Market Feedback: Who Learns What?*

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Abstract

We analyze information acquisition by a firm and traders in financial markets, when the firm conditions its investment decision on information revealed through stock prices and the firm is exposed to multiple sources of uncertainty. We highlight the delicate strategic interaction between both parties with regards to their incentives to acquire information. For positive NPV projects, there is a fundamental mismatch: traders want to collect the same information as the firm to maximize trading profits, but the firm optimally diversifies its information sources and acquires orthogonal information. For negative NPV projects, the agents' incentives are aligned, but multiple equilibria can lead to discontinuous jumps in firm values and stock prices. We connect the agents' information choices to real efficiency and price efficiency, and highlight an inherent discrepancy between these two efficiency measures.

Keywords: feedback effect; information acquisition; real efficiency; price efficiency.

JEL Classification: D83, G14, G31.

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1 Introduction

It has long been recognized that the informational content of stock prices ("price efficiency") relies crucially on the willingness of traders to acquire costly information (see e.g., Grossman and Stiglitz, 1980; Verrecchia, 1982). Similarly, it has been recognized that the profitability of firms' capital investment decisions ("real efficiency") depends on the quality of their private information and their ability to select optimal projects (see e.g., Lambert, 1986; Harris and Raviv, 1996). A more recent literature argues that there might be a connection between these two efficiency concepts because some of the information in prices might be useful for firms and can help them to invest more efficiently. This "feedback effect" (Bond et al., 2012) has received significant support from the recent empirical literature (see e.g., Luo, 2005; Chen et al., 2007; Foucault and Frésard, 2012).[1]

While existing theoretical work has focused on the information acquisition decision of either traders or firms in separation, our model allows both types to acquire information simultaneously. In particular, we show that the information acquisition decision of one type generates spill-overs for the other type. On the one hand, traders' acquisition of private information renders the equilibrium stock price more informative and allows the firm to extract valuable information about its investment opportunities. On the other hand, the firm's acquisition of private information is reflected in a more efficient investment decision, which affects the firm's future value and thus the traders' payoffs.

We study this joint information acquisition problem in a setting with multiple sources of uncertainty. More specifically, a firm can invest in a risky growth opportunity whose return is determined by two fundamentals (or "shocks"). All agents, traders and the firm, are ex ante uninformed about these shocks and not sure whether it is worthwhile to invest in the project. Both types can acquire private signals about these two shocks but it is too costly for them collect perfect information along both dimensions. Thus, all traders and the firm have to decide how to spend their limited resources most efficiently and what type of uncertainty they would like to reduce through learning. Importantly, each type has a comparative advantage with respect to one

[1] See also Edmans et al. (2012), Foucault and Frésard (2014), Edmans et al. (2017), and Dessaint et al. (2019) for empirical evidence of market feedback.

of the two shocks. Traders can learn more efficiently about one dimension of uncertainty (like e.g. product demand), while the firm can learn more efficiently about the other dimension (like e.g. its production cost).

Our first major insight is that this two-way interaction between the firm and traders might entail a fundamental tension of incentives. On the one hand, the firm can most efficiently learn from the stock price if all traders acquire private information about the shock it is not acquiring private information about itself. This way the firm can rely on price information regarding one shock and on private information for the other shock. The traders' strategic incentive, on the other hand, is more nuanced. In particular, the incentive depends crucially on the project's ex ante net present value (NPV). If the NPV is negative, and the project is less likely to be implemented, traders want to acquire information that differs from the information collected by the firm. If however the NPV is positive, the incentive is reversed and traders want to acquire the same information as the firm.

This intricate relationship between the incentives to acquire information leads to a very nuanced information choice equilibrium. We show that the traders' and the firm's information choice depends on the project's NPV and the firm's signal precisions along both dimensions. More specifically, the firm is able to specialize along one dimension only for moderately negative NPV projects. In this case, traders want to acquire different information such that the resulting price signal allows the firm to learn information about the other dimension. In other cases, however, we find that the firm also has to acquire information about the shock for which traders have a comparative advantage. In particular, for extremely low NPV projects, the firm is forced to acquire information along both dimensions even if the underlying signal becomes almost pure noise.

Our model abstracts from any agency conflicts and assumes that the firm only invests in the project if its expected return is positive. In particular, the firm's expectation comprises two types of signals. First, private signals about the two fundamental shocks that determine the project return. Second, an endogenous feedback signal based on its stock price. Importantly, we assume that the firm cannot acquire a perfect signal about both shocks, such that it has to rely on price information in some circumstances.

The firm's equilibrium stock price is an increasing step function of total order flow, i.e. the sum of informed traders' demand and that of liquidity traders. A competitive market maker observes total order flow and sets the stock price equal to the expected firm value. The market maker can more easily interpret variation in total order flow, if all traders acquire information about the same shock. In this case, particularly high values reveal a "high" shock value and vice versa for low values. If, however, traders acquire information about both shocks, order flow is affected by both fundamentals and it is not clear whether a slightly above-average observation is the result of a "high" fundamental for shock one and a "low" fundamental for shock two, or vice versa. Thus, the composition of traders has a direct effect on the informational content of total order flow and the equilibrium stock price.

Our main research question is whether the different agents acquire information in a way that leads to efficient real investment decisions. To answer this question, we introduce the concept of *real efficiency*, which is defined as the firm's expected long-term value. Perhaps surprisingly, we find that real efficiency could be decreasing in the firm's signal precision. This result arises through the indirect effect of the signal precision on the composition of informed trading. In particular, a higher precision about one shock can encourage traders to acquire more information along this dimension. This, in turn, reduces the informational content of the stock price along the other dimension, which is particularly valuable for the firm. We show that the net effect might be negative. In this case a better-informed firm will have a lower long-term value.

As a second efficiency measure, we consider *price efficiency*, which is defined as the reduction in payoff variance that can be achieved by conditioning on the stock price. This measure captures the degree to which asset prices predict future payoffs and is widely used in the empirical literature to gauge the informational content of stock prices. The conventional view is that price efficiency is a good proxy for real efficiency because a higher price-payoff correlation is expected to indicate that prices reveal more information to real-decision makers. We find that this conventional wisdom is generally incorrect when the underlying information environment is endogenized. For instance, we show that lower NPV projects generally have higher price efficiency but lower real efficiency.

For negative NPV projects, the firm's and the traders' information choice incentives are aligned. This means that both agents want to acquire information that differs from the information acquired by the other party. While this scenario might lead to a very efficient outcome with specialization, it can also lead to multiple equilibria. More specifically, there can be another (stable) information choice equilibrium in which the firm and traders acquire information about the shock for which the other party would be more qualified. In this scenario, we show that independent of the equilibrium that the agents' coordinate on, small shifts in model parameters can lead to discontinuous jumps in stock prices and our two efficiency measures. For instance, a small increase in the firm's precision or the project's NPV can lead to a discrete drop in real efficiency. Quite interestingly, the source of multiple equilibria in our model is the incentive of traders and the firm to acquire *different* information. This channel distinguishes our mechanism from the existing literature (see e.g., Barlevy and Veronesi, 2000; Garcia and Strobl, 2011; Goldstein et al., 2014; Mele and Sangiorgi, 2015) in which learning complementarities among traders lead to multiple equilibria.

The fundamental insight that the type of information in stock prices matters for market feedback is not new. Following Bond et al. (2012), who differentiate between forecasting price efficiency and revelatory price efficiency, several empirical papers have tried to separate the two concepts (see e.g., Bai et al., 2016; Edmans et al., 2017). Our contribution is to provide a formal framework that emphasizes the potential misalignment of incentives behind the information acquisition decisions that give rise to these efficiency measures.

The model makes three important assumptions. First, we consider a firm affected by multiple shocks that govern the return on its investment opportunity. Potential examples of these different dimensions of uncertainty are multinational firms that are exposed to different country-level shocks or conglomerates that are exposed to different industry-level shocks.[2] Second, we allow the firm and traders to acquire private information about the same set of fundamental shocks. Therefore, we do not preclude traders from certain types of shocks and consider all of them learnable. We do, however, assume that traders are relatively more skilled to learn about one dimension of uncertainty, while the firm is more skilled along the other dimension. Our third assumption is

[2] Goldstein and Yang (2015) discuss the importance of studying different dimensions of uncertainty in a model without market feedback.

that neither the firm nor informed traders have sufficient resources to collect perfect information about all shocks. This assumption is important for two reasons. First, it implies that the firm can learn additional information from the stock price. Second, it also creates a trade-off for the two types and renders the information acquisition decision non-trivial. The existing literature has highlighted several frictions that might lead to such a constraint (see e.g., Aghion and Stein, 2008; Mondria, 2010; Kacperczyk et al., 2016, among others).

Our paper contributes to two strands of the literature. First, it is related to the literature studying the real effects of financial markets, where trading and prices affect the firms' investment decisions, which in turn affect the firms' cash flows. This is known as the "feedback effect" and Bond et al. (2012) provide a review of this literature. The first theoretical contributions take the agents' information endowment as given and study market feedback with respect to a single fundamental.[3] Building on these models, Gao and Liang (2013) and Dow et al. (2017) endogenize the traders' signal precision in a single-shock setting, while Goldstein and Yang (2019) consider a setting with fixed information endowments but two sources of uncertainty. In contrast to these papers, we endogenize the information endowment of the firm and traders. As a result, we get several novel predictions relative to the existing work. In particular, our framework leads to an endogenous degree of information overlap, which in turn affects price efficiency and real efficiency. In related work, Benhabib et al. (2019) also study a feedback model with mutual learning by firms and traders. However, in contrast to our work, they assume that both types acquire information about different shocks. As a result, they do not focus on the strategic coordination between the firm and traders and the efficiency implications of this interaction, which lies at the core of our paper.

Second, our paper is also related to the economics and finance literature studying strategic information acquisition with multiple sources of uncertainty. Goldstein and Yang (2015) study a trading model in the spirit of Grossman and Stiglitz (1980) and show that the presence of multiple sources of uncertainty gives rise to strategic complementarities. Van Nieuwerburgh and Veldkamp

[3]See e.g., Dow and Gorton (1997), Subrahmanyam and Titman (2001), Goldstein and Guembel (2008), Goldstein et al. (2013), and Edmans et al. (2015).

(2009), Van Nieuwerburgh and Veldkamp (2010), and Kacperczyk et al. (2016) study trading models with multiple pieces of uncertainty and traders with limited attention capacity. Goldman (2004) and Goldman (2005) study a setting featuring a multi-division firm and endogenous information acquisition by traders. Our contribution relative to this literature is twofold. First, we allow for a real effect of the information collected by traders because of the feedback effect. Second, we also study the simultaneous information acquisition on the real side and its repercussions to the financial market. As a result, non of these papers addresses the potential firm-trader coordination problem and its efficiency implications.

The remainder of the paper is organized as follows. In Section 2 we provide the description of the model. Section 3 characterizes the equilibrium outcomes and Section 4 discusses model implications. Section 5 concludes and all proofs are contained in Appendix A.

2 Model Setup

The model considers three dates, $t \in \{0, 1, 2\}$, and a single firm. The firm's stock is traded in a secondary financial market and a benevolent manager can increase the firm's value through investment in a growth opportunity (or "project"). The financial market is populated by informed traders, uninformed noise traders, and a competitive market maker. At $t = 0$, informed traders and the firm's manager acquire private signals about individual components of the return on the growth opportunity. At $t = 1$, trading in the financial market occurs and subsequently the manager decides whether to invest in the growth opportunity. This decision may be influenced by the realization of the stock price P which creates a *feedback effect* (Bond et al., 2012). At $t = 2$, the firm's terminal value V is realized and paid out as a liquidating dividend. Figure 1 provides a timeline for the key events of the model.

2.1 The Firm's Decision

The firm is operated by a benevolent manager who maximizes the firm's expected long-term value V.[4] The firm has access to a growth opportunity with return, or net present value (NPV),

[4]For simplicity, we simply use "firm" instead of "firm manager" throughout.

t = 0

traders and firm manager acquire private information simultaneously

t = 1

1. traders submit market orders y_i
2. market maker sets stock price P
3. firm manager chooses investment K

t = 2

firm value V realized

Figure 1: Timeline for the main model.

equal to $x^{\theta_M} + x^{\theta_F}$, which is exposed to two dimensions of uncertainty represented by $j \in \{M, F\}$.[5] The firm can decide to invest in this growth opportunity by choosing $K \in \{0, 1\}$. The return on the growth opportunity depends on the two independent random variables θ_M and θ_F, which take on values "high" (H) and "low" (L) with equal probability.[6] We define the realization of each component by $x^H \equiv \frac{1}{2}(\mu + \sigma)$ and $x^L \equiv \frac{1}{2}(\mu - \sigma)$ and make the following assumption regarding the return on the growth opportunity.[7]

Assumption 1 (Return on growth opportunity) *The return on the growth opportunity satisfies the following condition:* $x^L \leq 0 < x^H$ *such that* $\mu \in (-\sigma, \sigma]$, *with* $\sigma > 0$.

Assumption 1 ensures that the firm's investment problem is non-trivial. If $\mu > \sigma$ ($\mu \leq -\sigma$), the return on the growth opportunity would be always positive (negative) such that the firm would always (never) invest and there would be no role for learning about θ_j, which is the focus of this paper. The ex ante NPV of the growth opportunity, $\mathbb{E}\left[x^{\theta_M} + x^{\theta_F}\right] = \mu \in (-\sigma, \sigma]$, can be negative or positive and we will show below that μ plays a crucial role in our model. Intuitively, a higher value of μ indicates that the growth opportunity is more profitable and could be interpreted as "good times" in which the firm has access to better investment opportunities. In the limit $\mu \to -\sigma$, the growth opportunity never generates a positive NPV, ex post, and the firm never invests. Vice versa, if $\mu \to \sigma$, the NPV is always positive, ex post, and it is always profitable to invest. The constant σ determines the variance of the return on the growth opportunity, $\text{Var}\left(x^{\theta_M} + x^{\theta_F}\right) = \frac{1}{2}\sigma^2$. We will

[5]Previous papers in the finance literature with a similar assumption include Kondor (2012), Goldstein and Yang (2015), and Goldstein and Yang (2019).

[6]Note that the more general assumption $\mathbb{P}(\theta_j = H) = \pi_H \in (0, 1)$ is isomorphic to adjusting μ because the firm's investment decision is only affected by the expected NPV.

[7]We draw θ_M and θ_F independently from the same distribution to avoid any baked-in asymmetries between the two dimensions of uncertainty.

oftentimes also refer to the risk-adjusted ex ante expected return, $\kappa \equiv \frac{\mu}{\sigma} \in (-1, 1]$.

The firm's terminal value is given by:

$$V = V_0 + K \left(x^{\theta_M} + x^{\theta_F} \right). \tag{1}$$

The constant V_0 represents the return on the firm's assets in place, $K \in \{0, 1\}$ the firm's investment choice, and $x^{\theta_M} + x^{\theta_F}$ the return on the growth opportunity. To ease the exposition, we set $V_0 = 0$ in the following. Including the assets in place explicitly will not affect our results.

The firm's investment decision is made to maximize the firm's expected value. As a result, the firm invests ($K = 1$) if its conditional expectation of $x^{\theta_M} + x^{\theta_F}$ is positive and does not invest ($K = 0$) otherwise.[8]

Next, we describe the two signal types that influence the firm's investment decision in more detail. The first signal type is an endogenous feedback signal from the financial market based on the stock price P. The informational content of this signal depends on the traders' aggregate information choice, as we will show below.

The second type comprises private signals about the two components that affect the return on the growth opportunity. We assume that the firm manager automatically observes a private signal $s_1 \in \{\theta_F, \emptyset\}$ with precision $\omega \equiv \mathbb{P}(s_1 = \theta_F) \in (0, 1]$. This signal can be interpreted as private information that the manager obtains as a result of running the firm. In addition to this signal, the firm can acquire another signal about either θ_F or θ_M. We denote the firm's information choice by $a \in \{0, 1\}$ with $a = 1$ corresponding to the choice of $F-$information and $a = 0$ to the choice of $M-$information. We write the firm's private signal as $s_2(a) \in \{H, L, \emptyset\}$, which is given as:

$$s_2(a) = \begin{cases} \theta_F & \text{w.p. } a \\ \theta_M & \text{w.p. } (1-a) \times \delta\omega \\ \emptyset & \text{otherwise.} \end{cases} \tag{2}$$

If the firm chooses $a = 1$, it receives a perfect signal about θ_F. If it chooses to learn about θ_M ($a = 0$), it receives a perfect signal about θ_M with probability $\delta\omega$.[9] The constant $\delta \in (0, 1]$

[8] Without loss of generality, we assume that the firm does not invest when indifferent.

[9] See, among others, Foucault and Frésard (2012), Foucault and Frésard (2014), or Dugast and Foucault (2018) for papers with a similar signal structure.

(inversely) captures the firm's comparative advantage with respect to $F-$information. If $\delta \to 0$, the firm cannot obtain an informative signal about θ_M, while $\delta \to 1$ implies that it can learn equally well about both shocks. Note that if the firm chooses $a = 0$, it will also observe a signal about θ_F through s_1.

The firm chooses whether to acquire information about θ_F or θ_M to maximize the expected firm value: $\max_{a\in\{0,1\}} \mathbb{E}[V]$, where V is defined in equation (1). It is important to note that the firm's terminal value depends on the optimal investment decision K. As we will show below, receiving informative signals allows the firm to invest more efficiently, which, in turn, raises the firm's expected value.

2.2 The Financial Market

Trading at $t = 1$ is modeled in the spirit of Kyle (1985). The financial market consists of the following three types of traders who trade claims to the firm's liquidating dividend V at a price P. First, a unit continuum of risk-neutral informed traders, indexed by $i \in [0, 1]$. Each trader can either buy up to one unit, sell up to one unit, or not trade at all, i.e. $y_i \in [-1, 1]$.[10] Because traders do not have price impact and are risk-neutral, they will always trade up to the limits if they decide to trade.[11] In addition to informed traders, noise traders collectively demand $z \sim U[-1, 1]$, which generates non-fundamental variation in total order flow and leads to a noisy price signal. Lastly, a risk-neutral, competitive market maker sets the stock price based on aggregate order flow $Y \equiv \int_0^1 y_i di + z$ to break even in expectation:

$$P = \mathbb{E}[V|Y]. \tag{3}$$

Informed traders face the same information choice problem as the firm. Each individual trader $i \in [0, 1]$ has to decide whether to acquire a private signal $\sigma_i \in \{H, L, \emptyset\}$ about θ_M or θ_F. We denote this decision by $b_i \in \{0, 1\}$ with $b_i = 1$ corresponding to the choice of $M-$information and $b_i = 0$ to

[10] A potential justification for this position limit are borrowing or short-sell constraints. See also Goldstein et al. (2013) and Goldstein and Yang (2019) for feedback models with the same assumption.

[11] Following the existing literature, we assume that traders do not trade if they are indifferent.

the choice of $F-$information.[12] We can thus write the private signal for trader i as:

$$\sigma_i(b_i) = \begin{cases} \theta_M & \text{w.p. } b_i\gamma \\ \theta_F & \text{w.p. } (1-b_i)\gamma \\ \emptyset & \text{otherwise.} \end{cases} \tag{4}$$

We assume that traders can acquire equally-precise signals about θ_M and θ_F.[13] Ex post, however, they specialize in one of the two shocks as in Goldstein and Yang (2015). To ease the exposition and to highlight the impact of δ and ω, we set γ, the precision of the traders' private signal, to unity for the rest of the paper. Hence, traders can receive a perfect signal about either θ_M or θ_F.

The crucial difference between traders and the firm is that traders can learn equally well about θ_M and θ_F, while the firm has a comparative advantage with respect to information about θ_F. It follows that traders have a *relative* advantage with respect to information about θ_M. We can therefore interpret θ_M as *external* information such as overall economic conditions or the firm's product demand. The other dimension of uncertainty (θ_F) can be interpreted as *internal* information such as the firm's production cost, which also affects the asset payoff V. The parameter δ captures the degree of *ex ante* information asymmetry between the firm and traders.

Trader i chooses whether to acquire information about θ_M or θ_F to maximize ex ante trading profits: $\max_{b_i \in \{0,1\}} \mathbb{E}[\Pi_i]$ where $\Pi_i \equiv y_i(V-P)$ and b_i determines the trader's private signal type in equation (4). It is worth noting that the expected trading profits also depend on the firm's real investment and information acquisition decision, which impact V. Thus, the traders' objective is not only affected by their own information choice but also by the firm's choice. We will elaborate more on this interaction below.

2.3 Equilibrium

Our equilibrium concept is Perfect Bayesian Equilibrium ("PBE"). We allow for a (symmetric) mixed-strategy information choice equilibrium such that each trader chooses $b_i = 1$ with proba-

[12] Thus ex ante identical traders might end up with different types of signals ex post, as in Grossman and Stiglitz (1980) or, more recently, Brunnermeier et al. (2020).

[13] In a previous version of the paper, we allowed for different signal precisions across the two shocks. Assuming equal precisions significantly simplifies the analytic expressions and does not change the main results significantly.

bility $\chi \in [0, 1]$. Since traders' private signals are conditionally independent, the mass of traders with a perfect signal about θ_M and θ_F is given by χ and $(1 - \chi)$, respectively. Similarly, the firm chooses $a = 1$ with probability $q \in [0, 1]$. Hence, the firm receives a perfect signal about θ_F with probability q. With probability $1 - q$, it observes an imperfect signal about θ_F (with precision ω) and an imperfect signal about θ_M (with precision $\delta\omega$).

Definition 1 (Perfect Bayesian Equilibrium) *A PBE consists of the following two sub-equilibria.*

1. *Trading and investment equilibrium at $t = 1$:*
   - *informed traders choose their asset demands, given their private signal, to maximize expected trading profits;*
   - *the market maker sets the price conditional on total order flow to break even in expectation;*
   - *the firm chooses capital investment, given its private signals and the stock price, to maximize its expected value.*
2. *Information choice equilibrium at $t = 0$:*
   - *traders acquire information to maximize expected profits anticipating the equilibrium at $t = 1$;*
   - *the firm acquires information to maximize its expected value anticipating the equilibrium at $t = 1$.*

We assume that all agents have rational expectations in that each player's belief about the other players' strategies is correct in equilibrium.

3 Model Solution

In this section, we characterize the different equilibria in the main model. In Section 3.1, we first discuss the trading and investment equilibrium at $t = 1$. Subsequently, in Section 3.2, we analyze the information choice equilibrium at $t = 0$.

3.1 Trading and Investment Equilibrium

As a first step, we take the firm's and the traders' information choices as given. Therefore, each individual trader receives a perfect signal about θ_M with probability χ and a perfect signal about

θ_F with probability $1-\chi$. Similarly, the firm receives a perfect signal about θ_F with probability q and an imperfect signal about θ_F and θ_M with probability $1-q$. Based on these signals, each trader has to decide whether to trade and, if yes, whether to buy or sell the asset. Furthermore, the market maker sets the equilibrium stock price based on the observed order flow and the firm decides whether to invest in the growth opportunity, given its private signals and the equilibrium stock price. We formally derive the trading and investment equilibrium at $t=1$ next.

Proposition 1 (Trading and investment equilibrium) *For given mixing probabilities $(q,\chi)\in[0,1]^2$, there is a trading and investment equilibrium in which:*

1. *Each trader buys on a "high" signal ($\sigma_i = H$) and sells on a "low" signal ($\sigma_i = L$);*
2. *The firm's stock price is an increasing step function of total order flow, $P = p(Y) \geq 0$;*
3. *The firm's investment decision satisfies:*
   (a) *If $\kappa \leq 0$, the firm invests if $s_1 \in \{\emptyset, H\}$, $s_2 \in \{\emptyset, H\}$, and $P \in \mathcal{P}$;*
   (b) *If $\kappa > 0$, the firm invests if $s_1 = H$ or if $s_2 = H$ or if $s_1 \in \{\emptyset, L\}$, $s_2 \in \{\emptyset, L\}$, and $P \in \mathcal{P}$.*

We provide the explicit expressions for $p(Y)$ and $\mathcal{P}$ in the Appendix.

Proof: *See Appendix A.1.1.*

Proposition 1 shows that each trader optimally buys (sells) on positive (negative) private information about the firm's fundamentals in anticipation of a higher (lower) payoff V. It follows that total order flow Y depends on the informed traders' private signals and therefore the two fundamentals θ_M and θ_F. We show in Appendix A.1.1 that there are six distinct intervals for Y which reveal different signals about the fundamentals to the market maker. For instance, a particularly high order flow indicates that θ_M and θ_F are "high" because it must be the result of buy orders from $M-$informed and $F-$informed traders. In this case, the market maker sets the equilibrium stock price equal to $\mathbb{E}\left[K|\theta_M = H, \theta_F = H\right](\mu+\sigma)$. As a result, P inherits the informational content of Y and the firm manager can learn additional information about the two fundamentals from this feedback signal.

The firm's investment decision depends on the private signal about θ_F and/or θ_M and the price signal, which can reveal additional information about θ_M or θ_F (or both). If the project's ex ante NPV is negative ($\kappa \leq 0$), the firm will never invest after a "low" private signal because the ex post project NPV is negative even if the price signal reveals that the other fundamental is "high." However, if the private signal is "high" or if the firm did not receive an informative signal, the firm invests based on a sufficiently high price realization.

If the project's ex ante NPV is positive ($\kappa > 0$), the firm will always invest based on a positive private signal ($s = H$), independent of P, because the ex post project NPV is positive even if the price reveals that the other shock is "low." If the private signal is "low" ($s = L$) or if the firm did not receive a private signal ($s = \emptyset$), it will invest based on a sufficiently high price realization. The explicit range of price realizations that trigger investment depends on the firm's signal types and realizations (a and $s = (s_1, s_2)$), the informational content of the price (χ), and the project's ex ante risk-adjusted NPV ($\kappa = \frac{\mu}{\sigma}$). We fully characterize $\mathcal{P}$ in Appendix A.1.1.

3.2 Information Choice Equilibrium

Next, we analyze the optimal information acquisition decisions at the initial date $t = 0$. To this end, we compute the firm's expected value and the traders' expected trading profits to find the optimal mixing probabilities q^* and χ^* from the intersection of the best-response functions $q(\chi)$ and $\chi(q)$. For the firm, we use the optimal investment policy in Proposition 1 together with the definition of V in equation (1) to compute the expected firm value, $\mathbb{E}[V]$. The firm then chooses $q \in [0, 1]$ to maximize $\mathbb{E}[V]$ which leads to a best-response function for a given mixing probability $\chi \in [0, 1]$ for each individual trader.

Lemma 1 (Best-Response Function: Firm) *Given the trading and investment equilibrium in Proposition 1, the firm's best-response function to a given $\chi \in [0, 1]$ is given by:*

$$q(\chi)\begin{cases} = 0 & \text{if } \chi < \overline{\chi}(\kappa, \delta, \omega) \\ \in [0, 1] & \text{if } \chi = \overline{\chi}(\kappa, \delta, \omega) \\ = 1 & \text{if } \chi > \overline{\chi}(\kappa, \delta, \omega); \end{cases}$$

with $\kappa = \frac{\mu}{\sigma} \in (-1, 1]$*. We provide the explicit expression for* $\overline{\chi}(\kappa, \delta, \omega)$ *in the Appendix.*

Proof: *See Appendix A.1.2.*

Lemma 1 formalizes the firm's best-response function. If traders are sufficiently likely to learn $M-$information, i.e. if χ is sufficiently large, the firm always specializes in $F-$information and chooses $q(\chi) = 1$. This result is very intuitive: a higher value of χ renders the stock price more informative about θ_M and the firm wants to avoid overlap in the information conveyed by the price signal and its private signal. Thus, the firm is better off acquiring a private signal about θ_F if the price signal is already more informative about θ_M. If, however, traders are more likely to learn θ_F, the firm might acquire imperfect signals about θ_F *and* θ_M. Overall, Lemma 1 shows that the firm's best-response q is an increasing step-function of χ. Hence, if traders are more likely to learn about θ_M, the firm becomes (weakly) more likely to learn only about θ_F: $q'(\chi) \geq 0$.

Next, we solve for the traders' best-response to a given mixing probability q. To this end, we compute the expected trading profits for an individual trader with a private signal about θ_M, $\mathbb{E}[\Pi_i | b_i = 1]$, and for an individual trader with a private signal about θ_F, $\mathbb{E}[\Pi_i | b_i = 0]$, using $\Pi_i = y_i(V - P)$ and the results in Proposition 1. We show below that the traders' best response to any $q \in [0, 1]$ is a $\chi(q)$ strictly between the two boundaries 0 and 1. Therefore, we can find the trader's best-response from the indifference condition, $\mathbb{E}[\Pi_i | b_i = 1; \chi = \chi(q)] = \mathbb{E}[\Pi_i | b_i = 0; \chi = \chi(q)]$.

Lemma 2 (Best-Response Function: Trader) *Given the trading and investment equilibrium in Proposition 1, the trader's best-response function to a given* $q \in [0, 1]$ *is given by:*

1. *If* $\kappa \in (-1, 0]$*:* $\chi(q) \in (0, 1)$ *and* $\chi'(q) \geq 0$*;*

2. *If* $\kappa \in (0, 1]$*:* $\chi(q) \in (0, 1)$ *and* $\chi'(q) \leq 0$*.*

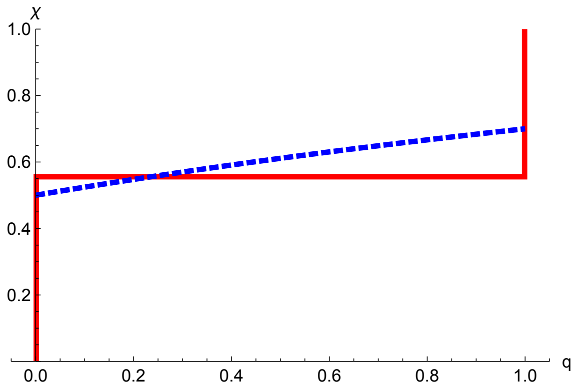
The exact expressions for $\chi(q)$ *are given in the Appendix.*

Proof: *See Appendix A.1.3.*

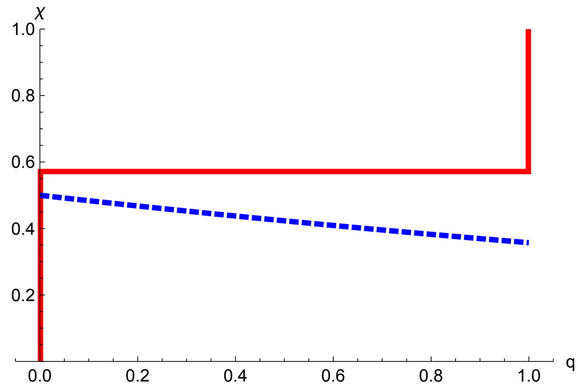
Lemma 2 characterizes the traders' best-response function to the firm's information choice $q \in [0, 1]$. In general, traders know that choosing a low (high) mixing probability χ when the firm chooses a high (low) mixing probability q, makes it more likely that they buy (sell) when the firm has received a "high" ("low") *private* signal about one of the two fundamentals. In other words, the information overlap (with regards to θ_F or θ_M) between the firm and traders is maximized at either $\{\chi = 0, q = 1\}$ or $\{\chi = 1, q = 0\}$. In the former case, the firm and all traders learn θ_F perfectly. In the latter case, traders learn θ_M perfectly, while the firm is most likely to learn θ_M (with probability $\delta\omega$). While it might appear that traders should always try to align their information choices with that of the firm, the results in Lemma 2 show that this intuition does not always hold in our setting.

If the project's ex ante NPV is negative ($\kappa \leq 0$), χ is increasing in q. As a result, traders are more (less) likely to learn about θ_M, if the firm is more (less) likely to specialize in θ_F. The intuition for the traders' incentive to acquire *different* information from the firm is as follows. Suppose a trader has received a "low" signal ($\sigma_i = L$) and (optimally) sold the firm's asset as shown in Proposition 1. If the firm learns about the same shock, it will optimally choose not to invest and the asset's payoff would be zero. If, however, the firm learns about the other shock, it might receive a "high" signal and set $K = 1$ if $P \in \mathcal{P}$. In this case, the asset's payoff is equal to $1 \times \mu \leq 0$, which is preferable for the trader who sold the asset. Alternatively, suppose a trader has received a "high" signal ($\sigma_i = H$) and bought the asset. The trader wants the firm to invest, but only if both components are "high" because the payoff would again be equal to μ, and thus negative, otherwise. It follows that it is again preferable for the trader if the firm acquires information about the other shock because the firm will never invest after a "low" private signal.

Perhaps surprisingly, the traders' incentives to acquire information are reversed for positive NPV projects ($\kappa > 0$). In this case, traders are more (less) likely to learn about $M-$information, if the firm becomes less (more) likely to specialize in $F-$information. The reason traders want to align their information choice with the firm is as follows. As before, suppose a trader has received a low signal $\sigma_i = L$ and sold the asset anticipating a low payoff. Now, the trader is worse off if the firm acquires information about the other shock because the firm might receive a "high" signal about the other shock and invest. The resulting payoff would be equal to $1 \times \mu > 0$ and thus greater than the payoff if the firm did not invest. Similarly, if the trader receives a "high" signal and buys, he wants the firm to also receive a high signal to make sure the firm will invest such that the asset



(a) $\kappa = -\frac{2}{5}$



(b) $\kappa = \frac{2}{5}$

Figure 2: Best-response functions for the firm (solid, red) and an individual trader (dashed, blue). Parameters: $\omega = \frac{1}{2}$ and $\delta = 1$.

payoff is positive.

Figure 2 plots the firm's and the traders' best-response functions for a set of parameter values. The solid, red line corresponds to the firm and the dashed, blue line to an individual trader. In both panels, the firm's best-response is an increasing step-function of χ: up to a threshold value $\overline{\chi}$, the firm sets $q = 0$, at the threshold value the firm is indifferent, and for $\chi > \overline{\chi}$, the firm sets $q = 1$. The traders' best-response, however, differs fundamentally across the two panels. In Panel (a), for $\kappa \leq 0$, the traders' best-response is increasing in q. As a result, there might be two stable equilibria in this case: $q^* = 0$ and $q^* = 1$.[14] Panel (b) corresponds to $\kappa > 0$ and shows that the the traders' best-response is decreasing in q in this range. Hence, there will be a unique information choice equilibrium for positive NPV projects. In the particular example in Figure 2 the unique equilibrium is $q^* = 0$.

Next, we will use these best-response functions to solve for the agents' equilibrium information choices q^* and χ^*.

Proposition 2 (Information Choice Equilibrium) *The firm's and the traders' equilibrium information choices are given by:*

1. *If $\kappa \in \left(-1, -\frac{1}{2}\right]$, there is a unique equilibrium with $\chi^* = \frac{1}{2}$ and $q^* = 0$;*

2. *If $\kappa \in \left(-\frac{1}{2}, 0\right]$, there is either a unique equilibrium or two equilibria with $\chi^* \in \left[\frac{1}{2}, 1\right)$ and $q^* \in \{0, 1\}$;*

[14] Note that the interior equilibrium is not stable: any small perturbation around $q^*(0, 1)$ would imply a convergence to one of the two corner equilibria. In the following, we will focus on stable equilibria. Section 4.2 discusses equilibrium multiplicity in more detail.

3. *If* $\kappa \in \left(0, \frac{1}{2}\right]$*, there is a unique equilibrium with* $\chi^* \in \left(0, \frac{1}{2}\right)$ *and* $q^* \in [0, 1]$*;*

4. *If* $\kappa \in \left(\frac{1}{2}, 1\right]$*, there is a unique equilibrium with* $\chi^* = \frac{1}{2}$ *and* $q^* = 1$*;*

We provide the explicit expressions for χ^* *and* q^* *in the Appendix.*

Proof: *See Appendix A.1.4.*

Proposition 2 characterizes the firm's and the traders' optimal information choices, q^* and χ^*. We can see that there are four distinct regions for the project's ex ante NPV κ. First, if κ is very small, the firm is forced to privately learn about θ_F and θ_M, i.e. to choose $q^* = 0$. If the firm specialized in $F-$information, traders would not have an incentive to trade because there would be no information rents. Intuitively, $q^* = 1$ would imply that the firm does not receive a private signal about θ_M. As a result, it will only invest in the project after observing a price signal that reveals $\theta_M = H$. However, in this case $M-$informed traders make zero profits because either $K = 0$, which implies that $V = 0$, or $K = 1$, but their private information is perfectly priced-in by the market maker. In equilibrium, traders split evenly between the two shocks: half of them become $M-$informed and the other half $F-$informed. This result is quite striking because it implies that the firm is willing to sacrifice a perfect signal about θ_F for noisy signals about both shocks *independent* of their signal precision, i.e. for all $(\delta, \omega) \in (0, 1]^2$.

Second, if the project's NPV is very large, we obtain the exact opposite outcome. In this case, the firm can afford to specialize in $F-$information because traders are incentivized to produce $M-$information in any case. Intuitively, the high project NPV implies that the firm will invest even without learning $M-$information from the price signal. Consequently, $M-$informed traders make positive trading profits and the firm can learn about θ_M from the stock price. In equilibrium traders again acquire information about both shocks evenly because they obtain an equally-precise signal along both dimensions.

If the NPV is moderately negative, the best-response functions for the firm and for traders are increasing, which can lead to multiple equilibria. In particular, the firm will always end up at a corner solution with $q^* = 0$ or $q^* = 1$, while the majority of traders becomes informed about $M-$information, i.e. $\chi^* > \frac{1}{2}$. As shown in Lemma 2, traders want to acquire different information from the firm in this case, which is thus compatible with the firm's preference for different information (Lemma 1). At both equilibria $q^* = 0$ and $q^* = 1$, the firm acquires more precise information about θ_F, because $\delta \leq 1$. Hence, it is more profitable for traders to tilt their information acquisition towards θ_M.

Finally, if the NPV is moderately positive, the two best-response functions have a unique intersection. In this case, there is a conflict between the strategic incentives of the firm and traders. Traders want to acquire similar information, while the firm wants to acquire different information. This mismatch leads to a unique equilibrium and, more specifically, to the possibility of an interior equilibrium for q^*. For all q, traders tilt their information choice towards θ_F such that $\chi^* < \frac{1}{2}$.

Corollary 1 (The impact of δ) *The equilibrium information choices satisfy:*

1. *If $q^* = 1$, χ^* does not depend on δ;*

2. *If $q^* = 0$, χ^* is decreasing in δ if $\kappa \leq 0$ and increasing in δ if $\kappa > 0$;*

3. *If $q^* \in (0, 1)$, χ^* is increasing in δ and q^* is decreasing in δ.*

Proof: *See Appendix A.1.5.*

Corollary 1 characterizes the dependence of χ^* and q^* on δ, which captures the firm's ability to learn $M-$information. An increase in δ indicates that the firm can obtain a more precise signal about θ_M. In the pure-strategy equilibrium with $q^* = 1$, the firm learns θ_F perfectly such that χ^* does not depend on δ. In the other pure-strategy equilibrium with $q^* = 0$, χ^* can either increase or decrease with δ. More specifically, it decreases with δ if the project's ex ante NPV is negative and it increases with it otherwise. Intuitively, an increase in δ increases the firm's overall signal precision with regards to $M-$information. As shown before, traders want to acquire different information for $\kappa \leq 0$ such that this increase is translated into a smaller share of $M-$informed traders. Vice versa, for $\kappa > 0$ traders want to acquire similar information such that an increase in δ is accompanied by an increase in χ^*. The same intuition applies to the mixed-strategy equilibrium, which only occurs for positive NPV projects.

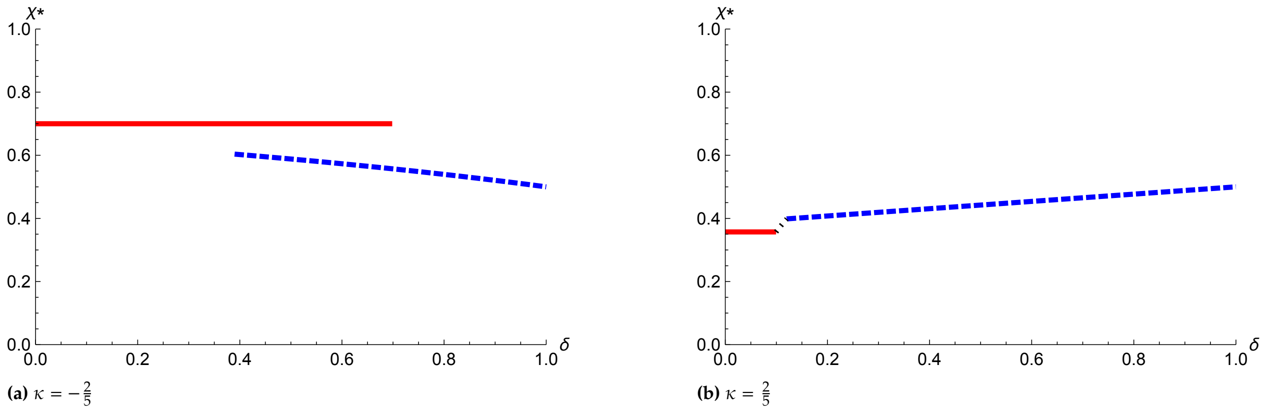


Figure 3: Equilibrium mixing probability for traders as a function of δ. Parameters: $\omega = \frac{3}{4}$. Solid red line: $q^* = 1$; Dashed blue line: $q^* = 0$; Dotted black line: $q^* \in (0, 1)$.

Figure 3 plots χ^*, the equilibrium share of $M-$informed traders, against δ. Panel (a) corresponds to a negative ex ante NPV project with $\kappa = -\frac{2}{5}$ and Panel (b) to a positive ex ante NPV project with $\kappa = \frac{2}{5}$. In both cases, the solid red line represents equilibria with $q^* = 1$, the dashed blue line those with $q^* = 0$, and the dotted black line interior equilibria with $q^* \in (0, 1)$. Panel (a) confirms the weakly negative relationship between χ^* and δ for negative NPV projects. It also emphasizes the possibility of multiple equilibria in this scenario. In Panel (b), there is a unique equilibrium and χ^* is weakly increasing in δ. For small values of δ, the unique equilibrium is $q^* = 1$. Then, any increase in δ leads to a decrease in q^*. This figure also emphasizes the result in Proposition 2 that a majority (minority) of traders acquires $M-$information if the ex ante NPV is negative (positive).

Corollary 2 (The impact of ω) *The equilibrium information choices satisfy:*

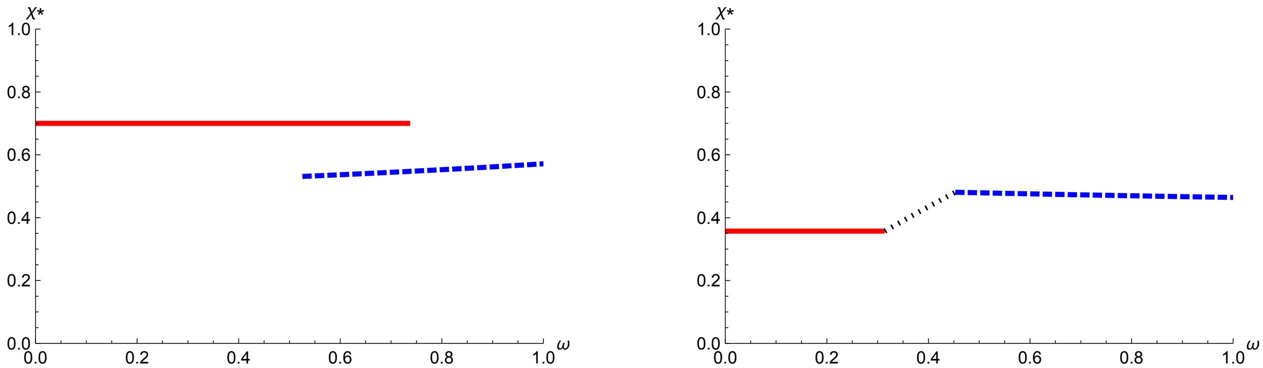
1. *If $q^* = 1$, χ^* does not depend on ω;*

2. *If $q^* = 0$, χ^* is increasing in ω if $\kappa \leq 0$ and decreasing in ω if $\kappa > 0$;*

3. *If $q^* \in (0, 1)$, χ^* is increasing in ω and q^* is decreasing in ω.*

Proof: *See Appendix A.1.6.*

In Corollary 2, we investigate the impact of ω, the precision of the firm's exogenous signal about θ_F. As before, χ^* does not depend on ω in the pure-strategy equilibrium with $q^* = 1$ because the firm acquires a perfect signal about θ_F. If $q^* = 0$, an increase in ω leads to more $M-$informed traders



(a) $\kappa = -\frac{2}{5}$

(b) $\kappa = \frac{2}{5}$

Figure 4: Equilibrium mixing probability for traders as a function of ω. Parameters: $\delta = \frac{3}{4}$. Solid red line: $q^* = 1$; Dashed blue line: $q^* = 0$; Dotted black line: $q^* \in (0, 1)$.

for negative NPV projects because traders have an incentive to acquire different information. It leads to less $M-$informed traders for positive NPV projects, because traders want to mimic the firm's information choice. In this specific case, an increase in ω lowers the informational content of the stock price with regards to $M-$information, which is particularly valuable for the firm because its private signal along this dimension is less precise. We will show below that this indirect effect mitigates, and in some cases even overturns, the positive impact of ω on the firm's expected value. Figure 4 evaluates the relationship between ω and χ^* numerically.

4 Model Implications

4.1 Efficiency Implications

Next, we analyze the efficiency implications of the agents' information acquisition decisions. We focus on two widely used efficiency measures, *real efficiency* and *price efficiency*. Both measures are formally defined next.

Definition 2 (Efficiency) *We define the following two measures of efficiency.*

1. *Real efficiency is defined as the ex ante expected firm value:*

$$RE \equiv \mathbb{E}[V].$$

2. *Price efficiency is defined as the relative reduction in payoff uncertainty that can be achieved by*

conditioning on the price:

$$PE \equiv 1 - \frac{\mathbb{E}[\text{Var}(V|P)]}{\text{Var}(V)}.$$

First, we define real efficiency (RE) as the ex ante expectation of the firm's realized long-term value V as in Goldstein et al. (2013) and Goldstein and Yang (2019). This measure captures the extent to which the firm *correctly* invests in the growth opportunity, i.e. it chooses $K = 1$ when the ex-post NPV, $x^{\theta_M} + x^{\theta_F}$, is positive.

Second, our definition of price efficiency (PE) captures the informational content of the price. It is equal to the proportion of the unconditional payoff variance that can be reduced by conditioning on the equilibrium stock price P.[15] Note that in our setting the conditional variance is a random variable that depends on the specific price realization. For example, if the price takes on its highest possible value $\text{Var}(V|P) = 0$ because the price reveals $\theta_M = \theta_F = H$ perfectly. This measure of price efficiency has been often used in the existing literature as a proxy for the information content of the stock price (see e.g., Peress, 2010; Edmans et al., 2016). Price efficiency is maximized at $PE = 1$, if the price always reveals the future payoff perfectly, and minimized at $PE = 0$, if observing the price does not add any information. Next, we will derive these two efficiency measures based on the equilibria in Proposition 1 and Proposition 2.

To compute real efficiency as a function of model parameters, we use the definition of the firm's terminal value $V = K\left(x^{\theta_M} + x^{\theta_F}\right)$, plug in the optimal investment policy derived in Proposition 1 together with the optimal information choices derived in Proposition 2, and take an unconditional expectation over the independent random variables (θ_M, θ_F, z).

Proposition 3 (Equilibrium RE) *Equilibrium real efficiency has the following properties:*

1. *If $q^* = 1$, real efficiency is increasing in κ; it does not depend on δ and ω;*

2. *If $q^* = 0$, real efficiency is increasing in κ and δ; it is either increasing or decreasing in ω;*

3. *If $q^* \in (0, 1)$, real efficiency is increasing in κ, δ, and ω.*

[15] Banerjee et al. (2018) and Frenkel et al. (2020) use a similar measure and define price efficiency as the *absolute* reduction in payoff variance. Our main results are robust to this alternative specification. In recent empirical work, Dávila and Parlatore (2018) and Dávila and Parlatore (2019) use price volatility and regression R-squareds to identify price efficiency.

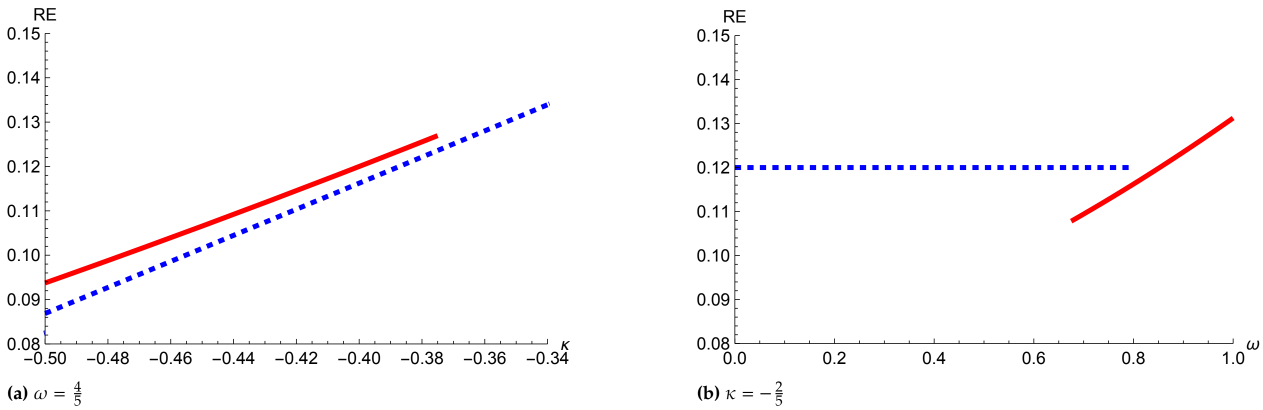


Figure 5: Real Efficiency as a function of κ and ω. Parameters: $\sigma = 1$ and $\delta = \frac{1}{2}$. Solid red line: $q^* = 1$; Dashed blue line: $q^* = 0$.

We provide the explicit expressions for RE in the Appendix.

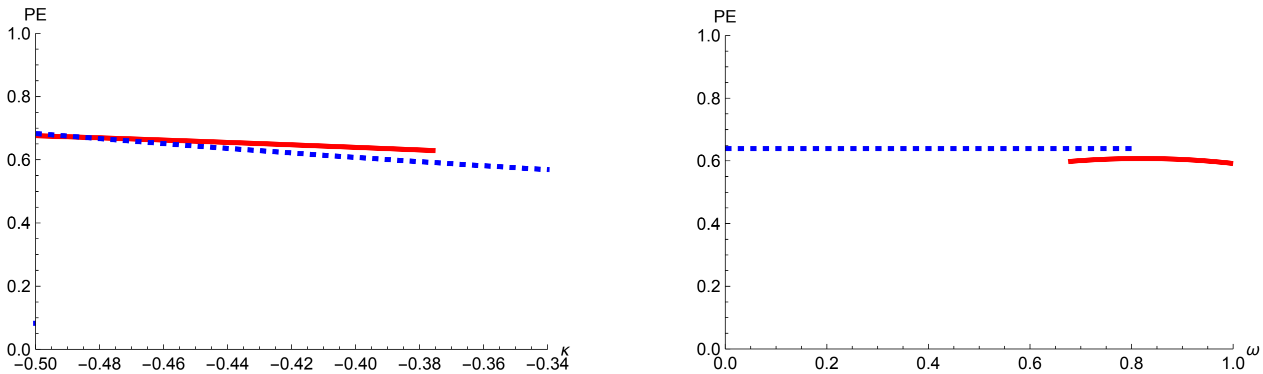
Proof: *See Appendix A.1.7.*

Proposition 3 describes the dependency of real efficiency on the model parameters κ, δ, and ω in the three different types of equilibria. For all three types, real efficiency is increasing in κ and δ, while it might be decreasing in ω. Intuitively, an increase in $\kappa = \frac{\mu}{\sigma}$ increases the ex ante NPV of the project and, as expected, translates into a higher ex ante firm value. It is worth noting, that this positive direct effect could be mitigated by the indirect effect through χ^*. For instance, in equilibria with $q^* = 0$, χ^* is always decreasing in κ, while $\mathbb{E}[V]$ is generally increasing in χ. Similarly, δ and ω have both a (positive) direct effect and a potentially negative indirect effect on $\mathbb{E}[V]$. On the one hand, an increase in both parameters increases the precision of the firm's private signals. On the other hand, it can lead to a reduction in χ^*, which in turn might lower real efficiency. For δ the positive direct effect always dominates, while for ω the negative indirect effect might dominate. In the latter case, real efficiency is decreasing in ω. Figure 5 plots real efficiency against κ and ω for a fixed set of model parameters.

Proposition 4 (Equilibrium PE) *Consider $\delta \to 1$, equilibrium price efficiency has the following properties:*

1. *If $q^* = 1$, price efficiency is decreasing in κ; it does not depend on ω;*

2. *If $q^* = 0$, price efficiency is decreasing in κ; it can be increasing or decreasing in ω;*



(a) $\omega = \frac{4}{5}$ **(b)** $\kappa = -\frac{2}{5}$

Figure 6: Price Efficiency as a function of κ and ω. Parameters: $\sigma = 1$ and $\delta = \frac{1}{2}$. Solid red line: $q^* = 1$; Dashed blue line: $q^* = 0$.

3. *If* $q^* \in (0, 1)$*, price efficiency can be increasing or decreasing in* κ *and* ω*.*

We provide the explicit expressions for PE *in the Appendix.*

Proof: *See Appendix A.1.8.*

In Proposition 4, we analyze the ramifications for price efficiency.[16] First, we show that price efficiency is decreasing in κ in the pure-strategy equilibria with either $q^* = 0$ or $q^* = 1$, which creates a stark mismatch between price efficiency and real efficiency because the latter was increasing in κ (see Proposition 3). The same is true, to some extent, for ω. In the equilibrium with $q^* \in (0, 1)$, ω has an ambiguous impact on price efficiency, while it had a positive impact on real efficiency. Figure 6 plots price efficiency against κ and ω. The figure confirms the negative relationship between PE and κ, and the ambiguous relationship between PE and ω.

4.2 Multiplicity and Jumps

An interesting implication of our setting is the possibility of multiple information choice equilibria for negative NPV projects (see Proposition 2). In this case, the firm and traders want to acquire information that differs from that acquired by the other party. Next, we analyze the two stable equilibria in this scenario more thoroughly.

Proposition 5 (Equilibrium Multiplicity) *If* $(\kappa, \delta, \omega) \in \mathcal{R}_{mult}$*, where* $\mathcal{R}_{mult}$ *is formally defined in the*

[16] To obtain analytical results, we consider the limit $\delta \to 1$. However, this assumption is not crucial as shown in the numerical example in Figure 6.

Appendix, there will be two stable information choice equilibria $(q_1^* = 0, \chi_1^*)$ *and* $(q_2^* = 1, \chi_2^*)$ *with* $\frac{1}{2} < \chi_1^* < \chi_2^* < 1$. *In this case, we obtain:*

1. *Expected trading profits are higher if* $(q_2^* = 1, \chi_2^*)$*;*
2. *Real efficiency can be higher in either stable equilibrium;*
3. *Price efficiency can be higher in either stable equilibrium.*

Proof: *See Appendix A.1.9.*

Proposition 5 formally characterizes the parameter space that leads to two stable equilibria. We know from the results in Proposition 2 that this scenario only arises for negative NPV projects with $\kappa \leq 0$ because the best-response functions for traders and the firm are jointly increasing. Moreover, the firm either chooses $q^* = 0$ or $q^* = 1$ and traders focus primarily on $M-$information, i.e. $\chi^* > \frac{1}{2}$.

Traders' expected profits are always higher if $q^* = 1$, i.e. if the firm fully specializes in $F-$information and acquires a perfect signal about θ_F. However, real efficiency and price efficiency could be higher in the equilibrium with $q^* = 0$. The two latter results can also be seen in Figure 5 and Figure 6. In both figures, the solid red line corresponds to $q^* = 1$ and the dashed blue line to $q^* = 0$. For instance in Panel (a) of Figure 5, real efficiency is higher for $q^* = 1$, while it is higher for $q^* = 0$ in Panel (b). Figure 6 shows a similar result for price efficiency.

These two figures also emphasize the possibility for jumps in price efficiency and real efficiency. Consider for instance Panel (b) in Figure 5. In this example, we obtain two stable equilibria for values of ω between ≈ 0.67 and ≈ 0.8. Independent of the equilibrium that traders and the firm coordinate on, a marginal shift in the model parameter ω will eventually lead to a discontinuous drop in real efficiency. A similar result arises for an increase in the project's NPV κ and Figure 6 illustrates the same effect for price efficiency. Quite interestingly, the source of multiple equilibria in our model is the incentive of traders and the firm to acquire *different* information. This channel distinguishes our mechanism from the existing literature (see e.g., Barlevy and Veronesi, 2000; Garcia and Strobl, 2011; Goldstein et al., 2014; Mele and Sangiorgi, 2015) in which learning complementarities among traders lead to multiple equilibria.

5 Conclusion

We consider a model in which a real-decision maker (the firm) and traders in a financial market are allowed to collect information simultaneously. The resulting information choice equilibrium highlights a potential coordination problem. For positive NPV projects, the firm wants traders to acquire information that differs from its own choice, while traders want to collect the same information. For negative NPV projects, the incentives are aligned, which can lead to multiple equilibria. We also show that the simultaneous choice of information by both parties can lead to a mismatch between price efficiency and real efficiency.

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A Appendix

A.1 Proofs

A.1.1 Proof of Proposition 1

We proceed as follows. We first take the traders' and the firm's information choice as given. It follows that a mass χ of traders receives a perfect signal about θ_M and a mass $(1-\chi)$ receives a perfect signal about θ_F. The firm observes a perfect signal about θ_F with probability $q+(1-q)\omega$ or a perfect signal about θ_M with probability $(1-q)\delta\omega$, and no signal otherwise. Moreover, we conjecture that each trader's optimal trading policy is to buy after a "high" signal and to sell after a "low" signal.

Given that noise traders trade a random amount $z \sim U[-1,1]$, we get the following four possibilities for total order flow Y:

1. if $\theta_M = H$ and $\theta_F = H$: $Y = X_{HH} + z$ with $X_{HH} \equiv 1$;
2. if $\theta_M = H$ and $\theta_F = L$: $Y = X_{HL} + z$ with $X_{HL} \equiv 2\chi - 1$;
3. if $\theta_M = L$ and $\theta_F = H$: $Y = X_{LH} + z$ with $X_{LH} \equiv 1 - 2\chi$;
4. if $\theta_M = L$ and $\theta_F = L$: $Y = X_{LL} + z$ with $X_{LL} \equiv -1$.

The market maker observes total order flow Y and sets the price equal to the expected firm value $\mathbb{E}[V|Y]$. Next, we distinguish between $\chi \geq \frac{1}{2}$ and $\chi < \frac{1}{2}$. In the first case, Y is more informative about θ_M, in the the second case it is more informative about θ_F. It follows from $z \sim U[-1,1]$ that the market maker learns the following information about θ_M and θ_F from Y:

1. If $\chi \geq \frac{1}{2}$:
   (a) $Y > X_{HL} + 1$: $(\theta_M = H, \theta_F = H)$;
   (b) $X_{HL} + 1 > Y > X_{LH} + 1$: $(\theta_M = H, \theta_F = H)$ or $(\theta_M = H, \theta_F = L)$;
   (c) $X_{LH} + 1 > Y > 0$: $(\theta_M = H, \theta_F = H)$, $(\theta_M = H, \theta_F = L)$, or $(\theta_M = L, \theta_F = H)$;
   (d) $0 > Y > X_{HL} - 1$: $(\theta_M = H, \theta_F = L)$, $(\theta_M = L, \theta_F = H)$, or $(\theta_M = L, \theta_F = L)$;

(e) $X_{HL} - 1 > Y > X_{LH} - 1$: $(\theta_M = L, \theta_F = L)$ or $(\theta_M = L, \theta_F = H)$;

(f) $X_{LH} - 1 > Y$: $(\theta_M = L, \theta_F = L)$.

2. If $\chi < \frac{1}{2}$:

   (a) $Y > X_{LH} + 1$: $(\theta_M = H, \theta_F = H)$;

   (b) $X_{LH} + 1 > Y > X_{HL} + 1$: $(\theta_M = H, \theta_F = H)$ or $(\theta_M = L, \theta_F = H)$;

   (c) $X_{HL} + 1 > Y > 0$: $(\theta_M = H, \theta_F = H)$, $(\theta_M = H, \theta_F = L)$, or $(\theta_M = L, \theta_F = H)$;

   (d) $0 > Y > X_{LH} - 1$: $(\theta_M = H, \theta_F = L)$, $(\theta_M = L, \theta_F = H)$, or $(\theta_M = L, \theta_F = L)$;

   (e) $X_{LH} - 1 > Y > X_{HL} - 1$: $(\theta_M = L, \theta_F = L)$ or $(\theta_M = H, \theta_F = L)$;

   (f) $X_{HL} - 1 > Y$: $(\theta_M = L, \theta_F = L)$.

Next, we derive the firm's optimal investment decision and the equilibrium stock price. Throughout, we will first derive the equilibrium stock price based on the conjectured investment policy. In a second step, we will verify the conjecture.

1. If $\kappa \in \left(-1, -\frac{1}{2}\right]$:

   (a) $\chi \geq \frac{1}{2}$: if $a = 1$, the firm invests if $s_2 = H$ and $p \in \{p_1, p_2\}$; if $a = 0$, the firm invests if $s_1 = s_2 = H$ or if $p = p_1$ or if $s_1 = H$ and $p = p_2$. The equilibrium stock price is given by $P = p(Y)$ with:

$$p(Y) = \begin{cases} p_1 = \mu + \sigma & \text{if } Y > X_{HL} + 1 \\ p_2 = \frac{1}{2}\left(q + (1-q)\omega\right)(\mu + \sigma) & \text{if } X_{HL} + 1 > Y > X_{LH} + 1 \\ p_3 = \frac{1}{3}(1-q)\delta\omega^2(\mu + \sigma) & \text{if } X_{LH} + 1 > Y > 0 \\ 0 & \text{if } Y < 0. \end{cases}$$

   If $P = p_1$, the firm knows that $\theta_M = \theta_F = H$ such that the expected NPV is equal to $\mu + \sigma > 0$. If $P = p_2$, the firm learns that $\theta_M = H$ and invests only if it also learns that $\theta_F = H$ from the private signal. If $P = p_3$, the firm learns no additional information and only invests if both private signals s_1 and s_2 are high, if $a = 0$.

(b) $\chi < \frac{1}{2}$: if $a = 1$, the firm invests if $s_2 = H$ and $p = p_1$; if $a = 0$, the firm invests if $s_1 = s_2 = H$ or if $p = p_1$ or if $s_2 = H$ and $p = p_2$. The equilibrium stock price is given by $P = p(Y)$ with:

$$p(Y) = \begin{cases} p_1 = \mu + \sigma & \text{if } Y > X_{LH} + 1 \\ p_2 = \frac{1}{2}(1-q)\delta\omega(\mu+\sigma) & \text{if } X_{LH} + 1 > Y > X_{HL} + 1 \\ p_3 = \frac{1}{3}(1-q)\delta\omega^2(\mu+\sigma) & \text{if } X_{HL} + 1 > Y > 0 \\ 0 & \text{if } Y < 0. \end{cases}$$

If $P = p_1$, the firm knows that $\theta_M = \theta_F = H$ such that the expected NPV is equal to $\mu + \sigma > 0$. If $P = p_2$, the firm learns that $\theta_M = H$ and invests only if it also learns that $\theta_M = H$ from the private signal. If $P = p_3$, the firm learns no additional information and only invests if both private signals s_1 and s_2 are high, if $a = 0$.

2. If $\kappa \in \left(-\frac{1}{2}, -\frac{1}{3}\right]$:

(a) $\chi \geq \frac{1}{2}$: if $a = 1$, the firm invests if $s_2 = H$ and $p \in \{p_1, p_2, p_3\}$; if $a = 0$, the firm invests if $s_1 = s_2 = H$ or if $p = p_1$ or if $s_1 \neq L$ and $p = p_2$ or if $p = p_3$ and either $s_1 = H$ and $s_2 = \emptyset$ or $s_2 = H$ and $s_1 = \emptyset$. The equilibrium stock price is given by $P = p(Y)$ with:

$$p(Y) = \begin{cases} p_1 = \mu + \sigma & \text{if } Y > X_{HL} + 1 \\ p_2 = \frac{1}{2}(\mu+\sigma) + \frac{1}{2}(1-q)(1-\omega)\mu & \text{if } X_{HL} + 1 > Y > X_{LH} + 1 \\ p_3 = \frac{1}{3}\left(\delta(q-1)\omega^2(3\mu+\sigma) - (\delta+1)(q-1)\omega(2\mu+\sigma) + q(2\mu+\sigma)\right) & \text{if } X_{LH} + 1 > Y > 0 \\ 0 & \text{if } Y < 0. \end{cases}$$

If $P = p_1$, the firm knows that $\theta_M = \theta_F = H$ such that the expected NPV is equal to $\mu + \sigma > 0$. If $P = p_2$, the firm learns that $\theta_M = H$ and invests as long as it does not learn that $\theta_F = L$. If $P = p_3$, the firm learns no additional information and only invests if either private signal is high and the other one is uninformative or high.

(b) $\chi < \frac{1}{2}$: if $a = 1$, the firm invests if $s_2 = H$ and $p \in \{p_1, p_2, p_3\}$; if $a = 0$, the firm invests if $s_1 = s_2 = H$ or if $p = p_1$ or if $s_2 \neq L$ and $p = p_2$ or if $p = p_3$ and either $s_1 = H$ or $s_2 = H$. The equilibrium stock price is given by $P = p(Y)$ with:

$$p(Y) = \begin{cases} p_1 = \mu + \sigma & \text{if } Y > X_{HL} + 1 \\ p_2 = \frac{1}{2}(\mu(\delta(q-1)\omega + 2) + \sigma) & \text{if } X_{HL} + 1 > Y > X_{LH} + 1 \\ p_3 = \frac{1}{3}\left(\delta(q-1)\omega^2(3\mu+\sigma) - (\delta+1)(q-1)\omega(2\mu+\sigma) + q(2\mu+\sigma)\right) & \text{if } X_{LH} + 1 > Y > 0 \\ 0 & \text{if } Y < 0. \end{cases}$$

If $P = p_1$, the firm knows that $\theta_M = \theta_F = H$ such that the expected NPV is equal to $\mu + \sigma > 0$. If $P = p_2$, the firm learns that $\theta_F = H$ and invests as long as it does not learn that $\theta_M = L$. If $P = p_3$, the firm learns no additional information and only invests if either private signal is high and the other one is uninformative or high.

3. If $\kappa \in \left(-\frac{1}{3}, 0\right]$:

   (a) $\chi \geq \frac{1}{2}$: if $a = 1$, the firm invests if $s_2 = H$ and $p \in \{p_1, p_2, p_3\}$; if $a = 0$, the firm invests if $s_1 = s_2 = H$ or if $p = p_1$ or if $s_1 \neq L$ and $p = p_2$ or if $p = p_3$ and either $s_1 = H$ or $s_2 = H$ or $s_1 = s_2 = \emptyset$. The equilibrium stock price is given by $P = p(Y)$ with:

$$p(Y) = \begin{cases} p_1 = \mu + \sigma & \text{if } Y > X_{HL} + 1 \\ p_2 = \frac{1}{2}(\mu + \sigma) + \frac{1}{2}(1 - q)(1 - \omega)\mu & \text{if } X_{HL} + 1 > Y > X_{LH} + 1 \\ p_3 = \frac{1}{3}(\mu((\delta + 1)(q - 1)\omega - q + 3) + \sigma) & \text{if } X_{LH} + 1 > Y > 0 \\ 0 & \text{if } Y < 0. \end{cases}$$

   If $P = p_1$, the firm knows that $\theta_M = \theta_F = H$ such that the expected NPV is equal to $\mu + \sigma > 0$. If $P = p_2$, the firm learns that $\theta_M = H$ and invests as long as it does not learn that $\theta_F = L$. If $P = p_3$, the firm learns no additional information and only invests if it does not learn that one shock is low.

   (b) $\chi < \frac{1}{2}$: if $a = 1$, the firm invests if $s_2 = H$ and $p \in \{p_1, p_2, p_3\}$; if $a = 0$, the firm invests if $s_1 = s_2 = H$ or if $p = p_1$ or if $s_2 \neq L$ and $p = p_2$ or if $p = p_3$ and either $s_1 = H$ or $s_2 = H$ or $s_1 = s_2 = \emptyset$. The equilibrium stock price is given by $P = p(Y)$ with:

$$p(Y) = \begin{cases} p_1 = \mu + \sigma & \text{if } Y > X_{HL} + 1 \\ p_2 = \frac{1}{2}(\mu(\delta(q - 1)\omega + 2) + \sigma) & \text{if } X_{HL} + 1 > Y > X_{LH} + 1 \\ p_3 = \frac{1}{3}(\mu((\delta + 1)(q - 1)\omega - q + 3) + \sigma) & \text{if } X_{LH} + 1 > Y > 0 \\ 0 & \text{if } Y < 0. \end{cases}$$

   If $P = p_1$, the firm knows that $\theta_M = \theta_F = H$ such that the expected NPV is equal to $\mu + \sigma > 0$. If $P = p_2$, the firm learns that $\theta_F = H$ and invests as long as it does not learn that $\theta_M = L$. If $P = p_3$, the firm learns no additional information and only invests if it does not learn that one shock is low.

4. If $\kappa \in \left(0, \frac{1}{3}\right]$:

   (a) $\chi \geq \frac{1}{2}$: the firm always invest if at least one private signal is equal to H; it also invests if $p \in \{p_1, p_2, p_3\}$;

$$p(Y) = \begin{cases} p_1 = \mu + \sigma & \text{if } Y > X_{HL} + 1 \\ p_2 = \mu + \frac{\sigma}{2} & \text{if } X_{HL} + 1 > Y > X_{LH} + 1 \\ p_3 = \mu + \frac{\sigma}{3} & \text{if } X_{LH} + 1 > Y > 0 \\ p_4 = \frac{1}{3}\mu(q - (\delta + 1)(q - 1)\omega) & \text{if } 0 > Y > X_{HL} - 1 \\ p_5 = \frac{1}{2}\mu(q(-\omega) + q + \omega) & \text{if } X_{HL} - 1 > Y > X_{LH} - 1 \\ p_6 = 0 & \text{if } X_{LH} - 1 > Y \end{cases}$$

   If one private signal is H, the expected project NPV cannot be negative. Similarly, $p > p_4$ rules out the possibility that both shocks are low.

   (b) $\chi < \frac{1}{2}$: $\chi \geq \frac{1}{2}$: the firm always invest if at least one private signal is equal to H; it also invests if $p \in \{p_1, p_2, p_3\}$;

$$p(Y) = \begin{cases} p_1 = \mu + \sigma & \text{if } Y > X_{LH} + 1 \\ p_2 = \mu + \frac{\sigma}{2} & \text{if } X_{LH} + 1 > Y > X_{HL} + 1 \\ p_3 = \mu + \frac{\sigma}{3} & \text{if } X_{HL} + 1 > Y > 0 \\ p_4 = \frac{1}{3}\mu(q - (\delta + 1)(q - 1)\omega) & \text{if } 0 > Y > X_{LH} - 1 \\ p_5 = -\frac{1}{2}\delta\mu(q - 1)\omega & \text{if } X_{LH} - 1 > Y > X_{HL} - 1 \\ p_6 = 0 & \text{if } X_{HL} - 1 > Y \end{cases}$$

   If one private signal is H, the expected project NPV cannot be negative. Similarly, $p > p_4$ rules out the possibility that both shocks are low.

5. If $\kappa \in \left(\frac{1}{3}, \frac{1}{2}\right]$:

   (a) $\chi \geq \frac{1}{2}$: the firm always invest if at least one private signal is equal to H; it also invests if $p \in \{p_1, p_2, p_3\}$; if $p = p_4$ the firm invests as long as it does not receive a low private signal.

$$p(Y) = \begin{cases} p_1 = \mu + \sigma & \text{if } Y > X_{HL} + 1 \\ p_2 = \mu + \frac{\sigma}{2} & \text{if } X_{HL} + 1 > Y > X_{LH} + 1 \\ p_3 = \mu + \frac{\sigma}{3} & \text{if } X_{LH} + 1 > Y > 0 \\ p_4 = \frac{1}{3}\left(\mu\left(-3\delta(q-1)\omega^2 + 2(\delta+1)(q-1)\omega - 2q + 3\right) + (q-1)\sigma(\omega-1)(\delta\omega-1)\right) & \text{if } 0 > Y > X_{HL} - 1 \\ p_5 = \frac{1}{2}\mu(q(-\omega) + q + \omega) & \text{if } X_{HL} - 1 > Y > X_{LH} - 1 \\ p_6 = 0 & \text{if } X_{LH} - 1 > Y \end{cases}$$

If one private signal is H, the expected project NPV cannot be negative. Similarly, $p > p_4$ rules out the possibility that both shocks are low. If $p = p_4$, the possibilities are $\{H, L\}$, $\{L, H\}$, and $\{L, L\}$ such that the NPV is positive if there is no low private signal.

(b) $\chi < \frac{1}{2}$: the firm always invest if at least one private signal is equal to H; it also invests if $p \in \{p_1, p_2, p_3\}$; if $p = p_4$ the firm invests as long as it does not receive a low private signal.

$$p(Y) = \begin{cases} p_1 = \mu + \sigma & \text{if } Y > X_{LH} + 1 \\ p_2 = \mu + \frac{\sigma}{2} & \text{if } X_{LH} + 1 > Y > X_{HL} + 1 \\ p_3 = \mu + \frac{\sigma}{3} & \text{if } X_{HL} + 1 > Y > 0 \\ p_4 = \frac{1}{3}\left(\mu\left(-3\delta(q-1)\omega^2 + 2(\delta+1)(q-1)\omega - 2q + 3\right) + (q-1)\sigma(\omega-1)(\delta\omega-1)\right) & \text{if } 0 > Y > X_{LH} - 1 \\ p_5 = -\frac{1}{2}\delta\mu(q-1)\omega & \text{if } X_{LH} - 1 > Y > X_{HL} - 1 \\ p_6 = 0 & \text{if } X_{HL} - 1 > Y \end{cases}$$

If one private signal is H, the expected project NPV cannot be negative. Similarly, $p > p_4$ rules out the possibility that both shocks are low. If $p = p_4$, the possibilities are $\{H, L\}$, $\{L, H\}$, and $\{L, L\}$ such that the NPV is positive if there is no low private signal.

6. If $\kappa \in \left(\frac{1}{2}, 1\right]$: the firm invests as long as it does not learn that both shocks are low.

   (a) $\chi \geq \frac{1}{2}$:

$$p(Y) = \begin{cases} p_1 = \mu + \sigma & \text{if } Y > X_{HL} + 1 \\ p_2 = \mu + \frac{\sigma}{2} & \text{if } X_{HL} + 1 > Y > X_{LH} + 1 \\ p_3 = \mu + \frac{\sigma}{3} & \text{if } X_{LH} + 1 > Y > 0 \\ p_4 = \frac{1}{3}\left(2\mu + (q-1)\left(\delta\omega^2 - 1\right)(\mu - \sigma)\right) & \text{if } 0 > Y > X_{HL} - 1 \\ p_5 = \frac{1}{2}(\mu + (q-1)(\omega-1)(\mu-\sigma)) & \text{if } X_{HL} - 1 > Y > X_{LH} - 1 \\ p_6 = 0 & \text{if } X_{LH} - 1 > Y \end{cases}$$

The only prices consistent with $\{L, L\}$ are p_4, p_5, and p_6. If $p = p_4$, it requires two negative private signals to choose $K = 0$. If $p = p_5$, it requires a low signal about θ_F not to invest and if $p = p_6$ the only possibility is that both shocks are low.

(b) $\chi < \frac{1}{2}$: the firm invests as long as it does not learn that both shocks are low.

$$p(Y) = \begin{cases} p_1 = \mu + \sigma & \text{if } Y > X_{LH} + 1 \\ p_2 = \mu + \frac{\sigma}{2} & \text{if } X_{LH} + 1 > Y > X_{HL} + 1 \\ p_3 = \mu + \frac{\sigma}{3} & \text{if } X_{HL} + 1 > Y > 0 \\ p_4 = \frac{1}{3}\left(2\mu + (q-1)\left(\delta\omega^2 - 1\right)(\mu - \sigma)\right) & \text{if } 0 > Y > X_{LH} - 1 \\ p_5 = \frac{1}{2}(\mu + (\mu - \sigma)(\delta(q-1)\omega + 1)) & \text{if } X_{LH} - 1 > Y > X_{HL} - 1 \\ p_6 = 0 & \text{if } X_{HL} - 1 > Y \end{cases}$$

The only prices consistent with $\{L, L\}$ are p_4, p_5, and p_6. If $p = p_4$, it requires two negative private signals to choose $K = 0$. If $p = p_5$, it requires a low signal about θ_M not to invest and if $p = p_6$ the only possibility is that both shocks are low.

Finally, we have to verify the conjectured trading policy. To this end, we first compute the expected stock price and firm value for the four different combinations of θ_M and θ_F. For the expected stock price, we can use the pricing function derived above together with the distribution of Y. For the expected firm value, we can use the conjectured pricing function and investment policy.

1. $\kappa \in \left(-1, -\frac{1}{2}\right]$:

   (a) $\chi \geq \frac{1}{2}$:

   i. $\theta_M = H$, $\theta_F = H$:

$$\mathbb{E}[P|HH] = \frac{1}{6}(\mu+\sigma)\left(2\delta(q-1)(\chi-1)\omega^2 - 3(q-1)(2\chi-1)\omega + q(6\chi-3) - 6\chi + 6\right)$$

and

$$\mathbb{E}[V|HH] = (\mu+\sigma)\left(\delta(q-1)(\chi-1)\omega^2 + (2\chi-1)(q(-\omega) + q + \omega) - \chi + 1\right)$$

ii. $\theta_M = H$, $\theta_F = L$:

$$\mathbb{E}[P|HL] = \frac{1}{6}(\mu+\sigma)\left(2\delta(q-1)(\chi-1)\omega^2 - 3(q-1)(2\chi-1)\omega + 3q(2\chi-1)\right)$$

and

$$\mathbb{E}[V|HL] = 0$$

iii. $\theta_M = L$, $\theta_F = H$:

$$\mathbb{E}[P|LH] = \frac{1}{3}\delta(q-1)(\chi-1)\omega^2(\mu+\sigma)$$

and

$$\mathbb{E}[V|LH] = 0$$

iv. $\theta_M = L$, $\theta_F = L$:

$$\mathbb{E}[P|LL] = 0$$

and

$$\mathbb{E}[V|LL] = 0$$

(b) $\chi < \frac{1}{2}$:

i. $\theta_M = H$, $\theta_F = H$:

$$\mathbb{E}[P|HH] =$$

and

$$\mathbb{E}[V|HH] = (\mu+\sigma)(\delta(-(q-1))\chi(\omega-2)\omega - \delta(q-1)\omega + \chi)$$

ii. $\theta_M = H$, $\theta_F = L$:

$$\mathbb{E}[P|HL] = -\frac{1}{3}\delta(q-1)\chi\omega^2(\mu+\sigma)$$

and

$$\mathbb{E}[V|HL] = 0$$

iii. $\theta_M = L$, $\theta_F = H$:

$$\mathbb{E}[P|LH] = -\frac{1}{6}\delta(q-1)\omega(\mu+\sigma)(2\chi(\omega-3)+3)$$

and

$$\mathbb{E}[V|LH] = 0$$

iv. $\theta_M = L$, $\theta_F = L$:

$$\mathbb{E}[P|LL] = 0$$

and

$$\mathbb{E}[V|LL] = 0$$

2. $\kappa \in \left(-\frac{1}{2}, -\frac{1}{3}\right]$:

   (a) $\chi \geq \frac{1}{2}$:

      i. $\theta_M = H, \theta_F = H$:

$$\mathbb{E}[P|HH] = \frac{1}{12}\big(12\mu\chi - 4\delta(q-1)(\chi-1)\omega^2(3\mu+\sigma) + (q-1)\omega(2\mu(4\delta(\chi-1)+10\chi-7)+4(\delta+1)\sigma(\chi-1)) - 20\mu q\chi + 14\mu q - 4q\sigma\chi + 4q\sigma + 6\sigma\big)$$

and

$$\mathbb{E}[V|HH] = (\mu+\sigma)((1-\chi)((q-1)\omega(\delta(\omega-1)-1)+q)+\chi)$$

      ii. $\theta_M = H, \theta_F = L$:

$$\mathbb{E}[P|HL] = \frac{1}{6}\big(12\mu\chi - 6\mu - 2\delta(q-1)(\chi-1)\omega^2(3\mu+\sigma) + (q-1)\omega(\mu(4\delta(\chi-1)+10\chi-7)+2(\delta+1)\sigma(\chi-1)) - 10\mu q\chi + 7\mu q - 2q\sigma\chi + 2q\sigma + 6\sigma\chi - 3\sigma\big)$$

and

$$\mathbb{E}[V|HL] = \mu(-(q-1))(\omega-1)(\delta(\chi-1)\omega - 2\chi + 1)$$

      iii. $\theta_M = L, \theta_F = H$:

$$\mathbb{E}[P|LH] = \frac{1}{3}(\chi-1)\big(\delta(-(q-1))\omega^2(3\mu+\sigma)+(\delta+1)(q-1)\omega(2\mu+\sigma)-q(2\mu+\sigma)\big)$$

and

$$\mathbb{E}[V|LH] = \mu(1-\chi)((q-1)\omega(\delta\omega-1)+q)$$

      iv. $\theta_M = L, \theta_F = L$:

$$\mathbb{E}[P|LL] = 0$$

and

$$\mathbb{E}[V|LL] = 0$$

   (b) $\chi < \frac{1}{2}$:

      i. $\theta_M = H, \theta_F = H$:

$$\mathbb{E}[P|HH] = \frac{1}{6}\big(2\delta(q-1)\chi\omega^2(3\mu+\sigma) - (q-1)\omega(2\chi(5\delta\mu+\delta\sigma+2\mu+\sigma)-3\delta\mu)+\mu((4q-6)\chi+6)+2q\sigma\chi+3\sigma\big)$$

and

$$\mathbb{E}[V|HH] = (\mu+\sigma)((q-1)\chi(\omega-1)(\delta\omega-1)+1)$$

ii. $\theta_M = H, \theta_F = L$:

$$\mathbb{E}[P|HL] = \frac{1}{3}\chi\left(\delta(q-1)\omega^2(3\mu+\sigma) - (\delta+1)(q-1)\omega(2\mu+\sigma) + q(2\mu+\sigma)\right)$$

and

$$\mathbb{E}[V|HL] = \delta\mu(q-1)\chi(\omega-1)\omega$$

iii. $\theta_M = L, \theta_F = H$:

$$\mathbb{E}[P|LH] = \frac{1}{6}\left(2\delta(q-1)\chi\omega^2(3\mu+\sigma) - (q-1)\omega(2\chi(5\delta\mu+\delta\sigma+2\mu+\sigma) - 3\delta\mu) + (2\mu+\sigma)(2(q-3)\chi+3)\right)$$

and

$$\mathbb{E}[V|LH] = \mu(\chi((q-1)\omega(\delta\omega-1)+q) - (2\chi-1)(\delta(q-1)\omega+1))$$

iv. $\theta_M = L, \theta_F = L$:

$$\mathbb{E}[P|LL] = 0$$

and

$$\mathbb{E}[V|LL] = 0$$

3. $\kappa \in \left(-\frac{1}{3}, 0\right]$:

(a) $\chi \geq \frac{1}{2}$:

i. $\theta_M = H, \theta_F = H$:

$$\mathbb{E}[P|HH] = \frac{1}{6}(\mu(-(q-1)\omega(2\delta(\chi-1) - 4\chi+1) - 4q\chi + q + 6) + \sigma(5-2\chi))$$

and

$$\mathbb{E}[V|HH] = \mu + \sigma$$

ii. $\theta_M = H, \theta_F = L$:

$$\mathbb{E}[P|HL] = \frac{1}{6}(6\mu\chi - \mu(q-1)\omega(2\delta(\chi-1) - 4\chi+1) - 4\mu q\chi + \mu q + 4\sigma\chi - \sigma)$$

and

$$\mathbb{E}[V|HL] = \mu(q-1)\chi(\omega-1)$$

iii. $\theta_M = L, \theta_F = H$:

$$\mathbb{E}[P|LH] = \frac{1}{3}(\chi-1)(\mu(-(\delta+1)(q-1)\omega + q - 3) - \sigma)$$

and

$$\mathbb{E}[V|LH] = -\mu(\chi-1)(\delta(q-1)\omega+1)$$

iv. $\theta_M = L, \theta_F = L$:

$$\mathbb{E}[P|LL] = 0$$

and

$$\mathbb{E}[V|LL] = 0$$

(b) $\chi < \frac{1}{2}$:

i. $\theta_M = H, \theta_F = H$:

$$\mathbb{E}[P|HH] = \frac{1}{6}(6\mu - \mu(q-1)\omega(\delta(4\chi - 3) - 2\chi) - 2\mu q\chi + 2\sigma\chi + 3\sigma)$$

and

$$\mathbb{E}[V|HH] = \mu + \sigma$$

ii. $\theta_M = H, \theta_F = L$:

$$\mathbb{E}[P|HL] = \frac{1}{3}\chi(\mu((\delta+1)(q-1)\omega - q + 3) + \sigma)$$

and

$$\mathbb{E}[V|HL] = \mu(q-1)\chi(\omega - 1)$$

iii. $\theta_M = L, \theta_F = H$:

$$\mathbb{E}[P|LH] = \frac{1}{6}(\mu(-2\chi((2\delta-1)(q-1)\omega + q + 3) + 3\delta(q-1)\omega + 6) + \sigma(3 - 4\chi))$$

and

$$\mathbb{E}[V|LH] = -\mu(\chi - 1)(\delta(q-1)\omega + 1)$$

iv. $\theta_M = L, \theta_F = L$:

$$\mathbb{E}[P|LL] = 0$$

and

$$\mathbb{E}[V|LL] = 0$$

4. $\kappa \in \left(0, \frac{1}{3}\right]$:

(a) $\chi \geq \frac{1}{2}$:

i. $\theta_M = H, \theta_F = H$:

$$\mathbb{E}[P|HH] = \mu + \frac{1}{6}\sigma(5 - 2\chi)$$

and

$$\mathbb{E}[V|HH] = \mu + \sigma$$

ii. $\theta_M = H$, $\theta_F = L$:

$$\mathbb{E}[P|HL] = \frac{1}{6}(6\mu\chi + 2(\delta+1)\mu(q-1)(\chi-1)\omega - 2\mu q\chi + 2\mu q + 4\sigma\chi - \sigma)$$

and

$$\mathbb{E}[V|HL] = \mu(\delta(q-1)(\chi-1)\omega + \chi)$$

iii. $\theta_M = L$, $\theta_F = H$:

$$\mathbb{E}[P|LH] = \frac{1}{6}(\mu((q-1)\omega(2\delta(\chi-1) - 4\chi + 1) + 4q\chi - q - 6\chi + 6) - 2\sigma(\chi-1))$$

and

$$\mathbb{E}[V|LH] = \mu + \mu\chi(q(-\omega) + q + \omega - 1)$$

iv. $\theta_M = L$, $\theta_F = L$:

$$\mathbb{E}[P|LL] = \frac{1}{6}(\mu(q-1)\omega(2\delta(\chi-1) - 4\chi + 1) + \mu q(4\chi - 1))$$

and

$$\mathbb{E}[V|LL] = 0$$

(b) $\chi < \frac{1}{2}$:

i. $\theta_M = H$, $\theta_F = H$:

$$\mathbb{E}[P|HH] = \mu + \frac{1}{6}\sigma(2\chi + 3)$$

and

$$\mathbb{E}[V|HH] = \mu + \sigma$$

ii. $\theta_M = H$, $\theta_F = L$:

$$\mathbb{E}[P|HL] = \frac{1}{6}(\mu(q-1)\omega(\delta(4\chi-3) - 2\chi) + 2\chi(\mu(q+3) + \sigma))$$

and

$$\mathbb{E}[V|HL] = \mu(\delta(q-1)(\chi-1)\omega + \chi)$$

iii. $\theta_M = L$, $\theta_F = H$:

$$\mathbb{E}[P|LH] = \frac{1}{6}(2\mu(\chi(-(\delta+1)(q-1)\omega + q - 3) + 3) + \sigma(3 - 4\chi))$$

and

$$\mathbb{E}[V|LH] = \mu + \mu\chi(q(-\omega) + q + \omega - 1)$$

iv. $\theta_M = L$, $\theta_F = L$:

$$\mathbb{E}[P|LL] = \frac{1}{6}(\mu(q-1)\omega(\delta(4\chi-3) - 2\chi) + 2\mu q\chi)$$

and

$$\mathbb{E}[V|LL] = 0$$

5. $\kappa \in \left(\frac{1}{3}, \frac{1}{2}\right]$:

(a) $\chi \geq \frac{1}{2}$:

i. $\theta_M = H, \theta_F = H$:

$$\mathbb{E}[P|HH] = \mu + \frac{1}{6}\sigma(5 - 2\chi)$$

and

$$\mathbb{E}[V|HH] = \mu + \sigma$$

ii. $\theta_M = H, \theta_F = L$:

$$\mathbb{E}[P|HL] = \frac{1}{3}\left(3\mu + \delta(q-1)(\chi-1)\omega^2(3\mu - \sigma) - (\delta+1)(q-1)(\chi-1)\omega(2\mu - \sigma) + 2\mu q\chi - 2\mu q - q\sigma\chi + q\sigma + 3\sigma\chi\right) - \frac{\sigma}{2}$$

and

$$\mathbb{E}[V|HL] = \mu(-(\chi-1)\omega(\delta\omega - 1) + q(\chi-1)(\omega(\delta\omega-1)+1)+1)$$

iii. $\theta_M = L, \theta_F = H$:

$$\mathbb{E}[P|LH] = \frac{1}{6}\left(\mu\left(6\delta(q-1)(\chi-1)\omega^2 - (q-1)\omega(4\delta(\chi-1)+10\chi-7)+10q\chi-7q-12\chi+12\right) + 2\sigma(\chi-1)(-(\omega-1)(\delta(q-1)\omega - q) - \omega)\right)$$

and

$$\mathbb{E}[V|LH] = \mu + \mu(q-1)(\omega-1)(\delta(\chi-1)\omega - 2\chi + 1)$$

iv. $\theta_M = L, \theta_F = L$:

$$\mathbb{E}[P|LL] = \frac{1}{6}\left(\mu\left(6\delta(q-1)(\chi-1)\omega^2 - (q-1)\omega(4\delta(\chi-1)+10\chi-7)+10q\chi-7q-6\chi+6\right) - 2(q-1)\sigma(\chi-1)(\omega-1)(\delta\omega-1)\right)$$

and

$$\mathbb{E}[V|LL] = (q-1)(\chi-1)(\omega-1)(\delta\omega-1)(\mu-\sigma)$$

(b) $\chi < \frac{1}{2}$:

i. $\theta_M = H, \theta_F = H$:

$$\mathbb{E}[P|HH] = \mu + \frac{1}{6}\sigma(2\chi + 3)$$

and

$$\mathbb{E}[V|HH] = \mu + \sigma$$

ii. $\theta_M = H, \theta_F = L$:

$$\mathbb{E}[P|HL] = \frac{1}{6}\left(-2\delta(q-1)\chi\omega^2(3\mu-\sigma) + (q-1)\omega(2(5\delta+2)\mu\chi - 3\delta\mu - 2(\delta+1)\sigma\chi) + 2\chi(q\sigma - 2\mu(q-3))\right)$$

and

$$\mathbb{E}[V|HL] = \mu(-(q-1)\chi(\omega(\delta\omega-1)+1) + \delta(q-1)(2\chi-1)\omega + \chi)$$

iii. $\theta_M = L, \theta_F = H$:

$$\mathbb{E}[P|LH] = \frac{1}{12}\left(12\mu + 4\delta(q-1)\chi\omega^2(\sigma-3\mu) + 4(\delta+1)(q-1)\chi\omega(2\mu-\sigma) - 8\mu q\chi + 4q\sigma\chi - 12\sigma\chi + 6\sigma\right)$$

and

$$\mathbb{E}[V|LH] = \mu + \delta\mu\chi\omega(q(-\omega) + q + \omega - 1)$$

iv. $\theta_M = L, \theta_F = L$:

$$\mathbb{E}[P|LL] = \frac{1}{6}\left(\mu\left(-6\delta(q-1)\chi\omega^2 + (q-1)\omega(\delta(10\chi-3)+4\chi) + (6-4q)\chi\right) + 2(q-1)\sigma\chi(\omega-1)(\delta\omega-1)\right)$$

and

$$\mathbb{E}[V|LL] = -(q-1)\chi(\omega-1)(\delta\omega-1)(\mu-\sigma)$$

6. $\kappa \in \left(\frac{1}{2}, 1\right]$:

(a) $\chi \geq \frac{1}{2}$:

i. $\theta_M = H, \theta_F = H$:

$$\mathbb{E}[P|HH] = \mu + \frac{1}{6}\sigma(5-2\chi)$$

and

$$\mathbb{E}[V|HH] = \mu + \sigma$$

ii. $\theta_M = H, \theta_F = L$:

$$\mathbb{E}[P|HL] = \frac{1}{6}\left(6\mu - 2\delta(q-1)(\chi-1)\omega^2(\mu-\sigma) + 2\mu q\chi - 2\mu q - 2q\sigma\chi + 2q\sigma + 6\sigma\chi - 3\sigma\right)$$

and

$$\mathbb{E}[V|HL] = \mu$$

iii. $\theta_M = L$, $\theta_F = H$:

$$\mathbb{E}[P|LH] = \frac{1}{6}\big(6\mu - 2\delta(q-1)(\chi-1)\omega^2(\mu-\sigma) + 3(q-1)(2\chi-1)\omega(\mu-\sigma) - 4\mu q\chi + \mu q + 4q\sigma\chi - q\sigma - 6\sigma\chi + 3\sigma\big)$$

and

$$\mathbb{E}[V|LH] = \mu$$

iv. $\theta_M = L$, $\theta_F = L$:

$$\mathbb{E}[P|LL] = \frac{1}{6}\big(6\mu\chi - 2\delta(q-1)(\chi-1)\omega^2(\mu-\sigma) + 3(q-1)(2\chi-1)\omega(\mu-\sigma) - 4\mu q\chi + \mu q + 4q\sigma\chi - q\sigma - 4\sigma\chi + \sigma\big)$$

and

$$\mathbb{E}[V|LL] = -(q-1)(\mu-\sigma)\big(\delta(\chi-1)\omega^2 - 2\chi\omega + \chi + \omega\big)$$

(b) $\chi < \frac{1}{2}$:

i. $\theta_M = H$, $\theta_F = H$:

$$\mathbb{E}[P|HH] = \mu + \frac{1}{6}\sigma(2\chi+3)$$

and

$$\mathbb{E}[V|HH] = \mu + \sigma$$

ii. $\theta_M = H$, $\theta_F = L$:

$$\mathbb{E}[P|HL] = \frac{1}{6}\big(2\delta(q-1)\chi\omega^2(\mu-\sigma) - 3\delta(q-1)(2\chi-1)\omega(\mu-\sigma) + \mu(6-2q\chi) + 2(q+3)\sigma\chi - 3\sigma\big)$$

and

$$\mathbb{E}[V|HL] = \mu$$

iii. $\theta_M = L$, $\theta_F = H$:

$$\mathbb{E}[P|LH] = \frac{1}{6}\big(6\mu + 2\delta(q-1)\chi\omega^2(\mu-\sigma) - 2\mu q\chi + 2q\sigma\chi - 6\sigma\chi + 3\sigma\big)$$

and

$$\mathbb{E}[V|LH] = \mu$$

iv. $\theta_M = L$, $\theta_F = L$:

$$\mathbb{E}[P|LL] = \frac{1}{6}\big(6\mu + 2\delta(q-1)\chi\omega^2(\mu-\sigma) - 3\delta(q-1)(2\chi-1)\omega(\mu-\sigma) - 2\mu(q+3)\chi + 2(q+2)\sigma\chi - 3\sigma\big)$$

and

$$\mathbb{E}[V|LL] = (\mu-\sigma)(\chi(-\delta(\omega-2)\omega + q(\delta(\omega-2)\omega-1)-1) + \delta(q-1)\omega + 1)$$

Next, we compute trading profits for $M-$informed and $F-$informed traders.

1. $\kappa \in \left(-1, -\frac{1}{2}\right]$:

   (a) $\chi \geq \frac{1}{2}$:

   $$\mathbb{E}[\Pi|b_i = 1] = \frac{1}{6}\delta(q-1)(\chi-1)\omega^2(\mu+\sigma)$$

   and

   $$\mathbb{E}[\Pi|b_i = 0] = \frac{1}{12}(\mu+\sigma)\left(2\delta(q-1)(\chi-1)\omega^2 - 3(q-1)(2\chi-1)\omega + q(6\chi-3)\right)$$

   (b) $\chi < \frac{1}{2}$:

   $$\mathbb{E}[\Pi|b_i = 1] = -\frac{1}{12}\delta(q-1)\omega(\mu+\sigma)(2\chi(\omega-3)+3)$$

   and

   $$\mathbb{E}[\Pi|b_i = 0] = -\frac{1}{6}\delta(q-1)\chi\omega^2(\mu+\sigma)$$

2. $\kappa \in \left(-\frac{1}{2}, -\frac{1}{3}\right]$:

   (a) $\chi \geq \frac{1}{2}$:

   $$\mathbb{E}[\Pi|b_i = 1] = \frac{1}{6}(\chi-1)\left((q-1)\omega(2\delta\mu+\delta\sigma-\mu+\sigma) + \delta(-(q-1))\sigma\omega^2 + q(\mu-\sigma)\right)$$

   and

   $$\begin{aligned}\mathbb{E}[\Pi|b_i = 0] = \frac{1}{12}&\left(-(q-1)\omega(2(\delta+1)\mu\chi - 2\delta\mu - 2(\delta+1)\sigma(\chi-1)+\mu)\right.\\ &\left.-2\delta(q-1)\sigma(\chi-1)\omega^2 + 2\mu q\chi + \mu q - 2q\sigma\chi + 2q\sigma + 6\sigma\chi - 3\sigma\right)\end{aligned}$$

   (b) $\chi < \frac{1}{2}$:

   $$\begin{aligned}\mathbb{E}[\Pi|b_i = 1] = &\frac{1}{12}(\mu(q-1)\omega(2(\delta+1)\chi-3\delta)\\ &+\sigma(2\chi((\omega-1)(\delta(q-1)\omega-q)+\omega-3)+3)-2\mu q\chi)\end{aligned}$$

   and

   $$\mathbb{E}[\Pi|b_i = 0] = \frac{1}{6}\chi\left((q-1)\omega((\delta-2)\mu-(\delta+1)\sigma) + \delta(q-1)\sigma\omega^2 + q(2\mu+\sigma)\right)$$

3. $\kappa \in \left(-\frac{1}{3}, 0\right]$:

(a) $\chi \geq \frac{1}{2}$:

$$\mathbb{E}[\Pi|b_i = 1] = \frac{1}{6}(\chi - 1)((2\delta - 1)\mu(q-1)\omega + \mu q - \sigma)$$

and

$$\mathbb{E}[\Pi|b_i = 0] = \frac{1}{12}(-\mu(q-1)\omega(2\delta(\chi - 1) + 2\chi + 1) + 2\mu q\chi + \mu q + 4\sigma\chi - \sigma)$$

(b) $\chi < \frac{1}{2}$:

$$\mathbb{E}[\Pi|b_i = 1] = \frac{1}{12}(\mu(q-1)\omega(2(\delta+1)\chi - 3\delta) - 2\mu q\chi + \sigma(3 - 4\chi))$$

and

$$\mathbb{E}[\Pi|b_i = 0] = \frac{1}{6}\chi((\delta - 2)\mu(q-1)\omega + 2\mu q + \sigma)$$

4. $\kappa \in \left(0, \frac{1}{3}\right]$:

(a) $\chi \geq \frac{1}{2}$:

$$\mathbb{E}[\Pi|b_i = 1] = \frac{1}{6}(\chi - 1)((2\delta - 1)\mu(q-1)\omega + \mu q - \sigma)$$

and

$$\mathbb{E}[\Pi|b_i = 0] = \frac{1}{12}(-\mu(q-1)\omega(2\delta(\chi - 1) + 2\chi + 1) + 2\mu q\chi + \mu q + 4\sigma\chi - \sigma)$$

(b) $\chi < \frac{1}{2}$:

$$\mathbb{E}[\Pi|b_i = 1] = \frac{1}{12}(\mu(q-1)\omega(2(\delta+1)\chi - 3\delta) - 2\mu q\chi + \sigma(3 - 4\chi))$$

and

$$\mathbb{E}[\Pi|b_i = 0] = \frac{1}{6}\chi((\delta - 2)\mu(q-1)\omega + 2\mu q + \sigma)$$

5. $\kappa \in \left(\frac{1}{3}, \frac{1}{2}\right]$:

(a) $\chi \geq \frac{1}{2}$:

$$\mathbb{E}[\Pi|b_i = 1] = \frac{1}{6}(\chi - 1)\left(-(q-1)\omega(-2\delta\mu + \delta\sigma + \mu + \sigma) + \delta(q-1)\sigma\omega^2 + \mu q + q\sigma - 2\sigma\right)$$

and

$$\mathbb{E}[\Pi|b_i = 0] = \frac{1}{12}\left(-(q-1)\omega(2\delta\mu(\chi - 1) + 2(\delta+1)\sigma(\chi - 1) + 2\mu\chi + \mu)\right.$$
$$\left. + 2\delta(q-1)\sigma(\chi - 1)\omega^2 + 2\mu q\chi + \mu q + 2q\sigma\chi - 2q\sigma + 2\sigma\chi + \sigma\right)$$

(b) $\chi < \frac{1}{2}$:

$$\mathbb{E}[\Pi|b_i = 1] = \frac{1}{12}\big((q-1)\omega(2(\delta+1)\chi(\mu+\sigma) - 3\delta\mu) \\ - 2\delta(q-1)\sigma\chi\omega^2 - 2\mu q\chi - 2q\sigma\chi - 2\sigma\chi + 3\sigma\big)$$

and

$$\mathbb{E}[\Pi|b_i = 0] = \frac{1}{6}\chi\left((q-1)\omega((\delta-2)\mu + \delta\sigma + \sigma) + \delta(-(q-1))\sigma\omega^2 + 2\mu q - q\sigma + 2\sigma\right)$$

6. $\kappa \in \left(\frac{1}{2}, 1\right]$:

(a) $\chi \geq \frac{1}{2}$:

$$\mathbb{E}[\Pi|b_i = 1] = \frac{1}{6}(\chi - 1)\left((\mu - \sigma)\left(\delta(q-1)\omega^2 - q\right) - 2\sigma\right)$$

and

$$\mathbb{E}[\Pi|b_i = 0] = \frac{1}{12}\big(2\delta(q-1)(\chi-1)\omega^2(\mu-\sigma) \\ - 3(q-1)(2\chi-1)\omega(\mu-\sigma) + (4\chi-1)(q(\mu-\sigma)+2\sigma)\big)$$

(b) $\chi < \frac{1}{2}$:

$$\mathbb{E}[\Pi|b_i = 1] = \frac{1}{12}\left(2\delta(q-1)\chi\omega^2(\sigma-\mu) + 3\delta(q-1)(2\chi-1)\omega(\mu-\sigma) + 2\mu q\chi - 2(q+4)\sigma\chi + 6\sigma\right)$$

and

$$\mathbb{E}[\Pi|b_i = 0] = \frac{1}{6}\chi\left((\mu-\sigma)\left(q - \delta(q-1)\omega^2\right) + 2\sigma\right)$$

We can show that the expected trading profits are positive such that it is optimal for traders to buy on $\sigma_i = H$ and sell on $\sigma_i = L$. It is easy to show that the traders' profits from the opposite strategy are negative.

A.1.2 Proof of Lemma 1

To solve for the firm's best-response function, we rely on the expressions for $\mathbb{E}[V]$ derived in the Proof of Proposition 1. We then take the derivative of $\mathbb{E}[V]$ with respect to q and show below that this derivative does not depend on q. Hence, we can check whether $\frac{\partial \mathbb{E}[V]}{\partial q}$ is (i) strictly positive, (ii) strictly negative, or (iii) equal to zero. In case (i) we can conclude that $q = 1$, in case (ii) that $q = 0$, and in case (iii) that $q \in [0, 1]$.

1. $\kappa \in \left(-1, -\frac{1}{2}\right]$: in this case $\frac{\partial \mathbb{E}[V]}{\partial q}$ is strictly positive if and only if $\chi > 1 - \frac{1-\omega}{2-\omega(2-\delta\omega)}$.

2. $\kappa \in \left(-\frac{1}{2}, -\frac{1}{3}\right]$: in this case $\frac{\partial \mathbb{E}[V]}{\partial q}$ is strictly positive if and only if $\chi > \frac{\kappa(\omega-1)}{\delta(3\kappa+1)\omega^2-\omega(2(\delta+2)\kappa+\delta+1)+4\kappa+1} + 1$, if the conjectured equilibrium is $\chi \geq \frac{1}{2}$ and if $\chi > -\frac{\delta\kappa\omega}{\kappa(\omega(\delta(3\omega-4)-2)+2)+(\omega-1)(\delta\omega-1)}$, if the conjectured equilibrium is $\chi < \frac{1}{2}$.

3. $\kappa \in \left(-\frac{1}{3}, 0\right]$: in this case $\frac{\partial \mathbb{E}[V]}{\partial q}$ is strictly positive if and only if $\chi > \frac{\delta\omega}{1-(1-\delta)\omega}$.

4. $\kappa \in \left(0, \frac{1}{3}\right]$: in this case $\frac{\partial \mathbb{E}[V]}{\partial q}$ is strictly positive if and only if $\chi > \frac{\delta\omega}{1-(1-\delta)\omega}$.

5. $\kappa \in \left(\frac{1}{3}, \frac{1}{2}\right]$: in this case $\frac{\partial \mathbb{E}[V]}{\partial q}$ is strictly positive if and only if $\chi > \frac{\kappa(\omega-1)}{\delta(3\kappa-1)\omega^2-2(\delta+2)\kappa\omega+\delta\omega+4\kappa+\omega-1} + 1$, if the conjectured equilibrium is $\chi \geq \frac{1}{2}$, and if $\chi > -\frac{\delta\kappa\omega}{\kappa(\omega(\delta(3\omega-4)-2)+2)-(\omega-1)(\delta\omega-1)}$, if the conjectured equilibrium is $\chi < \frac{1}{2}$.

6. $\kappa \in \left(\frac{1}{2}, 1\right]$: in this case $\frac{\partial \mathbb{E}[V]}{\partial q}$ is strictly positive if and only if $\chi > -\frac{\delta\omega}{\delta(\omega-2)\omega-1}$.

It follows that the firm chooses $q = 0$ if the mass of $M-$informed traders is sufficiently low and $q = 1$ if it is sufficiently large. At the cutoff, the firm is indifferent between any $q \in [0, 1]$.

A.1.3 Proof of Lemma 2

To solve for the traders' best-response functions, we use the expressions for expected trading profits that are derived in the Proof of Proposition 1. We then equate trading profits for $M-$informed and $F-$informed traders, and solve for χ.

1. $\kappa \in \left(-1, -\frac{1}{2}\right]$: in this case $\chi(q) = \frac{1}{2}$ for all $q \in [0, 1]$.

2. $\kappa \in \left(-\frac{1}{2}, -\frac{1}{3}\right]$: in this case, $\chi(q) = \frac{(2\delta-1)\kappa(q-1)\omega+\kappa q-1}{2\delta\kappa(q-1)\omega-2} \in \left(\frac{1}{2}, 1\right)$, which is increasing in q.

3. $\kappa \in \left(-\frac{1}{3}, 0\right]$: in this case, $\chi(q) = \frac{(2\delta-1)\kappa(q-1)\omega+\kappa q-1}{2\delta\kappa(q-1)\omega-2}$, which is increasing in q.

4. $\kappa \in \left(0, \frac{1}{3}\right]$: in this case, $\chi(q) = \frac{\delta\kappa(q-1)\omega-1}{-2\kappa\omega+2\kappa q(\omega-1)-2}$, which is decreasing in q.

5. $\kappa \in \left(\frac{1}{3}, \frac{1}{2}\right]$: in this case, $\chi(q) = \frac{\delta\kappa(q-1)\omega-1}{-2\kappa\omega+2\kappa q(\omega-1)-2}$, which is decreasing in q.

6. $\kappa \in \left(\frac{1}{2}, 1\right]$: in this case $\chi(q) = \frac{1}{2}$ for all $q \in [0, 1]$.

A.1.4 Proof of Proposition 2

For this proof, we rely on the results derived in Lemma 1 and Lemma 2.

1. $\kappa \in \left(-1, -\frac{1}{2}\right]$: in this case, we have shown before that $\chi^* = \frac{1}{2}$. Hence, $q^* = q(\frac{1}{2}) = 0$.

2. $\kappa \in \left(-\frac{1}{2}, -\frac{1}{3}\right]$: in this case, we know from the slope of the best-response functions that the optimal q is either $q^* = 0$ or $q^* = 1$. In particular $q^* = 0$ is an equilibrium if $\omega > \frac{2\kappa+1}{2\kappa+\sqrt{-\kappa(2\kappa+1)}+1}$ and $\delta > \frac{(\omega-1)(\kappa(4\kappa\omega+\omega+2)+1)}{\omega\left(\kappa\left(\kappa\left(3\omega^2-2\right)+\omega(\omega+2)-2\right)+\omega-1\right)}$. In this case, $\chi^* = \frac{(2\delta-1)\kappa\omega+1}{2\delta\kappa\omega+2}$. Moreover, $q^* = 1$ is an equilibrium if either (i) $\omega \leq \frac{3\kappa(\kappa+1)-\sqrt{-\kappa(\kappa(\kappa+1)(3\kappa+4)+1)}+1}{(\kappa+1)(3\kappa+1)}$ or if (ii) $\omega > \frac{3\kappa(\kappa+1)-\sqrt{-\kappa(\kappa(\kappa+1)(3\kappa+4)+1)}+1}{(\kappa+1)(3\kappa+1)}$ and $\delta < \frac{(\kappa(4\kappa+3)+1)(\omega-1)}{(\kappa+1)\omega(\kappa(3\omega-2)+\omega-1)}$. In this case, $\chi^* = \frac{1-\kappa}{2}$.

3. $\kappa \in \left(-\frac{1}{3}, 0\right]$: in this case, we know from the slope of the best-response functions that the optimal q is either $q^* = 0$ or $q^* = 1$. In particular, $q^* = 0$ is an equilibrium, if $\omega > \frac{1}{2}$ and $\delta > \frac{(\omega-1)(\kappa\omega-1)}{\kappa(3\omega-2)\omega+\omega}$. In this case, $\chi^* = \frac{(2\delta-1)\kappa\omega+1}{2\delta\kappa\omega+2}$. Moreover, $q^* = 0$ is an equilibrium if $\omega < \frac{1-\kappa}{(\delta-1)\kappa+\delta+1}$. In this case, $\chi^* = \frac{1-\kappa}{2}$.

4. $\kappa \in \left(0, \frac{1}{3}\right]$: in this case, we obtain a unique equilibrium. If $\omega > \frac{2}{\delta(-\kappa)+\sqrt{\delta(\delta(\kappa-6)\kappa+\delta+10\kappa+2)+1}+\delta+1}$ the equilibrium is $q^* = 0$ and $\chi^* = \frac{\delta\kappa\omega+1}{2\kappa\omega+2}$. If $\frac{1}{2\delta\kappa+\delta+1} < \omega < \frac{2}{\delta(-\kappa)+\sqrt{\delta(\delta(\kappa-6)\kappa+\delta+10\kappa+2)+1}+\delta+1}$, the equilibrium is $q^* = \frac{\omega(\delta((\delta-3)\kappa\omega+\kappa-1)-1)+1}{\delta\kappa\omega((\delta-3)\omega+3)}$ and $\chi^* = \frac{\delta\omega}{(\delta-1)\omega+1}$. Otherwise, the equilibrium is $q^* = 1$ and $\chi^* = \frac{1}{2\kappa+2}$.

5. $\kappa \in \left(\frac{1}{3}, \frac{1}{2}\right]$: in this case, we obtain a unique equilibrium. If ω is less than the cutoff:
$$\frac{\delta(-2(\kappa-1)\kappa-1)+\sqrt{(2\delta(\kappa-1)\kappa+\delta)^2-2\delta(2\kappa(\kappa(2\kappa+3)-3)+1)+(1-2\kappa)^2}+2\kappa-1}{2\delta(3\kappa-1)}$$
the equilibrium is $q^* = 1$ and $\chi^* = \frac{1}{2\kappa+2}$. If ω is greater than this cutoff but less than the positive, real root of the following cubic equation in w:
$$w^3\left(3\delta^2\kappa^2-\delta^2\kappa\right)+w^2\left(-4\delta^2\kappa^2+\delta^2\kappa+4\delta\kappa-\delta\right)+w\left(2\delta\kappa^2-3\delta\kappa+\delta-2\kappa+1\right)+2\kappa-1$$
the equilibrium is $q^* = \frac{1-2\kappa}{\delta\kappa\omega} + \frac{6\kappa(\omega-1)-2\omega+4}{\omega(\delta(3\kappa-1)\omega-4\delta\kappa+\delta+1)-1} + 1$ and $\chi^* = -\frac{\delta\kappa\omega}{\kappa(\omega(\delta(3\omega-4)-2)+2)-(\omega-1)(\delta\omega-1)}$. Otherwise, the equilibrium is $q^* = 0$ and $\chi^* = \frac{\delta\kappa\omega+1}{2\kappa\omega+2}$.

6. $\kappa \in \left(\frac{1}{2}, 1\right]$: in this case, we have shown before that $\chi^* = \frac{1}{2}$. Hence, $q^* = q(\frac{1}{2}) = 1$.

A.1.5 Proof of Corollary 1

These results follow directly from the expressions for χ^* and q^* derived in Proposition 2.

A.1.6 Proof of Corollary 2

These results follow directly from the expressions for χ^* and q^* derived in Proposition 2.

A.1.7 Proof of Proposition 3

To derive the expressions for real efficiency, we use the expressions for $\mathbb{E}[V]$ (for fixed information choices) in Proposition 1 and plug in the equilibrium values for χ^* and q^* in Proposition 2.

1. $\kappa \in \left(-1, -\frac{1}{2}\right]$:
$$RE = \frac{1}{8}(\kappa+1)\sigma\left(\delta\omega^2+1\right)$$

2. $\kappa \in \left(-\frac{1}{2}, -\frac{1}{3}\right]$: if $q^* = 1$,
$$RE = \frac{1}{8}(\kappa+1)(\kappa+2)\sigma$$
and if $q^* = 0$,
$$RE = \frac{\sigma\left(\kappa^2\omega\left(\delta\left(4-3\omega^2\right)+4\omega-3\right)+\kappa\omega(\delta(-\omega)(\omega+2)+4\delta+\omega+1)+\omega(\delta(-\omega)+\delta+1)+\kappa+1\right)}{8\delta\kappa\omega+8}$$

3. $\kappa \in \left(-\frac{1}{3}, 0\right]$: if $q^* = 1$,
$$RE = \frac{1}{8}(\kappa+1)(\kappa+2)\sigma$$
and if $q^* = 0$,
$$RE = \frac{\sigma(\kappa(\omega(\delta\kappa(4-3\omega)+\delta+\kappa\omega-1)+4)+2)}{8\delta\kappa\omega+8}$$

4. $\kappa \in \left(0, \frac{1}{3}\right]$: if $q^* = 1$,
$$RE = \frac{(\kappa(4\kappa+7)+2)\sigma}{8(\kappa+1)}$$
and if $q^* = 0$,
$$RE = \frac{\sigma(\kappa(\omega(-(\delta-3)\delta\kappa\omega+\delta+4\kappa+3)+4)+2)}{8\kappa\omega+8}$$
and if $q^* \in (0,1)$:
$$RE = \frac{\sigma(\omega(3\delta\kappa+\delta-2\kappa-1)+2\kappa+1)}{4(\delta-1)\omega+4}$$

5. $\kappa \in \left(\frac{1}{3}, \frac{1}{2}\right]$: if $q^* = 1$,

$$RE = \frac{(\kappa(4\kappa + 7) + 2)\sigma}{8(\kappa + 1)}$$

and if $q^* = 0$,

$$RE = \frac{\sigma\left(\kappa^2\omega(\delta(\delta\omega(3\omega - 4) + 3) + 4) + \kappa(\delta\omega(\omega(\delta(-\omega) + \delta + 4) - 3) + 7) + \omega(\delta(-\omega) + \delta + 1) + 1\right)}{8\kappa\omega + 8}$$

and if $q^* \in (0, 1)$:

$$RE = \frac{1}{4}\sigma\left(\kappa\left(2 - \frac{\delta\kappa\omega}{\kappa(\omega(3\delta\omega - 4\delta - 2) + 2) - (\omega - 1)(\delta\omega - 1)}\right) + 1\right)$$

6. $\kappa \in \left(\frac{1}{2}, 1\right]$:

$$RE = \frac{1}{4}(3\kappa\sigma + \sigma)$$

A.1.8 Proof of Proposition 4

It follows from the definition of PE and $P = \mathbb{E}[V|Y]$ that we can equivalently write $PE = 1 - \frac{\text{Var}(P)}{\text{Var}(V)}$. Below, we provide the expressions for $\text{Var}(P)$ and $\text{Var}(V)$ for a fixed χ and q. Plugging in the equilibrium values for χ and q yields to the equilibrium value of price efficiency.

1. $\kappa \in \left(-1, -\frac{1}{2}\right]$:

$$\begin{aligned}\text{Var}(P) = \frac{1}{48}(\kappa + 1)^2\sigma^2 \Big(&-4(q - 1)^2(\chi - 1)\omega^4 \\ &- 3\left(q\left(\chi((\omega - 2)\omega + 2) - \omega^2 + \omega - 1\right) - \chi(\omega - 1)^2 + \omega(\omega - 1) + 1\right)^2 \\ &+ 6(2\chi - 1)(q(-\omega) + q + \omega)^2 - 12(\chi - 1)\Big)\end{aligned}$$

$$\begin{aligned}\text{Var}(V) = \frac{1}{16}(\kappa + 1)^2\sigma^2 \Big(&4(q - 1)(\chi - 1)\omega^2 - \left((q - 1)(\chi - 1)\omega^2 + (2\chi - 1)(q(-\omega) + q + \omega) - \chi + 1\right)^2 \\ &- 4(q - 1)(2\chi - 1)\omega + 4q(2\chi - 1) - 4\chi + 4\Big)\end{aligned}$$

2. $\kappa \in \left(-\frac{1}{2}, -\frac{1}{3}\right]$:

$$\begin{aligned}\text{Var}(P) = \frac{1}{48}\Big(&-12(\chi - 1)(\kappa\sigma + \sigma)^2 - 3\sigma^2\left(3\kappa\chi - \kappa + (3\kappa + 1)(-(q - 1))(\chi - 1)\omega^2\right. \\ &\left.+ (q - 1)\omega(6\kappa\chi - 5\kappa + 2\chi - 2) - 4\kappa q\chi + 3\kappa q - q\chi + q + \chi\right)^2 \\ &+ 4(1 - \chi)\left((q - 1)\omega^2(3\kappa\sigma + \sigma) - 2(q - 1)\omega(2\kappa\sigma + \sigma) + q(2\kappa\sigma + \sigma)\right)^2 \\ &+ 6(2\chi - 1)(\kappa\sigma(q(\omega - 1) - \omega + 2) + \sigma)^2\Big)\end{aligned}$$

$$\mathrm{Var}(V) = \frac{1}{16}\sigma^2 \Big(4\left(3\kappa^2\chi - \kappa^2 + 2\kappa\chi - 4\kappa^2 q\chi + 3\kappa^2 q - (\kappa(3\kappa+2)+1)(q-1)(\chi-1)\omega^2 + (q-1)\omega(\kappa(6\kappa\chi - 5\kappa + 4\chi - 4) + 2(\chi-1)) - 2\kappa q\chi + 2\kappa q - q\chi + q + \chi\right) - \big(3\kappa\chi - \kappa + (3\kappa+1)(-(q-1))(\chi-1)\omega^2 + (q-1)\omega(6\kappa\chi - 5\kappa + 2\chi - 2) - 4\kappa q\chi + 3\kappa q - q\chi + q + \chi\big)^2\Big)$$

3. $\kappa \in \left(-\frac{1}{3}, 0\right]$:

$$\mathrm{Var}(P) = \frac{1}{48}\left(-12(\chi-1)(\kappa\sigma+\sigma)^2 - 3\sigma^2(\kappa(q(\chi-\omega)+\omega-2)-1)^2 + 6(2\chi-1)(\kappa\sigma(q(\omega-1)-\omega+2)+\sigma)^2 + 4(1-\chi)(\kappa\sigma(2(q-1)\omega-q+3)+\sigma)^2\right)$$

$$\mathrm{Var}(V) = \frac{1}{16}\sigma^2(3-\kappa(q(\chi-\omega)+\omega+2)(\kappa(q(\chi-\omega)+\omega-2)-2))$$

4. $\kappa \in \left(0, \frac{1}{3}\right]$:

$$\mathrm{Var}(P) = \frac{1}{48}\left(2\sigma^2\left(\kappa\left(\kappa\left(2q^2\chi + (q-1)^2(2\chi+3)\omega^2 - 8(q-1)q\chi\omega + 12\right) + 12\right) + 2\chi + 3\right) - 3(\kappa\sigma(q(\chi-\omega)+\omega+2)+\sigma)^2\right)$$

$$\mathrm{Var}(V) = \frac{1}{16}\sigma^2(3-\kappa(q(\chi-\omega)+\omega-2)(\kappa(q(\chi-\omega)+\omega+2)+2))$$

5. $\kappa \in \left(\frac{1}{3}, \frac{1}{2}\right]$:

$$\mathrm{Var}(P) = \frac{1}{48}\Big(12\chi(\kappa\sigma+\sigma)^2 + 4\chi(3\kappa\sigma+\sigma)^2 - 6(2\chi-1)(2\kappa\sigma+\sigma)^2 + 6\kappa^2(q-1)^2\sigma^2(1-2\chi)\omega^2 - 3\sigma^2\left(\kappa\left(q\chi(-3(\omega-2)\omega-2) - q\omega + 3\chi(\omega-1)^2 + \omega + 2\right) + (q-1)\chi(\omega-1)^2 + 1\right)^2 + 4\chi\left(\kappa\sigma(q((4-3\omega)\omega-2) + \omega(3\omega-4) + 3) + (q-1)\sigma(\omega-1)^2\right)^2\Big)$$

$$\mathrm{Var}(V) = \frac{1}{4}\left((\kappa\sigma+\sigma)^2 + \kappa^2\sigma^2\left(\chi\left(-q((\omega-3)\omega+1) + \omega^2 - 3\omega + 2\right) - q\omega + \omega\right) - \kappa^2\sigma^2((q-1)\chi(\omega-1)\omega - 1) + (\kappa-1)^2(-(q-1))\sigma^2\chi(\omega-1)^2\right) - \frac{1}{16}\sigma^2\left(\kappa\left(q\chi(-3(\omega-2)\omega-2) - q\omega + 3\chi(\omega-1)^2 + \omega + 2\right) + (q-1)\chi(\omega-1)^2 + 1\right)^2$$

6. $\kappa \in \left(\frac{1}{2}, 1\right]$:

$$\mathrm{Var}(P) = \frac{1}{48}\Big(12\chi(\kappa\sigma+\sigma)^2 + 4\chi(3\kappa\sigma+\sigma)^2 - 6(2\chi-1)(2\kappa\sigma+\sigma)^2 - 3\sigma^2\left(-\kappa\chi + 4\kappa + (\kappa-1)(q-1)\chi\omega^2 - (\kappa-1)(q-1)(2\chi-1)\omega - \kappa q\chi + q\chi + \chi\right)^2 + 6\sigma^2(1-2\chi)(\kappa + (\kappa-1)((q-1)\omega+1))^2 + 4\chi\left(2\kappa\sigma + (\kappa-1)(q-1)\sigma\left(\omega^2-1\right)\right)^2\Big)$$

$$\mathrm{Var}(V) = \frac{1}{4}\left((\kappa\sigma+\sigma)^2 + \kappa^2\sigma^2\left(\chi\left(-q((\omega-3)\omega+1) + \omega^2 - 3\omega + 2\right) - q\omega + \omega\right) - \kappa^2\sigma^2((q-1)\chi(\omega-1)\omega - 1) + (\kappa-1)^2(-(q-1))\sigma^2\chi(\omega-1)^2\right) - \frac{1}{16}\left(3\kappa\sigma + (\kappa-1)\sigma\left(q\chi((\omega-2)\omega-1) + (q-1)\omega - \chi(\omega-1)^2 + 1\right) + \sigma\right)^2$$

A.1.9 Proof of Proposition 5

1. If $\kappa \in \left(-\frac{1}{2}, -\frac{1}{3}\right]$: we obtain multiple equilibria if $\omega \in \left(\underline{\omega}, \overline{\omega}\right)$ where $\underline{\omega}$ is the solution to the following polynomial:

$$\omega^3 \left(3\delta\kappa^2 + \delta\kappa\right) + \omega^2 \left(2\delta\kappa + \delta - 4\kappa^2 - \kappa\right) + \omega \left(-2\delta\kappa^2 - 2\delta\kappa - \delta + 4\kappa^2 - \kappa - 1\right) + 2\kappa + 1 = 0$$

and

$$\overline{\omega} = \frac{1}{2}\left(\sqrt{\frac{\delta^2(\kappa+1)^2(2\kappa+1)^2 - 2\delta(\kappa+1)(4\kappa+1)(\kappa(4\kappa+3)+1) + (\kappa(4\kappa+3)+1)^2}{\delta^2(\kappa+1)^2(3\kappa+1)^2}} + \frac{2(\delta+2)\kappa^2 + 3(\delta+1)\kappa + \delta + 1}{\delta(\kappa+1)(3\kappa+1)}\right)$$

We can show that $0 < \underline{\omega} < \overline{\omega} < 1$ for $\delta \in (0, 1]$.

2. If $\kappa \in \left(-\frac{1}{3}, 0\right]$: we obtain multiple equilibria if either $\frac{1}{2} < \omega \leq \frac{1-\kappa}{2}$ and $\frac{(\omega-1)(\kappa\omega-1)}{\kappa(3\omega-2)\omega+\omega} < \delta < 1$ or $\frac{1-\kappa}{2} < \omega < 1$ and $\frac{(\omega-1)(\kappa\omega-1)}{\kappa(3\omega-2)\omega+\omega} < \delta < \frac{(\kappa-1)(\omega-1)}{(\kappa+1)\omega}$.

Based on the results in Proposition 1 and Proposition 2, we can show that the traders' expected profits are always higher if $q^* = 1$. For real efficiency and price efficiency the relationship is ambiguous, which can be seen in the two figures provided in the main text.