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Hedging house price risk in the presence of lumpy transaction costs ☆

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Abstract

This paper presents a life-cycle model of housing demand with uncertain house prices and lumpy transaction costs. The paper extends the (S, s) methodology to a non-stationary discrete time framework with multivariate stochastic price processes. This allows the characterization of a self-hedging mechanism in an incomplete housing market: households use earlier accumulated housing wealth to hedge against future housing cost risk. As a result, the direction of the effect of price uncertainty on housing demand depends critically on households' future housing consumption plans. When price uncertainty increases, households consume (and thereby invest in) less housing if they plan to realize the housing wealth gain. However, they will instead take a larger housing position if they plan to move to a bigger home in a correlated housing market in the future.

JEL classification: C61; D81; R21

Keywords: Housing demand under price uncertainty; Hedge; Life cycle

1. Introduction

This paper examines the effects of house price uncertainty on housing demand in a life-cycle framework. The study is motivated by the following considerations. First, for most households in the United States, housing is not only an important consumption good, but also the dominant financial asset in their portfolio. Second, like other financial assets, housing has substantial price risk. For example, as shown by Glaeser and Gyourko (2006), the standard deviation of three-year real changes in the average American metropolitan area house prices is $26,354 (in 2000 dollars), about one fifth of the median price level. Yet, unlike the markets for other financial assets, the housing market is highly incomplete (Englund et al., 2002). In addition, housing transactions involve large lumpy transaction costs (Haurin and Gill, 2002), which make it costly for households to adjust their housing positions in response to price risk. This paper asks how households make home purchase decisions in the presence of lumpy transaction costs, and how house price uncertainty affects their home purchase decisions and welfare.

To answer these questions, the paper presents a life-cycle housing demand model with stochastic house prices and lumpy transaction costs. The model follows the traditional (S, s) framework in which at each point in time households choose whether to transact and how much to purchase if transacting. The traditional (S, s) rule, as applied to durable goods by Grossman and Laroque (1990), requires an assumption that the optimization problem can be reduced to a problem with a single state variable. Although this assumption is convenient and useful in many (S, s) applications, it rules out interesting cases like models with multiple stochastic price processes. For example, households could benefit by recognizing the positive correlation between sequential home purchases and increasing housing demand to self-hedge against house price risk. This paper extends the traditional (S, s) approach by considering a finite horizon discrete time framework and by modeling multivariate house price process. In particular, house prices are correlated both over time and across markets. This allows us to confirm the hedging intuition in an intertemporal setting and to

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model the important role of households' expected future housing consumption plans when explaining the effects of house price uncertainty on housing demand.

To illustrate the sort of hedging, suppose a young couple plans to purchase a condominium now and to move into a single family house later. In a volatile housing market, both the condominium price and the single family house price are uncertain. According to classical economic theory, uncertainty adds to the risk associated with future housing capital gains. It therefore discourages their investment in the condominium. Thus, the risk averse couple would either delay the condominium purchase or purchase a smaller condominium. This is referred to as the effect of housing wealth risk.[1] However, the empirical observation indicates that the average within-metropolitan-area correlation between condo condominium prices and single family house prices is 0.9195.[2] The positive correlation between the condominium price and the single family house price implies that the ability to hold a condominium now and sell it in the future to finance a single family house has positive economic value. This value is often referred to as a real hedge. Like a financial hedge that allows one to purchase a security whose return is positively correlated with the cost of other future desired assets, a real hedge provides high returns when the future price of the single family house is high, and vice versa. House price uncertainty increases the value of hedging. As a result, the young couple may find it optimal to make an earlier and bigger condominium purchase even when the condominium price is volatile.

To formalize this intuition, the model in this paper incorporates two features. First, housing is illiquid. When prices are volatile, the presence of lumpy transaction costs can lead to a higher housing risk premium than would be required otherwise. This risk premium may fall sharply, however, once the second feature of the model is introduced: the possibility of a positive correlation between the price process of an earlier home and the price process of a home that the household plans to purchase later. This allows households to use earlier accumulated housing to hedge against the risk associated with future housing costs. This self-hedging mechanism is particularly important in the housing markets, given that conventional financial instruments cannot help households to insure against future housing cost risk.[3]

The key implication of this model is that the net effect of house price uncertainty on housing demand depends on the strength of the hedging incentive. This, in turn, depends on households' future housing consumption plans. For households who plan to move up the housing ladder and move to a correlated housing market, the hedging effect dominates the housing wealth risk effect. As a result, price uncertainty increases their housing investment (and thereby consumption). For households who plan to move down the housing ladder or to move to an uncorrelated housing market, the incentive to hedge diminishes and hence price uncertainty suppresses housing demand. Thus, our model predicts that the direction and magnitude of the effects of house price uncertainty on housing demand change across households, depending on their inter-market mobility, and vary across the stages of the life cycle, depending on whether households plan to move up or down the housing ladder.

Turning to welfare implications, our numerical exercise shows that the magnitude of the welfare cost under house price uncertainty is reduced when households have stronger hedging incentives. Thus, while families in an incomplete housing market are not able to access formal insurance financial instruments to diversify or insure against the house price risk, they do rely on private informal coping mechanisms to smooth housing consumption over the life cycle. If this is the case, the social insurance instruments proposed by Case et al. (1993) may be less efficient than prior studies suggest, as such insurance would serve partly to crowd out the self-hedging mechanism taken by certain households.

In addition to these economic implications, the analysis in this paper carries a small methodological lesson. By extending the traditional (S, s) methodology into the discrete time framework with multiple state variables, the paper provides an explicit characterization of the hedging incentive. Such an approach may be valuable when modeling the financial decisions for households who face uncertainties on multiple economic conditions. Furthermore, the optimal home purchase decision rules derived in this paper have an additional advantage of enabling us to learn about complex home purchase dynamics and hence providing theoretically sound instruments for testing the model in a repeat home purchase market in the future work.

The remainder of the paper is structured as follows. Section 2 briefly reviews the literature. Section 3 describes a life-cycle model of housing demand and Section 4 drives the optimal home purchase decision rules. Section 5 carries out a number of comparative static exercises by simulating and solving the model numerically. Section 6 discusses the welfare costs imposed by house price uncertainty. Section 7 concludes.

2. Literature review

This paper relates to two strands of the literature on economic dynamics. First, the paper builds on recent literature showing that homeownership can provide a hedge against fluctuations in future rent payments. The notion that housing provides a hedge against

[1] Henderson and Ioannides (1983) discuss this type of risk by showing that house price uncertainty makes home ownership less attractive, since risk averse households may otherwise invest in a safe asset whose fixed return may offset the renter's negative externality. Davidoff (2006) shows that labor income uncertainty can amplify housing risk because of a positive covariance between labor income and house price.

[2] Author's calculation, based on data from the National Association of Realtors Existing-Home Sales Series (http://www.realtor.org/Research.nsf/Pages/EHSdata).

[3] Flavin and Yamashita (2002) find a low correlation between housing and other financial assets, which provides evidence that housing markets are highly incomplete.

future housing cost risk was first laid out in Berkovec and Fullerton (1992). Sinai and Souleles (2005) formalize this intuition in a multi-stage model of housing tenure choices. They show that owning is more desirable when rental price risk is greater. They go on to estimate an empirical model based on the MSA-level price data, and the findings are consistent with their hedging theory. Ortalo-Magne and Rady (2002) develop an extended model that looks at the effects of rent uncertainty and income risk on housing tenure choices. Using the household level data from the US and UK, Banks et al. (2004) provide further empirical evidence on hedging. The focus of these studies is how the incentive to hedge could affect households' choice between renting and owning, which is most relevant for first time home buyers. In contrast, this paper restricts its attention to repeat home buyers. In particular, it endogenizes the decisions of the timing and size of housing consumption. By incorporating both transaction cost frictions and life-cycle considerations into a housing demand model, this paper generates testable predictions about how the price uncertainty affects housing demand by existing home owners.

A second strand of the literature relating to this paper is the theory of lumpy and infrequent adjustment. Within these models, discrete investment choices are typically driven by the presence of nonconvexities, and an (S, s) policy characterizes optimal investment behavior. See, for example, Grossman and Laroque (1990). The literature has almost exclusively focused on only one direction of the relationship: the implications of individual behavior on the aggregate investment. Largely ignored has been the other direction: the influence of market variables on individual behavior.[4] Taking house prices as exogenously given, this paper presents a formal characterization of the effects of market price uncertainty on individual behavior and welfare.[5]

3. Life-cycle model

3.1. Assumptions and notation

In this model, time is discrete. Households live for $T + 1$ periods. In each period t, households consume two consumption goods, namely housing services (H_t) and a non-durable good (C_t).[6] To change their consumption of housing services, households must sell their current houses and buy new ones. Each transaction generates lump-sum transaction costs, which consist of a fixed component (F) and a part proportional to the value of the sold house (δ).

Housing is modeled as continuous amount of housing services obtained in a specific period at a specific location. Households take location decisions as given. Each transaction requires a move from one location to another location. Locations differ only in their house prices. That is, when transaction occurs, households sell the existing house at one price and purchase a new house at another price.

House prices are exogenous. Let $\mathbf{P} \equiv (\mathbf{p_0}, \ldots, \mathbf{p_T})$ be a first order Markov house price process with conditional density $\phi(\mathbf{p_t}|\mathbf{p_{t-1}})$. We denote $\mathbf{p_t}$ as the J-tuple $(p_t^1, p_t^2, \ldots, p_t^J)$, where p_t^j denotes the unit price corresponding to the house at location j in period t $(J \leqslant T)$.[7] House prices vary across locations and change over time, reflecting the time-varying market condition tied to local amenities. At the beginning of period t, households observe the existing housing stock inherited from period $t-1$ at location $j-1$ and decide whether to make a transaction in period t. If a transaction occurs in period t, households sell the old house at price p_t^{j-1} and buy a new house at price p_t^j, anticipating that the next house will be bought at $p_{t'}^{j+1}$ at some point $t' > t$. Conditional on transacting in period t, households' decisions on the home purchase size depend on a set of conditional expectation terms: $E_t(p_{t+1}^j)$, $Var_t(p_{t+1}^j)$ and $Cov_t(p_{t+1}^j, p_{t+1}^{j+1})$.

With these assumptions about house price processes, we capture—in an admittedly stylized way—two realistic features: (i) time variation in house prices and (ii) heterogeneity in these prices at any point in time. It is the second feature that is new to the standard (S, s) type models. It will be apparent that this slight modification gives us an important degree of flexibility when characterizing the hedging incentive. In particular, the term $Cov_t(p_{t+1}^j, p_{t+1}^{j+1})$ measures the dependency between the return on the current house and the cost of future housing consumption. When p_{t+1}^j coincides with p_{t+1}^{j+1}, the model reduces to a standard housing demand model with a univariate house price process.

The markets for housing are incomplete in that there exists no financial asset whose return is correlated with the return on the housing asset. In each period $t \geqslant 1$, the household may borrow and lend between time $t-1$ and time t at interest rate r_t, which is assumed to be fixed and positive. In addition, we assume that households receive a deterministic income stream $(y_1, y_2, \ldots, y_T)$ and have a deterministic housing taste profile $(\theta_1, \theta_2, \ldots, \theta_T)$.

[4] As noted by Caplin and Leahy (1997), "One of the most limiting aspects of these models is that they focus exclusively on the impact that microeconomic inertial has on aggregate dynamics. They ignore the feedback from aggregate onto individual behavior."

[5] Using an intertemporal housing demand framework, Goodman (1990) and Haurin and Chung (1998) explicitly model lumpy transaction costs but do not consider house price risk.

[6] For ease of notation, we drop the household-specific subscript i.

[7] Here and in what follows, we will use the subscript to denote time and the superscript to denote location for housing stock H and house price p.

The period utility function consists of two components: a stochastic component $\tilde{u}(H_t, C_t; \theta_t)$, where θ_t is the housing taste in period t, and a stochastic utility shifter $\mathbf{s_t}$.

$$u(H_t, C_t; \theta_t, \mathbf{s_t}) = \begin{cases} \tilde{u}(H_t, C_t; \theta_t) + s_t^T, & \text{where} \quad H_t = H_t^j \text{ if } d_t = 1 \\ \tilde{u}(H_t, C_t; \theta_t) + s_t^N, & \text{where} \quad H_t = H_{t-1}^{j-1} \text{ if } d_t = 0 \end{cases} \tag{1}$$

The deterministic component $\tilde{u}(H_t, C_t; \theta_t)$ gives the period utility derived from consuming (H_t, C_t) for given housing taste θ_t. The stochastic component $\mathbf{s_t} \equiv [s_t^T, S_t^N]$ is a utility shifter, which is unobserved prior to period t. The superscripts T and N indicate the choice the household makes: to transact $(d_t = 1)$ or not to transact $(d_t = 0)$. As it will become clear in the next section, the stochastic utility shifter is a key element that allows us to derive an analytical solution to this complex intertemporal housing demand model.

We impose the following restrictions on the utility function.

Assumption 1. The function $\tilde{u}: R_+ \times R_+ \to R$ is bounded, additively separable, strictly increasing and strictly concave, twice continuously differentiable. It satisfies the Inada condition at the origin. In addition, $\tilde{u}_{CCC} = 0$ and $\tilde{u}_{HHH} = 0$.[8]

Assumption 2. The vector $\mathbf{s_t}$ is independent across households and over time. In particular, s_t^T and s_t^N are identically and independently distributed as type-I extreme value distribution $g(s_t^T, s_t^N)$.[9]

Households have a finite horizon. In the last period, they sell the house and derive utility from terminal wealth W_{T+1}.[10] At the beginning of period t prior to the last period, households observe the existing housing stock at location $j-1$ inherited from the previous period H_{t-1}^{j-1}, the accumulated financial wealth W_{t-1}, house prices up to time t $(\mathbf{p_0}, \ldots, \mathbf{p_t})$, and the current utility shifters (s_t^T, s_t^N). They then choose non-durable consumption (C_t), whether to transact (d_t), and the optimal size of the current house if transacting (H_t^j) in order to maximize the expected discounted remaining life time utility, before observing the next period's house prices and utility shifters.

$$\underset{(C_t, d_t, H_t^j)}{\text{Max}} \; E_t \sum_{\tau=t}^{T} \beta^{\tau-t} u(H_\tau, C_\tau; \theta_\tau, \mathbf{s_\tau}) + \beta^{T+1} \frac{W_{T+1}^{1-\gamma}}{1-\gamma}$$

subject to:

$$W_t = y_t + (1+r_t) W_{t-1} + d_t\big((1-\delta) p_t^{j-1} H_{t-1}^{j-1} - p_t^j H_t^j - F\big) - C_t \quad \text{for } t < T+1 \tag{2}$$

$$W_{T+1} = (1-\delta) p_{T+1}^J H_t^J + (1 + r_{T+1}) W_T - F \geqslant 0 \tag{3}$$

Eq. (2) is the household's budget constraint in period t prior to the last period. Expression (3) imposes a constraint on terminal wealth, where J indicates the location of the last house bought in the life cycle. It specifies that the house must be sold in the last period. The inequality rules out the possibility that households can leave with debt. The expectation $E_t(\cdot)$ indicates the expectation over future house prices and utility shifters conditional on information available in period t.

3.2. The Bellman equation

To establish the results that follow, it is convenient to work with the value functions of future discounted utility. For a household starting in period t with accumulated wealth W_{t-1} and housing stock H_{t-1}^{j-1}, we first define:

$$V_t^N\big(W_{t-1}, H_{t-1}^{j-1}; s_t^N, \mathbf{p_t}\big) = \underset{(C_t)}{\text{Max}}\, u\big(H_{t-1}^{j-1}, C_t; \theta_t, s_t^N\big) + \beta E_t V_{t+1}\big(W_t^N, H_{t-1}^{j-1}; \mathbf{s_{t+1}}, \mathbf{p_{t+1}}\big) \tag{4}$$

subject to:

$$W_t^N = y_t + (1+r_t) W_{t-1} - C_t$$

[8] The strict inequalities on $\tilde{u}_{CC}$ and $\tilde{u}_{HH}$ reflect aversion to housing and non-housing consumption risks. The boundedness of $\tilde{u}(H, C)$, together with its separability, implies that an optimum exists. The Inada condition states that a household must consume housing and non-housing consumption each period. Finally, the restrictions on the third derivatives are not necessary but help to simplify the technical proofs below.

[9] By allowing a two-state utility shifter, the model captures the empirical feature that certain households may gain additional utility from moving up the housing ladder because it signals an improvement in social status or represents the added convenience of proximity to work. Like the vector of unobserved state variables in structural demand models (see Rust, 1987), the vector of the utility shifter in this model must have at least as many components as the number of alternative choices. The assumption on type-I extreme distribution ensures that an analytical solution exists.

[10] This can also be interpreted as the bequest motive.

to denote the expected utility of households that decide to stay with the previous house, choose optimal non-durable consumption C_t and save the rest for the next period. The superscript N indicates that households do not make a transaction in period t. Next, define:

$$V_t^T\big(W_{t-1}, H_{t-1}^{j-1}; s_t^T, \mathbf{p_t}\big) = \underset{(H_t^j, C_t)}{\text{Max}}\ u\big(H_t^j, C_t; \theta_t, s_t^T\big) + \beta E_t V_{t+1}\big(W_t^T, H_t^j; \mathbf{s_{t+1}}, \mathbf{p_{t+1}}\big) \tag{5}$$

subject to:

$$W_t^T = y_t + (1+r_t)W_{t-1} + \big[(1-\delta)p_t^{j-1}H_{t-1}^{j-1} - p_t^j H_t^j - F\big] - C_t$$

to denote the expected utility of households transacting in period t, i.e., those that collect revenue $p_t^{j-1}H_{t-1}^{j-1}(1-\delta)$ from selling their old house, acquire a new house H_t^j at price p_t^j, and pay for fixed costs F. The superscript T indicates that households make a transaction in period t.

The finite horizon makes the value function inherently non-stationary. I solve the model backwards. In period $T+1$, households sell the house they own and collect the capital gains. Prior to the last period, the expected value function is written as:

$$V_t\big(W_{t-1}, H_{t-1}^{j-1}; \mathbf{s_t}, \mathbf{p_t}\big) = \underset{(d_t, H_t^j, C_t)}{\text{Max}}\ \big\{V_t^T\big(W_{t-1}, H_{t-1}^{j-1}; s_t^T, \mathbf{p_t}\big), V_t^N\big(W_{t-1}, H_{t-1}^{j-1}; s_t^N, \mathbf{p_t}\big)\big\} \tag{6}$$

All the relevant information in period t is captured by a set of state variables $(W_{t-1}, H_{t-1}^{j-1}, \theta_t, y_t, r_t, \mathbf{p_t}, \mathbf{s_t})$. Given the distributions of future house prices and utility shifters, the expected future utility term is defined as:

$$E_t V_{t+1}(W_t, H_t; \mathbf{s_{t+1}}, \mathbf{p_{t+1}}) = \iint V_{t+1}(W_t, H_t) g(\mathbf{s_{t+1}})\, \mathrm{d}\mathbf{s_{t+1}} \phi(\mathbf{p_{t+1}}|\mathbf{p_t})\, \mathrm{d}(\mathbf{p_{t+1}}|\mathbf{p_t}) \tag{7}$$

where the expectation is taken over future utility shifters and house prices. Households choose to transact if the benefit from transacting exceeds the cost associated with transacting. The optimal decision about whether to transact in period t depends on the differences in the current and expected utility associated with the two choices. Specifically, it takes the following form:

$$d_t = \begin{cases} 1 & \text{if } (s_t^N - s_t^T) \leqslant \Gamma(W_{t-1}, H_{t-1}^{j-1}, \mathbf{p_t}) \\ 0 & \text{if } (s_t^N - s_t^T) > \Gamma(W_{t-1}, H_{t-1}^{j-1}, \mathbf{p_t}) \end{cases} \tag{8}$$

with

$$\Gamma\big(W_{t-1}, H_{t-1}^{j-1}, \mathbf{p_t}\big) \equiv \big[\tilde{u}\big(H_t^j, C_t\big) - \tilde{u}\big(H_{t-1}^{j-1}, C_t\big)\big] + \beta E_t\big[V_{t+1}\big(W_t^T, H_t^j; s_{t+1}^T, \mathbf{p_{t+1}}\big) - V_{t+1}\big(W_t^N, H_{t-1}^{j-1}; s_{t+1}^N, \mathbf{p_{t+1}}\big)\big].$$

4. Solving the model

4.1. Optimal purchase size

To date, most of research on the infrequent home purchase model has relied on numerical solutions. The biggest challenge in solving the model analytically is the presence of lumpy transaction costs, which result in a kinked Bellman equation. In this section, I show that introducing the utility shifter term allows one to smooth the conditional value function without altering the mixed nature of the decision process. This yields an interior solution for the optimal housing demand even when lumpy transaction costs are taken into account.

Consider the value function conditional on transacting, as specified in Eqs. (5) and (7). The Bellman equation adds the current utility to the expected future discounted utility conditional on the current choices, under the presumption that future decisions are made optimally. The term $u(H_t^j, C_t; \theta_t, s_t^T)$ captures households' current utility from choosing (H_t^j, C_t) for given housing taste θ_t and utility shifter s_t^T. The Max operator implicit in the second term represents the households' choice between transacting and not transacting in period $t+1$. It is worth noting that without the stochastic utility shifters, conditional on knowing all the variables that affect the next period's home purchase decisions, households could have deduced at each point whether they would make a transaction during the next period. The resulting Bellman equation would remain kinked and could not be solved directly. With stochastic utility shifters, in each period t, conditional on the decision to transact, we can reduce this complex intertemporal model to a standard optimization problem.[11]

To show this, I first define the period-t probability of waiting (i.e., not transacting) in period $t+1$, $Q_t(W_t, H_t^j; \mathbf{p_{t+1}})$. By definition, this is equivalent to $Pr_t(d_{t+1} = 0 \mid d_t = 1)$. Conditional on transacting in period t and on knowing $\mathbf{p_{t+1}}$, provided that

[11] The role of the stochastic utility shifter in this model is very similar to the role of an unobserved state variable in estimating dynamics optimization models (see, for example, Hotz and Miller, 1993). Note that in order to make the problem analytically solvable, one must assume that utility shifter enters the utility function additively. Thus, one cannot simply replace the stochastic utility shifter by imposing similar assumptions on the taste parameter or house price processes.

the distribution of utility shifters (s^T_{t+1}, s^N_{t+1}) has a continuous and positive density, $Q_t(W_t, H^j_t; \mathbf{p_{t+1}})$ can be written as a smooth hazard function:

$$Q_t\left(W_t, H^j_t; \mathbf{p_{t+1}}\right) = 1 - \int_{-\infty}^{\Gamma(W_t, H^j_t, \mathbf{p_{t+1}})} \left(s^T_{t+1} - s^N_{t+1}\right) \mathrm{d}g\left(s^T_{t+1}, s^N_{t+1}\right) \tag{9}$$

Following Hotz and Miller (1993), there exists a mapping $f : [0, 1] \to R$ that allows us to express the conditional choice probability as a function of the difference between the conditional value functions:

$$Q_t\left(W_t, H^j_t; \mathbf{p_{t+1}}\right) = 1 - f\left(V^N_{t+1} - V^T_{t+1}\right) \tag{10}$$

where $f(\cdot)$ is a real valued, invertible function such that $V^N_{t+1} - V^T_{t+1} = f^{-1}(1 - Q_t)$. Under Assumption 2, this leads to:

$$V^N_{t+1}\left(W_t, H^j_t; \mathbf{p_{t+1}}\right) - V^T_{t+1}\left(W_t, H^j_t; \mathbf{p_{t+1}}\right) = f^{-1}\left(1 - Q_t\left(W_t, H^j_t; \mathbf{p_{t+1}}\right)\right) = \xi q_t\left(W_t, H^j_t; \mathbf{p_{t+1}}\right) \tag{11}$$

where $q_t(W_t, H^j_t; \mathbf{p_{t+1}}) = \log[\frac{Q_t(W_t, H^j_t; \mathbf{p_{t+1}})}{1 - Q_t(W_t, H^j_t; \mathbf{p_{t+1}})}]$ and ξ is a constant of proportionality. Conditional on period-t information on house prices, the expected option value of waiting (i.e., no-transaction) in period $t + 1$ is:

$$\Pi_{t+1}\left(W_t, H^j_t; \mathbf{p_{t+1}}\right) = Q_t\left(W_t, H^j_t; \mathbf{p_{t+1}}\right)\left(V^N_{t+1}\left(W_t, H^j_t; \mathbf{p_{t+1}}\right) - V^T_{t+1}\left(W_t, H^j_t; \mathbf{p_{t+1}}\right)\right) \tag{12}$$

Π_{t+1} embodies all the opportunity costs the household expects from transacting (i.e. being aggressive) in period $t + 1$. Note that it is the presence of lumpy transaction costs that gives rise to the option value of waiting. If transactions could occur at no cost, then waiting in the next period has no value, households would expect to transact each period.

Now the second term in Eq. (5) can be rewritten as the following smooth function:

$$E_t V_{t+1}\left(W_t, H^j_t; \mathbf{p_{t+1}}\right) = \int \left\{V^T_{t+1}\left(W_t, H^j_t; \mathbf{p_{t+1}}\right) + \Pi_{t+1}\left(W_t, H^j_t; \mathbf{p_{t+1}}\right)\right\} \phi(\mathbf{p_t}|\mathbf{p_{t+1}})\, \mathrm{d}(\mathbf{p_t}|\mathbf{p_{t+1}}) \tag{13}$$

where the expectation is taken over future prices only. Substituting Eq. (13) into the Bellman equation (5), we obtain a continuous optimization problem. In the absence of transaction costs, Π_{t+1} vanishes. When lumpy transaction costs are present, the term Π_{t+1} adjusts for the possibility of no transaction in the future. The Euler equation can now be derived in a standard way.

Proposition 4.1 *(Euler Equation). Under Assumptions* 1 *and* 2*, conditional on transacting in period t, the optimal conditional housing demand H^j_t is determined by the Euler equation*:

$$\frac{\frac{\partial \tilde{u}_t}{\partial H^j_t} + \beta \frac{\partial \bar{\Pi}_{t+1}}{\partial H^j_t}}{\frac{\partial \tilde{u}_t}{\partial C_t}} = \left(p^j_t - m_{t+1}(1-\delta) E_t p^j_{t+1}\right) + \gamma_{t+1}\left((1-\delta)^2 H^j_t Var_t\left(p^j_{t+1}\right) - (1-\delta) H^{j+1}_{t+1} Cov_t\left(p^j_{t+1}, p^{j+1}_{t+1}\right)\right) \tag{14}$$

where $m_{t+1} \equiv \frac{\frac{\beta \partial \tilde{u}_{t+1}}{\partial C_{t+1}}}{\frac{\partial \tilde{u}_t}{\partial C_t}}$ *is the stochastic discount factor and*

$$\gamma_{t+1} = -\frac{\frac{\partial^2 \overline{V}^T_{t+1}}{\partial W^2_{t+1}} + \frac{\partial^2 \overline{\Pi}_{t+1}}{\partial W^2_{t+1}}}{\frac{\partial \overline{V}^T_{t+1}}{\partial W_{t+1}} + \frac{\partial \overline{\Pi}_{t+1}}{\partial W_{t+1}}}$$

is a risk aversion factor of the value function. $\overline{V}^T_{t+1}$ and $\overline{\Pi}_{t+1}$ are evaluated at the conditional mean of future house prices.

Eq. (14) implicitly defines the optimal housing purchase size, conditional on transacting in period t. It says that the marginal substitution rate between housing and non-housing consumption at optimal (H^j_t, C_t) should be equal to the expected user cost of housing services at the time of home purchase. This result is consistent with the previous housing literature but brings additional dynamics. On the left-hand side, the marginal benefit from consuming one additional unit of housing services reflects the utility from both the current period and possible future periods. On the right-hand side, rather than using a static user cost, it presents a concept of user cost (UC_t) that combines transaction costs, price uncertainty, and households' risk attitude. The first term represents the traditional sense of housing user cost, i.e., the expected cost of purchasing a home in period t and selling it in period $t + 1$ adjusting for transaction costs.

The last bracketed term in (14) reflects the cost of house price uncertainty, measured by the increase in the housing cost that a household is willing to accept in exchange for price certainty. It indicates two simultaneous and offsetting effects of price uncertainty: a housing wealth risk effect and a hedging effect. On the one hand, high resale price uncertainty ($Var_t(p^j_{t+1})$) increases the risk associated with expected future housing wealth and hence reduces current housing demand. On the other hand, a positive correlation across houses ($Cov_t(p^j_{t+1}, p^{j+1}_{t+1})$) effectively reduces the cost of risk and generates positive hedging demand for housing. The bigger the expected size of the future desired house (H^{j+1}_{t+1}), the bigger the hedging effect. This is a somewhat surprising result for the housing market. It says that price uncertainty itself does not necessarily discourage housing demand. By explicitly incorporating the correlation between the currently owned house and the future desired house, this model provides a mechanism that produces positive feedback from housing market uncertainty to the demand for housing.

In the next section, we numerically simulate the net effect of price uncertainty. However, some of the insights about the effects of price uncertainty can be obtained by looking at the risk component of the user cost. To do this, we first define the right-hand side of Eq. (14) as user cost UC_t. We then define the hedging incentive index $HI_t \equiv \frac{H^{j+1}_{t+1}}{H^j_t}\rho^{j,j+1}_{t+1}$, with $\rho^{j,j+1}_{t+1}$ indicating the correlation between p^j_{t+1} and p^{j+1}_{t+1}. Intuitively, a positive correlation between the house price processes leads to a positive hedging incentive. The bigger the future desired house relative to the current house, the stronger the incentive to hedge.

Proposition 4.2 *(Effects of House Price Uncertainty). Under Assumptions* 1 *and* 2*,*

$$\frac{\partial UC_t}{\partial Var_t(p^{j+1}_{t+1})} < 0 \quad \text{if} \quad HI_t < 2(1-\delta)\sqrt{\frac{Var_t(p^j_{t+1})}{Var_t(p^{j+1}_{t+1})}} \tag{15}$$

$$\frac{\partial UC_t}{\partial Var_t(p^{j+1}_{t+1})} \geqslant 0 \quad \text{if} \quad HI_t \geqslant 2(1-\delta)\sqrt{\frac{Var_t(p^j_{t+1})}{Var_t(p^{j+1}_{t+1})}} \tag{16}$$

Proposition 4.2 emphasizes that a thorough understanding of how uncertainty in house prices affects housing choices should take into account the dynamics of the intertemporal aspects of the housing choices made over the life cycle. The function HI_t embodies two important aspects: (i) a relationship between the size of the currently desired house and that of the future desired house and (ii) a correlation between these two houses' price processes. By defining a threshold value for the hedging incentive HI_t, Proposition 4.2 provides conditions under which the hedging effect dominates the housing wealth gain effect.

The micro-level framework that was laid out in this paper also allows me to discuss the concept of the housing risk premium. In Appendix A.3, I show that the basic asset pricing framework for financial assets also holds for housing. The risk premium on any risky asset depends on the expected covariance between its returns and the stochastic discount factor. This is true even if housing is treated as a durable consumption good and featured as an argument in the utility function. As with other risky assets, the housing risk premium depends on whether housing provides returns in periods when they are mostly needed.

4.2. Optimal purchase timing

In the presence of lumpy transaction costs, households decide whether to transact by comparing the present discounted value of costs incurred by remaining in the current dwelling with the costs of transacting. The optimal decision rule takes the form of expression (8), which is consistent with a generalized (S, s) rule in the investment literature.

Let $\tilde{\eta}_t = p^{j-1}_t H^{j-1}_{t-1} - p^j_t H^j_t$ indicate the time-t market value of this housing stock imbalance. Using the method discussed in Section 4.1, I derive two boundary functions that represent the maximum and minimum values of the housing value imbalance for which a household with a $\tilde{\eta}_t$ within the region chooses not to transact. For any $\tilde{\eta}_t$ outside the region, the household transacts fully.

Proposition 4.3 *(Generalized* (S, s) *rule). Under Assumption* 1 *and Assumption* 2*, there exists an optimal housing adjustment policy: households transact if and only if the value-adjusted housing stock imbalance $\tilde{\eta}_t < \tilde{\eta}^A_t$ or $\tilde{\eta}_t > \tilde{\eta}^B_t$ where $\tilde{\eta}^A_t$ and $\tilde{\eta}^B_t$ are solutions to the following equation:*

$$-\frac{1}{2}\frac{\partial^2 \tilde{u}_t}{\partial C_t^2}\tilde{\eta}_t^2 - \frac{\partial^2 \tilde{u}_t}{\partial C_t^2}\left(C^N_t - C^T_t - TC_t\right)\tilde{\eta}_t + \lambda_t = 0 \tag{17}$$

where

$$\lambda_t = -\frac{1}{2}\frac{\partial^2 \tilde{u}_t}{\partial C_t^2}\left(TC_t - 2\left(C^N_t - C^T_t\right)TC_t\right) + \frac{\partial \tilde{u}_t}{\partial C_t}\left(\left(p^j_t - p^{j-1}_t\right)H^{j-1}_{t-1} + TC_t\right) + \frac{1}{2}\frac{\partial^2 \tilde{u}_t}{\partial C_t \partial H_t}p^j_t\left(H^{j-1}_{t-1} - H^j_t\right)^2 + \left(s^N_t - s^T_t\right)$$

and $TC_t \equiv \delta p^{j-1}_t H^{j-1}_{t-1} + F$.

Eq. (17) defines the levels of $\tilde{\eta}_t$ that leave a household indifferent between transacting and not transacting (i.e., simply staying in the current home). For the market values of the housing stock imbalance that fall within the region $(\tilde{\eta}^A_t, \tilde{\eta}^B_t)$, households stay with their current house. Once the market value of the housing stock imbalance falls outside the inaction region, households sell the current home and purchase the desired amount of home.

So far, I have shown that conditioning on current utility shifters information, the optimal policy resembles an (S, s)-band rule where the state variable $\tilde{\eta}_t$ takes into account both the quantity and the value of the stock imbalance. Once removing the conditioning on the current utility shifters, that is, prior to period t when uncertainty over time-t utility shifters has not been resolved yet, this model yields a probabilistic (S, s) rule where the distribution of the bands depends on the distribution of $(s_t^T - s_t^N)$.

5. Numerical analysis

In this section, I construct artificial economies with ex ante identical households facing different realizations of house price and compute their optimal housing demand and the associated welfare costs. The purpose is to confirm the predictions derived from the analytical solutions in Section 4.[12]

More specifically, I compare the optimal housing demand and associated welfare for two groups of households. They differ in whether households self-insure against future housing cost uncertainty. This is shown by considering two observable dimensions along which the degree of hedging incentive might be expected to vary. The first is variation in households' housing consumption plans during the life cycle. Given the model parameterization specified below, younger households move up the housing ladder while older households move down the housing ladder. This means that the hedging effect is more important at the earlier stage of the life cycle.

A second important source of variation is the underlying house price process. If the price of a household's current house is positively correlated with the price of the house that the household may buy in the future, then the household is able to use the current home purchase to hedge against future housing risk. In contrast, once the correlation parameter falls to or below zero, then the self-hedge mechanism is not in effect.

5.1. Simulation strategy

To make the simulation tractable, I assume all moves occur between two locations, i.e, there are only two price processes $((j, j') = \{(1, 2), (2, 1)\})$. In each transaction, households sell the existing house at one price and buy a different house at another price. The price vector follows a first order Markov process[13]:

$$\begin{pmatrix} p_t^1 \\ p_t^2 \end{pmatrix} = \mu + A \begin{pmatrix} p_{t-1}^1 \\ p_{t-1}^2 \end{pmatrix} + e_t$$

where $e_t \sim N\left(\binom{0}{0}, \begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}\right)$, μ is a 2×1 vector and A is a 2×2 matrix $\begin{pmatrix} q & \rho \\ \rho & q \end{pmatrix}$.

The distribution of prices is summarized by four sets of parameters: ρ, q, σ, and μ. The parameter ρ defines the extent to which housing prices are correlated across the current and future desired houses. The parameter q is a persistence parameter governing the relative importance of permanent and transitory components of the housing price. In addition, σ indicates the standard deviation of innovations in price processes and μ indicates the constant part of price processes.

To highlight the effect of an increase in house price uncertainty, I simulate two types of economies where $\sigma = 1\%$ corresponds to a relative stable market and $\sigma = 10\%$ corresponds to a highly volatile market.[14] For each type of economy, I simulate two types of households: households with strong hedging incentives and households with weak hedging incentives. In practice, it is hard for both researchers and households to know the exact correlation between the price process of the current house and that of future desired house. However, one can reasonably assume that households know whether they will move within the same housing market in the future, where a housing market could be defined as a local neighborhood, or more broadly, a metropolitan area. If households plan to move within the same market, then the current house and the future house are highly correlated. If households plan to move out of the current market but cannot predict which market to move to, then the current market and the future market are assumed

[12] A simple numerical exercise gives freedom in generating the model predictions. In particular, the parameters on the age-varying taste profile and house price processes are chosen to derive the comparative statics regarding the effects of price risk and housing consumption plan. This cannot be easily done in a formal calibration exercise.

[13] It is probably more realistic to consider a more general price vector. However, the introduction of additional state variables would not be computationally tractable. For example, if an n-point grid is used for one price process, then an n^m grid is required for price vector of length m. Computational time increases exponentially.

[14] σ refers to the 1-year-ahead forecast of standard deviation in housing return. To understand the magnitude of σ, I generate time series estimates of expected return and risk based on movements in the metropolitan-level quarterly house price appreciation rates in 1978–2004 reported by the Office of Federal Housing Enterprise Oversight (OFHEO). For each metropolitan area in the sample, I fit a GARCH-M model and use the resulting estimates to derive the 1-year-ahead forecasts of housing return and risk in each year. That is, households are assumed to make their housing decisions at time t based on their forecasts of the risk and return over the coming year, where the risk is measured by the forecasted standard deviation σ. The resulting statistics show that the estimated σ has a mean of 3% with a standard deviation of 7%. In our comparative static exercise, $\sigma = 1\%$ corresponds to the 25%th percentile in the distribution of σ and $\sigma = 10\%$ corresponds to the 95%th percentile. Since the model's focus is on the effect of changes in price uncertainty, the values of the μ and q are chosen to first guarantee that AR(1) process is stationary and second, to ensure that the unconditional mean of the return process is the same across different specifications of ρ.

to be uncorrelated by expectation. In our numerical example, we use $\rho = 0.9$ to illustrates the first case and $\rho = 0$ to illustrates the second case.[15]

Finally, I assume that households consider a 30-year housing planning horizon starting from age 30. For computational convenience, income and taste are assumed to evolve deterministically with age. For example, households are assumed to have an increasing taste profile up to age 45 and then a declining taste profile after that. Intuitively, this describes a typical family's life-cycle pattern: young families have a preference for bigger houses as they anticipate getting married and having children, while middle-aged families are likely to plan on downsizing as they may anticipate children growing up and leaving for college. In addition, the stochastic utility shifters are set to zero.[16]

Given the finite nature of the problem, a solution exists. The model can be solved backwards by standard numerical methods.[17] I examine an average household's average life-cycle housing behavior through simulation. To do so, I first simulate 500 households that enter the economy at age 30 with an existing home and exit at age 60. For each household, I simulate a time series of age-dependent tastes and a time series of house prices according to their respective governing stochastic processes. I then solve their optimal housing decisions at all ages. Finally, average life-cycle housing behavior is generated by taking the average of the optimal solutions in these 500 simulations.

5.2. Effects of price uncertainty on conditional housing demand

The main prediction of the model is that the hedging incentive offsets the negative effect of house price risk on housing demand. To confirm this, this section restricts the attention on the household's conditional housing demand if a household were to transact at a given age. That is, conditional on transacting in a given period, how does an anticipated increase in price uncertainty change the average home purchase size?[18] It would be interesting to examine the implication of the hedging incentive for the timing of home purchase. However, since the model does not generate clear prediction for this, I explore the issue in separate empirical research.

Figs. 1–3 plot the life-cycle path of optimal conditional housing demand under different parameterizations. Given the taste-age profile specified above, it is not surprising that the optimal life-cycle consumption path attains a pronounced hump shape. The theoretical discussion in the previous section provides a basis for understanding the concept of the hedging incentive. More formally, hedging demand is defined as the additional amount of housing services households will purchase attributed to the positive correlation (versus zero) between prices of the current property and the next property.

I first fix σ at 10%. Fig. 1 plots the levels of optimal conditional purchase size for different values of ρ: $\rho = 0$ and $\rho = 0.9$. When $\rho = 0$, future housing cost is independent of current housing return. Hence σ translates fully into housing wealth risk. When $\rho = 0.9$, the current purchase is positively correlated with the future purchase. In this case, the same source of price uncertainty not only implies the wealth risk but also determines the extent to which the current property can serve as a hedge against future housing cost fluctuations. In addition, Fig. 1 shows that the hedging effect is more dominant in the first half of the life cycle, when households anticipate upgrading.

Figs. 2–3 explore the effects of an anticipated increase in price uncertainty under different hedging incentives. In Fig. 2, the assumption $\rho = 0$ implies the absence of any hedging incentive. Optimal conditional housing demand is simulated for low and high levels of σ ($\sigma = 1\%$ and $\sigma = 10\%$ respectively). Not surprisingly, high uncertainty in this case represents large housing wealth risk and leads to lower housing demand. The resulting difference between the housing demand in Fig. 2 reveals the following message: housing wealth risk crowds out housing investment.

To investigate the interaction effects of hedging incentive and price uncertainty, Fig. 3 fixes $\rho = 0.9$ and repeats the same exercise as in Fig. 2. Comparing Fig. 3 with Fig. 2, the positive correlation (versus zero correlation) makes an important difference in the region where households plan on upgrading. In particular, at the early stages of the life cycle, under strong hedging incentive, high price uncertainty actually increases the level of housing demand. In contrast, the hedging incentive is much weaker in the later stages of the life cycle. This is because when households start to move down the housing ladder, the value of the existing house is sufficient to offset future housing cost risk, hence there is no need to buy additional housing to hedge. The simulation result delivers the second message: the negative effect of housing wealth risk decreases as the strength of the hedging incentive increases.

This result is not surprising. Consider a household that prepares to roll over in the future and has the option between holding a risk-free bond and investing in housing. Housing investment, although more risky in its returns, offers a hedge against future

[15] In reality, even the cross-market mobility may not be perfectly known in advance. To the extent that households over-predict or under-predict ρ, our model could overstate or understate the amount of the hedging demand for housing. For example, an unexpected job reallocation may cause a household to have over-predicted its hedging incentive and hence to have over-invested in housing when making earlier home purchase decisions.

[16] As noted earlier, the stochastic utility shifters are necessary for solving the model analytically. Since the numerical solution does not rely on the analytical approach, I set the utility shifters to zero to simplify the computation.

[17] Additional details on the model parameters and the solution method are provided in Appendix A.5.

[18] This is different from the case in which households optimally choose to transact each period, as in the no-lumpy transaction-cost case. In particular, the model endogenizes the decision of whether to transact. Hence the derived conditional demand takes explicitly into account the alternative choice the household faces in the future as a consequence of lumpy transaction costs.

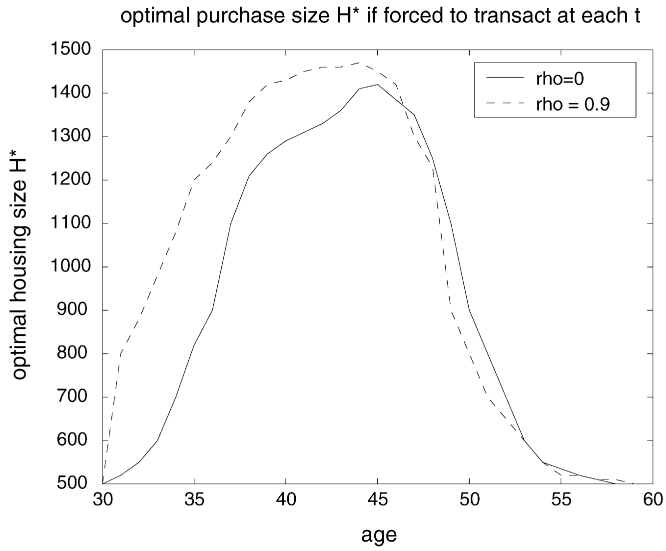

Fig. 1. Housing demand at low and high levels of correlation ($\sigma = 10$).

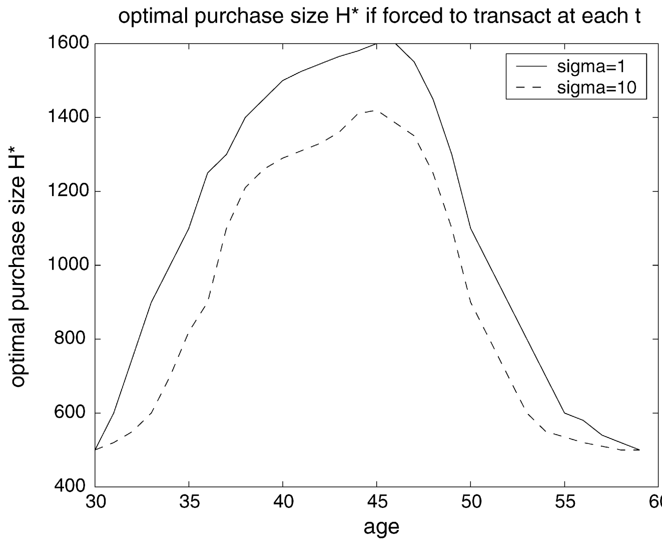

Fig. 2. Housing demand at low and high levels of price volatility ($\rho = 0$).

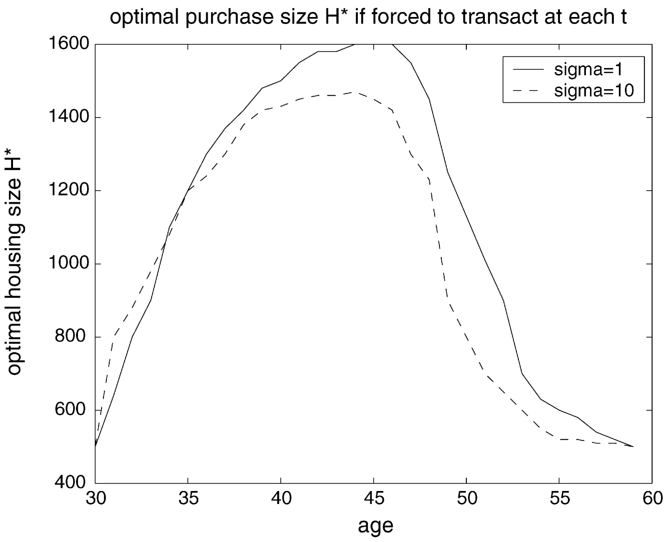

Fig. 3. Housing demand at low and high levels of volatility ($\rho = 0.9$).

housing cost variations. In this sense, housing investment is actually a "safer" strategy than bond investment. The higher the is the level of house price uncertainty, the more attractive is the insurance role of current home purchase.

While the simulation exercise confirms the predictions from the model, some care must be taken when interpreting the magnitude of the results. In particular, three points should be noted to ensure proper understanding of the results. First, given that the parameters in the simulation exercise are not calibrated from the real economy, the simulated difference in housing demand for households with and without hedging incentives should not be taken as quantitative evidence. To have an idea about the fraction of households with positive hedging incentives, one can apply the concept of the hedging incentive to the Panel Study of Income Dynamics (PSID). Among all the households that have bought a house in the PSID sample (1968–1997), 62% of them traded up later by buying a more expensive house in real terms. Among households that traded up, 71.3% of them moved within the same metropolitan area. If houses within the same metropolitan area are considered as highly positive correlated, then at least 28.7% of homeowners in the PSID have strong positive hedging incentives as defined in our simulation exercise.[19]

Second, the simulation analysis has exclusively focused on the effect of house price uncertainty on housing demand. In doing so, I have assumed away the uncertainties in future income. In reality, households may also face risk in their future income.

[19] Note that the PSID sample may significantly under-represent the true proportion of households that trade up and over-represent the true proportion of households that move within the local neighborhood. This is attribute to the fact that households that move out of the current residence are difficult to track continuously in a longitudinal data set.

Depending on whether their future income moves together with future house prices, the fluctuations in house prices may provide additional hedge against future income risk (Davidoff, 2006). In particular, for households belonging to occupations or sectors in which earnings are negatively correlated with future housing price, current home purchase provides a hedge not only against future housing cost risk but also against future income risk. In this case, hedging demand for housing in under-predicted by our model. In contrast, for households that are likely to experience negative shocks to labor income and house prices at the same time, current home purchase adds to the wealth risk. In this case, the amount of hedging demand for housing is over-predicted by our model.

Finally, the model has assumed away the impact of housing capital gains tax on home purchase behavior. The taxation of housing capital gains is important for home purchase decisions (Hoyt and Rosenthal, 1990) and may distort the hedging incentive in our model. Prior to 1997, rollover previsions in the US tax code discouraged households from buying down in nominal terms. In addition, homeowners above age 55 or older qualified for a one-time exclusion of \$125,000 in calculating taxable gains. After 1997, under the Taxpayer Relief Act of 1997 (TRA97), all homeowners qualified for capital gains exclusion. On the one hand, by eliminating the differential treatment of housing capital gains for all households, TRA97 significantly improved the mobility to trade down for households under age 55 (Cunningham and Engelhardt, in press), leading to a weaker hedging incentive on average. On the other hand, the tax code prior to 1997 could have had a negative effect on the housing demand for young homeowners, as these households might have invested less in housing to avoid trading down in the future. By eliminating the tax incentive to avoid trading down, TRA97 potentially increased the average housing demand for households in the earlier stages of their housing ladders. The net effect of TRA97 on hedging demand is not immediately clear.

Bearing these caveats in mind, a natural extension of this research would be to empirically investigate the quantitative importance of the hedging incentive for home purchase decisions in a volatile market. While this approach is not explored here, it is worth noting that the model presented above generates several testable empirical implications.

The most robust prediction of the model is that, absent the hedging incentive, an exogenous rise in price uncertainty decreases housing demand. The magnitude of such an effect decreases as hedging incentives rise. The strength of the hedging incentive is governed by two key factors in a household's future housing consumption plan: (1) the size of the future desired house relative to the current house and (2), the spatial correlation in prices across the current and future desired houses. Since most surveys used for housing demand analysis report both current home value and previous home value, it should be possible to control for whether a household trades up by buying a more expensive house or by moving from an area with low house prices to an area with high house prices. While the spatial correlation across the current and future houses can rarely be perfectly predicted, one could infer the relative strength of the correlation by comparing households with different levels of geographical mobility: those moving within the same neighborhood; those moving across neighborhoods but within the same city; those moving across cities but within the same state; those moving across states but within the same country; and those moving out of the country. Given these two key measures of the hedging incentive, our model predicts that the more likely a household is to move up the housing ladder, and the lower the household's geographical mobility, the stronger its hedging incentive is, and the less negative the impact of house price risk on the current housing demand. In addition, one could further examine the impact of housing capital gains taxation on hedging demand for housing by comparing home purchase behavior before and after the TRA97. These issues are interesting and are currently explored in a related research project.

6. Welfare analysis

In this section, I study the potential welfare costs from increasing price uncertainty and examine how sensitive these costs are to the presence of the self-hedging mechanism. The economic experiment performed in this paper involves the comparison of the welfare cost of price uncertainty in an economy in which self-hedging is not permitted to the welfare cost of price uncertainty in a counterpart economy in which self-hedging is permitted. For each economy, the welfare cost of price uncertainty is computed as the proportional increase in average annual housing asset holdings to compensate households for the loss of expected lifetime utility due to increasing price uncertainty. This is similar to the welfare cost measure adopted in Li and Yao (2007).

The issue, then, is what to use as a benchmark to provide an economy without self-hedging. I provide two different cases: (i) an economy in which the price correlation between the two alternative housing markets is 0 and (ii) an economy in which the price correlation between the two alternative housing markets is 0.9. The self-hedging mechanism is made available by increasing the price correlation from 0 to 0.9.

6.1. Welfare measure

To obtain an informative measure of welfare costs, we ask the following question: how much more housing and non-housing consumption, as well as bequeathed wealth, would a household facing high price uncertainty ($\sigma = 10\%$) need to have to be as happy as it would be in a low uncertainty environment ($\sigma = 1\%$)? In other words, how much would $p_t H_t$, C_t, and W_{T+1} have to increase in percentages to compensate households for the loss of expected remaining lifetime utility due to increasing price uncertainty?

Expected lifetime utility is computed by employing the same simulation framework as described in Section 5. Specifically, using the parameterized model described in Section 5, I simulate 500 households with each simulation generating a time series of first order Markov housing price processes. For each household, I compute its value functions when it enters the economy at age 30 and average these values over 500 simulations. The value function is specified as follows:

$$\overline{V}^j = \frac{1}{500}\sum_{i=1}^{500} E_0\left\{\sum_{\tau=0}^{T}\beta^\tau u(H_\tau, C_\tau)(p_\tau)^{\theta_\tau(1-\gamma)} + \beta^{T+1}\frac{W_{T+1}^{1-\gamma}}{1-\gamma}\right\} \tag{18}$$

where j indicates the state of housing price uncertainty and i is the index for the simulated household in state j, with $j \in \{low, high\}$ where *low* refers to the state of low price uncertainty when $\sigma = 1\%$ and *high* refers to the state of high price uncertainty when $\sigma = 10\%$. The utility cost measure can then be calculated as:

$$\Omega = \left(\frac{\overline{V}^{low}}{\overline{V}^{high}}\right)^{\frac{1}{1-\gamma}} - 1 \tag{19}$$

where Ω is interpreted as the proportional increase in annual consumption (housing and non-housing) and end-of-life bequest. The proof is given in Appendix A.6.

6.2. Findings

Table 1 summarizes the welfare cost estimates for various values of ρ and γ. ρ is the correlation between the return on the current home and that on the future home. Everything else held equal, ρ reflects the strength of hedging incentives. γ indicates households' risk aversion. Below, I start with the case where $\gamma = 3$, which is a standard assumption from the literature.

For the case where $\rho = 0$, in an economy with uncorrelated housing price processes, reducing price uncertainty from $\sigma = 10\%$ to $\sigma = 1\%$ is equivalent in utility terms to increasing average consumption by 0.3 percent. Households are better off with less price uncertainty because there are no positive self-hedging effects to counter the negative housing wealth risk effect when housing prices are uncorrelated. According to the Economic Report of the President, in 2005, housing expenditures, including the imputed rent of owner-occupied houses, were \$1114.6 billion. The expenditures on non-durable goods consumption, including food, clothing, shoes, gasoline, oil, fuel and coal, were \$2539.3 billion. This implies an annual cost of \$11.5 billion for the US economy, which is \$3.9 billion in housing expenditures and \$7.6 billion in non-durable consumption.

The potential welfare effects of self-hedging benefits are determined by comparing the welfare cost derived above with the welfare cost when the self-hedging mechanism becomes available ($\rho > 0$). With $\rho = 0.9$, an increase in average housing and non-housing consumption of 0.07 percent is needed to compensate for households in the high price uncertainty situation. This implies a cost of \$2.69 billion for the economy per year. This cost estimate is four times smaller than the one in an economy without the self-hedging mechanism operating.

As already noted, when ρ becomes positive, the fact that households take greater advantage of the self-hedging mechanism in the earlier stages of the life cycle causes welfare to be less sensitive to price changes. Yet even when ρ is equal to 0.9, the welfare cost still remains positive. This is because earlier home asset, although almost perfectly correlated with future housing cost, serves at best as an incomplete hedge for households on the upgrading housing ladder. The optimal self-hedging motivated mechanism is not able to make agents as well off as under the situation where there is little price uncertainty.

To illustrate how these results would change with higher risk aversion, I report results for $\gamma = 10$ in Table 1. I use this value since it is taken to be close to an upper bound for an empirically plausible degree of risk aversion. The results are not surprising: as households become more risk averse, price uncertainty leads to greater welfare loss. The important result, however, is that in a more risk-averse economy, these potential benefits generated from self-hedging are more substantial and may be realized for relatively lower values of ρ.

6.3. Caveats

The welfare results presented so far are computed under the assumption that the housing market is incomplete. Compared with stock market risk, house price risk has a much bigger impact on most households' financial portfolio. In light of this, economists

Table 1
Welfare cost of price uncertainty as a percentage of consumption

Risk aversion parameter	For economies with uncorrelated housing price processes	For economies with highly correlated price processes
$\gamma = 3$	0.3	0.07
$\gamma = 10$	3.8	1.8

The cost estimates reported here are computed as $\Omega = (\frac{\overline{V}^{low}}{\overline{V}^{high}})^{\frac{1}{1-\gamma}} - 1$, where *high* refers to the state $\sigma = 10$ and *low* refers to the state $\sigma = 1$. All estimates are percentages.

have been proposing different housing-related financial products to improve the completeness of the housing market. For example, Case et al. (1993) propose a market in which futures contracts are tied to regional house price indexes, allowing households to hedge by taking short positions in these derivatives contracts.[20] However, mitigating house price risk could be costly. As suggested by Englund et al. (2002), the attractiveness of a derivatives market in price index futures depends on the quality and integrity of the indexes. Furthermore, for most homeowners, housing is the largest component of their wealth. A typical homeowner cannot be well diversified across asset classes such as equities, bonds, and property. In other words, the lumpiness of housing would substantially reduce the diversification or hedging benefits of financial instruments even if the market were complete.

More recently, a home equity insurance program based on the ideas of Shiller and Weiss (1999) has been offered as an experiment in certain neighborhoods in Chicago and Syracuse. Such insurance is valuable not only for local community but also for homeowners. On the one hand, it helps to stabilize and revitalize the local community by mitigating self-reinforcing downward spirals in which homeowners pull out of a community in fear of declining prices. On the other hand, it provides a unique opportunity to protect homeowners against possible future house price declines. However, the extent to which individual homeowners are benefited from such program depends on the strength of their hedging incentives. For households that plan to trade up within the same local housing market, a public housing insurance program could simply serve to crowd out the housing demand through self-hedging mechanism.

7. Conclusion

This paper aims to better understand the extent to which house price uncertainty affects households' home purchase behavior. To this end, I have built and solved a theoretical model of an individual household's home purchase decision problem over its life cycle.

The theoretical model has the following ingredients: risk-averse households, a life-cycle framework, stochastic multivariate price processes, lumpy transaction costs, and incomplete markets. There are several benefits from using such a theoretical framework. First, this rich model allows us to characterize optimal decisions about both the timing and the size of home purchase in a market featuring lumpy transaction costs. Second, the life-cycle framework makes it clear that the future housing consumption plan needs to be taken into account when examining the effect of house price uncertainty on housing demand. Third, the assumptions on the multivariate house prices and incomplete markets enable us to distinguish between the two incentives under price uncertainty: an incentive to avoid the risk associated with housing wealth and an incentive to self-hedge against future housing cost risk.

The optimal home purchase behavior derived from this model supports the basic implications of the (S, s) type models: lumpy and infrequent housing transactions and a threshold decision rule in the presence of lumpy transaction costs. Unlike the standard (S, s) literature, the model in this paper explicitly accounts for stochastic multivariate house price processes and a non-stationary framework. The resulting decision rule requires less restrictive assumptions.

Treating housing as not only a consumption but also an investment, the model predicts that, under house price uncertainty, the risk associated with housing wealth reduces housing demand. This negative effect can be mitigated by the incentive to self-hedge against future house price uncertainty. The net effect of price uncertainty on housing demand depends critically on a household's expected future housing path, which is characterized by the correlation between the household's current and future desired houses and by the probability of moving up the housing ladder. The more likely the household is to move up the housing ladder within the same housing market, the stronger the hedging effect.

Consistent with these findings, the paper shows that the incentive to hedge also reduces the welfare cost imposed by house price uncertainty. This result suggests that ignoring the role of future housing consumption plans may lead us to overestimate the negative impact of house price uncertainty on households' home purchase decisions and to underestimate the proportion of housing wealth accumulated under hedging incentives. The test of empirical implications generated from this model is left for future research.

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[20] Using $S\&P$/Case-Shiller Home Price Indices (currently available for 20 MSAs), the Chicago Mercantile Exchange (CME), in partnership with MacroMarkets, successfully launched Housing Futures and Options in May 2006.

Appendix A

A.1. Proof of Proposition 4.1

Substituting Eq. (13) into the Bellman equation (5), I obtain a continuous optimization problem. To highlight the role of price uncertainty, I approximate the expectation by applying the second order Taylor expansion.

$$\begin{aligned} V_t^T\big(W_{t-1}, H_{t-1}^{j-1}\big) &= \max \tilde{u}\big(H_t^j, C_t; \theta_t, s_t^T\big) + \beta E_{\mathbf{p_{t+1}}|\mathbf{p_t}}\big(V_{t+1}^T\big(W_t^T, H_t^j\big) + \Pi_{t+1}\big(W_t^T, H_t^j\big)\big) \\ &= \max \tilde{u}\big(H_t^j, C_t; \theta_t, s_t^T\big) + \beta\big(\overline{V}_{t+1}^T + \overline{\Pi}_{t+1}\big) + \frac{\beta}{2}\left(\frac{\partial^2 \overline{V}_{t+1}^T}{\partial^2 W_{t+1}} + \frac{\partial^2 \overline{\Pi}_{t+1}}{\partial^2 W_{t+1}}\right) Var_t(W_{t+1}) \end{aligned}$$

where $\overline{V}_{t+1} = V_t^T(W_t^T, H_t^j; \overline{W}_{t+1})$ and $\overline{\Pi}_{t+1} = \Pi_{t+1}(W_t^T, H_t^j; \overline{W}_{t+1})$. Note that W_{t+1} indicates the end-of-period wealth in period $t+1$. Given the conditional distribution of house prices (the time-t expectation is summarized by the following terms: $E_t(p_{t+1}^j)$, $Var_t(p_{t+1}^j)$ and $Cov_t(p_{t+1}^j, p_{t+1}^{j+1})$) and the budget constraint, we have

$$\begin{aligned} \overline{W}_{t+1} &\equiv E_t(W_{t+1}) = y_{t+1} + (1 + r_{t+1})W_t + \big((1-\delta)E_t\big(p_{t+1}^j\big)H_t^j - E_t\big(p_{t+1}^{j+1}\big)H_{t+1}^{j+1} - F\big) - C_{t+1} \\ Var_t(W_{t+1}) &= (1-\delta)^2\big(H_t^j\big)^2 Var_t\big(p_{t+1}^j\big) + \big(H_{t+1}^{j+1}\big)^2 Var_t\big(p_{t+1}^{j+1}\big) - 2(1-\delta)H_t^j H_{t+1}^{j+1} Cov_t\big(p_{t+1}^j, p_{t+1}^{j+1}\big) \end{aligned}$$

The first order condition can be taken in a standard way.

$$\frac{\partial \tilde{u}_t}{\partial C_t} = \beta\left(\frac{\partial \overline{V}_{t+1}^T}{\partial W_t} + \frac{\partial \overline{\Pi}_{t+1}}{\partial W_t}\right) \tag{A-1}$$

$$\frac{\partial \tilde{u}_t}{\partial H_t^j} + \beta\left(\frac{\partial \overline{V}_{t+1}^T}{\partial H_t^j} + \frac{\partial \overline{\Pi}_{t+1}}{\partial H_t^j}\right) = -\beta\left(\frac{\partial \overline{V}_{t+1}^T}{\partial W_t} + \frac{\partial \overline{\Pi}_{t+1}}{\partial W_t}\right)\frac{\partial W_t}{\partial H_t^j} - \frac{\beta}{2}\left(\frac{\partial^2 \overline{V}_{t+1}^T}{\partial W_{t+1}^2} + \frac{\partial^2 \overline{\Pi}_{t+1}}{\partial W_{t+1}^2}\right)\frac{\partial Var_t(W_{t+1})}{\partial H_t^j} \tag{A-2}$$

Applying the Envelop Theorem,

$$\frac{\partial V_t^T}{\partial H_{t-1}^{j-1}} = \beta\left(\frac{\partial \overline{V}_{t+1}^T}{\partial W_t} + \frac{\partial \overline{\Pi}_{t+1}}{\partial W_t}\right)\frac{\partial W_t^T}{\partial H_{t-1}^{j-1}} \tag{A-3}$$

Updating Eq. (A-3) one period, we obtain

$$\frac{\partial \overline{V}_{t+1}^T}{\partial H_t^j} = \beta\left(\frac{\partial \overline{V}_{t+2}^T}{\partial W_{t+1}} + \frac{\partial \overline{\Pi}_{t+2}}{\partial W_{t+1}}\right)\frac{\partial \overline{W}_{t+1}}{\partial H_t^j} \tag{A-4}$$

Updating Eq. (A-1) one period, we obtain

$$\frac{\partial \tilde{u}_{t+1}}{\partial C_{t+1}} = \beta\left(\frac{\partial \overline{V}_{t+2}^T}{\partial W_{t+1}} + \frac{\partial \overline{\Pi}_{t+2}}{\partial W_{t+1}}\right) \tag{A-5}$$

It follows from the budget constraint that

$$\frac{\partial W_t}{\partial H_t^j} = -p_t^j \tag{A-6}$$

$$\frac{\partial \overline{W}_{t+1}}{\partial H_t^j} = (1-\delta)E_t\big(p_{t+1}^j\big) \tag{A-7}$$

$$\frac{\partial Var_t(W_{t+1})}{\partial H_t^j} = (1-\delta)^2 2H_t^j Var_t\big(p_{t+1}^j\big) - 2(1-\delta)H_{t+1}^{j+1} Cov_t\big(p_{t+1}^j, p_{t+1}^{j+1}\big) \tag{A-8}$$

Combining Eqs. (A-1)–(A-8), we have

$$\frac{\frac{\partial \tilde{u}_t}{\partial H_t^j} + \beta\frac{\partial \bar{\Pi}_{t+1}}{\partial H_t^j}}{\frac{\partial \tilde{u}_t}{\partial C_t}} = \big(p_t^j - m_{t+1}(1-\delta)E_t p_{t+1}^j\big) + \gamma_{t+1}\big((1-\delta)^2 H_t^j Var_t\big(p_{t+1}^j\big) - (1-\delta)H_{t+1}^{j+1} Cov_t\big(p_{t+1}^j, p_{t+1}^{j+1}\big)\big) \tag{A-9}$$

where $m_{t+1} \equiv \frac{\frac{\beta\partial \tilde{u}_{t+1}}{\partial C_{t+1}}}{\frac{\partial \tilde{u}_t}{\partial C_t}}$ is the stochastic discount factor and

$$\gamma_{t+1} = -\frac{\frac{\partial^2 \overline{V}_{t+1}^T}{\partial W_{t+1}^2} + \frac{\partial^2 \overline{\Pi}_{t+1}}{\partial W_{t+1}^2}}{\frac{\partial \overline{V}_{t+1}^T}{\partial W_{t+1}} + \frac{\partial \overline{\Pi}_{t+1}}{\partial W_{t+1}}}$$

is a risk aversion factor of the value function.

A.2. Proof of Proposition 4.2

Define time-t expected user cost as

$$UC_t \equiv \left(p_t^j - m_{t+1}(1-\delta)E_t p_{t+1}^j\right) + \gamma_{t+1}\left((1-\delta)^2 H_t^j Var_t\left(p_{t+1}^j\right) - (1-\delta)H_{t+1}^{j+1} Cov_t\left(p_{t+1}^j, p_{t+1}^{j+1}\right)\right)$$
$$= \left(p_t^j - m_{t+1}(1-\delta)E_t p_{t+1}^j\right) + \gamma_{t+1}\left((1-\delta)^2 H_t^j Var_t\left(p_{t+1}^j\right) - (1-\delta)H_{t+1}^{j+1} \rho_{t+1}^{j,j+1} \sqrt{Var_t(p_{t+1}^j) Var_t(p_{t+1}^{j+1})}\right)$$

where $\rho_{t+1}^{j,j+1} \equiv \frac{Cov_t(p_{t+1}^j, p_{t+1}^{j+1})}{Var_t(p_{t+1}^j) Var_t(p_{t+1}^{j+1})}$. It follows that

$$\frac{\partial UC_t}{\partial Var_t(p_{t+1}^{j+1})} = \frac{\gamma_{t+1}}{1+r_{t+1}}\left((1-\delta)^2 H_t^j - (1-\delta)H_{t+1}^{j+1}\rho_t^{j,j+1}\sqrt{\frac{Var_t(p_{t+1}^{j+1})}{Var_t(p_{t+1}^{j+1})}}\right) \tag{A-10}$$

Define hedging incentive index $HI_{t+1} \equiv \frac{H_{t+1}^{j+1}}{H_t^j}\rho_{t+1}^{j,j+1}$.

$$\frac{\partial UC_t}{\partial Var_t(p_{t+1}^{j+1})} < 0 \quad \text{if} \quad HI_{t+1} < 2(1-\delta)\sqrt{\frac{Var_t(p_{t+1}^j)}{Var_t(p_{t+1}^{j+1})}} \tag{A-11}$$

$$\frac{\partial UC_t}{\partial Var_t(p_{t+1}^{j+1})} \geqslant 0 \quad \text{if} \quad HI_{t+1} \geqslant 2(1-\delta)\sqrt{\frac{Var_t(p_{t+1}^j)}{Var_t(p_{t+1}^{j+1})}} \tag{A-12}$$

A.3. Derivation of housing risk premium

Define the one-period return on the jth house between time $t+1$ and time t as $R_{t+1}^j \equiv \frac{p_{t+1}^j - p_t^j}{p_t^j}$. Then $E_t\{(1-\delta)(1+R_{t+1}^j) - (1+r^f)\}$ indicates the expected housing risk premium, conditional on time-t information and adjusted for the transaction costs.

Proposition 7.1 *(The Consumption-Based Housing Risk Premium). Under Assumption* 1*, the housing market risk premium is determined by two factors*: (i) *the covariance between the stochastic discount factor and the expected housing return and* (ii) *the expected consumption value of future housing service.*

$$E_t\left\{(1-\delta)\left(1+R_{t+1}^j\right) - \left(1+r_t^f\right)\right\} = -\frac{Cov_t(m_{t+1}, (1-\delta)(1+R_{t+1}))}{E_t m_{t+1}} - \frac{\frac{\partial u_t}{\partial H_t} + E_t\frac{\partial \pi_{t+1}}{\partial H_t}}{\frac{\partial u_t}{\partial C_t}p_t^j} \tag{A-13}$$

Proof. Taking the first order conditions with respect to C_t and H_t^j yields:

$$\frac{\partial u_t}{\partial C_t} = \beta E_t\left(\frac{\partial V_{t+1}^T}{\partial W_t^T} + \frac{\partial \Pi_{t+1}^T}{\partial W_t^T}\right) \tag{A-14}$$

$$\frac{\partial u_t}{\partial H_t^j} = -\beta E_t\left\{\left(\frac{\partial V_{t+1}^T}{\partial H_t^j} + \frac{\partial \Pi_{t+1}^T}{\partial H_t^j}\right) + \left(\frac{\partial V_{t+1}^T}{\partial W_t^T} + \frac{\partial \Pi_{t+1}^T}{\partial W_t^T}\right)\frac{\partial W_t^T}{\partial H_t^j}\right\} \tag{A-15}$$

Applying envelop theorem and updating one period yields:

$$\frac{\partial V_{t+1}^T}{H_t^j} = E_t\left\{\left(\frac{\partial V_{t+2}}{\partial W_{t+1}^T} + \frac{\partial \Pi_{t+2}}{\partial W_{t+1}^T}\right)(1-\delta)P_{t+1}^j\right\} \tag{A-16}$$

Combining first order conditions and equation above yields

$$\frac{\frac{\partial u_t}{\partial H_t^j}}{\frac{\partial u_t}{\partial C_t}} + \frac{\beta E_t\frac{\partial \Pi_{t+1}}{\partial H_t^j}}{\frac{\partial u_t}{\partial C_t}} = p_t^j\left\{1 - E_t(1-\delta)\frac{p_{t+1}^j}{p_t^j}\frac{\frac{\beta\partial u_{t+1}}{\partial C_{t+1}}}{\frac{\partial u_t}{\partial C_t}}\right\} \tag{A-17}$$

Define $m_{t+1} \equiv \frac{\frac{\beta\partial u_{t+1}}{\partial C_{t+1}}}{\frac{\partial u_t}{\partial C_t}}$ and $R_{t+1}^j \equiv \frac{p_{t+1}^j - p_t^j}{p_t^j}$, we then have

$$E_t\left\{m_{t+1}(1-\delta)\left(1+R_{t+1}^j\right)\right\} = 1 - \frac{\frac{\partial u_t}{\partial H_t^j} + \beta E_t\frac{\partial \Pi_{t+1}}{\partial H_t^j}}{p_t^j\frac{\partial u_t}{\partial C_t}} \tag{A-18}$$

Expanding this equation gives

$$E_t m_{t+1} E_t\{(1-\delta)(1+R^j_{t+1})\} + Cov_t(m_{t+1}, (1-\delta)(1+R^j_{t+1})) = 1 - \frac{\frac{\partial u_t}{\partial H^j_t} + \beta E_t \frac{\partial \Pi_{t+1}}{\partial H^j_t}}{p^j_t \frac{\partial u_t}{\partial C_t}} \tag{A-19}$$

The gross return on a risk free rate satisfies

$$(1+r^f_{t+1})E_t(m_{t+1}) = 1 \tag{A-20}$$

The housing market risk premium can then be written as

$$RP^h_{t+1} \equiv E_t[(1-\delta)R^j_{t+1}] - r^f_{t+1} = -\frac{Cov_t[(1-\delta)(1+R^j_{t+1}), m_{t+1}]}{E_t[m_{t+1}]} - \frac{\frac{\partial u_t}{\partial H^j_t} + \beta E_t \frac{\partial \Pi_{t+1}}{\partial H^j_t}}{p^j_t \beta \frac{\partial u_{t+1}}{\partial C_{t+1}}} \qquad \Box \tag{A-21}$$

A.4. *Proof of Proposition 4.3*

Households choose to transact if the benefit from transacting exceeds the cost associated transacting. The optimal decision on whether to transact in period t depends on the differences in the current and expected utility associated with the two alternative choices. Specifically, it takes the following form:

$$d_t = \begin{cases} 1 & \text{if } (s^N_t - s^T_t) \leqslant \Gamma(W_{t-1}, H^{j-1}_{t-1}, \mathbf{p_t}) \\ 0 & \text{if } (s^N_t - s^T_t) > \Gamma(W_{t-1}, H^{j-1}_{t-1}, \mathbf{p_t}) \end{cases} \tag{A-22}$$

where $\Gamma(W_{t-1}, H^{j-1}_{t-1}, \mathbf{p_t}) \equiv [\tilde{u}(H^j_t, C_t; \theta_t) - \tilde{u}(H^{j-1}_{t-1}, C_t; \theta_t)] + \beta E_t[V_{t+1}(W^T_t, H^j_t; s^T_t, \mathbf{p_t}) - V_{t+1}(W^N_t, H^{j-1}_{t-1}; s^N_t, \mathbf{p_t})]$.

For given (W_{t-1}, H^{j-1}_{t-1}), define the benefit from waiting as $D_t(W_{t-1}, H^{j-1}_{t-1})$.

$$\begin{aligned}
D_t(W_{t-1}, H^{j-1}_{t-1}) &\equiv V^N_t(W_{t-1}, H^{j-1}_{t-1}) - V^T_t(W_{t-1}, H^{j-1}_{t-1}) \\
&= (\tilde{u}(H^{j-1}_{t-1}, C^N_t; \theta_t) + s^N_t) - (\tilde{u}(H^j_t, C^T_t; \theta_t) + s^T_t) + \beta E_{\mathbf{p_{t+1}}|\mathbf{p_t}}\Big((V^T_{t+1}(W^N_t, H^{j-1}_{t-1}; \mathbf{p_{t+1}}) \\
&\quad - V^T_{t+1}(W^T_t, H^j_t; \mathbf{p_{t+1}})) + \beta E_{\mathbf{p_{t+1}}|\mathbf{p_t}}(\Pi_{t+1}(W^N_t, H^{j-1}_{t-1}; \mathbf{p_{t+1}}) - \Pi_{t+1}(W^T_t, H^j_t; \mathbf{p_{t+1}}))\Big) \\
&= \left(\frac{\partial \tilde{u}_t}{\partial H} + \beta \frac{\partial E_t V^T_{t+1}}{\partial H} + \beta \frac{\partial E_t \Pi_{t+1}}{\partial H}\right)\Big|_{H=H^j_t}(H^{j-1}_{t-1} - H^j_t) \\
&\quad + \frac{1}{2}\left(\frac{\partial^2 \tilde{u}_t}{\partial H^2} + \beta \frac{\partial^2 E_t V^T_{t+1}}{\partial H^2} + \beta \frac{\partial^2 E_t \Pi_{t+1}}{\partial H^2}\right)\Big|_{H=H^j_t}(H^{j-1}_{t-1} - H^j_t)^2 \\
&\quad + \frac{\partial \tilde{u}_t}{\partial C}\Big|_{C=C^T_t}(C^N_t - C^T_t) + \frac{1}{2}\frac{\partial^2 \tilde{u}_t}{\partial C^2}\Big|_{C=C^T_t}(C^N_t - C^T_t)^2 \\
&\quad + \beta\left(\frac{\partial E_t V^T_{t+1}}{\partial W} + \frac{\partial E_t \Pi_{t+1}}{\partial W}\right)\Big|_{W=W^T_t}(W^N_t - W^T_t) \\
&\quad + \frac{1}{2}\beta\left(\frac{\partial^2 E_t V^T_{t+1}}{\partial W^2} + \frac{\partial^2 E_t \Pi_{t+1}}{\partial W^2}\right)\Big|_{W=W^T_t}(W^N_t - W^T_t)^2 + (s^N_t - s^T_t) \\
&= \frac{1}{2}\frac{\partial^2 \tilde{u}_t}{\partial C^2}(\eta_t - TC_t)^2 + \left(-\frac{\partial \tilde{u}_t}{\partial C} + \frac{\partial^2 \tilde{u}_t}{\partial C^2}(W^N_t - W^T_t)\right)(\eta_t - TC_t) \\
&\quad + \left(-\frac{\partial \tilde{u}_t}{\partial C} + \beta \frac{\partial E_t V^T_{t+1}}{\partial W_t} + \beta \frac{\partial E_t \Pi_{t+1}}{\partial W_t}\right)(W^N_t - W^T_t) \\
&\quad + \left(\frac{\partial^2 \tilde{u}_t}{\partial C^2} + \beta \frac{\partial^2 E_t V^T_{t+1}}{\partial W^2_t} + \beta \frac{\partial^2 E_t \Pi_{t+1}}{\partial W^2_t}\right)(W^N_t - W^T_t)^2 \\
&\quad \times \left(\frac{\partial \tilde{u}_t}{\partial H} + \beta \frac{\partial E_t V^T_{t+1}}{\partial H} + \beta \frac{\partial E_t \Pi_{t+1}}{\partial H}\right)\Big|_{H=H^j_t}(H^{j-1}_{t-1} - H^j_t) \\
&\quad + \frac{1}{2}\left(\frac{\partial^2 \tilde{u}_t}{\partial H^2} + \beta \frac{\partial^2 E_t V^T_{t+1}}{\partial H^2} + \beta \frac{\partial^2 E_t \Pi_{t+1}}{\partial H^2}\right)\Big|_{H=H^j_t}(H^{j-1}_{t-1} - H^j_t)^2 \\
&\quad + (s^N_t - s^T_t)
\end{aligned} \tag{A-23}$$

where the last equality has used the fact that $C_t^N - C_t^T = -(W_t^t - W_t^N) - (\eta_t - TC_t)$. Note that $\eta_t \equiv p_t^{j-1} H_{t-1}^{j-1} - p_t^j H_t^j$ and $TC_t \equiv \delta p_t^{j-1} H_{t-1}^{j-1} + F$.

Conditional on transacting, the first order conditions are:

$$\frac{\partial \tilde{u}_t}{\partial C} - \beta\left(\frac{\partial E_t V_{t+1}^T}{\partial W} + \frac{\partial E_t \Pi_{t+1}}{\partial W}\right) = 0$$

$$\frac{\partial \tilde{u}_t}{\partial H} + \beta \frac{\partial E_t V_{t+1}^T}{\partial H} + \beta \frac{\partial E_t \Pi_{t+1}}{\partial H} = \frac{\partial \tilde{u}_t}{\partial C_t} p_t^j$$

This also leads to

$$\frac{\partial^2 \tilde{u}_t}{\partial C^2} + \beta \frac{\partial^2 E_t V_{t+1}^T}{\partial W_t^2} + \beta \frac{\partial^2 E_t \Pi_{t+1}}{\partial W_t^2} = 0$$

$$\frac{\partial^2 \tilde{u}_t}{\partial H^2} + \beta \frac{\partial^2 E_t V_{t+1}^T}{\partial H^2} + \beta \frac{\partial^2 E_t \Pi_{t+1}}{\partial H^2} = \frac{\partial^2 \tilde{u}_t}{\partial C_t \partial H_t} p_t^j$$

Substituting Eqs. (A-24)–(A-24) into Eq. (A-23), we get

$$D_t\big(W_{t-1}, H_{t-1}^{j-1}\big) = -\frac{1}{2}\frac{\partial^2 \tilde{u}_t}{\partial C_t^2}\tilde{\eta}_t{}^2 - \frac{\partial^2 \tilde{u}_t}{\partial C_t^2}\big(C_t^N - C_t^T - TC_t\big)\tilde{\eta}_t + \lambda_t \tag{A-24}$$

where $\lambda_t = -\frac{1}{2}\frac{\partial^2 \tilde{u}_t}{\partial C_t^2}(TC_t - 2(C_t^N - C_t^T)TC_t) + \frac{\partial \tilde{u}_t}{\partial C_t}((p_t^j - p_t^{j-1})H_{t-1}^{j-1} + TC_t) + \frac{1}{2}\frac{\partial^2 \tilde{u}_t}{\partial C_t \partial H_t} p_t^j (H_{t-1}^{j-1} - H_t^j)^2 + (s_t^N - s_t^T)$.

Define $\Delta_t = (\frac{\partial^2 \tilde{u}_t}{\partial C_t^2}(C_t^N - C_t^T - TC_t))^2 + 2\frac{\partial^2 \tilde{u}_t}{\partial C_t^2}\lambda_t$.

Since $-\frac{1}{2}\frac{\partial^2 \tilde{u}_t}{\partial C_t^2} > 0$. As long as $\Delta_t \geqslant 0$, there exist $\tilde{\eta}_t^A$ and $\tilde{\eta}_t^B$, such that $d_t = 1$ if $\tilde{\eta}_t \leqslant \tilde{\eta}_t^A$ or if $\tilde{\eta}_t \geqslant \tilde{\eta}_t^B$, and $d_t = 0$ if $\tilde{\eta}_t^A < \tilde{\eta}_t < \tilde{\eta}_t^B$

A.5. Numerical method

The Bellman equation for this problem is written as:

$$V_t\big(W_{t-1}, H_{t-1}, p_t^1, p_t^2\big) = \frac{(H_\tau^{\theta_\tau} C_\tau^{1-\theta_\tau})^{1-\gamma}}{1-\gamma} + \beta E_t V_{t+1}\big(W_t, H_t, p_t^2, p_t^1\big) \quad t = 1, \ldots, T$$

The control variables are $\{H_t, C_t\}_{t=1}^T$. The state variables are $\{t, W_{t-1}, H_{t-1}, p_t^1, p_t^2\}$.

The numerical solution follows the standard approach from Judd (1998). In practice, the additional dimensionality of stochastic price processes poses difficult computational problems. For computational ease, I assume two price processes. At each transaction, households sell the old house at one price and buy the new house at another price. Following Tauchen and Hussey (1991), I replace the continuous price processes by a discrete approximation. That is, I form a first order Markov process, giving transition probabilities from p_{t-1} to p_t, which mimics the underlying continuous AR(1) process. Wealth, previous housing stock and optimal housing stock are also discretized, with the range of the grid chosen such that no extrapolation is used. The numerical solution is found explicitly at a finite set of modes in the state space. In solving for the optimal solution, it is necessary to calculate conditional expectation over each stochastic variable. When the stochastic variable is discretized, the transition probabilities are explicitly given.

I then optimize over the different choices using the grid search. This gives the optimal level of housing consumption and non-housing consumption for each age with each possible combination of state variables.

Parameterization:

- $\beta = 0.9$ $\gamma = 3$ $\delta = 0.1$ $F = 1000$ $t = 30 \cdots 60$
- Taste-age profile: $\theta_t = \frac{1}{400}(250 - (t-45)^2)$
- Price process:

$$\begin{pmatrix} p_t^1 \\ p_t^2 \end{pmatrix} = \mu + A\begin{pmatrix} p_{t-1}^1 \\ p_{t-1}^2 \end{pmatrix} + e_t \quad \text{where} \quad e_t \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}\right)$$

Hedging incentive	Small volatility	Large volatility
	$\sigma = 1\%$	$\sigma = 10\%$
weak ($\rho = 0$)	$\mu = \begin{pmatrix} 19.92 \\ 19.96 \end{pmatrix}$	$A = \begin{pmatrix} 0.8977 & 0 \\ 0 & 0.8977 \end{pmatrix}$
strong ($\rho = 0.9$)	$\mu = \begin{pmatrix} 10 \\ 11 \end{pmatrix}$	$A = \begin{pmatrix} 0.5 & 0.4733 \\ 0.4733 & 0.5 \end{pmatrix}$

The choice of parameter values guarantees that (1) the AR(1) process is stationary and (2), for the given value of σ, the unconditional mean of the price processes is the same across the specifications on ρ.

A.6. *Proof of Eq. (19)*

Let $j = (low, high)$, where *low* refers to "low price uncertainty" ($\sigma = 1$) and *high* refers to "high price uncertainty" ($\sigma = 10$). Combine Eqs. (18) and (19)

$$\begin{aligned}
\overline{V}^{low} &= \overline{V}^{high}(1+\Omega)^{1-\gamma} \\
&= E_0\left\{\sum_{\tau=0}^{T} \beta^{\tau} u(H_\tau, C_\tau)(p_\tau)^{\theta_\tau(1-\gamma)} + \beta^{T+1}\frac{W_{T+1}^{1-\gamma}}{1-\gamma}\right\}(1+\Omega)^{1-\gamma} \\
&= E_0\left\{\sum_{\tau=0}^{T} \beta^{\tau}\frac{[[(p_\tau H_\tau)(1+\Omega)]^{\theta_\tau}[C_\tau(1+\Omega)]^{1-\theta_\tau}]^{1-\gamma}}{1-\gamma} + \beta^{T+1}\frac{(W_{T+1}(1+\Omega))^{1-\gamma}}{1-\gamma}\right\}
\end{aligned}$$

Therefore, Ω can be interpreted as the compensation in terms of proportional increase in housing consumption, non-housing consumption and bequest for the remaining life time periods that are necessary to bring the household's expected lifetime utility to the mean utility of households experiencing no price uncertainty.

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