Collective Activism

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February 1, 2021

Abstract

We provide a theoretical framework to conceptualize "investor collective action organization" (ICAO). The ICAO overcomes the free-rider problem within ICAO members by sharing information and activism costs, but it worsens the free-rider problem between the ICAO and solo activists. An exogenous increase in the ICAO size raises the average firm value and the total payoff of all activists, at the expense of worsening market liquidity in financial markets. Without coordination costs, an ICAO will always form, but solo activists will also persist, free-riding on the activism efforts of ICAO members. With coordination costs, the model can produce multiple equilibria, implying that small reduction in coordination costs can spur significant ICAO formation.

Keywords: Investor collective action, free-rider problems, coordination costs

JEL: D82, G14, G18

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1 Introduction

Activism by institutional investors plays an important role in corporate governance. To enhance value, activists can monitor management, influence governance policies, and mitigate agency costs that arise from the separation of ownership and control. However, many investors have weak incentives to engage in activism on their own. They incur all of the associated costs but receive benefits proportional only to their fractional stakes. Moreover, they can benefit from the activism efforts of other investors. One way investors can increase activism incentives is to coordinate their actions.

In this paper, we provide a formal analysis and framework to understand the scope and limits of efforts by investors to increase governance activism through collective action. In an investor collective action organization, or ICAO, investors come together to jointly determine their activism efforts. Investors that join the ICAO can enforce agreements amongst themselves, but interact noncooperatively with other investors (Ray and Vohra (2015)). Our objective is to understand the scope for an ICAO to form and create value via activism, but also to understand the factors that constrain ICAO formation. Both aspects are important to capture the empirical experience, i.e., there are some recent examples of successful ICAOs (Doidge, Dyck, Mahmudi, and Virani (2019) and Dimson, Karakas, and Li (2020), but such organizations remain the exception rather than the rule.

We address the following questions: What are the defining features of an ICAO? Does the existence an ICAO lead to increased activism and firm value? Will an ICAO form when investors can benefit from the activism efforts of others without paying for them? If so, what is the optimal size of the ICAO, i.e., will all investors join the ICAO so that there are no activists working alone or will both types of activists exist? How do coordination costs impact the nature of ICAO formation? To address these questions, we build on the model of Back, Colin-Dufresne, Fos, Li, and Ljundqvist (2018) that generalizes a dynamic version of the Kyle (1985) model to study the links between liquidity, activism, and firm value. They focus on the actions of a single strategic trader, or activist, while we allow for multiple activists and an (endogenous) ICAO.

We model a single firm in which informed investors are endowed with an initial stake. These investors can exert costly effort to increase the value of the firm by correcting agency problems or other sub-optimal policies (see e.g., Cuñat, Giné, and Guadalupe (2012) for related evidence). Each informed investor can work alone as a "solo activist" or join a group of investors to work as an "ICAO activist". In anticipation of this choice, activists can trade in a financial market to alter their initial holdings. There are also uniformed investors, or noise traders. They trade in the financial market but are not activists.

In the model set up, we recognize that regulations limit the extent to which investors can coordinate. For example, proxy rules allow communications between investors when an investor is not seeking proxy authority, but also place significant obligations on groups with sufficient voting rights that are deemed to be acting "jointly or in concert". Investors that join the ICAO do not cede the power to vote their shares. Rather, they share otherwise private information about initial endowments, and therefore activism intentions. With more information, ICAO members trade more aggressively.

There are three potential free-rider problems to overcome. First, ICAO members can free ride on the activism effort of other members. Following prior work that focuses on coalition formation, we assume that the problem of coordination is solved once the ICAO is formed (Ray and Vohra (2015)). Second, the ICAO and solo activists can free ride on the activism effort of others. The presence of a large ICAO makes this free rider problem worse. Third, when there are costs of forming the ICAO, there must be a cost sharing mechanism among potential members. If investors can free ride on these costs, the ICAO may not form in the first place.

ICAO members benefit from cost sharing and from the potential to increase trading profits from sharing information about initial stakes. Offsetting these benefits is the decrease in market liquidity in the financial market due to more informed trading, which harms uninformed investors. In the baseline model, ICAO members can costlessly share information, and more generally, there are no coordination costs. Taking the number of ICAO members as given, activism effort and firm value increase with the number of ICAO members.

To endogenize ICAO formation and size, we consider a process that asks whether a solo activist would be better off if it remains on its own compared to joining the ICAO. If no ICAO exists, the benefits of forming one are sufficiently large that any two solo activists are better off joining together to form an ICAO. The next solo activist compares its expected payoff if it joins the ICAO with its expected payoff if it does not join. The process continues until the ICAO reaches its optimal size, i.e., in equilibrium no activist, either an ICAO member or solo activist wants to deviate from its status quo. ICAO formation is smooth in the sense that the ICAO can start with as few as two members and it grows until it reaches its optimal size. When we calibrate the model with reasonable parameters, the ICAO forms with a limited number of ICAO members (about 50) that have substantial investor ownership (about 25 percent ownership), similar to the ICAO studied in Doidge et al. (2019). That is, an ICAO will always form, but solo activists will also persist, free-riding on the actions of the ICAO.

To enrich the model we add coordination costs, both fixed and variable. Fixed costs for the ICAO include the costs of creating a legal organization that can satisfy regulatory rules and establish communication capabilities to facilitate information sharing across members. Variable costs increase with the number of members, e.g., communication costs, competitive costs, and costs incurred to mitigate regulatory concerns about investors acting in concert.

To study ICAO formation with coordination costs, we again consider a process in which solo activists compare the payoff of joining the ICAO to remaining on their own. Not surprisingly, with sufficiently high fixed and/or variable coordination costs no ICAO forms. More interestingly, within a range of more moderate coordination costs, the model produces multiple equilibria, i.e., ICAO formation is not smooth. Either no ICAO forms or the ICAO forms only if a sufficiently large number of activists simultaneously join. With few members, coordination costs overwhelm the benefits. A sufficiently large number of initial members is needed so that the benefits of joining the ICAO exceed the costs and the regular dynamics of ICAO formation of the baseline model take over. These results can be driven by either fixed costs alone or by variable costs alone.

One benefit of joining the ICAO is the potential to increase trading profits from sharing information about initial stakes. At the same time, ICAO members are competitors. To address the concern that investors that compete for trading profits may not want to join the ICAO, we show that the results hold when there is no financial market, and thus, no trading. These results also highlight that the model has broader applicability. For example, the model applies to investors that do not trade on information, i.e., indexers such as Blackrock. It also applies to companies such as Hermes EOS that provide third party engagement services. Hermes coordinates the actions of its clients (investors) to engage with companies. Thus costs are shared, but there is no information sharing and there is no possibility for trading gains.

The model focuses on the actions of private investors interested in improving firms' governance. It can be extended to other settings. For example, one way to reduce coordination costs is to have a third party that facilitates formation and information sharing, similar to the role played by "conditional cooperators" described by Ostrom (2000). The United Nations played this role to establish the UN Principles for Responsible Investment (PRI) with a group of large institutional investors. The UN PRI aims to achieve an economically efficient, sustainable global financial system. Investors coordinate engagements on sustainability issues via the UN PRI's Collobaration Platform (see Dimson et al. (2020)).

The paper provides a new, parsimonious theoretical framework to understand the scope and limits of collective activism by investors. The ICAO overcomes the free-rider problem within ICAO members by sharing information and activism costs, but it worsens the free-rider problem between the ICAO and solo activists. However, if free-riding is the only obstacle, an ICAO will always form and will coexist with solo activists that do not join.

In practice, ICAOs do form to address governance concerns (e.g., Doidge et al. (2019)) but despite recent interest, e.g., the Investor Stewardship Group and the Investor Forum, remain quite rare. The model highlights the importance of coordination costs as the key obstacle to ICAO formation, consistent with the evidence in Bradley, Brav, Goldstein, and Jiang (2010). An important source of coordination costs, both fixed and variable, is the regulatory and legal framework. In a survey of institutional investors McCahery, Sautner, and Starks (2016) find that legal concerns are an important impediment to coordinated actions. Therefore, one path to increase investor activism via ICAOs is to reduce regulatory costs (see also Black and Coffee (1994)). Calls to further regulate the actions of institutional investors (e.g., Elhauge (2016) and Posner, Morton, and Weyl (2017)) should be balanced by considering the benefits of coordinated investor activism.

Other mechanisms to coordinate investor actions include proxy advisors and hedge fund wolfpacks. Proxy advisors are for-profit organizations that react to issues raised by others. They provide information and voting recommendations to clients for a fee. The issues of interest are not on how these organizations form, but rather their impact. For example, Malenko and Malenko (2020) theoretically examine how proxy advisors affect the quality of corporate decision making. Brav, Dasgupta, and Matthews (2020) study how wolfpacks, temporary coalitions among investors, form and engage in activism. Their model features a larger, lead activist that implicitly coordinates with smaller, follower activists. While the trading profits primarily go to the lead activist, the smaller activists participate with the goal of enhancing their reputation and attracting fund flows. Thus, their model utilizes a different mechanism to overcome the free-rider problem.

Finally, our paper is related to the theoretical literature that studies blockholder governance. Winton (1993), Noe (2002, and Edmans and Manso (2011) all study a firm with multiple investors (blockholders) but are interested in different questions. For example, Edmans and Manso (2011) focus on multiple blockholders that cannot coordinate to determine the optimal blockholder structure. With multiple blockholders governance by voice is weaker, but governance by exit is stronger. Our focus is understanding investor coordination via ICAOs.

2 A Model of Collective Activism

2.1 Setting

We consider the static setting of Back et al. (2018), but we extend the analysis to allow for multiple activists and an ICAO. The ICAO has two defining characteristics, namely, that its members share information and activism costs so that the ICAO effectively coordinates the actions of its members.[1]

There is one firm, owned by uninformed investors (i.e. noise traders) and informed investors. We label informed investors as activists. There are K activists, where K is a positive integer. To consider collective activism, we assume $K \geq 2$. There are three periods, $t = 0, 1$, and 2. We describe the order of events in Panel A of Figure 1. Activists are endowed with shares in the firm and can accumulate additional firm shares in an anonymous market. They can then expend costly effort to change the firm value by correcting agency problems.

The K activists are divided into two groups:

- *ICAO activists*: I activists belong to the ICAO and act collectively, as defined below. Without loss of generality, we label these activists as $1, 2, \ldots, I$. The ICAO is assumed to maximize the joint profits of its members.
- *Solo activists*: J activists act independently, where $J \equiv K - I$. Each solo activist maximizes her own profits.

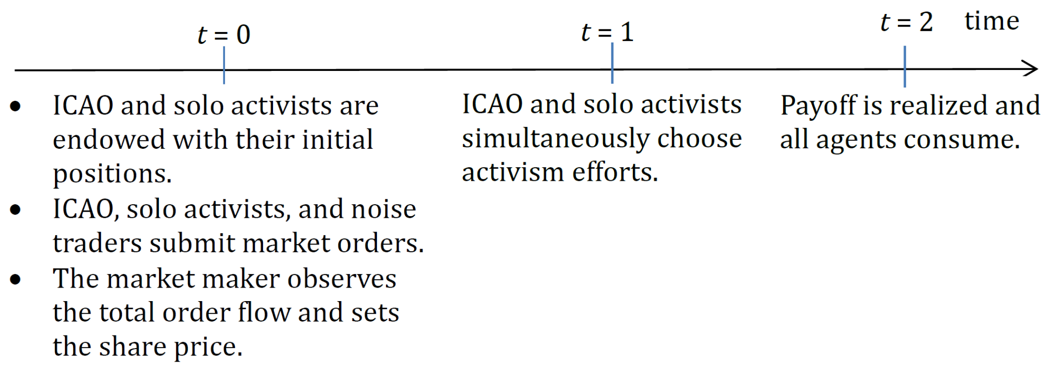
When $I = 1$, the ICAO has only one member and the setting effectively degenerates to a benchmark economy without an ICAO. When $I = K$, all activists belong to the ICAO and act in a collective way. Rather than investigate these two polar economies, we consider more general cases by allowing I to take any integer value between 1 and K. We initially take ICAO size I as exogenously given to gain insights on the impact of an ICAO on trading, activism, and value. Later, we endogenize ICAO size I to address whether an ICAO will form, what conditions support and/or discourage collective action through the ICAO, and the impact of ICAO formation on firm value.

At the beginning of period 0, activist k—either an ICAO member or a solo activist—receives an initial position, $x_k \sim N(\mu_x, \sigma_x^2)$, where $\mu_x > 0$ and $\sigma_x > 0$. Following Back et al. (2018), we assume that $\{x_k\}_{k=1}^K$ is independent and identically distributed and that x_k

[1]The ICAO can be formally understood as a "coalition" whose defining idea is "that of a group which can enforce agreements among its members, while it interacts noncooperatively with other nonmember individuals and the outside world in general." (Ray and Vohra, 2015, p. 240)

Figure 1: The Economy

Panel A: Timeline



Panel B: Players, Information, and Actions

	ICAO	Solo Activists	Noise Traders
Activists	$\{1, 2, ..., I\}$	$\{I+1, I+2, ..., I+J\}$	
Information	$\{x_1, x_2, ..., x_I\}$	Activist j: $\{x_{I+j}\}$	
Actions	$\{\Theta, v_1, ..., v_I\}$	Activist j: $\{\theta_j, v_{I+j}\}$	z

is private information to activist k. A solo activist j keeps its initial position, x_j, secret until the end of the economy. To facilitate information sharing each ICAO member i immediately reports its initial position, x_i, to the ICAO. Therefore, each ICAO member is endowed with information $\{x_1, \ldots, x_I\}$.[2] We summarize the information and actions of activists in Panel B of Figure 1.

A Kyle (1985) market operates in period 0. Starting with initial position x_{I+j},[3] the j^{th} solo activist trades θ_j shares so that its position at the beginning of period 1 becomes

$$y_j = x_{I+j} + \theta_j, \text{for } j = 1, \ldots, J. \tag{1}$$

ICAO members also trade and their net trading position is Θ . Thus, after trading, their overall ownership position is

$$Y = x_1 + \ldots + x_I + \Theta. \tag{2}$$

[2]Doidge et. al. (2019) document that information sharing is an important aspect of the activities of the Canadian Coalition of Good Governance. In the baseline model, we assume that information sharing is costless. In Section 6, we consider an extension with coordination costs.

[3]Recall that we have labeled ICAO activists by $1, 2, \ldots, I$. So, the j^{th} solo activist refers to activist $I + j$, and hence its initial position is x_{I+j}.

It is important to note that ICAO members execute their own trades and retain their voting rights for legal reasons, i.e., to avoid concerns about "acting in concert" which can trigger public disclosure obligations and a potentially costly public formal solicitation. For brevity, we subsequently refer to ICAO members' trading as "ICAO trading" and their aggregate ownership position as "ICAO ownership." In our setting, ICAO members can perfectly infer the trades of other members (each member knows the initial ownership positions and can infer expected activism choices as all members use the same activism technology) so that all members can infer Θ and therefore Y. All members have the same information so that the ICAO can fully coordinate its members' behavior. Therefore, the ICAO acts as a single player that observes all information of its members and maximizes the total profits of its members.

There is also noise trading, $z \sim N(0, \sigma_z^2)$ with $\sigma_z > 0$. The total order flow, which is what the market maker observes, is

$$\omega = \sum_{I+j=1}^{J} \theta_j + \Theta + z. \tag{3}$$

As usual, the period-0 asset price is determined by a competitive market maker who sets the asset price according to the weak-efficiency rule:

$$p(\omega) = E(V|\omega), \tag{4}$$

where V is the firm value in period 2, which is affected by activism and introduced shortly.

2.2 Investor Activism

2.2.1 The Value and Costs of Activism

Each activist k has access to an independent activism technology through which it can exert effort v_k in period 1 which affects the firm's final value in period 2 by v_k. The firm's final value in period 2 is thus

$$V = \sum_{k=1}^{K} v_k. \tag{5}$$

We could introduce asset-in-place values without qualitatively changing the analysis. We assume that activism effort v_k is a continuous variable. In some contexts, it may appear better to model activism effort as a discrete choice (e.g., replacing the CEO of the firm). In fact, our setting qualitatively captures such situations, with the following interpretation: Variable v_k represents the amount of evidence that activists collect to convince the board of directors to replace the CEO. With more evidence, the replacement request is more likely to be approved, which is captured by the linear firm value function

(5). Activist k can exert effort v_k, paying a variable cost

$$c(v_k) = \frac{v_k^2}{2\psi}, \tag{6}$$

and this level of activism changes the firm's period-2 per-share value by an amount v_k. The ψ term in (6) captures the activists' productivity, with more productive activists facing lower costs of activism. Without loss of generality, we normalize ψ at 1. In Section 6, we modify the cost function to consider ICAO coordination costs.[4]

ICAO members share activism costs. When activist i joins the ICAO, it contributes activism technology v_i which incurs a cost of $c(v_i)$. This technology costs $c(v_i)$ regardless of whether or not activist i is an ICAO member. However, with the ICAO, costs are shared and v_i can be done at a cost of $\frac{c(v_i)}{I}$ per member (see (11)). In effect, each member pays a fraction of the costs of its own activism technology and a fraction of the costs of the activism technology of other ICAO members. Such cost sharing is valuable in general as joining the ICAO effectively enhances the activist's productivity. It is particularly valuable for activists that have business relationships with the firm and might be punished for their activism. If an activist is a member of the ICAO, the firm will blame all members of the ICAO, and thus will spread this punishment cost over all the members.

2.2.2 The Optimal Value of Activism

A solo activist j accumulates y_j shares. These stakes depend upon its initial endowment of shares, x_{I+j}, and its trading, θ_j , in period 1. Given that solo activist j has accumulated y_j number of shares after trading, it chooses effort v_{I+j} to maximize its conditional expected profits, $E[y_j V - c(v_{I+j}) | x_{I+j}, y_j]$, taking as given the effort choices of other solo activists and the ICAO. In principle, activist j needs to forecast other activists' effort levels. But the linear specification of V makes this forecast unnecessary. The expected value of an investors' activism depends on its stakes y_j multiplied by the value V of the firm after the investor (and others) engage in activism, net of its activism cost $c(v_{I+j})$. As a result, the optimal value to the j^{th} solo activist is

$$g(y_j) = \max_{v_{I+j}} E[y_j V - c(v_{I+j}) | x_{I+j}, y_j]. \tag{7}$$

It is clear from this equation that our setup captures what Edmans and Holderness (2017) describe as the "standard free rider problem." That is, the j^{th} solo activist engages in

[4]When employing an activism technology, we could also assume that the ICAO is more forceful than a solo activist so that the impact of a given level of effort is greater for an ICAO. Our main conclusions do not change with such an assumption. See Appendix B for details.

too little activism if y_j is less than 1 on average (i.e., the j^{th} solo activist bears the full activism cost $c(v_{I+j})$ but enjoys only a proportion y_j of the benefits of that activism on firm value). Given that V is linear in v_k in (5), the j^{th} solo activist's effect choice problem in (7) is equivalent to the following:

$$\max_{v_{I+j}} \left[y_j v_{I+j} - c(v_{I+j}) \right]. \tag{8}$$

To define the optimal value of activism to an ICAO we need to specify how ICAO members consider their activism choices. As noted above, all ICAO members have the same information and the ICAO can fully coordinate its members' behavior. Therefore, we assume that the ICAO makes choices as if it is a single investor with position Y, which equals the sum of the y's for all I members of the ICAO.

The ICAO has I activism technologies, with each member contributing one technology. Each activism technology incurs costs as in (6). The total cost is the sum of the costs across all I technologies. As a result, the optimal activism value to the ICAO is

$$G(Y) = \max_{(v_1,\ldots,v_I)} E\left[YV - \sum_{i=1}^{I} c(v_i) \middle| x_1,\ldots,x_I, Y \right]. \tag{9}$$

It is clear from this equation, that with the ICAO the standard free rider problem still remains if Y is less than 1 on average.

Note, that this expression is equivalent to maximizing each ICAO member's profit as follows:

$$\max_{(v_1,\ldots,v_I)} \left[\frac{Y}{I} \sum_{i=1}^{I} v_i - \frac{1}{I} \sum_{i=1}^{I} c(v_i) \right]. \tag{10}$$

Intuitively, each member has an average position $\frac{Y}{I}$, which is used to scale the activism benefit in the first term. The second term says that each member has to equally share the total activism costs. From (10), the ICAO's activism effort for each activism technology, v_i, is determined by

$$\max_{v_i} \left[\frac{Y}{I} v_i - \frac{c(v_i)}{I} \right]. \tag{11}$$

If a solo activist and an ICAO member have the same stakes and activism technology (i.e., $y_j = \frac{Y}{I}$ and $v_{I+j} = v_i$), comparing (8) with (11) makes it clear that the ICAO invests more in activism technology v_i because its members equally share the activism cost ($\frac{c(v_i)}{I}$ in (11) vs. $c(v_{I+j})$ in (8)).

2.2.3 Activists' Trading in Period-0

At the period-0 trading game, the j^{th} solo activist chooses to purchase θ_j shares to maximize its objective function

$$E\left[g(y_j) - \theta_j p(\omega) \,|x_{I+j} \right], \tag{12}$$

taking as given the trading rules of other solo activists and of the ICAO, as well as the pricing rule of the market maker. The ICAO chooses to purchase Θ shares to maximize its objective function

$$E\left[G\left(Y\right)-\Theta p\left(\omega\right)\left|x_{1},\ldots,x_{I}\right.\right]. \tag{13}$$

Denote by π_j and Π, the period-0 optimal values to the j^{th} solo activist and the ICAO.

2.3 Discussion: ICAO Coordination and Externalities

The ICAO in our model has two defining characteristics that allows it to coordinate actions of its members in both periods. First, the ICAO collects and shares information among its members. This information sharing facilitates the members of the ICAO's trading and activism decisions. Second, the ICAO shares activism costs. This helps to internalize the externality across its members.

Note that in an ICAO context, there are two types of externalities (and associated free rider problems) of activism activities: (1) the ICAO-members externality, which is the fact that ICAO members, without coordination, could free ride on each other's activism activities; and (2) the ICAO-members/solo-activist externality, which is the fact that the ICAO as a single player and the remaining solo activists can free ride on each other's activism activities. The ICAO can only coordinate its members and thus overcome the first free rider problem. Doing so directly benefits the ICAO members (through internalizing the ICAO-members externality). Incidentally, ICAO activism indirectly benefits solo activists (through the positive ICAO-members/solo-activist externality). The ICAO internalizes the within-ICAO externality effectively through cost sharing across its members. To see this, consider how an investor would choose their level of activism if they had a fixed stake, and considered acting independently or collectively. By joining an ICAO, an activist has become more productive, facing a lower cost function ($\frac{c(v_i)}{I}$ vs. $c\left(v_{I+j}\right)$), and as a result will invest more in activism.

In Section 5, we will explore how, given the presence of an ICAO with these two defining characteristics, activists will decide whether to join the ICAO, and what will determine ICAO size. After having established results using this baseline model, in Section 6 we will enrich the discussion by considering additional costs and benefits of collective action. In that section we first allow for coordination costs for the ICAO (e.g., it is costly to set up the ICAO with an ability to share information and costs of activism). In Appendix B, we also consider an additional benefit of the ICAO, that activism through the ICAO may be more impactful (i.e., the ICAO is usually a large stakeholder of the firm and thus more forceful in implementing its activism effort than a solo activist in affecting the firm value).

3 The Equilibrium

We consider a subgame perfect equilibrium, which is composed of two subequilibria: (1) In period 0, the ICAO and J solo activists choose trading policies and the market maker sets the pricing rule to form an equilibrium in the Kyle trading game; and (2) In period 1, the ICAO and solo activists choose activism effort policies to form a Bayesian Nash equilibrium. As usual, we compute the equilibrium backward.

3.1 The Equilibrium in the Period-1 Activism Game

In period 1, the j^{th} solo activist chooses its activism level according to (7). The ICAO has I activism technologies and chooses the effort levels for these technologies according to (9). Using the expressions of V and $c(v_i)$ in (5) and (6), we can compute the optimal effort of the ICAO and the optimal effort of solo activist j as follows:

$$\text{Activist } j : v_{I+j}\left(y_j\right) = y_j, \text{ for } j = 1, \ldots, J; \tag{14}$$

$$\text{ICAO} : v_i\left(Y\right) = Y, \text{ for } i = 1, \ldots, I. \tag{15}$$

Inserting the above optimal activism policies into (7) and (9), we can compute

$$g\left(y_j\right) = \frac{y_j^2}{2} + \left(I^2 + J - 1\right)\mu_x y_j, \tag{16}$$

$$G\left(Y\right) = \frac{IY^2}{2} + J\mu_x Y. \tag{17}$$

In (16), the first term $\frac{y_j^2}{2}$ represents the effect of the j^{th} solo activist's own effort, while the second term $\left(I^2 + J - 1\right)\mu_x y_j$ captures the positive externality due to the efforts of the other solo activists and the ICAO. A similar interpretation applies to equation (17). We show in Section 5 the positive externality effect is one key factor in determining the equilibrium ICAO size.

3.2 The Equilibrium in the Period-0 Trading Game

As in most Kyle models, we consider a linear equilibrium in the trading game, in which the trading policies and the pricing rule linearly depend on the agents' private information. In Appendix A, we show that in a linear equilibrium, the trading policies and the pricing rule must take the following forms:

$$\text{ICAO} : \Theta = \delta \sum_{i=1}^{I} (x_i - \mu_x), \tag{18}$$

$$\text{Activist } j : \theta_j = \beta (x_{I+j} - \mu_x), \text{for} j = 1, \ldots, J; \tag{19}$$

$$\text{Market maker} : p = p_0 + \lambda\omega, \tag{20}$$

where (δ, β, λ) are endogenous coefficients and

$$p_0 = \left(I^2 + J\right) \mu_x. \tag{21}$$

Equations (18) and (19) say that the ICAO and solo activist j trade based on their information advantage, which is measured by the difference between their private information and the public prior about the private information. Equation (20) is similar to the traditional Kyle model, in which the price is driven by order flows. The value of p_0 in (21) is simply the prior mean $E(V)$ of V (noting that $E(V)$ is endogenous).

The ICAO takes θ_j and p in (19) and (20) as given and chooses Θ to maximize (13). The first-order condition (FOC) delivers

$$\Theta = \frac{I}{2\lambda - I} \sum_{i=1}^{I} (x_i - \mu_x). \tag{22}$$

Comparing with the conjectured policy (18), we have

$$\delta = \frac{I}{2\lambda - I}. \tag{23}$$

The second-order (SOC) of the ICAO's problem is

$$\lambda \geq \frac{I}{2}. \tag{24}$$

The j^{th} solo activist takes $\Theta, \theta_{j'}$ (for $j' \neq j$), and p in (18), (19), and (20) as given and chooses θ_j to maximize (12). The FOC delivers

$$\theta_j = \frac{x_{I+j} - \mu_x}{2\lambda - 1}, \tag{25}$$

which is compared with (19), yielding

$$\beta = \frac{1}{2\lambda - 1}. \tag{26}$$

The SOC of activist j's problem is

$$\lambda \geq \frac{1}{2}. \tag{27}$$

Comparing (23) with (26), we observe that the ICAO trades more aggressively on its information than a solo activist (i.e., $\delta \geq \beta$). This is understandable, as the ICAO has more information than a solo activist.

The market maker uses the total order flow ω to forecast firm value, V. We can compute:

$$
\begin{aligned}
V &= IY + \sum\nolimits_{j=1}^{J} y_j \text{ (by(5), (15), and (14))} \\
&= I\left(x_1 + \ldots + x_I + \Theta\right) + \sum\nolimits_{j=1}^{J} \left(x_{I+j} + \theta_j\right) \text{ (by (1) and (2))} \\
&= I\left[\sum\nolimits_{i=1}^{I} x_i + \delta \sum\nolimits_{i=1}^{I} \left(x_i - \mu_x\right)\right] + \sum\nolimits_{j=1}^{J} \left[x_{I+j} + \beta\left(x_{I+j} - \mu_x\right)\right] \text{ (by (18) and (19))}
\end{aligned}
$$

and

$$
\begin{aligned}
\omega &= \Theta + \sum\nolimits_{j=1}^{J} \theta_j + z \text{ (by (3))} \\
&= \delta \sum\nolimits_{i=1}^{I} \left(x_i - \mu_x\right) + \sum\nolimits_{j=1}^{J} \beta\left(x_{I+j} - \mu_x\right) + z \text{ (by (18) and (19))}.
\end{aligned}
$$

Hence, using Bayes' theorem, we have

$$
\lambda = \frac{Cov\left(V, \omega\right)}{Var\left(\omega\right)} = \frac{I\left(1+\delta\right)\delta I \sigma_x^2 + \left(1+\beta\right)\beta J \sigma_x^2}{\delta^2 I \sigma_x^2 + \beta^2 J \sigma_x^2 + \sigma_z^2}. \tag{28}
$$

Equations (23), (26), and (28), together with two SOCs (24) and (27), form the system to determine the three unknowns (λ, δ, β). Summarizing, we have the following proposition.

Proposition 1 *In period* 1*, there exists a Bayesian Nash equilibrium in the activism game, with activism policies of the ICAO and solo activist* j *respectively given by*

$$
\begin{aligned}
v_i &= Y, \text{ for } i = 1, \ldots, I, \\
v_{I+j} &= y_j, \text{ for } j = 1, \ldots, J,
\end{aligned}
$$

where

$$
\begin{aligned}
Y &= x_1 + x_2 + \ldots + x_I + \Theta, \\
y_j &= x_{I+j} + \theta_j, \text{ for } j = 1, \ldots, J.
\end{aligned}
$$

In period 0*, there exists an equilibrium in the Kyle trading game, in which the trading rules of the ICAO and solo activist* j*, and the pricing rule are, respectively,*

$$
\begin{aligned}
\Theta &= \delta \sum\nolimits_{i=1}^{I} \left(x_i - \mu_x\right), \\
\theta_j &= \beta\left(x_{I+j} - \mu_x\right), for j = 1, \ldots, J, \\
p &= p_0 + \lambda \omega,
\end{aligned}
$$

with

$$\omega = \sum\nolimits_{j=1}^{J} \theta_j + \Theta + z,$$

and $\lambda \geq \frac{I}{2}$ *is the real root of the following fourth-order polynomial:*

$$16Q\lambda^4 - 16Q(I+1)\lambda^3 - 4\left[I^3 + J - Q\left(4I + I^2 + 1\right)\right]\lambda^2$$
$$+4I\left[I^2 + J - Q(I+1)\right]\lambda - I^2(I + J - Q) = 0, \text{ with } Q \equiv \frac{\sigma_z^2}{\sigma_x^2},$$

and

$$\delta = \frac{I}{2\lambda - I}, \beta = \frac{1}{2\lambda - 1} \text{ and } p_0 = \left(I^2 + J\right)\mu_x.$$

In the above proposition, the trading game equilibrium is not fully analytical. That is, variables (λ, δ, β) are not in closed form. We can further characterize the values of (λ, δ, β) in some limiting economies. The following corollary summarizes the results in two special economies: (1) $I = 1$, without an ICAO; (2) $I = K$, with an all-inclusive ICAO.

Corollary 1 *(a) Suppose that* $I = 1$ *so that all activists are effectively solo activists. Then,*

$$\lambda = \frac{1}{2}\left(1 + \sqrt{K}\frac{\sigma_x}{\sigma_z}\right) \text{ and } \delta = \beta = \frac{1}{\sqrt{K}}\frac{\sigma_z}{\sigma_x}.$$

(b) Suppose that $I = K$ *so that all activists belong to the ICAO. Then,*

$$\lambda = \frac{K}{2}\left(1 + \sqrt{K}\frac{\sigma_x}{\sigma_z}\right) \text{ and } \delta = \frac{1}{\sqrt{K}}\frac{\sigma_z}{\sigma_x}.$$

For most of what follows, we focus on economies with financial trading, because in these economies, we can speak to issues such as how the ICAO affects activists' trading activities and financial market liquidity. It is also of interest to understand what would happen if there were no noise trading. Corollary 2 characterizes the limiting economy with degenerating noise trading in the financial market. This limiting economy effectively diminishes the difference in ICAO's trading behavior and solo activists' trading behavior. As Corollary 2 shows, the financial trading of activists vanishes as noise trading σ_z disappears (i.e., $\lim_{\sigma_z \to 0} Var(\Theta) = \lim_{\sigma_z \to 0} Var(\theta_j) = 0$). In this sense, the limiting economy with $\sigma_z \to 0$ is equivalent to an economy without a financial market. Analyzing this limiting economy allows us to transparently characterize many of our results.

Corollary 2 *Fix* (K, I, σ_x) *and let* $\sigma_z \to 0$*. Then,*

$$\lambda \propto \frac{\sqrt{I^3 + J}}{2}\frac{\sigma_x}{\sigma_z}, \beta \propto \frac{1}{\sqrt{I^3 + J}}\frac{\sigma_z}{\sigma_x}, \text{ and } \delta \propto \frac{I}{\sqrt{I^3 + J}}\frac{\sigma_z}{\sigma_x}$$

where "$X \propto Y$*" means* $\lim_{\sigma_z \to 0} \frac{X}{Y} = 1$.

4 Implications of Collective Activism

To gain additional insight into the impact of the ICAO on value, activism effort, and trading, we conduct comparative statics analysis with respect to parameter I, the number of members that join the ICAO, i.e., the degree of collective activism. In this section we take I as exogenously given. In the next section we determine the optimal number of members that join the ICAO.

4.1 Activism Effort, Firm Value, and Trading

To start, we convey the intuition about the driving forces of the model with a numerical example. It is based on the following empirically plausible parameter values: $\sigma_z = 30\%, \sigma_x = 2\%, \mu_x = 0.5\%$, and $K = 100$.[5] Peress and Schmidt (2018) estimate that the standard deviation of noise trading constitutes anywhere from one third to three quarters of the standard deviation of total trades in the market. We therefore set $\sigma_z = 30\%$. Ben-David et al. (2018) document that the aggregate of the top 10 institutional owners is 13.8% of the average stock. The top 50 on average own in aggregate 33.3%. This implies that the average ownership of big institutional investors ranges from 0.67% to 1.38%. Since these numbers are about really big institutions, we set $\mu_x = 0.5\%$ for average institutional ownership, and set $\sigma_x = 2\%$ to allow for some heterogeneity among institutions. Doidge et al. (2019) find that the Canadian Coalition for Good Governance (CCGG) has on average about 47 institutional investor members during their sample period from 2005 to 2015. This corresponds to an equilibrium ICAO size $I^* = 47$ in Section 5, where we endogenize the ICAO formation. We thus set $K = 100$.

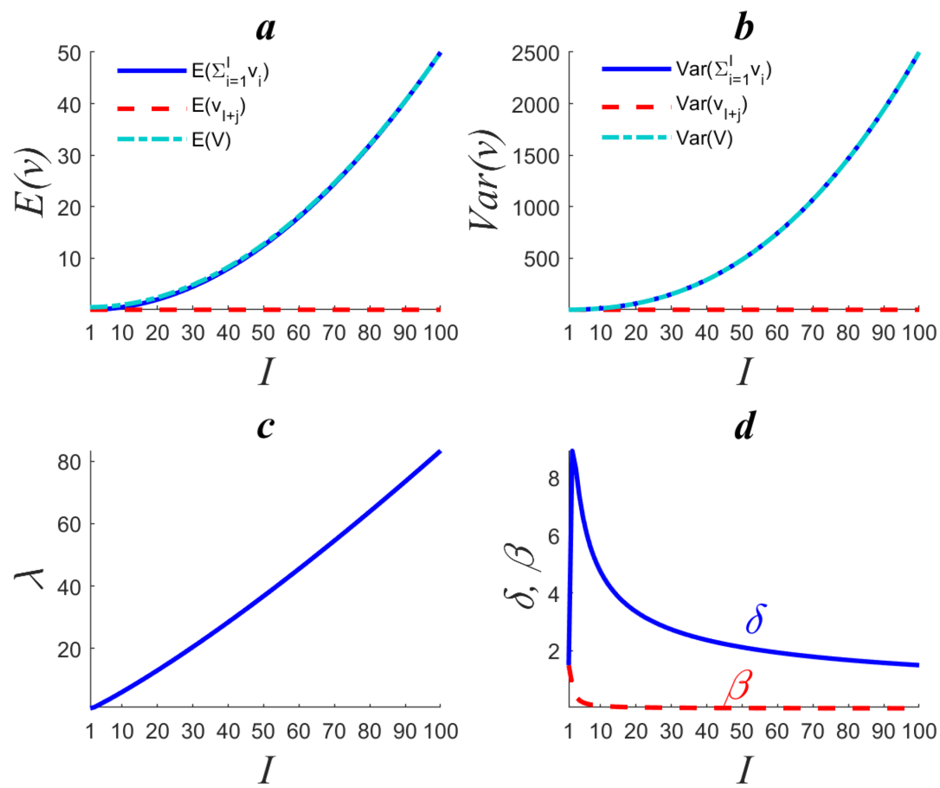
We report the results in Figure 2. The plots start from $I = 1$, which serves as a benchmark economy without an ICAO. Comparing the case of $I = 1$ with the case of $I > 1$ allows us to understand the general impact of the introduction the ICAO.

Panels a and b of Figure 2 plot activism effort and firm value. In Panel a, the average level of activism effort by the ICAO increases with ICAO size, while the average level of activism effort of solo activists is unaffected by ICAO size.[6] The firm's final value, V, is the sum of the activism activities of the ICAO and the solo activists, and the average firm value increases with ICAO size. The intuition for this result is that in the period-1 activism game, the ICAO chooses effort on behalf of all its members, which endogenizes the positive within-ICAO externality effect on value. As a result, a larger

[5] The results are robust to parameter choices.

[6] The activism technology allows activists to create or destroy firm value (i.e., v_k can be positive or negative). We find that activists create value on average (i.e., $E[v_k] > 0$) because they are endowed with long positions on average (i.e., $E[x_k] = \mu_x > 0$).

Figure 2: Activism Effort and Trading Equilibrium



This figure plots the activism effort and trading equilibrium against the ICAO size I. Parameter values are: $\sigma_z = 30\%, \sigma_x = 2\%, \mu_x = 0.5\%$, and $K = 100$.

ICAO improves economy efficiency. This is consistent with Doidge et al. (2019) who find that the creation of CCGG improves governance outcomes and firm valuation.

In Panel b of Figure 2, increasing ICAO size increases the volatility of the ICAO's activism. As the ICAO grows in size, with shared costs, there is more investment in each activism technology, increasing overall volatility. In contrast, a solo activist's volatility declines with ICAO size, although this decline is almost negligible in the figure. A solo activist's effort decreases with I because it trades less aggressively in the trading game, resulting in a less volatile position at the start of the activism game. Overall, the effect of the ICAO's activism activity dominates and so the total volatility of V increases with ICAO size. Panels a and b of Figure 2 suggest that in the presence of the ICAO, the activism activities of all activists, measured by both average and volatility, are primarily determined by the activism activity by the ICAO.

Panels c and d of Figure 2 show the endogenous market parameters (λ, β, δ) in the period-0 trading game. [7] The parameter λ ("Kyle's lambda") captures the price impact of noise trading. It is an inverse measure of market liquidity: A higher λ corresponds to a less liquid financial market. The figure shows that a larger ICAO size I worsens market liquidity.[8] Intuitively, the ICAO has more private information because it observes all information of its members. Therefore, a larger ICAO brings more information asymmetry into the period-0 trading game which worsens market liquidity. In Panel d, we plot β and δ, the trading aggressiveness of the solo activists and the ICAO, respectively. As liquidity worsens with an increase in the ICAO size I, a solo activist's price impact increases and hence reduces trading aggressiveness, i.e., β decreases with I. In contrast, the ICAO's trading aggressiveness δ is hump-shaped in I. By Corollary 1, δ takes the same value at $I = 1$ and $I = K$, which suggests that the pattern of δ is non-monotonic. The hump-shaped pattern of δ is due to the interactions between two offsetting forces. On the one hand, an increase in the ICAO size directly scales up the ICAO's activism value $G(Y)$ in equation (17), which therefore increases the ICAO's incentive to trade. On the other hand, the worse liquidity also causes the ICAO to trade less aggressively on its information.

The results of this numerical example in large part generalize. We focus on the value impact of ICAO size, and how this result is affected by the extent of noise trading, in the following proposition.

Proposition 2 *(a) Collective activism improves firm valuation so long as activists have long positions on average. That is, for fixed $(K, \sigma_x, \sigma_z, \mu_x) \in \mathbb{R}^4_{++}$, we have $\frac{\partial E(p)}{\partial I} = \frac{\partial E(V)}{\partial I} > 0$.*

(b) Fix $(K, I, \sigma_x) \in \mathbb{R}^3_{++}$ and let $\sigma_z \to 0$. We have

$$\begin{aligned}
\frac{\partial E(\sum_{i=1}^{I} v_i)}{\partial I} &> 0, \frac{\partial E(v_{I+j})}{\partial I} = 0, \text{ and } \frac{\partial E(V)}{\partial I} > 0; \\
\frac{\partial Var(\sum_{i=1}^{I} v_i)}{\partial I} &> 0, \frac{\partial Var(v_{I+j})}{\partial I} < 0, \text{ and } \frac{\partial Var(V)}{\partial I} > 0; \\
\frac{\partial \lambda}{\partial I} &> 0, \frac{\partial p_0}{\partial I} > 0, \text{ and } \frac{\partial \beta}{\partial I} < 0; \\
\frac{\partial \delta}{\partial I} &< 0 \text{ if and only if } I^3 + I > 2K.
\end{aligned}$$

[7] We do not plot the relationship between the period-0 value of the firm and ICAO size. Due to the weak-efficiency pricing rule (4), $p_0 = E(V) = \sum_{k=1}^{K} E[v_k]$ so this plot is identical to the plot for firm value in the top row.

[8] This worsening market liquidity may adversely affect the (unmodeled) firm valuation before period 0, representing a cost of a larger ICAO. For instance, in period -1, if the long-and-hold investors expect that they will experience liquidity shock in period 0 and the market then is less liquid, then they will require a higher compensation for holding the stock in period -1.

4.2 Activism Payoff

In this subsection, we compute the unconditional expected profits of the ICAO, each ICAO member, and the solo activists. These measures are used in the next section to pin down ICAO size in equilibrium.

For the ICAO, we have

$$\Pi \equiv E(G - \Theta p) = E\left(\frac{IY^2}{2} + J\mu_x Y\right) - Cov(\Theta, p) \text{ (by (17))}$$
$$= \underbrace{\frac{1}{2} I \left[I^2 \mu_x^2 + (1+\delta)^2 I \sigma_x^2\right]}_{\text{ICAO's own activism effort}} + \underbrace{JI\mu_x^2}_{\text{solo activists' efforts}} - \underbrace{\lambda \delta^2 I \sigma_x^2}_{\text{trading costs}}. \tag{29}$$

The ICAO's payoff Π comes from three sources. First, its activism effort directly changes the firm value in the best interests of the ICAO members. This effect is captured by $\frac{IE(Y^2)}{2} = \frac{1}{2} I \left[I^2 \mu_x^2 + (1+\delta)^2 I \sigma_x^2\right]$, where $I^2 \mu_x^2$ is related to the average activism effort of the ICAO and $(1+\delta)^2 I \sigma_x^2$ is related to the volatility of its activism. Activism volatility benefits the ICAO because the ICAO has the flexibility to adjust its activism effort in response to different values of its share position Y. Second, the activism of solo activists creates value ($E(v_{I+j}) = \mu_x > 0$) and has a positive externality on the ICAO's payoff, since on average, the ICAO holds a positive position ($E(Y) > 0$). This effect is captured by $J\mu_x E(Y) = JI\mu_x^2$. Third, the ICAO trades in the financial market to achieve a payoff-maximizing share position. Because the ICAO is large, trading has price impact, generating trading costs of $E(\Theta p) = \lambda \delta^2 I \sigma_x^2$.

Inserting the expression of δ in (23) into (29), we can compute

$$\Pi = I\left(\frac{I^2}{2} - I + K\right)\mu_x^2 + \frac{I^2 \lambda}{2\lambda - I}\sigma_x^2. \tag{30}$$

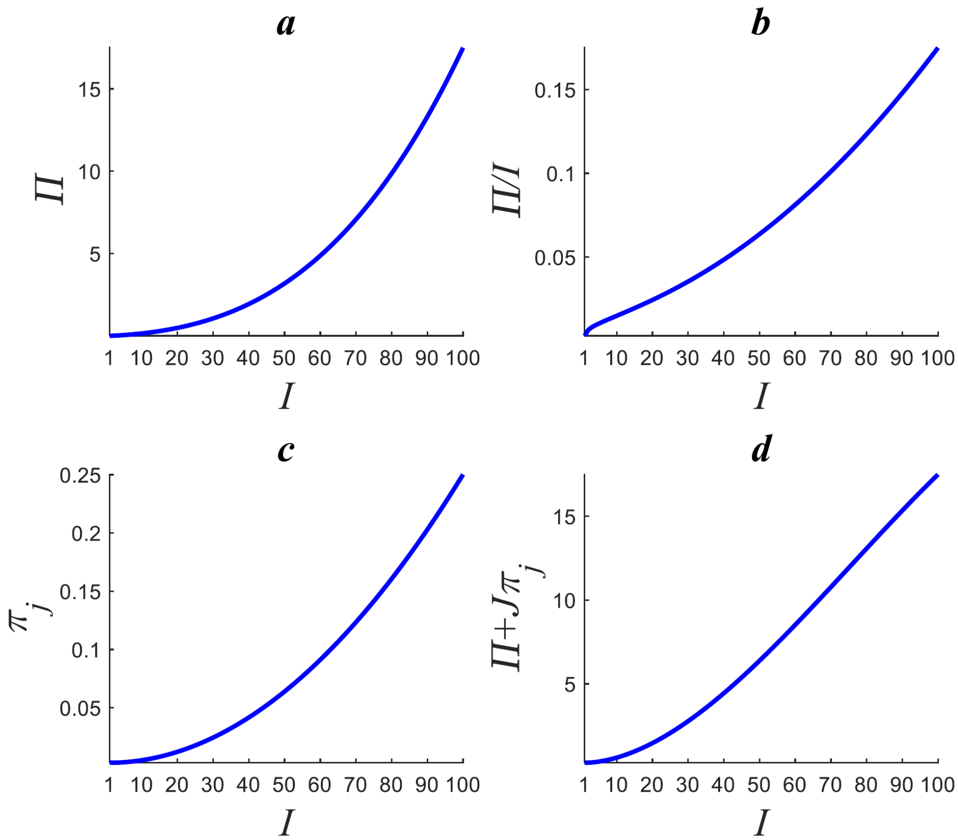
This implies that the payoff of each ICAO member is

$$\frac{\Pi}{I} = \left(\frac{I^2}{2} - I + K\right)\mu_x^2 + \frac{I\lambda}{2\lambda - I}\sigma_x^2. \tag{31}$$

For the j^{th} activist, we can compute its payoff as follows:

$$\pi_j \equiv E(g_j - \theta_j p) = E\left[\frac{y_j^2}{2} + \left(I^2 + J - 1\right)\mu_x y_j\right] - Cov(\theta_j, p) \text{ (by(16))}$$
$$= \underbrace{\frac{1}{2}\left[\mu_x^2 + (1+\beta)^2 \sigma_x^2\right]}_{\text{activist} j\text{'s effort}} + \underbrace{\left(I^2 + J - 1\right)\mu_x^2}_{\text{ICAO and other activists' efforts}} - \underbrace{\lambda \beta^2 \sigma_x^2}_{\text{trading costs}}. \tag{32}$$

Figure 3: Activism Payoff



This figure plots activist payoff against the ICAO size I. Parameter values are: $\sigma_z = 30\%, \sigma_x = 2\%, \mu_x = 0.5\%$, and $K = 100$.

Again, the j^{th} activist's payoff is determined by three factors: its own period-1 activism effort, the period-1 activism effort of the ICAO and other solo activists, and its own trading costs in the period-0 trading game. Inserting the expressions of δ and β in (23) and (26) into (32), we can further compute

$$\pi_j = \left(I^2 - I - \frac{1}{2} + K\right)\mu_x^2 + \frac{\lambda}{2\lambda - 1}\sigma_x^2. \tag{33}$$

Figure 3 plots the ICAO's payoff Π, an ICAO member's payoff $\frac{\Pi}{I}$, a solo activist's payoff π_j, and the total activism payoff $(\Pi + J\pi_j)$ against the ICAO size I. Increasing the ICAO size helps to improve the ICAO's payoff, an ICAO member's payoff and the total payoff. This is primarily driven by the fact that the ICAO internalizes the positive externality across its members (the within-ICAO externality). In addition, we can show that π_j increases with I for high values of I. Intuitively, increasing the ICAO size has two effects on π_j. First, a larger ICAO invests more in activism activities, which, through the positive ICAO-solo-activists externality, benefits a solo activist. Second, the presence of a larger ICAO implies more informed trading in the financial market, which worsens market liquidity and harms a solo activist's trading profits. When the ICAO size is very large, the first positive effect is strong and dominates.

Proposition 3 *(a) Collective activism improves each ICAO member's payoff and the ICAO's total payoff. That is, both $\frac{\Pi}{I}$ and I increase with the ICAO size I.*

(b) Collective activism benefits a solo activist when the ICAO size is sufficiently large or when the noise trading is sufficiently small in the financial market. That is, π_j increases with I for sufficiently large I or for sufficiently small σ_z.

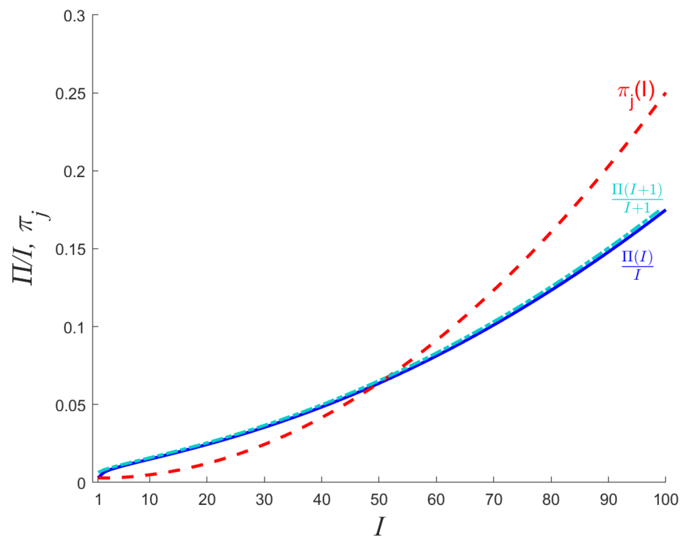
5 Endogenous ICAO Formation

In this section, we endogenize ICAO size, I. This exercise also speaks to the endogenous degree of coordination among activists. Our idea of endogenizing I is similar to that of endogenizing the population size of informed traders in the Grossman-Stiglitz (1980) model. At the very beginning of the economy, each activist is identical and decides whether to join the ICAO or remain independent. An ICAO member expects to receive payoff $\frac{\Pi}{I}$ while solo activists expect to receive payoff π_j. The marginal ICAO member compares the expected payoff if it joins the ICAO and size is $I+1$ with its expected payoff if it does not join the ICAO and size is I. That is, the general idea is to compare $\frac{\Pi(I+1)}{I+1}$ with $\pi_j(I)$. We consider a process that asks whether an activist wants to become an ICAO member, and in equilibrium produces a result that no activist, either an ICAO member or a solo activist, would like to deviate from its status quo. To sharpen intuition, we first show results using a numerical example, and then offer a formal analysis.

5.1 A Numerical Example

In Figure 4, we plot an ICAO member's payoff and a solo activist's payoff against ICAO size I in the top panel, and report their values in the bottom panel. The parameter values are the same as in previous figures: $\sigma_z = 30\%, \sigma_x = 2\%, \mu_x = 0.5\%$ and $K = 100$. To emphasize that π_j and $\frac{\Pi}{I}$ depend on I, we denote them as $\pi_j(I)$, $\frac{\Pi(I)}{I}$, and $\frac{\Pi(I+1)}{I+1}$. To pin down the choice of an activist faced with the choice to join the ICAO or not, we focus on the difference between the curves $\frac{\Pi(I+1)}{I+1}$ with $\pi_j(I)$. Consider the following thought experiment for forming an ICAO. Start at $I = 1$. Now all activists are effectively solo activists, and in this numerical example, an activist's payoff is $\pi_j(1) = 0.0029$.

Figure 4: ICAO Formation



I	1	2	3	4	…	49	50	51	**52**	53	54	…	98	99	100
Π/I	0.0029	0.0065	0.0081	0.0093	…	0.0621	0.0637	0.0654	**0.0670**	**0.0687**	0.0704	…	0.1694	0.1722	0.1750
π_j	0.0029	0.0029	0.0029	0.0030	…	0.0615	0.0639	**0.0664**	**0.0690**	0.0716	0.0742	…	0.2403	0.2452	0.2502

This figure reports the payoff of an ICAO member and the payoff of a solo activist as a function of the ICAO size I. Parameter values are: $\sigma_z = 30\%, \sigma_x = 2\%, \mu_x = 0.5\%$, and $K = 100$.

Suppose that an activist, say, activist 2, deviates from being solo. It approaches activist 1 and proposes to form an ICAO. By doing so, both activists 1 and 2 enjoy a payoff of $\frac{\Pi(2)}{2} = 0.0065$, which is higher than the original payoff, $\pi_j(1) = 0.0029$. Therefore, activist 1 accepts activist 2's proposal to form an ICAO—activists 1 and 2 become ICAO members 1 and 2. Facing this newly formed ICAO with $I = 2$, each of the remaining 98 solo activists receives a payoff of $\pi_j(2) = 0.0029$.

At $I = 2$, solo activists still have incentives to deviate. Say, activist 3, approaches the existing ICAO composed of members 1 and 2, and proposes to join the ICAO. If they accept activist 3's offer, the ICAO's size increases from 2 to 3, generating a payoff of $\frac{\Pi(3)}{3} = 0.0081$ to each member. Activist 3 will want to join the ICAO as its expected payoff is greater than the payoff it expects if the ICAO remains at size $I = 2$ ($\frac{\Pi(3)}{3} = 0.0081 > \pi_j(2) = 0.0029$). ICAO members 1 and 2 will agree to admit acitivst 3 as member 3 as the payoff is higher than the payoff if the ICAO remains at size $I = 2$ ($\frac{\Pi(3)}{3} = 0.0081 > \frac{\Pi(2)}{2} = 0.0065$). The remaining 97 activists are still solo and each receives payoff $\pi_j(3) = 0.0029$.

The above process continues until the ICAO reaches its equilibrium size, $I^* = 52$. At this size, neither solo activists nor ICAO members want to deviate. At $I = 52$, each of the remaining 48 solo activists receives a payoff of $\pi_j(52) = 0.0690$. They have no incentives to join the ICAO as the 53^{rd} member because joining the ICAO results in a lower expected payoff, $\frac{\Pi(53)}{53} = 0.0687$. Similarly, the 48 ICAO members have no incentives to leave the ICAO: Each ICAO member expects a payoff of $\frac{\Pi(52)}{52} = 0.0670$. If a member withdraws, the ICAO size shrinks from 52 to 51. The deviating member becomes a solo activist and expects a lower payoff of $\pi_j(51) = 0.0664$.

In this example, at the equilibrium ICAO size of $I^* = 52$, a solo activist receives a payoff of $\pi_j(52) = 0.0690$, which is higher than an ICAO member's payoff $\frac{\Pi(52)}{52} = 0.0670$. This result is generally true in our setting. We formalize it in Corollary 3 in Section 5.3.

5.2 Equilibrium Concept

We define an equilibrium ICAO size I^* based on non-deviations of both types of activists. Consider an interior ICAO size $I^* \in \{2, \ldots, K-1\}$. An existing ICAO member enjoys a payoff $\frac{\Pi(I^*)}{I^*}$. If it withdraws from the ICAO, the ICAO is still active (since no member is pivotal) but its size shrinks by one. The deviating ICAO member becomes solo, receiving a payoff of $\pi_j(I^* - 1)$. Thus, the non-deviation condition of an ICAO activist is:

$$\text{No deviation of ICAO members: } \frac{\Pi(I^*)}{I^*} \geq \pi_j(I^* - 1). \tag{34}$$

An existing solo activist enjoys a payoff of $\pi_j(I^*)$. If it deviates and proposes to join the ICAO, the ICAO will accept its proposal. This is because a larger ICAO generates a higher payoff to its members: $\frac{\Pi(I^*+1)}{I^*+1} > \frac{\Pi(I^*)}{I^*}$ (see Part (a) of Proposition 3). As a result, the deviating activist receives payoff $\frac{\Pi(I+1)}{I+1}$. This implies the following non-deviation condition of an independent activist:

$$\text{No deviation of solo activists: } \pi_j(I^*) \geq \frac{\Pi(I^*+1)}{I^*+1}. \tag{35}$$

For corner equilibria, only one of the above two conditions need to hold, since only one type of activists is active. Specifically, all activists join the ICAO, i.e., $I^* = K$, if and only if condition (34) holds at $I^* = K$. No ICAO is formed, i.e., $I^* = 1$, if and only if condition (35) holds at $I^* = 1$. That is,

$$I^* = K \Longleftrightarrow \frac{\Pi(K)}{K} \geq \pi_j(K-1); \tag{36}$$

$$I^* = 1 \Longleftrightarrow \pi_j(1) \geq \frac{\Pi(2)}{2}. \tag{37}$$

5.3 Equilibrium Characterization

Conditions (34) and (35) essentially compare payoff functions $\frac{\Pi(I+1)}{I+1}$ and $\pi_j(I)$. Using equations (31) and (33), we can compute the benefit of a solo activist switching to an ICAO member as follows:

$$\eta(I) \equiv \frac{\Pi(I+1)}{I+1} - \pi_j(I) = \frac{(I+1)\lambda_I + \lambda_{I+1}(2\lambda_I I - I - 1)}{(2\lambda_I - 1)(2\lambda_{I+1} - I - 1)}\sigma_x^2 - \frac{I(I-2)}{2}\mu_x^2, \quad (38)$$

where λ_I and λ_{I+1} are the Kyle's lambda in economies in which the ICAO size is I and $I+1$, respectively.

To understand the above benefit function, let us consider the trade-off faced by a solo activist. Staying out of the ICAO offers it two benefits. First, in the period-0 trading game, it is small relative to the ICAO (in terms of information) and thus, its trading cost is smaller than that of an ICAO member. Second, in the period-1 activism game, relative to an ICAO member, a solo activist can enjoy a larger positive externality from the existence of a large ICAO (through the ICAO-solo-activists externality). Relative to an ICAO member, a solo activist suffers two costs as well. First, in the period-0 trading game, the solo activist has less information, which reduces its trading aggressiveness. Second, in the period-1 activism game, the solo activist has to bear all of its activism costs, while the ICAO members share activism costs (i.e., the ICAO internalizes the within-ICAO externality). The cost-benefit interactions determine the behavior of $\eta(I)$.

Conditions (34) and (35) characterizing an interior ICAO size $I^* \in \{2, \ldots, K-1\}$ are equivalent to the following:

$$\text{No deviation of ICAO members: } \eta(I^* - 1) \geq 0; \quad (39)$$

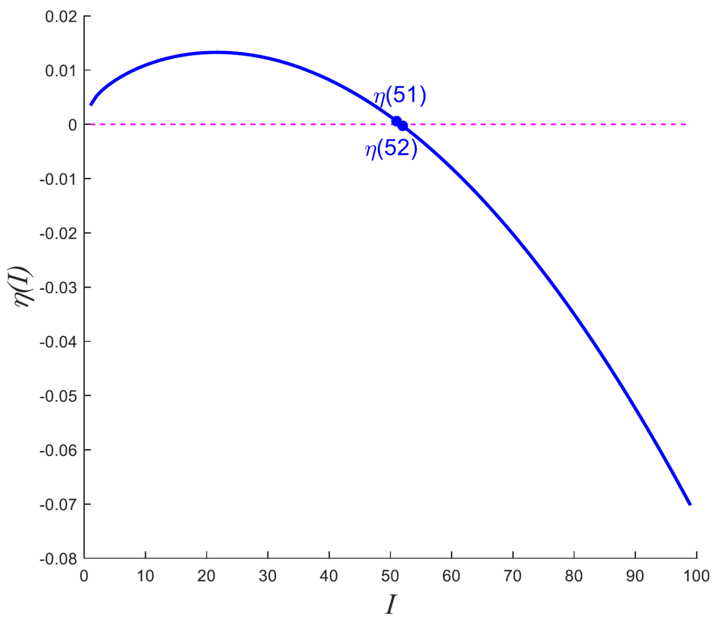
$$\text{No deviation of solo activists: } \eta(I^*) \leq 0. \quad (40)$$

The two corner equilibria are defined similarly. That is, $I^* = K$ if and only if $\eta(K-1) \geq 0$; $I^* = 1$ if and only if $\eta(1) \leq 0$.

We can show that $\eta(1) > 0$ and $\eta(2) > 0$. This implies that an equilibrium ICAO contains at least three members in economies with more than two activists (i.e., if $K \geq 3$, then $I^* \geq 3$).[9] In addition, we can show that $\eta(I)$ is negative for large values of I. Intuitively, a large ICAO implies that the ICAO-solo-activists externality is particular strong, so that a solo activist enjoys a higher payoff than an ICAO member. Thus, when there are many activists in the economy, they naturally divide into two groups, ICAO members and solo activists (i.e., if K is large, then $I^* \in (3, K)$).

[9] In Section 6, we introduce coordination costs and show that it is possible that no ICAO is formed in equilibrium.

Figure 5: Benefit Function



This figure plots the benefit function $\eta(I)$ of joining the ICAO. Parameter values are: $\sigma_z = 30\%, \sigma_x = 2\%, \mu_x = 0.5\%$, and $K = 100$.

Figure 5 plots function $\eta(I)$ for the same parameter configuration as Figure 4. In fact, both Figure 4 and Figure 5 present the same information. In the previous subsection, we use Figure 4 to illustrate that $I^* = 52$. Now, in Figure 5, we see that $\eta(I)$ changes signs at $I = 52 : \eta(51) > 0$ and $\eta(52) < 0$. Hence, by conditions (39) and (40), the equilibrium ICAO size is $I^* = 52$.

Proposition 4 *In economies with endogenous ICAO formation:*

(a) There exists an equilibrium ICAO size I^*, which is determined by $(K, \sigma_z/\sigma_x, \mu_x/\sigma_x)$.

(b) If there are two activists, then the two activists form an ICAO in equilibrium. If there are more than two activists, then an equilibrium ICAO contains at least three members. That is, $I^* \geq \min\{3, K\}$.

(c) When there are sufficiently many activists in the economy, in equilibrium, they are endogenously divided into two types, ICAO members and solo activists.

Next, we consider how results change as we reduce the importance of noise traders, and hence date-0 trading activity.

Proposition 5 *Fix* (K, μ_x, σ_x) *and let* $\sigma_z \to 0$. *Then:*

(a) The optimal ICAO size is

$$I^* \propto \min \left\{ K, 3 + \left[\sigma_x^2/\mu_x^2\right] \right\},$$

where $[\sigma_x^2/\mu_x^2]$ *is the integer part of* σ_x^2/μ_x^2.

(b) The optimal ICAO size (weakly) increases with K *and* $\frac{\sigma_x^2}{\mu_x^2}$. *That is,* $\frac{\partial I^*}{\partial K} \geq 0$ *and* $\frac{\partial I^*}{\partial(\sigma_x^2/\mu_x^2)} \geq 0$.

Part (b) of Proposition 5 suggests that the equilibrium ICAO size I^* decreases with μ_x and increases with σ_x. Intuitively, a higher μ_x means that a solo activist enjoys more benefits by free-riding the ICAO activism and so is more likely to stay out of the ICAO. A higher σ_x means that an ICAO member enjoys more benefits from sharing information and acting collectively via the ICAO.

In equilibrium, solo activists enjoy a higher payoff than ICAO members. To see this result, note that condition (35) says that $\pi_j(I^*) \geq \frac{\Pi(I^*+1)}{I^*+1}$. Since $\frac{\Pi(I)}{I}$ is increasing in I (by Part (a) of Proposition 3), we must have $\pi_j(I^*) \geq \frac{\Pi(I^*+1)}{I^*+1} > \frac{\Pi(I^*)}{I^*}$.

Corollary 3 *In economies with endogenous ICAO formation, the equilibrium payoff of a solo activist's payoff is higher than that of an ICAO member. That is,* $\pi_j(I^*) > \frac{\Pi(I^*)}{I^*}$.

6 An Extension with Costly Coordination

6.1 Setting

The baseline model in Section 2 takes ICAO size, I, as given. The extended model in Section 5 that considers endogenous ICAO formation predicts an ICAO will always form. While there are settings where ICAOs form and create value (see e.g., Doidge et. al. (2019)) and there is interest in forming new ICAOs (e.g., cite to add here, UK working group), ICAOs remain quite rare. What discourages ICAO formation, and, are there ways to overcome barriers to ICAO formation? We address these questions in this section.

So far, we assumed that it is costless for the ICAO to coordinate its members. In practice, there are coordination costs, some of which are fixed and some of which are variable and increase with ICAO size. With the introduction of coordination costs, most of our results continue to hold and some new results emerge. Some of the new results are intuitive, e.g., an ICAO may not form in equilibrium when coordination costs are high. Other results are more subtle. For instance, small changes in underlying parameters can cause large changes in the equilibrium ICAO size. This implies that either an ICAO does not exist (when coordination costs are high), or if an ICAO forms (when coordination costs are low), then it has more than two members.

We model the ICAO's coordination cost Φ having two elements: (1) a fixed cost such as the cost of setting up an ICAO; and (2) a variable cost which positively depends on the ICAO size I. In our analysis, we consider a linear coordination cost function:

$$\Phi(I) = \begin{cases} 0, & if I = 1, \\ \phi_0 + \phi_1 I, & if I \geq 2, \end{cases} \tag{41}$$

where $\phi_0 \geq 0$ and $\phi_1 \geq 0$ are two constants. Trivially, there is no coordination cost for a solo activist, so that $\Phi(1) = 0$. It costs ϕ_0 to form an ICAO, and once the ICAO is formed, it costs the ICAO ϕ_1 per member to coordinate the actions and thus, the total variable cost is $\phi_1 I$. The baseline model corresponds to $\phi_0 = \phi_1 = 0$.

For simplicity, we assume that coordination costs depend only on the ICAO size I but not on activism intensity $(v_1, \ldots, v_I)$. This assumption seems reasonable: First, this activism-intensity-related coordination cost may be partly captured by the activism cost function $c(v_k)$ in (6), because $c(v_k)$ is convex and the ICAO on average invests more in activism than independent activists. Second, even if the activism-intensity-related coordination cost can be conceptually different from the activism cost $c(v_k)$ specified in (6), this coordination cost may be negligible relative to $c(v_k)$.

For a given ICAO size, the coordination cost Φ is fixed and it does not interact with activists' trading and activism activities. Thus, the trading game equilibrium and the activism game equilibrium are still characterized by Proposition 1. As a result, the implications of collective activism remain qualitatively unchanged from the baseline model as long as the ICAO size is exogenous. However, once the ICAO size becomes endogenous, new findings emerge.

6.2 Endogenous ICAO Formation with Coordination Costs

We can follow the same steps as in the baseline model and compute the benefit of a solo activist switching to an ICAO member as follows:

$$\begin{aligned} \eta(I) &\equiv \frac{\Pi(I+1)}{I+1} - \pi_j(I) \\ &= \frac{(I+1)\lambda_I + \lambda_{I+1}(2\lambda_I I - I - 1)}{(2\lambda_I - 1)(2\lambda_{I+1} - I - 1)}\sigma_x^2 - \frac{I(I-2)}{2}\mu_x^2 - \left(\frac{\phi_0}{I+1} + \phi_1\right). \end{aligned} \tag{42}$$

Equation (42) is a direct extension of equation (38) by considering the coordination cost Φ. An interior ICAO size $I^* \in \{2, \ldots, K-1\}$ is still characterized by conditions (39) and (40): $\eta(I^*-1) \geq 0$ and $\eta(I^*) \leq 0$. An all-inclusive ICAO exists (i.e., $I^* = K$) if and only if $\eta(K-1) \geq 0$. No ICAO is formed ($I^* = 1$) if and only if $\eta(1) \leq 0$.

Non-existence of an ICAO There are two new findings. First, in the presence of sufficiently large coordination costs, no ICAO is formed in equilibrium. Second, and of more interest, there can be multiple equilibrium ICAO sizes under the non-deviation equilibrium concept specified in Section 5.2.

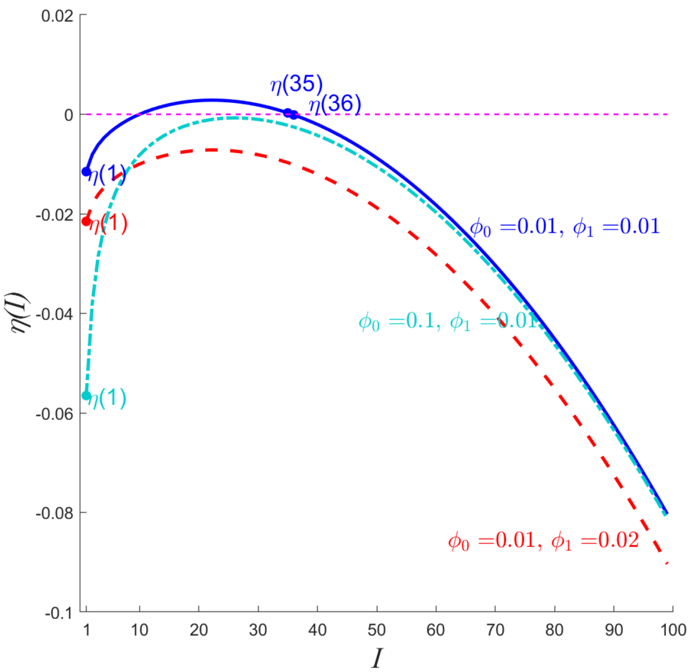
We illustrate these two findings in Figure 6, which plots function $\eta(I)$ for different parameter values of (ϕ_0, ϕ_1). Other parameter values are the same as Figure 4. We report three combinations of the fixed coordination cost ϕ_0 and the variable coordination cost ϕ_1. When $\phi_0 = 0.01$ and $\phi_1 = 0.01$, there are two equilibrium values of I^*: (1) $I^* = 1$ (i.e., no ICAO is formed); and (2) $I^* = 36$ (i.e., an ICAO with 36 members arises). In the other two cases, we either raise coordination costs by increasing the variable cost ϕ_1 from 0.01 to 0.02 while leaving the fixed coordination cost ϕ_0 alone, or raise coordination costs by increasing the fixed cost ϕ_0 from 0.01 to 0.1 while leaving the variable cost unchanged at ϕ_1. In both cases, no ICAO is formed in equilibrium (i.e., $I^* = 1$), which is intuitive since the coordination cost is high.

Multiple equilibrium I^* and equilibrium selection We focus our attention on the multiple equilibria example with $\phi_0 = \phi_1 = 0.01$. We argue that the equilibrium with the positive I^* is more reasonable and stable. We illustrate our point using a narrative similar to Figure 4. Similar to Figure 4, Figure 7 plots $\frac{\Pi(I)}{I}$, $\frac{\Pi(I+1)}{I+1}$, and $\pi_j(I)$ against the ICAO size I in the top panel and reports their values in the bottom panel. Based on the non-deviation equilibrium concept, there are two equilibrium values of I^*: (1) $I^* = 1$, since $\eta(1) < 0$; and (2) $I^* = 36$, since $\eta(35) > 0$ and $\eta(36) < 0$.

Let us consider the following thought experiment for ICAO formation. Start from $I = 1$. Now all activists are effectively solo activists, and in this case, an activist's payoff is 0.0029. As before, if activist 2 approaches activist 1 and proposes to form an ICAO, then both activists 1 and 2 can only receive a payoff of -0.0085. Thus, activist 1 would not accept activist 2's proposal (and in fact, activist 2 should not even propose in the first place). Thus, according to the (local) non-deviation equilibrium concept, $I = 1$ is an equilibrium.

To break out of the no ICAO equilibrium, we need a group of investors of sufficient size to simultaneously propose to form an ICAO. As opposed to letting activist 2 approach

Figure 6: Benefit Function in the Extended Economy

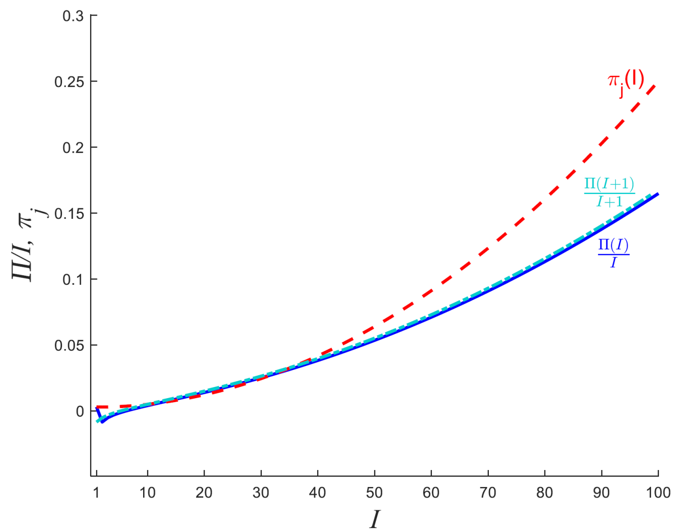


This figure plots the benefit function $\eta(I)$ of joining the ICAO. Parameter values are: $\sigma_z = 30\%, \sigma_x = 2\%, \mu_x = 0.5\%$, and $K = 100$.

activist 1, we assign activist 11 to move, who approaches activists 1, 2,..., and 10 and proposes the ICAO formation. This time, each of the activists 1, 2, ..., and 11 would enjoy a payoff of 0.0050 if an ICAO is formed, which is higher than the alternative if each says no and the ICAO doesn't form where the payoff to being a solo activist is simply 0.0029. This move kicks off the start, and an ICAO would be formed. The game then proceeds as in Section 5.2.1: Activist 12 proposes to join the ICAO, and she would be welcomed since each ICAO member's payoff would increase from 0.0050 to 0.0059. This same logic also applies to the 13th, 14th, ..., and 36th activist. When there are 36 members in the ICAO, the 37th activist would stop proposing to join the ICAO since the payoff of being solo is 0.0342, higher than that of joining the ICAO (0.0341). The ICAO members would not want to quit since the payoff of being in the ICAO (= 0.0327) is higher than that of leaving the ICAO (= 0.0324).

This argument essentially suggests that the non-ICAO equilibrium is unstable but that a stable equilibrium ICAO size I^* is 36.

Figure 7: ICAO Formation in the Extended Economy



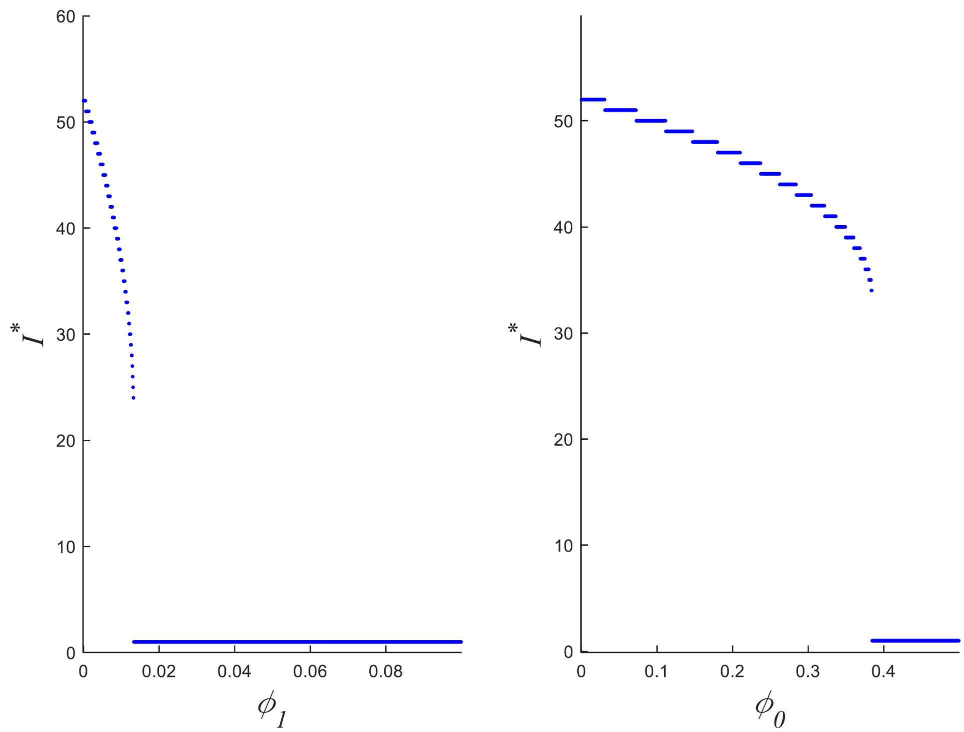
I	**1**	2	3	…	11	12	13	…	33	34	35	**36**	37	38	…	98	99	100
Π/I	0.0029	**-0.0085**	-0.0052	…	0.0050	0.0059	0.0069	…	0.0288	0.0301	0.0314	**0.0327**	**0.0341**	0.0354	…	0.1593	0.1621	0.1649
π_j	**0.0029**	0.0029	0.0029	…	0.0055	0.0060	0.0066	…	0.0291	0.0307	**0.0324**	**0.0342**	0.0360	0.0742	…	0.2403	0.2452	0.2502

This figure plots the benefit function $\eta(I)$ of joining the ICAO. Parameter values are: $\sigma_z = 30\%, \sigma_x = 2\%, \mu_x = 0.5\%, \phi_0 = 0.01, \phi_1 = 0.01$, and $K = 100$.

Discontinuous ICAO formation We conduct comparative statics with respect to coordination cost parameters ϕ_0 and ϕ_1 and report the results in Figure 8. To isolate the role of variable cost and fixed cost, we set $\phi_0 = 0$ in the left panel and $\phi_1 = 0$ in the right panel. Other parameter values are the same as previous figures. In Figure 8, we first employ the non-deviation equilibrium concept to identify the candidate equilibrium ICAO size, and if there are multiple equilibrium values, we use the above thought experiment to refine the equilibrium. One interesting result is that small changes in underlying parameters can cause large changes in the equilibrium ICAO size. For instance, in the left panel, when variable coordination cost ϕ_1 is higher than 0.0132, no ICAO is formed (i.e., $I^* = 1$). However, as ϕ_1 becomes slightly lower than 0.0128, an ICAO is formed, and its equilibrium size jumps suddenly to $I^* = 24$, rather than gradually to 2. In the right panel, there exists a similar abrupt jump of I^* at fixed coordination cost ϕ_0: When ϕ_0 slightly decreases below 0.3839, an ICAO endogenously arises with 34 members.

This is an interesting result. It says that even if there are no ICAOs in a particular market, relatively small changes in coordination costs that motivate a group of investors to act, can lead quickly to large groups of investors acting collectively.

Figure 8: Equilibrium ICAO Size in the Extended Economy



This figure plots the equilibrium ICAO size I^* against the variable coordination cost ϕ_1 and the fixed coordination cost ϕ_0. In the left panel, we set $\phi_0 = 0$. In the right panel, we set $\phi_1 = 0$. Parameter values are: $\sigma_z = 30\%, \sigma_x = 2\%, \mu_x = 0.5\%$, and $K = 100$.

ICAO size limit and ICAO member payoff The other results in the baseline model continue to hold in the extended economy. When an ICAO endogenously arises, as the number of activists become high, the activists will naturally be divided into two groups: ICAO members and solo activists. This is because a large ICAO benefits solo activists through the ICAO-solo-activists externality, which limits the ICAO size. Also, for the same reason, ICAO members receive a lower payoff than solo activists.

Proposition 6 *Suppose that an ICAO endogenously arises in the extended economy. Then:*

(a) When there are sufficiently many activists in the economy, in equilibrium, they are endogenously divided into two types, ICAO members and solo activists.

(b) The equilibrium payoff of an ICAO member is lower than a solo activist's payoff.

7 Conclusion

We develop a framework to study the implications of collective activism of institutional investors through an ICAO and the endogenous formation of an ICAO. In our setting, ICAO members can effectively share their private information and coordinate their activities to pursue a common goal. A larger ICAO increases the average firm value through effective coordination but worsens market liquidity by bringing more private information into the financial market. As the ICAO includes more members, each ICAO member enjoys a higher payoff, but a nonmember activist's payoff may be affected in a non-monotonic way. Nonetheless, the total payoffs of all activists (members and nonmembers) become higher in the presence of a larger ICAO.

We endogenize the creation of an ICAO to study the equilibrium degree of coordination among activists. Our framework allows us to consider various trade-offs faced by activists in determining whether they would like to join the ICAO. When there are many activists in the economy, they endogenously divide into two groups: one group of activists form an ICAO, while the other group of activists are solo activists. Surprisingly, in equilibrium, an ICAO member's payoff is lower than a solo activist's. This is because solo activists can free-ride the value-creation benefit brought by the large player, ICAO. In the presence of coordination costs, an ICAO may fail to exist, but once it exists, it endogenously includes more than two members.

Appendix A: Proofs

Proof of Proposition 1

We first prove that in any linear equilibrium, the trading policies and the pricing rule must take the forms given by equations (18)–(21), and then prove that the equilibrium is characterized by the solution of λ to the 4^{th} order polynomial given in Proposition 1.

The FOC of the ICAO's trading problem is

$$G'(Y) - E\left[p + \Theta\lambda \mid x_1, \ldots, x_I\right] = 0. \tag{A1}$$

By the envelope theorem of the ICAO's activism problem (9), we have

$$G'(Y) = E\left[V \mid x_1, \ldots, x_I\right]. \tag{A2}$$

Taking unconditional expectations yields

$$E\left[G'(Y)\right] = E\left[V\right] = E\left[p\right], \tag{A3}$$

where the second equality follows from the pricing rule (4) of the market maker.

Taking unconditional expectations on equation (A1), we have:

$$E\left[G'(Y)\right] - E\left[p\right] - \lambda E\left[\Theta\right] = 0 \Rightarrow E\left[\Theta\right] = 0,$$

by (A3). Since Θ on average is equal to zero, the optimal policy must take the form of (18). Similarly, we can use the FOC of the j^{th} solo analyst's trading problem and the envelope theorem of its activism problem to show that θ_j must take the form of (19).

We next compute the value of p_0. Again, using (A1) and the expression of (20), we have

$$\begin{aligned} G'(Y) - p_0 - E\left[\omega + \lambda\Theta \mid x_1, \ldots, x_I\right] = 0 \Rightarrow \\ E\left[G'(Y)\right] - p_0 - E\left[\omega + \lambda\Theta\right] = E\left[G'(Y)\right] - p_0 = 0 \Rightarrow \\ p_0 = E\left[G'(Y)\right]. \end{aligned} \tag{A4}$$

Using the expression of $G(Y)$ in (17) and the definition of Y in (2), we can compute

$$E\left[G'(Y)\right] = E\left[IY + J\mu_x\right] = II\mu_x + J\mu_x = \left(I^2 + J\right)\mu_x.$$

Combining the above expression with (A4), we can compute $p_0 = \left(I^2 + J\right)\mu_x$.

Now let us compute the values of λ, β, and δ. Inserting (23) and (26) into (28), we can obtain the 4^{th} order polynomial in terms of λ. The SOCs (24) and (27) require that the solution must be greater than $\max\left\{\frac{I}{2}, \frac{1}{2}\right\} = \frac{I}{2}$ since $I \geq 1$. To establish existence, let us define the 4^{th} order polynomial as

$$\begin{aligned} H(\lambda) \equiv\ & 16Q\lambda^4 - 16Q(I+1)\lambda^3 - 4\left[I^3 + J - Q\left(4I + I^2 + 1\right)\right]\lambda^2 \\ & + 4I\left[I^2 + J - Q(I+1)\right]\lambda - I^2(I + J - Q) = 0. \end{aligned}$$

Direct computation shows

$$H(I/2) = -I^3(I-1)^2 \leq 0.$$

Clearly,

$$\lim_{\lambda \to \infty} H(\lambda) = \infty > 0.$$

Hence, by the intermediate value theorem, there exists a solution $\lambda \in [I/2, \infty)$ to $H(\lambda) = 0$.

Once we compute λ, we can use (23) and (26) to figure out δ and β. QED.

Proof of Corollary 1

Proof of Part (a). Setting $I = 1$ and $J = K - 1$ in the 4^{th} order polynomial determining λ in Proposition 1, we have:

$$4\lambda^2 - 4\lambda + 1 - K\frac{\sigma_x^2}{\sigma_z^2} = 0.$$

Solving the above quadratic equation, we have:

$$\lambda = \frac{1}{2}\left(1 + \sqrt{K}\frac{\sigma_x}{\sigma_z}\right),$$

which satisfies the SOC $\lambda \geq \frac{1}{2}$. Using the expressions of δ and β in Proposition 1, we can compute

$$\delta = \beta = \frac{1}{\sqrt{K}}\frac{\sigma_z}{\sigma_x}.$$

Proof of Part (b). Setting $I = K$ and $J = 0$ in the 4^{th} order polynomial determining λ in Proposition 1, we have:

$$4\lambda^2 - 4K\lambda - K^3\frac{\sigma_x^2}{\sigma_z^2} + K^2 = 0.$$

Solving the above equation and considering the SOC $\lambda \geq \frac{K}{2}$, we obtain

$$\lambda_K = \frac{K}{2}\left(1 + \sqrt{K}\frac{\sigma_x}{\sigma_z}\right).$$

Inserting this expression into the expression of δ in Proposition 1, we have

$$\delta = \frac{1}{\sqrt{K}}\frac{\sigma_z}{\sigma_x}.$$

QED.

Proof of Corollary 2

Suppose $I = 1$. Then, by Part (a) of Corollary 1, the results in Corollary 2 hold.

Now suppose $I > 1$ and consider $\sigma_z \to 0$. First, let us prove $\lambda \to \infty$. Suppose not. Then, λ must approach to a finite value which is larger than $\frac{I}{2}$. The left-hand-side (LHS) of the fourth-order polynomial in Proposition 1 will approach to

$$-4\left(I^3 + J\right)\lambda^2 + 4I\left(I^2 + J\right)\lambda - I^2(I + J),$$

which is negative since its discriminant is $-16JI^3(I-1)^2$. Thus, a contradiction.

Next, let us prove the order of λ. Retaining the highest order of the LHS of the fourth-order polynomial in Proposition 1, we have:

$$16Q\lambda^4 - 4\left(I^3 + J\right)\lambda^2 \to 0.$$

This implies

$$\lambda \propto \frac{\sqrt{I^3 + J}}{2}\frac{\sigma_x}{\sigma_z}.$$

Inserting the above equation into the expressions of δ and β in Proposition 1, we have:

$$\delta \propto \frac{I}{\sqrt{I^3 + J}}\frac{\sigma_z}{\sigma_x}\text{and}\beta \propto \frac{1}{\sqrt{I^3 + J}}\frac{\sigma_z}{\sigma_x}.$$

QED.

Proof of Proposition 2

Proof of Part (a). By Proposition 1, we can compute

$$E\left(p\right) = E\left(V\right) = p_0 = \mu_x\left(I^2 + J\right) = \mu_x\left(I^2 + K - I\right).$$

Thus,

$$\frac{\partial E\left(p\right)}{\partial I} = \frac{\partial E\left(V\right)}{\partial I} = \frac{\partial p_0}{\partial I} = \mu_x\left(2I - 1\right) > 0(\text{by}I \geq 1).$$

Proof of Part (b). By Corollary 2, we have

$$\begin{aligned}
\lambda &\propto \frac{\sqrt{I^3 + J}}{2}\frac{\sigma_x}{\sigma_z} = \frac{\sqrt{I^3 - I + K}}{2}\frac{\sigma_x}{\sigma_z}, \\
\beta &\propto \frac{1}{\sqrt{I^3 + J}}\frac{\sigma_z}{\sigma_x} = \frac{1}{\sqrt{I^3 - I + K}}\frac{\sigma_z}{\sigma_x}, \\
\delta &\propto \frac{I}{\sqrt{I^3 + J}}\frac{\sigma_z}{\sigma_x} = \frac{I}{\sqrt{I^3 - I + K}}\frac{\sigma_z}{\sigma_x}.
\end{aligned}$$

Since $\frac{\partial\left(I^3 - I + K\right)}{\partial I} = 3I^2 - 1 > 0$ (by $I \geq 1$), we have $\frac{\partial\lambda}{\partial I} > 0$ and $\frac{\partial\beta}{\partial I} < 0$.

Direct computation shows $\frac{\partial}{\partial I}\ln\frac{I}{\sqrt{I^3 - I + K}} = -\frac{I^3 + I - 2K}{2I(I^3 - I + K)}$. Thus,

$$\frac{\partial\delta}{\partial I} < 0 \Longleftrightarrow I^3 + I - 2K > 0 \Longleftrightarrow I^3 + I > 2K.$$

By $p_0 = \mu_x\left(I^2 + J\right) = \mu_x\left(I^2 + K - I\right)$, we have $\frac{\partial p_0}{\partial I} = \mu_x\left(2I - 1\right) > 0$.

Next, we prove the properties of activism effort. From Proposition 1, we have

$$\begin{aligned}
E\left(v_1 + ... + v_I\right) &= E\left(IY\right) = IE\left(\Theta + \sum\nolimits_{i=1}^{I} x_i\right) = I^2\mu_x, \\
E\left(v_{I+j}\right) &= E\left(y_j\right) = \mu_x, \text{ for } j = 1, ..., J.
\end{aligned}$$

Thus,

$$\begin{aligned}
\frac{\partial E\left(v_1 + ... + v_I\right)}{\partial I} &= 2I\mu_x > 0, \\
\frac{\partial E\left(v_{I+j}\right)}{\partial I} &= 0, \text{ for } j = 1, ..., J,
\end{aligned}$$

Again, from the expressions of v_k in Proposition 1, we can compute

$$\begin{aligned} Var\left(v_1+...+v_I\right) &= I^2 Var\left(Y\right) = I^2\left(1+\delta\right)^2 Var\left(\sum_{i=1}^{I} x_i\right) = I^3\left(1+\delta\right)^2\sigma_x^2, \\ Var\left(v_{I+j}\right) &= Var\left(y_j\right) = \left(1+\beta\right)^2\sigma_x^2, \\ Var\left(V\right) &= I^3\left(1+\delta\right)^2\sigma_x^2 + \left(K-I\right)\left(1+\beta\right)^2\sigma_x^2. \end{aligned}$$

Using the expressions of δ and β in Corollary 2, we have

$$\begin{aligned} Var\left(v_1+...+v_I\right) &\propto I^3\sigma_x^2, \\ Var\left(v_{I+j}\right) &\propto \left(1+2\beta\right)\sigma_x^2 = \left(1+2\frac{1}{\sqrt{I^3-I+K}}\frac{\sigma_z}{\sigma_x}\right)\sigma_x^2, \\ Var\left(V\right) &\propto \sigma_x^2\left(I^3-I+K\right). \end{aligned}$$

Thus, $\frac{\partial Var(v_1+...+v_I)}{\partial I} > 0$, $\frac{\partial Var\left(v_{I+j}\right)}{\partial I} < 0$ and $\frac{\partial Var(V)}{\partial I} > 0$. QED.

Proof of Proposition 3

Proof of Part (a). Using (31), we can compute

$$\begin{aligned} & \frac{\Pi\left(I+1\right)}{I+1} - \frac{\Pi\left(I\right)}{I} \\ = & \left(\frac{\left(I+1\right)^2}{2} - \left(I+1\right) + K\right)\mu_x^2 - \left(\frac{I^2}{2} - I + K\right)\mu_x^2 \\ & + \frac{\left(I+1\right)\lambda_{I+1}}{2\lambda_{I+1} - \left(I+1\right)}\sigma_x^2 - \frac{I\lambda_I}{2\lambda_I - I}\sigma_x^2 \\ = & \left(I - \frac{1}{2}\right)\mu_x^2 + \frac{\lambda_{I+1}\left[2\lambda_I - I\left(I+1\right)\right] + I\left(I+1\right)\lambda_I}{\left(2\lambda_I - I\right)\left(2\lambda_{I+1} - I - 1\right)}\sigma_x^2, \end{aligned}$$

where λ_I and λ_{I+1} are the Kyle's lambda when the ICAO size is I and $I+1$, respectively. So, in order to show $\frac{\Pi(I+1)}{I+1} > \frac{\Pi(I)}{I}$, it suffices to show that $\lambda_{I+1}\left[2\lambda_I - I\left(I+1\right)\right] + I\left(I+1\right)\lambda_I > 0$.

By the SOC (24), we have $\lambda_{I+1} > \frac{I+1}{2}$, and thus

$$\begin{aligned} & \lambda_{I+1}\left[2\lambda_I - I\left(I+1\right)\right] + I\left(I+1\right)\lambda_I \\ > & \frac{I+1}{2}\left[2\lambda_I - I\left(I+1\right)\right] + I\left(I+1\right)\lambda_I \\ = & \frac{1}{2}\left(I+1\right)^2\left(2\lambda_I - I\right) > 0, \text{ by (24).} \end{aligned}$$

Proof of Part (b). By the expression π_j in (33), we have

$$\begin{aligned} & \pi_j\left(I+1\right) - \pi_j\left(I\right) \\ = & \left[\left(I+1\right)^2 - \left(I+1\right) - \left(I^2 - I\right)\right]\mu_x^2 + \left(\frac{\lambda_{I+1}}{2\lambda_{I+1} - 1} - \frac{\lambda_I}{2\lambda_I - 1}\right)\sigma_x^2 \\ = & 2I\mu_x^2 + \left(\frac{\lambda_{I+1}}{2\lambda_{I+1} - 1} - \frac{\lambda_I}{2\lambda_I - 1}\right)\sigma_x^2. \end{aligned}$$

As I approaches ∞, λ_I and λ_{I+1} become close to each other, and so $\left(\frac{\lambda_{I+1}}{2\lambda_{I+1}-1} - \frac{\lambda_I}{2\lambda_I-1}\right)\sigma_x^2$ approaches 0. The term $2I\mu_x^2$ approaches ∞ as I approaches ∞. Thus, for large values of I, we must have $\pi_j(I+1) > \pi_j(I)$.

Using the expression π_j in (33) and the expression of λ in Corollary 2, we can show that for sufficiently small σ_z,

$$\pi_j \propto \left(I^2 - I - \frac{1}{2} + K\right)\mu_x^2 + \frac{\sigma_x^2}{2}.$$

Thus, by continuity, we must have $\pi_j(I+1) > \pi_j(I)$ for small values of σ_z. QED.

Proof of Proposition 4

Proof of Part (a)

Suppose $\pi_j(1) \geq \frac{\Pi(2)}{2}$. Then, $I^* = 1$ is an equilibrium. Otherwise, check whether $\pi_j(2) \leq \frac{\Pi(3)}{2}$. If yes, then $I^* = 2$ is an equilibrium. Otherwise, continue to check whether $\pi_j(3) \leq \frac{\Pi(4)}{2}$. This process continues until $I = K - 1$, and if $\pi_j(K-1) \leq \frac{\Pi(K)}{K}$, then $I^* = K$. In this process, we are ensured to find an equilibrium ICAO size I^*.

We can rewrite equation (38) as follows:

$$\eta(I) = \frac{I(I-2)}{2\sigma_x^2}\Delta(I),$$

where

$$\Delta(I) \equiv \underbrace{\frac{2\left[(I+1)\lambda_I + \lambda_{I+1}(2\lambda_I I - I - 1)\right]}{I(I-2)(2\lambda_I - 1)(2\lambda_{I+1} - I - 1)}}_{\equiv\rho\left(I,K,\frac{\sigma_z^2}{\sigma_x^2}\right)} - \frac{\mu_x^2}{\sigma_x^2}. \tag{A5}$$

The term $\rho\left(I, K, \frac{\sigma_z^2}{\sigma_x^2}\right)$ in (A5) depends only on the values of $\left(I, K, \frac{\sigma_z^2}{\sigma_x^2}\right)$, because both λ_I and λ_{I+1} are solely determined by $\left(I, K, \frac{\sigma_z^2}{\sigma_x^2}\right)$ in Proposition 1. Thus, the value of I^* depends only on $\left(K, \frac{\sigma_z^2}{\sigma_x^2}, \frac{\mu_x^2}{\sigma_x^2}\right)$.

Proof of Part (b)

To prove Part (b), we need to prove

$$\frac{\Pi(2)}{2} > \pi_j(1), \tag{A6}$$

$$\frac{\Pi(3)}{3} > \pi_j(2). \tag{A7}$$

Let us first prove (A7). Setting $I = 2$ in equation (38), we have

$$\frac{\Pi(3)}{3} - \pi_j(2) = \frac{3\lambda_2 + \lambda_3(4\lambda_2 - 3)}{(2\lambda_3 - 3)(2\lambda_2 - 1)}\sigma_x^2 > 0$$

because $\lambda_2 > 1$ and $\lambda_3 > \frac{3}{2}$ by Proposition 1.

Now, let us prove (A6). Setting $I = 1$ in equation (38) and using the expression of λ_1 in Part (a) of Corollary 1, we have

$$\frac{\Pi(2)}{2} > \pi_j(1) \Longleftrightarrow \frac{\lambda_2}{\lambda_2 - 1} > \frac{1}{2}\left(1 + \frac{1}{\sqrt{KS}}\right) \Longleftrightarrow \frac{1}{\sqrt{KS}} + 1 > \left(\frac{1}{\sqrt{KS}} - 1\right)\lambda_2, \quad \text{(A8)}$$

where $S \equiv \frac{\sigma_x^2}{\sigma_z^2}$, and λ_2 is determined by the following fourth-order polynomial in Proposition 1 with $I = 2$:

$$4\lambda_2^4 - 12\lambda_2^3 + (13 - KS - 6S)\lambda_2^2 + (4S + 2KS - 6)\lambda_2 + (1 - KS) = 0. \quad \text{(A9)}$$

If $KS > 1$, then condition (A8) is automatically satisfied, since the right-hand side is negative and the left-hand side is positive. If $KS < 1$, then condition (A8) is equivalent to

$$\lambda_2 < \frac{1 + \sqrt{KS}}{1 - \sqrt{KS}},$$

which holds true according to the following lemma. QED.

Lemma A1 *Suppose $KS < 1$ and $I = 2$. Then, in the trading game, there is an equilibrium and the Kyle's lambda λ is in the range of $\left(1, \frac{1+\sqrt{KS}}{1-\sqrt{KS}}\right)$.*

Proof of Lemma A1. Define the fourth-order polynomial in (A9) by $F(\lambda)$. The idea of proving Lemma A1 is to show (1) that $F(1) < 0$ and $F(\frac{1+\sqrt{KS}}{1-\sqrt{KS}}) > 0$ (so that there is a solution in $\left(1, \frac{1+\sqrt{KS}}{1-\sqrt{KS}}\right)$); and (2) that $F(\lambda) > 0$ for $\lambda > \frac{1+\sqrt{KS}}{1-\sqrt{KS}}$ (so that there is no solution in $\left(\frac{1+\sqrt{KS}}{1-\sqrt{KS}}, \infty\right)$).

Direct computation shows

$$F(1) = -2S < 0.$$

Define $t \equiv \sqrt{KS} \in (0, 1)$ and $S = \frac{t^2}{K}$. Then, we can show that $F(\frac{1+\sqrt{KS}}{1-\sqrt{KS}})$ has the same sign as

$$(2K - 1) + t\left[2K\left(1 - t^2\right) + 5\left(K - t^2\right) + 3(K - 1) + 3t(2K + 3)\right] > 0.$$

Thus, $F(\frac{1+\sqrt{KS}}{1-\sqrt{KS}}) > 0$.

We next show that $F(\lambda)$ is increasing in $\left(\frac{1+\sqrt{KS}}{1-\sqrt{KS}}, \infty\right)$ so that $F(\lambda) > 0$ for $\lambda > \frac{1+\sqrt{KS}}{1-\sqrt{KS}}$. To do so, we will show that $f(\lambda) = F'(\lambda) > 0$ for $\lambda > \frac{1+\sqrt{KS}}{1-\sqrt{KS}}$. Direct computation shows

$$f(\lambda) = 16\lambda^3 - 36\lambda^2 + 2(13 - KS - 6S)\lambda + (4S + 2KS - 6).$$

We proceed in two steps. First, we show $f(\frac{1+\sqrt{KS}}{1-\sqrt{KS}}) > 0$. Second, we show that $f(\lambda)$ is increasing in $\left(\frac{1+\sqrt{KS}}{1-\sqrt{KS}}, \infty\right)$ so that $f(\lambda) > 0$ for $\lambda > \frac{1+\sqrt{KS}}{1-\sqrt{KS}}$.

First, $f(\frac{1+\sqrt{KS}}{1-\sqrt{KS}})$ has the same sign as

$$A(t) \equiv -(K+4)t^4 + 2(K+3)t^3 + 20Kt^2 + 2(5K-1)t + K \text{with} t \equiv \sqrt{KS} \in (0,1).$$

Note that $A''(t) = -12(K+4)t^2 + 12(K+3)t + 40K$ is concave in t. Also, $A(0) = 40K > 0$ and $A(1) = 4(10K-3) > 0$. Thus, $A''(t) > (1-t)A''(0) + tA''(1) > 0$. Therefore, $A'(t)$ is increasing in t. Since $A'(0) = 2(5K-1) > 0$, we have $A'(t) > 0$ for $t \in (0,1)$. As a result, $A(t)$ is increasing in $t \in (0,1)$. Hence, $A(t) > A(0) = K > 0$. This implies that $f(\frac{1+\sqrt{KS}}{1-\sqrt{KS}}) > 0$.

Second, let us show $f'(\lambda) > 0$ for $\lambda > \frac{1+\sqrt{KS}}{1-\sqrt{KS}}$. Direct computation shows

$$f'(\lambda) = 48\lambda^2 - 72\lambda + 2(13 - KS - 6S).$$

Note that for $\lambda > \frac{1+\sqrt{KS}}{1-\sqrt{KS}}$, only the right increasing branch of $f'(\lambda)$ is relevant. So, it suffices to show $f'(\frac{1+\sqrt{KS}}{1-\sqrt{KS}}) > 0$. We can show that $f'(\frac{1+\sqrt{KS}}{1-\sqrt{KS}})$ has the same sign as

$$B(t) \equiv -6t^4 + (12-K)t^3 + (75K-6)t^2 + 21Kt + K.$$

Again, $B''(t) = -72t^2 + 6(12-K)t + 2(75K-6)$ is concave. So, $B''(t) > (1-t)B(0) + tB(1) = (1-t)K + t12(12K-1) > 0$. Hence, $B'(t)$ is increasing, and $B'(t) > B'(0) = 21K > 0$. Therefore, $B(t)$ is increasing, and $B(t) > B(0) = K$ for all $t \in (0,1)$. QED.

Proof of Part (c)

Inserting $I = K$ into (38), we have

$$\begin{aligned}
&\frac{1}{\sigma_x^2}\left[\frac{\Pi(K)}{K} - \pi_j(K-1)\right] \\
= &-\frac{(K-1)(K-3)}{2}\frac{\mu_x^2}{\sigma_x^2} + \frac{K\lambda_{K-1} + \lambda_K(2\lambda_{K-1}(K-1) - K)}{(2\lambda_{K-1}-1)(2\lambda_K - K)}.
\end{aligned}$$

By Part (b) of Corollary 1, we have

$$\lambda_K = \frac{K}{2}\left(1 + \sqrt{KS}\right) \propto \frac{K\sqrt{KS}}{2},$$

where "$X \propto Y$" means that $\lim_{K\to\infty} \frac{X}{Y} = 1$. Also note that $\lambda_{K-1} \propto \lambda_K$. Thus,

$$\begin{aligned}
&\frac{1}{\sigma_x^2}\left[\frac{\Pi(K)}{K} - \pi_j(K-1)\right] \\
\propto &-\frac{(K)(K)}{2}\frac{\mu_x^2}{\sigma_x^2} + \frac{K\frac{K\sqrt{KS}}{2} + \frac{K\sqrt{KS}}{2}\left(2\frac{K\sqrt{KS}}{2}K\right)}{\left(2\frac{K\sqrt{KS}}{2}\right)2\frac{K\sqrt{KS}}{2}} \\
\propto &-\frac{K^2}{2}\frac{\mu_x^2}{\sigma_x^2} + \frac{K}{2} < 0.
\end{aligned}$$

In consequence, as K becomes sufficiently large, we have $\frac{\Pi(K)}{K} < \pi_j(K-1)$, which means that $I^* < K$. QED.

Proof of Proposition 5

Part (b) immediately follows from Part (a). So, our proof focuses on Part (a). Fix (K, I, σ_x) and let $\sigma_z \to 0$. By Proposition 2, we know $\lambda \to \infty$. Thus, the net benefit $\frac{\Pi(I+1)}{I+1} - \pi_j(I)$ of joining an ICAO given by equation (38) approaches

$$\frac{I\sigma_x^2}{2}\left[1 - (I-2)\frac{\mu_x^2}{\sigma_x^2}\right],$$

which is downward sloping in I and crosses 0 at $\hat{I} = 2 + \frac{\sigma_x^2}{\mu_x^2}$. So, the equilibrium ICAO size is $I^* = 2 + \left[\frac{\sigma_x^2}{\mu_x^2} + 1\right] = 3 + \left[\frac{\sigma_x^2}{\mu_x^2}\right]$ as long as $K \geq 3$. When $K \leq 3$, we know $I^* = K$. Taken together, we have $I^* = \min\{K, 3 + [\sigma_x^2/\mu_x^2]\}$. QED.

Proof of Proposition 6

The proof of Part (a) of Proposition 6 is similar to the proof of Part (c) of Proposition 4. The proof of Part (b) of Proposition 6 is similar to the proof of Corollary 3. The details are thus omitted. QED.

Appendix B: An Extension with ICAO Enagement Benefits and Coordination Costs

When employing an activism technology, the ICAO can be more forceful than a solo activist. In this appendix, we provide an extension that allows for this ICAO engagement benefits. Consider activism technology k. An activism level v_k still costs $c(v_k)$ given by (6). But how this level of activism influences the firm value depends on who is engaging in activism. Specifically, if the activism effort v_k is implemented by a solo activist, then it will add firm value by $\Gamma(1)v_k$, and if v_k works through the ICAO, then it will add firm value by $\Gamma(I)v_k$. Here, $\Gamma(I)$ is an increasing function of I, so that $\Gamma(I) \geq \Gamma(1)$. In our analysis, we specify the following Γ function:

$$\Gamma(I) = \frac{I}{I+\gamma} \in (0, 1] \text{ with } \gamma \geq 0. \tag{A10}$$

Accordingly, the firm value in (5) changes to

$$V = \Gamma(I)\sum_{i=1}^{I} v_i + \Gamma(1)\sum_{j=1}^{J} v_{I+j}. \tag{A11}$$

For instance, we can interpret $\Gamma(I)$ as the probability that an activism proposal is accepted by the board of directors. Let us consider an example with three activists. Activists 1 and 2 form an ICAO, and activist 3 is a solo activist. Let $\Gamma(1) = \frac{1}{2}$ and $\Gamma(2) = \frac{2}{3}$. Consider activist 3 first. Its effort level v_3 is positively related to the number of proposals.

For instance, if $v_3 = \$50$, this can mean that it provides 50 proposals, and each proposal can add firm value by \$1, but only $\Gamma(1) = \frac{1}{2}$ of these proposals are approved by the board. So, the effective activism level is $\Gamma(1)v_3 = \frac{1}{2} \times \$50 = \$25$. By contrast, the ICAO can press the board more so that its proposals are more likely to be accepted.[10] As a result, the acceptance rate of the ICAO's proposals increases from $\Gamma(1) = \frac{1}{2}$ to $\Gamma(2) = \frac{2}{3}$. In this case, the firm value becomes $V = \frac{1}{2}(v_1 + v_2) + \frac{2}{3}v_3$, as in (A11).

Except the addition of coordination costs, the other features of our baseline model in Section 2 remain unchanged. The baseline model corresponds to $\phi_0 = \phi_1 = \gamma = 0$. We can follow the same procedure as Section 2 and compute the equilibrium in this extended economy.

Proposition 7 *In period 1, there exists a Bayesian Nash equilibrium in the activism game, with activism policies of the ICAO and solo activist j respectively given by*

$$\begin{aligned} v_i &= \Gamma(I)Y, \text{ for } i = 1, 2, ..., I, \\ v_{I+j} &= \Gamma(1)y_j, \text{ for } j = 1, 2, ..., J, \end{aligned}$$

where

$$\begin{aligned} Y &= x_1 + x_2 + \cdots + x_I + \Theta, \\ y_j &= x_{I+j} + \theta_j, \text{ for } j = 1, 2, ..., J. \end{aligned}$$

In period 0, there exists an equilibrium in the Kyle trading game, in which the trading rules of the ICAO and solo activist j, and the pricing rule are, respectively,

$$\begin{aligned} \Theta &= \delta \sum_{i=1}^{I} (x_i - \mu_x), \\ \theta_j &= \beta(x_{I+j} - \mu_x), \text{ for } j = 1, 2, ..., J, \\ p &= p_0 + \lambda\omega, \end{aligned}$$

with

$$\omega = \sum\nolimits_{j=1}^{J} \theta_j + \Theta + z,$$

and $\lambda \geq \frac{\Gamma(I)^2 I}{2}$ *is the real root of the following fourth-order polynomial:*

$$\begin{aligned} & 16Q\lambda^4 - 16Q(\Gamma(1)^2 + I\Gamma(I)^2)\lambda^3 \\ & -4[I^2(I-Q)\Gamma(I)^4 - 4IQ\Gamma(I)^2\Gamma(1)^2 + (J-Q)\Gamma(1)^4]\lambda^2 \\ & +4I\Gamma(I)^2\Gamma(1)^2[I(I-Q)\Gamma(I)^2 + (J-Q)\Gamma(1)^2]\lambda \\ & -I^2(I+J-Q)\Gamma(I)^4\Gamma(1)^4 \\ = {} & 0, \text{ with } Q \equiv \sigma_z^2/\sigma_x^2, \end{aligned}$$

[10] "It's one thing to feel the scorn of a 3% shareholder; it's another to face down 10 institutions holding half your float." ("Stephen Jarislowsky has Every Right to Say 'I told you so'," The Globe and Mail, October 25, 2002.)

and

$$\delta = \frac{\Gamma(I)^2 I}{2\lambda - \Gamma(I)^2 I}, \beta = \frac{\Gamma(1)^2}{2\lambda - \Gamma(1)^2} \text{ and } p_0 = \left[I^2\Gamma(I)^2 + J\Gamma(1)^2\right]\mu_x.$$

Proposition 7 is a direct extension of Proposition 1. We can show that all of our results in the main text go through in this extended setting with ICAO engagement.

References

Back, K., P. Collin-Dufresne, V. Fos, T. Li, and A. Ljungqvist, 2018. Activism, strategic trading, and liquidity. *Econometrica* 86, 1431–1643.

Bradley, M., A. Brav, I. Goldstein, and W. Jiang, 2010. Activist arbitrage: A study of open-ending attempts of closed-end funds. *Journal of Financial Economics* 95, 1–19.

Collin-Dufresne, P. and V. Fos , 2015. Shareholder activism, informed trading, and stock prices. Working Paper.

Dimson, E., O. Karakis, and X. Li, 2020, Coordinated Engagements.,SSRN.

Doidge, C., A. Dyck, H. Mahmudi, and A. Virani, 2019. Collective action and governance activism. *Review of Finance* 23, 893–893.

Grossman, S. J. and J. E. Stiglitz, 1980. On the impossibility of informationally efficient markets. *American Economic Review* 70, 393–408.

Kyle, A., 1985. Continuous auctions and insider trading. *Econometrica* 53, 1315–1335.

Peress, J. and D. Schmidt, 2018. Noise Traders Incarnate: Describing a Realistic Noise Trading Process, Working Paper.

Ray, Debraj and Rajiv Vohra, 2015, "Coalition Formation," Handbook of Game Theory with Economic Applications, Volume 4, 2015, 239-326.