Long-Term Contracting with Formal and Relational Contracts

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Abstract

This paper explores the interaction between formal and relational contracts in settings in which trading partners engage in a series of overlapping interactions requiring specific investment. The value of investments are not realized or observed until later periods, by which time additional investments need to have been made to maximize the value of the trading relationship. There is in this model therefore a role for long-term, multi-period contracts within an ongoing relationship, whereas most models of repeated contracting consider only repeated one-period contracting problems. In addition, firms may engage in hybrid contracting that employs formal contracts to support self-enforceability of a relational contract. I show that relational and formal contracts are complementary to each other and to contract length in several senses. (1) Given the length of the contract, a purely relational contract is self-enforceable for patient enough players, but incorporating a formal contract to create a hybrid relational contract lowers the threshold discount factor. (2) Exogenous increases in the length of the contracting problem (2a) require higher discount factors to achieve efficient investment; (2b) increase the likelihood that a formal contract is required to achieve self-enforceability of the relational contract; and (2c) require increased dependence on formally contracted payments and decreased reliance on relationally enforced bonuses in the optimal hybrid relational contract. As a result, (3) if agents choose from among projects with varying contract lengths, both higher discount factors and the use of formal contracts to support a relational contract allow the parties to choose longer-term contracts if desirable.

1 Introduction

This paper considers a repeated buyer-seller principal-agent model in which efficiency (and thus joint profit maximization) requires that the seller make a relationship-specific investment in each period; the benefit of this investment accrues to the buyer and may not be realized or observed until later periods. Investment is costly enough that the anticipated outcome of ex post bargaining absent a contract will not generate enough seller surplus to induce investment, and a formal contract cannot be used to induce investment because seller investment is assumed to be unverifiable and therefore noncontractible. However, as seller investment is observable to the parties, self-enforcing relational contracts may be able to induce investment for sufficiently patient players. The contribution of this paper is to extend this relatively familiar contracting setting to the case in which investments are observed, and generate their benefits, in later periods rather than in the same period as the investment. This creates a need for multi-period long-term contracts, and it creates

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complexities in the relational contracting environment since the seller must continue to invest in subsequent periods before knowing whether the relational contract governing prior period investments will be honored. The central research question is how formal and relational contracts optimally work together to support such long-term investments. The analysis shows that formal and relational contracts are complementary, with formal contracts enabling longer-term relational contracts and with longer-term relational contracts relying on a larger proportion of the compensation delivered through the formal contract.

This paper contributes to the literature that explores interaction between formal and relational contracts. Seminal papers include Baker, Gibbons, and Murphy (1994) and Levin (2003); accessible overviews include Malcomson (2013) and Corts (2018). In general, this literature emphasizes that formal and relational contracts may be substitutes–that is, alternative ways of governing a set of transactions–or complements. In particular, formal contracts may complement relational contracts when they reduce temptations to deviate by limiting deviation possibilities, in particular by limiting the ability to reneg on promised payments by making at least a portion of the incentive compensation conditional on verifiable, even if imperfect, performance measures. No paper in that literature considers multi-period long-term contracting. This paper seeks to make a first contribution to filling that gap. In addition, this paper may shed theoretical light on empirical work in this area. In seemingly the only paper to address contract length in the context of relational contracting, Corts and Martinez (2017) find that increased frequency of interaction between contracting parties in the Costa Rican coffee market, which would support stronger relational contracting, leads to longer-term contracts between those parties, a finding that is consistent with the theoretical results obtained here.

In the benchmark case in this paper, in which the investment is short-term–meaning that investment, trade of the intermediate good, observation of investment, and settlement of the contract (whether formal or relational) all occur within a single period–the contracting problem reduces to a standard problem in the literature and the results mirror those in, say, Levin (2003). In particular, a formal contract cannot induce investment; a purely relational contract can induce investment for high enough discount factors; and a hybrid formal-relational contract can induce investment for a wider range of discount factors.

I then proceed to consider the possibility that investments are long-term investments, meaning specifically that the investment made by the seller in period t affects buyer profits and is observed by the buyer in period $t + z$, for $z \geq 1$. This changes the contracting problem because it means that in order to sustain efficient investment in every period, the seller must continue to make investments in periods between t and $t + z$ before observing whether the buyer has honored its commitment to the promised bonus for the investment in period t. I show that relational and formal contracts are complementary to each other and to contract length in several senses. (1) Given the length of the contract, a purely relational contract is self-enforceable for patient enough players, but incorporating a formal contract to create a hybrid relational contract lowers the threshold discount factor. (2) Exogenous increases in the length of the contracting problem (2a) require higher discount factors to achieve efficient investment; (2b) increase the likelihood that a formal contract is required to achieve self-enforceability; and (2c) require increased dependence on formally contracted payments and decreased reliance on relationally enforced bonuses in the optimal hybrid relational contract. As a result, (3) if agents choose from among projects with varying contract lengths, higher discount factors and the use of formal contracts to support a relational contract allow the parties to choose longer-term contracts if desirable.

The intuition for these results is clear: relationally enforced bonuses that are enforced upon observation of effort or investment at the end of a long-term contract have weaker power to induce seller effort as contracts lengthen, while the short-run gain for the buyer to withhold the bonus remains strong. As a result, relationally enforced bonuses become less effective contracting tools as contract terms lengthen. The bonus in the optimal hybrid relational contract therefore falls, and the formally contracted payment rises, as contract terms lengthen. These results relate closely to existing work on the interaction of formal and relational contracts, while extending this analysis into the previously unstudied realm of repeated contracting relationships characterized by overlapping long-term investments.

2 Model

A single buyer and a single seller interact repeatedly. Both parties are risk-neutral and discount the future at a discount factor of δ. The seller creates, at a cost normalized to 0 but for the investment specified below, an intermediate good that it sells to the buyer. If the seller makes a specific investment in period t in adapting its input to the buyer's needs, which it can do at cost $k > 0$, and the buyer acquires this input in period t, the gross surplus accruing to the buyer in period $t + z$, for $z \geq 0$, is $v + w$, where $v, w > 0$. If the buyer acquires the intermediate good from the seller in period t without the investment, the gross surplus accruing to the buyer in period $t + z$ is v. In both cases, this gross surplus accrues directly to the buyer and is unverifiable and noncontractible. Also in both cases, the ownership of the intermediate good must be transferred to the buyer in period t; the lag between period t investment and period $t + z$ surplus realization represents a period during which the buyer is processing and working with the intermediate good.[1] Both seller and buyer have access to an outside option yielding surplus of $v/2$. For the buyer, this is an alternative seller from which the buyer can acquire the intermediate input (without the specific investment) at a price of $v/2$. For the seller, this is an alternative buyer that is willing to pay $v/2$ for the intermediate input (without specific investment).[2]

The seller's payment to the principal and the seller's delivery of the intermediate good are the two verifiable actions that can be formally contracted on. Seller investment in period t, or lack thereof, is observed by the buyer in period $t + z$, which is the period in which the surplus accruing to the buyer reflects the period t investment in the intermediate good. This is meant to capture a case in which a firm buys an intermediate good from a supplier and then earns profit from the sale of a final product created using this input. Because the profit of the seller results from complex manipulation of the input and may be embedded in a large organization, the buyer's profit resulting from the acquisition of the intermediate good (which, if known, suffices to infer the seller's investment) is assumed to be unverifiable.

To focus on the role of relational and hybrid contracts in inducing efficient investment, I make additional assumptions on w and k.[3]

(A1) Investment is efficient: $k < w$.

(A2) Ex post bargaining absent any contract will not induce investment: $k > w/2$.

The timing of a typical period in steady state is as follows.[4]

[1] It is also interesting to consider the possibility that the seller need not take possession at this stage, and that the formal contract can specify trade of the intermediate good in some future period. This is a different contracting problem with different possibilities for holdup.

[2] It is also possible to consider the possibility that the buyer is the one who makes a specific investment that increases the value of pair-specific trade. Because the seller is the claimant on the ultimate value created, one can show that in fact the contractability of trade itself suffices to ensure efficient investment. A fixed (unconditional) price for the trade of the intermediate good suffices to induce investment because it transfers the full claim on the incremental surplus to the investing party. This is true regardless of the length of the delay in observing outcomes and all other parameters of the model.

[3] In principle, given the lag between timing of investment and surplus realization, these constraints should be $k < \delta^z w$ and $k > \delta^z w/2$. However, because I am considering only the steady state in which ongoing investments are made and surplus realized, it suffices to consider the undiscounted costs and benefits. This needs further elaboration.

[4] I will address the start-up phase of the repeated contracting relationship in an appendix.

Stage 0. Bargaining over contracts. This results, per the usual assumptions, in the adoption of a joint-profit maximizing contract that divides the net surplus evenly between the parties.

Stage 1. The seller chooses whether or not to incur the investment cost k for the good to be transacted in this period t.

Stage 2. The intermediate good is traded; contracts conditioned on the trade of the good are settled.

Stage 3. The buyer realizes its gross profit for period t, and observes the seller's investment level in $t-z$, which is the investment that determines period t gross buyer surplus (ie, the buyer's final market outcomes using the input acquired in period $t-z$); contracts conditioned on observation of investments in period $t-z$ are settled.

Note that this ordering of stages is not arbitrary if one wishes to nest the $z = 0$ case, the standard case considered in the literature. When $z = 0$, every transaction in a repeated contracting relationship is self-contained and fully concluded prior to contracting for the next transaction. To accommodate this case, Stage 1 must precede Stage 2 (the investment must be made before the good changes hands) and Stage 2 must precede Stage 3 (the good must change hands before the final market outcome can be realized). If one restricts attention to cases in which $z \geq 1$, one can in principle reverse Stages 2 and 3; that is, the buyer might observe its period t profit and the $t - z$ investment prior to taking possession of the period t good when the period t intermediate good is not involved in the production of the buyer's period t final market output. I will show in an appendix that making this reversal does not meaningfully change the results.[5]

3 Analysis: short-term contracts (z=0)

First consider the case in which $z = 0$ as in the existing literature. In this case, this period's investments are observed in-period, prior to contracting over or investing in goods to be traded in subsequent periods. By assessing the incentive constraints, it is easy to establish the benchmark results on formal, purely relational, and hybrid contracts in this model.

3.1 Formal contracts only

First note that formal contracts alone cannot induce investment. They can be conditioned only on delivery of the good and not on investment. Thus, the seller's payment from the buyer is always flat with respect to whether the seller undertakes the costly investment. For any positive cost k, the seller will not invest. If the parties agree to a formal contract, it will specify only that the intermediate good (with no reference to investment) be traded at $v/2$, yielding both parties an outcome exactly equivalent to the outside option.

Proposition 1 *Assume a short-term contracting environment ($z = 0$). Formal contracts alone cannot induce efficient investment for any discount factor.*

3.2 Purely relational contracts

Second, it is expositionally useful to develop the results by first considering a purely relational contract before examining hybrid relational contracts that incorporate formal contracts as part of the agreement. In a "purely relational" contract, no formal contract is used, meaning that there is not even formal contracting

[5] Follow-up needed.

on delivery of the good; even that payment is only relationally enforced.[6] The literature demonstrates that in simple stationary models such as this one, it suffices to consider relational contracts of the following form: the buyer pays the seller a price p conditional on investment; the seller invests; and both parties continue with this agreement until one deviates, at which point both parties revert to the outside option. Note that in order to yield the investing seller half the net surplus, the buyer must set the price p equal to the cost of investment k plus half the net surplus ($(v+w-k)/2$): thus, $p=(v+w+k)/2$.[7]

In this case, the incentive constraints of the parties are as follows. The seller must not prefer to shirk, forfeit the bonus, and receive the outside option payoff forever rather than to invest, receive the bonus, and receive half the net surplus forever. Let $IC_z^x(y)$ denote the incentive constraint for player y ($y=B,S$) under contracts of form x ($x=PR$ denotes purely relational contracts; the absence of superscripts denotes the case of hybrid relational contracts) in a setting with investment lag of z.

$$IC_0^{PR}(S): 0+\frac{\delta}{1-\delta}\frac{v}{2} \leq (p-k)+\frac{\delta}{1-\delta}\frac{v+w-k}{2}.$$

Note that this holds for all discount factors δ since both components of the RHS are larger than their corresponding components on the LHS. Note that $p-k>0$ because $p-k$ reduces to $(v+w-k)/2$, which is positive by (A1), and that $(v+w-k)/2>v/2$ reduces to $w-k>0$, which is also true by (A1). Intuitively, the agent has no incentive to shirk on the investment because such shirking is detected immediately and punished harshly, both in the withholding of the current period bonus and through reversion to the outside option payoff in future periods.

The buyer must not prefer to withhold the bonus in this period and revert to the outside option payoff rather than to pay the bonus and continue with investments in the future.

$$IC_0^{PR}(B): (v+w)+\frac{\delta}{1-\delta}\frac{v}{2} \leq (v+w-p)+\frac{\delta}{1-\delta}\frac{v+w-k}{2}.$$

Note that, unlike the seller, the buyer does have some positive short-run incentive to deviate from this relational contract (ie, to withhold the bonus payment and expropriate the investment); $IC_0^{PR}(B)$ will therefore not hold for all discount factors. Rearranging, $IC_0^{PR}(B)$ can be simplified to

$$IC_0^{PR}(B): p \leq \frac{\delta}{1-\delta}\frac{w-k}{2}.$$

The short-run temptation to withhold the bonus must be less than the discounted future value of the relationship that will be lost if the relational contract is terminated. By (A1), the discounted quantity $\frac{w-k}{2}>0$; thus the inequality holds for $\delta \to 1$ and this constraint will be satisfied for all δ above some threshold discount factor $\tilde{\delta}_0^{PR}$. Substituting $p=(v+w+k)/2$ and rearranging for the threshold discount factor for the purely relational contract yields $\tilde{\delta}_0^{PR}=\frac{v+w+k}{v+2w}$. Note that $\tilde{\delta}_0^{PR}<1$ because $k<w$ by (A1).

[6] This might be the case if the seller is working on or contributing to an asset owned by the buyer and the work on or contribution to that asset is unverifiable; in such cases the "trade" of the intermediate good may not be verifiable. More commonly, this assumption that no formal contracts are possible is made when considering contracting environments with weak legal institutions and ineffective contract enforcement.

[7] Throughout the paper I will constrain the posited relational contract to achieve equal surplus division as dictated by ex ante bargaining with equal outside options. Therefore, the results apply to contracts constrained in this way, even if I describe them as applying to all contracts that induce efficient investment. For example, there may be in this subsection's model of pure relational contracts the possibility to induce efficient investment in a relational contract in which the seller gets less than half the surplus at a lower discount rate than the one that ensures self-enforceability of the relational contract in which the seller gets half the surplus.Considering such possibilities complicates the analysis without enriching the insights; I therefore restrict attention to contracts that both induce efficient investment and achieve equal surplus division.

Thus, the purely relational contract can always be induced by a self-enforcing purely relational contract if the players are patient enough. In fact, the threshold discount factor is lower (it is easier to sustain the purely relational contract) the more value is created by the relationship–that is, the greater the gap between w and k.

Proposition 2 *Assume a short-term contracting environment ($z = 0$). Pure relational contracts can induce efficient investment for patient enough players ($\delta > \widetilde{\delta}_0^{PR}$; $\widetilde{\delta}_0^{PR} < 1$).*

3.3 Hybrid relational contracts

Now consider hybrid relational contracts. In a hybrid relational contract the buyer compensates the agent both through a formally contracted fixed price f for the verifiable delivery of the good and a relationally enforced bonus b upon the (unverifiable) observation of seller investment. To achieve the equal division of net surplus, $f + b = (v + w + k)/2$. It will be convenient to analyze the optimal bonus because it appears in both $IC(S)$ and $IC(B)$, and to treat the fixed price f as derivative of the bonus: $f = (v + w + k)/2 - b$.

The seller's incentive constraint is changed since the seller now stands to lose only the relationally contracted bonus and not the entire payment if it shirks on the investment.

$$IC_0(S) : f + \frac{\delta}{1-\delta}\frac{v}{2} \leq (f + b - k) + \frac{\delta}{1-\delta}\frac{v + w - k}{2}.$$

It is possible now that the seller could have a short-run incentive to deviate, saving the investment cost but forgoing only the smaller relationally contracted bonus portion of the agreed compensation. Rearranging,

$$IC_0(S) : b \geq k - \frac{\delta}{1-\delta}\frac{w-k}{2}.$$

Similarly, the buyer's incentive constraint is changed because withholding the bonus is less attractive than withholding the entire payment; committing to a portion of the payment through a formal contract reduces incentives to expropriate the investment by withholding the bonus.

$$IC_0(B) : (v + w - f) + \frac{\delta}{1-\delta}\frac{v}{2} \leq (v + w - f - b) + \frac{\delta}{1-\delta}\frac{v + w - k}{2}.$$

Rearranging,

$$IC_0(B) : b \leq \frac{\delta}{1-\delta}\frac{w-k}{2}.$$

Comparing these incentive constraints $IC_0(S)$ and $IC_0(B)$, one can see that the bonus b must be large enough to induce seller investment ($IC_0(S)$ is a lower bound on the bonus b) and yet small enough to induce the buyer to pay the bonus ($IC_0(B)$ is an upper bound on the bonus b). This analysis is most easily understood graphically.

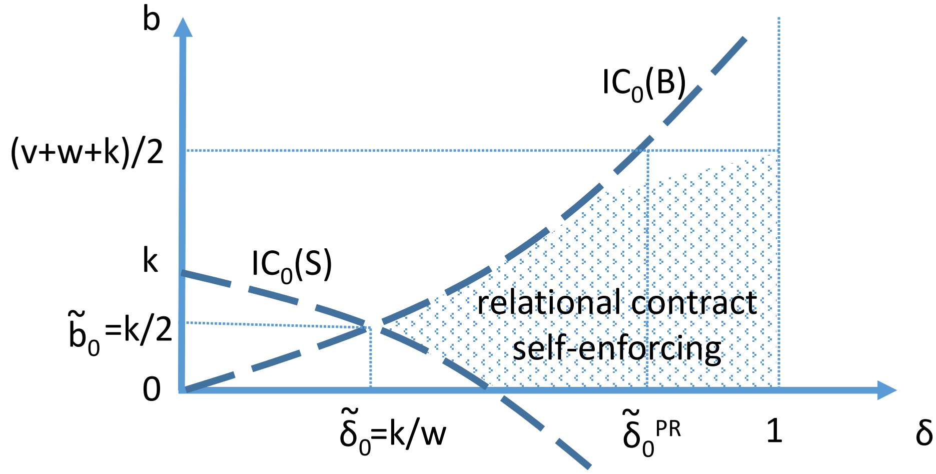

Figure 1: Short-term contracts (z=0)

Figure 1 depicts the $IC_0(B)$ and $IC_0(S)$ constraints in δ, b space. Note that in fact the bonus b is constrained (if the fixed component is to be non-negative) to lie between 0 and $\frac{v+w+k}{2}$. The horizontal axis ($b = 0$) represents the "all-wage" contract, in which the entire payment is formally contracted, while the $b = \frac{v+w+k}{2}$ line represents the purely relational "all-bonus" contract, in which the entire payment is relationally contracted and the formal contract is not required for self-enforceability.[8]

Because $IC_0(S)$ decreases without bound as $\delta \to 1$ and $IC_0(B)$ increases without bound as $\delta \to 1$, and because $IC_0(S)$ lies above $IC_0(B)$ at $\delta = 0$, these constraints must intersect at some $\widetilde{\delta}_0 \in (0, 1)$ and some $\widetilde{b}_0 \in (0, \frac{v+w+k}{2})$. Combining the incentive constraints, one can derive the threshold discount factor under the optimal hybrid contract: $\widetilde{\delta}_0 = \frac{k}{w}$, with the hybrid relational contract being self-enforcing at this threshold discount factor for $\widetilde{b}_0 = \frac{k}{2}$. In a slight abuse of terminology, I will refer to this bonus $\widetilde{b}_0$ as the "optimal bonus" for a particular k, w, z, even though the bonus is fully determined only at this threshold discount factor $\widetilde{\delta}_0$ where the bonus is is dictated by intersection of the $IC(B)$ and $IC(S)$ constraints.

For any $\delta \geq \widetilde{\delta}_0$, the hybrid relational contract inducing efficient investment is in fact self-sustaining for any bonus b in the shaded range. Note that the bonus may be zero for patient enough players ($\delta > \frac{2k}{w+k}$), with the entire payment made through the formal contract (but with the relational contract's threat to terminate the relationship still playing a very important role). Similarly, for patient enough players ($\delta > \widetilde{\delta}_0^{PR} = \frac{v+w+k}{v+2w}$), the entire payment may be made without any use of a formal contract, as shown in the prior subsection. It is easy to show that $\widetilde{\delta}_0 = \frac{k}{w} < \frac{2k}{w+k} < \widetilde{\delta}_0^{PR} = \frac{v+w+k}{v+2w} < 1$, meaning that the order of the intersections and thresholds shown in the figure is not arbitrary but is in fact determined by the model. This yields the following propositions.

Proposition 3 *Assume a short-term contracting environment ($z = 0$). Hybrid relational contracts induce efficient investment for patient enough players ($\delta > \widetilde{\delta}_0 = \frac{k}{w}$) and strictly increase the range of discount factors for which relational contracts induce efficient investment relative to purely relational contracts ($\widetilde{\delta}_0 < \widetilde{\delta}_0^{PR}$).*

With respect to the overarching research question of whether formal and relational contracts are complements or substitutes, this is clearly a model in which the answer for this short-run model is "complements".

[8] Recall that the "all-wage" contract is not a purely formal contract, because continued contracting is still conditioned (relationally) on the observed investment and payment behavior of the parties.

Formal contracts used in conjunction with relational contracts strictly increase the range of discount factors for which the relational contract is self-enforcing.[9] This sets the stage for the analysis of the long-term contracting problem that is central to this paper.

4 Analysis: long-term contracts (z=1)

Now consider the case in which $z = 1$, indicating that there is a long-term component to each individual transaction, not only to the relationship as a whole. Investment in each period affects profits in the future, after subsequent investments have been made, and only then is investment observed by the buyer. For simplicity, this section analyzes the case of $z = 1$ in order to establish the basic intuition for the results, with the following section exploring the generalization to the case of $z > 1$. In this section and the next, I focus exclusively on hybrid relational contracts in the formal analysis (and, for this reason, I drop the superscripts in the notation). It is straightforward to see that, as before, formal contracts alone cannot induce investment. They can be conditioned only on delivery of the good and not on investment. Thus, the seller's payment from the buyer is always flat with respective to whether it undertakes this costly investment and the seller will not invest if there is any positive investment cost. The purely relational contract will be discussed, as above with respect to Figure 1, as a special case of the hybrid relational contract.

As before, the hybrid relational contract involves a formal contract specifying the trade of the intermediate good in period t for a payment of f, and a relationally enforced agreement that the seller will invest in every period; that the buyer pays a bonus b in each period conditional on the observation of investment that is realized that period, which is now the realization from period $t - z$; and that the parties will revert to their outside options if either party ever deviates.

In the $z = 1$ case, the incentive constraints are different from the $z = 0$ case because the consequences of reneging play out differently over the periods t and $t + 1$. Consider now the relevant incentive constraints for the seller and the buyer.

If the seller shirks in period t, the seller does not incur the investment cost in period t or in period $t+1$. The seller continues to deliver the intermediate good to the buyer in period t and $t + 1$ and receives the formally contracted price f in periods t and $t + 1$. The seller receives the bonus b (based on investments made in prior periods, still being realized in period t) in period t. In period $t + 1$, the buyer observes the lack of investment in period t and does not make the bonus b payment to the seller. From the next period $t + 2$ forward, there is no relational contracting and the buyer's payoff reverts to the outside option. Thus, the seller's incentive constraint for $z = 1$ becomes:

$$IC_1(S) : (f + b) + \delta f + \frac{\delta^2}{1-\delta}\frac{v}{2} \leq (f + b - k) + \delta(f + b - k) + \frac{\delta^2}{1-\delta}\frac{v + w - k}{2}.$$

That is, the seller gets both payments in the present period and incurs no investment cost; with this still undetected, the seller gets the fixed payment f in the next period; during period $t+1$, the deviation will be detected, and the bonus b will not be paid, so the seller will receive only the formally contracted payment

[9] The levels of formally contracted and relationally enforced payments themselves are substitutes in the sense that, if the discount factor is such that the associated set of self-enforcing bonuses (in the shaded region in Figure 1) is not a singleton, any combination of formally contracted and relationally enforced payments summing to the desired surplus transfer sustains the hybrid relational contract.

f; from $t+2$ forward, the seller receives $v/2$ and incurs no investment costs. Rearranging, this becomes

$$IC_1(S): k + \delta(k-b) \leq \frac{\delta^2}{1-\delta}\frac{w-k}{2}.$$

Here, the right-hand side represents the long-run loss from deviation, which is balanced against the now multi-period gain from deviation: saving the investment cost with no penalty in the present period, and saving the investment cost but losing the bonus in the subsequent period. The seller in effect saves the investment cost for two periods, loses the bonus for only one period (in the future), and of course forfeits the future value of the relationship, but beginning one period further in the future. This increases the temptation to deviate relative to the case of $z = 0$. Rearranging again yields:

$$IC_1(S): b \geq \frac{1+\delta}{\delta}k - \frac{\delta}{1-\delta}\frac{w-k}{2}.$$

Compared to $IC_0^H(S)$, the first RHS term of $IC_1(S)$ is a larger positive value and the second term has not changed, resulting in an unambiguous increase in the lower bound on b.

Now consider the buyer's incentive constraint. If the buyer renegs on a promised bonus payment in period t, then in period t and period $t+1$ the buyer continues to profit from the seller's investments in prior periods (since the buyer has already taken possession of the intermediate good), even though beginning in period $t+1$ both players revert to the outside option. The buyer's incentive constraint for $z = 1$ becomes:

$$IC_1(B): (v+w-f) + \delta(v+w-\frac{v}{2}) + \frac{\delta^2}{1-\delta}\frac{v}{2} \leq (v+w-f-b) + \delta(v+w-f-b) + \frac{\delta^2}{1-\delta}\frac{v+w-k}{2}.$$

That is, in the current period the buyer pays only f while reaping gross profits of $v+w$ thanks to the seller's prior investment in $t-1$. In the next period the buyer benefits from the seller's prior investment in period t even though it is now taking advantage of the outside option for procurement of the intermediate good. From $t+1$ forward the profits and payments revert to those associated with the outside option. Rearranging $IC_1(B)$ yields

$$IC_1(B): b + \delta(\frac{v+w+k}{2} - \frac{v}{2}) \leq \frac{\delta^2}{1-\delta}\frac{w-k}{2}.$$

Here, the long long-run loss (period $t+2$ and beyond) on the right-hand side is weighed against the now multi-period gain from deviation, consisting of the in-period appropriation of the bonus plus the next-period gain associated with replacing the payment of the full $f+b$ with the acquisition of the intermediate good on the market at the outside option value. This increases the temptation to deviate relative to the case of $z = 0$. Rearranging again for b yields:

$$IC_1(B): b \leq -\delta(\frac{w+k}{2}) + \frac{\delta^2}{1-\delta}\frac{w-k}{2}.$$

Compared to $IC_0^H(B)$, the second RHS term of $IC_1(B)$ is a smaller version of the comparable term and there now appears the additional negative first RHS term, resulting in an unambiguous lowering of the upper bound on b.

Having developed the intuition and analysis for the $z = 0$ case, it is easiest to understand the $z = 1$ case by considering how it changes that analysis. As described above, comparing $IC_1(S)$ and $IC_1(B)$ to $IC_0^H(S)$ and $IC_0^H(B)$, it is clear that both incentive constraints have tightened. Through a combination of increases in deviation payoffs and delays in punishment payoffs, the net temptation to deviate has increased for both parties at any given discount factor. For the seller, the increased deviation gains come from continuing to receive the bonus payment during periods when it has stopped investing but this has not yet been detected. For the buyer, the increased deviation gains come from continuing to reap the benefits of prior seller investments after the buyer has ceased making bonus payments.

In order to graph these new incentive constraints to understand how the analysis has changed relative to the case of $z = 0$ depicted in Figure 1, additional analysis is required before constructing the analogous Figure 2 for the case of $z = 1$. First, note that $IC_1(B)$'s upper bound on b (understood as $b = f(\delta)$, as graphed in the figure) has shifted down relative to $IC_0(B)$, and that $IC_1(S)$'s lower bound on b has shifted up relative to $IC_0(S)$. In addition, it is possible to show that both $IC_1(S)$ and $IC_1(B)$ are monotonic in δ. This is straightforward for $IC_1(S)$: the first term becomes smaller and remains positive as δ increases, while the second term becomes larger and remains negative, ensuring that $IC_1(S)$ is decreasing in δ. For $IC_1(B)$, seeing this requires differentiating the constraint with respect to δ, which demonstrates after some manipulation that $IC_1(B)$ is monotonically increasing in δ for all parameter values. Thus, the downward shift in $IC_1(B)$ and upward shift in $IC_1(S)$ also translate into rightward shifts in these two constraints. As the discount factor goes to 0, $IC_1(B)$ continues to approach 0 as in the $z = 0$ before, but $IC_1(S)$ now increases without bound. As the discount factor goes to 1, $IC_1(S)$ decreases without bound, while $IC_1(B)$ increases without bound, both as before in the $z = 0$ case.

The algebraic results derived above imply the constraints take the general form portrayed in Figure 2, which depicts the incentive constraints for $z = 1$ overlaid on the $z = 0$ case from Figure 1.

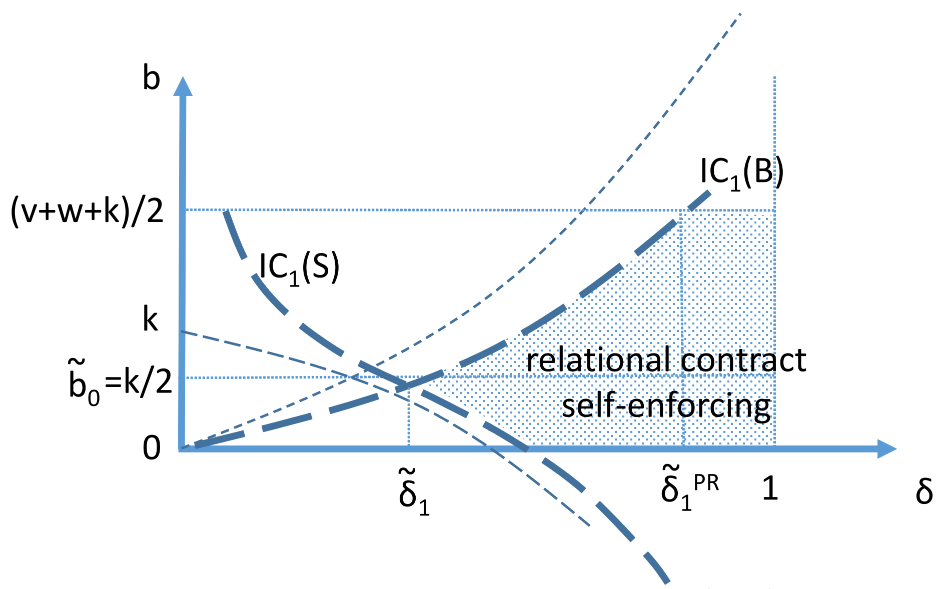

Figure 2: Long-term contracts (z=1)

Note that these results on the behavior of $IC_1(S)$ and $IC_1(B)$ as δ approaches 1 together imply that the $z = 1$ threshold discount factor $\widetilde{\delta}_1$ defined by the intersection of the $IC_1(S)$ and $IC_1(B)$ constraints is smaller than 1 but higher than for the case of $z = 0$: $\widetilde{\delta}_1 < 1$ and $\widetilde{\delta}_1 > \widetilde{\delta}_0^H$, yielding the following result.

Proposition 4 *Assume a long-term contracting environment ($z = 1$). Optimal hybrid relational contracts are self-enforceable for patient enough players ($\delta > \widetilde{\delta}_1$; $\widetilde{\delta}_1 < 1$); inducing efficient investment through a* *relational contract requires more patient players for long-term contracts ($z = 1$) than for short-term contracts ($z = 0$): $\widetilde{\delta}_1 > \widetilde{\delta}_0^H$.*

The algebraic results above also imply that $IC_1(B)$ intersects the "all-bonus" $b = \frac{v+w-k}{2}$ line at a discount factor strictly lower than 1 and at a higher discount factor than in the $z = 0$ case: that is, $\widetilde{\delta}_1^{PR} < 1$ and $\widetilde{\delta}_1^{PR} > \widetilde{\delta}_0^{PR}$. This implies further results.

Proposition 5 *Assume a long-term contracting environment ($z = 1$). Pure relational contracts can induce efficient investment for patient enough players ($\widetilde{\delta}_1^{PR} < 1$), albeit it with a higher discount factor than in a short-term contracting environment ($\widetilde{\delta}_1^{PR} > \widetilde{\delta}_0^{PR}$).*

It is important to establish two more algebraic results relating to where the optimal bonus $\widetilde{\delta}_1$ lies in the $z = 1$ case relative to the case of $z = 0$ and also relative to the "all-bonus" line. While it is not possible to derive a tractable closed from solution by combining these two equations in two unknowns, due to the presence of higher order terms, it is nonetheless possible to determine the *relative* value of the optimal bonus. By multiplying $IC_1(S)$ through by δ and then subtracting $IC_1(B)$ from $IC_1(S)$, the discounted term representing the future value of the relationship falls out and leaves a simple equation in two unknowns that provides insight on potential combinations of b, δ that may simultaneously solve $IC_1(S)$ and $IC_1(B)$: $\widehat{b}_1(\delta) = k - \frac{\delta}{1+\delta}\frac{w+k}{2}$. Again, this (and any subsequent expression involving a $\widehat{b}$) is not a solution for b as a function of δ, but rather a set of *possible* solutions, giving the values of b that would correspond to various discount factors if those discount factors were in fact the threshold discount factor. Note that this expression is monotonically decreasing in δ. Recall that the relevant discount factors are the range $\delta \in (\widetilde{\delta}_0^H = \frac{k}{w}, 1)$ because it is shown above that the threshold discount factor increases as z changes from 0 to 1: $\widetilde{\delta}_1 > \widetilde{\delta}_0^H$. This expression $\widehat{b}_1(\delta)$ falls from $\frac{k}{2}$ to $\frac{3k-w}{2} > 0$ over this range. Thus, it lies everywhere in the permitted range, both above the "non-negative bonus constraint" at the horizontal axis and below the "all-bonus constraint" at $b = \frac{v+w+k}{2}$. This ensures that the intersection of $IC_1(B)$ and $IC_1(S)$ determines the threshold discount factor rather than the intersection of one of the incentive constraints with one of the boundary constraints.

Knowing that the threshold discount factor is determined by the intersection of the incentive constraints implies that the intersection of the upsloping $IC_1(B)$ constraint with the all-bonus line lies strictly to the right of the threshold discount factor: $\widetilde{\delta}_1 > \widetilde{\delta}_1^{PR}$. This yields further results.

Proposition 6 *Assume a long-term contracting environment ($z = 1$). The use of formal contracts as part of a hybrid relational contract strictly increases the range of discount factors for which relational long-term contracts induce efficient investment ($\widetilde{\delta}_1 > \widetilde{\delta}_1^{PR}$).*

Proposition 7 *Assume a long-term contracting environment ($z = 1$). For any given discount factor, inducing efficient investment with a relational contract is weakly more likely (and strictly more likely for some non-empty set of discount factors) to require complementary use of formal contracts for a long-term contract than for a short-term contract (formal contracts are required for long-term contracts but not for short-term contracts for δ such that $\max(\widetilde{\delta}_0^{PR}, \widetilde{\delta}_1) < \delta < \widetilde{\delta}_1^{PR}$, and this interval is non-empty).*

Note that, in addition, this analysis of $\widehat{b}_1(\delta)$ implies that it lies below $\widetilde{b}_0 = k/w$ for all relevant discount factors. Thus, whatever the threshold discount factor $\widetilde{\delta}_1$ (given that $\widetilde{\delta}_1 > \widetilde{\delta}_0$), it must be that $\widetilde{b}_1 < \widetilde{b}_0$. That is, the proportion of the payment made through the bonus in the optimal hybrid contract at the threshold discount factor falls as the contract lengthens from $z = 0$ to $z = 1$.

Proposition 8 *The optimal hybrid relational contract in the long-term contract ($z = 1$) has a lower relationally contracted bonus and relies more heavily on formally contracted payments than in the short-run ($z = 0$) case:* $\widetilde{b}_1 < \widetilde{b}_0$.

The above results take the contract length as an endogenous fact of the model, with $z = 1$. The first and third propositions of this section together yield another proposition, if one imagines a model in which players choose from among projects with different contract lengths.

Proposition 9 *Assume the players choose from potential projects with short- and long-term contracting environments ($z = 0$ and $z = 1$). Longer-term contracts become feasible as the discount factor increases and if the players incorporate formal contracts into hybrid relational contracts. If there are opportunities for profitable trade that require long-term investment, they are more likely to be undertaken as the discount factor increase and if the firms are able to complement their relational contracts with formal contracts.*

To recap results in the context of Figure 2. In the long-term contracting problem, it remains true that formal contracts cannot induce efficient investments, but that relational contracts can, though players must be even more patient than in the short-term contracting problem; for a purely relational contract, this is true as long as $\delta > \widetilde{\delta}_1^{PR} > \widetilde{\delta}_0^{PR}$. It remains true that using a formal contract in a hybrid relational contract allows efficient investment to be induced at a lower discount factor than in a purely relational contract $\widetilde{\delta}_1 < \widetilde{\delta}_1^{PR}$, although it too requires more patient players than in the short-term contracting problem: $\widetilde{\delta}_1 > \widetilde{\delta}_0$. The long-run hybrid relational contract is more weakly more likely to require the use of a formal contract for any given discount factor (because $\widetilde{\delta}_1^{PR} > \widetilde{\delta}_0^{PR}$, meaning that purely relational contracts suffice over a smaller range of discount factors). The long-run contract will also have a lower relationally enforced bonus payment and a higher formally contracted fixed payment: $\widetilde{b}_1 < \widetilde{\delta}_0^{H}$.

The intuition for the fact that the optimal bonus payment (at the threshold discount factor) falls as the contract lengthens is clear and robust. For this reason, it will be clear in the next section that the results extend to further lengthening of the contract. Graphically, it is clear that the optimal bonus (defined by the intersection of $IC_z(B)$ and $IC_z(S)$ will decrease if $IC_z(B)$ shifts right faster than $IC_z(S)$ does as z increases. That is, if increases in contract length tighten the buyer's incentive constraint faster than the seller's, then the optimal bonus (which is problematic from the buyer's point of view, since it is the buyer who is tempted to reneg on paying the bonus) must fall. Knowing this, consider the effect of changes in contract length on the two parties' incentives to deviate. The seller's gains from avoiding investment costs increase with the number of periods required for the buyer to detect cheating. The penalty of losing the bonus gets pushed further into the future as the contract lengthens. Thus, the loss of the bonus becomes less important as a deterrent to the seller because it gets pushed further into the future. Lowering the bonus is therefore less problematic to the seller's incentives the longer the contract. For the buyer, in contrast, appropriating the full bonus in the present period is always a key element of the incentive to deviate; it is not pushed off into the future regardless of contract length. Thus, lowering the bonus remains a very powerful way to satisfy the buyer's incentive constraint, even as contracts lengthen and this makes it relatively less important to the seller's incentive constraint. Thus, lowering the bonus is an efficient rebalancing of incentives when the contract lengthens and the bonus becomes less important in the seller's incentive constraint.

With respect to the overarching research question of whether relational contracting, formal contracting, and long-term contracting are complements or substitutes, this is a model in which the answer is unambiguously "complements". Longer-term contracting makes relational contracting more difficult, requiring more patient players, more use of formal contracting, and a larger portion of the payment being made through formally contracted payments.

5 Analysis: longer-term contracts (z>1)

Now consider the case in which $z > 1$, extending the overlapping contracting problem to more than one period into the future. This analysis closely follows that of the case of $z = 1$. The logic of the short- and medium-run payoffs from deviation and the long-run losses all remain the same. In translating these into the general $z > 1$ constraints, one must be careful to count up the periods in which the deviation gains accrue before reversion to the outside option sets in, modifying the incentive constraints accordingly. Here again I consider analytically only the case of the optimal hybrid relational contract and consider purely relational contracting as a special case. The incentive constraints are generalizations of the expressions for the $z = 1$ case, and it is easy to check that both $IC_z(S)$ and $IC_z(B)$ derived below reduce when $z = 1$ to the constraints defined in the prior section. These expressions can be rearranged to demonstrate that both incentive constraints become tighter as the delay in detecting and punishing deviations becomes longer.

For the seller, the incentive constraint becomes

$$IC_z(S) : \sum_{t=0}^{z} \delta^t k - \delta^z b \leq \frac{\delta^{z+1}}{1-\delta} \frac{w-k}{2}.$$

For every period up to period $t + z$, the deviating seller gains the investment cost; the deviation is only detected in period z, which is therefore the only period in which the bonus b is withheld by the buyer. Beginning in period $z+1$ the players revert to their outside options. $IC_z(S)$ can be re-written to solve for b:

$$IC_z(S) : b \geq \frac{1}{\delta^z} \sum_{t=0}^{z} \delta^t k - \frac{\delta}{1-\delta} \frac{w-k}{2}.$$

As z increases, it is easy to see that the first right-hand side term increases (the fraction grows as the denominator shrinks, and the sum grows by the addition of new positive terms) while the second right-hand side term does not change. Thus, increases in z raise the lower bound on b, tightening $IC_z(S)$ and moving it everywhere up and to the right in the figure as z increases.

For the buyer, the incentive constraint becomes

$$IC_z(B) : b + \sum_{t=1}^{z} \delta^t \left(\frac{v+w+k}{2} - \frac{v}{2}\right) \leq \frac{\delta^{z+1}}{1-\delta} \frac{w-k}{2}.$$

The in-period gain to deviation is the gain of the full bonus payment b. For the subsequent z periods the buyer continues to benefit from already-acquired intermediate goods that reflect seller investment in prior periods, while only paying the outside option price for new purchases of the intermediate good. Beginning in period $z + 1$ the players revert to their outside options. $IC_z(B)$ can be re-written to solve for b:

$$IC_z(B) : b \leq -\sum_{t=1}^{z} \delta^t \frac{w+k}{2} + \frac{\delta^{z+1}}{1-\delta} \frac{w-k}{2}.$$

As z increases, it is easy to see that the first right-hand side term decreases (the sum grows with the addition of new positive terms, but the entire sum is preceded by a negative sign) while the second right-hand side term decreases as well (the discounting term shrinks as the numerator falls with higher powers of the discount factor). Thus, increases in z lower the upper bound on b, tightening $IC_z(B)$ and moving it everywhere down and to the right in the figure. As a result, the threshold discount factor $\widetilde{\delta}_z$ must be increasing in z, as must the purely relational contracting threshold discount factor defined by the intersection of $IC_z(B)$ and the "all-bonus constraint". This yields the following propositions.

Proposition 10 *Longer term hybrid relational contracts require higher discount factors in order to be self-enforcing (the threshold discount factor $\widetilde{\delta}_z$ is increasing in z).*

Proposition 11 *Longer term purely relational contracts require higher discount factors in order to be self-enforcing (the threshold discount factor $\widetilde{\delta}_z^{PR}$ is increasing in z).*

As before in the case of $z = 0$, it is possible to manipulate the incentive constraints to cancel some higher-order terms and to arrive at a relatively simple expression giving a relationship between b and δ in potential optimal hybrid contracts. In this case, for general z, the expression becomes $\widehat{b}_z(\delta) = \frac{1}{1+\delta^z}[k - \frac{\delta}{1-\delta}(1-\delta^z)\frac{w-k}{2}]$. As before, $\widehat{b}_z(\delta)$ does not give the optimal bonus as a function of the discount factor, but only a range of bonuses that would be optimal if a particular discount factor were the threshold discount factor. Analysis of this new $\widehat{b}_z(\delta)$ function yields one general result: $\widehat{b}_z(\delta)$ is decreasing in z. This is shown by evaluating the difference $\widehat{b}_{z+1}(\delta) - \widehat{b}_z(\delta)$. Substituting the above expression for $\widehat{b}_z(\delta)$, this yields $\widehat{b}_{z+1}(\delta) - \widehat{b}_z(\delta) = \frac{\delta^z(k-\delta w)}{(1+\delta^z)(1+\delta^{z+1})}$, which is negative for all relevant discount factors, $\delta > \widetilde{b}_0 = \frac{k}{w}$. Thus, $\widehat{b}_z(\delta)$ shifts down for all relevant δ as z increases. Because it is shown above that $\widehat{b}_1(\delta) < \widetilde{b}_0 = \frac{k}{w}$ for all δ, and because $\widehat{b}_1(\delta)$ shifts down with increases in z, it is clear that the all-bonus upper bound will never, for any z, come into play in determining the threshold discount factor $\widetilde{\delta}_z$; $\widetilde{\delta}_z$ will always be determined by the intersection of the two incentive constraints or by the intersection of the $IC_z(S)$ constraint with the $b = 0$ axis. Thus, $\widetilde{\delta}_z$ must always be strictly below $\widetilde{\delta}_z^{PR}$. This yields the following propositions.

Proposition 12 *Regardless of contract length z, the use of formal contracts as part of a hybrid relational contract strictly increases the range of discount factors for which relational long-term contracts induce efficient investment ($\widetilde{\delta}_z < \widetilde{\delta}_z^{PR}$).*

Proposition 13 *For any given discount factor, inducing efficient investment with a relational contract is weakly more likely (and strictly more likely for some non-empty set of discount factors) to require complementary use of formal contracts as contract term lengthens (formal contracts are required for long-term contracts of length $z+1$ but not for contracts of length z for δ such that $\max(\widetilde{\delta}_z^{PR}, \widetilde{\delta}_{z+1}) < \delta < \widetilde{\delta}_{z+1}^{PR}$, and this interval is non-empty).*

It remains to explore whether increases in z lead to decreasing use of the bonus payment, as in the case of increasing from $z = 0$ to $z = 1$. At present, I can show that this is true for increases in z up to $z = 3$, though it seems likely to be true in general. Note that to show this for all z, it would suffice to show that $\widehat{b}_z(\delta)$ is downsloping in δ.[10] If so, the hypothesized result ($\widetilde{b}_{z+1} < \widetilde{b}_z$) holds because it is shown above that $\widehat{b}_z(\delta)$ decreases in z and that $\widetilde{\delta}_z$ increases in z. Thus, $\widetilde{b}_{z+1} = \widehat{b}_{z+1}(\widetilde{\delta}_{z+1}) < \widehat{b}_z(\widetilde{\delta}_{z+1}) < \widehat{b}_z(\widetilde{\delta}_z) = \widetilde{b}_z$, where the first inequality follows from the fact that $\widehat{b}_z(\delta)$ falls in z, and the second inequality follows from the facts that $\widetilde{\delta}_z$ increases in z and that $\widehat{b}_z(\delta)$ is decreasing in δ. For both inequalities to hold strictly requires that $\widehat{b}_z(\delta) > 0$

[10] Note that it would suffice to show that $\widehat{b}_{z+1}(\delta) - \widehat{b}_z(\delta) < 0$ is decreasing in δ–that is, that the gap between $\widehat{b}_{z+1}(\delta)$ and $\widehat{b}_z(\delta)$ widens. This, combined with the already established fact that $\widehat{b}_1(\delta)$ is downsloping, would prove that $\widehat{b}_z(\delta)$ is downsloping for all z.

for all relevant δ, ensuring that this $\widehat{b}_z(\delta)$ derived from the intersection of $IC_z(S)$ and $IC_z(B)$ continues to define the set of potential optimal bonuses, rather than the intersection of $IC_z(S)$ with the $b = 0$ boundary condition.[11] Thus, the hypothesized result holds up to $z' + 1$ if $\widehat{b}_z(\delta)$ can be shown to be both (a) decreasing in δ for z up to z' and (b) strictly positive for all relevant δ at z', two results that remain to be proved. That is, the proof outlined above holds for one contract period longer than the highest value of z for which these two conditions on $\widehat{b}_z(\delta)$ can be shown to hold.

In fact, it is possible to show $\widehat{b}_z(\delta)$ is downsloping in δ for $z = 1$ and $z = 2$. That $\widehat{b}_1(\delta)$ is decreasing in δ follows immediately from the algebraic expression for it derived in the last section. That $\widehat{b}_2(\delta)$ is decreasing in δ follows from additional manipulation of the expression for $\widehat{b}_2(\delta)$, which can be simplified to $\widehat{b}_2(\delta) = \frac{1}{(1+\delta^2)}[k - \delta(1+\delta)\frac{w-k}{2}]$. This expression can be differentiated to show that $\frac{\partial \widehat{b}_2(\delta)}{\partial \delta} < 0$. In addition, one can show that it falls from k to $\frac{2k-w}{2} > 0$ (this is positive by (A2)) over $\delta \in (0, 1)$, proving that $\widetilde{b}_2$ is always determined by the intersection of $IC_2(S)$ and $IC_2(B)$ and lies strictly interior to the boundary constraints.[12] This yields the final proposition.

Proposition 14 *As the length of the lag in investment, and therefore the effective length of contract, increases anywhere within the range of $z = 0$ to $z = 3$, the optimal bonus strictly decreases: $0 \le \widetilde{b}_3 < \widetilde{b}_2 < \widetilde{b}_1 < \widetilde{b}_0$. Longer contracts rely less on relationally contracted bonuses and more on formally contracted payments.*

6 Conclusion

This paper considers long-term contracting environments in which players in repeated contracting relationships would like to support long-term commitments but cannot rely on formal contracts alone. I show that formal contracts can be powerful complements to relational contracting, allowing players to sustain long-term relational contracts that would not otherwise be self-enforcing, and that the importance of the formal contract increases as contract terms lengthen. This is a first step toward understanding the interaction of formal and relational contracts in multi-period long-term contracting environments.

7 References

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[11] If the $b = 0$ boundary condition is reached for some z, the optimal bonus $\widetilde{b}_z$ becomes constrained to 0 and the threshold discount factor $\widetilde{\delta}_z$ becomes defined by the intersection of $IC_z(S)$ and the horizontal axis. Further increases in z continue to increase $\widetilde{\delta}_z$ strictly but do not further reduce the optimal bonus $\widetilde{b}_z$.

[12] Note that this does not prove that $\widehat{b}_3(\delta)$ is strictly positive for all relevant δ; however, even if the $b = 0$ binds for $\widetilde{b}_3$, it remains true that $\widetilde{b}_3 < \widetilde{b}_2$ because $\widetilde{b}_2 > 0$.