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Marketing agencies, media experts and sales agents: Helping competitive firms improve the effectiveness of marketing

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ABSTRACT

A major challenge for marketers is how to spend a marketing budget such that the impact on a target market is maximized. There are numerous organizations including marketing agencies, media experts and sales agents that promise to deliver more impact for the money a firm allocates to marketing. The service that these organizations offer are to organize and coordinate spending such that a higher fraction of the potential market is effectively reached by a firm's marketing effort. My objective is to understand the optimal strategy that marketing agencies, media experts and sales agents should use to sell these services. In particular, I analyze how these services should be priced and whether a seller gains by selling such services exclusively. The model consists of a seller of marketing services and two symmetric firms that compete in a differentiated market. A downstream firm that purchases the services reaches a higher fraction of the potential market due to the efficiency provided by the services. The analysis shows that the optimal selling strategy for the services is a function of three factors: a) the degree of differentiation between firms, b) the fraction of the target that is reached by firms (prior to using the seller's services) and c) the increase in "reach" provided by the seller's services. Non-exclusive selling is likely to be optimal, the less that downstream firms compete with each other due to strong differentiation in the downstream market or a low level of overlap in the customers reached by the marketing of each downstream firm. In contrast, exclusive selling is advantageous when many customers have been reached by the marketing of both firms or the level of differentiation between the firms is low. Surprisingly, in many situations, the seller's profit is inversely related to the level of differentiation.

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1. Introduction

1.1. Background

Today, increased competition, media fragmentation and an increase in the types of channels where consumers shop, make it increasingly difficult for marketing managers to achieve "measurable impact" with marketing activities. In fact, maintaining impact with marketing spending is a major challenge for managers. As a result, there are numerous organizations, from marketing agencies to media experts, which offer services to provide clients with a higher level of impact from their spending on marketing. The service provided by these organizations is that of assisting clients to organize and coordinate marketing activity such that the "effective reach" of a client's marketing activity is increased. A significant problem for firms are potential customers who are a) unaware of a firm's product or b) aware of the product yet lack complete and accurate information on its characteristics and pricing. In general, most customers will not buy unless they know what they buying. The services are assumed to increase the fraction of the market that has complete and accurate information about the firm's product.

Many market research agencies build databases on the behavior and media habits of consumers that can improve the effectiveness of marketing spending. Companies such ICOM, Experian, Acxiom and Donelley Marketing have databases on millions of households, companies or individuals that can be used to identify high potential customers in many categories.[1] For example, ICOM collects information on the social issues that respondents most identify with (choices include animal welfare, environmental issues, the arts, health and religious causes). This information is then used by organizations (the sellers of various products and services) to improve the performance of their spending. To be specific, the symphony orchestra might use ICOM to prepare a mailing list based on the level of interest that a household has in the arts. Targeted direct mail campaigns such as this

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[1] Frequently, market research companies and media experts possess monopoly-like positions for the type of information that they sell. ICOM is the only market research company in Canada that owns a database containing millions of households. Experian is the only market research company in the US that has an extensive database on the credit worthiness and financial status of individuals. Donelley Marketing is the only research company that possesses information on the characteristics of firms and individuals based on their involvement in Yellow Pages telephone listings.

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Table 1
Examples of services that increase the impact of marketing

Marketing expert[a]	Target	Source of marketing efficiency	Year
O&M Direct London, U.K.	High potential buyers of phone cards	Identification of UC Admission Service to reach parents whose children are about to go to university	1995
Real Media Inc. New York, NY	Men over 40 with income >$90,000	Ad-targeting technology using NY Times database	1997
On Target Solutions Cincinnati, OH	Families with an asthma sufferer	Direct to Patient media: reaches prospects in physicians' offices	1999
VentureDirect Worldwide New York, NY	B2B customers in 23 separate segments	Integrated DM, e-mail and web communication	2000
Starcom MediaVest Group Chicago, IL	Mainstream customers of cars (GM)	Ad targeting technology based on proprietary optimizers	2003
Mediacom Latino New York, NY	Hispanic Media Buying Services	Proprietary tools and information on the media habits, culture and language of Hispanics	2003

[a] Sources available on request from the author.

have a much higher response rate than standard campaigns based on geographic targeting. Clients find that information sold by companies like ICOM to improve targeting can increase spending effectiveness by more than 50% (Bush, 1998).

Another important phenomenon is a trend away from integrated advertising agencies to organizations that specialize in creative development and media buying respectively (Horsky, 2006). In particular, many of the world's largest advertising organizations have evolved into advertising holding groups comprised of several creative shops and a media buying service. The media buying service of these groups offer services that assist clients to choose media vehicles, programmes, and timing that maximize the impact of a given marketing budget. Beyond these media buying services, there are consultants that describe themselves as media experts (see “Star Turn”, The Economist, March 9, 2000). Examples of media-expert companies include Carat, Mediacom and Starcom. These firms commit significant resources to collecting information that links the media habits of viewers to their consumption patterns and lifestyles. Moreover as media decisions have become more complex, the value of media experts has risen (see Pfanner, 2005). As noted by Ms. Tassaro of Media Edge, “Anyone can come up with a cookie-cutter media plan, but leading-edge media planning has become a multi-layered process demanding specialized knowledge and relationships” (Fitzgerald, 1999).

In industrial markets, sales agents and manufacturer's reps frequently offer increased access to a target market: they allow a firm to reach customers that are difficult for company's internal salesforce to reach and inform. Table 1 summarizes several examples of companies that provide services to increase the impact of a client's marketing spend.

The one thing these examples have in common is that they allow a downstream firm to get more for its investment in marketing.

I build a model with three objectives in mind. The first is to increase our understanding of how the services provided by marketing agencies, media experts and sales agents affect competition in downstream markets. The second is to provide normative insight about how these services will be a priced as a function of downstream market characteristics. The third is to understand the incentives that sellers of such services might have to limit their provision within a market (by granting exclusivity).

1.2. Framework and results

The model consists of a seller of marketing services and two downstream firms that compete in a differentiated market. In the first stage, the seller offers its services to the downstream firms. The services increase the efficiency of a firm's marketing effort. This implies that a firm which accepts the seller's offer reaches a higher fraction of the potential target market due to the efficiency the services provide.

After deciding whether to accept the seller's offer, the firms set prices for their products. The marketing of each firm activates customers in the target market by informing them of the characteristics and price of the firm's product.[2] Marketing thus creates a second dimension of customer heterogeneity: the degree to which consumers are informed about products in the market. From the perspective of each firm, there are two key groups of consumers created by marketing: consumers who are aware of both products in the market and consumers who are only aware of the focal firm's product. Marketing does not guarantee that a customer will buy the firm's product but it does guarantee that a customer will evaluate the company's offer and make a decision.

The setting of prices by the seller of these services is not straightforward. It is driven by how the services ultimately affect competition in the downstream market. As expected, the optimal price for the information is positively related to the added impact provided by the services. However, the relationship between the optimal prices and a) the degree of differentiation between the downstream firms and b) the fraction of the target market that is reached by firms without the seller's services is complex.

In many situations, the seller's price and profit are negatively related to the level of differentiation. One would think that differentiation might limit the degree to which competition dissipates benefits created by more efficient marketing. For example, there is evidence that increases in category demand, improvements in distribution efficiency and product modifications are more valuable when differentiation insulates firms from each other and allows higher prices.[3] However, in a market where demand is driven by reaching customers with marketing, this is not necessarily the case.

I also examine the seller's incentive to sell its services exclusively. The optimal strategy depends on how the demand-enhancing effect of the services compares to their competition-increasing effect. Since increases in efficiency invariably lead to higher demand, the natural strategy is for the seller to sell to both firms (for exclusive selling to be attractive, the exclusive price must be more than two times the price that can be charged when selling to both firms). Non-exclusive selling is likely to be optimal the less that downstream firms compete with each other due to a) a high level of differentiation between the firms and b) a low level of overlap in the customers that are reached by the marketing of each firm. In contrast, exclusive selling is advantageous when many customers have been reached by the marketing of both firms and the level of differentiation between the firms is low. To implement an exclusive selling strategy, the seller needs to provide a contractual guarantee to the buyer.

In the following section, I review the literature that is relevant to the selling of marketing services that allow firms to increase the impact of their marketing spend.

[2] Without marketing, consumers are inactive. This representation follows models proposed by Butters (1977) and Grossman and Shapiro (1984).

[3] As noted by Hermalin (1993) and Cabral and Villas-Boas (2005), there are situations where differentiated firms are unable to capture the benefits of reduced costs of production or marketing. Nevertheless, in a standard spatial model, higher prices due to higher differentiation make increases in the density of demand more valuable (Salop, 1979). Iyer and Soberman (2000) also find that the value of product modifications can be positively related to the level of differentiation.

1.3. Related research

A rich stream of research examines the optimal selling strategies for monopolists that are endowed with knowledge or expertise that is valuable to downstream buyers. This literature has its origins in financial markets where an informed agent sometimes has valuable information about the uncertain value of a risky asset (such as a stock, bond, or option). The informed agent can sell the information to uninformed investors who then use the information to make advantageous trades with investors who have not purchased it (Admati & Pfleiderer, 1986). As demonstrated by Grossman and Stiglitz (1980), the problem is interesting because market prices are themselves informative about the private information of traders.[4]

These ideas have been extended to the marketing arena where sellers (or consultants) sell reports that are valuable because they provide buyers with better estimates of a stochastic demand parameter (Sarvary & Parker, 1997). The authors find that an information seller can sometimes be better off when he faces competition than when he is the only seller. In fact, when buyers need two reports, a recent study suggests that information sellers will split the market to sell only first or second opinions to customers (Sarvary, 2002).

There is also literature that considers the selling of syndicated marketing information that identifies relationships between brand loyalty and the preferences or demographic characteristics of consumers. Iyer and Soberman (2000) consider information that facilitates targeted product modifications. The primary finding of this paper is that the nature of the information is the main determinant of the information seller's optimal strategy. For example, when the information seller has information on attractive product modifications for the loyal customers of two competing firms, the information seller will sell a complete set of information to both firms. In addition, the value of the information is highly dependent on the impact of the modifications relative to the level of differentiation between firms.

Several other papers consider the impact or use of information in a marketing context. Shaffer and Zettelmeyer (1999) consider the use of value-adding information in the context of distribution channels. The authors study how this information affects the division of profits in a market with two competing manufacturers and a common retailer. Raju and Roy (2000) consider the value of information on uncertain demand to firms that are *ex ante* of different sizes (the authors focus on the value of information and not on its selling or provision). Pasa and Shugan (1996) model expertise as an ability to interpret information about demand and they are interested in characterizing the value of this expertise within companies. Finally, Villas-Boas (1994) studies the transmission of information between competing firms through an advertising agency and proposes an explanation for why competing firms might use a common advertising agency. While the context of Villas-Boas is similar to the context of this paper, the focus is different. In Villas-Boas, the agency is a mechanism for reducing firm-specific asymmetric information. In contrast, the services provided by the seller in my analysis are equally valuable to both potential buyers: the services of the marketing agencies, media experts and sales agents allow any firm that purchases the services to increase the impact of its marketing effort.

This leads to the critical element that distinguishes this analysis from the existing literature. The services provided by the seller in this model do not provide a better estimate of a stochastic parameter. The services allow a client-firm to increase the fraction of potential consumers who are fully informed about its product characteristics and price. The analysis is related to the diffusion of innovation literature (Arrow, 1962; Kamien & Tauman, 1984, 1986). However, services that increase the awareness of a firm's offering are different than innovations that reduce the cost of production primarily because marketing itself creates a second dimension of customer heterogeneity.

The article proceeds as follows. The model is presented in the next section. In Section 3, I analyze the downstream market equilibrium as a function of the level of differentiation and the fraction of the potential market that is aware of each firm's offer. In Section 4, I determine the relationship between the pricing of these services and downstream market differentiation. and examine the incentives of the seller to sell the services exclusively. Finally, I conclude in Section 5.

2. The model

The model consists of a marketing agency, media expert or sales agent that offers services to two potential clients who compete in a downstream market. The game has two stages. The first stage is the selling of services to the downstream firms. In second stage, the downstream firms choose prices (these prices depend on the decisions taken in the first stage). Finally consumers make decisions whether or not to buy either of the products offered by the competing firms. I next describe the downstream market and then explain the mechanism by which marketing creates demand for products.

2.1. The downstream market

The downstream market consists of two firms located at either end of a unitary Hotelling market. The firm at the left end of the market is Firm 1 and at the right end is Firm 2. Each firm sells a single product and the unit cost of production, c, is assumed to be constant. The products differ with respect to an attribute and consumers are uniformly distributed along the attribute with a density of one. This implies that any consumer is identified by an ideal point along the attribute corresponding to her preferred brand.

A consumer buys at most one unit of product and places a value v on her ideal product. Because the firms are located at either end of the market, a consumer does not obtain her ideal product. A consumer located a distance x from Firm i ($i=1, 2$) obtains a surplus $v-tx-p_i$ by consuming Firm i's product, where t is the "preference" cost per unit distance and p_i is the price charged by Firm i. The parameter t measures the sensitivity of consumers to the product attribute (it serves as a measure of the degree of differentiation in the market). Marketing informs consumers about the characteristics and prices of products but does not affect v (the consumer's willingness to pay) or t (the preference cost).[5]

A consumer only buys if she knows of a product that offers positive surplus i.e. $v-tx-p_i>0$. Without marketing, consumers are assumed to be uninformed about the characteristics or prices of products; the *only* way a consumer becomes informed about a firm's offer (product characteristics and price) is through that firm's marketing. As in the model of Butters (1977), marketing provides truthful information about the firm's offer and customers do not experiment with products they do not know. If a consumer is aware of more than one product offering positive surplus, she buys the product offering the greatest surplus.

This model is a 2-firm version of the model proposed by Grossman and Shapiro (1984) where marketing activates consumers. The model has been used in marketing in papers such as Soberman (2004, 2005), Dukes and Gal-Or (2003), Dukes (2004) and Gal-Or and Dukes (2006). Typically marketing effort is treated as a decision variable in these

[4] There are many approaches that an informed agent can use to profit from an informational advantage in financial markets. For example, an informed agent can sell information directly to uninformed investors, can trade with them or can sell shares in a portfolio of assets that have been constructed using the information (Admati & Pfleiderer, 1988, 1990).

[5] There is en extensive literature on the numerous paths by which marketing can affect the behaviour and decisions of consumers (Tirole, 1988). Nevertheless, the idea that marketing provides information about products and their key attributes is relatively uncontroversial.

models. Here however, the base level of each firm's marketing effort is assumed to be an exogenous parameter. This allows me to focus on the upstream selling of marketing services that increase the reach of a firm's marketing effort.

2.2. The impact of marketing

The marketing effort of Firm i (i=1, 2) is assumed to reach a fraction ϕ_i of the potential market and this fraction is assumed to be uniformly distributed along the market. This means that marketing creates a second dimension of consumer heterogeneity based on the information consumers have about products. Following the structure described in Fershtman and Muller (1993), 4 distinct groups of consumers are formed when the firms market at levels ϕ_1 and ϕ_2 respectively. First, there are consumers who have been reached by the marketing of both firms (a fraction $\phi_1\phi_2$ of the market). Second, there are consumers who were not reached by the marketing of either firm (a fraction $(1-\phi_1)(1-\phi_2)$ of the market). Finally, there are two groups of consumers been reached by the marketing of one of the two competing firms (given by $\phi_1(1-\phi_2)$ and $\phi_2(1-\phi_1)$ respectively).

Without the seller's services, the firms are assumed to be symmetric and their marketing has a base reach of $\underline{\phi}$ (i.e. $\phi_1=\phi_2=\underline{\phi}$). In order for the seller to be able to improve the reach of a firm's marketing effort, I restrict $\underline{\phi}<1$.

2.3. Stage one: The selling and buying of marketing services

Before describing the selling stage, I clarify the precise benefit that the marketing agent, media expert or sales agent offers to downstream firms. The seller is assumed to possess information or knowledge that allows a firm to obtain a ρ% increase in the effective reach of its marketing effort. As explained earlier, the services offered by the seller allow the client to increase the fraction of the market that has complete and accurate information about the firm's product: this increases the number of customers who seriously consider buying the firm's product. In the context of the model, this implies that a firm that engages the seller will increase the impact of its marketing effort from $\underline{\phi}$ to $\underline{\phi}(1+\rho)$.[6]

A further issue regards the viability of using an exclusive strategy to sell marketing services. In this model, the seller always has an incentive to violate an exclusive agreement. As a result, the seller must make a credible commitment for the exclusive strategy to be viable. This seems reasonable as industry interviews reveal that many marketing agencies offer category exclusivity for their services (at least for a specified period of time). For example, the services of ICOM are often sold with category exclusivity for a specific period (between 6 months and one year).[7] The loss of both reputation and related business (many clients use marketing services companies across categories) were the violation of an exclusive contract detected appears to make exclusivity commitments both frequent and highly credible. In the game presented here, a violation would be detected since the second firm acquiring the services would have marketing impact in excess of its initial levels. Conversely, in markets where a downstream firm cannot tell whether an exclusive agreement has been violated, exclusive selling may not be possible.

The structure of the game is as follows. The seller of marketing services decides whether or not to offer exclusivity before it contacts downstream firms. The game tree for the first stage of the game is shown in Fig. 1. The downstream profits are denoted by π_y and $y=a$, d, b, n (a implies the firm is the sole purchaser of the marketing services, d implies that the firm faces a competitor that has purchased the marketing services, b implies that both firms have purchased the services and n refers to the situation where neither firm has purchased them).

The left side of the tree represents the choice of exclusive selling. The prices P_{x1} and P_{x2} are chosen by the seller in the context of exclusive selling. Second, Fig. 1 shows that Firm 1 receives the exclusive offer first; however, this is arbitrary since the downstream firms are *ex ante* symmetric. If Firm 2 is contacted by the seller, Firm 2 knows that Firm 1 has rejected the seller's offer. However, when an exclusive strategy is chosen this does not happen. The seller chooses a price P_{x1} such that Firm 1 is strictly better off by accepting the seller's offer. The right side of the tree represents the choice of non-exclusive selling and P_b is the non-exclusive price chosen by the seller. The downstream firms are assumed to make simultaneous decisions about accepting the seller's offer under non-exclusive selling.

Several comments are worth noting. On the one hand, there are many ways to model exclusive selling: the services could be sold through an auction, buyers could be asked to submit two bids simultaneously for the services or, in contrast to Fig. 1, the price for the second buyer could be chosen *a priori* and announced to the first firm. On the other hand, the steps shown in Fig. 1 are a straightforward way to represent a process where a) the seller of marketing services has market power (the seller moves first) and b) the “exclusive offer” has bite (a firm that refuses the seller's exclusive offer knows that it will *probably* a face a competitor who accepts a similar offer from the seller). One could also model the selling of services as independent negotiations between the seller and each firm (as a bargaining process). However, the optimal downstream arrangement (non-exclusive or exclusive selling) would not be affected because a) the game is one of complete information and b) there is no coordination problem.[8]

2.4. Stage two: Competition between downstream firms

The second stage of the game entails the simultaneous pricing decisions by downstream firms and finally those of informed consumers. The profit of a candidate firm is a function of the price that it charges and the demand that it realizes from the two segments it serves (consumers who are aware of the candidate firm x_i and consumers who are aware of both firms y_i).

$$\pi_1 = p_1(\phi_1(1-\phi_2)x_1 + \phi_1\phi_2 y_1) \tag{1}$$

$$\pi_2 = p_2(\phi_2(1-\phi_1)x_2 + \phi_1\phi_2 y_2) \tag{2}$$

These functions represent the profits net of expenditures on marketing. I use these functions because the role of the seller's services is to provide more impact for the money that each firm allocates to marketing (the amount each firm allocates is assumed identical).

The demand from each segment follows the reasoning of Section 2.1. In the group of consumers who have been reached by

[6] The analysis will show that different combinations of ϕ and t are associated with different types of equilibria. When ρ is high enough, the introduction of the services can change the type of equilibrium observed in the market. My analysis focuses on efficiency increases that do not lead to a change in the type of equilibrium. Later in the paper, I discuss how the seller's optimal selling strategy is affected when ρ is high enough to lead to such a change. An interesting extension to this model suggested by an anonymous reviewer would be that of making ρ, a costly decision for the seller of services (the impact of the services would be a function of *ex ante* investment made by the seller). I do not investigate this idea; nevertheless, the selling sub-game in such an extension would be analogous to the selling game presented here.

[7] This information is based on personal interviews with the managers of ICOM. Violating an exclusive contract is easy to detect since many of ICOM's clients are competitors in multiple categories.

[8] When coordination is important, cooperative or non-cooperative bargaining provide insight into the types of arrangements that are observed empirically. For example, in vertical channels when manufacturers set wholesale prices and retailers choose retail prices, there is a “coordination problem” (Dukes, Gal-Or, & Srinivasan, 2006; Iyer & Villas-Boas, 2003).

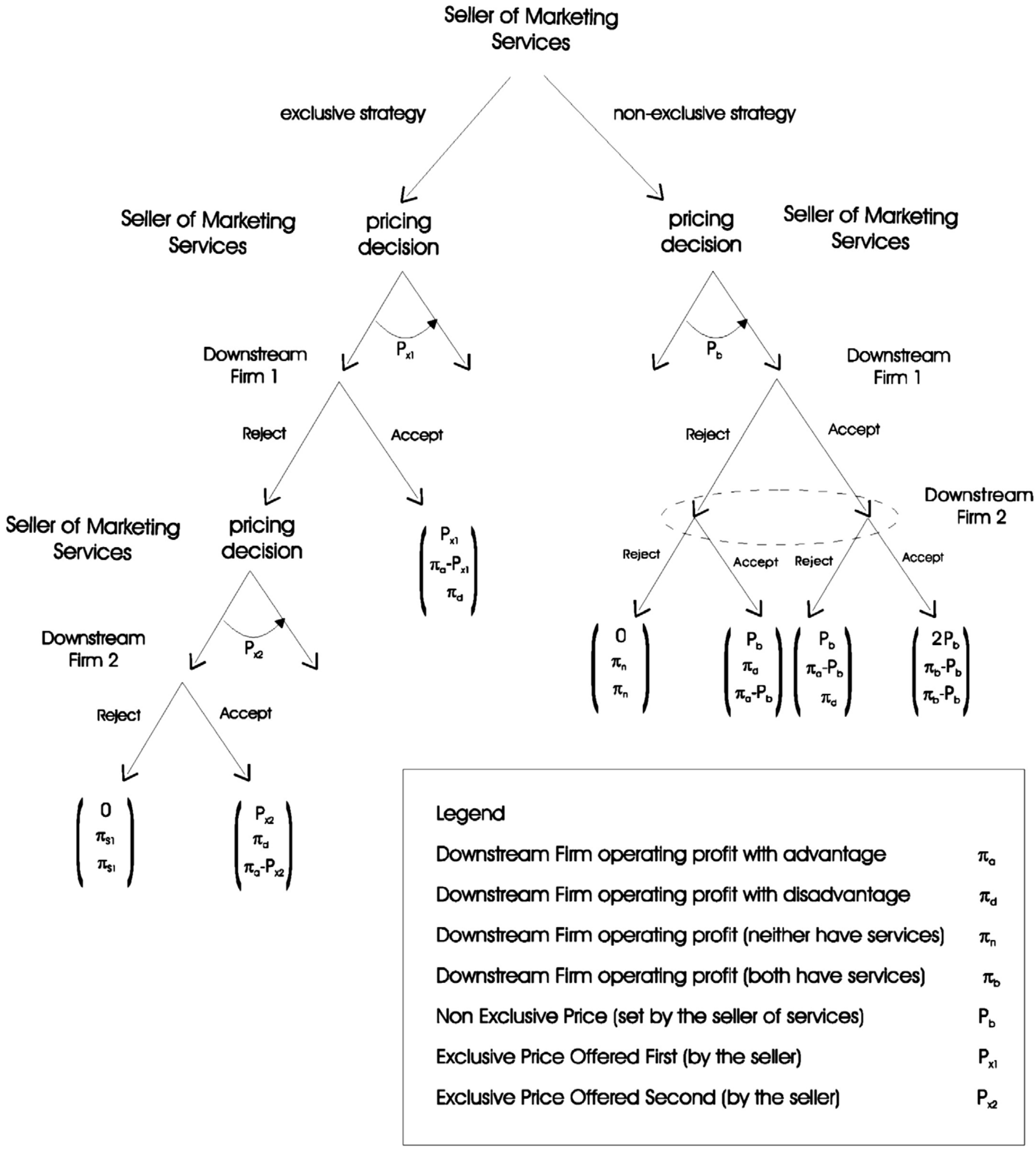

Fig. 1. Stage one of the game.

Firm i's marketing only and not the competitor's, demand is determined by individual rationality i.e. all consumers in the segment who obtain positive surplus in the segment from Firm i's product will buy from Firm i. This implies that demand from this segment is $x_i = \frac{v-p_i}{t}$ except if $v-p_i>t$ in which case $x_i=1$. The derivation of demand from consumers who have been reached by the marketing of both firms depends on the location of the indifferent consumer given prices p_1 and p_2. It is straightforward to show that $y_1 = \frac{p_2-p_1+t}{2t}$ and $y_2 = \frac{p_1-p_2+t}{2t}$. These expressions hold except when $p_2-p_1>t$ or $p_1-p_2>t$. In these cases, Firm 1 or Firm 2 respectively capture the entire group of fully informed consumers.

The extensive form of stage two is summarized as follows:

Step 1: Firms choose prices p_i $(i=1, 2)$ as a function of the fraction of the market that is reached by each firm's marketing ϕ_i $(i=1, 2)$.

Step 2: If a consumer has been reached by the marketing of one or more firms, she purchases the product that provides her with maximum surplus assuming that her participation constraint is satisfied.

To simplify the analysis, I make two normalizations. The actual level of differentiation between downstream firms is a function of the total surplus created when a consumer consumes her ideal product ($v-c$) relative to the level of differentiation t. Thus, without loss of generality, I set the marginal cost equal to zero and the reservation utility to 1. This allows me to analyze a complete range of differentiation conditions by varying t.

In addition, I restrict the analysis to conditions where all consumers are potential customers of either firm $\left(t<\frac{1}{2}\right)$. When all consumers in the market have been reached by the marketing of both firms, the equilibrium price is t. When $t<\frac{1}{2}$ and $p_1=p_2=t$, any consumer in the market realizes positive surplus from either firm's product.[9]

I now proceed to the analysis of the downstream game. This provides the basis for determining the optimal selling strategy for the seller in Section 4.

3. The downstream market and equilibrium decisions

Given the range of conditions I examine $\left(t<\frac{1}{2}\right)$, the analysis reveals that there are two distinct regions based on the types of equilibria that occur in the downstream market. The first region is $t\in\left(\frac{1}{5},\frac{1}{2}\right)$ which I call strong differentiation. The second region is $t\in\left(0,\frac{1}{5}\right)$ which I call weak differentiation. I first present the equilibrium outcomes for the region of strong differentiation.

3.1. Strong differentiation: $t\in\left(\frac{1}{5},\frac{1}{2}\right)$

The objective functions for the downstream firms when they both choose prices less than $1-t$ are a function of demand from the two groups of consumers that have been reached by each firm's marketing.

$$\pi_1=p_1\left(\phi_1(1-\phi_2)+\phi_1\phi_2\frac{p_2-p_1+t}{2t}\right) \tag{3}$$

$$\pi_2=p_2\left(\phi_2(1-\phi_1)+\phi_1\phi_2\frac{p_1-p_2+t}{2t}\right) \tag{4}$$

In contrast, when the prices are marginally greater than $1-t$, the demand from consumers who have only been reached by the marketing of one firm is downward sloping in the price that the firm charges (the first term in large parenthesis in Eqs. (5) and (6) is negatively related to p_1 and p_2 respectively).

$$\pi_1=p_1\left(\phi_1(1-\phi_2)\frac{1-p_1}{t}+\phi_1\phi_2\frac{p_2-p_1+t}{2t}\right) \tag{5}$$

$$\pi_2=p_2\left(\phi_2(1-\phi_1)\frac{1-p_2}{t}+\phi_1\phi_2\frac{p_1-p_2+t}{2t}\right) \tag{6}$$

Both sets of objective functions are needed to identify the equilibrium outcome in the downstream market. First, I consider the case where $\phi_1=\phi_2$.

3.1.1. Strong differentiation: Symmetric competitors

The downstream firms are symmetric in terms of marketing reach when a) neither firm accepts the seller's offer $\phi_1=\phi_2=\underline{\phi}$ or b) both of them do $\phi_1=\phi_2=\underline{\phi}(1+\rho)$. Proposition 1 summarizes the downstream equilibrium as a function of ϕ (the marketing reach of the firms when they are symmetric). All proofs are provided in Appendix A.

Proposition 1.

1. *When $\phi<2t$, $p_1=p_2=1-t$ and firms earn profits of $\frac{1}{2}\phi(1-t)(2-\phi)$.*
2. *When $\phi>2t$, $p_1=p_2=\frac{1}{\phi}(2t-t\phi)$ and firms earn profits of $\frac{1}{2}t(2-\phi)^2$.*

When $\phi<2$t Proposition 1 shows that each firm sets price such the informed consumer at the maximum distance from the firm is indifferent between buying and not buying. Firms effectively charge the maximum feasible price to the market: were firms to raise price above $1-t$, then the captive consumer located at the maximum distance would find the product too expensive.[10]

In essence, firms choose not to compete for consumers who are aware of both firms (demand from this segment is evenly split between the firms). The potential gain in demand from a price reduction is less than the loss in profit created by giving existing consumers a subsidy.[11] In this situation, prices and profits are negatively related to the level of differentiation because a higher t reduces the profit earned from each consumer (i.e. $p=1-t$). In addition, profits are positively related to the marketing reach of the firms $\left(\frac{\partial\pi}{\partial\phi}=(1-t)(1-\phi)>0\right)$.

When there are enough customers in the market who are aware of the offerings of both firms, the firms have an incentive to deviate from the equilibrium strategies described in Proposition 1. When $\phi>2t$, each firms sets price in order to a) extract profit from the consumers who are reached by its marketing effort (and not the competitor's) and b) compete for the consumers who are reached by the marketing of both firms. This leads to an equilibrium in pure pricing strategies where prices are positively related to the level of differentiation $\left(\frac{\partial p}{\partial t}=\frac{2-\phi}{\phi}>0\right)$. Profits are also positively related to the level of differentiation $\left(\frac{\partial\pi}{\partial t}=\frac{1}{2}(2-\phi)^2>0\right)$. In contrast, profits are negatively related to the reach delivered by each firm's marketing $\left(\frac{\partial\pi}{\partial\phi}=t(\phi-2)<0\right)$. These relationships are the opposite of those observed when $\phi<2t$.

3.1.2. Strong differentiation: Asymmetric competitors

The downstream firms are asymmetric when only one firm engages the seller of marketing services. If only Firm 1 accepts the offer of the seller, this implies that $\phi_1=\underline{\phi}(1+\rho)$ and $\phi_2=\underline{\phi}$. Proposition 2 summarizes the downstream equilibrium as a function of $\underline{\phi}$ and ρ.

Proposition 2.

1. *When $\underline{\phi}<\frac{2t}{1+\rho}$, $p_1=p_2=1-t$. Firm 1 earns profits of $\frac{1}{2}\underline{\phi}(t-1)(\underline{\phi}-2)(\rho+1)$ and Firm 2 earns profits of $\frac{1}{2}\underline{\phi}(t-1)(\underline{\phi}+\underline{\phi}\rho-2)$.*
2. *When $\underline{\phi}>\frac{2t}{1+\rho}$, $p_1=\frac{6t-3t\underline{\phi}+4t\rho-3t\underline{\phi}\rho}{3\underline{\phi}(1+\rho)}$ and $p_2=\frac{6t-3t\underline{\phi}+2t\rho-3t\underline{\phi}\rho}{3\underline{\phi}(1+\rho)}$. Firm 1 earns profits of $\frac{(3\underline{\phi}-4\rho+3\underline{\phi}\rho-6)^2t}{18(1+\rho)}$ and Firm 2 earns profits of $\frac{(3\underline{\phi}-2\rho+3\underline{\phi}\rho-6)^2t}{18(1+\rho)}$.*

Proposition 2 demonstrates that the equilibria when the competitors are asymmetric (in terms of marketing reach) follow a similar pattern to those observed for the symmetric case. For example, when the level of marketing reach is sufficiently low, the pricing equilibrium involves both firms choosing the maximum feasible price. Not surprisingly, the firm with higher marketing reach earns higher profits. Proposition 2 also shows that Firm 2 prices more aggressively

[9] This assumption ensures fully competitive conditions between the downstream firms. When $t>\frac{1}{2}$, the externalities between the downstream firms are reduced and when $t>1$, there is no interaction between the firms.

[10] Captive consumers have only been reached by a focal firm's marketing i.e., either they buy from the focal firm or not at all.

[11] This outcome is competitive even though it looks collusive.

than Firm 1 when $\underline{\phi} > \frac{2t}{1+\rho}$. The explanation for this is that when Firm 1's marketing reach is higher than Firm 2's, a greater fraction of Firm 2's demand comes from consumers who have been reached by marketing from both firms. Thus, Firm 2 has a stronger incentive than Firm 1 to choose a price that is close to the optimum for the "competitive segment". This means that Firm 2's price is lower than Firm 1's. Surprisingly, Firm 2 captures more than 50% of the competitive segment in spite of having "less effective marketing" $\left(x = \frac{1-2\rho}{2} < \frac{1}{2}\right)$.[12]

3.2. Weak differentiation: $t \in \left(0, \frac{1}{5}\right)$

When the level of differentiation is sufficiently low, the cost to compete for consumers who have been reached by marketing from both firms increases. The reason the cost is higher is that the competitive price for consumers who are informed about the products of both firms is t and the monopoly price for consumers who have only been reached by one firm is $1-t$.[13] As the level of differentiation increases, the difference between these two prices increases. When $t < \frac{1}{5}$, the equilibrium where firms price at $1-t$ (when marketing reach levels are low) and the equilibrium where prices are a function of marketing reach (when the levels of marketing reach are higher) are observed. However, for a range of $\underline{\phi}$, the equilibrium in pure pricing strategies breaks down. I examine this issue for symmetric competitors and then for asymmetric competitors.

3.2.1. Weak differentiation: Symmetric competitors

When the levels of marketing reach are either too high or too low, the firms have an incentive to defect from the competitive price of $\frac{1}{\phi}(2t - t\phi)$ to the highest feasible price $1-t$ in order to charge the maximum price to consumers who have only been reached by one firm's marketing (these consumers do not make a comparison between the firms). However, if Firm 1 increases price to $1-t$ then Firm 2 has an incentive to increase its price to $1-2t$ and earn higher profit on *all* consumers who have seen Firm 2's advertising. But when Firm 2 chooses a price of $1-2t$, Firm 1 has an incentive to undercut Firm 2 and choose a price of $1-3t$.

Similar to Shilony (1977) and Narasimhan (1988), this implies the non-existence of an equilibrium in pure strategies. This game (with continuous action spaces) also has discontinuous payoffs (when $|p_1 - p_2| > t$, the firm with a lower price obtains the entire segment of consumers who have been reached by the marketing of both firms). In games such as this the existence of a mixed strategy equilibrium depends on two theorems from Dasgupta and Maskin (1986). The model satisfies the conditions for a mixed strategy equilibrium and this is discussed in Appendix A.

Undercutting for the fully informed segment does not reduce profits from the fully informed segment to zero: a firm will not reduce its price such that it earns less than it would earn were the competitive price of t charged to the fully informed segment. This implies that the guaranteed profit that either firm can earn by selling to the fully informed segment is $\frac{\phi_1 \phi_2 t}{2}$. The outcome when firms are symmetric and a pure strategy equilibrium does not exist is summarized in Lemma 1.

Lemma 1. *When* $\phi \in \left(\frac{1+t-\sqrt{-6t+5t^2+1}}{2-t}, \frac{1+t+\sqrt{-6t+5t^2+1}}{2-t}\right)$*, the equilibrium involves mixed pricing strategies. The firms earn profits of* $\phi(1-t)(1-\phi) + \frac{1}{2}\phi^2 t$.

Using Lemma 1, I derive Proposition 3 which identifies the equilibria for the feasible range of ϕ when differentiation is weak.

[12] The feasibility condition on x implies that ρ must be less than $\frac{1}{2}$ for the equilibrium to hold.

[13] The monopoly price when $t > \frac{1}{2}$ is $\frac{1}{2}$, however, when $t < \frac{1}{2}$, a monopoly maximises profit by charging the reservation price to the most distant consumer.

Proposition 3.

1. *When* $\phi < 2t$, $p_1 = p_2 = 1-t$ *and firms earn profits of* $\frac{1}{2}\phi(1-t)(2-\phi)$.
2. *When* $\phi \in \left(2t, \frac{1+t-\sqrt{-6t+5t^2+1}}{2-t}\right)$ $p_1 = p_2 = \frac{1}{\phi}(2t - t\phi)$ *and firms earn profits of* $\frac{1}{2}t(2-\phi)^2$.
3. *When* $\phi \in \left(\frac{1+t-\sqrt{-6t+5t^2+1}}{2-t}, \frac{1+t+\sqrt{-6t+5t^2+1}}{2-t}\right)$*, the equilibrium entails mixed pricing strategies and firms earn profits of* $\phi(1-t)(1-\phi) + \frac{1}{2}\phi^2 t$.
4. *When* $\phi \in \left(\frac{1+t+\sqrt{-6t+5t^2+1}}{2-t}, 1\right)$, $p_1 = p_2 = \frac{1}{\phi}(2t - t\phi)$ *and firms earn profits of* $\frac{1}{2}t(2-\phi)^2$.

Note that the interval $\left(\frac{1+t-\sqrt{-6t+5t^2+1}}{2-t}, \frac{1+t+\sqrt{-6t+5t^2+1}}{2-t}\right)$ exists if an only if $t < \frac{1}{5}$. In other words, the level of differentiation between firms needs to be sufficiently low for mixed pricing strategies to be possible. The pattern of outcomes for the feasible range of ϕ when $t = \frac{1}{6}$ is shown in Fig. 2. Fig. 2 shows that as ϕ increases from 0 to 1, the type of equilibria observed changes three times. Moreover, because profits are negatively related to differentiation $\left(\frac{\partial \pi}{\partial t} = \frac{1}{2}\phi(3\phi - 2)\right)$ when $\phi < \frac{2}{3}$ and price strategies are mixed, the relationship between profits and differentiation changes three times as ϕ increases from 0 to 1. At low levels of ϕ, the relationship is negative, it then becomes positive, it then becomes negative and then positive again.

When price strategies are mixed, the relationship between marketing reach and profits depends on the level of marketing reach $\left(\frac{\partial \pi}{\partial \phi} = 3t\phi - 2\phi - t + 1\right)$. When the level of reach is less than $\frac{1-t}{2-3t}$, the relationship is positive and when the level of reach is greater than $\frac{1-t}{2-3t}$, the relationship is negative. The relationship changes because increasing marketing reach has two effects. The first is to raise the level of demand for each firm (this has a positive effect on profits). The second is to increase the fraction of consumers reached by the marketing of both firms. These consumers compare the offers from each firm and this creates an incentive for firms to cut prices (this has a negative effect on profits). When the level of marketing reach exceeds $\frac{1-t}{2-3t}$, the second effect is larger. This explains why the relationship between marketing reach and firm profits is negative when $\phi > \frac{1-t}{2-3t}$.

3.2.2. Weak differentiation: Asymmetric competitors

As earlier, I assume that Firm 1 accepts the offer of the seller. This implies that $\phi_1 = \underline{\phi}(1+\rho)$ and $\phi_2 = \underline{\phi}$. Similar to the case of symmetric firms, when the levels of marketing reach are either too high or too low, the pure price strategy equilibrium breaks down and the pricing equilibrium is in mixed strategies. The outcome when firms are asymmetric and a pure strategy equilibrium does not exist is summarized in Lemma 2 (the profits of Firm 1 and Firm 2 are summarized as a function of a function of $\underline{\phi}$ and ρ).

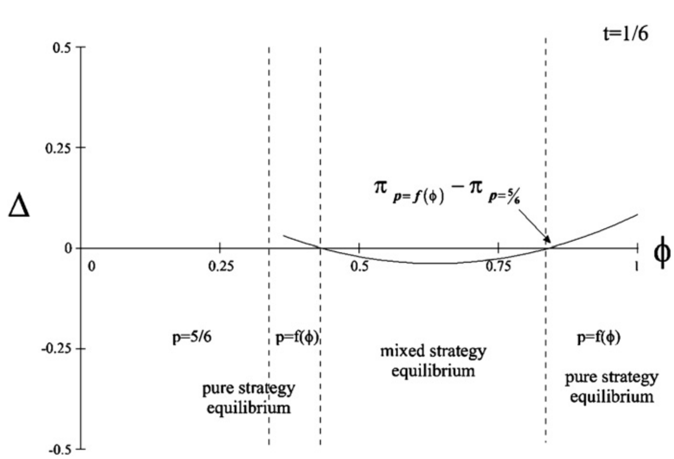

Fig. 2. Different equilibrium zones for the entire range of ϕ.

Lemma 2. *When* $\underline{\phi}\in\left(\frac{1+t-\sqrt{-6t+5t^2+1}}{2-t},\frac{1+t+\sqrt{-6t+5t^2+1}}{2-t}\right)$, *and only Firm 1 accepts the seller's offer, the profits earned by Firms 1 and 2 respectively are* $\pi_1=(1+\rho)\underline{\phi}(1-\underline{\phi})(1-t)+\frac{\underline{\phi}^2(1+\rho)t}{2}$ and $\pi_2=\underline{\phi}(1-\underline{\phi})(1-t)+\frac{\underline{\phi}^2 t}{2}$.

Using Lemma 2, I derive Proposition 4 which identifies the equilibria for the feasible range of $\underline{\phi}$ and ρ when differentiation is weak.

Proposition 4.

1. *When* $\underline{\phi}<\frac{2t}{1+\rho}$, $p_1=p_2=1-t$. *Firm 1 earns profits of* $\frac{1}{2}\underline{\phi}(t-1)(\underline{\phi}-2)(\rho+1)$ *and Firm 2 earns profits of* $\frac{1}{2}\underline{\phi}(t-1)(\underline{\phi}+\underline{\phi}\rho-2)$.
2. *When* $\underline{\phi}\in\left(\frac{2t}{1+\rho},\frac{1}{t-2}\left(-t+\sqrt{-6t+5t^2+1}-1\right)\right)$, $p_1=\frac{6t-3t\underline{\phi}+4t\rho-3t\underline{\phi}\rho}{3\underline{\phi}(1+\rho)}$ and $p_2=\frac{6t-3t\underline{\phi}+2t\rho-3t\underline{\phi}\rho}{3\underline{\phi}(1+\rho)}$. *Firm 1 earns profits of* $\frac{(3\underline{\phi}-4\rho+3\underline{\phi}\rho-6)^2t}{18(1+\rho)}$ *and Firm 2 earns profits of* $\frac{(3\underline{\phi}-2\rho+3\underline{\phi}\rho-6)^2t}{18(1+\rho)}$.
3. *When* $\underline{\phi}\in\left(\frac{1+t-\sqrt{-6t+5t^2+1}}{2-t},\frac{1+t+\sqrt{-6t+5t^2+1}}{2-t}\right)$, *the equilibrium entails mixed pricing strategies and Firms 1 and 2 earn profits of* $(1+\rho)\underline{\phi}(1-\underline{\phi})(1-t)+\frac{\underline{\phi}^2(1+\rho)t}{2}$ *and* $\underline{\phi}(1-\underline{\phi})(1-t)\frac{\underline{\phi}^2t}{2}$ *respectively.*
4. *When* $\underline{\phi}\in\left(\frac{1+t+\sqrt{-6t+5t^2+1}}{2-t},1\right)$, *the outcome is identical to that described in point 2 (above).*

The equilibria described in Proposition 4 show that the pattern and relationships observed for asymmetric firms are similar to those observed for the symmetric case. In particular, the relationship between differentiation and profits is negative when price strategies are mixed and marketing reach is low and becomes positive at higher levels of marketing reach. The relationship between marketing reach and profits depends on the level of marketing reach (detailed comparative statics are provided in Appendix A). Fig. 3 is a convenient summary of the equilibrium conditions derived in Sections 3.1 and 3.2. Note that in all cases, I assume that ρ (the efficiency increase provided by the seller of marketing services) is sufficiently low such that after the services are introduced, the equilibrium type is unchanged. It is interesting to note that in some parameter conditions, the profits of the downstream firms are inversely related to the level of differentiation between firms.

This completes Section 3 where the outcomes in the downstream market as a function of the possible marketing reach levels that firms have (depending on whether they buy services from the marketing agency or not) are derived. These outcomes are the raw materials needed to solve the first stage of the game, that of determining the optimal strategy for the marketing agency.

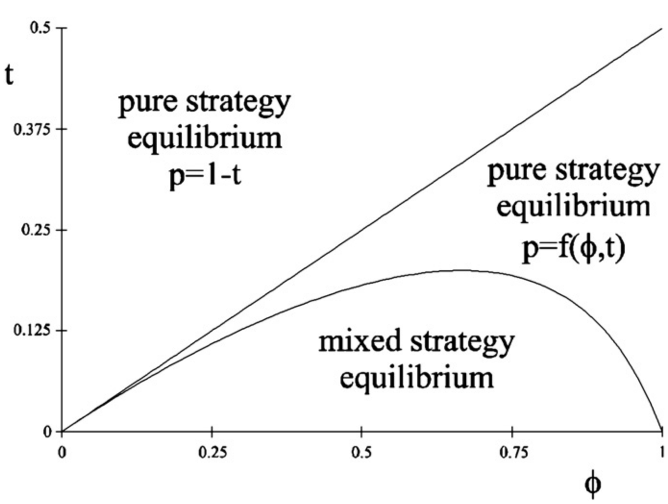

Fig. 3. The downstream equilibrium as a function of ϕ and t.

4. Optimal pricing and selling strategies for marketing services

First, I examine how the optimal price is determined when the seller would like to sell to a) both firms (non-exclusively) or b) just one firm (exclusively). I then determine which of these strategies is going to optimize the seller's profit for the parameter regions shown in Fig. 3.

4.1. Key comparisons for the seller of marketing services

A first challenge for the seller who wishes to sell the marketing services non-exclusively (i.e. to both firms) is to identify the highest price at which both firms will engage the seller. Referring back to Fig. 1, $\pi_a-P_b\geq\pi_n$ and $\pi_b-P_b\geq\pi_d$ are necessary conditions for both downstream firms to accept the seller's offer. This implies that $P_b\leq\min(\pi_a-\pi_n,\ \pi_b-\pi_d)$. In other words, the maximum price must lead to an increase in profit for a firm when neither firm has access to the services *and* a firm must also realize an increase in profit by purchasing the services to compete with a competitor that already has them. The profit earned by the seller of marketing is $2P_b$.

In order to maximize the profit associated with selling the services to only one downstream firm, the seller needs to provide the sole buyer with a guarantee of exclusivity. The guarantee is required because in all conditions, the services have value for the firm that did not engage the seller. When the guarantee is provided, Fig. 1 shows that the maximum price the seller can charge is the difference in profit that the marketing services $\pi_a-\pi_d$ create *ex post*. As long as $\pi_a>\pi_n$, any P_{x2} (acceptable to Firm 2) is sufficient to ensure that Firm 1 accepts the seller's offer.[14]

4.2. Optimal action for the seller when differentiation is strong: $t\in\left\{\frac{1}{5},\frac{1}{2}\right\}$

Because there are two regimes when the differentiation between firms is strong, I present the optimal selling strategy for both situations in Proposition 5. As noted earlier, ρ is assumed small enough such that a regime change does not occur.

Proposition 5.

1. *When* $\underline{\phi}<2t$, *the optimal strategy for the seller is to sell the services non-exclusively at a price of* $P_b=\frac{1}{2}\rho\underline{\phi}(1-t)(2-(1+\rho)\underline{\phi})$ *Profits are* $\rho\underline{\phi}(1-t)(2-(1+\rho)\underline{\phi})$.
2. *When* $\underline{\phi}>2t$, *the optimal strategy is to sell the services exclusively at a price of* $P_{x1}=\frac{2t\rho}{3(1+\rho)}(2+\rho-(1+\rho)\underline{\phi})$.

Proposition 5 underlines how the nature of downstream competition affects the optimal strategy for the seller. When $\underline{\phi}>2t$, the fraction of the market that is has been reached by the marketing of both firms is significantly higher. As a result, a greater fraction of potential consumers make direct comparisons of the surplus offered by the products in the market. When the marketing services are acquired by both firms, the fraction of consumers making comparisons increases further. The competition-increasing effect of these comparisons outweighs the increase in demand generated by the marketing services. Accordingly, the seller of services extracts higher rents from the downstream market when she offers a contract of exclusivity to one firm. In this situation, the seller maximizes the increase in demand generated by the marketing services (for one firm) but limits the degree to which the marketing services exacerbate the level of competition in the market. In contrast, when $\underline{\phi}<2t$, the fraction of potential consumers making direct comparisons of the surplus offered by the competing products is significantly smaller. In addition, there are more consumers in the market who have not been reached by the marketing of either firm. As a

[14] If $\pi_a<\pi_n$, Firm 1 can refuse the seller's offer and make itself better off (Firm 2 will also refuse the offer for any $P_{x2}>0$).

result, the optimal strategy for the marketing agency is to sell its services to both downstream firms. In this way, it is able to capitalize on the primary effect of the marketing services in this situation, that of enhancing primary demand for both firms.

4.3. Optimal action for the seller when differentiation is weak: $t<\frac{1}{5}$

When differentiation is weak, there are three regime changes as $\underline{\phi}$ increases from zero to one. The optimal strategy for each regime is described in Proposition 6.

Proposition 6.

1. *When $\underline{\phi}<2t$, he optimal strategy for the seller is to sell the services non-exclusively at a price of $P_b=\frac{1}{2}\rho\underline{\phi}(1-t)(2-(1+\rho)\underline{\phi})$.*
2. *When $\underline{\phi}\in\left(2t,\frac{1+t-\sqrt{-6t+5t^2+1}}{2-t}\right)$, the optimal strategy is to sell the services exclusively at a price of $P_{x1}=\frac{2t\rho(2+\rho-(1+\rho)\underline{\phi})}{3(1+\rho)}$.*
3. *When $\underline{\phi}\in\left(\frac{1+t-\sqrt{-6t+5t^2+1}}{2-t},\frac{1+t+\sqrt{-6t+5t^2+1}}{2-t}\right)$ and*
   (a) *$\rho>\frac{2t+6\underline{\phi}-9t\underline{\phi}-2}{6t\underline{\phi}-4\underline{\phi}}$, the optimal strategy is to sell exclusively at a price of $\rho\underline{\phi}(1-t)(1-\underline{\phi})+\frac{1}{2}t\underline{\phi}^2\rho$.*
   (b) *$\rho<\frac{2t+6\underline{\phi}-9t\underline{\phi}-2}{6t\underline{\phi}-4\underline{\phi}}$, the optimal strategy is to sell non-exclusively at a price of $\frac{\rho\underline{\phi}(6t\underline{\phi}-4\underline{\phi}-2t-2\underline{\phi}\rho+3t\underline{\phi}\rho+2)}{2}$. This is only possible if $t\lesssim 0.14777$.*
4. *When $\underline{\phi}\in\left(\frac{1+t+\sqrt{-6t+5t^2+1}}{2-t},1\right)$, the optimal strategy is to sell the services exclusively at a price of $P_{x1}=\frac{2t\rho(2+\rho-(1+\rho)\underline{\phi})}{3(1+\rho)}$.*

When differentiation is low, exclusive selling is more likely because the level of competition between firms is high. Nevertheless, Proposition 6 shows that the optimal strategy does not have a simple relationship to the level of marketing reach. When the level of marketing reach is low ($\underline{\phi}<2t$), it always better for the seller of marketing services to sell non-exclusively. Conversely, at high levels of marketing reach (i.e. $\underline{\phi}>\frac{2-2t}{6-9t}$), it is optimal for the seller to sell the services exclusively. However, when the levels of marketing reach are in an intermediate range, i.e. $\underline{\phi}\in\left(\frac{1+t-\sqrt{-6t+5t^2+1}}{2-t},\frac{1+t+\sqrt{-6t+5t^2+1}}{2-t}\right)$, a seller may earn higher profit by selling non-exclusively. In this region, when the impact of the services, ρ, is below the threshold of $\frac{2t+6\underline{\phi}-9t\underline{\phi}-2}{6t\underline{\phi}-4\underline{\phi}}$, non-exclusive selling is optimal. Above the threshold, exclusive selling yields higher profits.

The analysis shows that the optimal strategy depends on how fiercely the downstream firms compete with each other. When competition is fierce, it is better to sell exclusively. In contrast, when competition is muted, it is better to sell the services non-exclusively. Most importantly, the level of competition is not a simple function of how differentiated the downstream firms are. It also depends on the marketing reach of firms. The level of overlap in the consumers reached by the marketing of the two competitors is as important a determinant of competitive intensity as differentiation. When there is a high level of overlap in the consumer groups reached by each firm, a seller will find that exclusive selling yields higher profits.

A second point relates to the relationship between seller profits and the exogenous parameters (ρ, $\underline{\phi}$ and t). Invariably, the profits the seller earns are positively related to the efficiency (ρ) provided by the services. Whether the services are sold exclusively or to both downstream firms, the more impactful the services are, the more profit that is earned. The relationship between profits and the base level of marketing reach is more nuanced. When the base level of marketing reach is low, profits are positively related to the base level of marketing reach. However, when the regime involves a) pure pricing strategies that are a function of marketing reach or b) marketing reach levels that are high; the seller's profits are negatively related to the base level of marketing reach.

What is most interesting is the relationship between the seller's profit and differentiation. The seller makes more money when the level of differentiation between the firms is low for a significant fraction of the parameter space. This can be the case when the size of the segment reached by the marketing of both firms (relative to the fraction of the potential market reached by the marketing effort of at least one firm) is either low or high. When the size of the segment that has been reached by the marketing of both firms is low, the profits of the seller are negatively related to differentiation. Here, the equilibrium price is determined by the maximum price that can be charged to captive consumers: this price is negatively related to the level of differentiation. In contrast, when the size of the segment that has been reached by the marketing of both firms is high, the equilibrium is in mixed strategies. Here, the profits of downstream firms are affected by the maximum profit that can be earned from captive consumers. This too is inversely related to the level of differentiation in the market. In fact, there are only two situations in which the relationship between seller profits and the level of differentiation is positive. The first is when the equilibrium outcome involves pure price strategies. In this situation, equilibrium pricing is driven by the desire of the downstream firms to compete for customers who are reached by the marketing effort of both firms. This is the regime that is similar to the standard price equilibrium in a spatial model where consumers have full information. The second is when the equilibrium entails mixed pricing strategies and the levels of reach are high $\left(\underline{\phi}>\frac{2}{3}\right)$.

4.4. Optimal selling strategies when the services lead to a regime change

The results of Section 4.3 relate to services where the efficiency increase associated with the services *do not change* the equilibrium outcome. However, a seller's service will change the type of equilibrium observed in the market when ρ is high enough. To obtain, a qualitative idea of how regime-changing services would be sold, I summarize the strategy prescriptions for low levels of ρ in Fig. 4. Fig. 4 reveals information that is useful to infer the optimal strategy for selling marketing services that lead to a regime change. Independent of the level of differentiation in the market, as one moves from the left to the right (as is the case with increases in ρ), one moves from zones where non-exclusive selling is optimal to ones where exclusive selling is optimal. As argued earlier, high levels of marketing reach increase the level of competition between downstream firms. This favours exclusive selling. If the base level of marketing reach is such that the firms are in a region where non-exclusive selling is optimal (for sufficiently low levels of ρ), a higher level of ρ will ultimately push the equilibrium into a region where exclusive selling is optimal. In other words, while a regime change does not automatically imply a change in the optimal selling strategy (from non-exclusive to exclusive), once ρ is above a threshold, the optimal selling strategy is exclusive selling.

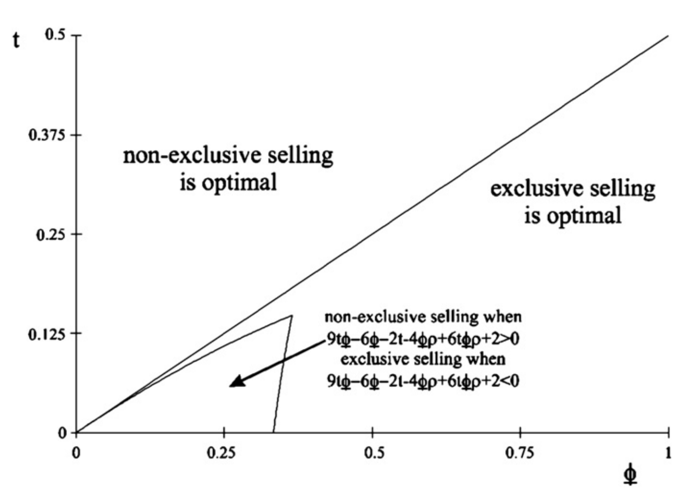

Fig. 4. Summary of the optimal strategies for the seller without regime changes.

As one moves from the left side of Fig. 4 to the right side, the main effect of higher symmetric levels of marketing reach changes from demand enhancement to exacerbating competition. When the efficiency gain is sufficient to move the symmetric equilibrium significantly to the right, the seller limits the exacerbation of competition by selling the services exclusively.

5. Conclusion

The major message of the analysis is that the optimal strategy for a seller of marketing services depends on how fiercely the downstream firms compete with each other. When competition is fierce, it is better to sell exclusively. This echoes the finding of Goldfarb and Yang (2007) who suggest that a B2B marketer can benefit by targeting just one firm when the firms are sophisticated and competition between them is intense.

In contrast, when competition is muted, a seller earns more by selling her services non-exclusively. It is important to note that the level of competition is not a simple function of the level of differentiation between the potential buyers of the services. It also depends on how extensive the marketing reach of firms before the seller's services are used. The analysis shows that the level of overlap in the consumers reached by the marketing of the two firms is a key determinant of competitive intensity. The model thus provides guidance about the type of contracts a seller of marketing services should use. For example, over a period of two years, Otis Sauter Partners Inc. (a marketing agency in Toronto, Canada) developed an electronic list of tea drinkers using information collected through several consumer contests. The marketing agency then sold the list *exclusively* to Red Rose (Unilever) as a basis to conduct direct marketing with high potential tea drinkers (the main competitors in the Canadian tea market are Unilever and Tetley). The list is valuable to a tea marketer like Red Rose because it allows Red Rose to increase the impact of its marketing by reaching tea drinkers for whom Red Rose is not top of mind. Note that the mainstream tea market in Canada is characterized by low differentiation and frequent price promotions. These observations explain the exclusive arrangement that was negotiated between Otis Sauter Partners and Red Rose.

Another important message of the analysis is to clarify the relationship between a) the profits of the seller of marketing services and b) the level of differentiation in the downstream market. Surprisingly, for a significant fraction of the parameter space, the model shows that differentiation is negatively related to seller profits. This has an important implication for marketing agencies, media experts and sales agents. A key decision for these organizations is to choose where to focus their own marketing efforts. Given the well-accepted belief that differentiation is a strong indicator of profits, these organizations might gravitate towards categories where products are well differentiated. The model shows that markets where products are not well differentiated can be just as attractive (if not more profitable) for sellers of marketing services. In other words, it may be a mistake to only look at "the degree to which firms are differentiated from each other" as a basis for deducing the value of services that increase the impact of marketing effort.

Two limitations of the analysis need to be highlighted. A first limitation relates to the assumption of independence for the incremental reach added to the base level of marketing reach for each downstream firm when the marketing agency contracts with both firms. When additional reach provided by a seller is not independent (for example, the additional customers are disproportionately customers who are reached by neither firm's marketing activity), the findings would be different. A lack of independence in the incremental reach provided by the marketing agency would increase the competition-exacerbating effect of the marketing services. This would cause exclusive selling to be more attractive at higher levels of differentiation and lower base levels of marketing reach.

A second limitation of this study relates to the types of marketing services that are represented by the model. As noted earlier, this model applies to those services for which the effect is to increase in the number of customers reached "effectively" by a firm's marketing effort. Clearly, there are marketing services that do more than increase a firm's effective marketing reach. For example, internet services which allow baseball fans to purchase and print their tickets online do more than increase the reach of a baseball team's marketing effort. They also increase the value of attending the baseball game: baseball fans appreciate how the online printing of tickets eliminates the need to queue at the ticket counter to obtain reserved tickets. Similarly, a recent promotion developed in Toronto involved making a beer brand the title sponsor for *Hip-Hop/Rap* bars in the trendy Queen Street West corridor. The promotion appealed to breweries because it was a vehicle to increase distribution and presence for a beer brand in Toronto. But the promotion also allowed a beer brand with tired positioning to refresh its image and improve "its fit" with young white collar beer drinkers. Some agencies also sell information that allows a firm to implement "consumer addressability" such that individual consumers can be offered customized products and pricing (Blattberg & Deighton, 1991; Chen & Iyer, 2002). The model I present does not address situations where a marketing service repositions a product, adds value to the consumption experience for the customer or allows a firm to implement consumer addressability. Services that have these effects are important and need to be explored in future research.

Appendix A

Proof of Proposition 1. When $t\in\left(\frac{1}{5},\frac{1}{2}\right)$, the objective functions when prices are marginally less than $1-t$ are:

$$\pi_1 = p_1\left(\phi_1(1-\phi_2)+\phi_1\phi_2\frac{p_2-p_1+t}{2t}\right) \tag{i}$$

$$\pi_2 = p_2\left(\phi_2(1-\phi_1)+\phi_1\phi_2\frac{p_1-p_2+t}{2t}\right) \tag{ii}$$

The first order conditions for prices are:

$$\frac{\partial\pi_1}{\partial p_1} = -\frac{1}{2}\phi_1\frac{-2t+\phi_2 t-\phi_2 p_2+2p_1\phi_2}{t} = 0 \tag{iii}$$

when $p_1 = p_2 = 1-t \Rightarrow \frac{\partial\pi_1}{\partial p_1} = -\frac{1}{2}\phi_1\frac{-2t+\phi_2}{t}$

$$\frac{\partial\pi_2}{\partial p_2} = \frac{1}{2}\phi_2\frac{2t-\phi_1 t+\phi_1 p_1-2p_2\phi_1}{t} = 0 \tag{iv}$$

when $p_1 = p_2 = 1-t \Rightarrow \frac{\partial\pi_2}{\partial p_2} = \frac{1}{2}\phi_2\frac{2t-\phi_1}{t}$. Therefore $\frac{\partial\pi_1}{\partial p_1}, \frac{\partial\pi_2}{\partial p_2} > 0$ for ϕ sufficiently low. The objective functions when prices are marginally greater than $1-t$ are:

$$\pi_1 = p_1\left(\phi_1(1-\phi_2)\frac{1-p_1}{t}+\phi_1\phi_2\frac{p_2-p_1+t}{2t}\right)-\alpha_1\phi_1^2 \tag{v}$$

$$\pi_2 = p_2\left(\phi_2(1-\phi_1)\frac{1-p_2}{t}+\phi_1\phi_2\frac{p_1-p_2+t}{2t}\right)-\alpha_2\phi_2^2 \tag{vi}$$

The first order conditions for prices are:

$$\frac{\partial\pi_1}{\partial p_1} = \frac{1}{2}\phi_1\frac{2-4p_1-2\phi_2+2p_1\phi_2+\phi_2 p_2+\phi_2 t}{t} = 0 \tag{vii}$$

$$\frac{\partial\pi_2}{\partial p_2} = \frac{1}{2}\phi_2\frac{2-4p_2-2\phi_1+2p_2\phi_1+\phi_1 p_1+\phi_1 t}{t} = 0 \tag{viii}$$

when $p_1 = p_2 = 1-t \Rightarrow \frac{\partial\pi_1}{\partial p_1} = -\frac{1}{2}\phi_1\frac{2-4t-\phi_2+2\phi_2 t}{t}$. It is straightforward to show that $-\frac{1}{2}\phi_1\frac{2-4t-\phi_2+2\phi_2 t}{t} < 0$ for all $t < \frac{1}{2}$. As long as ϕ is low enough and $p_1 = p_2 = 1-t$, $\frac{\partial\pi}{\partial p}^{+} < 0$ and $\frac{\partial\pi}{\partial p}^{-} > 0$.

For a corner solution, $\frac{\partial \pi_1}{\partial p_1} = -\frac{1}{2}\phi_1 \frac{2-4t-\phi_2+2\phi_2 t}{t} > 0$. For $\frac{\partial \pi_1}{\partial p_1} > 0$ when $\phi_1 = \phi_2$, I need $\phi(2t-\phi) > 0$ which implies that $\phi < 2t$. Therefore the equilibrium when $\phi < 2t$ is $p_1 = p_2 = 1-t$ and firms earn profits of $\frac{1}{2}\phi(1-t)(2-\phi)$.

However, when $\phi < 2t$, the equilibrium price is less than $1-t$ and using Eqs. (iii) and (iv), I have two equations and two unknowns (p_1 and p_2). Solving the equations yields the symmetric solution of $p_1 = \frac{1}{\phi}(2t-t\phi)$ and $p_2 = \frac{1}{\phi}(2t-t\phi)$. At these prices, firms earn profits of $\frac{1}{2}t(\phi-2)^2$. □

Proof of Proposition 2. When only Firm 1 engages the seller, the derivatives with respect to price for the two firms (at prices slightly less than $1-t$) must be positive for the corner solution of $p_1 = p_2 = 1-t$ to be an equilibrium. The derivatives when $\phi_1 = (1+\rho)\underline{\phi}$, $\phi_2 = \underline{\phi}$ and $p_1 = p_2 = 1-t-\epsilon$ are (ϵ is an arbitrarily small number):

$$\frac{\partial \pi_1}{\partial p_1} = \frac{1}{2t}\left(2t\underline{\phi} + 2t\rho\underline{\phi} - \underline{\phi}^2 - \rho\underline{\phi}^2\right) \tag{ix}$$

$$\frac{\partial \pi_2}{\partial p_2} = \frac{1}{2t}\left(2t\underline{\phi} - \underline{\phi}^2 - \rho\underline{\phi}^2\right) \tag{x}$$

Therefore $\frac{\partial \pi_1}{\partial p_1} - \frac{\partial \pi_2}{\partial p_2} = 2t\rho\phi > 0$. This implies that $\frac{\partial \pi_1}{\partial p_1} > \frac{\partial \pi_2}{\partial p_2}$ and the first condition to be violated will be $\frac{\partial \pi_2}{\partial p_2} > 0$. This occurs when $2t-(1+\rho)\underline{\phi} < 0 \Rightarrow \underline{\phi} < \frac{2t}{1+\rho}$. When this condition is satisfied, the profits obtain easily by substituting, the values for ϕ_i and p into the firm objective functions. When $\underline{\phi} > \frac{2t}{1+\rho}$, both firms set prices at the maximum which occurs when $\frac{\partial \pi_1}{\partial p_1} = \frac{\partial \pi_2}{\partial p_2} = 0$. Substituting into Eqs. (ix) and (x), two equations in two unknowns are obtained.

$$\frac{1}{2t}\left(2t - t\underline{\phi} - 2\underline{\phi}p_1 + \underline{\phi}p_2\right)(\rho+1)\underline{\phi} = 0 \tag{xi}$$

$$\left(-\frac{1}{2t}\right)\left(t\underline{\phi} - 2t + t\underline{\phi}\rho - \underline{\phi}p_1 + 2\underline{\phi}p_2 - \underline{\phi}\rho p_1 + 2\underline{\phi}\rho p_2\right)\underline{\phi} = 0 \tag{xii}$$

Solving the equations yield the equilibrium prices of

$$p_1 = \frac{6t - 3t\underline{\phi} + 4t\rho - 3t\underline{\phi}\rho}{3\underline{\phi}(1+\rho)} \text{ and } p_2 = \frac{6t - 3t\underline{\phi} + 2t\rho - 3t\underline{\phi}\rho}{3\underline{\phi}(1+\rho)} \tag{xiii}$$

The profits of Firms 1 and 2 obtain easily by substituting the equilibrium prices into the objective functions for Firms 1 and 2. □

Proof of Lemma 1. In order for the pure strategies to constitute an equilibrium, the profits earned by choosing the pure price strategy must exceed the profits that can be earned by defecting to $1-t$. At a price of $1-t$, the firms would sell to consumers who were reached by its marketing and not by that of the competitor. When the marketing reach of both firms is ϕ, profits are $\frac{1}{2}t(2-\phi)^2$ as per Proposition 1. The profits for a firms that defect are $\phi(1-\phi)(1-t)$. These profits are equal when $\phi = \frac{1}{t-2}\left(-t \pm \sqrt{-6t+5t^2+1}-1\right)$. This implies that the defect profits are greater than pure strategy profits when $\phi \in \left(\frac{1+t-\sqrt{-6t+5t^2+1}}{2-t}, \frac{1+t+\sqrt{-6t+5t^2+1}}{2-t}\right)$. This interval exists if an only if $t < \frac{1}{5}$ because $\sqrt{-6t+5t^2+1}$ has real roots if and only if $(5t-1)(t-1) > 0$. When the "defect profits" are greater than the profits from choosing a price of $\frac{1}{\phi}(2t-t\phi)$, suppose Firm 1 defects to $1-t$. Then Firm 2 will have an incentive to increase its price to $1-2t$ and earn higher profit on *all* consumers who are reached by Firm 2's marketing. But when Firm 2 chooses a price of $1-2t$, Firm 1 has an incentive to undercut Firm 2 and choose a price of $1-3t$ (each firm effectively faces demand from two discrete groups of consumers with different reservation prices). In this game, the existence of a mixed strategy equilibrium depends on two existence theorems from Dasgupta and Maskin (1986). First, the sum of the payoff functions needs to be upper hemi-continuous. This implies that the sum of the individual payoffs not jump down in the limit of the equilibrium strategies.[15] Second, the individual payoff functions need to be weakly semi-continuous. Both of these properties are satisfied. Thus, a mixed strategy equilibrium exists where the two firms undercut each other to capture demand from the fully informed segment.

However, undercutting for the fully informed segment does not reduce profits earned from the fully informed segment to zero (in the symmetric model of Narasimhan, 1988, the profit earned from the switching segment in a mixed pricing equilibrium is zero). A firm will not reduce its price such that it earns less than it would earn were the competitive price of t charged to the fully informed segment (when both firms charge the competitive price, the profits earned from the fully informed segment are $\frac{\phi_1\phi_2 t}{2}$). This implies that the guaranteed profit that either firm can earn by selling to the fully informed segment is $\frac{\phi_1\phi_2 t}{2}$. Therefore, the guaranteed profits for firms in this region are $(1-t)\phi(1-\phi) + \frac{\phi^2 t}{2}$.

The equilibrium pricing strategy entails the firms randomizing over an interval between $(p, 1-t)$: p is the minimum price and $1-t$ is the maximum price in the support for the mixed strategy.

Note that $F(p)$ and $f(p)$ be the c.d.f. and p.d.f. respectively of the symmetric mixed pricing strategy. The objective functions for each firm are:

$$\pi_1 = p_1\left(\phi_1(1-\phi_2) + \phi_1\phi_2 \int_{p_1-t}^{p_1+t}\left(\frac{p_2-p_1+t}{2t} f(p_2)\right)dp_2 + \phi_1\phi_2(1-F(p_1+t))\right) \tag{xiv}$$

$$\pi_2 = p_2\left(\phi_2(1-\phi_1) + \phi_1\phi_2 \int_{p_2-t}^{p_2+t}\left(\frac{p_1-p_2+t}{2t} f(p_1)\right)dp_1 + \phi_1\phi_2(1-F(p_2+t))\right) \tag{xv}$$

The mixed strategy which cannot be described analytically satisfies Eqs. (xiv) and (xv). □

Proof of Proposition 3. Part 1 of the proposition follows the reasoning of Proposition 1. Parts 2, 3 and 4 follow the reasoning of Lemma 1. When $\phi > 2t$ and the mixed price strategy is not an equilibrium i.e. $\phi \notin \left(\frac{1+t-\sqrt{-6t+5t^2+1}}{2-t}, \frac{1+t+\sqrt{-6t+5t^2+1}}{2-t}\right)$, the equilibrium is that both firms choose a price of $\frac{1}{\phi}(2t-t\phi)$. □

Proof of Lemma 2. When $\underline{\phi} \in \left(\frac{1}{t-2}\left(-t+\sqrt{-6t+5t^2+1}-1\right), \frac{1}{t-2}\left(-t-\sqrt{-6t+5t^2+1}-1\right)\right)$ and Firm 1 accepts the seller's offer (but not Firm 2) such that $\underline{\phi}(1+\rho) < \frac{1}{t-2}\left(-t-\sqrt{-6t+5t^2+1}-1\right)$, then the equilibrium will be in mixed strategies.[16]

In this situation, the group of consumers reached by Firm 1's marketing effort, $\phi_1(1-\phi_2)$, is larger than the group of consumers reached by Firm 2, $\phi_2(1-\phi_1)$. In fact, the group of customers reached by Firm 1 is $\underline{\phi}(1-\underline{\phi})(\rho+1)$ whereas for Firm 2, it is $\underline{\phi}(1-\underline{\phi}(1+\rho))$. Following the reasoning of Narasimhan (1988), Firm 1 has less incentive to reduce price from $1-t$ than Firm 2 because it loses more *guaranteed* profit than Firm 2. Firm 1's guaranteed profit is $(1-t)\underline{\phi}(1+\rho)(1-\underline{\phi}) + \frac{\underline{\phi}^2(1+\rho)t}{2}$. As a result, Firm 1's pricing strategy includes a mass point at $1-t$ and Firm 2's equilibrium profit exceeds the profit it earns by serving its "captive" segment at a price of $1-t$ and capturing half of the segment that has been reached by the marketing

[15] In this game, where the payoffs are the sum of profits from a Hotelling game and two discrete segments (reserved for each of the two players respectively), this property is satisfied.

[16] I present sufficient conditions for the equilibrium to be in mixed strategies when the firms are asymmetric. Because I focus on levels of ρ that do not lead to a change in regime, this is sufficient to characterize the equilibrium zones. It is straightforward to calculate necessary conditions for the equilibrium to be in mixed strategies by determining the point at which Firm 1 (the firm with an advantage) has an incentive to defect to a price of $1-t$. I do not present this condition because it is long and does not yield additional insight.

of both firms. Let w be the probability that Firm 1 sets price at $1-t$ and let $F(p)$ and $f(p)$ be the c.d.f. and p.d.f. of Firm 1's mixed pricing strategy. Conversely let $G(p)$ and $g(p)$ be the c.d.f. and p.d.f. of Firm 2's mixed pricing strategy. It proves useful to define $\hat{F}(p)$ and $\hat{f}(p)$ as the c.d.f. and p.d.f. of Firm 1's mixed pricing strategy *conditional* on not choosing a price of $1-t$.

The objective functions for the firms are:

$$\pi_1 = p_1\left(\phi_1(1-\phi_2)+\phi_1\phi_2\int_{p_1-t}^{p_1+t}\left(\frac{p_2-p_1+t}{2t}g(p_2)\right)dp_2+\phi_1\phi_2(1-G(p_1+t))\right. \tag{xvi}$$

Firm 2's objective function if $p_2\in(1-2t, 1-t)$ is:

$$\pi_2 = wp_2\left(\phi_2(1-\phi_1)+\phi_1\phi_2\frac{1-p_2}{2t}\right)+p_2(1-w)$$
$$\left(\phi_2(1-\phi_1)+\phi_1\phi_2\int_{p_2-t}^{p_2+t}\left(\frac{p_1-p_2+t}{2t}\hat{f}(p_1)\right)dp_1+\phi_1\phi_2\left(1-\hat{F}(p_2+t)\right)\right. \tag{xvii}$$

When $p_2<1-2t$, Firm 2's objective function is:

$$\pi_2 = wp_2\phi_2+p_2(1-w)\left(\phi_2(1-\phi_1)+\phi_1\phi_2\int_{p_2-t}^{p_2+t}\left(\frac{p_1-p_2+t}{2t}\hat{f}(p_1)\right)dp_1\right.$$
$$\left.+\phi_1\phi_2\left(1-\hat{F}(p_2+t)\right)\right) \tag{xviii}$$

Let $W(p)=\phi_1\phi_2\int_{p1-t}^{p1+t}\left(\frac{p_2-p_1+t}{2t}g(p_2)\right)dp_2+\phi_1\phi_2(1-G(p_1+t)$. Using Eq. (xvi) and the guaranteed profit of the firm, it is straightforward to show that:

$$W(p)=\frac{\underline{\phi}(\rho+1)(3t\underline{\phi}-2\underline{\phi}-2t-2p+2\underline{\phi}p+2)}{2p} \tag{xix}$$

Following Narasimhan (1988), the conditional distributions of the two firms will be identical because the only difference between the two firms is the size of the captive segment. I then rewrite Eq. (xviii) replacing for $W(p)$.

$$\pi_2 = wp_2\underline{\phi}+p_2(1-w)\left(\underline{\phi}(1-(1+\rho)\underline{\phi})+\frac{\underline{\phi}(\rho+1)(3t\underline{\phi}-2\underline{\phi}-2t-2p_2+2\underline{\phi}p_2+2)}{2p_2}\right) \tag{xx}$$

Because the support for p_2 is continuous, π is constant so $\frac{\partial\pi_2}{\partial p_2}=0$.[17]

$$\frac{\partial\pi_2}{\partial p_2}=\underline{\phi}(w-\rho+w\rho)=0\Rightarrow w=\frac{\rho}{1+\rho}$$

I now substitute into Eq. (xx) and evaluate at a price in the support ($p_2=1-2t$) to identify Firm 2's equilibrium profits: $\pi_2=(1-t)\underline{\phi}(1-\underline{\phi})+\frac{1}{2}t\underline{\phi}^2$.[18] I cannot explicitly identify the mixed pricing strategy; however, the profits earned by Firms 1 and 2 respectively are $\pi_1=(1+\rho)\underline{\phi}(1-\underline{\phi})(1-t)+\frac{\underline{\phi}^2(1+\rho)t}{2}$ and $\pi_2=\underline{\phi}(1-\underline{\phi})(1-t)+\frac{\underline{\phi}^2t}{2}$. □

Proof of Proposition 4. Part 1 of the proposition follows the reasoning of Proposition 1. Parts 2, 3 and 4 follow the reasoning of Lemma 2. When $\underline{\phi}>\frac{2t}{1+\rho}$ and the mixed price strategy is not an equilibrium i.e. $\underline{\phi}\notin\left(\frac{1+t-\sqrt{-6t+5t^2+1}}{2-t},\frac{1+t+\sqrt{-6t+5t^2+1}}{2-t}\right)$, the equilibrium is that the firms choose prices of $p_1=\frac{6t-3t\underline{\phi}+4t\rho-3t\underline{\phi}\rho}{3\underline{\phi}(1+\rho)}$ and $p_2=\frac{6t-3t\underline{\phi}+2t\rho-3t\underline{\phi}\rho}{3\underline{\phi}(1+\rho)}$. □

A.1. Comparative statics for the asymmetric case when pricing strategies are mixed

$$\frac{\partial\pi_1}{\partial t}=\frac{1}{2}\underline{\phi}(3\underline{\phi}-2)(\rho+1)<0 \text{ if } \underline{\phi}<\frac{2}{3} \tag{xxi}$$

$$\frac{\partial\pi_2}{\partial t}=\frac{1}{2}\underline{\phi}(3\underline{\phi}-2)<0 \text{ if } \underline{\phi}<\frac{2}{3} \tag{xxii}$$

Eqs. (xxi) and (xxii) show that the relationship between profits and differentiation depends on the initial level of marketing reach.

$$\frac{\partial\pi_1}{\partial\underline{\phi}}=(3t\underline{\phi}-2\underline{\phi}-t+1)(\rho+1) \tag{xxiii}$$

$$\frac{\partial\pi_2}{\partial\underline{\phi}}=3t\underline{\phi}-2\underline{\phi}-t+1 \tag{xxiv}$$

Eqs. (xxiii) and (xxiv) show that the relationship between profits and the base level of marketing reach is positive for $\underline{\phi}<\frac{1-t}{2-3t}$ and negative when $\underline{\phi}>\frac{1-t}{2-3t}$.

Proof of Proposition 5. When $t\in\left\{\frac{1}{5},\frac{1}{2}\right\}$ and $\phi<2t$, Propositions 1 and 2 imply that $\Pi_n=\frac{1}{2}\underline{\phi}(1-t)(2-\underline{\phi})$, $\Pi_b=\frac{1}{2}\underline{\phi}(t-1)(\rho+1)(\underline{\phi}+\underline{\phi}\rho-2)$, $\Pi_a=\frac{1}{2}\underline{\phi}(t-1)(\underline{\phi}-2)(\rho+1)$ and $\Pi_d=\frac{1}{2}\underline{\phi}(t-1)(\underline{\phi}+\underline{\phi}\rho-2)$. As discussed in the paper, the exclusive payoff is equal to $P_{\text{exc}}=\Pi_a-\Pi_d=\rho\underline{\phi}\,(1-t)$.

The non-exclusive price is the minimum of $(\Pi_b-\Pi_d, \Pi_a-\Pi_n)$. $\Pi_b-\Pi_d=\frac{1}{2}\rho\underline{\phi}(t-1)(\underline{\phi}+\underline{\phi}\rho-2)$ and $\Pi_a-\Pi_n=\frac{1}{2}\rho\underline{\phi}(t-1)(\underline{\phi}-2)$. $\Pi_b-\Pi_d<\Pi_a-\Pi_n$ because $(\Pi_b-\Pi_d)-(\Pi_a-\Pi_n)=\frac{1}{2}\rho^2\underline{\phi}^2(t-1)<0$. Therefore $\Pi_b-\Pi_d$ is the maximum price allowed for firms to sell non-exclusively. This implies that the seller's profits are $2(\Pi_b-\Pi_d)$ if she sells non-exclusively. As a result, $\Pi_{\text{non exc}}=\rho\underline{\phi}\,(t-1)(\underline{\phi}+\underline{\phi}\rho-2)$ and $\Pi_{\text{exc}}=\rho\underline{\phi}(1-t)$.

$\Pi_{\text{non exc}}-\Pi_{\text{exc}}=\rho\underline{\phi}(t-1)(\underline{\phi}+\underline{\phi}\rho-1)>0$ because a) $t-1<0$ and b) $\underline{\phi}+\underline{\phi}\rho-1<0$ since $\underline{\phi}(1+\rho)<1$ (the seller cannot offer an efficiency gain that leads to reach level that exceeds 1). Therefore, when marketing reach levels are low and the efficiency gain is relatively small i.e. $\rho<2\frac{t}{\underline{\phi}}-1$, the seller earns more by selling non-exclusively. The profit of the seller is $\rho\underline{\phi}(t-1)(\underline{\phi}+\underline{\phi}\rho-2)$.

When $t\in\left\{\frac{1}{5},\frac{1}{2}\right\}$ and 03D5>2t, Propositions 1 and 2 imply that $\Pi_n=\frac{1}{2}t(\underline{\phi}-2)^2$, $\Pi_b=\frac{1}{2}t(\underline{\phi}+\underline{\phi}\rho-2)^2$, $\Pi_a=\frac{(3\underline{\phi}-4\rho+3\underline{\phi}\rho-6)^2t}{18(\rho+1)}$ and $\Pi_d=\frac{(3\underline{\phi}-2\rho+3\underline{\phi}\rho-6)^2t}{18(\rho+1)}$. The profit earned from exclusive selling in this situation is $\Pi_a-\Pi_d=\frac{2t\rho}{3(1+\rho)}(2+\rho-(1+\rho)\underline{\phi})$.

As discussed in the text, the non-exclusive price is the smaller of two differences:

$$\Pi_b-\Pi_d=\frac{1}{18}(\rho+1)^{-1}(9\underline{\phi}^2-4\rho-24\underline{\phi}\rho-24\underline{\phi}+18\underline{\phi}^2\rho+9\underline{\phi}^2\rho^2+12)t\rho$$

$$\Pi_a-\Pi_n=\frac{1}{18}(\rho+1)^{-1}(16\rho-24\underline{\phi}-24\underline{\phi}\rho+9\underline{\phi}^2+9\underline{\phi}^2\rho+12)t\rho$$

$(\Pi_b-\Pi_d)-(\Pi_a-\Pi_n)=\frac{t\rho^2}{18(1+\rho)}(9\underline{\phi}^2+9\underline{\phi}^2\rho-20)<0$ because a) the first term is positive and b) the second term is negative (even at the maximum $\phi(=1)$ and maximum $\rho(=1)$, the second term equals $-2<0$). Thus, the maximum price that can be charged for non-exclusive selling is Πb − Πd. Summarizing, $\Pi_{\text{exc}}=\frac{2t\rho}{3(1+\rho)}(2+\rho-(1+\rho)\underline{\phi})$ and $\Pi_{\text{non exc}}=\frac{1}{9\rho+9}(12t\rho-24t\underline{\phi}\rho-4t\rho^2-24t\underline{\phi}\rho^2+9t\underline{\phi}^2\rho+18t\underline{\phi}^2\rho^2+9t\underline{\phi}^2\rho^3)$. This implies that $\Pi_{\text{exc}}-\Pi_{\text{non exc}}=\left(-\frac{1}{9}\right)(\rho+1)^{-1}(9\underline{\phi}^2-10\rho-18\underline{\phi}\rho-18\underline{\phi}+18\underline{\phi}^2\rho+9\underline{\phi}^2\rho^2)t\rho$. The first term $\left(-\frac{1}{9}\right)(\rho+1)^{-1}t\rho$ is negative. The first derivative of the second term is:

$$\frac{\partial}{\partial\rho}(9\underline{\phi}^2-10\rho-18\underline{\phi}\rho-18\underline{\phi}+18\underline{\phi}^2\rho+9\underline{\phi}^2\rho^2)=18\underline{\phi}^2-18\underline{\phi}+18\underline{\phi}^2\rho-10 \tag{xxv}$$

The second derivative of the second term is:

$$\frac{\partial}{\partial\rho}(18\underline{\phi}^2-18\underline{\phi}+18\underline{\phi}^2\rho-10)=18\underline{\phi}^2>0 \tag{xxvi}$$

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17 The support for Firm 2's pricing strategy does not include the segment $(1-2t, 1-t)$ when Firm 1 chooses $1-t$ with positive probability. Thus the condition $\frac{\partial\pi_2}{\partial p_2}=0$ holds for $p_2<1-2t$.

18 As noted earlier, Firm 2 (the weaker firm) benefits from Firm 1's reduced incentive to reduce price. Recall that Firm 2's captive segment is $\underline{\phi}(1-(1+\rho)\underline{\phi})$ and not $\underline{\phi}(1-\underline{\phi})$.

This implies that with regards to ρ, the second term is convex. As a result, the maximum value of the function is found at one endpoint of the allowable interval for ρ. The minimum endpoint is $\rho=0$. At the minimum endpoint, the second term equals $9\underline{\phi}(\underline{\phi}-2)<0$. The maximum endpoint occurs at the maximum value for ρ which is $\frac{1}{\underline{\phi}}-1$ since $\underline{\phi}(1+\rho)\leqq 1$. At $\rho=\frac{1}{\underline{\phi}}-1$, the second term equals $1-\frac{10}{\underline{\phi}}-<0$. Since the first term and the second term are negative, $\Pi_{\text{exc}}-\Pi_{\text{non exc}}>0$ and the optimal strategy in these conditions is exclusive selling. This implies that the seller's optimal profit is $\Pi_{\text{exc}}=\frac{2t\rho}{3(1+\rho)}\left(2+\rho-(1+\rho)\underline{\phi}\right)$. □

Proof of Proposition 6. The proof of Parts 1, 2 and 4 (the regions where the pricing strategies are pure) are identical to the proof provided for Proposition 5.

When $\underline{\phi}\in\left(\frac{1}{t-2}\left(-t+\sqrt{-6t+5t^2+1}-1\right),\frac{1}{t-2}\left(-t-\sqrt{-6t+5t^2+1}-1\right)\right)$, $\Pi_n=(1-t)\underline{\phi}(1-\underline{\phi})+\frac{\underline{\phi}^2 t}{2}$, $\Pi_b=\underline{\phi}(1-t)(\rho+1)(1-\underline{\phi}(1+\rho))+\frac{(1+\rho)^2\underline{\phi}_t^2}{2}$, $\Pi_a=(1+\rho)\underline{\phi}(1-\underline{\phi})(1-t)+\frac{\underline{\phi}^2(1+\rho)t}{2}$ and $\Pi_d=\underline{\phi}(1-\underline{\phi})(1-t)+\frac{\underline{\phi}^2 t}{2}$. This implies that $\Pi_{\text{exc}}=\Pi_a-\Pi_d=\rho\underline{\phi}(1-t)(1-\underline{\phi})+\frac{1}{2}t\underline{\phi}^2\rho$.

As discussed in the text, the non-exclusive price is the smaller of two differences:

1. $\Pi_b-\Pi_d=\underline{\phi}(1-t)(\rho+1)(1-\underline{\phi}(1+\rho))+\frac{(1+\rho)^2\underline{\phi}^2 t}{2}-\left(\underline{\phi}\left(1-\underline{\phi}\right)(1-t)+\frac{\underline{\phi}^2 t}{2}\right)=\frac{1}{2}\rho\underline{\phi}(6t\underline{\phi}-4\underline{\phi}-2t-2\underline{\phi}\rho+3t\underline{\phi}\rho+2)$
2. $\Pi_a-\Pi_n=(1+\rho)\underline{\phi}\left(1-\underline{\phi}\right)(1-t)+\frac{\underline{\phi}^2(1+\rho)t}{2}-\left((1-t)\underline{\phi}(1-\underline{\phi})+\frac{\underline{\phi}^2 t}{2}\right)=\frac{1}{2}\rho\underline{\phi}(3t\underline{\phi}-2\underline{\phi}-2t+2)$

$(\Pi_b-\Pi_d)-(\Pi_a-\Pi_n)=\frac{1}{2}\rho\underline{\phi}^2(3t-2)(\rho+1)<0$ because $t<\frac{1}{5}$. Thus, the maximum price that can be charged for non-exclusive selling is $\Pi_b-\Pi_d$. This implies that the profits under non-exclusive selling are $\Pi_{non\ exc}=\rho\underline{\phi}(6t\underline{\phi}-4\underline{\phi}-2t-2\underline{\phi}\rho+3t\underline{\phi}\rho+2)$. To determine the optimal strategy, I examine the difference between Π_{exc} and $\Pi_{\text{non exc}}$.

$$\Pi_{\text{exc}}-\Pi_{\text{non exc}}=\left(-\frac{1}{2}\right)\rho\underline{\phi}(9t\underline{\phi}-6\underline{\phi}-2t-4\underline{\phi}\rho+6t\underline{\phi}\rho+2)$$

Note that $(-\frac{1}{2})\rho\underline{\phi}<0$. When $\underline{\phi}<\frac{1-t}{(2-3t)(2\rho+3)}$, $9t\underline{\phi}-6\underline{\phi}-2t-4\underline{\phi}\rho+6t\underline{\phi}\rho+2>0$ and when $\underline{\phi}>\frac{1-t}{(2-3t)(2\rho+3)}$, $9t\underline{\phi}-6\underline{\phi}-2t-4\underline{\phi}\rho+6t\underline{\phi}\rho+2<0$. Thus for low levels of $\underline{\phi}$, non-exclusive selling is optimal and for high levels, the optimal strategy is to sell exclusively.

For non-exclusive selling to be optimal, $9t\underline{\phi}-6\underline{\phi}-2t-4\underline{\phi}\rho+6t\underline{\phi}\rho+2>0$. This implies that both ρ and $\underline{\phi}$ need to be sufficiently small; in fact, $\underline{\phi}<\frac{2-2t}{6-9t}$ is a necessary condition for the inequality to be satisfied since 0 is the minimum value for ρ. However, these conditions must occur when a) the corner solution of $1-t$ is not the equilibrium ($\underline{\phi}>2t$) and b) the pure strategy pricing is not an equilibrium. Since the maximum value of $\underline{\phi}$ for exclusive selling to be possible is $\frac{2-2t}{6-9t}$, $\underline{\phi}>2t=0.4$ implies that $t<\frac{1}{18}\sqrt{13}-\frac{7}{18}\approx 0.18858$ is a necessary condition for non-exclusive selling to be optimal. Second $\underline{\phi}>\frac{1}{t-2}\left(-t+\sqrt{-6t+5t^2+1}-1\right)$ and $\underline{\phi}<\frac{2-2t}{6-9t}$ must also be satisfied for non-exclusive selling to be possible:

$$\frac{1}{t-2}\left(-t+\sqrt{-6t+5t^2+1}-1\right)<\frac{2-2t}{6-9t}\Rightarrow 4(t-2)(40t-98t^2+71t^3-4)>0$$

Numerically, this condition is satisfied if and only if $t\lessapprox 0.14777$ (the polynomial has two real roots). Because $0.14777<0.18858$, the limiting condition for non-exclusive selling to be optimal is $t\lessapprox 0.14777$. To summarize, when $\underline{\phi}\in(\frac{1}{t-2}(-t+\sqrt{-6t+5t^2+1}-1),\frac{1}{t-2}\left(-t-\sqrt{-6t+5t^2+1}-1\right))$ and $9t\underline{\phi}-6\underline{\phi}-2t-4\underline{\phi}\rho+6t\underline{\phi}\rho+2<0$, exclusive selling is optimal and the seller earns $\Pi_{\text{exc}}=\rho\underline{\phi}(1-t)(1-\underline{\phi})+\frac{1}{2}t\underline{\phi}^2\rho$. If $\underline{\phi}$ falls in the allowable range and $9t\underline{\phi}-6\underline{\phi}-2t-4\underline{\phi}\rho+6t\underline{\phi}\rho+2>0$, non-exclusive selling is optimal and the seller earns $\Pi_{\text{non exc}}=\rho\underline{\phi}(6t\underline{\phi}-4\underline{\phi}-2t-2\underline{\phi}\rho+3t\underline{\phi}\rho+2)$. This is only possible if $t\lessapprox 0.14777$. □

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