The term structure of expected recovery rates*

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Abstract

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JEL Classification: G01, G12
Keywords: credit default swaps (CDS); no-arbitrage; stochastic recovery rate; seniority; term structure

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There is widespread agreement that corporate debts' recovery rates are time-varying, but empirical work in this area is limited. We show that the joint information from the term structure of senior and subordinate CDS is able to identify the level and the dynamics of recovery rates. We estimate a reduced form no-arbitrage model on 46 firms across different industries. We find that the term structure of expected recovery rates is, on average, downward sloping. However, an inversion occurs during the 2008 crisis, suggesting that the market expects higher recoveries conditional on short-term survival. The inverted shape in the recovery term structure is more pronounced for distressed industries.

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1 Introduction

Credit spreads consist of two main components: the default probability, and the loss given default. The loss given default (LGD) is often expressed in terms of the debt's recovery rate, which is defined as one minus the LGD. While much is known on the modeling of default probabilities, the same cannot be said for risk-neutral recovery rates. The lack of empirical work on the modeling of risk-neutral recovery rates is primarily due to the fact that the two spread components are difficult to identify using single-seniority financial instruments. Instead, the standard practice in the literature is to assume a constant recovery rate. Unfortunately, this assumption is not realistic given the mounting evidence in support of time-varying recovery rates in the credit risk literature.[1]

In this paper, we use information from both senior and subordinate credit default swaps (CDS) to isolate the recovery rate component. The senior CDS are insurance contracts for the senior unsecured bonds, while the subordinate CDS are insurance contracts for the subordinate or lower tier-2 bonds. In the event of default, the expected loss is larger for the subordinate debt. Consequently, the spread of subordinate CDS contracts is larger than the spread of their corresponding senior contracts despite having the *same* default probability.

Our empirical approach consists of jointly modeling the senior and subordinate CDS contracts in a five-factor reduced form no-arbitrage model. We apply our modeling framework to 46 firms across industries. The data set spans the period from January 1, 2001 to May 31, 2012. We exploit the differences in senior and subordinate CDS spreads to estimate the dynamics of their recovery term structure. The estimation results show that both the default intensity as well as the recovery components vary significantly over time. The average expected risk-neutral recovery rate across firms and default horizons is approximately 34 % for senior contracts, and 20 % for subordinate contracts. Our estimates of recovery rates are economically plausible and consistent with realized recovery rates reported in the literature. In addition, we study time-series dynamics of several financial firms and show that recovery rates implied by the CDS contracts are highly responsive to news events that have significant impact on the lender's ability to recover the debt. For instance, in section 4.5, we study the time-series dynamic of recovery rates for Fannie Mae and Freddie Mac and document a volatile drop in their recovery levels in the beginning of 2007, which were later reverted to a historically high level after their U.S. government-led bailout.

To our knowledge, this is the first paper that utilizes information from credit default swap contracts with multiple seniorities in order to estimate the stochastic dynamics of risk-neutral recovery. The model allows for stochastic interest rates, stochastic default intensity, and

[1] For instance, Altman, Brady, Resti and Sironi (2005) examine recovery rates for corporate bond defaults between 1982 and 2002 and find that they vary significantly over time and are negatively correlated with default rates. Using an extensive set of firms, Acharya, Bharath and Srinivasan (2007) also document large cross sectional and time series variations in realized recovery rates.

stochastic loss given default. We model the short rate dynamics, the default intensity as well as the loss given default using a quadratic specification. This approach ensures that the short rate and the default intensity is always positive and that the loss given default is bounded between zero and one.

We obtain a number of important findings about the recovery rate dynamic. First, we document a sharp decline in recovery rates during the financial crisis. More specifically, we show the average recovery rate falls dramatically from mid-2007 onwards, which marks the onset of the financial crisis. Second, we find that the term structure of expected recovery implied by CDS contracts significantly changes shape during the 2008 financial crisis. On average, the term structure of expected recovery is downward sloping. However, it becomes upward sloping during the 2008 financial crisis. Our findings suggest that during good economic times, there is more uncertainty about the future state of the economy. Risk averse investors therefore command a recovery risk premium for investing in longer term debt contracts, resulting in the downward sloping recovery rate term structure. On the other hand, an upward sloping term structure during the crisis period shows the market expects firms to recover less if they were to default in the midst of the recession than if they were to survive and default at future dates when the economy improves. We find that the inversion of expected recovery term structure during the crisis is more pronounced for firms in distressed industries, e.g. financial firms. Our finding that the term structure of expected recovery is upward sloping during bad economic times is in line with Zhang (2009) who shows that realized recovery rates are negatively correlated with lagged macroeconomic conditions.

Third, we find that increases in CDS spreads during the financial crisis are mainly due to the increase in default probabilities. Although the increase in LGD is small relative to the change in default probabilities, its magnitude is economically significant. Importantly, we find the increase in LGD during the financial crisis is much larger at the short end of the term structure. The relatively larger increase in LGD at shorter default horizons explains why the term structure of recovery flattens or inverts during the financial crisis period.

Fourth, we find that industry characteristics are an important determinant of the CDS implied recovery rates, consistent with the theoretical argument in Shleifer and Vishny (1992) and the finding in Acharya, Bharath and Srinivasan (2007) who study realized recovery rates on an extensive set of firms. That is, we show that our recovery rates decrease most dramatically for financial firms during the crisis period. While we find that industry characteristics impact firms' recovery rates, we do not find that firms' ratings monotonically explain the cross-sectional differences in recovery rates. This finding is consistent with Altman and Kishore (1996) who show that bond ratings have no impact on the recovery rates once the seniority of the bonds is taken into consideration. Credit ratings are solely assigned based on the firm's default probability and we find a clear monotonic relationship between the risk-neutral default probabilities and the credit ratings.

We show that our estimation approach using multiple-seniority CDS term structures allows for the identification of the recovery rate dynamic. Specifically, we perform a large-scale simulation exercise and show that the parameters in our model are well identified. We also show that the identification of the recovery level of a subordinate (or senior) contract comes from its CDS term structure, while identification of the senior and subordinate recovery dynamics requires the panel data of their relative CDS term structures.[2] Finally, our simulation exercise shows that the estimated default probabilities are severely biased if the recovery rates are stochastic and if we use single-seniority CDS term structure in the estimation.

There exists relatively few studies that consider a stochastic recovery model for valuing CDS.[3] Christensen (2007) estimates a stochastic recovery model using CDS data for the Ford Motor Corporation. Our study differs from this paper in at least two ways. First the scale of our empirical exercise is much larger (46 firms instead of one), which allow us to investigate differences in the term structure of expected recovery across industries and rating categories. Second, and more importantly, we use the term structure of CDS contracts with multiple seniorities (senior and subordinated) which results in an improved identification. Consequently, our estimations lead to new findings about the term structure of expected recovery rates.

Bakshi, Madan and Zhang (2006) examine time-varying recovery rate models on a sample of BBB-rated bonds. Their modeling approach differs from ours in that their recovery rate and default intensity dynamics are governed solely by the factors driving the risk-free rate while in our approach, they depend on latent firm-specific factors in addition to the risk-free term structure. In other words, we allow firm-specific factors to influence the time-varying default and recovery risks. Further, we study the time-varying dynamic of expected recovery term structure, thereby generating a complementary contribution.

The rest of this paper is organized as follows. Section 2 introduces the model and Section 3 discusses the data and the estimation method. Section 4 presents the empirical results. Section 5 illustrates the model's identification of the recovery rate dynamic. Section 6 shows that our findings are robust to liquidity-related concerns in CDS market. Finally, Section 7 concludes.

[2] Pan and Singleton (2008) show that the constant recovery rate is identified using the term structure of CDS spreads. Our identification builds on their strategy and shows that the term structure of CDS spreads with more than one seniority allows for the identification of the stochastic dynamic of recovery rates. Schneider, Sögner and Veža (2010) and Elkamhi, Jacobs and Pan (2014) estimate constant recovery rate models for corporate CDS using the insights of Pan and Singleton (2008).

[3] Other papers that use information from more than one type of security to study recovery rates include Carr and Wu (2009), Das and Hanouna (2009), Jarrow (2001), Madan, and Guntay and Unal (2003), and Le (2015). However, none of these papers allow for stochastic recovery rates.

2 Model

2.1 Default-free model

We start with the model for default-free bonds. Let r_t denote the instantaneous default-free interest rate. Following Duffee (1999), we assume the short rate dynamics are described by two latent factors. We assume that r_t has a quadratic specification given by

$$r_t = (\delta_0 + \delta_1 X_{1,t} + \delta_2 X_{2,t})^2 , \tag{1}$$

where $X_{1,t}$ and $X_{2,t}$ are latent factors that drive the short rate dynamics.[4] The risk-neutral dynamics of these latent factors are given by

$$X^r_{t+1} = \mu^r + \rho^r X^r_t + \Sigma^r \varepsilon^r_{t+1}, \tag{2}$$

where X^r_t and μ^r are 2×1 vectors, and ρ^r and Σ^r are 2×2 diagonal matrices. We assume the 2×1 vector of residual shocks in the short rate dynamic is generated by $\varepsilon^r_{t+1} \sim Normal(0, I)$, where I is the identity matrix. Note that we apply the superscript r to variables in (2) to indicate that they are specific to the short rate dynamics.

The price of a zero-coupon bond at time t that matures in h periods is given by

$$B(t, t+h) = E^Q_t \left[\exp(-\sum_{j=0}^{h-1} r_{t+j}) \right], \tag{3}$$

where E^Q_t indicates the expectation under the risk-neutral measure. Given the dynamics in equations (1) and (2), the price of a default-free zero coupon bond can be written as

$$B(t, t+h) = \exp(A_h + B'_h X^r_t + X^{r\prime}_t C_h X^r_t), \tag{4}$$

where the coefficients A_h, B_h and C_h are given by recursive relations derived in Appendix A.

2.2 Credit default swap valuation

2.2.1 Default intensity and loss given default

Following the framework set forth by Jarrow and Turnbull (1995), Duffie and Singleton (1997, 1999), and Lando (1998), we model default as a surprise event driven by a Poisson process. The risk-neutral intensity for the Poisson process at time t is defined as λ_t. The probability

[4] See for example Longstaff (1989), Constantinides (1992), Ahn, Dittmar and Gallant (2002), Leippold and Wu (2002),Ang, Boivin, Li and Zhao (2006), Dong and Loo-Kung (2011), and Doshi, Ericsson, Jacobs, and Turnbull (2013) for quadratic term structure models. See Gourieroux and Monfort (2008) for an application of a discrete time quadratic factor model to mortality intensity modeling.

of surviving at least h periods conditional on no default up until time t is

$$E_t^Q[1_{\tau>t+h}] = E_t^Q\left[\exp(-\sum_{j=0}^{h-1}\lambda_{t+j})\right]. \tag{5}$$

where $1_{\tau>t+h}$ is the indicator function that is equal to 1 if default happens after time $t+h$ and 0 otherwise.

The default intensity is assumed to depend on the same latent factors that drive the short rate dynamics and two additional latent factors which are credit-risk specific. Similar to the short rate dynamics, we assume a quadratic specification for the default intensity

$$\lambda_t = (\alpha_0 + \alpha_1 X_{1,t} + \alpha_2 X_{2,t} + \alpha_3 X_{3,t} + \alpha_4 X_{4,t})^2. \tag{6}$$

The above equation shows the default intensity dynamic depends on four latent factors. Factors, $X_{1,t}$ and $X_{2,t}$, are estimated from the risk-free term structure (see equation (1)), while $X_{3,t}$ and $X_{4,t}$, are firm-specific factors driving the default intensity.

We assume that when the firm defaults, its loss rate dynamic at time t (i.e., loss given default) for senior and subordinate CDS contracts is given by

$$LGD_t^s = \exp\left(-(\beta_0^s + \beta_1^s X_{1,t} + \beta_2^s X_{2,t} + \beta_5^s X_{5,t})^2\right), \tag{7}$$

for $s \in \{sen, sub\}$. We assume the loss given default for both senior and subordinate CDS contracts are driven by the same dynamic in (7), but with different factor loadings. That is, we let β_i^{sen} and β_i^{sub}, for $i = 0, 1, 2, 5$, denote the factor loadings in the LGD dynamic for the senior and subordinate contracts, respectively. The above equation shows the loss-given-default dynamic depends on the latent factors describing the short rate dynamic (i.e., $X_{1,t}$ and $X_{2,t}$), plus an additional latent factor $X_{5,t}$ that is specific to the LGD_t. Our modeling assumptions in (6) and (7) imply that the default intensity and recovery rate dynamics are driven by independent latent factors. The correlation between their dynamics are generated through their dependence on the risk-free term structure dynamic.

Overall, our modeling approach consists of five independent latent factors that can be grouped into two components. The first component contains the factors specific to the short rate factors and is denoted by $X_t^r = [X_{1,t} \quad X_{2,t}]'$. The second component is denoted by $X_t^c = [X_{3,t} \quad X_{4,t} \quad X_{5,t}]'$, and consists of credit-risk specific factors. The five latent factors in the model can be represented more compactly by $X_t = [X_t^{r\prime} \quad X_t^{c\prime}]'$, which is a 5×1 vector. We assume that X_t follows the following dynamics

$$X_{t+1} = \mu + \rho X_t + \Sigma\varepsilon_{t+1}, \tag{8}$$

where μ is a 5×1 vector, and ρ and Σ are 5×5 diagonal matrices. Shocks to the latent factors

are represented by $\varepsilon_{t+1} \sim Normal(0, I)$, where ε_{t+1} is a 5×1 vector.

2.2.2 Valuation

For the valuation of a credit default swap, we first consider the payments by the protection buyer. Let CDS denote the annual CDS spread. The protection buyer promises to make payments $CDS\Delta$ on each coupon date, conditional on no default by the reference obligor, where Δ is the time between successive payment dates in years. For simplicity, we assume that the payments are equally spaced. If a credit event occurs, the protection buyer receives a payment from the protection seller and the contract terminates. The present value of the payment by the protection buyer on a payment date that is h periods ahead is

$$PB(t, t+h) = E_t^Q \left[CDS\Delta \times \exp(-\sum_{j=0}^{h-1} r_{t+j}) \times 1_{\tau > t+h} \right], \tag{9}$$

where $1_{\tau > t+h}$ is the indicator function that is equal to 1 if default happens after time $t+h$ and 0 otherwise.

Next, we consider the cash flow of the protection seller. When default occurs between time interval $t+h-1$ and $t+h$ for $h > 0$, the protection seller makes a payment of LGD_{t+h-1}. The expected value of this cash flow from the perspective of protection seller is as follows,

$$\begin{aligned} PS(t, t+h) &= E_t^Q \left[LGD_{t+h-1} \times \exp(-\sum_{j=0}^{h-1} r_{t+j}) \times 1_{\tau > t+h-1} \right] \\ &\quad - E_t^Q \left[LGD_{t+h-1} \times \exp(-\sum_{j=0}^{h-1} r_{t+j}) \times 1_{\tau > t+h} \right]. \end{aligned} \tag{10}$$

In Appendix B, we show the expectations in equations (9) and (10) can be solved recursively. Their solutions are given by

$$PB(t, t+h) = \exp(F_h + G_h' X_t + X_t' H_h X_t) \tag{11}$$

$$PS(t, t+h) = \exp(M_h + N_h' X_t + X_t' O_h X_t) - \exp(J_h + K_h' X_t + X_t' L_h X_t), \tag{12}$$

where the recursive relations of the exponential coefficients in equations (11) and (12) are given in Appendix B.

Finally, the CDS spread level is set such that the present value of the payments by the protection buyer is equal to the present value of the payments by the protection seller. That is,

$$\sum_{h=1}^{T/\Delta} PB(t, t+h\Delta N) = \sum_{k=1}^{N \cdot T} PS(t, t+k), \tag{13}$$

where N refers to the number of trading days in a year, Δ refers to the time period between two successive premium payments, and T refers to the maturity in years of the CDS contract. The time period for payments between two successive premium payments is 0.25 years. This corresponds to the payment frequency of the U.S. credit default swaps used in our study.

3 Data and estimation

3.1 Data

We obtain data used for estimating the risk-free term structure model from Bloomberg. We use the six-month LIBOR rates and the swap rates with maturities of one, two, three, four, five, seven and ten years. The CDS data are obtained from Markit. Our sample consists of single name CDS contracts that are denominated in US dollars between January 1, 2001 and May 2012. We keep daily CDS spreads on senior and subordinate debts. We eliminate firms that have less than one year of data for both senior and subordinate contracts before December 2007, which is the start of the recession due to the financial crisis according to the National Bureau of Economic Research's (NBER) website. This filter ensures that firms in our sample have a sufficiently long time series of CDS data prior to the financial crisis, thereby enabling their recovery rates' comparison before and during the financial crisis. The final sample consists of 46 firms.

Table 1 describes the firms in our sample. We report the sample averages and standard deviations of daily CDS spreads for senior and subordinate contracts. To save space, summary statistics are reported only for CDS contracts with one-, five-, and ten-year maturities. The columns next to each firm name report the firm's ticker, its overall entity ratings, and its start and end dates of CDS data. The overall entity rating corresponds to the firm's sample monthly average of S&P long-term entity credit ratings from COMPUSTAT. For each firm-month, we assign an integer value to each rating class from 1 (AAA) to 6 (B). The monthly ratings are then averaged over the sample and rounded off to the nearest integer, then translated back to character rating.

Table 1 shows firms in our sample are distributed across AAA to B rating categories, with a relatively higher concentration in the A and BB categories. Table 1 also indicates a substantial variation in the CDS spreads across firms. Freddie Mac and Fannie Mae have the lowest average spreads for the senior and subordinate contracts in our sample, which reflect their higher credit worthiness relative to the other firms. In most cases, we find that average CDS spreads increase with maturity, indicating an upward sloping term structure of credit spreads for both the senior and subordinate contracts. The exceptions are Bear Stearns, Countrywide Financial and Lehman Brothers, which have data starting only after December 2006. We find that these firms have exceptionally large spreads at short maturities starting

in mid-2007, suggesting that they were in distress. The start and end periods of CDS data varies across firms as dictated by the availability of both senior and subordinate contracts.

3.2 Estimation

We use the unscented Kalman filter to filter the latent state variables. Consider the following nonlinear state-space system

$$Y_t = G(X_t) + u_t, \tag{14}$$

where Y_t is the D-dimensional vector of observables, and X_t is a vector of latent state variables. Equation (14) represents the measurement equation. We assume additive D-dimensional normally distributed error term, u_t, with unconditional covariance matrix, i.e. $E\left[u_t^T u_t\right]$, that is diagonal.

The Radon–Nikodym derivative takes the form

$$\frac{\Delta P}{\Delta Q} = \frac{\exp\left(-\Lambda_t' \varepsilon_{t+1}\right)}{E_t[\exp\left(-\Lambda_t' \varepsilon_{t+1}\right)]}.$$

We assume time-varying prices of risk that depend on the state variables, i.e. $\Lambda_t = \lambda_0 + \lambda_1 X_t$. Therefore, the dynamics of state variables under the physical measure are given by

$$\begin{aligned} X_{t+1} &= \mu^P + \rho^P X_t + \Sigma \varepsilon_{t+1} && (15) \\ \mu^P &= \mu - \Sigma \lambda_0 && (16) \\ \rho^P &= \rho - \Sigma \lambda_1, && (17) \end{aligned}$$

where ε_{t+1} is the normally distributed state noise. The superscript P denotes physical-measure parameters. The λ_0 is a 5×1 vector and λ_1 is a diagonal 5×5 matrix, which characterize the market price of risks. We normalize the state propagation equation such that the physical-measure drift, i.e. μ^P, of state variables X_3, X_4, and X_5 are zero. In our setup, the state propagation equation (15) is Gaussian, while the measurement equation (14), which is determined by (13) is nonlinear. We therefore use the unscented Kalman filter, which is suitable for the nonlinear filtration. We use the square-root unscented Kalman filter proposed by Van der Merwe and Wan (2001), which is found to be numerically stable and computationally feasible. Other studies that apply the unscented Kalman filter to estimate the risk-free term structure and credit risk models include Carr and Wu (2009), Chen, Cheng, Fabozzi, and Liu (2008), and Christoffersen, Dorion, Jacobs and Karoui (2014).

For ease of estimation and parsimony, we make further normalizations to the CDS model parameters. Specifically, we normalize the factor loadings of the intensity specific latent factors X_3 and X_4 (see equation (6)) to one, i.e., we set $\alpha_3 = \alpha_4 = 1$. Similarly, we normalize $\beta_5^{sen} = 1$, which is the senior CDS contracts' factor loading for X_5 that drives the recovery dynamic

(see equation (7)). All other loadings in the model are estimated as model parameters. We emphasize that this normalization does not impact our conclusions.[5]

Both the risk-free model and the credit risk model are estimated using the standard maximum-likelihood together with the unscented Kalman filter. In case of the risk-free term structure model, the measurement equation (14) include the 6-month LIBOR and a panel of swap rates. While in case of the credit risk model, the measurement equation (14) are daily CDS spreads on senior and subordinate debts across 5 different maturities. In both cases, we construct the log-likelihood value at time t assuming normally distributed forecasting errors,

$$l_t(\Theta) = -\frac{1}{2}\log\left(\overline{\Omega}_t\right) - \frac{1}{2}\left(Y_t - \overline{Y_t}\right)^T \left(\overline{\Omega}_t\right)^{-1} \left(Y_t - \overline{Y_t}\right) \tag{18}$$

where $\overline{Y_t}$ and $\overline{\Omega}_t$ denote the ex-ante forecasts conditional on time $t-1$ of Y_t and the conditional covariance matrix, Ω_t, obtained from the unscented Kalman filter. The model parameters are chosen to maximize the log likelihood of the data series,

$$\Theta = \arg\max_{\Theta} \textstyle\sum_{t=1}^{N} l_t(\Theta).$$

The parameter set Θ for the risk-free model contains δ_0, δ_1, δ_2, μ^r, μ^{rP}, ρ^r, ρ^{rP}, Σ^r and the measurement error standard deviations of LIBOR and swap rates for the risk-free term structure model. The superscript r denotes the factors specific to the risk-free term structure and the superscript P indicates the dynamics under the physical measure.

Following Duffee (1999), we first estimate the parameters describing the short rate dynamic. We assume that the estimated short rate dynamic is the true process. The dynamic of the risk-free term structure factors is estimated only once and assumed to be the same across all firms. Therefore, the parameter set Θ for the credit risk model contains α_0, α_1, α_2, β_0^{sen}, β_1^{sen}, β_2^{sen}, β_0^{sub}, β_1^{sub}, β_2^{sub}, β_5^{sub}, μ^c, ρ^c, ρ^{cP}, Σ^c and the measurement error standard deviations of the senior and subordinate CDS spreads. The superscript c indicates the factors specific to the credit risk (i.e., $X_t^c = [X_{3,t} \quad X_{4,t} \quad X_{5,t}]'$). Therefore, μ^c is a 3×1 vector, and ρ^c, ρ^{cP}, and Σ^c are 3×3 diagonal matrices. Recall that we normalize $\alpha_3 = \alpha_4 = \beta_5^{sen} = 1$ and $\mu^{cP} = 0$.

[5] By normalizing the values of α_3 and α_4, their values absorbed by (and identified with) the structural dynamics of the latent factor X_3 and X_4. Similarly, normalizing β_5^{sen}, its effect will be reflected in the estimates of the structural dynamic of the latent factor X_5.

4 Empirical results

4.1 Risk-free term structure

Table 2 presents estimation results for the risk-free term structure. Panel A reports the parameter estimates describing the dynamic of the two latent term structure factors, i.e. X_1 and X_2, as well as their loadings on the instantaneous risk-free rate (see equations (1)-(2)). All estimates in Table 2 are statistically significant. The first factor is closely related to the long term interest rates. The correlation between the 10-year zero and the first factor is around 90%, while the correlation between the changes in the 10-year zero rates and the changes in the first factor is around 83.16%. The second factor is closely related to the difference between the long-term zero rates and the short-term zero rates. The correlation between the difference in 10-year zero rates and six-month zero rates with the second factor is -91%. Overall, our estimates suggest the first factor is closely related to the level of the yield curve, while the second factor is closely related to the slope of the yield curve, which is consistent with Duffee (1999). The two term structure factors are highly persistent under the risk-neutral measure. Looking at the autoregressive parameter, ρ^r, in Panel A of Table 2, we find the first factor (X_1) is slightly more persistent than the second factor (X_2). Panel B of Table 2 reports root-mean-squared errors (RMSEs) for different maturity yields. The RMSEs range from 7.15 to 21.03 basis points across different maturity contracts.

4.2 CDS model estimates and fit

Table 3 presents the distribution of the parameter estimates across the 46 firms in our sample. Panel A reports the distribution of factor loadings for the default intensity, the loss-given-default (LGD) of senior contracts, and the LGD of the subordinate contracts. Recall that our CDS valuation model consists of five latent factors. The first two factors, i.e. X_1 and X_2, are estimated in the first stage using the 6-month LIBOR and a panel of swap rates. Factors X_3 and X_4 are specific to the dynamic of default intensity, while factor X_5 affects the dynamic of LGD only and is referred to as the recovery factor. For each factor loading, Panel A reports the cross-firm mean, standard deviations, and distribution at 25, 50, and 75 percentiles. Recall that we normalize the factor loadings of the intensity dynamic on X_3 and X_4 to one (i.e., $\alpha_3 = \alpha_4 = 1$ in equation (6)), and the factor loading of the senior LGD is normalized to one (i.e., $\beta_5^{sen} = 1$ in equation (7)). Therefore, their distributions are not reported. The last row in Panel A reports the percentage of cross-firm estimates that are statistically significant at the five percent level.

Panel A shows a significant variation in the factor loadings of the latent factors X_1 and X_2 (i.e., α_1 and α_2 in equation (6)). This is partly because we have a large cross-section of firms, and it is the cross-sectional difference in spreads that drives the variation in their

loadings. The intensity dynamic loads significantly on the risk-free latent variables X_1 and X_2, suggesting a strong dependence of default risk on the risk-free term structure. Similarly, the LGD dynamics significantly load on X_1 and X_2 for most of the firms in our sample. Looking at the estimates of the constant term for LGDs (i.e., β_0 in equation (7)), we find they are substantially larger for senior contracts relative to subordinate contracts. This finding is in line with our expectation of larger expected recovery for senior contracts. The larger constant in the LGD dynamic for senior contracts implies that it has a lower loss-given-default rate relative to subordinate contracts.

In Panel B, we report the distribution of parameter estimates that drive the dynamic of factors X_3, X_4, and X_5 (see equation (8)). Parameter estimates for factors X_1 and X_2 are estimated using the risk-free term structure and reported in Table 2. The parameter estimates are reported under the physical and risk-neutral measures. Recall that the physical-measure drifts, μ^P, of latent factors $X_3, X_4,$ and X_5 are normalized to zero, and hence not reported. Panel B shows the LGD-specific latent factor, X_5, is highly persistent with most estimates of ρ being close to one.

Panel C reports the distribution of standard deviations of the measurement error. The measurement error's standard deviation is the highest for three-year CDS contracts, while for the remaining maturities, they are relatively comparable. Nevertheless, the measurement error's standard deviation is generally low.

Table 4 reports the model's performance in terms of the relative root-mean-squared errors and the mean absolute percentage errors. We report the average relative RMSE and absolute percentage error for the overall sample as well as the breakdown for different rating classes and industries. To save space, we report the model fit only for contracts with one, five, and ten years to maturity. The relative root-mean-squared error (RMSE) for maturity h contracts is calculated as

$$\text{Relative RMSE} = \sqrt{\frac{1}{G}\sum_{j=1}^{G}\left(\frac{CDS(j,h) - CDS^M(j,h)}{CDS(j,h)}\right)^2},$$

where G is the number of observations used, and $CDS(j,h)$ and $CDS^M(j,h)$ are the market-observed and model-implied spreads of the jth contract respectively. Similarly, the absolute percentage error is calculated as

$$\text{Absolute percentage error} = \frac{1}{G}\sum_{j=1}^{G}\frac{\left|CDS(j,h) - CDS^M(j,h)\right|}{CDS(j,h)}.$$

Table 4 shows the RMSE values of senior and subordinate contracts with the same maturity are very comparable. This finding suggests our CDS model is able to fit senior and subordinate contracts equally well. We find that one-year CDS contracts have the largest RMSE, while for

CDS contracts at longer default horizons, their RMSE values are comparable. Looking at the overall relative RMSE, the average relative RMSEs across 46 firms are around 10-11% for the five and ten-year contracts, while it is about 19% for one-year contracts. We find similar results when looking at the absolute percentage error. Table 4 shows that on average, the relative RMSE as well as the mean absolute percentage error are lower for firms in the higher rating categories. The breakdown based on the industry categories shows that the model performs equally well in fitting the term structure of CDS spreads across industries. Importantly, the model fits the term structure of senior and subordinate contracts equally well, suggesting that it is not biased towards senior over subordinate contracts, and vice versa.

4.3 The term structure of recovery rates

This section explores the recovery rate dynamic estimated from CDS contracts. We first examine cross-sectional differences in expected recovery rates. After, we examine the time-varying dynamic of expected recovery rates with a particular focus on financial firms.

4.3.1 Expected recovery across firms

Using the parameters and the factor dynamics estimated from the CDS valuation model for each firm, we calculate the time series of *expected recovery*, which is defined as

$$R^{EXP}(t,h) = 1 - E_t^Q[LGD_{t+h-1}], \tag{19}$$

where $E_t^Q[LGD_{t+h-1}]$ follows from equation (7). The expected recovery, $R^{EXP}(t,h)$, represents the market's expectation of the recovery rate at time t, if the firm were to default at time $t+h$ for $h \geq 0$. We calculate expected recovery rates for various horizons. However, to save space, we report results for three different horizons: 1-, 5-, and 10-year, which together comprehensively represent the term structure of expected recovery.

Table 5 reports the time-series averages of expected recovery and default probability for the overall sample and for firms grouped by different ratings and industries. The results are reported separately for senior and subordinate contracts. The table also reports the time-series averages of one-year *unconditional default probabilities* implied by the model.[6] The one-year unconditional default probability at time t is calculated as $1 - E_t^Q[1_{\tau>t+h}]$, where $E_t^Q[1_{\tau>t+h}]$ is the survival probability given in (5), and evaluated at $h = 1$ year. To see how the default probability is related to the expected recovery rates, we also report the time-series correlations between one-year expected recovery and one-year default probability for senior and subordinate contracts.

[6] To save space, we report time-series averages for each firm in our sample in Table I.A.1 of the Internet Appendix.

We find the expected recovery for the senior contracts is between 33.29% and 39.54%. As for the subordinate CDS contracts, the overall expected recovery falls between 20.64% and 24.65%, which is slightly more than half of the levels implied by the senior contracts. It is useful to compare our estimates of expected recovery which are evaluated under the risk-neutral measure to the realized estimates found in the literature. Altman and Kishore (1996) back out the realized recovery from a sample of 696 defaulted bonds between 1971 and 1995. They find an average realized recovery of 47.65% for the senior unsecured bonds and an average realized recovery of 31.34% for the subordinate bonds. In comparison to Altman and Kishore (1996), our results suggest that there is a *positive* recovery risk premium of 8% to 14% for senior contracts and 7% to 11% for subordinate contracts. The positive risk premium for the recovery rate that we find supports the view that bond investors are risk averse to how much they can recover after a firm defaults.

Table 5 shows the correlations between one-year expected recovery and one-year unconditional default probability are negative, suggesting that on average, the recovery rate at default decreases when the likelihood of default rises. This finding is consistent with the literature documenting the decline in recovery rates during the economic downturn, i.e. when default is more likely.

We also find that one-year unconditional default probability, on average, is larger for firms with worse credit ratings, reflecting their higher likelihood of default. However, we do not find that firms' ratings monotonically explain the cross-sectional differences in recovery rates. This finding is consistent with Altman and Kishore (1996) who show that the bond ratings have no impact on the recovery rates once the seniority of the bonds is taken into consideration. Credit ratings, on the other hand, are solely assigned based on the firm's default probability, which explains why we observe a clear monotonic relationship between the risk-neutral default probabilities and the credit ratings.

4.3.2 Time-varying expected recovery

We next examine the time-varying dynamic of expected recovery rates. Figure 2 reports time-series properties of the expected recovery dynamic for the 46 firms in our sample. The top two panels plot the time series of one-year expected recovery rate averaged across firms for senior (left) and subordinate (right) contracts. We calculate daily one-year expected recovery rates for each firm over its sample period. The results in Figure 2 represent their cross-sectional averages through time. We require that data on at least five firms are available to plot the daily cross sectional averages. The time-series dynamics of expected recovery at other horizons are qualitatively similar and are not plotted here due to space constraint. As expected, the top two panels in Figure 2 show the expected recovery rate for senior contracts is always higher than that for subordinate contracts. The average expected recovery rates for both senior and subordinate contracts are at their lowest level near the end of 2008, corresponding with the

midst of the credit risk crisis.[7] We also note the levels of expected risk-neutral recovery rates that we observe before and after the credit risk crisis are in line with realized recovery rates reported by Moody's. In Moody's reports, realized recovery rates for senior unsecured bonds before the crisis were around 55%. It then falls to 37.5% during the crisis and increases back to about 49% in 2010.[8]

Figure 2 shows the expected recovery rates of senior and subordinate contracts are highly correlated. Nevertheless, there is a subtle difference between their dynamics. Looking at the top-right panel, we find the expected recovery rates of subordinate contracts steadily decline between 2004 and 2008, and slowly increase back to pre-crisis levels between 2010 and 2012. On the other hand, for the senior contracts (top-left), the expected recovery rate decreases most rapidly in the beginning of 2007 when the U.S. subprime mortgage market crisis started to unfold. The significant decline in the recovery rates during the financial crisis is not dominated by a single firm. In an unreported analysis, we find a similar sharp drop in expected recovery rates of senior contracts across firms in different rating groups and industries. Further, the top-left panel of Figure 2 shows the expected recovery rate of senior contracts in 2009-2012 does not return to pre-crisis levels as in the case for subordinate contracts.

The middle panels in Figure 2 plot cross-sectional averages of the slope of expected recovery. For each day t, we calculate the average value of expected recovery's slope by averaging the *relative* slope of expected recovery across firms. The relative slope of expected recovery is defined as

$$\frac{R^{EXP}(t, 10 \text{ years}) - R^{EXP}(t, 1 \text{ years})}{R^{EXP}(t, 10 \text{ years})}, \tag{20}$$

where $R^{EXP}(t, 1 \text{ years})$ and $R^{EXP}(t, 10 \text{ years})$ are expected recovery rates (see equation (7)) with horizons of 1 and 10 years, respectively. We use the relative slope of expected recovery instead of the raw slope so that the results are comparable across firms, which is important for calculating the daily cross-sectional averages.

The middle panels in Figure 2 show that the slope of expected recovery's term structure is also time-varying and mostly negative throughout our sample period. Thus, on average, the term structure of recovery rate is downward sloping. This finding suggests a market's expectation that a smaller fraction of CDS's notional value can be recovered as the default horizon increases. Our finding for the downward sloping term structure of expected recovery suggest that the recovery risk is priced in the CDS contracts. Longer maturity CDS contracts are exposed to more uncertainty about the firm's LGD. Investors are risk averse about this uncertainty and hence command a recovery risk premium for investing in longer term debt

[7] There is a sharp drop in the cross-firm expected recovery in 2003. This result is driven by the fall in expected recovery of Fannie Mae and Freddie Mac when the accounting practices of these two firms were under intense scrutiny. We discuss the recovery rate dynamics of Fannie Mae and Freddie Mac in Section 4.5.

[8] For Moody's report of the recovery rates prior to the crisis, see Emery, Ou, and Tennant (2008). For Moody's report during and after the credit risk crisis, see Chiu, Metz, and Ou (2011).

contracts.

We compare the time-varying dynamics of expected recovery term structure to that of CDS spreads. This exercise helps us answer whether changes in the term structure of recovery rate can be explained by changes in the term structure of CDS spreads. The bottom-left panel of Figure 2 plots daily average slopes of the market observed CDS spreads across firms. Daily average slope of market spreads is calculated as the mean of the *relative* slope of CDS spreads across firms. Similar to the average slope of expected recovery, we define the relative slope of CDS spreads as

$$\frac{CDS\left(t, 10 \text{ years}\right) - CDS\left(t, 1 \text{ years}\right)}{CDS\left(t, 10 \text{ years}\right)}, \tag{21}$$

where $CDS\left(t, 1 \text{ years}\right)$ and $CDS\left(t, 10 \text{ years}\right)$ are market-observed CDS spread levels with 1 and 10 years to maturity, respectively. To save space, we only report the average slope of senior CDS spreads. Looking at the middle-left and bottom-left panels, we do not find a clear relationship between the slope of expected recovery and the slope of CDS spreads on an aggregate level. Further, the time-series correlation between these two slopes is approximately -51%. We find that the average slope of CDS spreads is positive and fairly stable up until the start of the subprime crisis. Afterward, there is a sharp drop in the average slope, resulting in a negatively sloped CDS term structure in the beginning of 2009.

An important observation from the middle panels of Figure 2 is that during the 2008 crisis, the slope of expected recovery is close to zero. The relatively flat term structure of expected recovery during the crisis implies the market prices the short-term and long-term CDS in such a way that they have equally low recovery rates. The low levels of short-term expected recovery support the prevailing view that fire-sale and liquidity effects may depress the firm's ability to recover its fair value if it were to default in the midst of economic distress (see Schuermann (2004) and Altman (2015)). We discuss the term structure of expected recovery during the 2008 subprime crisis in Section (4.4).

4.3.3 Financial versus non-financial firms

Firms in the financial industry played an important role in the 2008 crisis. We examine their expected recovery dynamic further in this subsection. Out of the 46 firms in our sample, 17 of them are classified as financial firms. Most of the financial firms we study have a sample average rating of "A" or higher. We define financial firms as those belonging to the finance, insurance, and real estate (FIR) sectors following the classification on Kenneth French's website. The remaining 29 firms in our sample are classified as non-financial firms. In Table 1, we denote financial firms with an asterisk next to the company's name.

Panels in the left (right) column of Figure 3 plot time-series properties of expected recovery for financial (non-financial) firms. The values shown in each panel represent their daily cross-sectional averages implied by their senior CDS contracts. The top panels plot the time-series

averages of one-year expected recovery. We find the expected recovery of firms in the financial sector falls sharply in mid-2007, which marks the beginning of the subprime crisis. We obtain similar conclusions when looking at the recovery dynamics for subordinate contracts.

Next, we look at the average slope of expected recovery for financial and non-financial firms. The middle panels of Figure 3 plot the results. The middle-left panel shows that the slope of expected recovery for financial firms started increasing rapidly at the beginning of 2007 and stayed mostly positive from mid-2007 through 2008. This finding suggests that the expected recovery's term structure of financial firms were upward sloping during the financial crisis. Importantly, the inversion of the expected recovery's term structure is only evident for financial firms. For non-financial firms (middle-right panel), the average slope of their expected recovery did not invert but were relatively flat from mid-2007 through 2009.

The two bottom panels in Figure 3 plot the average slope of CDS spreads for financial and non-financial firms. The average slope of the CDS spreads for financial firms are fairly stable up until the beginning of the subprime crisis. It then changes sharply from positively-sloped to negatively-sloped in 2007. In fact, the term structure of CDS spreads for financial firms remained negatively sloped for most of 2008 through 2009. During this period, it was more expensive to insure a financial firm through a 1-year senior CDS contract than through a 10-year senior CDS contract. The bottom-right panel shows the slope of non-financial firms' CDS spreads also started decreasing in 2007. However, it did not fall as quickly compared to that of financial firms and became inverted for only a few months in 2009. For both financial and non-financial firms, we find the term structure of CDS spreads reverted back to being positively-sloped after the subprime crisis.

Overall, Figure 3 shows that investors expected the recovery rate to be lower during the subprime crisis, which in turn raised the spread of the short-term CDS contracts relative to those at the longer horizons. Our results suggest that the fear of illiquidity and of the fire-sale effect is significantly priced in CDS spreads between 2007 and 2008, and this effect is significantly stronger for firms in the financial industry.

4.4 The impact of the financial crisis

4.4.1 Changes in the term structure of expected recovery

In this section, we examine the impact of the 2008 subprime crisis. The top two panels in Figure 4 plot the average term structure of expected recovery for senior and subordinate contracts in three periods: February 2007 (pre-crisis), February 2008 (during-the-crisis), and February 2011(post-crisis). The results shown are averages of firms for which we have data available in each period. For comparisons, we also plot the average CDS spreads (bottom-left panel) as well as the average default probabilities (bottom-right panel) implied by the model.

The top two panels of Figure 4 show the average expected recovery rate in February 2007

(solid line), on average, decreases as the horizon increases. The expected recovery rate, on the other hand, exhibits a U-shaped pattern during the 2008 subprime crisis (dotted line). The observed U-shaped pattern in the expected recovery provides an important economic insight on the market's expectation of the recovery level in 2008. That is, during the crisis, investors expect firms will recover more of their debt's value if they were to default after the crisis. These findings apply to both senior and subordinate contracts. We find the U-shape pattern observed during the crisis disappears or becomes less evident in the 2011 post-crisis period (dashed line). Thus, we find that, on average, the term structure of expected recovery in 2011 returns to their pre-crisis shape.

The bottom-left panel of Figure 4 shows the CDS term structure flattens during the financial crisis. The change in shape of CDS term structure somewhat mirrors that of the expected recovery. Before the subprime crisis, the slope of credit spreads is clearly upward sloping. However, in February 2008, the CDS spreads' term structure has a hump shape; the credit spreads increase from three to five years, and then slightly decrease from five to ten years.

Next we look at the term structure of default probabilities shown in the bottom-right panel of Figure 4. We find the shape of default probability is slightly convex before the financial crisis, while during the crisis, it becomes concave. This result suggests that the relative increase in default probabilities during the financial crisis period is higher at the short horizon. Therefore, during the financial crisis, we observe changes in the term structure of default probabilities, as well as for the expected recovery.

As shown earlier in Figure 3 and Table 6, the subprime crisis affected the recovery dynamics of financial firms more severely than others. We next examine the recovery term structure of individual financial firms before (February 2007), and during (February 2008), the subprime crisis. Figure 5 plots the expected recovery term structure for senior contracts. Each panel corresponds to an individual firm denoted by its ticker in the title.[9] The general patterns in Figure 5 is that the recovery term structure are, on average, upwardly sloped in February 2008, even at extremely short horizons. This finding also applies to subordinate contracts, which are not reported here to save space.

The changing shape of expected recovery term structure during the crisis period emphasizes the need to incorporate stochastic recovery when pricing credit instruments. The assumption of a constant recovery commonly used in the literature has two important implications. First, it implies that the recovery rate is constant through time, and second, it implies the same risk-neutral recovery rate across all future horizons regardless of when the default happens. Clearly, our results demonstrate that these two assumptions are not supported by the data. Finally, Zhang (2009) shows the realized recovery rates are negatively associated with the

[9] Although we have 17 financial firms, we plot the results for 16 firms in order to save space and conserve the 4-by-4 structure of the panels. We exclude Capital One Bank in Figure 5, though its results are qualitative similar to the other financial firms.

macroeconomic conditions at the origination of the loans. Our results of upward sloping term structure of risk-neutral recovery during the financial crisis are in line with his findings.

4.4.2 Changes in expected loss given default

We next examine the impact of the 2008 crisis on the level of expected loss given default. We focus on two periods. The first is February 2007, which is the period *before* the crisis. The second is February 2008, which is the period *during* the financial crisis.

For each firm-day observation in February 2008, we calculate the logarithmic changes (i.e., percentage changes) in LGDs and in CDS spreads relative to their one-year prior values. The monthly averaged values in February 2008 are then calculated for each firm. Table 6 reports the results cross-sectionally averaged over different rating classes and industries.[10] The level of CDS spread depends on two sources of risk: the firm's default risk, and the LGD to the CDS's notational value. Thus, by looking at CDS spread changes in relation to changes in the firm's expected LGD, we can qualitatively examine to what extent do CDS spread changes during the 2008 crisis is due to changes in the LGD.

Table 6 shows the expected LGD increases significantly at short-term horizons, i.e. from one to five years. Looking at the overall sample, the change in expected LGD for senior contracts is about 50% at the one-year horizon, while it is negligibly small at the ten-year horizon. The largest increase in expected LGD is for the Finance, Insurance and Real estate sectors (F,I,R). The breakdown based on credit ratings shows the increase in LGD is larger for higher rated firms. This finding is not surprising because most of the financial firms in our sample have AAA/AA ratings. We find similar results when looking at the change in LGD term structure for subordinate firms.

Table 6 also reports the average logarithmic change in CDS spreads from the pre-crisis to during-the-crisis periods. We find that market-observed CDS spreads increase significantly across maturities, seniority, rating classes, and industries, reflecting the higher cost of insuring all types of firms during the crisis. Similar to the change in term structure of LGD, the increase in CDS spreads is larger at shorter horizons for both seniorities. For the overall sample, 1-year senior CDS spreads increase by 260.9%, which is large relative to the 121% increase for 10-year senior CDS. The magnitude of change in CDS spreads is much larger relative to change in expected LGD at all horizons. This finding suggests the increase in CDS spreads during the crisis is largely due to the increase in default probabilities. Nevertheless, we observe a substantially large increase in LGD at short-term horizons, particularly for highly rated firms. This suggests that a sizable increase in short-term CDS spreads can be attributed to the change in LGD. We do not observe a significant change in LGD for subordinate contracts among B-rated firms, which is the lowest rating category in our sample.

[10] D.R. Horton Inc. and Selectron Corp. do not have CDS data in February 2008 and are excluded from Table 11. As a result, 44 firms are reported in the Table.

Overall, Table 6 shows the subprime crisis shock increases the LGD substantially at the short-term horizons, which then raises the level of CDS spreads. However, the change in LGD at the longer horizons are small and even negative for some firms, suggesting a significant change in the term structure of expected recovery rates during the crisis.

4.5 Time-varying recovery: Case studies

This section examines how important news arrivals impact the time-series dynamic of expected recovery rate at an individual firm level. The objective of this empirical exercise is twofold. First, we show that our recovery dynamics estimated using a panel of CDS spreads are economically meaningful in that they respond to important news arrivals. Second, by showing that the recovery rate responds quickly to important economic news, we further underscore the importance of time-varying recovery rate for pricing of CDS contracts.

We focus our study on the two government sponsored entities, Fannie Mae and Freddie Mac. We chose these two firms because they were severely impacted by the subprime crisis as well as drawing widespread media attention. Both Fannie Mae and Freddie Mac shared similar economic fates during our sample period; they were subject to accounting scandals in 2003, and their revenues are strongly tied to the U.S. housing market. The top two panels in Figures 6 and 7 plot the time series of one-year expected recovery for Fannie Mae and Freddie Mac, respectively. The panels below plot their time series of market-observed and model-implied five-year CDS spreads, model-implied one-year default probabilities, and one-year trailing stock return. Looking at the second-row panels, the market-observed CDS spread (black line) and the model-implied CDS spread (grey line) have almost identical time-series dynamics. These results show that our CDS model performs well in fitting the spreads of senior and subordinate contracts.

Figures 6 and 7 show the expected recovery of Fannie Mae and Freddie Mac vary substantially over time. The expected recovery dynamics of these two firms are fairly similar. The time-series average of their expected recovery on a five-year senior contract is 70.15% for Fannie Mae, and 64.70% for Freddie Mac. These values are relatively large compared to other firms in our sample, which reflect their sponsorship by the U.S. government. In 2003, we observe a few negative shocks to the expected recovery of Fannie Mae and Freddie Mac, while their unconditional default probabilities (third-row-left panels) were relatively stable. These findings indicate the increases in CDS spreads of Fannie Mae and Freddie Mac in 2003 were primarily due to the fall in their debts' expected recovery.

An important news item that significantly impacted both Fannie Mae and Freddie Mac occurred on June 9, 2003. Figures 6 and 7 show the expected recovery of Fannie Mae and Freddie Mac drop significantly in mid-2003, before climbing back to their conventional values. On that day, Freddie Mac fired three of its top executives because they refused to cooperate in

the investigation of the firm's accounting practices. Freddie Mac was re-audited for three prior years because its previous auditor, Arthur Andersen, misapplied accounting rules. As a result of the news, Freddie Mac's stock plunged 16% on that day, and the option-implied volatility jumped from 24% to 50%. This accounting malpractice received wide media attention and its effect spread to Fannie Mae. The fact that a firm's accounting-error news substantially impacts its debt's recovery rate is not surprising since financial statements are an important source of information for lenders (see also Bharath, Sunder and Sunder (2008) for empirical evidence).

On March 15, 2007, the United States House committee on financial services put forward the bill intended to avoid a repeat of the financial scandals that affected both Fannie Mae and Freddie Mac.[11] On this date, the expected one-year recovery of senior contracts for Fannie Mae and Freddie Mac dropped by about 4.4 and 4.8 %. This event, which marks the beginning of the subprime crisis, initiated a series of volatile changes in CDS spreads for Fannie Mae and Freddie Mac from mid-2007 through 2008 when their CDS data ends. Also, during this period, we observe a rapid rise in the unconditional default probabilities and option-implied volatilities of Fannie Mae and Freddie Mac, reflecting the fear of their default.

Although the expected recovery of these two firms fell by as much as 50% in 2007 relative to their pre-crisis levels, their levels started increasing in early 2008 and eventually exceeded their pre-crisis period values. Interestingly, while the expected recovery increased, CDS spreads of Fannie Mae and Freddie Mac widened, and their unconditional default probability rose sharply (bottom-left panels). In other words, the default risk of these two firms became positively correlated with their recovery rate during the crisis. The rising recovery rates of Fannie Mae and Freddie Mac during the subprime crisis can be linked to the government bail out. In March 2008, JP Morgan Chase and the federal government bailed out Bear Stearns as it neared collapse.[12] This first bailout of the subprime-crisis era signaled the willingness of the U.S. government to rescue other ailing financial companies, including Fannie Mae and Freddie Mac.[13] A a result, their CDS spreads fell in the end of March 2008, and in the mean time, their one-year expected recovery rose back to its pre-crisis level. The second burst of positive news for Fannie Mae and Freddie Mac came on July 24, 2008, when the United States Congress passed the Housing and Economic Recovery Act of 2008 (HERA).[14] This bill was intended to restore confidence in Fannie Mae and Freddie Mac by strengthening regulations and injecting capital into their mortgage funding. The enactment of the HERA led to the government conservatorship of Fannie Mae and Freddie Mac on September 7, 2008. This is when the

[11] http://archives.financialservices.house.gov/hearing110/htsyron031507.pdf

[12] http://www.federalreserve.gov/newsevents/reform_bearstearns.htm

[13] For some evidence in the media, see "Bear Stearns bailout may signal time to buy bank stocks" by Richard Barley and Natalie Harrison, NY Times, March 20, 2008.

[14] The bill authorized the Federal Housing Administration to guarantee up to $300 billion in new fixed rate mortgages for subprime borrowers, if lenders would write-down principal loan balances to 90 percent of current appraisal value.

expected recovery levels of the two firms rose, surpassing their pre-crisis levels.

As shown in Figures 6 and 7, we argue that the increase in expected recovery in 2008 is due to the government bail out. The model-implied default probability of Fannie Mae and Freddie Mac increase by roughly 400% from mid-2007 to September 2008. However, their CDS spreads only increase by about 100%, suggesting the recovery rates of their debts must increase in order to balance the rapid rise in their default probability. Our results in Figures 6 and 7 provide an economic insight linking the effect of government bailouts to the borrowing costs in the debts market.

5 Identification

This section illustrates that the recovery dynamic can be identified and well estimated. First, we report a full-scale simulation study of our model. Second, using a static analysis, we show how the recovery levels of senior and subordinate debts are identified using the term structures of senior and subordinate CDS spreads jointly. Third, we explain the importance of using multiple-seniority CDS spreads to robustly estimate the default intensity and recovery rate dynamics.

5.1 Simulation study

We illustrate the identification of the recovery rate dynamic using an extensive simulation study. We simulated CDS spreads data using the "true" *Base case parameters* reported in Table 7. The baseline parameters are chosen such that they match the average pre-crisis spreads and expected recovery rates of the most representative industry (financial) in our sample.[15] Panel A reports factor loadings on the intensity dynamic and the subordinate LGD dynamic; see equations (6) and (7). Panel B reports parameters driving the latent factors X_3, X_4, and X_5; see equations (15)-(17).

We generate 100 sample paths of daily simulated CDS spreads using the "true" base case parameters in Table 7. We allow for normally distributed noise in the simulated CDS spreads. Each simulation path consists of 1,500 daily observations, which is approximately equal to 6 years. The noise term's standard deviation level is set equal to five percent of the simulated CDS spread level. The means and standard deviations of simulated CDS spreads at various maturities are reported in Panel A of Table 8.

We estimate the full CDS valuation model on 100 simulated sample paths of senior and subordinate CDS spreads. This estimation exercise yields 100 sets of parameter estimates, as

[15] The average spreads are calculated by pooling data from all financial firms. The average CDS spreads before December 2007 (pre-crisis period) are 22, 35 and 45 basis points for 1-, 5-, and 10-year senior contracts, respectively. The expected recovery rates for senior contracts are 50, 44, and 41 percent for 1-, 5-, and 10-year horizons, respectively.

well as the time series of latent state variables. We report the means and standard deviations of the parameters estimated from the simulated samples under the *Estimated parameters* section in Table 7. We find the parameters estimated from the simulation samples are very close to their true values. Importantly, any differences between the means and the true parameter values can be reconciled within two standard deviations. This finding suggests that the parameters in the full CDS valuation model can be estimated and identified.

We next examine the estimated time-series dynamics of the recovery rates. The top panel of Figure 8 plots the time-series of one-year expected recovery for senior contracts. The solid line displays daily averaged results generated by true parameters, and the dotted line displays daily averaged results based on the model estimates. One standard deviation band ("S.D. band") around the model estimates are represented by the shaded-grey area. We do not plot the standard deviation band around daily averaged results for the simulated data, i.e. solid line, to avoid visual clustering. Overall, we find the means of one-year expected recovery that we calculated from the data always fall within the shaded area, indicating that deviations between the data generated ("true") recovery and the estimated recovery can be reconciled within one standard deviation. We obtain similar conclusions when looking at expected recovery at other horizons.

The middle panel of Figure 8 plots the average term structure of expected recovery calculated using the simulated data and the model estimates. Average term structure of expected recovery is calculated by first averaging the term structure of expected recovery across time for each simulation path, and then again across the 100 samples. We plot the results calculated from the model estimates as well as from the true simulated data. As before, the shaded area represents one standard deviation band calculated from the estimation results. We observe a small downward bias in the expected recovery's term structure obtained from the estimations. However, such bias from the true expected recovery's term can be reconciled within one standard deviation, which could arise due to the noise that we add to the simulated CDS data. The bottom panel of Figure 8 plots the average percentage errors for five-year senior CDS spreads. We plot the means and standard deviations calculated across all paths at each time point. We find the time-series bias in pricing five-year senior CDS is very small, suggesting the model performs well in capturing the time-varying recovery rate and default intensity dynamics. We obtain similar conclusions when looking at the pricing errors for other maturities and for subordinate contracts.

Finally, we evaluate the model performance in fitting the simulated data by examining their pricing errors against their "true" values along three dimensions: CDS spreads, expected recovery rates, and binary CDS spreads. Table 8 reports the results. In Panel A, we report the means and standard deviations of simulated CDS spreads, expected recovery rates, and binary CDS spreads. A binary CDS spread is priced similarly to a standard CDS contract but with a zero recovery rate when the firm defaults, i.e. the LGD is 100 %. There is no

recovery risk in binary CDS contracts due to the certainty of zero percent recovery, and hence their spreads reflect only the default risk. We use binary CDS spreads as our benchmark for evaluating the robustness of the estimates for the default intensity dynamic. Overall, the simulated samples show an increasing term structure of CDS and binary CDS spreads, while expected recovery rates decrease as the default horizon increases.

Panel B of Table 8 summarizes the means and standard deviations of absolute percentage pricing errors for CDS spreads, expected recovery rates, and binary CDS spreads grouped by maturity and seniority levels. Results in Panel B of Table 8 show the pricing error is slightly higher for shorter-maturity CDS contracts. Nevertheless, the absolute percentage errors in pricing CDS spreads, on average, are small and fall under five percent. Expected recovery rates and binary CDS spreads are also reasonably well estimated with absolute errors below 11 % relative to their true values, respectively. These relatively small pricing errors for expected recovery rates and binary CDS spreads suggest that we can estimate the recovery and intensity dynamic fairly well.

5.2 Intuitive analysis

Using a static analysis, this section illustrates how the LGD levels of senior and subordinate CDS contracts are identified using their CDS term structures jointly. Our analysis is based on the base case parameters reported in Table 7.

We recall the LGD dynamics of senior and subordinate CDS share a common structure described by equation (7). The coefficients β_i^{sub} (β_i^{sen}) for $i = 1, 2, 5$ denote the loadings of subordinate (senior) LGD dynamic to the three state variables X_1, X_2, and X_5, respectively. However, X_5 is the recovery-specific factor, while X_1 and X_2 are identified with the risk-free term structure. Therefore, for ease of exposition, we set the factor loadings on X_1 and X_2 to zero. Further, recall that we normalize β_5^{sen} to one to facilitate the econometric identification. The relative difference between LGD_t^{sub} and LGD_t^{sen} in the model is therefore captured through β_5^{sub} and the level of $X_{5,t}$. For the static analysis, we examine how the level of $X_{5,t}$ and the value of its factor loading β_5^{sub} are identified from the term structures of senior and subordinate CDS.

Figure 9 plots various combinations of $X_{5,t}$ and β_5^{sub} that keep the ratio of senior to subordinate CDS spreads with the same maturity constant.[16] Other parameters except $X_{5,t}$ and β_5^{sub} are held fixed equal to the base case parameters in Table 7. We plot the results for five CDS maturities: one, three, five, seven, and ten-year CDS spreads. Looking at CDS spread ratios for each maturity, the relationship between $X_{5,t}$ and its factor loading β_5^{sub}

[16] The default intensity does not impact the ratio of senior to subordinate CDS term structures. In the Internet Appendix (Figure I.A.1), we show that default intensity factors, i.e., X_3 and X_4, do not affect the relative CDS spreads between senior and subordinate contracts, and hence do not impact their CDS spread ratios.

is non-monotonic, reflecting the quadratic specification of the LGD dynamic (see equation (7)). There are many possible combinations of $X_{5,t}$ and β_5^{sub} that keep the ratio of senior to subordinate CDS spreads constant. We therefore cannot identify a unique combination of $X_{5,t}$ and β_5^{sub} using *single-maturity* senior and subordinate CDS spreads.

Figure 9, however, shows that a unique pair of $X_{5,t}$ and β_5^{sub}can be identified when using *multiple-maturity* senior and subordinate CDS spreads. The constant-CDS-ratio curves of different maturities intersect at one location, which corresponds to the base case parameters $X_{5,t}$ and β_5^{sub}. Overall, Figure 9 illustrates, in a simple static setting, how the LGDs of senior and subordinate CDS contracts are identified from their CDS term structures.

5.3 Identifying time-varying intensity and recovery

This section discusses the importance of using multiple-seniority CDS contracts to identify the recovery rate and default intensity dynamics. We argue that the term structure of *multiple-seniority* CDS contracts is required to separately identify the time-varying default intensity dynamic from the time-varying recovery rate dynamic.

To support our argument, we estimate a constant recovery model on CDS data generated by the model with stochastic recovery and stochastic default intensity.[17] That is, we estimate a model that misspecifies the recovery rate dynamic but not the default intensity dynamic. Our objective is to show that if we use single-seniority CDS contracts in the estimation, the average recovery level can be recovered, but errors from misspecifying the recovery dynamic will cause systemetic biases in the default intensity estimates. However, if multiple-seniority CDS contracts are used, errors from misspecifying the recovery dynamic will not severely impact the default intensity estimates because such estimation approach imposes a strict identification mechanism on the default risk.

Using the CDS data simulated in Section 5.1, we estimate the constant recovery rate model on two distinct samples. The first estimation sample relies on simulated CDS spreads of senior *and* subordinate contracts (multiple seniority). In the second estimation sample, we estimate the model only on simulated senior CDS spreads (single seniority). The simulated data that we use consists of 100 paths each with 1500 observations; see Panel A of Table 8 for their summary statistics.

Table 9 reports absolute percentage pricing errors from the two estimation samples. We report the means and standard deviations of pricing errors for senior CDS spreads, binary CDS spreads, and expected recovery rates of senior contracts. We examine pricing errors only for senior contracts to facilitate a fair comparison between the two estimation samples. Absolute pricing error for senior recovery rates is calculated against the time-series averages of their simulated values (Panel A of Table 8).

[17] Under our modeling framework, the constant recovery rate model is obtained by setting the parameters β_1, β_2 and β_5 in equation (7) to zero.

As expected, Table 9 shows the pricing errors for senior CDS spreads are substantially smaller for the estimation sample that uses only senior contracts. This finding is not surprising because the model is optimized to fit only senior CDS spreads. However, this relatively small senior CDS pricing errors come at the expense of systemetic biases in the default intensity estimates. Table 9 shows the errors from pricing binary CDS spreads are much smaller when the model is estimated using both senior and subordinate CDS contracts. Because binary CDS spreads are only affected by default risk, we use them as our benchmark for evaluating the robustness of the default intensity estimates. The absolute pricing errors of binary CDS spreads are between 6.8–12.5% when estimated using multiple-seniority contracts (Estimation sample (1)), and 17.7–23.7% when estimated using single-seniority contracts (Estimation sample (2)).

Table 9 also reports the absolute percentage errors for senior recovery rates calculated against the time-series average of their simulated values, which is 50%. We find the errors are smaller when the model is estimated using both multiple-seniority CDS contracts. Nevertheless, the errors from the two estimation samples are fairly small, suggesting the model can roughly identify the average recovery level even when the true data generating process is a stochastic recovery model.

We next analyze the source of pricing errors for senior CDS spreads in the two estimation samples. Because the model that we estimate misspecifies the recovery rate dynamic to be constant, we expect the model's CDS pricing errors to vary with the time-varying recovery rates, which are absent from the model. Failure to find a strong relationship between the pricing errors and the time-varying recovery rates would suggest that this misspecification is absorbed as biases in other parameter estimates of the model. To test this conjecture, we estimate the following pooled regression of daily log CDS pricing errors on daily log implied LGDs:

$$log(CDS_{i,t}^{\tau}) \; - log(\widehat{CDS}_{i,t}^{\tau}) = a^{\tau} + b \log\left(Implied \;\; LGD_{i,t}^{\tau}\right) + \varepsilon_{i,t}, \tag{22}$$

where $CDS_{i,t}^{\tau}$ and $\widehat{CDS}_{i,t}^{\tau}$ are the model-generated and "true" simulated τ-year CDS spreads of simulation path i on day t. The $Implied \;\; LGD_{i,t}^{\tau}$ is the LGD level for τ-year CDS contracts.

For each of the 100 simulation paths, we estimate the model in equation (22) across five CDS maturities.[18] We find the regression adjusted R^2 value for the estimation sample (1) is, on average, 65.5%, while for the estimation sample (2), it is only 2.6%. These results show that when the model is estimated on multiple-seniority CDS spreads, CDS pricing errors correctly reflect time variations in the LGD which is absent from the constant recovery rate model.

[18] It is calculated as the ratio of CDS spreads to binary CDS spread with the same maturity. This definition is drawn from the fact that binary CDS assume zero percent debt recovery upon default and hence must have bigger spreads relative to CDS contracts of the same maturity. Therefore, the ratio of CDS to binary CDS spreads of the same maturity must be between zero and one, which reflects the implied loss rate on the debt value insured by the CDS contract. We use the implied LGD as our instrument that proxies for time-varying recovery dynamics.

However, if we estimate the constant recovery model using single-seniority CDS spreads, time variations in the recovery rate is incorrectly picked up by the intensity dynamic, suggesting that the default intensity is severely biased. In this latter case, the intensity dynamic no longer reflects only the default risk, but also the recovery risk, which explains why binary CDS spreads in Table 9 are severely mispriced for the estimation sample (1).

Overall, these results confirm the importance of using the term structure of senior and subordinate CDS spreads when estimating time-varying recovery model. That is, the use of multiple-seniority CDS contracts imposes a stricter identification mechanism on the default intensity, making it less likely to pick up changes in CDS spreads that are due to time-varying recovery rates.

6 Robustness check

6.1 Accounting for bid-ask spreads

We show that our parameter estimates are robust to the inclusion of daily CDS bid-ask spreads in the estimation. We collected daily bid and ask quotes from Bloomberg. We require that the bid-ask quotes from Bloomberg are available for at least four-consecutive-overlapping years with Markit on both senior and subordinate contracts. This requirement leaves us with sufficient data for six financial firms in our sample. These financial firms are: Freddie Mac (FRE), JP Morgan (JPM), Washington Mutual (WM), Well Fargo & Co. (WFC), and Wachovia Corp (WB). The bid-ask spreads of senior and subordinate CDS contracts for these 6 firms are highly correlated. Their correlation coefficients are greater than 0.85 suggesting that their relative spreads always move in tandem. We re-estimate the model incorporating daily bid and ask quotes for the six financial firms.

Following Pan and Singleton (2008), we account for CDS bid-ask spreads by assuming that τ-maturity senior contracts on day t are priced with normally distributed errors with conditional standard deviations of $\sigma_u^{SEN}(\tau) \cdot \left| Bid_t^{SEN}(t,\tau) - Ask_t^{SEN}(t,\tau) \right|$. We assume that $\sigma_u^{SEN}(\tau)$ is a constant, and $Bid_t^{SEN}(\tau)$ and $Ask_t^{SEN}(\tau)$ refer to day t's bid and ask spreads of the τ-maturity senior contracts. Similarly, we assume that τ-maturity subordinate contracts are priced with normally distributed errors with conditional standard deviations $\sigma_u^{SUB}(\tau) \cdot \left| Bid_t^{SUB}(\tau) - Ask_t^{SUB}(\tau) \right|$, where the variables are defined analogously.

The above error assumption affects our estimation in two ways. First, it affects the measurement equation (14), which is used to filter the latent state variables. Second, it affects the conditional covariance matrix of CDS pricing errors, $\overline{\Omega}_t$, which enters the log-likelihood equation (18).

We conduct the robustness exercise on the six financial firms. Due to the space consideration, we only report the expected recovery rates and the parameter estimates for Fannie Mae.

Results for the remaining five financial firms are summarized in Figure I.A.2 of the Internet Appendix.

Figure 10 plots the expected recovery rate for one-year horizon from the two estimation methods. We do not find that the recovery dynamics of Fannie Mae are significantly affected when including its daily bid-ask spreads in the estimation. This finding is further confirmed in Table 10, which reports the parameter estimates from the two estimation exercises for Fannie Mae, i.e. with and without daily bid-ask errors. Overall, we find that our estimates do not substantially differ when including daily bid-ask information in the estimation. We obtain similar results for the remaining five financial firms with bid-ask spread data.

6.2 Price discovery during the financial crisis

In this section, we explore the relative liquidity between senior and subordinate CDS spreads by examining their relative contribution to price discovery. Norden and Weber (2012) examine the equilibrium relation of senior and subordinate CDS spreads written on 20 large European banks. They find that during the financial crisis, price discovery in the CDS market occurs in senior CDS and that the long-run equilibrium relationship between senior and subordinate CDS spreads breaks down. We show that the price discovery mechanism in the CDS market of U.S. entities in our sample was not significantly affected during the crisis, suggesting that our main conclusions are not driven by structural changes in the relative liquidity between senior and subordinate spreads.

Following the method in Blanco, Brennan and Marsh (2005), we estimate a two-stage vector error correction model (VECM) examining the equilibrium relationship between senior and subordinate CDS spreads of 46 firms in our sample. Due to the space consideration, we describe the estimation procedure in the Internet Appendix and the results are reported in Table I.A.2. In summary, we show that the price discovery mechanism in the CDS market of U.S. entities in our sample was not significantly affected during the crisis, suggesting that our main conclusions are not driven by structural changes in the relative liquidity between senior and subordinate spreads.

7 Conclusion

Existing empirical work on the dynamic of risk-neutral recovery rates is limited due to the econometric difficulty in isolating the default risk component from the recovery component. In this paper, we circumvent this identification issue using joint information in the term structures of senior and subordinate credit default swaps (CDS). Our empirical approach consists of jointly modeling senior and subordinate CDS spreads in a reduced form framework. We illustrate our model's identification of the recovery dynamic using a simple static analysis,

and provide extensive simulation results showing that the model is econometrically identifiable.

We estimate the model on a large sample of daily senior and subordinate CDS data between 2001 and 2012. The estimated recovery rates are economically plausible in the sense that they imply a positive recovery risk premium when compared to prior studies that estimated the realized recovery rates. In addition, we show that recovery rates implied by the CDS contracts are highly responsive to important corporate events such as accounting news with significant impact on the lender's ability to recover the debt.

We find that the expected recovery rates decline substantially during the 2008 financial crisis period. This decline in expected recovery rates is significantly larger at the shorter horizons relative to the longer horizons. This disproportionate change in the expected recovery rates between short and long horizons results in the inversion of the recovery rate term structure: from downward sloping before the financial crisis period to upward sloping during the financial crisis period. Furthermore, this inversion is more prominent for firms which are in distress during the crisis period, i.e. financial firms.

Overall, the use of joint information from senior and subordinate CDS significantly improves the identification of the recovery rates. Our model is able to capture the stylized facts and provide new insights about the term structure of expected recovery rates. Lastly, we provide strong evidence for the importance of time-varying recovery rates in credit risk models.

Appendix A: Default-free model

We want to price a default free zero coupon bond

$$B(t, t+h) = E_t^Q \left[\exp(-\sum_{j=0}^{h-1} r_{t+j}) \right] \tag{A.1}$$

We can re-write equation (1) in the form

$$\begin{aligned} r_t &= (\delta_0 + \delta_1 X_{1,t} + \delta_2 X_{2,t})^2 \\ &= \delta_0^2 + 2\delta_0 \delta' X_t^r + X_t^{r\prime} \delta \delta' X_t^r \end{aligned} \tag{A.2}$$

where $\delta = (\delta_1, \delta_2)'$ and $X_t^r = (X_{1,t}, X_{2,t})'$ are 2×1 vectors. Recall that from equation (2), the dynamics of the state variables are

$$X_{t+1}^r = \mu^r + \rho^r X_t^r + \Sigma^r \varepsilon_{t+1}^r, \tag{A.3}$$

where ρ^r and Σ^r are 2×2 matrices.

We compute the expectation in equation (A.1) using law of iterative expectations. Recall

that the price of a one period ahead zero coupon risk-free bond at time $t+h-1$ is

$$B(t+h-1,t+h) = E^Q_{t+h-1}[\exp(-r_{t+h-1})] = \exp(-r_{t+h-1}) \tag{A.4}$$

Substituting expression (A.2) we have

$$\begin{aligned} B(t+h-1,t+h) &= \exp[-(\delta_0^2 + 2\delta_0\delta' X^r_{t+h-1} + X^{r\prime}_{t+h-1}\delta\delta' X^r_{t+h-1})] \\ &= \exp(A_1 + B_1' X^r_{t+h-1} + X^{r\prime}_{t+h-1} C_1 X^r_{t+h-1}) \end{aligned} \tag{A.5}$$

where

$$A_1 = -\delta_0^2;\ B_1 = -2\delta_0\delta';\ C_1 = -\delta\delta' \tag{A.6}$$

The price of a two period ahead zero coupon bond at time $t+h-2$ is

$$\begin{aligned} B(t+h-2,t+h) &= E^Q_{t+h-2}[\exp(-r_{t+h-1} - r_{t+h-2})] \\ &= \exp(-\delta_0^2 - 2\delta_0\delta' X^r_{t+h-2} - X^{r\prime}_{t+h-2}\delta\delta' X^r_{t+h-2}) \times \\ &\quad \exp(A_1 + B_1'\left(\mu^r + \rho^r X^r_{t+h-2}\right) + \left(\mu^r + \rho^r X^r_{t+h-2}\right)' C_1 \left(\mu^r + \rho^r X^r_{t+h-2}\right)) \times \\ &\quad E^Q_{t+h-2}\left[B_1'\Sigma^r\varepsilon^r_{t+h-1} + 2\left(\mu^r + \rho^r X^r_{t+h-2}\right)' C_1\Sigma^r\varepsilon^r_{t+h-1} + \varepsilon^{r\prime}_{t+h-1}\Sigma^{r\prime}C_1\Sigma^r\varepsilon^r_{t+h-1}\right] \end{aligned} \tag{A.7}$$

Now, the expectation of an exponential of a quadratic Gaussian random variable can be computed using the following property. Let

$$Q = \epsilon' V \epsilon + a'\epsilon + d, \tag{A.8}$$

then the expectation of the exponential of Q is given by

$$E[\exp(tQ)] = \exp(-\frac{1}{2}\ln(\det(I - 2t\ \Gamma\ V)) + td + \frac{1}{2}ta'(\Gamma^{-1} - 2tV)^{-1}at) \tag{A.9}$$

where ϵ is an $N \times 1$ vector described by a multi-variate normal distribution $\epsilon \sim N(0,\Gamma)$ and det indicates determinant.[19]

Comparing equation (A.9) and the expectation E^Q_{t+h-2} in the last line in equation (A.7) we have, $\epsilon = \varepsilon^r_{t+h-1}$, $V = \Sigma^{r\prime}C_1\Sigma^r$, $a' = (B_1'\Sigma^r + 2\left(\mu^r + \rho^r X^r_{t+h-2}\right)' C_1\Sigma^r)$, $\Gamma = I$, $t = 1$ and $d = 0$. Using this equivalence and organizing the common terms together, the expectation in equation (A.7) can be written as below.

$$B(t+h-2,t+h) = \exp(A_2 + B_2' X^r_{t+h-2} + X^{r\prime}_{t+h-2} C_2 X^r_{t+h-2}) \tag{A.10}$$

$$\begin{aligned} A_2 = \quad & -\delta_0^2 + (A_1 + B_1'\mu^r + \mu^{r\prime}C_1\mu^r) \\ & +\tfrac{1}{2}(\Sigma^{r\prime}B_1 + 2\Sigma^{r\prime}C_1\mu^r)'(I - 2\Sigma^{r\prime}C_1\Sigma^r)^{-1}(\Sigma^{r\prime}B_1 + 2\Sigma^{r\prime}C_1\mu^r) \\ & -\tfrac{1}{2}\ln[\det(I - 2\Sigma^{r\prime}C_1\Sigma^r)] \end{aligned} \tag{A.11}$$

$$B_2' = \quad -2\delta_0\delta' + (B_1 + 2C_1\mu^r)'\rho^r + 2(\Sigma^{r\prime}B_1 + 2\Sigma^{r\prime}C_1\mu^r)'(I - 2\Sigma^{r\prime}C_1\Sigma^r)^{-1}\Sigma^r C_1\rho^{r\prime} \tag{A.12}$$

$$C_2 = \quad -\delta\delta' + \rho^{r\prime}C_1\rho^r + 2\left(\Sigma^{r\prime}C_1\rho^r\right)'(I - 2\Sigma^{r\prime}C_1\Sigma^r)^{-1}\Sigma^{r\prime}C_1\rho^r \tag{A.13}$$

This process is repeated and the expectation in equation (A.1) is

$$B(t,t+h) = \exp(A_h + B_h' X^r_t + X^{r\prime}_t C_h X^r_t) \tag{A.14}$$

[19] The proof is given in Mathai and Provost (1992, p. 40).

$$A_h = \begin{array}{c} -\delta_0^2 + (A_{h-1} + B_{h-1}'\mu^r + \mu^{r\prime} C_{h-1}\mu^r) \\ +\frac{1}{2}(\Sigma^{r\prime} B_{h-1} + 2\Sigma^{r\prime} C_{h-1}\mu^r)'(I - 2\Sigma^{r\prime} C_{h-1}\Sigma^r)^{-1}(\Sigma^{r\prime} B_{h-1} + 2\Sigma^{r\prime} C_{h-1}\mu^r) \\ -\frac{1}{2}\ln[\det(I - 2\Sigma^{r\prime} C_{h-1}\Sigma^r)] \end{array} \tag{A.15}$$

$$B_h' = -2\delta_0\delta' + (B_{h-1} + 2C_{h-1}\mu^r)'\rho^r + 2(\Sigma^{r\prime} B_{h-1} + 2\Sigma^{r\prime} C_{h-1}\mu^r)'(I - 2\Sigma^{r\prime} C_{h-1}\Sigma^r)^{-1}\Sigma^r C_{h-1}\rho^{r\prime} \tag{A.16}$$

$$C_h = -\delta\delta' + \rho^{r\prime} C_{h-1}\rho^r + 2\left(\Sigma^{r\prime} C_{h-1}\rho^r\right)'\left(I - 2\Sigma^{r\prime} C_{h-1}\Sigma^r\right)^{-1}\Sigma^{r\prime} C_{h-1}\rho^r \tag{A.17}$$

Appendix B: Credit default swap valuation

The derivation of equation (11)

The sum of default intensity and the short rate can be re-written as

$$\begin{aligned} r_t + \lambda_t &= (\delta_0 + \delta' X_t^r)^2 + (\alpha_0 + \alpha^{r\prime} X_t^r + \alpha^{c\prime} X_t^c)^2 \\ &= \gamma_0 + \gamma_1' X_t + X_t'\Omega X_t \end{aligned} \tag{B.1}$$

where $X_t = [X_t^r, X_t^c]'$, with $X_t^r = [X_{1,t}, X_{2,t}]'$ denoting the short-rate specific factors, and $X_t^c = [X_{3,t}, X_{4,t}, X_{5,t}]'$ denoting the credit-risk specific factors. The coefficients in (B.1) are given by

$$\gamma_0 = \delta_0^2 + \alpha_0^2 \tag{B.2}$$

$$\gamma_1 = \begin{bmatrix} 2(\delta_0\delta' + \alpha_0\alpha^{r\prime}) \\ 2\alpha_0\alpha^{c\prime} \end{bmatrix} \tag{B.3}$$

$$\Omega = \begin{bmatrix} \delta\delta' + \alpha^r\alpha^{r\prime} & \alpha^r\alpha^{c\prime} \\ \alpha^c\alpha^{r\prime} & \alpha^c\alpha^{c\prime} \end{bmatrix} \tag{B.4}$$

where $\delta = [\delta_1\ \delta_2]'$, $\alpha^r = [\alpha_1\ \alpha_2]'$, $\alpha^c = [\alpha_3\ \alpha_4\ 0]'$. Recall from equation (8), the dynamics of the state variables are as below

$$X_{t+1} = \mu + \rho X_t + \Sigma\varepsilon_{t+1}, \tag{B.5}$$

where ρ and Σ are 5×5 matrices while μ and X_t are 5×1 vectors. Notice that the expectation in equation (A.1) is same as the equation (11) and the dynamics of the state variables in equation (A.3) are also similar to the dynamics in equation (B.5). Therefore, the derivation of the expectation in equation (11) follows that in Appendix A. We provide the results below

$$E_t^Q\left[\exp(-\sum_{j=0}^{h-1} r_{t+j} + \lambda_{t+j})\right] = \exp(F_h + G_h' X_t + X_t' H_h X_t), \tag{B.6}$$

where F_h, G_h, and H_h are computed using the recursions defined as follows with the initial conditions $F_1 = -\gamma_0$, $G_1 = -\gamma_1$ and $H_1 = -\Omega$.

$$F_h = \begin{array}{c} -\gamma_0 + (F_{h-1} + G_{h-1}'\mu + \mu' H_{h-1}\mu) \\ +\frac{1}{2}(\Sigma' G_{h-1} + 2\Sigma' H_{h-1}\mu)'(I - 2\Sigma' H_{h-1}\Sigma)^{-1}(\Sigma' G_{h-1} + 2\Sigma' H_{h-1}\mu) \\ -\frac{1}{2}\ln[\det(I - 2\Sigma' H_{h-1}\Sigma)] \end{array} \tag{B.7}$$

$$G_h' = -\gamma_1' + (G_{h-1} + 2H_{h-1}\mu)'\rho + 2(\Sigma' G_{h-1} + 2\Sigma' H_{h-1}\mu)'(I - 2\Sigma' H_{h-1}\Sigma)^{-1}\Sigma H_{h-1}\rho' \tag{B.8}$$

$$H_h = -\Omega + \rho' H_{h-1}\rho + 2\left(\Sigma' H_{h-1}\rho\right)'\left(I - 2\Sigma' H_{h-1}\Sigma\right)^{-1}\Sigma' H_{h-1}\rho \tag{B.9}$$

The derivation of equation (12)

We rewrite equation (10), by splitting them into two components as follows

$$PS(t,t+h) = E_t^Q \begin{bmatrix} LGD_{t+h-1} \times \exp(-\sum_{j=0}^{h-1} r_{t+j}) \times \exp(-\sum_{j=0}^{h-2} \lambda_{t+j}) \\ -LGD_{t+h-1} \times \exp(-\sum_{j=0}^{h-1} r_{t+j}) \times \exp(-\sum_{j=0}^{h-1} \lambda_{t+j}) \end{bmatrix}. \tag{B.10}$$

We derive each of the two terms in the expectation (10) separately. We start with the first term in the expectation i.e.,

$$Y^1(t,t+h) = E_t^Q \left[LGD_{t+h-1} \times \exp(-\sum_{j=0}^{h-1} r_{t+j}) \times \exp(-\sum_{j=0}^{h-2} \lambda_{t+j}) \right] \tag{B.11}$$

where $Y^1(t,t+h)$ denotes the expectation of the first term in equation (B.10) at time t for horizon h, and LGD_{t+h-1} follows from equation (7) and re-written below.

$$\begin{aligned} LGD_{t+h-1} &= \exp\left(-\left(\beta_0 + \beta_1 X_{1,t+h-1} + \beta_2 X_{2,t+h-1} + \beta_5 X_{5,t+h-1}\right)^2\right) && \text{(B.12)} \\ &= \exp\left(-\left(\beta_0 + \beta^{r\prime} X^r_{t+h-1} + \beta^{c\prime} X^c_{t+h-1}\right)^2\right) && \text{(B.13)} \end{aligned}$$

$\beta^r = [\beta_1 \ \ \beta_2]'$ is the vector of coefficients associated with the term structure factors, and $\beta^c = [0 \ 0 \ \beta_5]'$ is the vector of coefficients associated with the credit-risk specific factors, i.e. $X^c_t = [X_{3,t}, X_{4,t}, X_{5,t}]'$.

The expectation in equation (B.11) can be re-written as

$$Y^1(t,t+h) = E_t^Q \left[LGD_{t+h-1} \times \exp(-r_{t+h-1}) \times \exp(-\sum_{j=0}^{h-2} r_{t+j} + \lambda_{t+j}) \right] \tag{B.14}$$

As before, we compute the expectation in the above expression using the law of iterative expectation. We start with the conditional expectation at time $t+h-1$ and work backwards,

$$\begin{aligned} Y^1(t+h-1,t+h) &= E^Q_{t+h-1}\left[LGD_{t+h-1} \times \exp(-r_{t+h-1})\right] && \text{(B.15)} \\ &= \exp(M_1 + N_1' X_{t+h-1} + X_{t+h-1}' O_1 X_{t+h-1}) \end{aligned}$$

where

$$M_1 = -\left(\delta_0^2 + \beta_0^2\right) \tag{B.16}$$

$$N_1 = -\begin{bmatrix} 2(\delta_0\delta' + \beta_0\beta^{r\prime}) \\ 2\beta_0\beta^{c\prime} \end{bmatrix} \tag{B.17}$$

$$O_1 = -\begin{bmatrix} \delta\delta' + \beta^r\beta^{r\prime} & \beta^r\beta^{c\prime} \\ \beta^c\beta^{r\prime} & \beta^c\beta^{c\prime} \end{bmatrix} \tag{B.18}$$

From here, the derivation of the expectation in equation (B.14) is similar to the derivation of

the equation (B.6) and (A.1). More specifically, the conditional expectation at time t is

$$Y^1(t, t+h) = \exp(M_h + N_h' X_t + X_t' O_h X_t) \tag{B.19}$$

where

$$\begin{aligned} M_h = \quad & \begin{array}{c} -\gamma_0 + (M_{h-1} + N_{h-1}'\mu + \mu' O_{h-1}\mu) \\ +\frac{1}{2}(\Sigma' N_{h-1} + 2\Sigma' O_{h-1}\mu)'(I - 2\Sigma' O_{h-1}\Sigma)^{-1}(\Sigma' N_{h-1} + 2\Sigma' O_{h-1}\mu) \\ -\frac{1}{2}\ln[\det(I - 2\Sigma' O_{h-1}\Sigma)] \end{array} \end{aligned} \tag{B.20}$$

$$N_h' = \quad -\gamma_1' + (N_{h-1} + 2O_{h-1}\mu)'\rho + 2(\Sigma' N_{h-1} + 2\Sigma' O_{h-1}\mu)'(I - 2\Sigma' O_{h-1}\Sigma)^{-1}\Sigma O_{h-1}\rho' \tag{B.21}$$

$$O_h = \quad -\Omega + \rho' O_{h-1}\rho + 2\left(\Sigma' O_{h-1}\rho\right)'\left(I - 2\Sigma' O_{h-1}\Sigma\right)^{-1}\Sigma' O_{h-1}\rho \tag{B.22}$$

with the initial conditions M_1, N_1 and O_1 from equations (B.16), (B.17) and (B.18) respectively.

In what follows, we show the derivation of the second term in the expectation of equation (B.10).

$$Y^2(t, t+h) = E_t^Q\left[LGD_{t+h-1} \cdot \exp\left(-\sum_{j=0}^{h-1} r_{t+j} + \lambda_{t+j}\right)\right] \tag{B.23}$$

The superscript 2 in $Y^2(t, t+h)$ indicates the expectation of the second term. Again, we derive the expectation by working backwards.

$$Y^2(t+h-1, t+h) = E_{t+h-1}^Q\left[LGD_{t+h-1} \times \exp\left(-r_{t+h-1} - \lambda_{t+h-1}\right)\right] \tag{B.24}$$

Using equation (B.1) and equation (B.13),

$$\begin{aligned} Y^2(t+h-1, t+h) &= \exp\left(-\left(\beta_0 + \beta^{r\prime} X_{t+h-1}^r + \beta^{c\prime} X_{t+h-1}^c\right)^2\right) \times \\ &\quad \exp\left(-\left(\gamma_0 + \gamma_1' X_{t+h-1} + X_{t+h-1}'\Omega X_{t+h-1}\right)\right) \\ &= \exp(J_1 + K_1' X_{t+h-1} + X_{t+h-1}' L_1 X_{t+h-1}) \end{aligned} \tag{B.25}$$

where

$$J_1 = -(\delta_0^2 + \beta_0^2 + \alpha_0^2) \tag{B.26}$$

$$K_1 = -\begin{bmatrix} 2(\delta_0\delta' + \beta_0\beta^{r\prime} + \alpha_0\alpha^{r\prime}) \\ 2\left(\beta_0\beta^{c\prime} + \alpha_0\alpha^{c\prime}\right) \end{bmatrix} \tag{B.27}$$

$$L_1 = -\begin{bmatrix} \delta\delta' + \beta^r\beta^{r\prime} + \alpha^r\alpha^{r\prime} & \beta^r\beta^{c\prime} + \alpha^r\alpha^{c\prime} \\ \beta^c\beta^{r\prime} + \alpha^c\alpha^{r\prime} & \beta^c\beta^{c\prime} + \alpha^c\alpha^{c\prime} \end{bmatrix} \tag{B.28}$$

As before, from this point, the derivation of the expectation in equation (B.23) is similar to the derivation of the equation (B.6) and (A.1). More specifically, the conditional expectation at time t is

$$Y^2(t, t+h) = \exp(J_h + K_h' X_t + X_t' L_h X_t) \tag{B.29}$$

where

$$\begin{aligned} J_h = \quad & \begin{array}{c} -\gamma_0 + (J_{h-1} + K_{h-1}'\mu + \mu' L_{h-1}\mu) \\ +\frac{1}{2}(\Sigma' K_{h-1} + 2\Sigma' L_{h-1}\mu)'(I - 2\Sigma' L_{h-1}\Sigma)^{-1}(\Sigma' K_{h-1} + 2\Sigma' L_{h-1}\mu) \\ -\frac{1}{2}\ln[\det(I - 2\Sigma' L_{h-1}\Sigma)] \end{array} \end{aligned} \tag{B.30}$$

$$K_h' = -\gamma_1' + (K_{h-1} + 2L_{h-1}\mu)'\rho + 2(\Sigma' K_{h-1} + 2\Sigma_{h-1}' L_{h-1}\mu)'(I - 2\Sigma' L_{h-1}\Sigma)^{-1}\Sigma L_{h-1}\rho' \tag{B.31}$$

$$L_h = -\Omega + \rho' L_{h-1}\rho + 2\left(\Sigma' L_{h-1}\rho\right)'\left(I - 2\Sigma' L_{h-1}\Sigma\right)^{-1}\Sigma' L_{h-1}\rho \tag{B.32}$$

with the initial conditions J_1, K_1, and L_1 from equation (B.26), equation (B.27) and equation (B.28) respectively.

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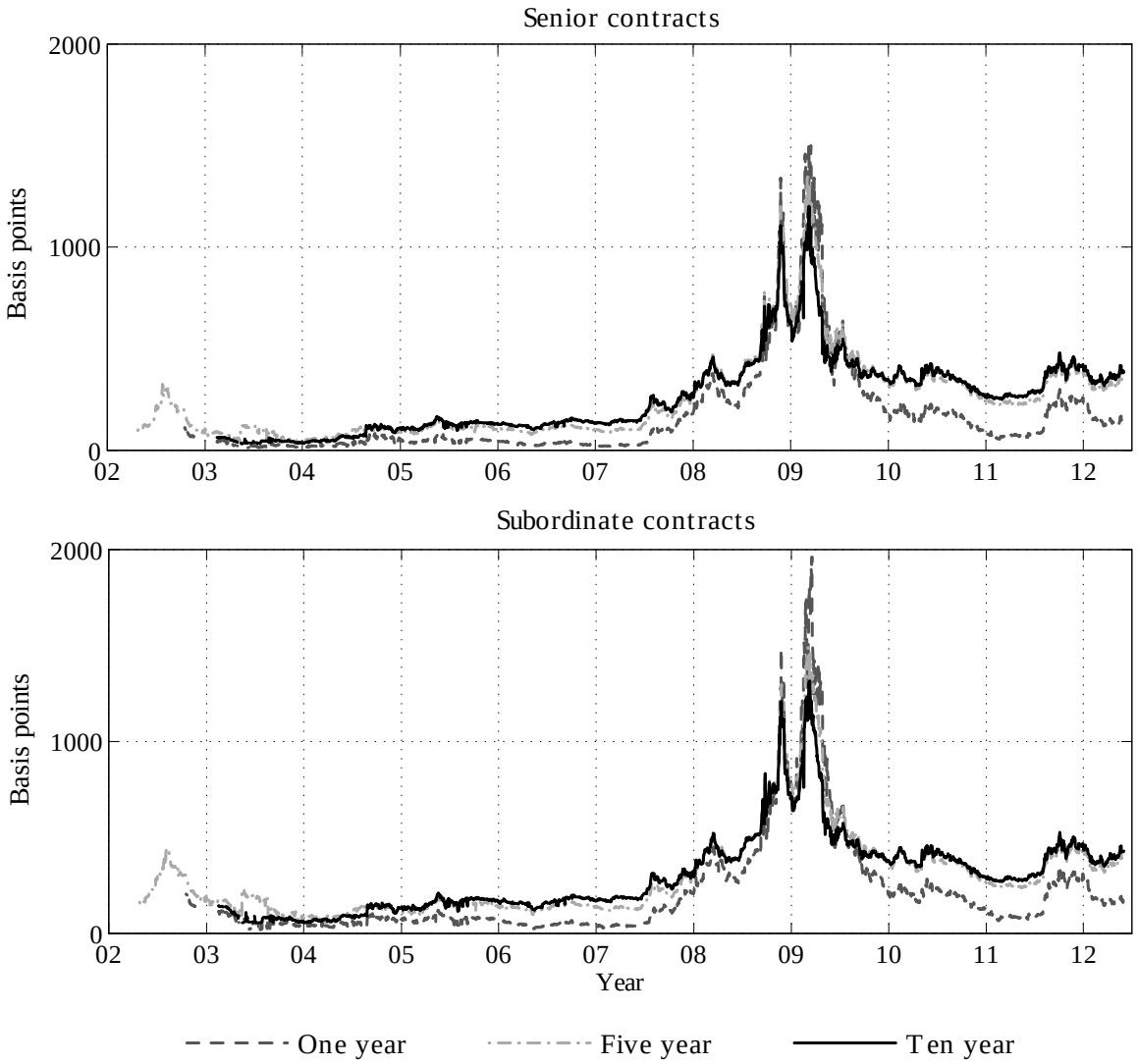

Figure 1. *Average market-observed spreads.* We plot the daily average CDS spreads (in basis points) across firms in our sample (see Table 1). The top (bottom) panel plots daily time-series averages of the market spread for senior (subordinate) CDS contracts with maturities of 1, 5, and 10 years. We obtain daily CDS spreads data from MARKIT. The data availability of CDS spreads differs across firms. Thus, the number of firms used to calculate the time-series averages varies through time. It increases over time as more firms have CDS trading on their debts in the later part of the sample period. The start of the time-series averages differ between different maturities and seniorities. We require that CDS data is available on at least five firms to calculate the daily average.

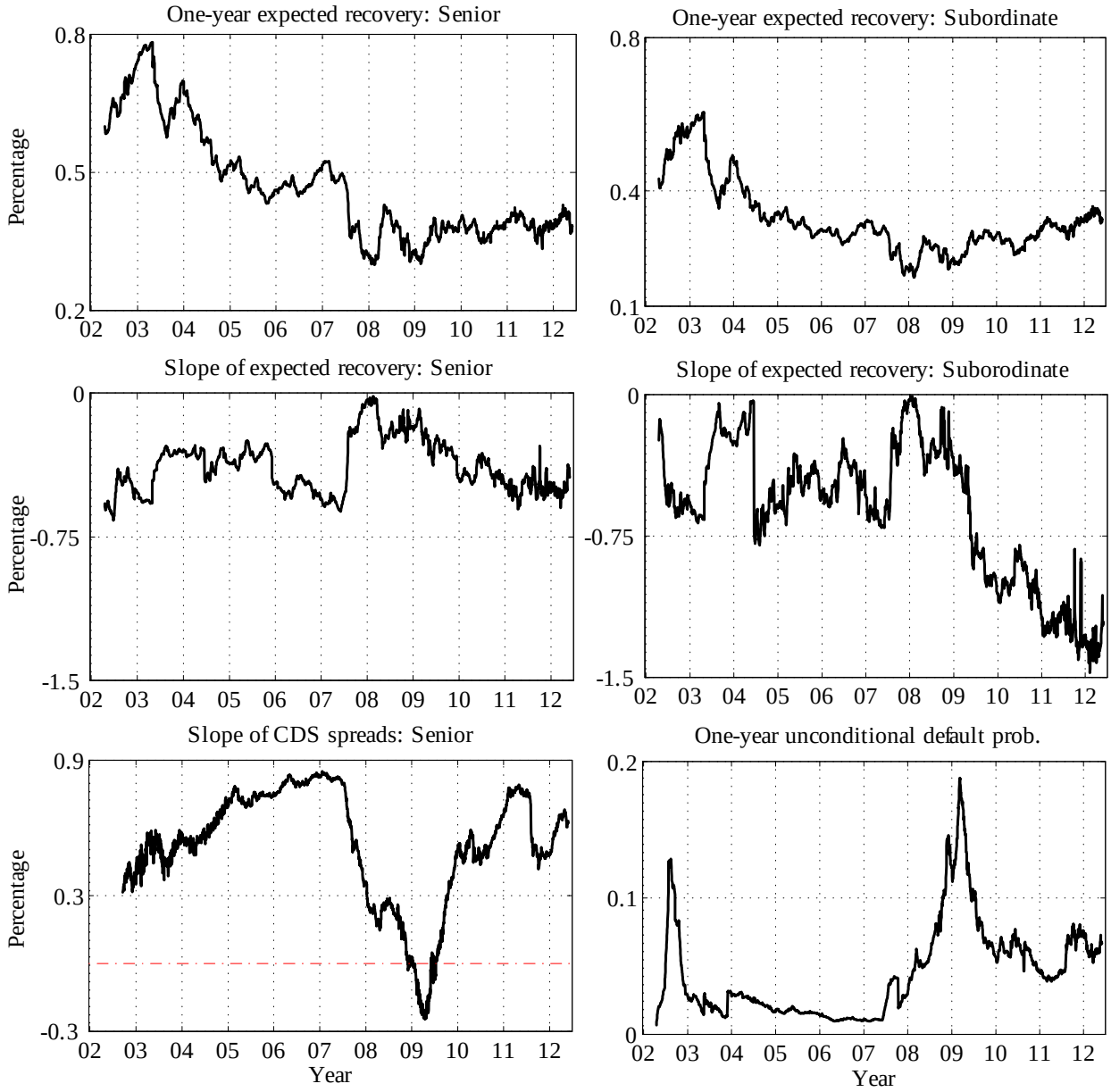

Figure 2. *Recovery dynamics: cross-firm averages.* The sample consists of 46 single name securities described in Table 1. The top two panels plot the time series of one-year expected recovery for senior (top-left) and subordinate (top-right) CDS contracts. The middle-left and middle-right panels plot the time series of relative slope of expected recovery for the senior and subordinate CDS contracts, respectively. The plotted results are average values across firms, and we require that data on at least five firms are available to plot the daily averages. The relative slope of expected recovery is defined in equation (20). The bottom-left panel plots the time series of relative slope of market observed CDS spreads. The relative slope of market observed CDS spreads is defined in equations (21). Finally, the bottom-right panel plots daily averages of one-year unconditional default probabilities implied by the model. The number of firms used to calculate the time-series averages varies through time.

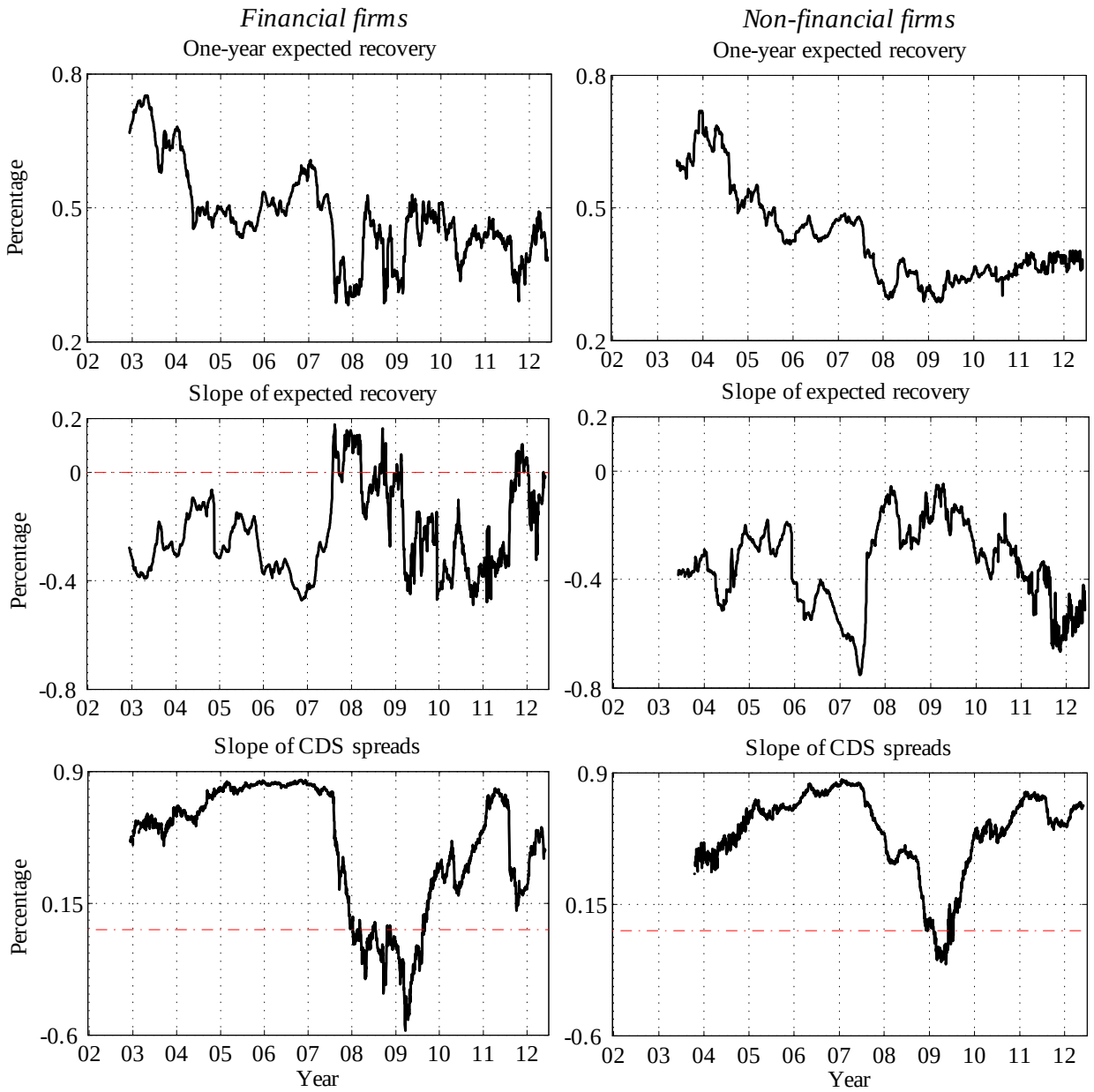

Figure 3. *Recovery dynamics of senior contracts: financial versus non-financial firms.* We plot the time series of average recovery dynamics of senior CDS contracts for financial and non-financial firms (see Table 1). Panels in the left column plot the results averaged across 17 financial firms, while panels in the right column plot the results averaged across the other 29 non-financial firms. Financial firms are those categorized under the finance, insurance, and real estate sectors following the classification on Kenneth French's website. The top two panels plot the average one-year expected recovery for senior contracts. The middle two panels plot the average relative slope of expected recovery for senior contracts. The bottom panels plot the time-series average of the relative slope of market-observed senior CDS spreads. The relative slope of expected recovery and the relative slope of market observed CDS spreads are calculated according to equations (20) and (21), respectively. We require that data on at least five firms are available to plot the daily averages.

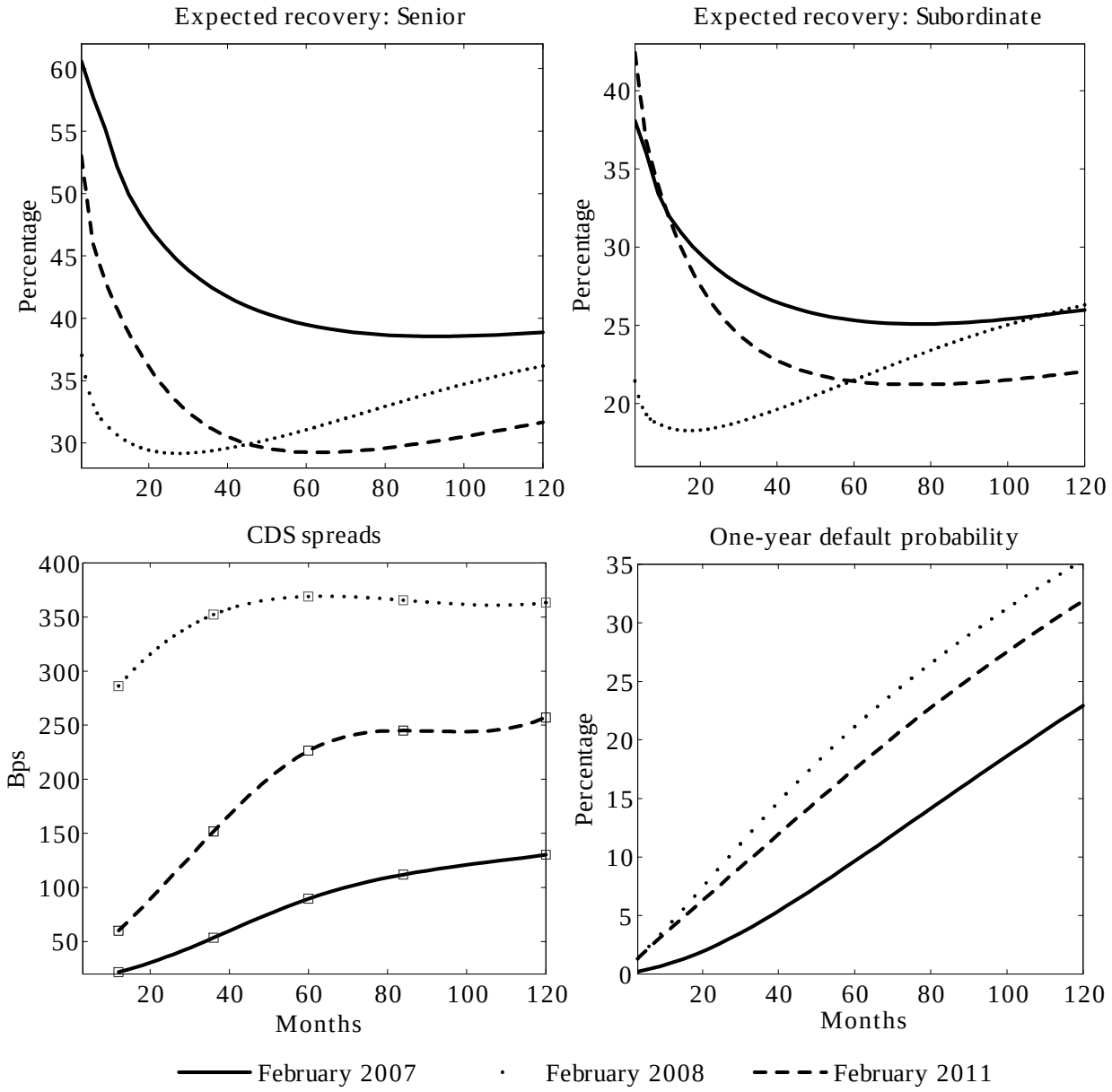

Figure 4. *Term structure of expected recovery: before, during, and after the 2008 crisis.* In each panel, we plot cross-sectional average results for three different periods: February 2007 (pre-crisis), February 2008 (during crisis), and February 2011 (post-crisis). The top two panels plot the average term structure of expected recovery across firms for the senior and subordinate CDS contracts. The bottom-left and bottom-right panels plot the average senior CDS spreads observed in the market (in basis points) and their average unconditional default probability implied by our model at various horizons. The x-axis in each panel indicates the number of months. We plot the results for the pre-crisis period using solid line, and the post-crisis period using a dashed line. Results for the during-crisis period are plotted using a dotted line. The sample of firms for each period differs due to CDS data availability as summarized in Table 1.

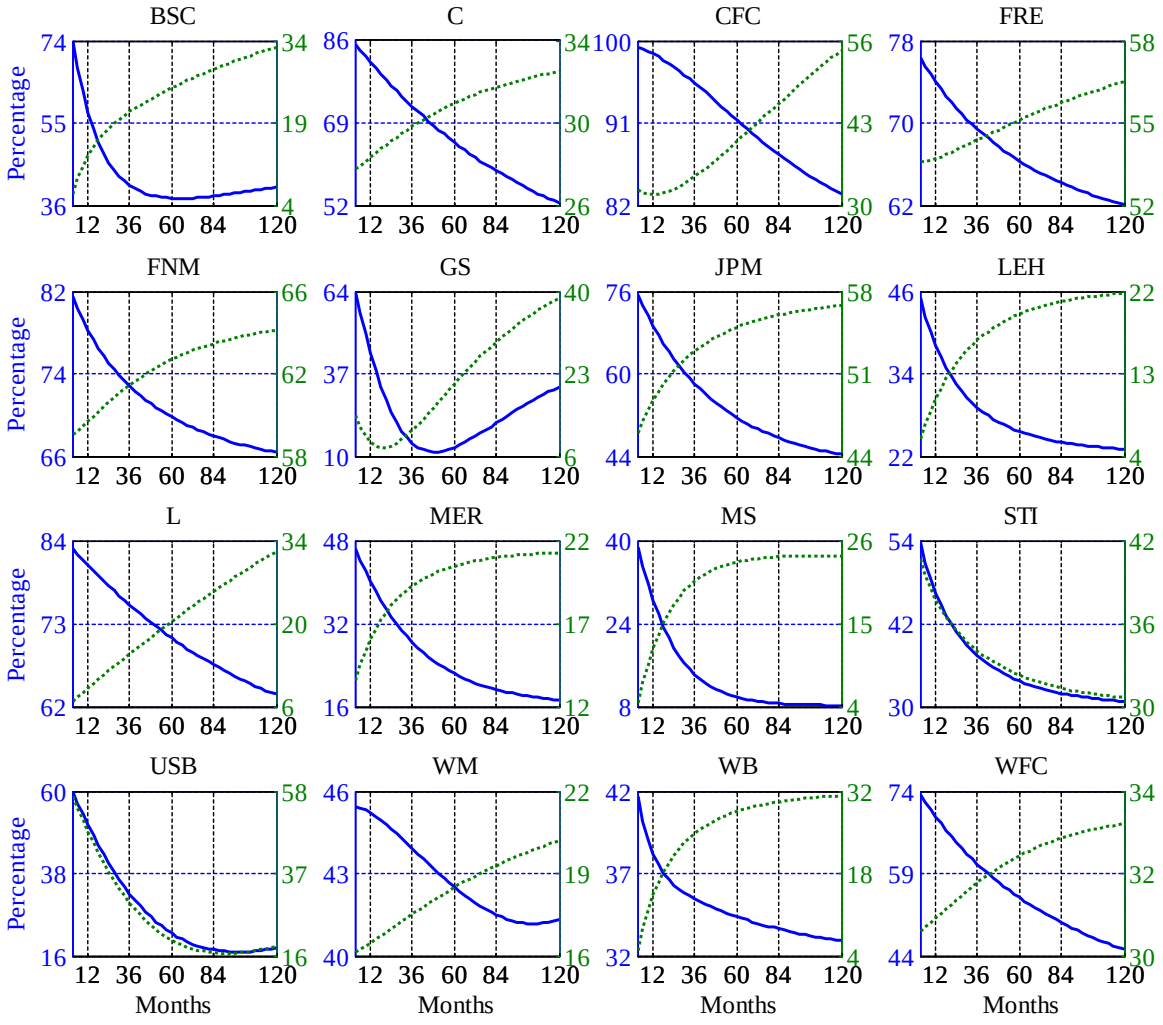

Figure 5. *Term structure of expected recovery for individual financial firms: before and during the 2008 crisis.* The sample consists of 16 financial firms described in Table 1. We plot the average term structure of expected recovery for senior contracts of each financial firms in February 2007 (pre-crisis), and in February 2008 (during crisis). We plot the results for February 2007 using a solid line against the left y-axis, and plot the results for February 2008 using a dotted line against the right y-axis. The expected recovery levels are in percentage terms and the x-axis in each panel indicates the number of months. The title of each panel represents the ticker of the corresponding financial firm.

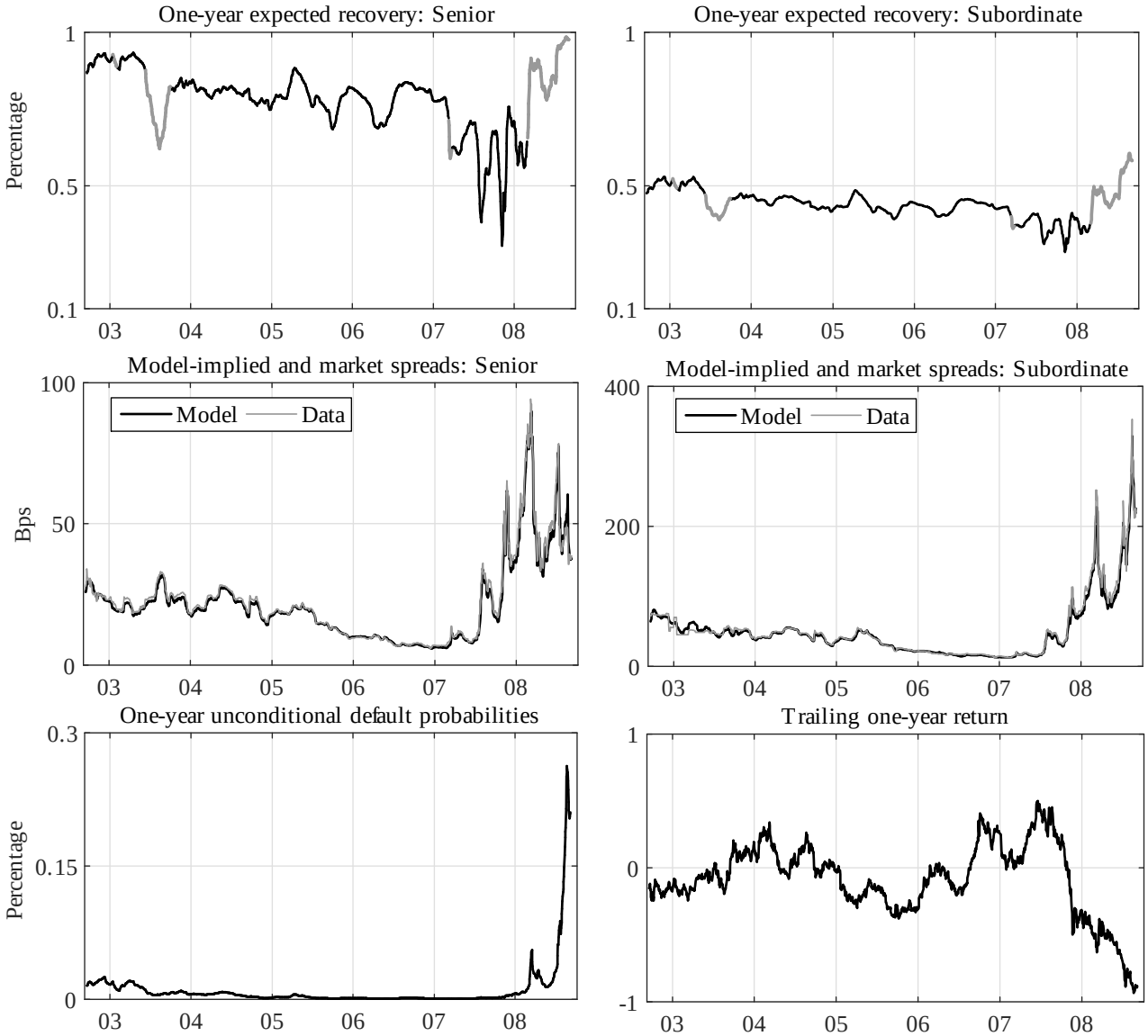

Figure 6. *Fannie Mae: A case study.* This figure plots various time-series properties and estimates for Fannie Mae. The top two panels plot time series of expected recovery at the one-year horizon for senior and subordinate contracts. Panels in the second row plot the market observed (in gray) and model-implied CDS spreads (in black) for senior and subordinate contracts with five-year maturity. Panels in the third row plot the model-implied one-year unconditional default probability and the trailing one-year cumulative return of the firm. The bottom-left panel plots the time series of option-implied volatilities calculated using at-the-money put options with 30 days to maturity. The bottom-right panel plots the trailing one-year cumulative return of the firm. We highlight certain portions of the plot in the top two panels using a grey-colored line to indicate the events discussed in the text.

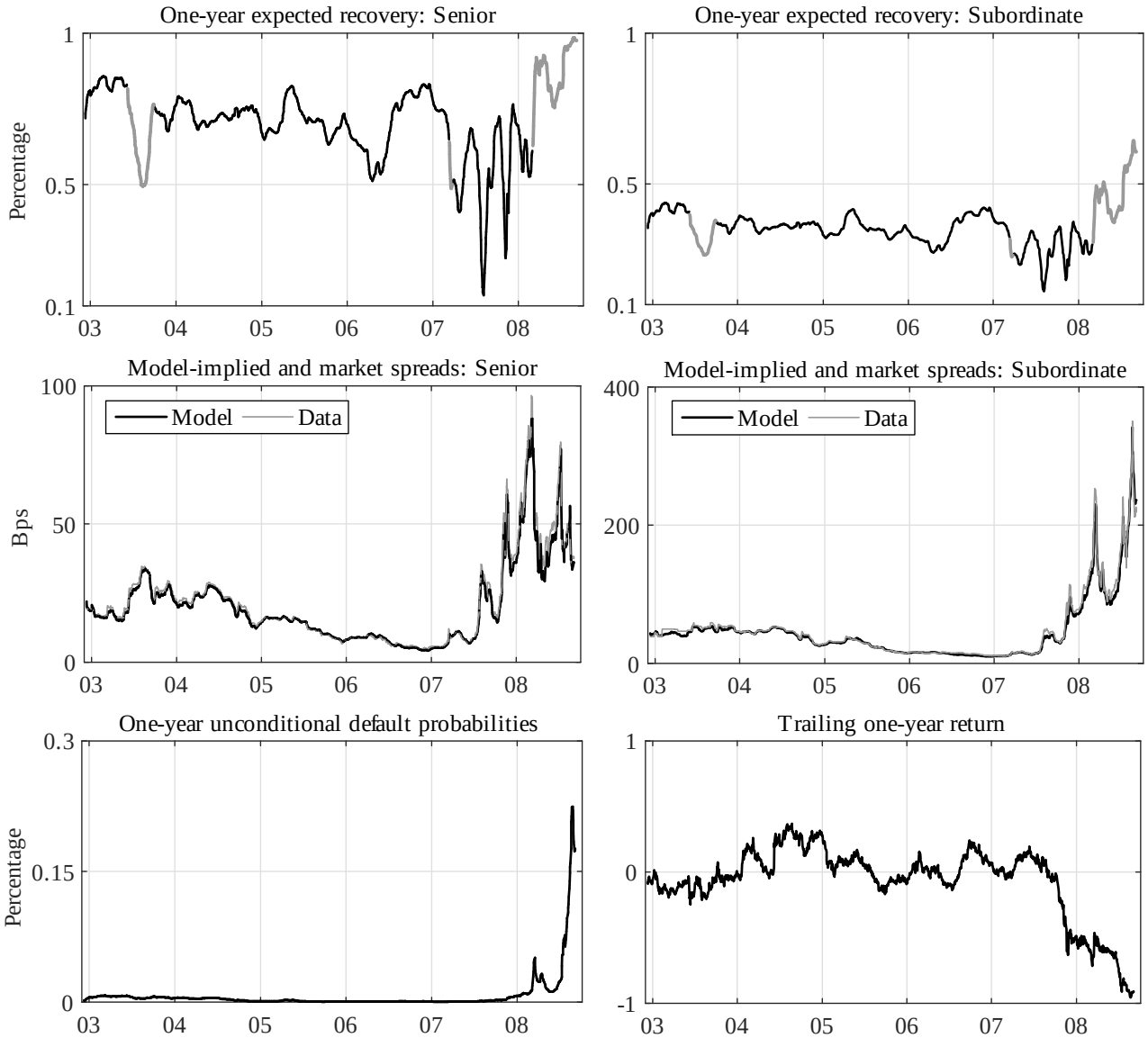

Figure 7. *Freddie Mac: A case study.* This figure plots various time-series properties and estimates for Freddie Mac. The top two panels plot time series of expected recovery at the one-year horizon for senior and subordinate contracts. Panels in the second row plot the market observed (in gray) and model-implied CDS spreads (in black) for senior and subordinate contracts with five-year maturity. Panels in the third row plot the model-implied one-year unconditional default probability and the trailing one-year cumulative return of the firm. The bottom-left panel plots the time series of option-implied volatilities calculated using at-the-money put options with 30 days to maturity. The bottom-right panel plots the trailing one-year cumulative return of the firm. We highlight certain portions of the plot in the top two panels using a grey-colored line to indicate the events discussed in the text.

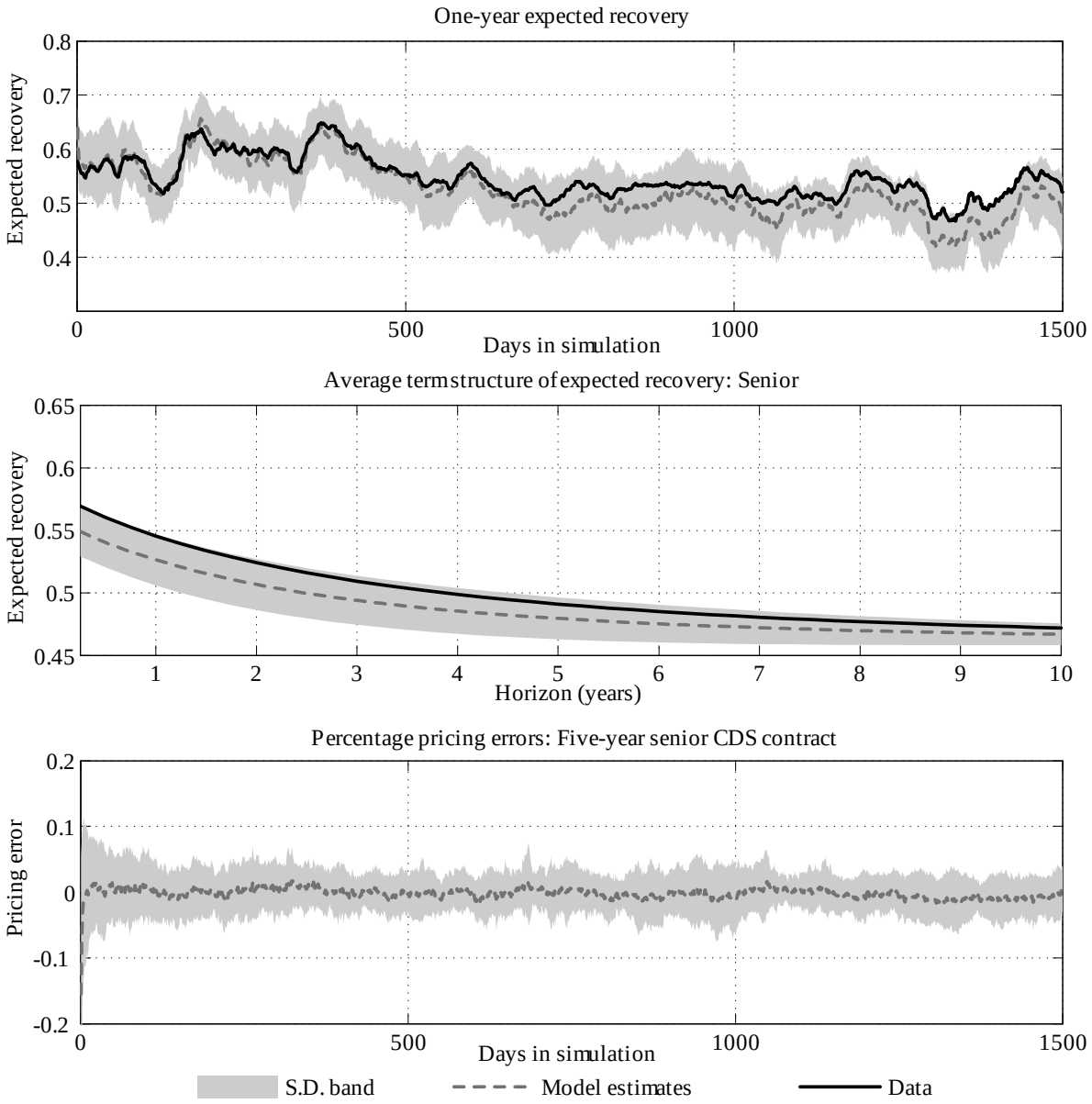

Figure 8. *Simulation results.* We plot results from estimating the term structure of CDS spreads simulated using parameters reported in Table 7. We simulate 100 sample paths of daily CDS spreads with 1, 3, 5, 7, and 10 years to maturity. CDS data are simulated with normally distributed noise. We set the noise term's standard deviation equal to five percent of the model-implied spread level. We estimate the model on each simulation path using the method described in Section **??**. The top panel reports results for the expected one-year recovery, and the middle panel reports results for the average term structure of recovery. We plot the mean of the model estimates using a grey-dashed line. One standard deviation band (S.D band) is plotted on top of the mean of model estimates. Solid lines in the top two panels plot the mean values of one-year expected recovery and term structure of expected recovery, calculated using the "true" simulated data. We do not plot the standard deviation band around the results calculated using the simulated data to avoid visual clustering. The bottom panel plots the sample mean and standard deviation of daily percentage pricing errors for five-year senior contract.

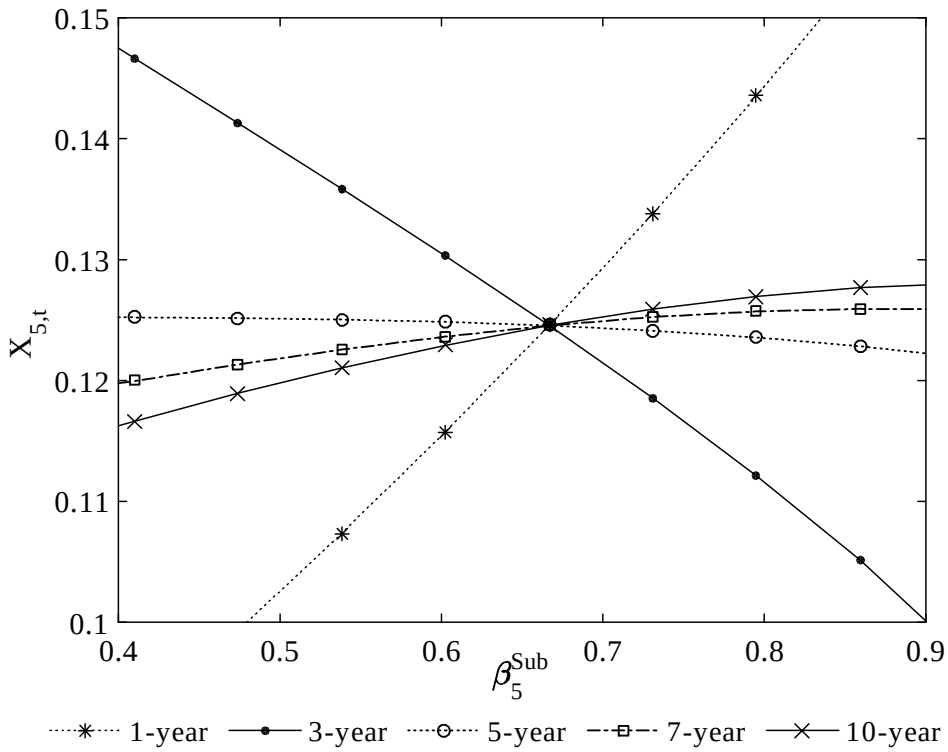

Figure 9. *Sensitivity of CDS term structures to loss-given-default loadings.* This figure shows the combinations of factor loading β_5^{sub} and the level of recovery-specific factor $X_{5,t}$ in equation (7) that result in a constant CDS spread ratio between senior and subordinate contracts. We plot the combinations of $\left(\beta_5^{sub}, X_{5,t}\right)$ that keep senior to subordinate CDS ratios constant for 1-,3-,5-,7-, and 10-year maturity contracts. The x-axis corresponds to the value of β_5^{sub}, while the y-axis corresponds to the value of $X_{5,t}$. The coefficient β_5^{sen} is normalized to one in our model. For simplicity, we remove the effect of risk-free term structure on the LGD by setting the factor loadings on the state variables $X_{1,t}$ and $X_{2,t}$ to zero. All other parameters except β_5^{sub} and $X_{5,t}$ are held fixed equal to the base case parameters reported in Table 7. Each $\left(\beta_5^{sub}, X_{5,t}\right)$ combination determines a unique term structure of recovery rates. The model used for generating senior and subordinate CDS spreads is described in Section 2.2. The true data is generate using the initial state for the state variables are as follows: $X_{3,t} = 0.00093$, $X_{4,t} = -0.00077$ and $X_{5,t} = 0.1246$.

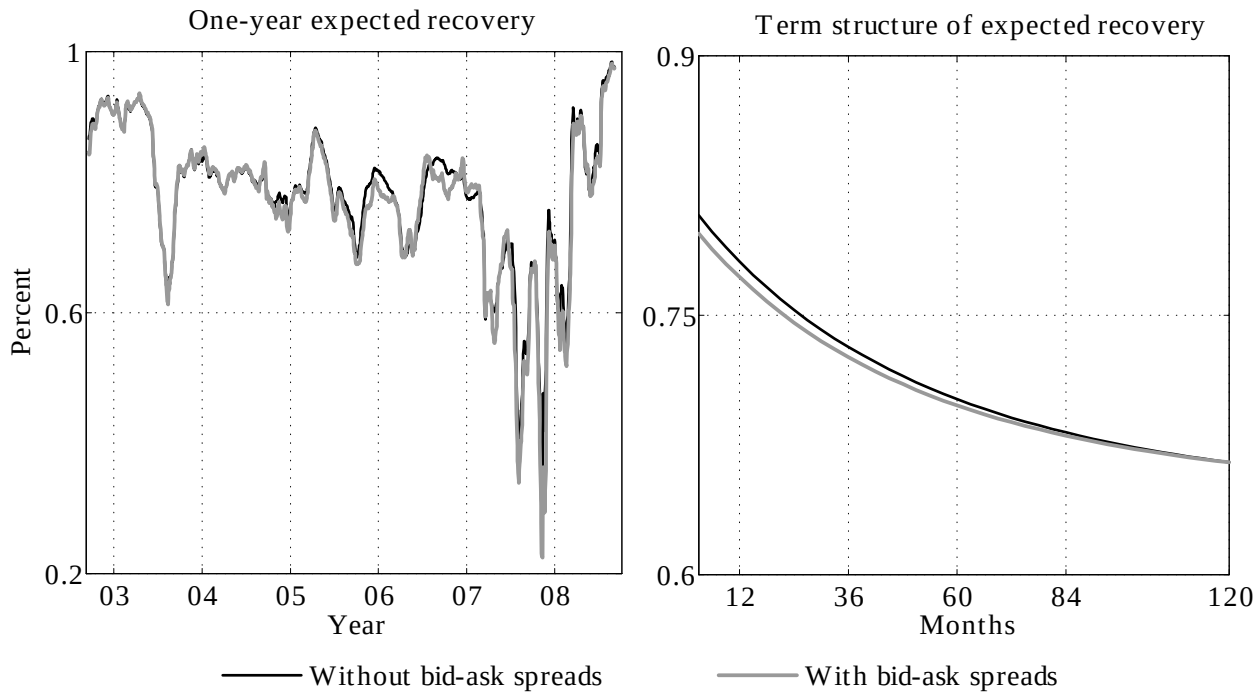

Figure 10. *Impact of daily bid-ask spreads: Fannie Mae.* We estimate the CDS valuation model using time-series data of senior and subordinate CDS contracts written on Fannie Mae with maturities of 1, 3, 5, 7, and 10 years. We plot results from two estimation methods. The first estimation method, labeled *Without bid-ask spread,* does not use information from daily CDS bid and ask quotes. In the second estimation method, labeled *With bid-ask spreads*, we include daily bid and ask information of Fannie Mae in the unscented Kalman filter and in the model's optimization function. Daily bid and ask data of Fannie Mae are collected from Bloomberg. We account for CDS bid and ask spreads by assuming that the τ-maturity senior contracts on day t are priced with normally distributed errors with conditional standard deviations of $\sigma_u^{SEN}(\tau)\left|Bid_t^{SEN}(\tau) - Ask_t^{SEN}(t,\tau)\right|$, where $\sigma_u^{SEN}(\tau)$ is a constant, and $Bid_t^{SEN}(\tau)$ and $Ask_t^{SEN}(\tau)$ refer to day t's bid and ask spreads of τ-maturity senior contracts. We also assume τ-maturity subordinate contracts are priced with normally distributed errors with conditional standard deviations $\sigma_u^{SUB}(\tau)\left|Bid_t^{SUB}(\tau) - Ask_t^{SUB}(\tau)\right|$; the variables are defined analogously. The left panel plots the time series of one-year expected recovery rate. The right panel plots the average term structure of expected recovery. Parameters estimates for Fannie Mae used to generate the plots are reported in Table 10.

Table 1. *Summary statistics: individual firms*

This table describes the 46 firms in our sample. We obtain daily CDS spreads data from MARKIT. The column labeled *Data availability* indicates the start and end dates of the CDS spreads data in our sample in mm/yy format. For each firm, we report the sample mean and standard deviation (S.D.) of daily CDS spreads (in basis points) on senior and subordinate debts for three maturities: 1-year, 5-year, and 10-year. An asterisk next to the firm's name indicates that it is a financial firm. We define financial firms as those categorized under the finance, insurance, and real estate sectors following the classification on Kenneth French's website. We report, next to each firm, its official ticker and its corresponding overall credit rating level. We obtain monthly S&P long-term entity credit ratings from COMPUSTAT.

Firm Name	Ticker	Entity Ratings	Data Availability	Senior - Market Spreads (bps)						Subordinate - Market Spreads (bps)					
				1 Yr		5 Yr		10 Yr		1 Yr		5 Yr		10 Yr	
				Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.
Amkor Tech Inc	AMKR	B	05/03-08/10	408	277	641	250	689	193	548	325	808	302	866	278
Bear Stearns Cos Inc*	BSC	A	12/06-07/08	135	175	113	101	109	84	152	187	133	112	130	94
Best Buy Co Inc	BBY	BBB	04/03-05/12	66	79	144	137	168	141	79	83	160	137	184	146
Boyd Gaming Corp	BYD	BB	12/05-05/12	567	666	863	617	813	485	590	725	874	633	834	521
Cap One Bk*	COF	BBB	04/04-05/12	88	123	101	91	101	72	110	154	121	108	121	83
Citigroup Inc*	C	AA	02/01-05/12	83	141	102	116	111	104	121	212	141	172	151	153
Cox Comms Inc	COX	BBB	03/01-05/12	62	93	96	76	115	68	100	165	136	147	151	156
Ctrywde Finl Corp*	CFC	BBB	12/06-11/08	523	590	299	266	252	206	576	636	322	275	280	208
D R Horton Inc	DHI	BB	03/06-01/08	157	180	190	134	232	102	194	200	229	145	273	111
Fed Home Ln Mtg Corp*	FRE	AAA	12/02-09/08	10	9	21	15	28	14	29	57	48	46	55	40
Fed Natl Mtg Assn*	FNM	AAA	09/02-09/08	10	9	22	14	29	13	30	57	51	44	60	39
Goldman Sachs Gp Inc*	GS	A	12/06-05/12	150	130	158	97	158	88	189	147	203	123	200	114
Harrahs Entmt Inc	HET	BB	07/04-11/10	672	1515	1051	1560	965	1298	705	1541	1061	1494	1002	1308
Health Mgmt Assoc Inc	HMA	BB	01/05-05/12	177	204	331	250	353	229	210	236	371	292	388	258
Iron Mtn Inc	IRM	BB	05/05-05/12	131	70	334	94	374	90	133	73	344	98	387	95
JPMorgan Chase & Co*	JPM	A	01/01-05/12	38	39	64	42	74	39	58	58	85	60	97	55
KB Home	KBH	BB	06/03-05/12	224	181	356	212	371	203	265	196	399	212	415	202
Kerr Mcgee Corp	KMG	BB	03/06-05/12	23	20	51	49	67	44	31	25	63	58	81	51
Kohls Corp	KSS	BBB	01/04-05/12	41	52	86	65	104	59	47	58	87	64	105	60
L 3 Comms Corp	LLL	BBB	08/05-05/12	61	33	146	44	176	43	63	32	152	44	179	43
Lehman Bros Hldgs Inc*	LEH	A	01/07-09/08	198	206	143	104	129	81	216	217	164	122	147	96
Loews Corp*	L	A	08/03-05/12	29	22	57	32	72	31	37	35	69	41	86	40

Table 1. *Summary statistics: individual firms (continued...)*

Firm Name	Ticker	Entity Ratings	Data Availability	Senior - Market Spreads (bps)						Subordinate - Market Spreads (bps)					
				1 Yr		5 Yr		10 Yr		1 Yr		5 Yr		10 Yr	
				Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.	Mean	S.D.
(continued....)															
Merrill Lynch & Co Inc*	MER	A	03/06-05/12	174	166	180	125	175	112	233	230	239	182	232	165
MGM Mirage	MGM	BB	06/04-06/10	662	1586	705	1048	688	898	807	1896	799	1080	763	855
Morgan Stanley*	MS	A	06/04-05/12	162	246	156	155	154	131	194	260	193	188	190	163
Navistar Intl Corp	NAV	BB	03/05-08/10	305	284	465	258	502	215	310	290	483	248	519	210
Neiman Marcus Gp Inc	NMG	B	01/06-05/12	363	532	606	468	603	369	421	589	659	448	657	343
Office Depot Inc	ODP	BB	12/03-05/12	351	671	388	469	402	375	358	666	391	444	404	378
Omnicare Inc	OCR	BB	08/04-05/12	117	82	269	115	305	107	120	78	282	116	315	105
Sanmina SCI Corp	SANM	B	01/05-05/12	217	284	472	357	490	309	486	912	730	616	728	485
Sinclair Broadcast Gp Inc	SBGI	BB	10/04-05/12	479	850	554	501	547	362	483	845	564	465	550	321
Solectron Corp	SLR	BB	04/06-10/07	39	18	157	56	211	56	69	25	278	81	349	84
Sungard Systems	SNDT/SDS	B	08/06-08/10	229	187	462	201	508	158	334	255	571	219	636	187
SunTrust Bks Inc*	STI	A	09/04-03/11	81	92	101	97	101	92	151	169	157	157	150	149
Tesoro Corp	TSO	BB	11/04-05/12	127	121	278	175	306	163	131	117	295	163	319	149
Time Warner Inc	TWX	A	04/02-05/12	24	22	103	107	91	29	37	32	149	151	121	49
TJX Cos Inc	TJX	A	08/03-07/11	21	24	48	30	63	30	31	34	59	32	79	33
Toll Bros Inc	TOL	BBB	04/04-05/12	105	100	158	73	179	63	127	113	183	77	204	62
Triad Hosps Inc	TRI	BB	09/04-03/08	71	37	161	61	220	62	119	62	228	81	296	68
Tribune Co	TRB	BBB	10/03-12/08	827	1787	908	1670	895	1540	832	1723	917	1667	899	1554
TRW Automotive Inc	TRW	BBB	10/05-08/10	452	873	537	570	529	439	459	801	552	512	556	403
U S Bancorp*	USB	AA	11/04-03/12	36	39	65	56	78	66	66	64	92	76	99	80
Utd Rents Inc	URI	BB	07/04-04/11	393	306	567	278	587	232	422	246	627	243	662	224
WA Mut Inc*	WM	A	05/04-09/08	284	935	188	540	169	409	295	908	223	575	214	522
Wachovia Corp*	WB	A	03/02-12/08	39	94	52	86	61	83	53	125	67	107	79	107
Wells Fargo & Co*	WFC	AA	10/03-05/12	45	59	68	57	75	51	69	89	94	86	102	78

Table 2. *Risk-free term structure's estimates and properties*
This table reports the estimation results of the risk-free term structure. We apply the unscented Kalman filter to estimate a two-factor latent model on the term structure of risk-free rate. The factor dynamics and short-rate loadings are estimated using the 6-month LIBOR rate, together with swap rates with maturities of 1, 2, 3, 4, 5, 7, and 10 years. Panel A reports parameter estimates of the risk-free term structure factors. The standard error is reported (in bracket) beneath each estimate. Panel B reports the root-mean-squared errors (RMSEs) and the standard deviation of the measurement error equation (ME SD) for the risk-free term structure.

Panel A: Risk-free term structure factor loadings and dynamics

	Factor 1	Factor 2
δ_0	0.043	
	(9.94E-04)	
δ_1	0.345	
	(1.89E-03)	
δ_2		1.005
		(3.71E-03)
ρ	0.9994	0.9979
	(2.25E-04)	(5.57E-04)
ρ^P	0.9971	0.9970
	(3.22E-03)	(8.33E-03)
μ x 100	-0.002	-0.004
	(1.81E-04)	(1.43E-05)
μ^Px 100	-0.012	-0.006
	(2.44E-03)	(2.06E-03)
σ x 100	0.035	0.009
	(3.17E-04)	(9.85E-05)

Panel B: RMSE and measurement error standard deviation (bps)

	6 months	1 year	2 year	3 year	4 year	5 year	7 year	10 year
RMSE	21.03	7.51	7.43	6.17	4.68	4.67	7.56	12.52
ME SD	22.28	7.77	9.12	8.88	7.94	8.78	10.12	13.93

Table 3. *Parameter estimates from the CDS valuation model*

This table reports the estimation results of the CDS valuation model on 46 individual firms. Table 1 describes the firms and the sample period used in the estimation. For each firm, we estimate the CDS valuation model using time-series data of senior and subordinate CDS contracts with maturities of 1, 3, 5, 7, and 10 years. Panels A-B report the distribution of parameter estimates across firms. In Panel A, we report the factor loadings of the default intensity dynamic, the loss given default (LGD) of senior CDS contract, and the LGD of junior CDS contract on the five latent factors. Factors X_1 and X_2 describe the dynamic of the risk-free term structure, which are estimated using the 6-month LIBOR and swap rates; their structural parameter estimates are reported in Table 2. The default intensity dynamic and the LGD dynamic both depend on X_1 and X_2. Factors X_3 and X_4 are specific to the dynamic of default intensity. We normalize the factor loadings of the default intensity dynamic on X_3 and X_4 to one (i.e., $\alpha_3 = \alpha_4 = 1$ in equation (6)). Factor X_5 loads only on the dynamic of LGDs, and is referred to as the recovery factor. The factor loading of the senior LGD is normalized to one (i.e., $\beta_5^{sen} = 1$ in equation (7)). For each estimate, we report the cross-firm mean, standard deviations, and distributional percentiles at 25, 50, and 75 % levels. The last row in Panel A reports the percentage of cross-firm estimates that are statistically significant at the five percent level. In Panel B, we report the distribution of parameter estimates that drive the factor dynamic of X_3, X_4, and X_5. We report the distribution of the drift, μ, and the autoregressive parameter, ρ, under the risk-neutral measure. We also report the distribution of the autoregressive parameter, ρ^{P}, under the physical measure. We recall that the off-diagonal elements in ρ and σ are zero, and hence, the reported values are their diagonal estimates. Finally, in Panel C, we report the distribution of the measurement errors from fitting the senior and subordinate CDS contracts across different maturities.

Panel A: Estimates of the factor loadings

Statistics	Intensity loadings × 100					Senior LGD loadings				Subordinate LGD loadings			
	Constant × 100	Risk-free factors × 100		Intensity factors		Constant	Risk-free factors		Recovery factor	Constant	Risk-free factors		Recovery factor
		X_1	X_2	X_3	X_4		X_1	X_2	X_5		X_1	X_2	X_5
Mean	0.590	-1.656	-6.105	1	1	0.979	-0.429	4.991	1	0.549	-0.437	1.156	0.648
25%	0.076	-5.128	-13.179			0.496	-0.997	-5.602		0.072	-2.558	-6.181	0.235
50%	0.436	-1.048	-1.639			0.879	-0.206	-1.580		0.433	-0.101	0.091	0.683
75%	1.417	4.329	12.587			1.126	0.527	0.424		0.806	1.905	1.712	0.925
Std deviation	1.311	19.669	48.198			0.824	3.750	44.795		0.625	4.254	19.588	0.789
Pct significant	89%	98%	90%			100%	80%	93%		87%	87%	96%	91%

Table 3. *Parameter estimates from the CDS valuation model* (Continued...)

Panel B: Estimates of the factor dynamics

Statistics	Under risk-neutral measure									Under physical measure		
	Intensity factor (X_3)			Intensity factor (X_4)			Recovery factor (X_5)			X_3	X_4	X_5
	ρ	$\mu\times100$	$\sigma\times100$	ρ	$\mu\times100$	$\sigma\times100$	ρ	$\mu\times100$	$\sigma\times100$	ρ^P	ρ^P	ρ^P
Mean	0.983	-0.005	0.038	0.963	-0.001	0.016	0.995	-0.044	2.042	0.967	0.974	0.965
25%	0.994	-0.001	0.010	0.970	0.000	0.008	0.997	-0.040	0.841	0.984	0.983	0.986
50%	0.998	0.000	0.020	0.998	0.000	0.011	0.999	-0.013	1.611	0.995	0.993	0.998
75%	0.999	0.000	0.029	1.000	0.000	0.021	1.000	-0.001	2.497	1.000	1.000	1.000
Std deviation	0.032	0.018	0.073	0.040	0.003	0.014	0.011	0.100	1.633	0.035	0.032	0.036
Pct significant	100%	90%	100%	98%	72%	100%	100%	80%	100%	100%	100%	100%

Panel C: Standard deviation (bps) of the measurement error equation

Statistics	Senior CDS contracts					Subordinate CDS contracts				
	1 Yr	3 Yr	5 Yr	7 Yr	10 Yr	1 Yr	3 Yr	5 Yr	7 Yr	10 Yr
Mean	26.6	17.6	16.4	15.9	15.4	25.8	17.8	15.5	14.7	14.9
25%	3.9	3.2	3.2	3.7	3.7	4.6	4.8	4.4	4.6	5.1
50%	8.2	7.3	6.5	6.2	6.8	11.4	8.2	7.2	7.8	8.6
75%	31.7	17.7	16.4	15.0	12.6	30.9	20.8	19.3	17.9	22.1
Std Dev	14.7	7.4	7.0	6.0	4.4	13.4	8.5	7.6	6.8	8.9

Table 4. *Model fit*

This table reports the average in-sample errors of the CDS valuation model. We report two measures of average in-sample fit: the relative root-mean-squared-error (RMSE), and the mean absolute percentage error. We report the average values of relative RMSE and mean absolute percentage error in percentage terms for three maturities: 1-year, 5-year and 10-year. The results are reported for the overall sample, as well as for subsamples grouped by credit ratings and industry sectors. The last column reports the number of firms in each category. We obtain monthly S&P long-term entity credit ratings from COMPUSTAT. Industry classifications are obtained from Kenneth French's website. F,I,R refers to firms in the Finance, Insurance and Real estate sectors. Mining/Contr. includes firms in the Mining and Construction businesses.

Category	Relative RMSE (%)						Mean Absolute Percentage Error (%)						Number of firms
	Senior			Subordinate			Senior			Subordinate			
	1 Yr	5 Yr	10 Yr	1 Yr	5 Yr	10 Yr	1 Yr	5 Yr	10 Yr	1 Yr	5 Yr	10 Yr	
Overall	18.9	9.9	11.2	19.1	10.3	11.3	14.4	7.6	8.5	14.8	7.9	8.6	46
AAA/AA	18.6	8.8	11.1	19.0	8.0	7.8	14.4	6.9	8.3	14.8	6.4	5.8	5
A	18.5	11.5	12.6	19.6	12.0	14.4	13.8	8.9	9.6	15.0	9.2	11.0	12
BBB	18.9	10.6	12.0	19.4	10.9	12.1	14.5	8.0	9.2	15.0	8.3	9.1	9
BB	19.0	9.1	10.5	18.4	9.3	9.9	14.5	6.8	7.7	14.3	7.1	7.5	16
B	19.7	8.6	8.9	19.4	10.1	10.9	15.6	6.8	6.9	15.5	8.0	8.8	4
Mining/Contr	15.9	9.0	10.7	18.7	9.9	9.8	11.4	7.0	8.1	14.3	8.0	7.5	4
Manufacturing	19.7	9.9	11.4	18.4	11.5	10.4	15.0	7.4	8.5	14.2	8.8	8.2	8
Communications	19.6	11.0	10.3	19.7	10.0	10.8	15.5	9.0	7.7	15.6	7.6	7.9	3
Retail Trade	19.4	8.1	10.7	19.2	10.4	11.5	15.0	6.4	7.9	15.1	8.2	8.6	6
F,I,R	18.4	11.1	12.0	19.4	10.0	12.0	13.8	8.4	9.1	14.9	7.5	9.1	17
Services	20.0	9.0	10.6	18.8	9.8	11.8	15.8	6.8	8.0	14.7	7.4	8.9	8

Table 5. *Expected recovery, default probability grouped by ratings and industries*
This table reports the time-series averages of expected recovery and default probability implied by the CDS valuation model. The results are reported for the overall sample as well as for subsamples grouped by the overall credit ratings and industry sectors. Average expected recovery for senior and subordinate CDS contracts are reported at 1-,5-, and 10-year horizons. *Default prob.* reports the average unconditional default probability for 1-year horizon. The two columns labeled under *Correlation* report the averages of time-series correlations between the 1-year expected recovery and 1-year default probability for senior and subordinate contracts. The last column reports the number of firms in each category. We obtain monthly S&P long-term entity credit ratings from COMPUSTAT. Industry classifications are obtained from Kenneth French's website. F,I,R refers to firms in the Finance, Insurance and Real estate sectors. Mining/Contr. includes firms in the Mining and Construction businesses.

Category	Default prob. (%)	Expected recovery (%)						Correlation (%)		Number of firms
		Senior			Subordinate			Senior	Subordinate	
	1 Yr	1 Yr	5 Yr	10 Yr	1 Yr	5 Yr	10 Yr	1 Yr	1 Yr	
Overall	3.90	39.54	34.02	33.25	24.65	21.16	20.64	-20.85	-18.64	46
AAA/AA	1.17	61.85	49.94	45.84	33.77	30.42	28.76	-27.83	-21.31	5
A	2.03	36.08	32.01	30.34	20.35	16.12	15.01	-14.61	-14.46	12
BBB	4.22	39.25	34.31	35.32	27.42	24.38	24.74	-19.71	-13.73	9
BB	5.04	36.04	29.91	29.30	24.68	20.94	20.63	-22.45	-23.03	16
B	7.61	36.73	35.92	37.37	19.74	18.34	18.17	-27.06	-21.27	4
Mining/Contr	2.91	36.79	29.83	30.06	19.05	19.99	21.74	-25.62	-23.79	4
Manufacturing	5.41	41.34	39.40	42.48	26.09	26.72	28.91	-22.19	-19.43	8
Communication	6.67	36.07	23.27	19.01	31.14	19.45	15.12	-9.78	-8.00	3
Retail Trade	3.27	40.63	30.85	27.82	31.86	22.22	18.42	-0.42	-4.25	6
F,I,R	2.18	42.34	36.63	35.34	22.92	19.24	18.21	-27.80	-23.64	17
Services	5.97	33.66	31.59	30.59	21.83	20.10	20.69	-21.85	-19.41	8

Table 6. *Relative change in loss given default during the 2008 crisis*
We report the average logarithmic change in expected loss given default and market CDS spreads in February 2008 relative to their one-year prior values. For each day in February 2008, we calculate the logarithmic change in the expected loss given default and the market CDS spread relative to their one-year prior values. The monthly averages in February 2008 are then calculated for each firm. This table reports the average results for the overall sample as well as for subsamples grouped by the overall credit ratings and industry sectors. We report the change in expected loss given default and market CDS spreads (in %) for senior and subordinate CDS contracts at 1-,5-, and 10-year horizons. The last column reports the number of firms in each category. We obtain monthly S&P long-term entity credit ratings from COMPUSTAT. Industry classifications are obtained from Kenneth French's website. F,I,R refers to firms in the Finance, Insurance and Real estate sectors. Mining/Contr. includes firms in the Mining and Construction businesses.

Category	Change in expected loss given default (%)						Change in market CDS spreads (%)						Number of firms
	Senior			Subordinate			Senior			Subordinate			
	1 Yr	5 Yr	10 Yr	1 Yr	5 Yr	10 Yr	1 Yr	5 Yr	10 Yr	1 Yr	5 Yr	10 Yr	
Overall	49.6	16.1	-1.8	29.6	7.2	-6.7	260.9	161.1	121.3	247.9	146.9	112.5	44
AAA/AA	74.1	34.3	15.9	51.2	21.4	8.9	293.6	231.5	181.9	312.3	221.0	164.5	5
A	49.4	14.6	-5.4	28.4	0.9	-14.0	309.9	200.2	145.6	280.4	180.8	135.3	12
BBB	79.1	26.7	4.8	43.1	11.7	-3.1	300.4	195.0	137.2	283.9	174.6	130.2	9
BB	28.5	7.3	-10.0	21.1	6.4	-10.4	201.8	99.3	81.6	197.1	94.7	77.2	14
B	27.2	4.8	0.8	5.4	0.9	0.3	175.8	95.8	66.0	154.0	72.7	53.9	4
Mining/Contr	55.3	5.1	-0.7	42.5	4.3	-0.9	241.8	104.7	45.1	220.3	107.1	62.5	3
Manufacturing	24.5	-11.6	-21.8	2.7	-14.0	-23.2	184.4	124.0	92.0	160.2	103.0	79.6	7
Communications	21.8	7.0	1.6	13.1	3.9	0.8	119.9	83.0	63.0	143.7	85.4	62.2	3
Retail Trade	48.3	25.0	15.8	39.6	12.6	9.0	242.9	166.5	115.1	256.3	153.4	116.1	6
F,I,R	70.7	28.9	6.3	41.9	17.8	3.0	328.2	224.0	169.8	314.8	213.0	157.4	17
Services	36.1	14.0	-16.3	20.9	1.4	-29.8	257.7	106.2	95.8	222.2	77.9	76.3	8

Table 7. *Monte Carlo simulation study: True vs. Estimates*

This table reports results from the Monte Carlo simulation study. *Base case parameters* reports the "true" parameter values used in the simulation study. We simulate 100 paths of daily CDS spreads over 1500 days using the base case parameters. CDS spreads data are simulated with normally distributed noise. We assume the noise term's standard deviation is five percent of the model-implied spread level. *Estimated parameters* reports the means and standard deviations (S.D.) of parameters estimated from the 100 simulated samples. Panel A reports the factor loadings of the default intensity dynamic, the loss given default (LGD) of senior CDS, and the LGD of junior CDS. Factors X_1 and X_2 describe the dynamic of the risk-free term structure, which are estimated using the 6-month LIBOR and swap rates. Factors X_3 and X_4 are specific to the dynamic of default intensity. We normalize the factor loadings of the default intensity dynamic on X_3 and X_4 to one (i.e., $\alpha_3 = \alpha_4 = 1$ in equation (6)). Factor X_5 loads only on the dynamic of LGDs, and is referred to as the recovery factor. The factor loading of the senior LGD is normalized to one (i.e., $\beta_5^{sen} = 1$ in equation (7)). Panel B reports the structural parameters that drive the factor dynamics of state variables X_3, X_4, and X_5 under the physical and risk-neutral measures. The off-diagonal elements in ρ and σ are zero, and hence, the reported values are their diagonal estimates. The values of σ are identical under the risk-neutral and physical measures.

Panel A: Factor loadings

	Intensity loadings					Senior LGD loadings				Subordinate LGD loadings			
	Constant × 100	Risk-free factors × 100		Intensity factors		Constant	Risk-free factors		Recovery factor	Constant	Risk-free factors		Recovery factor
		X_1	X_2	X_3	X_4		X_1	X_2	X_5		X_1	X_2	X_5
Base case parameters													
True	0.270	-0.342	-0.680	1	1	1.094	8.641	-9.676	1	0.730	5.761	-6.450	0.667
Estimated parameters													
Mean	0.275	-0.341	-0.685	1	1	1.034	8.788	-9.630	1	0.728	5.896	-6.499	0.671
S.D.	0.014	0.011	0.027			0.047	0.471	0.633		0.060	0.287	0.233	0.036

Panel B: Factor dynamics

	Under risk-neutral measure									Under physical measure		
	Intensity factor (X_3)			Intensity factor (X_4)			Rec. factor (X_5)			X_3	X_4	X_5
	ρ	$\mu\times100$	$\sigma\times100$	ρ	$\mu\times100$	$\sigma\times100$	ρ	$\mu\times100$	$\sigma\times100$	ρ^P	ρ^P	ρ^P
Base case parameters												
True	0.9997	0.0001	0.0176	0.9970	0.0002	0.0062	0.9993	-0.0149	1.4912	0.9986	0.9981	0.9831
Estimated parameters												
Mean	0.9998	0.0001	0.0183	0.9868	0.0002	0.0060	0.9908	-0.0152	1.3925	0.9957	0.9918	0.9764
S.D.	0.0001	0.0000	0.0011	0.0092	0.0000	0.0005	0.0058	0.0011	0.1865	0.0084	0.0054	0.0090

Table 8. *Monte Carlo simulation study: Pricing errors*
We simulate 100 paths of daily CDS and binary CDS spreads over 1500 days using the "true" base case parameters reported in Table 7. Panel A reports the means and standard deviations of simulated CDS spreads, as well as their corresponding values of expected recovery rates and binary CDS spreads. CDS spreads data are simulated with normally distributed noise. We assume the noise term's standard deviation is five percent of the model-implied spread level. CDS are priced using the valuation model in Section 2.2. Binary CDS are priced similarly to standard CDS contracts with the exception of zero percent recovery when the firm defaults. Consequently, only the default risk is priced in binary CDS spreads. All the beta coefficients in equation (7) are set to zero such that $LGD = 1$ when we price binary CDS contracts. In Panel B, we summarize pricing errors from estimating the full stochastic recovery model on each of the 100 simulated CDS samples. The estimation exercise yields 100 sets of parameter estimates. Using these parameter estimates, we price senior and subordinate CDS contracts and calculate their absolute percentage pricing errors with respect to their "true" values implied by the base case parameters. We report the means and standard deviations of absolute percentage errors for CDS spreads, expected recovery rates, and binary CDS spreads.

Panel A: Data simulated using the base case parameters

Simulated data	Senior contracts					Subordinate contracts				
	1 Yr	3 Yr	5 Yr	7 Yr	10 Yr	1 Yr	3 Yr	5 Yr	7 Yr	10 Yr
CDS spreads (bps)										
Mean	18.9	30.5	39.3	46.0	53.3	29.3	45.3	57.0	65.7	75.1
Std deviation	10.1	11.1	11.3	11.1	10.6	15.4	16.4	16.4	16.0	15.0
Recovery rates (%)										
Mean	54.6	51.0	49.1	48.1	47.2	31.0	29.5	28.6	28.1	27.6
Std deviation	0.9	0.5	0.4	0.2	0.1	0.6	0.4	0.3	0.2	0.1
Binary CDS spreads (bps)										
Mean	42.6	65.0	81.0	92.7	105.0					
Std deviation	22.0	23.3	23.2	22.5	21.1					

Panel B: Pricing errors from estimating the full stochastic recovery model

Absolute percentage error	Senior contracts					Subordinate contracts				
	1 Yr	3 Yr	5 Yr	7 Yr	10 Yr	1 Yr	3 Yr	5 Yr	7 Yr	10 Yr
CDS spreads (bps)										
Mean	7.8%	4.2%	2.8%	1.9%	2.1%	3.1%	3.0%	2.9%	1.9%	2.1%
Std deviation	1.9%	2.0%	1.9%	1.1%	0.9%	2.0%	1.0%	1.0%	1.0%	1.0%
Recovery rates (%)										
Mean	10.3%	8.4%	6.2%	4.2%	2.9%	10.7%	9.1%	7.2%	6.6%	4.9%
Std deviation	4.1%	3.0%	3.4%	1.9%	1.0%	4.0%	3.1%	3.2%	3.0%	2.0%
Binary CDS spreads (bps)										
Mean	10.1%	7.1%	5.2%	5.7%	5.5%					
Std deviation	4.0%	3.4%	2.9%	3.1%	3.4%					

Table 9. *Robust identification of the recovery level and default risk: A simulation study*
This table illustrates the importance of using multiple-seniority CDS term structures to robustly identify the dynamics of recovery rate and default risk. We report errors from estimating a constant recovery model on CDS data simulated by a stochastic recovery model. We simulated 100 samples of CDS data, each with 1500 daily observations, using the base case parameters reported in Table 7. CDS spreads data are simulated with normally distributed noise. We assume the noise term's standard deviation is five percent of the model-implied spread. The means and standard deviations of the simulated data are summarized in Panel A of Table 8. We estimate a constant recovery rate model on simulated CDS data using two different samples. In the first estimation sample, we use simulated CDS spreads for senior and subordinate contracts. In the second estimation sample, only CDS spreads for senior contracts are used. The constant recovery rate model that we use is obtained by setting parameters β_1, β_2 and β_5 in equation (7) to zero. Our simulated data consists of 100 sample paths, and therefore each estimation sample yields 100 sets of parameter estimates. Using the parameters estimated from these two estimation samples, we price senior CDS and binary CDS contracts and calculate their absolute percentage pricing errors with respect to "true" values implied by the base case parameters. Binary CDS are priced similarly to standard CDS contracts with the assumption of zero percent recovery on the underlying security when the firm defaults, i.e. LGD is one. We report the absolute percentage pricing errors for senior CDS spreads, binary CDS spreads, and expected recovery rates of senior contracts.

Absolute percentage error	Estimation sample (1)					Estimation sample (2)				
	Senior and subordinate CDS spreads					Senior CDS spreads only				
	1 Yr	3 Yr	5 Yr	7 Yr	10 Yr	1 Yr	3 Yr	5 Yr	7 Yr	10 Yr
Senior CDS spreads										
Mean	13.2%	6.1%	4.7%	4.0%	5.4%	11.1%	5.2%	3.8%	3.4%	2.9%
Std deviation	4.0%	2.1%	1.6%	1.5%	1.3%	3.7%	2.4%	1.8%	1.6%	1.5%
Binary CDS spreads										
Mean	12.5%	9.0%	7.3%	7.3%	6.8%	23.7%	20.7%	19.1%	18.5%	17.7%
Std deviation	4.9%	4.3%	3.6%	3.6%	3.8%	8.0%	10.8%	13.0%	14.3%	14.9%
Recovery of senior contracts										
Mean			2.7%					7.1%		
Std deviation			2.9%					6.2%		

Table 10. *Accounting for bid-ask spreads in the estimation: Fannie Mae*

We estimate the CDS valuation model using time-series data of senior and subordinate CDS contracts written on Fannie Mae with maturities of 1, 3, 5, 7, and 10 years. We report results from two estimation methods. The first method, labeled *Without bid/ask,* does not account for CDS bid and ask spreads. In the second method, labeled *With bid/ask*, we include daily bid and ask information of Fannie Mae when filtering state variables and constructing the optimization function. Daily bid and ask data are collected from Bloomberg. We account for CDS bid ask spreads by assuming that the τ-maturity senior contracts on day t are priced with normally distributed errors with mean zero and standard deviations $\sigma_u^{SEN}(\tau)\left|Bid_t^{SEN}(t,\tau) - Ask_t^{SEN}(t,\tau)\right|$, where $\sigma_u^{SEN}(\tau)$ is a constant, and $Bid_t^{SEN}(\tau)$ and $Ask_t^{SEN}(\tau)$ refer to day $t's$ bid and ask spreads of the τ-maturity senior contracts. We also assume τ-maturity subordinate contracts are priced with normally distributed errors with mean zero and standard deviations $\sigma_u^{SUB}(\tau)\left|Bid_t^{SUB}(\tau) - Ask_t^{SUB}(\tau)\right|$, where the variables are defined analogously. Panel A reports the factor loadings of the default intensity dynamic, the loss given default (LGD) of senior CDS contracts, and the LGD of junior CDS contracts on the five latent factors. Factors X_1 and X_2 describe the dynamic of the risk-free term structure. Factors X_3 and X_4 are specific to the dynamic of default intensity. We normalize the factor loadings of the default intensity dynamic on X_3 and X_4 to one (i.e., $\alpha_3 = \alpha_4 = 1$ in equation (6)). Factor X_5 loads only on the dynamic of LGDs, and is referred to as the recovery factor. The factor loading of the senior LGD is normalized to one (i.e., $\beta_5^{sen} = 1$ in equation (7)). Panel B reports the structural parameters that drive the factor dynamics of state variables X_3, X_4, and X_5 under the physical and risk-neutral measures. We recall that the off-diagonal elements in ρ and σ are zero, and hence, the reported values are their diagonal estimates. Standard error is reported (in bracket) beneath each estimate.

Panel A: Estimates of the factor loadings for Fannie Mae

	Intensity loadings					Senior LGD loadings				Subordinate LGD loadings			
	Constant × 100	Risk-free factors × 100		Intensity factors		Constant	Risk-free factors		Recovery factor	Constant	Risk-free factors		Recovery factor
		X_1	X_2	X_3	X_4		X_1	X_2	X_5		X_1	X_2	X_5
Without bid/ask	-0.176	-3.364	-37.343	1	1	1.219	-7.715	-1.702	1	0.773	-0.216	-11.403	0.668
	(0.001)	(0.019)	(0.045)			(0.007)	(0.032)	(0.026)		(0.066)	(0.063)	(0.078)	(0.050)
With bid/ask	-0.193	-3.728	-32.565	1	1	1.225	-7.669	-2.024	1	0.755	-0.239	-12.850	0.623
	(0.007)	(0.068)	(0.033)			(0.002)	(0.026)	(0.022)		(0.081)	(0.070)	(0.065)	(0.032)

Panel B: Estimates of the factor dynamics for Fannie Mae

	Under risk-neutral measure									Under physical measure		
	Intensity factor (X_3)			Intensity factor (X_4)			Recovery factor (X_5)			X_3	X_4	X_5
	ρ	$\mu\times100$	$\sigma\times100$	ρ	$\mu\times100$	$\sigma\times100$	ρ	$\mu\times100$	$\sigma\times100$	ρ^P	ρ^P	ρ^P
Without bid/ask	9.996E-01	1.029E-04	1.240E-02	9.976E-01	-9.868E-05	8.804E-03	9.992E-01	-3.214E-02	1.749E+00	9.986E-01	9.956E-01	9.872E-01
	(2.45E-06)	(2.95E-06)	(5.46E-05)	(1.04E-04)	(1.16E-05)	(3.21E-05)	(4.46E-05)	(6.56E-04)	(1.55E-02)	(1.95E-01)	(6.68E-03)	(1.95E-02)
With bid/ask	9.996E-01	7.968E-05	1.040E-02	9.981E-01	-1.375E-04	1.135E-02	9.991E-01	-3.591E-02	1.751E+00	9.684E-01	9.934E-01	9.958E-01
	(2.15E-06)	(3.03E-06)	(7.85E-05)	(5.82E-05)	(7.22E-06)	(6.90E-05)	(5.72E-05)	(4.81E-04)	(1.28E-02)	(1.23E-01)	(1.18E-02)	(1.36E-02)