in the MLP \& Small prizes condition preferred the $\$ 1.00$ bottle with sweepstakes. We plot the sweepstakes choice probabilities in Figure 1, where we classify the subjects with brand preference rating higher than 4 (lower than 4) as High-brand Valuation (Low-brand Valuation). Consistent with our hypotheses, for High-brand Valuation consumers, MLP works better. In contrast, for Low-brand Valuation consumers, MLP \& Small is more effective. The results indicate an interaction between brand preference and sweepstakes type. To further examine the impact of sweepstakes design, we estimated the equation below.

> Sweepstakes Pr eference $=\beta_{0}+\beta_{1} *$ Sweepstakes_Type $+\beta_{2} *$ Brand_Pr eference $+\beta_{3} *$ Sweepstakes_Type $*$ Brand_Pr eference

The variable Sweepstakes_Type is a dummy variable that reflects the condition (1 for the MLP and 0 for the MLP \& Small prizes). Brand_Preference is the preference for diet Coke. The key is the interaction between Sweepstakes _Type and Brand_Preference. A positive interaction coefficient indicates that high brand valuation consumers prefer large sweepstakes and small prize sweepstake are preferred more by consumers with low brand valuations.

The results are provided in Table 6. The results show that subjects' choices for participating in the sweepstakes were significantly influenced by the type of sweepstakes with greater preference for the MLP \& Small prize structure ( $p<.05$ ). There is no main effect of brand preferences: high brand preference for diet coke did not increase the propensity to participate in the sweepstakes. Most important, as predicted by Hypothesis 2 (b), is the significant positive interaction effect between sweepstakes type MLP and brand preference $(p<.05)$ indicating that sweepstakes with the MLP structure is preferred more by consumers with stronger brand valuations while consumers with lower brand preferences are more likely to participate in sweepstakes with a MLP \& Small prize structure.

## Experiment 3

The first objective of this experiment is to test the Hypothesis 3 which states that consumers are likely to prefer sweepstakes where there is a single prize at every level. The secondary objective is to examine whether manipulating higher subadditivity leads to preference for larger prizes. Eighty six subjects participated in the experiment for extra course credit. The design was a single factor (subadditivity) between subjects with two levels (relatively low versus relatively high). Participants were given 50 cents. They were provided the option of either keeping the money or using it to purchase a pack of Lemon Lime flavour of Orbit chewing gum which was a new product introduction at the time the experiment was conducted. If they purchased the chewing gum, they had an opportunity to participate in one
of two sweepstakes. Subjects were informed that there would be two winners in each sweepstakes. The single level prize structure consisted of two prizes of $\$ 100$ each while the other sweepstakes consisted of two levels, the first prize of which was $\$ 115$ and the second prize $\$ 85$.

Subadditivity was manipulated by varying the illusion of control. This manipulation has been suggested by Gonzalez and Wu (1999). For both sweepstakes, subjects were told that the experimenter would draw a random number between 0 and 99 and that the subject holding the number closest to the random number would win the sweepstake. If multiple players held the same number, a tie breaker number would be drawn. In the low subadditivity condition, the sweepstakes and the tie breaker number was predetermined by the experimenter and written on the sweepstakes entry form. In the high subadditivity condition, subjects were asked to select their own number as well as the tie breaker number and write it on the entryform. After making their choices of either keeping the 50 cents or participating in one of the sweepstakes, their relative preference for their sweepstakes choice was measured on a fifteen point scale anchored between "Strongly Preferred Sweepstakes 1 and "Strongly preferred Sweepstakes 2".

## Results

Since four subjects elected to keep the money and not participate in the sweepstakes, we analyse the remaining 82 subjects. Overall, the results find support for the hypothesis that there should be a single prize at each level. Across the entire sample, a significant percentage of the subjects ( $63.41 \%$ ) preferred the two level sweepstakes prize structure over the single level structure ( $\chi_{1}^{2}=5.90, p<.05$ ). Thus, Hypothesis 3 is supported.

The secondary objective of the experiment was to examine if higher subadditivity increases the preference for the larger prizes. $65.85 \%$ of the subjects in the high subadditivity condition preferred the higher prize option while $60.98 \%$ who selected in the high prize
option in the low subadditivity condition. Though directionally correct, the proportion is not significantly different ( $\chi_{1}^{2}=0.21$, n.s.). We next examined the intensity of preference for the two sweepstakes. In the high subadditivity condition, the preference intensity for the high prize sweepstakes is significantly greater than in the low subadditivity condition $\left(X_{H i g h}\right.$ Sub $=$ $\left.10.05, X_{\text {Low Sub }}=8.58 ; F_{1,80}=3.88, p<.05\right)$. The results provide some support for the notion that relatively higher subadditivity increases preferences for bigger prizes. We suspect that the manipulation of prize levels dampened the effect of illusion of control. Future research is needed to examine how subadditivity can be better manipulated and managed.

## CONCLUSION AND DISCUSSION

This article examines sweepstakes reward structure that maximizes consumers' valuations. Such consumer value-maximizing sweepstakes should effectively motivate consumers' participation and thus generate additional sales. Our analysis shows that the sweepstakes reward structure should be based on three factors: the promotional objectives, the risk aversion of the customers and the level of subadditivity.

Our results prescribe that a firm should begin by setting an objective to either attract switchers or target current users. If the current users are risk neutral, the consumer valuemaximizing award is a single grand prize. If the current users are risk averse, then the award should consist of multiple "large" prizes. When the objective is to target current users, then fewer prizes should be awarded than when the targets are switchers. For example, E.W. Scripps follows this approach in their sweepstakes design for their television channels. For their well established channel HGTV, their 2008 sweepstakes reward consisted of a grand prize worth $\$ 850,000$. Following the prescriptive strategy recommended by the model, their sweepstakes to attract new customers to their smaller Do It Yourself (DIY) channel, the prizes structure consisted of 1 grand prize of $\$ 100,000,5$ prizes of $\$ 10,000$ and 42 prizes of
\$1000. Similarly, Microsoft offered a grand prize of a trip to space and 490 small prizes for their Vanishing Point sweepstakes in 2007 for their established operating system. In 2008, the prize structure consisted of a grand prize of $\$ 100,000$ and 30,000 smaller prizes for their launch of Microsoft Office Live. If the non-current user segment is risk neutral with respect to gains but sufficiently loss averse, then the prescribed reward structure is a single grand prize but also to include several small prizes which ideally should be close to the customers' opportunity cost. If the non-loyal customers are risk averse in gain and loss averse, then the best prize allocation is to have both multiple large prizes as well as several small prizes.

An implicit assumption in the model is that the opportunity cost of switching brands is homogenous. In the presence of heterogeneity, the low-brand valuation segments should be further subdivided based on opportunity costs. If the segment size is skewed towards high (low) opportunity costs, the size of the smaller prizes needs to be increased (decreased). Another assumption in the analysis is that the level of customer risk aversion is homogenous. If the customers are heterogeneous, then the firm should measure the extent of risk aversion and the degree of heterogeneity. Under certain conditions such as when there is a large segment of risk averse consumers, it is profitable to increase the level of prizes.

Another factor that impacts the value-maximizing reward structure is the degree of subadditivity. It is important that future research investigate the sweepstake design factors that affect subadditivity. We provide some evidence that creating illusion of control by increasing the effort required to participate may be a fruitful avenue for future research. The effort levels can be thought of as either being incorporated in the game itself or in terms of effort required to submit the entry forms.

Future research should also consider alternative theoretical framework other than Cumulative Prospect Theory (CPT). Recently, Birnbaum (2008) demonstrates some potential problems associated with CPT, and proposes the configural weight models which have more
general weighting functions that allows for features such as splitting effect. To illustrate, consider a sweepstakes consisting of two prizes (e.g., an iPod and a digital camera), each worth $\$ 400$ to consumers with odds of winning 1:2,000 for each prize. Since the value of the prizes is the same, under CPT, these prizes can be combined as $2 \$ 400$ prizes with odds of 2:2,000. But a configural weight model allows for a separate weight assigned to each prize, effectively splitting a branch of $2 \$ 400$ prizes into 2 branches of $\$ 400$. According to Birnbaum (2008), such splitting may change the weights assigned to these prizes and hence subjects' valuations for the prospects. Thus, using alternative models such as the configural model may change consumers' valuations for multiple prize sweepstakes.

Several other formats of sweepstakes also deserve attention in future research. One format is a combination of risk prospects and uncertainty where the odds of winning $a$ prize is provided but the odds of winning specific prizes are not. For example, in 2008, Dr Pepper conducted a sweepstake where the odds of winning a prize were 1 in 6 . Another interesting format has been used by Hershey in 2006 where customers obtain points by making purchases but have to obtain prizes by bidding against other customers. A format being used increasingly is a sequential contest where the initial prize is a small fixed amount but the winners have the chance to play for larger amounts in subsequent rounds as a sweepstake or based on skill (e.g. 2008 "The Dunlop Million Dollar Slam" where prizes depend on the performances of preselected professional tennis players in specific tournaments).

Though sweepstakes are used very frequently, there is no evidence how this promotional tool is more effective than others. Clearly, sweepstakes do not possess the disadvantage of reducing brand equity through price discounts and likely are also more attention getting. However, little is known regarding the relative benefits of sweepstakes and the conditions and/or segments they are likely to have more impact. Given the large expenditures incurred in sweepstakes, these issues merit future research.

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## FOOTNOTES

1. http://promomagazine.com/contests/
2. http://promomagazine.com/games/pennington sweeps 082405/index.html.
3. http://www.promomagazine.com/ar/marketing_winning_odds/
4. http://www.yourenvoy.com/news.htm
5. We thank the Editor for suggesting this experiment.

Table 1: Summary of Value Maximizing Sweepstakes Design

|  | High Brand Valuation | Low Brand Valuation |
| :---: | :---: | :---: |
| Risk Aversion in Gain (Concavity of Gain Function) | - Number of prizes increases with risk aversion in gain. <br> - Risk neutral case: Winner take all. | - Number of large prizes increases with risk aversion in gain. <br> - Number of small prizes (=switching cost $\tau$ ) increases with risk aversion in gain. <br> - Risk neutral case: 1 Grand Prize + many small prizes ( $\tau$ ), number of small prizes increasing with risk aversion. |
| Loss Aversion (Loss Aversion Parameter) | N/A | - Number of small prizes ( $\tau$ ) increases with loss aversion. |
| Subadditivity <br> (Extent of Overweighing Small Winning Probability) | - Number of prizes decreases with subadditivity. | - Number of small prizes ( $\tau$ ) decreases with subadditivity. |

Table 2: Sweepstakes Choices in Experiment 1

| Cookies Sweepstakes |  | Mint Sweepstakes |  |
| :---: | :---: | :---: | :---: |
| Choice | Prizes and Odds of Winning | Choice | Prizes and Odds of Winning |
| Buy Fig <br> Newtons |  | Buy Altoids |  |
| M\&Ms <br> Pack 1 | Grand Prize: $\$ 1000$ (Odds of Winning 1:1000) | Lifesavers Pack 1 | Grand Prize: $\$ 500$ (Odds of Winning 1:500) |
| M\&Ms <br> Pack 2 | Grand Prize: $\$ 500$ <br> (Odds of Winning 1:1000) <br> Second Prize: 250 prizes of $\$ 2$ (Odds of Winning 1:4) | Lifesavers Pack 2 | Grand Prize: \$300 <br> (Odds of Winning 1:500) <br> Second Prize: 200 prizes of $\$ 1$ (Odds of Winning 2:5) |
| M\&Ms Pack 3 | First Prize: 10 prizes of $\$ 50$ (Odds of Winning 1:100) <br> Second Prize: 20 prizes of $\$ 25$ <br> (Odds of Winning 1:50) | Lifesavers Pack 3 | First Prize: \$200 (Odds of Winning 1:500) Second Prize: \$ 150 (Odds of Winning 1:500) Third Prize: \$ 100 (Odds of Winning 1:500) Fourth Prize: \$50 (Odds of Winning 1:500) |
| M\&Ms <br> Pack 4 | First Prize: 20 prizes of $\$ 50$ (Odds of Winning 1:50) <br> Second Prize: 250 prizes of $\$ 2$ (Odds of Winning 1:4) | Lifesavers Pack 4 | First Prize: 20 prizes of $\$ 15$ (Odds of Winning 1:25) <br> Second Prize: 200 prizes of $\$ 1$ (Odds of Winning 2:5) |
| No <br> Purchase |  | No <br> Purchase |  |

Table 3: Risk Aversion Prospects for Experiment 1

| Risk <br> question <br> Number | Probability <br> $\left(\boldsymbol{p}_{\mathbf{i}}\right)$ | Amount <br> $\left(\boldsymbol{y}_{\boldsymbol{i}}\right)$ | Certainty <br> equivalent $\left(\boldsymbol{x}_{\boldsymbol{i}}\right)$ |
| :--- | :---: | :---: | :---: |
| 1 | $0.01 \%$ | $\$ 100,000$ | $\$ 10$ |
| 2 | $25 \%$ | $\$ 200$ | $\$ 50$ |
| 3 | $80 \%$ | $\$ 500$ | $\$ 400$ |
| 4 | $0.50 \%$ | $-\$ 5,000$ | $-\$ 2500$ |
| 5 | $5 \%$ | -400 | $-\$ 20$ |
| 6 | $1 \%$ | $\$ 5,000$ | $\$ 5$ |
| 7 | $10 \%$ | $\$ 400$ | $\$ 40$ |
| 8 | $99 \%$ | $-\$ 1,000$ | $-\$ 999$ |
| 9 | $20 \%$ | $\$ 300$ | $\$ 60$ |
| 10 | $95 \%$ | $\$ 1,000$ | $\$ 950$ |

Table 4: Choice of Sweepstakes in Experiment 1

|  | Sweepstakes design | Sweepstakes Choices |  | Difference in choice (Low Brand Valuation - High Brand Valuation) |
| :---: | :---: | :---: | :---: | :---: |
|  |  | High Brand Valuation | Low Brand Valuation |  |
| Low Risk Aversion | Only Grand Prize | 55.00 \% | 12.07 \% | -43\% |
|  | Grand Prize \& Small | 6.67 \% | 46.55\% | 40\% |
|  | Multiple Large | 33.33 \% | 15.52 \% | -17\% |
|  | Multiple \& Small | 5.00 \% | 25.86 \% | 20\% |
| High Risk Aversion | Only Grand Prize | 11.86 \% | 13.46 \% | 2\% |
|  | Grand Prize \& Small | 23.73 \% | 9.62 \% | -14\% |
|  | Multiple Large | 38.98 \% | 28.85 \% | -10\% |
|  | Multiple \& Small | 25.42 \% | 48.08 \% | 23\% |
| Difference in choices (High risk aversion low risk aversion) | Only Grand Prize | -43\% | 1\% |  |
|  | Grand Prize \& Small | 17\% | -37\% |  |
|  | Multiple Large | 6\% | 13\% |  |
|  | Multiple \& Small | 20\% | 23\% |  |

Table 5: Summary of Hypotheses Tests ( $\sqrt{ }$ indicates empirical confirmation)*

|  | Dependent Variable | Sweepstakes Pair | Sigma ( $\sigma$ ) <br> (-Risk Aversion) | Alpha ( $\alpha$ ) (-Subadditivity) | I(LowValue) <br> LossAversion |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | MLP\& Small vs GP | $\begin{aligned} & (20, \$ 25 ; 250, \$ 2)> \\ & (1, \$ 1000)^{* *} \end{aligned}$ | -( $\sqrt{ }$ ) | $+(\sqrt{ })$ | $+(\sqrt{ })$ |
|  | MLP\& Small vs MLP | $\begin{aligned} & (20, \$ 25 ; 250, \$ 2)> \\ & (10, \$ 50 ; 20, \$ 25) \\ & \hline \end{aligned}$ | -( $\sqrt{ }$ ) | $+(\sqrt{ })$ | + ( ${ }^{\text {) }}$ |
|  | MLP\&Small vs GP\& Small | $\begin{aligned} & (20, \$ 25 ; 250, \$ 2)> \\ & (1, \$ 500 ; 250, \$ 2) \\ & \hline \end{aligned}$ | $-(\sqrt{ })$ | $+(\sqrt{ })$ | ? |
|  | $\begin{aligned} & \hline \text { MLP vs } \\ & \text { GP } \end{aligned}$ | $\begin{aligned} & (10, \$ 50 ; 20, \$ 25)> \\ & (1, \$ 1000) \end{aligned}$ | - | + | ? |
|  | $\begin{aligned} & \text { GP\&Small vs } \\ & \text { GP } \\ & \hline \end{aligned}$ | $\begin{aligned} & (1, \$ 500 ; 250, \$ 2)> \\ & (1, \$ 1000) \end{aligned}$ | - | + (V) | + (V) |
|  | $\begin{aligned} & \text { GP\&Small vs } \\ & \text { MLP } \\ & \hline \end{aligned}$ | $\begin{aligned} & (1, \$ 500 ; 250, \$ 2)> \\ & (10, \$ 50 ; 20, \$ 25) \\ & \hline \end{aligned}$ | ? | ? | + |
|  | MLP\&Small vs GP | $\begin{aligned} & \hline(20, \$ 25 ; 250, \$ 2)> \\ & (1, \$ 1000) \end{aligned}$ | $-(\sqrt{ })$ | $+(\sqrt{ })$ | $+(\sqrt{ })$ |
|  | MLP\&Small vs MLP | $\begin{aligned} & (20, \$ 25 ; 250, \$ 2)> \\ & (10, \$ 50 ; 20, \$ 25) \end{aligned}$ | $-(\sqrt{ })$ | + | + (V) |
|  | MLP\&Small vs GP\&Small | $\begin{aligned} & (20, \$ 25 ; 250, \$ 2)> \\ & (1, \$ 500 ; 250, \$ 2) \\ & \hline \end{aligned}$ | $-(\sqrt{ })$ | + ( ${ }^{\text {) }}$ | ? |
|  | $\begin{aligned} & \text { MLP vs } \\ & \text { GP } \end{aligned}$ | $\begin{aligned} & (10, \$ 50 ; 20, \$ 25)> \\ & (1, \$ 1000) \end{aligned}$ | $-(\sqrt{ })$ | + (V) | ? |
|  | $\begin{aligned} & \text { GP\&Small vs } \\ & \text { GP } \\ & \hline \end{aligned}$ | $\begin{aligned} & (1, \$ 500 ; 250, \$ 2)> \\ & (1, \$ 1000) \end{aligned}$ | - ( $\sqrt{ }$ ) | + ( ${ }^{\text {) }}$ | + (V) |
|  | $\begin{aligned} & \text { GP\&Small vs } \\ & \text { MLP } \\ & \hline \end{aligned}$ | $\begin{aligned} & (1, \$ 500 ; 250, \$ 2)> \\ & (10, \$ 50 ; 20, \$ 25) \end{aligned}$ | ? | ? | + (V) |
|  | MLP\&Small vs GP | $\begin{aligned} & (20, \$ 15 ; 200, \$ 1)> \\ & (1, \$ 500) \end{aligned}$ | - ( $\sqrt{ }$ ) | + ( ${ }^{\text {) }}$ | + |
|  | MLP\&Small vs MLP | $\begin{aligned} & (20, \$ 15 ; 200, \$ 1)>(1, \$ 200 ; \\ & 1, \$ 150 ; 1, \$ 100 ; 1, \$ 50) \end{aligned}$ | -( $\sqrt{ }$ ) | $+(\sqrt{ })$ | + |
|  | MLP\&Small vs GP\&Small | $\begin{aligned} & (20, \$ 15 ; 200, \$ 1)> \\ & (1, \$ 300 ; 200, \$ 1) \\ & \hline \end{aligned}$ | $-(\sqrt{ })$ | $+(\sqrt{ })$ | ? |
|  | $\begin{aligned} & \text { MLP vs } \\ & \text { GP } \end{aligned}$ | $\begin{aligned} & (1, \$ 200 ; 1, \$ 150 ; 1, \$ 100 ; \\ & 1, \$ 50)>(1, \$ 500) \\ & \hline \end{aligned}$ | - | + | ? |
|  | GP\&Small vs GP | $\begin{aligned} & (1, \$ 300 ; 200, \$ 1)> \\ & (1, \$ 500) \end{aligned}$ | -( $\sqrt{ }$ ) | $+(\sqrt{ }$ ) | $+(\sqrt{ })$ |
|  | $\begin{aligned} & \text { GP\&Small vs } \\ & \text { MW } \\ & \hline \end{aligned}$ | $\begin{aligned} & (1, \$ 300 ; 200, \$ 1)>(1, \$ 200 ; \\ & 1, \$ 150 ; 1, \$ 100 ; 1, \$ 50) \\ & \hline \end{aligned}$ | ? | ? | + ( $\left.{ }^{( }\right)$ |
| 000.0000000000 | MLP\&Small vs GP | $\begin{aligned} & (20, \$ 15 ; 200, \$ 1)> \\ & (1, \$ 500) \end{aligned}$ | - | $+(\sqrt{ })$ | + |
|  | MLP\&Small vs MLP | $\begin{aligned} & (20, \$ 15 ; 200, \$ 1)>(1, \$ 200 ; \\ & 1, \$ 150 ; 1, \$ 100 ; 1, \$ 50) \end{aligned}$ | - | $+(\sqrt{ })$ | $+(\sqrt{ })$ |
|  | MLP\&Small vs GP\&Small | $\begin{gathered} (20, \$ 15 ; 200, \$ 1)> \\ (1, \$ 300 ; 200, \$ 1) \end{gathered}$ | -( $\sqrt{ }$ ) | $+(\sqrt{ })$ | ? |
|  | MLP vs GP | $\begin{aligned} & (1, \$ 200 ; 1, \$ 150 ; 1, \$ 100 \\ & 1, \$ 50)>(1, \$ 500) \end{aligned}$ | $-(\sqrt{ })$ | + | ? |
|  | GP\&Small vs GP | $\begin{aligned} & (1, \$ 300 ; 200, \$ 1) \\ & >(1, \$ 500) \\ & \hline \end{aligned}$ | - | + (V) | + |
|  | GP\&Small vs MW | $\begin{aligned} & (1, \$ 300 ; 200, \$ 1)>(1, \$ 200 ; \\ & 1, \$ 150 ; 1, \$ 100 ; 1, \$ 50) \end{aligned}$ | ? | ? | + (V) |

*GP denotes Grand Prize
MLP denotes Multiple Large Prizes
GP\&Small denotes Grand Prize and Small Prizes
MW denotes Multiple Winners
** For example $(20, \$ 25 ; 250, \$ 2)>(1, \$ 1000)$ denotes that structure of 20 prizes of $\$ 25$ and 250 prizes of $\$ 2$ is preferred to 1 prize of $\$ 1000$.

Table 6: Regression Analysis of Sweepstakes Choice (Experiment 2)

| Variable | Para <br> meter | Estimate | Standard <br> Error | $\chi^{2}$ |
| :--- | :--- | :--- | :--- | :--- |
| Intercept | $\beta_{0}$ | 0.3523 | 0.6587 | 0.28 |
| Sweepstakes_Type <br> (=1 for MLP, $=0$ for MLP\&Small) | $\beta_{1}$ | -2.1995 | 1.1768 | $3.40^{* *}$ |
| Brand_Preference <br> (7-point preference scale, 7 being <br> highest) | $\beta_{2}$ | 0.0323 | 0.1588 | 0.04 |
| Sweepstakes_Type $\times$ Brand_Prefere <br> nce | $\beta_{3}$ | 0.6540 | 0.2903 | $5.07^{* *}$ |

Dependent variable: ChooseSweepstakes $=1$ if choosing a brand with sweepstakes.
** $p<.05$

Figure 1: Sweepstakes Choices and Brand Valuation (Experiment 2)


MLP sweepstakes: (\$300, 1:600; \$200, 1:400)
MLP \& Small Sweepstakes: (\$25, 1:50; \$2, 1:4)

## Web Appendix

## Consumer Value-Maximizing Sweepstakes \& Contests

Ajay Kalra, Mengze Shi

## Appendix 1: Sweepstakes for High-brand valuation Segment

## Optimality condition

(P1) $\operatorname{Max}_{s} V_{H}(S)=\omega\left(\frac{m_{1}}{N}\right) g\left(r_{1}\right)+\sum_{j=2}^{n}\left(\omega\left(\frac{\sum_{k=1}^{j} m_{k}}{N}\right)-\omega\left(\frac{\sum_{k=1}^{j-1} m_{k}}{N}\right)\right) g\left(r_{j}\right)+\left(1-\omega\left(\frac{\sum_{j=1}^{n} m_{j}}{N}\right)\right) g(0)$
s.t. $\sum_{j=1}^{n} m_{j} r_{j}=R$

We characterize value-maximizing sweepstakes based on optimality conditions. Let the value-maximizing sweepstakes be $S^{*}=\left\{r_{1}^{*}, m_{1}^{*} ; r_{2}^{*}, m_{2}^{*} ; \ldots r_{n}^{*}, m_{n}^{*}\right\}$. Consider a very small amount of prize $(\nabla r)$ reduced from $r_{k}^{*}$ and allocated to $r_{j}^{*}(1 \leq j, k \leq n, j \neq k)$, while prizes of all other ranks remains the same. With total budget $R$ fixed, $r_{k}^{*}$ should decrease by ${ }^{m} \nabla r / m_{k}$. Therefore, ${ }^{d r_{k}} / d r_{j}=-m_{j} / m_{k}$. Such an allocation should not change anticipated value from sweepstakes, that is,

$$
\begin{equation*}
\frac{d V_{H}\left(S^{*}\right)}{d r_{j}}=\omega_{j} g^{\prime}\left(r_{j}^{*}\right)+\omega_{k} g^{\prime}\left(r_{k}^{*}\right) \frac{d r_{k}}{d r_{j}}=m_{j}\left[\frac{\omega_{j}}{m_{j}} g^{\prime}\left(r_{j}^{*}\right)-\frac{\omega_{k}}{m_{k}} g^{\prime}\left(r_{k}^{*}\right)\right]=0 \tag{A1}
\end{equation*}
$$

Equation (A1) leads to optimality condition for value-maximizing sweepstakes of (P1):

$$
\begin{equation*}
\frac{\omega_{j}}{m_{j}} g^{\prime}\left(r_{j}^{*}\right)=M_{H}^{*}(j=1,2, . . n) \tag{A2}
\end{equation*}
$$

## Proof that there is only one winner at each rank when consumers are risk averse in gain

 Suppose there are two winners at $j^{\text {th }}$ rank for prize $r_{j}$. Consumer's anticipated value from this rank of prize is equal to$$
\begin{aligned}
& \left(\omega\left(\left(\sum_{k=1}^{j-1} m_{k}+2\right) / N\right)-\omega\left(\left(\sum_{k=1}^{i-1} m_{k}\right) / N\right)\right) g\left(r_{j}\right)= \\
& \left.\left(\sum_{\left(\frac{k=1}{j-1} m_{k}+1\right.}^{N}\right)-\omega\left(\frac{\sum_{k=1}^{j-1} m_{k}}{N}\right)\right) g\left(r_{j}\right)+\left(\omega\left(\frac{\sum_{k=1}^{j-1} m_{k}+2}{N}\right)-\omega\left(\frac{\sum_{k=1}^{j-1} m_{k}+1}{N}\right)\right) g\left(r_{j}\right)
\end{aligned}
$$

For a positive and sufficiently small $\sigma$, we can reallocate prize $\left(2 r_{j}\right)$ into $\left(r_{j}-\sigma\right)$ and $\left(r_{j}+\sigma\right)$, keeping other prizes and their associated decision weights unchanged. With such a change in prize structure, the anticipated value from these two prizes become

$$
\left(\omega\left(\frac{\sum_{k=1}^{j-1} m_{k}+1}{N}\right)-\omega\left(\frac{\sum_{k=1}^{j-1} m_{k}}{N}\right)\right) g\left(r_{j}+\sigma\right)+\left(\omega\left(\frac{\sum_{k=1}^{j-1} m_{k}+2}{N}\right)-\omega\left(\frac{\sum_{k=1}^{j-1} m_{k}+1}{N}\right)\right) g\left(r_{j}-\sigma\right)
$$

Then the change in anticipated value of sweepstakes resulting from prize reallocation is

$$
\begin{align*}
& \binom{\sum_{k=1}^{j-1} m_{k}+1}{N}-\omega\left(\frac{\sum_{k=1}^{j-1} m_{k}}{N}\right)\left(g\left(r_{j}+\sigma\right)-g\left(r_{j}\right)\right)-\left(\omega\left(\frac{\sum_{k=1}^{j-1} m_{k}+2}{N}\right)-\omega\left(\frac{\sum_{k=1}^{j-1} m_{k}+1}{N}\right)\right) \times \\
& \left(g\left(r_{j}\right)-g\left(r_{j}-\sigma\right)\right) \tag{A3}
\end{align*}
$$

In (A3), according to equation (6), $\omega\left(\left(\sum_{k=1}^{j-1} m_{k}+1\right) / N\right)-\omega\left(\sum_{k=1}^{j-1} m_{k} / N\right)$ is larger than $\omega\left(\left(\sum_{k=1}^{j-1} m_{k}+2\right) / N\right)-\omega\left(\left(\sum_{k=1}^{i-1} m_{k}+1\right) / N\right)$. Moreover, the difference is strictly positive and independent of $\sigma$. On the other hand, when $\sigma$ becomes smaller, the difference between $\left(g\left(r_{j}\right)-g\left(r_{j}-\sigma\right)\right)$ and $\left(g\left(r_{j}+\sigma\right)-g\left(r_{j}\right)\right)$ decreases and eventually approaches to zero. Therefore, there exists a positive and sufficiently small $\sigma^{*}$ so that (A3) is positive for any $0<\sigma<\sigma^{*}$. In other words, the firm can increase the anticipated value of sweepstakes promotion by reallocating $\left(2 r_{j}\right)$ into $\left(r_{j}-\sigma\right)$ and $\left(r_{j}+\sigma\right)$ as long as $\sigma$ is small enough. Thus, it is not value-maximizing to have a rank with more than one winner. Instead, only a single winner should be awarded for every level of prize.

## Appendix 2: Sweepstakes for the Low-brand valuation Segment

$$
\begin{align*}
\operatorname{Max}_{s}(S) & =\omega\left(\frac{m_{1}}{N}\right) g\left(r_{1}-\tau\right)+\sum_{j=2}^{J}\left(\omega\left(\sum_{k=1}^{i} m_{k} / N\right)-\omega\left(\sum_{k=1}^{i-1} m_{k} / N\right)\right) g\left(r_{j}-\tau\right)  \tag{P2}\\
& -\sum_{j=J+1}^{n}\left(\omega\left(\sum_{k=1}^{j} m_{k} / N\right)-\omega\left(\sum_{k=1}^{j-1} m_{k} / N\right)\right) l\left(\tau-r_{j}\right)-\left(1-\omega\left(\sum_{j=1}^{n} m_{j} / N\right)\right) l(\tau)
\end{align*}
$$

s.t. $\sum_{j=1}^{n} m_{j} r_{j}=R$

## Proof that the lowest prize should be at least as large as opportunity cost $\tau$.

Suppose a sweepstake $S$ includes a prize smaller than opportunity cost $\tau$. Without loss of generality, we let $S=\left\{r_{1}, m_{1} ; r_{2}, m_{2} ; \ldots ; r_{n}, m_{n}\right\}$ where $r_{n} m_{n}=R_{n}, \sum_{j=1}^{n} m_{j} r_{j}=R$, and $r_{n}<\tau$.
Now we show that under the same budget $(R)$, we can enhance the valuation of sweepstakes with an increase in lowest reward $r_{n}$. Keeping $\left\{r_{1}, m_{1} ; r_{2}, m_{2} ; \ldots ; r_{n-1}, m_{n-1}\right\}$ the same, we let $r_{n}$ increase by a very small amount while keeping $R_{n}$ (hence total expense $R$ ) unchanged. With a constant $R_{n}$, since $m_{n}=R_{n} / r_{n}$, an increase in $r_{n}$ implies a decrease in $m_{n}$. An increase
in $r_{n}$ will lead to following changes in the low-brand valuation consumers' anticipated value from sweepstakes participation:

$$
\begin{align*}
& \frac{\partial V_{L}(S)}{\partial r_{n}}=\frac{\partial \omega_{n}}{\partial r_{n}}\left[-l\left(\tau-r_{n}\right)\right]+\omega_{n} l^{\prime}\left(\tau-r_{n}\right)+\frac{\partial \omega_{n}}{\partial r_{n}}[l(\tau)] \\
& =\frac{m_{n}}{N}\left[\frac{\omega\left(\sum_{j=1}^{n} m_{j} / N\right)-\omega\left(\sum_{j=1}^{n-1} m_{j} / N\right)}{m_{n} / N} l^{\prime}\left(\tau-r_{n}\right)-\omega^{\prime}\left(\sum_{j=1}^{n} m_{j} / N\right) \frac{l(\tau)-l\left(\tau-r_{n}\right)}{r_{n}}\right]>0  \tag{A4}\\
& \text { The above inequality always holds because a) } \frac{\omega\left(\sum_{j=1}^{n} m_{j} / N\right)-\omega\left(\sum_{j=1}^{n-1} m_{j} / N\right)}{m_{n} / N}>\omega^{\prime}\left(\sum_{j=1}^{n} m_{j} / N\right)
\end{align*}
$$ due to the $s$-shaped decision weighting function, and b) $l^{\prime}\left(\tau-r_{n}\right)>\frac{l(\tau)-l\left(\tau-r_{n}\right)}{r_{n}}$ due to the concavity of the loss function. Since $\frac{\partial V_{L}(S)}{\partial r_{n}}>0$, sweepstakes valuation can be enhanced with an increase in $r_{n}$. Therefore a sweepstakes $S=\left\{r_{1}, m_{1} ; r_{2}, m_{2} ; \ldots ; r_{n}, m_{n}\right\}$ with $r_{n}<\tau$ is not value-maximizing. The lowest prize in value-maximizing sweepstakes should be at least as large as opportunity cost $\tau$.

## Optimality Condition

As in Appendix 1, we characterize value-maximizing sweepstakes based on optimality conditions. Let the value-maximizing sweepstakes be $S^{*}=\left\{r_{1}^{*}, m_{1}^{*} ; r_{2}^{*}, m_{2}^{*} ; \ldots r_{n}^{*}, m_{n}^{*}\right\}$. Since the smallest prize should be as large as $\tau$, we let $r_{n}^{*}=\tau$. We now consider a very small amount of prize $(\nabla r)$ deducted from $r_{k}^{*}$ and allocated to $r_{j}^{*}(1 \leq j, k \leq n, j \neq k)$, while prizes of all other ranks remain the same. With total budget $R$ fixed, $r_{k}^{*}$ should increase by $m_{j} \nabla r / m_{k}$. Therefore, ${ }^{d r_{k}} / d r_{j}=-m_{j} / m_{k}$. Anticipated value from sweepstakes would then change by:

$$
\begin{gather*}
\frac{d V_{L}\left(S^{*}\right)}{d r_{j}}=\omega_{j} g^{\prime}\left(r_{j}^{*}-\tau\right)+\omega_{k} g^{\prime}\left(r_{k}^{*}-\tau\right) \frac{d r_{k}}{d r_{j}}=m_{j}\left[\frac{\omega_{j}}{m_{j}} g^{\prime}\left(r_{j}^{*}-\tau\right)-\frac{\omega_{k}}{m_{k}} g^{\prime}\left(r_{k}^{*}-\tau\right)\right]=0 \quad(j, k \neq n)  \tag{A5}\\
\frac{d V_{L}\left(S^{*}\right)}{d r_{n}}=\omega_{n} g^{\prime}\left(r_{n}^{*}-\tau\right)+\omega_{k} g^{\prime}\left(r_{k}^{*}-\tau\right) \frac{d r_{k}}{d r_{n}}=m_{n}\left(\frac{\omega_{n}}{m_{n}} g^{\prime}(0)-\frac{\omega_{k}}{m_{k}} g^{\prime}\left(r_{k}^{*}-\tau\right)\right),(j=n)  \tag{A6}\\
\frac{d V_{L}\left(S^{*}\right)}{d r_{j}}=\omega_{j} g^{\prime}\left(r_{j}^{*}-\tau\right)+\omega_{n} l^{\prime}\left(r_{k}^{*}-\tau\right) \frac{d r_{n}}{d r_{j}}=m_{j}\left(\frac{\omega_{j}}{m_{j}} g^{\prime}\left(r_{j}^{*}-\tau\right)-\frac{\omega_{n}}{m_{n}} l^{\prime}(0)\right),(k=n) \tag{A7}
\end{gather*}
$$

Equation (A5) characterizes optimality condition for first ( $n-1$ ) prizes,

$$
\begin{equation*}
\frac{\omega_{j}}{m_{j}} g^{\prime}\left(r_{j}^{*}-\tau\right)=M_{L}^{*}(j=1,2, . . n-1) \tag{A8}
\end{equation*}
$$

Similar to (A2), (A8) requires identical anticipated value generating ability $M_{L}^{*}$ from the top $(n-1)$ prizes. At the bottom prize that equals to switching cost $\tau$, anticipated value is not differentiable because of loss aversion. The value-maximizing sweepstakes contain a bottom prize equal to $\tau$ if

$$
\begin{equation*}
\frac{\omega_{n}}{m_{n}} l^{\prime}(0) \geq M_{L}^{*} \geq \frac{\omega_{n}}{m_{n}} g^{\prime}(0) \tag{A9}
\end{equation*}
$$

Condition (A9) ensures that reducing the lowest prize will reduce the anticipated value.
When low-brand valuation consumers are risk-averse in gain, (A8) and (A9) indicate that the value-maximizing sweepstakes is to have multiple big prizes in addition to the bottom prize equal to $\tau$. Following exactly the same logic as given in second part of Appendix 1, we can show that number of winners for each rank of big prizes should be equal to one. Then the optimality condition (A8) becomes:

$$
\begin{equation*}
\left(\omega\left(\frac{j}{N}\right)-\omega\left(\frac{j-1}{N}\right)\right) g^{\prime}\left(r_{j}^{*}-\tau\right)=M_{L}^{*}(j=1,2, . . n-l) \tag{A10}
\end{equation*}
$$

## Value-maximizing number of lowest-prize winners

We now analyse the value-maximizing number of winners for the lowest prize $(\tau)$. Consider a very small increase $(\nabla m)$ in number of last-prize winners ( $m_{n}^{*}$ ) and a decrease in $r_{j}^{*}(1 \leq j \leq$ $n-1$ ), while keeping prizes of all other ranks the same. To maintain the same total budget $R$, $r_{j}^{*}$ should decrease by $\tau \nabla m$. Therefore, $\partial r_{j} / \partial m_{n}=-\tau$. Such a reallocation should not change the low-brand valuation consumers' anticipated value of sweepstakes:

$$
\begin{align*}
\frac{\partial V_{L}\left(S^{*}\right)}{\partial m_{n}} & =\omega_{j} g^{\prime}\left(r_{j}^{*}-\tau\right) \frac{\partial r_{j}}{\partial m_{n}}+\frac{\partial \omega\left(n-1+m_{n}^{*} / N\right)}{\partial m_{n}}(l(\tau)-l(0)) \\
& =\tau\left(\frac{l(\tau)}{\tau} \frac{\omega^{\prime}\left(n-1+m_{n}^{*} / N\right)}{N}-\omega_{j} g^{\prime}\left(r_{j}^{*}-\tau\right)\right)=0 \tag{A11}
\end{align*}
$$

Combining condition (A11) with (A10), we have the following optimality condition:

$$
(\omega(j / N)-\omega(j-1 / N)) g^{\prime}\left(r_{j}^{*}-\tau\right)=\frac{l(\tau)}{\tau} \frac{\omega^{\prime}\left(n-1+m_{n}^{*} / N\right)}{N}(j=1,2, \ldots n-1)
$$

which can be rewritten as

$$
\begin{equation*}
\omega^{\prime}\left(n-1+m_{n}^{*} / N\right)=\frac{\omega(j / N)-\omega(j-1 / N)}{1 / N} \frac{g^{\prime}\left(r_{j}-\tau\right)}{l(\tau) / \tau} \quad(j=1,2, \ldots n-1) \tag{A12}
\end{equation*}
$$

In the special case of low-brand valuation consumers being risk-neutral in gain, the valuemaximizing sweepstakes only offers one big prize; that is, $n=2$. Then condition (A12) simplifies to:

## Appendix 3

Table A: Parameter Estimates for Pairwise Logit Models (Cookies)*

| M\& M Sweepstakes |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{n}=76$ | Sweepstakes Pair | Sigma ( $\sigma$ ) |  | Alpha ( $\alpha$ ) |  | I(LowValue) |  |
|  |  | Coeff | $T$ | coeff | $t$ | coeff | $t$ |
|  <br> Small vs GP | $(20, \$ 25 ; 250, \$ 2)>(1, \$ 1000)$ | -4.340 | -1.82 | 5.553 | 3.03 | 1.432 | 2.33 |
| MLP\& Small vs MLP | $(20, \$ 25 ; 250, \$ 2)>(10, \$ 50 ; 20, \$ 25)$ | -4.823 | -1.99 | 5.834 | 3.02 | 2.645 | 3.95 |
| MLP\&Small vs <br> GP\& Small | $(20, \$ 25 ; 250, \$ 2)>(1, \$ 500 ; 250, \$ 2)$ | -9.124 | -3.54 | 3.447 | 2.15 | 0.058 | 0.1 |
| MLP vs GP | $(10, \$ 50 ; 20, \$ 25)>(1, \$ 1000)$ | -0.541 | -0.27 | 1.693 | 1.14 | -0.509 | -0.9 |
| GP\&Small vs GP | $(1, \$ 500 ; 250, \$ 2)>(1, \$ 1000)$ | -0.234 | -0.12 | 3.560 | 2.41 | 0.855 | 1.54 |
| GP\&Small vs MLP | $(1, \$ 500 ; 250, \$ 2)>(10, \$ 50 ; 20, \$ 25)$ | 3.881 | 1.75 | 0.492 | 0.32 | 2.185 | 3.72 |

Fig Newtons Sweepstakes

| MLP\&Small <br> vs GP | $(20, \$ 25 ; 250, \$ 2)>(1, \$ 1000)$ | $-\mathbf{- 7 . 1 1 4}$ | $-\mathbf{3}$ | $\mathbf{4 . 2 7 5}$ | $\mathbf{2 . 6 3}$ | $\mathbf{1 . 2 3 3}$ | $\mathbf{2 . 2 8}$ |
| :--- | :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| MLP\&Small <br> vs MLP | $(20, \$ 25 ; 250, \$ 2)>(10, \$ 50 ; 20$, <br> $\$ 25)$ | $-\mathbf{4 . 8 9 5}$ | $-\mathbf{- 2 . 2 8}$ | 2.435 | 1.69 | $\mathbf{1 . 5 1 2}$ | $\mathbf{2 . 8 4}$ |
| MLP\&Small <br> vs |  |  |  |  |  |  |  |
| GP\&Small | $(20, \$ 25 ; 250, \$ 2)>(1, \$ 500 ; 250, \$ 2)$ | $\mathbf{- 1 0 . 5 8 3}$ | $-\mathbf{- 3 . 8}$ | $\mathbf{5 . 1 2 3}$ | $\mathbf{2 . 9 4}$ | 0.241 | 0.44 |
| MLP vs GP | $(10, \$ 50 ; 20, \$ 25)>(1, \$ 1000)$ | $-\mathbf{6 . 6 3 0}$ | $-\mathbf{- 2 . 7 9}$ | $\mathbf{3 . 2 2 7}$ | $\mathbf{2 . 0 3}$ | -0.337 | -0.62 |
| GP\&Small <br> vs GP | $(1, \$ 500 ; 250, \$ 2)>(1, \$ 1000)$ | $\mathbf{- 5 . 9 4 8}$ | $-\mathbf{- 2 . 5 8}$ | $\mathbf{3 . 1 7 6}$ | $\mathbf{2}$ | $\mathbf{1 . 7 1 4}$ | $\mathbf{3 . 1}$ |
| GP\&Small <br> vs MLP | $(1, \$ 500 ; 250, \$ 2)>(10, \$ 50 ; 20, \$ 25)$ | -1.403 | -0.73 | -0.197 | 0.15 | $\mathbf{1 . 5 9 6}$ | $\mathbf{3 . 0 6}$ |

*Numbers in bold indicate significant estimates.

Table B: Parameter Estimates for Pairwise Logit Models (Mints)*

| $\mathrm{n}=76$ | Sweepstakes Pair | Sigma ( $\sigma$ ) |  | Alpha ( $\alpha$ ) |  | I(LowValue) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Coeff | $t$ | coeff | $t$ | coeff | $t$ |
| Lifesaver Sweepstakes |  |  |  |  |  |  |  |
| $\begin{aligned} & \hline \text { MLP\&Small } \\ & \text { vs GP } \\ & \hline \end{aligned}$ | $(20, \$ 15 ; 200, \$ 1)>(1, \$ 500)$ | -14.704 | -3.9 | 7.037 | 3.29 | -0.263 | -0.41 |
| MLP\&Small vs MLP | $\begin{aligned} & (20, \$ 15 ; 200, \$ 1)>(1, \$ 200 ; 1, \\ & \$ 150 ; 1, \$ 100 ; 1, \$ 50) \\ & \hline \end{aligned}$ | -12.763 | -3.78 | 5.783 | 2.98 | 0.326 | 0.55 |
| MLP\&Small vs GP\&Small | $(20, \$ 15 ; 200, \$ 1)>(1, \$ 300 ; 200, \$ 1)$ | -11.094 | -3.64 | 4.690 | 2.67 | 0.266 | 0.46 |
| MLP vs GP | $\begin{aligned} & (1, \$ 200 ; 1, \$ 150 ; 1, \$ 100 ; 1, \$ 50) \\ & >(1, \$ 500) \end{aligned}$ | -0.656 | -0.34 | 1.394 | 1.05 | 1.142 | 2.08 |
| GP\&Small vs GP | $(1, \$ 300 ; 200, \$ 1)>(1, \$ 500)$ | -7.126 | -2.84 | 4.281 | 2.66 | 0.900 | 1.62 |
| GP\&Small vs MW | $\begin{aligned} & (1, \$ 300 ; 200, \$ 1)>(1, \$ 200 ; 1, \\ & \$ 150 ; 1, \$ 100 ; 1, \$ 50) \\ & \hline \end{aligned}$ | -5.428 | -2.4 | 2.350 | 1.64 | 0.762 | 1.42 |
| Altoids Sweepstakes |  |  |  |  |  |  |  |
| $\begin{aligned} & \text { MLP\&Small } \\ & \text { vs GP } \\ & \hline \end{aligned}$ | $(20, \$ 15 ; 200, \$ 1)>(1, \$ 500)$ | -1.341 | -0.63 | 2.543 | 1.83 | 0.747 | 1.26 |
| MLP\&Small vs MLP | $\begin{array}{\|l\|} \hline(20, \$ 15 ; 200, \$ 1)>(1, \$ 200 ; 1, \\ \$ 150 ; 1, \$ 100 ; 1, \$ 50) \\ \hline \end{array}$ | -2.946 | -1.29 | 4.008 | 2.58 | 1.501 | 2.39 |
| MLP\&Small vs <br> GP\&Small | $\begin{aligned} & (20, \$ 15 ; 200, \$ 1)>(1, \$ 300 ; \\ & 200, \$ 1) \end{aligned}$ | -7.236 | -2.96 | 3.354 | 2.24 | -0.175 | -0.3 |
| MLP vs GP | $\begin{aligned} & (1, \$ 200 ; 1, \$ 150 ; 1, \$ 100 ; 1, \$ 50) \\ & >(1, \$ 500) \end{aligned}$ | -4.437 | -1.92 | -0.502 | 0.35 | -1.592 | -2.71 |
| $\begin{aligned} & \text { GP\&Small } \\ & \text { vs GP } \end{aligned}$ | $(1, \$ 300 ; 200, \$ 1)>(1, \$ 500)$ | -1.419 | -0.66 | 3.797 | 2.55 | 0.978 | 1.62 |
| GP\&Small vs MW | $\begin{aligned} & (1, \$ 300 ; 200, \$ 1)>(1, \$ 200 ; 1, \\ & \$ 150 ; 1, \$ 100 ; 1, \$ 50) \end{aligned}$ | -1.495 | -0.61 | 4.908 | 2.77 | 2.233 | 3.22 |

*Numbers in bold indicate significant estimates.

