A Theory of Market Pioneers, Dynamic Capabilities and Industry Evolution*

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Abstract

We analyze a model of industry evolution where the number of active submarkets is endogenously determined by pioneering innovation from incumbents and entrants. Incumbent pioneers enjoy an advantage at additional pioneering innovation via a dynamic capability, which takes the form of an improved technology for innovation in young submarkets. Entrants are motivated in part by a desire to acquire the dynamic capability. We show that dynamic capabilities increase total innovation, but whether the capability confers an advantage in terms of marginal or average cost is important in determining how the impact of dynamic capabilities is distributed across incumbent and entrant innovation rates. We complement the existing literature - that focuses on exogenous arrival of submarkets or the steady state of a model with constant submarkets - by describing how competition, free entry, and the dynamic capability of incumbents drives the evolution of an industry. The shift from immature to mature submarkets can lead to a shakeout in firm numbers, and eventually leads to a reduction in total dynamic capabilities in an industry.

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1 Introduction

Helfat et al. [2007] describes dynamic capabilities as “the capacity of an organization to purposefully create, extend or modify its resource base.” In this paper we model both the impact of dynamic capabilities on incumbents (who get the benefits of such a resource base) and entrants (who strive to obtain it). Dynamic capabilities foster innovation, especially pioneering innovation in new submarkets. We show that whether that innovation comes from incumbents using the dynamic capability, or entrants seeking the dynamic capability, depends on whether the dynamic capabilities contribute to marginal or average productivity. We show how this mix of innovation drives industry evolution, offering a theory of industry evolution coming from the foundation of dynamic capabilities.

A relevant example of dynamic capabilities is IBM. IBM has its origins in a firm that was in several markets including coffee grinders and meat slicers, but also in the punch-card business. The punch-card experience was central to the eventual development of IBM as a computer firm. Computers and punch cards are related, but not the same; being in the punch card business was neither necessary nor sufficient for IBM’s success with computers. Understanding the process by which some firms become IBM and others end up falling behind is critically related to entry decisions across markets and submarkets, and at the heart of this paper.

Another example of a company that used dynamic capabilities as a source of sustained competitive advantage is 3M. Originally a mining company, 3M eventually became a company that staked its competitive advantage on continuously pioneering new submarkets. Their ability was specifically related to being able to enter new submarkets, take a leadership position, and then move into different, often related areas, eventually including new submarkets in areas like the paper industry. Their identity stresses the need to constantly enter new areas to maintain this capability.

In our model, the dynamic capability is defined by a technology, available only to some incumbents, that offers superior innovation opportunities. This means a lower cost for a given level of innovation, and possibly a higher return to additional investment. We model these dynamic capabilities as related to the type of submarket the incumbent is engaged in. King and Tucci [2002] and Helfat and Lieberman [2002] stress the benefit of experience
in generating dynamic capabilities, in keeping with the IBM example.\(^1\) In particular, firms’ entry decisions are highly driven by experience in similar submarkets.\(^2\) Both the dynamic capabilities literature such as King and Tucci [2002], and industrial organization literature such as Franco and Filson [2006] have emphasized that firms that are early entrants are typically ones that have been operating in other relatively recent submarkets. \(^3\)

We model this by introducing two types of submarkets, mature and immature, where mature submarkets are older on average. We associate innovation in immature industries with market pioneering and dynamic capabilities, as in King and Tucci [2002]: leaders in immature submarkets (i.e. pioneers) have an access to the technology that improves additional pioneering innovation.\(^4\) In addition to conferring the dynamic capability, short term profits and innovation costs can differ across the two types of submarkets. Dynamic capabilities are therefore related to the Innovator’s Dilemma (Christensen [1997]): choosing between pioneering and innovation into mature submarkets may involve a trade-off between short run gains and the firm’s long run position, since pioneering leads to a dynamic capability that has long run benefits, but possibly at a cost to current profitability.\(^5\)

Innovation into mature submarkets results in overtaking a leadership position of an existing mature submarket, while innovation into immature markets can result in either overtaking in an existing submarket or in the creation of a new immature submarket. This innovation in immature submarkets is what we associate with market pioneering. The two types of submarkets

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\(^1\)For a general discussion, see Helfat et al. [2007], Teece and Pisano [1994], and Teece [2009].

\(^2\)Outside of the literature on dynamic capabilities, the notion that firms diversify into related product areas has long been documented. Gort [1962] showed that diversifying firms choose related areas. This basic fact has both motivated a variety of models (for instance Mitchell [2000]) and led to a wide variety of papers studying the forces behind the phenomenon.

\(^3\)In King and Tucci [2002], for instance, entry into a new market niche by an incumbent is strongly related with the incumbents activity in the most recent existing market niche. Franco and Filson [2006] document that entrants into new submarkets disproportionately come from the most recent, high tech firms, suggesting that production in the most recent submarkets is relevant to entry into new submarkets.

\(^4\)It is easy to add to the model the possibility that leadership in mature submarkets confers dynamic capabilities in these markets too without changing our qualitative results.

\(^5\)This notion of dynamic capabilities being related to a sort of “ambidexterity” in producing current and future profits is highlighted in the Organizational Behavior literature in O’Reilly and Tushman [2008].
capture the idea that innovation contributes not only to competition in existing products but also to the evolution of the industry, and that new ideas usually go through a process of gradual improvement before they establish themselves as viable product categories. More generally, our model is consistent with the assumption that new submarkets are technologically similar to more recent, immature submarkets.\(^6\) Over time, the immature submarkets can either die-off or turn into mature submarkets.\(^7\)

We show that the form of the dynamic capability has important implications for the way dynamic capabilities impact innovation across incumbents and entrants. We differentiate between marginal dynamic capabilities, which can be thought of as reductions in the marginal cost of innovation from the dynamic capability, and average dynamic capabilities, which lower the total cost. To see how this difference matters, suppose the dynamic capabilities lower a fixed cost of innovation\(^8\). The dynamic capability raises the return to being an incumbent with the capability, but does not impact the innovation rate of incumbents; instead, the value of incumbency is offset by greater competition from entrants seeking out the returns from the dynamic capability. The greater the dynamic capability, in this case, the higher the flow rate of profits for incumbents, since they save on the fixed cost of innovation, but the sooner those profits are eroded by entrants. In other words, the impact of dynamic capabilities is not seen in the innovation rate of incumbents, but rather in the innovation rate of entrants. Moreover, the proportion of innova-

\(^6\)Helfat [1997] notes that R&D in a new submarket is buttressed by knowledge in similar submarkets. Scott Morton [1999] found that entry into a new drug is tied to a firm’s experience with drugs having similar characteristics. Kim and Kogut [1996] use this notion to drive “technological trajectories,” where a firm’s prior experience determines its future decisions. De Figueiredo and Kyle [2006] show that, more generally, innovative firms with a greater stock of knowledge are more likely to introduce new products. We therefore assume that innovation in new and recent submarkets is related to participation in the current stock of recent submarkets, which we take as a measure of the sort of knowledge stock highlighted in De Figueiredo and Kyle [2006]. This feature is natural when one assumes that technology advances over time, as in the hard drive industry studied by King and Tucci [2002] or the case of IBM, where experience with punch cards was more important to their subsequent computer business than was their meat slicing experience. But more generally, new submarkets are likely to be similar.

\(^7\)In our baseline model only firms with the best product in each submarket make positive profits, but we discuss how the model can be easily extended to allow for richer submarket competition. To capture cross-submarket competition we assume that profits are decreasing in the total number of submarkets of each kind.

\(^8\)Or, more generally, any inframarginal costs.
tion done by incumbents is actually declining in the dynamic capability. On the other hand, when dynamic capabilities lower marginal cost for incumbents, dynamic capabilities increase incumbents’ innovation. This effect may be at least partially offset, however, by increased competition from entrants seeking the dynamic capability.

Our modeling of dynamic capabilities and submarket dynamics allows us to connect the literature on industry innovation with the literature that asks about the direction of innovation. Our paper builds on papers of industry innovation where the number of submarkets is exogenous and market forces affect the quantity of innovation only. This literature has shown that new submarkets are an important driver of industry evolution: for example, both Klepper and Thompson [2006] and Sutton [1998] show that, taking arrivals of new submarkets as exogenous, such a model can help explain firm and industry dynamics. On the other hand, Klette and Kortum [2004] show that the steady state of a model with a constant set of submarkets can generate predictions about the cross section of firm size and innovative behavior consistent with empirical evidence. In Klette and Kortum [2004], every submarket is identical, and the set of submarkets is fixed, so there is no sense in which what a firm is doing now impacts the sorts of markets it might enter in the future, as stressed by the dynamic capabilities literature. Our paper endogenizes the arrival of new submarkets through market pioneering. As we show, early in the life cycle, when submarkets are disproportionately immature, pioneering innovation is the focus, with new submarkets generated at a relatively fast rate, a rate that slows down as the industry grows larger.

Besides describing the effect of dynamic capabilities on innovation, the main results of this paper are the characterization of the evolution of the industry. The model predicts the following patterns: a) In the early stages of the industry there is innovation into immature markets by both incumbents and new entrants; pioneering innovation first increases but later decreases and at some point may drop discontinuously as new entrants give up. b) The sudden drop of pioneering can cause a shake-out in the industry, i.e., a drop in the number of firms. c) The number of immature submarkets first increases and then falls, while the number of mature submarkets monotonically increases towards a steady-state. d) The total size of the industry (measured...
by the total number of submarkets) follows an S-shape, it first increases at an increasing rate, but at some point the rate of growth decreases (in fact, the total number of submarkets can decrease at some later point in the life of the industry if the immature submarkets die off sufficiently fast). e) The stock of dynamic capabilities per incumbent follows an inverted-U shape, peaking relatively early in the industry life cycle.

One interpretation is that process innovations are disproportionately non-pioneering, while product innovations represent, at least partially, pioneering of new submarkets. With that interpretation, we can compare the model’s predictions on pioneering to well known evidence on product innovations over the course of an industry life cycle. This evidence was documented first by Utterback and Abernathy [1975], and has been further discussed in papers including Cohen and Klepper [1996] and Klepper [1996a]. Innovations move from product to process innovations, with product innovations steadily falling and process innovations rising. Moreover, our model is consistent with the depiction of industry evolution driven by a changing standard product contained in Klepper [1996a]. One can interpret mature submarkets as variants of the “standard” product with a particular unique feature; immature submarkets are variants that are not yet accepted as a standard, and may never be. Maturity reflects a submarket’s integration into the standard under that interpretation.

Our model of the shake-out is related to the one that derives from Klepper [1996b], further applied in Klepper [2002], Buenstorf and Klepper [2010]. In that framework, prices fall, eventually making entry unattractive. Here, competition makes entry difficult because only the incumbents have the requisite dynamic capability to efficiently enter under the more competitive circumstances. As suggested in Klepper [1997] and Klepper and Simons [2005], early entrants generate a capability that helps them to survive even after entry falls. Our model is therefore broadly consistent with the evidence in Buenstorf and Klepper [2010], that new submarkets might be associated with a shift toward innovation by “leading incumbents.”

The model allows immature submarkets to differ from mature submarkets in terms of current profitability as well as the dynamic capability they bring. Our model, therefore, incorporates various sorts of implications of early entry

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10Moreover, the notion that new submarkets strengthen incumbents positions relative to entrants is consistent with the message of Buenstorf et al. [2012], who show that new submarkets for multipurpose tractors in Germany benefited incumbents with related market experience.
as described in Lieberman and Montgomery [1988]. We show, in fact, that in some cases the measured returns to early movers are entirely generated on the supply side by the relative cost of de novo entry. Put another way, differences in capabilities of the firms that follow the early movers determine the return to early moving, and not necessarily the capabilities of the early movers themselves.

The next section introduces the model. Section 3 derives the equilibrium. Section 4 discusses the dynamics implied by the equilibrium. Section 5 relates these dynamics to the experience of industries. Extensions and proofs are contained in appendices.

2 Model

At any given time $t$ there is an industry made up many small submarkets. We take each submarket to be one of a continuum of mass $N_t$ of total submarkets. Of the $N_t$ submarkets, $M_t$ are mature, and $I_t$ remain immature (i.e. young), so $I_t + M_t = N_t$. The industry is long lived (although submarkets may not be), with time continuous and future payouts discounted at the interest rate $r$.

We take each submarket to be characterized by a profit making leader, and follower firms who earn zero profits, as in the canonical quality ladder models of Klette and Kortum [2004] and Grossman and Helpman [1991]. Each leader of a mature submarket earns $\pi(M_t, I_t)$ any time they are the leader, and each leader of an immature submarket earns $\alpha \pi(M_t, I_t)$ per instant, from the submarket leadership. These returns do not include any costs of innovation, which will be determined endogenously. Although no entry into immature submarkets will ever occur for $\alpha$ sufficiently negative, we do allow for $\alpha < 0$. Such a case would correspond to the situation where immature submarkets earn losses but they promise profits when they mature. When $\alpha < 1$ leadership in immature submarkets is less profitable, per instant, than mature submarkets; there is therefore an “innovators dilemma” in the

\footnote{In the appendix we show that the model is amenable to allowing several profit making firms per submarket at only the cost of notational complexity.}

\footnote{Alternatively, one could capture negative profits in immature submarkets by subtracting a constant from $\pi(M, I)$ rather than by using $\alpha<0$. Assuming $\alpha<0$ combined with our assumption that $\pi(M, I)$ is decreasing means that profits in immature industries increase (become less negative) as competition increases. Nothing material would be changed by making this alternative assumption for profits in immature submarkets.}
sense that mature submarkets generate higher current returns, but immature submarkets generate dynamic capabilities that we describe formally below.

We assume that $\pi$ is continuous and decreasing in both arguments, reflecting the notion that there is elasticity of substitution between submarkets, and therefore more submarkets lead to less profits per submarket. This simple model of profits by submarket is analogous to assumptions in both Klette and Kortum [2004] and Klepper and Thompson [2006]; Klette and Kortum use the term "goods" and Klepper and Thompson use "submarkets."

Industry evolution comes via innovation. There are two types of innovation: one focused on immature submarkets, which we term pioneering innovation, and innovation focused on mature submarkets. Mature submarket innovation works exactly like the quality ladder structure in Klette and Kortum [2004] or Grossman and Helpman [1991]: a successful innovation into mature submarkets generates a new, higher quality version of some submarket, and therefore makes the innovator become the new submarket leader.

Innovative effort in immature submarkets is the source of both improved products in those immature submarkets, as well as new designs that generate new submarkets. This pioneering research, therefore, is a form of early moving, either as a first mover, or as one of the firms which enters the submarket soon after the first mover. A fraction (or, identically, probability) $1 - \phi$ of innovations from research into immature submarkets generates an improvement to an existing immature submarket, resulting in a simple changing of leadership in that immature area. This matches the notion that immature areas still attract commercial competition, but perhaps in different amounts from mature submarkets. The remaining fraction of successes generate an entirely new immature submarket.

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13 In the appendix we introduce an explicit model of consumer preferences and show that it delivers this structure for profits; however, we suppress it here since all of the fundamentally new analysis does not require a specific interpretation of the origin of profits. The key here is determining how those profits translate into valuations for submarket leaders, and in turn innovation rates.

14 One can take this research to be undirected or directed across submarkets. Incumbents would never want to innovate in a submarket they led in due to the Arrow replacement effect; as a result, in a symmetric equilibrium, every submarket would be researched equally, and never by its current leader, just as under undirected research.

15 The model could be extended to allow research in both types of submarkets to generate new submarkets. We choose to focus on the role of immature submarket research in generating new submarkets because it fits with the notion that immature submarkets are more similar to undiscovered submarkets, than mature submarkets are.
In order to model the dynamic capability of incumbents, we will assume that the technology for generating innovations differs across incumbents and entrants. We focus our attention, in keeping with the evidence, on the possibility that incumbents near the frontier, here defined by immature submarkets, have a special ability to produce further innovations. For incumbents in immature submarkets, the arrival rate of innovation in immature submarkets is determined by the production function \( F(K, L) \).\(^{16}\) Here \( L \) is the number of leadership positions the firm has in immature submarkets, and \( K \) is all other inputs in the production of innovation (and can potentially be multidimensional). We make the following assumptions on \( F \):

Assumption 1. The innovation technology for incumbents satisfies:

(a) \( F \) is constant returns to scale
(b) \( F \) is concave
(c) \( F(0, L) = 0 \)
(d) The per-unit cost of \( K \) is normalized to one, i.e. it is denominated in dollars of the input

The form of this production function follows Klette and Kortum [2004] and is the key feature that embodies the nature of the dynamic capability that we assume. Leadership in immature submarkets generates the capability needed to operate the production function for innovation in those submarkets. A firm with more of the dynamic capability conferred by \( L \) generates more innovations for a given amount of \( K \). As a result, firms with entry into recent, immature submarkets are assumed to have a resource that generates additional innovations in immature submarkets, in keeping with the literature on entry into related areas.\(^{17}\) The dynamic capability conferred by immature submarket leadership might come in different forms, sometimes favoring developing leadership positions in other immature submarkets, and other times generating a capability in developing entirely new submarkets. In that case \( \phi \) represents the fraction of that leads to moving first in new markets, while the remaining fraction \( 1 - \phi \) is associated with entering immature

\(^{16}\)Here we suppress the \( t \) subscripts to streamline the presentation.

\(^{17}\)The technology operated by incumbents need not be interpreted as generating entry into new submarkets solely by incumbents themselves; indeed, papers including Franco and Filson [2006] stress the role of spin-outs in generating entry into technologically advanced submarkets. Our model allows the incumbent innovation which benefits from the dynamic capability of the firm as being executed by employees who leave the firm. As in Franco and Filson [2006] we will assume that rents coming from such activity are captured by the parent firm.
markets as an early mover.\textsuperscript{18}

Because of the constant returns assumption, firms will optimally choose employment of $K$ in proportion to their dynamic capability $L$, as in Klette and Kortum [2004]. Optimization can be done for each submarket; the firms entire payoff is the number of submarkets it operates of a given type, times the return to a leadership position of that type. To see this rewrite $F(K, L) = Lf(k)$ where $f(k) = F(k, 1)$ The interpretation is that a firm with $L$ leadership positions produces, with $k = K/L$ units of input per leadership position, and therefore $L$ times as much of the input $K$ overall, exactly $L$ times as many innovations per unit of time. A firm with $L$ leadership positions can choose $k$ to maximize the return per submarket, since its optimization problem in $k$ is independent of $L$, as $L$ represents a multiplicative factor. The firm with any number of immature submarkets then employs $k$ units of input in each immature submarket it operates in, for a total of $K = kL$ units over all of its immature submarkets.

The important feature, as in Klette and Kortum [2004], is that linearity allows everything to be determined at the submarket level. A firm with $L$ leadership positions in immature industries employs exactly $L$ times as much of the input as one that holds only one leadership position, and generates improvements exactly $L$ times as often. The payoff from this is $LW_t$, where $W$ is the return per immature submarket, which is exactly $L$ times the payoff of holding one leadership position. The key development that follows determines the return to a single submarket, for instance for an immature submarket, $W$. Moreover, note that the division of $W$ between future profits on the given submarket, and the return that comes indirectly via the value of the dynamic capability stemming from the submarket, is both hard to compute and not necessary for determining the equilibrium outcomes. This is the key feature that makes the constant returns to scale case tractable both here and in Klette and Kortum [2004].

There are a continuum of firms, each with a finite number of leadership positions. Firms come in many sizes, corresponding to a different number of leadership positions; the distribution of firms is over the number of mature

\textsuperscript{18}The model can allow for dynamic capabilities for mature submarkets by letting innovation be a function of mature submarket leadership positions. Those innovations could be modeled as only relevant to mature submarkets (in keeping with the notion that dynamic capabilities pertain to related areas) or to all submarkets. Since the evidence suggests that a key form of dynamic capabilities deteriorates as the firm falls behind the frontier, and for notational simplicity, we focus all dynamic capabilities on immature areas.
and immature submarket leadership positions it holds. A firm can hold a portfolio that includes leadership in submarkets of both types. The preceding discussion implies that a firm with twice as many leadership positions in immature submarket type will hire twice as much of the other inputs that describe the dynamic capability, and generate twice as many innovations of that type. We therefore analyze the decision of incumbents on a per-submarket basis; the same decisions apply to any incumbent, regardless of how many leadership positions it holds. We return to the distribution of firms only after equilibrium has been characterized at the submarket level.

In addition to the incumbent innovation technology there is a *de novo* entry technology, which has constant cost normalized to 1 for mature submarkets and \( c \) for immature submarkets.\(^{19}\) This normalization implies that all profits and costs are expressed in terms of the cost of de novo innovation in mature markets. This technology can also be operated by incumbents, although they have no competitive advantage relative to de novo entrants. We assume that \( c > 1/f'(0) \) so that, at least for small \( k \), the marginal cost of innovating is lower for incumbents, consistent with the dynamic capability.

In order to characterize innovation levels and the value of dynamic capabilities, we need to describe the present value of being a submarket leader. Denote by \( V_t \) the present discounted value of a leadership position at time \( t \) in a mature submarket when the current state is \((M_t, I_t)\), and by \( W_t \) the value of a leadership position in an immature submarket. Consider a de novo entrant investing 1 unit in the mature innovation technology for \( dt \) units of time. This generates a payoff of

\[
V_t dt - dt
\]

while investing \( c \) for \( dt \) units of time in the immature technology generates

\[
W_t dt - c dt
\]

On the other hand, the incumbents in an immature submarket choose \( k \) to maximize

\[
W_t f(k) dt - k dt
\]

Denote this optimal choice by \( k^*(W) \). The dynamic capability generates extra innovation beyond what would be obtained by the same investment in

\(^{19}\)Note that this need not be the only entry technology; the model could allow for exogenous entry from other sources, including spin outs.
the *de novo* entry technology that is freely available. In turn this generates a return for the incumbent firm. Its return to innovation is

\[ DC(W) = W f(k^*(W)) - k^*(W) \]  

(1)

By free entry, the net return from *de novo* innovation will never be strictly positive, so \( DC(W) \) is entirely excess return beyond what the return from the innovation technology of *de novo* firms.

Optimization by the incumbent firms implies that

\[ f'(k) = 1/W \]

Substituting, therefore, \( DC(W) = f(k)/f'(k) - k \), which is positive since \( f \) is concave, confirming that the dynamic capability leads to additional value. In section 3.3.1 we discuss how a shift in \( f \) changes the return to the dynamic capability, as well as the equilibrium innovation level.

The final part of the model to describe is the evolution of the aggregates \( M \) and \( I \) over time. Not all immature submarkets eventually become viable, mature submarkets. Immature submarkets sometimes fail to become viable, dying at exogenous Poisson rate \( \lambda \). Immature submarkets mature at Poisson rate \( \mu \), at which point they are viable and permanent.\(^{20}\) Denoting total innovation in immature submarkets as a rate of \( i_t \) units per immature submarket, the change over time in \( I \) is the new arrivals from innovation \( (i\phi I) \) minus the maturing \( (\mu I) \) and failing \( (\lambda I) \) submarkets:

\[ \dot{I} = i\phi I - \mu I - \lambda I \]  

(2)

Here we take intensity to be symmetric across immature submarkets, since they are identical; this corresponds to a symmetric equilibrium.

New mature submarkets come from maturing immature submarkets. The change over time in the measure of mature submarkets is therefore

\[ \dot{M} = \mu I \]  

(3)

\(^{20}\)Stochastic maturation of submarkets is meant to capture the fact that the viability of a submarket is not guaranteed after a fixed amount of time, but the probability of viability is an increasing function of how long the submarket has survived. The Poisson assumption is merely for tractability, so that immature submarkets can be described without regard to their age; while the total probability of viability is increasing in age, the hazard rate into maturity is constant.
Denote the total rate of innovation in mature submarkets as $m_t$ per mature submarket. Note that this does not change $M$, since these innovations are entirely generating new leaders among the mature submarkets.\footnote{Below we relax the assumption that mature submarkets survive forever.} Next we define and characterize the equilibrium levels of $i_t$ and $m_t$.

3 Equilibrium

In equilibrium, if the resource incumbents have is insufficient to deter entrants, de novo entry occurs. An important variable in the equilibrium construction is the quantity of innovation per immature submarket leader, when de novo entrants are making zero profits, i.e. at times when $W_t = c$; this determines the maximum amount of innovation the incumbents will ever generate, when de novo entry occurs and the net return to such entry is exactly zero. Let $\bar{k} = k^*(c)$, and $\bar{i} = f(\bar{k})$. Total innovation in immature submarkets is $i_t$, of which $\bar{i}$ is accounted for by incumbents; the net amount of innovation contributed by de novo entrants is therefore $i - \bar{i}$, if the total innovation rate exceeds what the incumbents offer.

An industry equilibrium for some initial $M_0, I_0$ is a sequence $\{m_t, i_t, V_t, W_t, M_t, I_t\}$ such that

1. $m_t > 0$ implies $V_t = 1$ (profit maximization and free entry for de novo mature entrants)
2. $i_t > \bar{i}$ implies $W_t = c$ (profit maximization and free entry for de novo immature entrants)
3. $i_t \leq \bar{i}$ implies $i_t = f(k^*(W_t))$ (profit maximization for incumbents in immature submarkets)
4. $V_t$ and $W_t$ satisfy (4) and (6) (Bellman equations described below)
5. $M_t$ and $I_t$ satisfy (2) and (3)

A key task of this section is to characterize the values $V_t$ and $W_t$ described in the final equilibrium condition for all possible combinations of $M$ and $I$, which in turn determines innovation rates.
3.1 Measuring the Benefits of Early Entry

The model gives an immediate insight into the sources of returns for early movers (i.e. innovators in immature submarkets) and late movers (i.e. new leaders in mature submarkets), such as that described in Lieberman and Montgomery [1988]. It also points to the difficulty in identifying such advantages in the data. There are two sorts of possible ways one could describe an early mover advantage here, if one takes entry in the immature stage as early entry. In the model it is assumed that per instant profits differ between the two types of submarkets by a factor of $\alpha$. So, measured by the flow rate of profits for the firm, $\alpha$ being greater than one would define a first mover advantage. However, when de novo entry in both areas is positive (i.e. $m_t > 0$ and $i_t > 0$), the relative gross return to entry in the two areas $W/V$ is exactly $c$; $\alpha$ is irrelevant. So if one measures early mover advantage not in flow terms but in present discounted terms (i.e. stock value), whether or not their is an early mover advantage is a question of whether $c$ is bigger or smaller than one. Early movers could make more profits per instant than late movers, but have lower present discounted value at the time of entry, or vice versa.

Intuitively, the capabilities of future entering firms in each of the two areas, and not the current profitability of the areas, determines any measured “early mover advantage” of early versus late movers, as measured by discounted return to successful entry in immature versus mature submarkets. This shows the difficulty in assessing the inherent benefit from being an early mover, as defined by the relative flow rate of rewards for early entrants (i.e. $\alpha$) compared to the realized discounted returns to early moving, which depends on the endogenous response of other firms.

3.2 Mature Submarkets: Perpetual Innovation by Incumbents and Entrants

Mature submarkets behave as all submarkets do in Klette and Kortum [2004], with perpetual innovation and changing leadership. This benchmark characterization is not the key prediction of the model; on the contrary, this section merely shows the sense in which the model follows the line of previous work: an industry populated with a constant set of mature submarkets would behave exactly as in Klette and Kortum [2004], and deliver the same predictions about innovation that they deliver. We then build an endogenous evolution of the number and type of submarkets, including immature submarkets, in
the theory of market pioneering that follows.\footnote{Since immature submarkets may eventually mature, we must compute the return in mature submarkets first, since it is part of the expected return in an immature submarket.}

We can characterize the return to mature innovation in terms of a simple Bellman equation, familiar from pricing equations in finance:

\begin{equation}
    rV_t = \pi(M_t, I_t) - m_t V_t + \dot{V}_t
\end{equation}

Mature submarket leadership generates a flow payoff of \( \pi \), and has a risk \( m_t \) of losing all value. Finally, in such a valuation, one must take account of the possibility that the value of leadership in a mature submarket might change over time due to changes in \( M_t \) and \( I_t \), which we denote \( \dot{V}_t \), for the time derivative of \( V \). To insure that innovation in mature industries is perpetual, and therefore an industry populated by mature industries will behave like Klette and Kortum [2004], we assume

Assumption 2. For all \( M, I \), \( \pi(M, I)/r > 1 \)

This assumption simply implies that profits are always high enough to attract de novo entrants to mature submarkets, since if no innovation were taking place, an innovator would receive return greater than the cost of entry.\footnote{This assumption can be weakened to only hold on the “relevant” range of \( M \) and \( I \) that is generated in equilibrium.} As a result, entry always pins down the value of incumbency in mature submarkets at the entry cost: \footnote{Proofs are contained in the appendix.}

Lemma 1. For all \( t \), \( V_t = 1 \)

The fact that \( V_t = 1 \), so the value is unchanged when \( M \) and \( I \) changes, implies \( \dot{V}_t = 0 \); we can substitute this and \( V = 1 \) into (4) to compute the rate of innovation:

**Proposition 1.** The rate of innovation in mature submarkets is

\begin{equation}
    m_t = \pi(M_t, I_t) - r
\end{equation}

Note the “demand side” characterization of \( m \): it changes as the returns to innovation change, through the impact on \( \pi \). The model has both the free entry characteristics of Klette and Kortum [2004] and demand side mechanics in the spirit of Adner and Levinthal [2001]. The Klette and Kortum characterization of innovation rates across firms is perfectly compatible with
our model if $M$ and $I$ are constant. We will show below that, in fact, in the long-run, $M$ and $I$ converge to constant values, and therefore innovation in our equilibrium converges to the one in Klette and Kortum with constant innovation per submarket. We characterize pioneering innovation next.

### 3.3 Immature Submarket Innovation with De Novo Entry

In this section we evaluate market pioneering given that mature submarkets generate a constant payoff of 1, according to Lemma 1. In Section 4 we put the pieces together and examine the model’s predictions for the evolution of the stock of mature and immature submarkets implied by innovation in immature submarkets, and the eventual maturity of those submarket.

An immature submarket has discounted returns $W$ determined by the recursion

$$rW = \alpha\pi(M, I) - i(1 - \phi)W - \lambda W - \mu(W - V) + DC(W) + \dot{W}$$  \hspace{1cm} (6)

The first term is the current profits generated from leadership; the second and third terms are the expected capital loss from either an improvement which displaces the current leader or failure of the entire submarket, either of which ends that dividend payment. The fourth term is the capital gain or loss when the submarket matures, accounting for the loss of $W$ and the gain of a mature leadership position valued at $V$. Note that $V = 1$ so this term simplifies further. The next term is the value of the dynamic capability. The final term is the time derivative of $W$, which governs how the value changes when no event takes place for the submarket, but external market forces evolve.

We first explore how the value function is determined when there is entry by de novo firms. In that case, $W = c$. On the interior of any such region, $W$ is therefore constant so $\dot{W} = 0$. Therefore we can rewrite (6) as

$$rc = \alpha\pi(M, I) - i(1 - \phi)c - \lambda c - \mu(c - 1) + (\tilde{i}c - \tilde{k})$$

so that

$$i = \frac{1}{(1 - \phi)c}(\alpha\pi(M, I) + \mu - (\mu + \lambda + r - 1 - \tilde{i})c - \tilde{k})$$ \hspace{1cm} (7)
As a result, \( i \) varies continuously in the range since \( \pi \) is continuous.\footnote{Expression (7) is simplified due to our previous observation in Lemma 1 that the value to the mature leadership is constant in equilibrium. If it was not constant, (7) would contain also a \( V_t \), but the qualitative features of our model would remain unchanged.}

### 3.3.1 Dynamic Capabilities and Innovation

The definition of the value of dynamic capabilities in (1) is precisely the excess value for the incumbent’s technology when de novo profits are exactly zero, i.e. when \( W = c \). We therefore now address comparative statics in this valuation of the dynamic capability \( DC(c) \).

Suppose that we increase dynamic capabilities of incumbents for all \( k \) by shifting out \( f \). The following proposition shows that this increases total innovation when both technologies are active.

**Proposition 2.** Suppose that the function \( f(k) \) is replaced by \( \hat{f}(k) \) with \( \hat{f}(k) > f(k) \) for all \( k \). Then, for every \( M, I \) such that \( i > \bar{i} \), the equilibrium \( i \) is higher under the function \( f \). Moreover the value generated by the dynamic capability, \( DC(c) \), must be greater under \( \hat{f} \).

More dynamic capabilities means more innovation and a greater return from dynamic capabilities. The additional innovation comes from potentially two sources: more innovation from incumbents who hold the dynamic capability, and more innovation from entrants seeking to acquire the dynamic capability.

From the first order condition for the incumbent’s choice of \( k \) it is evident that the incumbent’s innovation is related to marginal, and not total, dynamic capabilities:

**Proposition 3.** Suppose that the function \( f(k) \) is replaced by \( \hat{f}(k) \) with \( \hat{f}'(k) > f'(k) \) for all \( k \). Then, for every \( M, I \) such that \( i > \bar{i} \), the equilibrium innovation by incumbents is higher under the function \( f \).

In the extreme case where, at the optimum \( \tilde{k} = k^*(c) \) for the production function \( f \), the marginal product \( \hat{f}'(k) < f'(k) \), incumbents may have a greater total dynamic capability (since \( \hat{f} > f \)) but utilize it less. Competition from entrants seeking to gain the dynamic capability can, paradoxically, result in less innovation by incumbents even with a greater capability. The incumbents return to the dynamic capability must be higher, however, any
time the dynamic capability is higher; that higher return can come either from more innovation or from less inputs being employed in innovation as \( f \) shifts out. This distinction has important implications for the measurement of dynamic capabilities: the effect of dynamic capabilities might show themselves through innovation rates or simply returns of incumbents, or both; it definitely shows itself in the total innovation rate in the immature submarkets which generate dynamic capabilities. For production functions such as power function \( Ak^\alpha \), an increase in \( A \) increases both incumbent and total innovation, since both marginal and total dynamic capabilities increase with \( A \).

The model therefore offers separate drivers of entry by incumbents and entrants. More dynamic capabilities increase the return to incumbency, and therefore attract more entrants; more marginal dynamic capabilities increase innovation by incumbents at the expense of entrants. These separate forces in the model provide a natural rationale for the non-monotonicity in the relationship between entry and incumbent innovation as described in Aghion et al. [2007], since entry is driven by the heterogeneous distribution of dynamic capabilities across industries, and the rate of innovation by incumbents is driven by the distribution of marginal dynamic capabilities. These drivers are different from the ones emphasized in Aghion et al. [2007].

### 3.4 Innovation by Incumbents Only

Alternatively, it could be the case that there is only innovation by incumbents, so \( i = f(k^*(W)) \). Characterizing \( W \) in this case, however, is more difficult. In that case (6) can be rewritten as

\[
rW = \alpha \pi(M, I) + \phi_i W - \lambda W - \mu(W - 1) - k^*(W) + \dot{W}
\]

To make sure this value is well-defined, we assume that

**Assumption 3.** \( r + \lambda + \mu > \phi \bar{i} \)

Assumption 3 guarantees that discounting and eventual exit from immaturity (either through death or maturity) is sufficient to keep the dynamic capability in immature industries from replicating itself so rapidly that a leadership position has infinite value, generating additional leadership positions faster than the value depreciates.

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\(^{26}\)Other papers about competition and innovation are harder to compare to the model, since there are many firms at every point in time.
The analysis of our model simplifies in case where the maturation process of firms makes it more difficult for other incumbent firms to profit. This natural assumption is consistent with evidence that prices decline as firms mature, documented in many papers, including Gort and Klepper [1982]. We capture this effect by the following assumption (which we maintain for the remainder of the paper):

**Assumption 4.**

\[
\mu \frac{d\pi(M, I)}{dM} - (\mu + \lambda) \frac{d\pi(M, I)}{dI} < 0
\]

Assumption 4 ensures that an industry with immature submarkets both dying and maturing becomes more competitive, other things equal, in the sense that profits for each submarket decline, over time; the first term is the impact of the gain in mature submarkets, while the second is the impact of the decline in immature submarkets. The assumption is not essential for our model's life cycle predictions, but it simplifies the analysis greatly while being consistent with evidence. If we take profits to be a function of output, and output to be linear in the two types of submarkets, i.e. \( \pi(M, I) \equiv \pi(M + \gamma I) \), then Assumption 4 simplifies to:

\[
\frac{\mu}{\mu + \lambda} > \gamma
\]

Assumption 4 implies that profits per submarket, and as a result innovation per submarket, decrease over the lifetime of the industry:

**Lemma 2.** \( \dot{\pi} \leq 0 \), strictly if \( I > 0 \).

Since profits are falling, once de novo entry is unprofitable, it remains unprofitable forever after:

**Lemma 3.** If \( i < \bar{i} \) at \( t \), \( i < \bar{i} \) for all \( s > t \).

Intuitively, industry conditions are becoming more competitive under Assumption 4, so value is declining; once value is too low for de novo entrants, it never recovers.

Once immature innovation is limited to incumbents, we can construct \( W \) directly. Let \( \hat{W}(M, I) \) be the present discounted value of a firm with one immature submarket leadership position in state \( M, I \), given that incumbent innovation will be the only innovation in immature submarkets forever after, i.e. taking innovation to be \( i = f(k^*(\hat{W})) \). Since \( M \) and \( I \) are greater at every
future state starting from a greater initial $M$ or $I$, the resulting $\hat{W}(M, I)$ is strictly decreasing in both arguments.\textsuperscript{27} The equilibrium value function $W$ is, therefore, either $\hat{W}$ (if free entry does not bind) or $c$ (if it does). In other words

$$W(M, I) = \min\{c, \hat{W}(M, I)\}$$

We can describe the set of points where de novo entry ends by first describing the set of points $I = g_0(M)$ defined by

$$\hat{W}(M, g_0(M)) = c$$

For $I \geq g_0(M)$, $i \leq \bar{i}$. For $I < g_0(M)$, $i \geq \bar{i}$. Since $\hat{W}$ is decreasing in both arguments, $g_0$ must be decreasing.

We can therefore further characterize innovation in the range where all innovation is by incumbents. First we show formally that $W$ declines in the region where de novo entry has ceased.

**Lemma 4.** Suppose $i < \bar{i}$. Then $\dot{W} \leq 0$.

For incumbents, concavity of $f$ implies that falling $W$ leads to falling $k(W)$. Combined with the fact that $i$ is decreasing when the free entry condition binds from (7), we conclude that

**Proposition 4.** $i$ is decreasing over time.

Innovation in immature submarkets declines over time. We can make a further characterization: if $i$ reaches $\bar{i}$ at some finite date $T$, it does not do so continuously; it jumps down. From (6), $i$ must move discontinuously to keep continuity of $W$ at the boundary between the two regions, since the slope of $W$ jumps.

**Proposition 5.** $\lim_{\epsilon \downarrow 0} i_{T-\epsilon} > \bar{i}$

To understand the discontinuity in $i$, consider the time just before and after free entry condition binds, where $W$ is approximately $c$. Consider at any point in time expected profits over the next $dt$ units of time. These expected dividends are forever strictly declining after free entry stops binding as competition gets more and more fierce. If expected dividends were roughly the same before and after the change, and continuation value went from

\textsuperscript{27}A formal statement of this would follow the same argument as Lemma 4
constant (when free entry binds) to strictly declining (after), then \( W \) would jump down. But \( W \) is continuous; to equate the present discounted value just before and just after free entry stops binding, expected dividends must therefore jump up discontinuously, to offset the fact that they will decline from then on. This upward jump comes through the probability of losing your market leadership: only it can change discontinuously, and so it must decline discontinuously to make the expected dividends jump up, keeping the discounted sum of expected dividends constant.

The discontinuity result, in particular, contrasts with the result for pioneering innovation when the free entry condition starts binding; it can be shown, were one to dispense with Assumption 4, and at some point free entry went from not binding to binding, the evolution of \( i \) is continuous.\(^{28}\) Innovation by entrants ends suddenly, even though the model would have it begin smoothly if such a case were to arise. We take this “crash” of innovation by entrants, then, to be a characteristic of the model of dynamic capabilities and free entry applied to pioneering innovation.

This feature naturally connects the forces of the model to the shakeout: when entry goes down suddenly, there is a strong force toward contraction in firm numbers. In order to show this formally, we develop the dynamics of the model in more detail in the next section.

### 3.5 Innovation and the Rate of Maturity, Death, and Cost of Entry

If submarkets become entrenched quickly, is pioneering innovation encouraged or discouraged? Maturity is tied to dynamic capabilities because the dynamic capabilities degrade when submarkets mature; the tradeoff between staying “current” and reaping profits is directly impacted by maturity and death of immature submarkets.

One might imagine that the answer depends on the relative per-instant profits of the different types of submarkets, measured by \( \alpha \), since that reflects the change in earnings with maturity. However, when entry is occurring in both mature and immature submarkets, this is irrelevant: all that matters are the entry costs. It is direct from equation (7) that:

**Proposition 6.** Suppose \( c < (>)1 \). Then for any \( M, I \) where \( i > \bar{i} \), \( di/d\mu > (<?)0 \).

\(^{28}\): The argument is identical to the argument in the proof of Lemma 7 in the Appendix.
If $c > 1$ then maturity reflects a net loss for the holder of the leadership position since $W > V$. On the other hand, if $c < 1$, maturity leads to higher value. Higher value leads to faster innovation, as more entrants chase the rents of incumbency. When $c > 1$, both the rate of formation of new immature submarkets, and the rate of maturity, increase as $\mu$ increases, so growth of immature submarkets is decreasing in $\mu$. On the other hand, if $c < 1$, faster maturity encourages entry of new immature submarkets; it is even possible that faster maturity leads to an increase in the growth of immature submarkets because the innovation effect outweighs the direct effect of maturity lowering the number of immature submarkets. Since $i$ is independent of $\mu$, all of the change in innovation comes from de novo entrants; the amount of innovation by incumbents is independent of $\mu$.

The higher is $c$, for fixed $\alpha$ and $\pi$, the higher is the value (or stock price) of a firm with one immature submarket leadership position. In other words: when immature submarkets are associated with high price-earnings ratios (high $c$ for a given $\alpha$), faster maturity slows innovation; when mature submarkets generate relatively high price-earnings ratios for firms, faster maturity speeds pioneering innovation as firms seek to develop leadership positions that eventually lead to valuable leadership positions in mature submarkets.\footnote{The rate of maturity has no impact on innovation in mature areas, for a fixed $M$ and $I$.}

For comparison to the impact of $\mu$ on $i$, we can ask whether or not instability in immature submarkets' future, as measured by the death rate $\lambda$, increases or decreases innovation. Here the answer is unambiguous: it is immediate from (7) that higher $\lambda$ implies lower innovation. The reason is that the value of incumbency is lower for any $i$ as $\lambda$ grows.

One can also think of the cost $c$ as the cost for an outsider to acquire the dynamic capability. There is no direct comparative static of innovation rates in $c$; as $c$ goes up, the entrant technology becomes less attractive, which makes the incumbent’s dynamic capability more attractive. Therefore the net effect on innovation depends on the relative importance of incumbent and entrant innovation in the industry.

4 Life Cycle Dynamics

We now characterize the evolution of $M$ and $I$. We accomplish this by studying the derivative of the two state variables $I$ and $M$ as a function of
their current levels. We therefore are especially concerned with the set of points where $\dot{I}$ goes from positive to negative, so that the industry goes from rising immature submarkets to declining. We denote this set of combinations of $I$ and $M$ by $I = g(M)$. This is defined by a level of immature innovation sufficient to offset submarkets that either become mature or fail; if there are more submarkets, innovation is less attractive and therefore insufficient to maintain $I$. Since $\pi$ is decreasing in both arguments, the greater is $M$, the less is $I$ to sustain the same level of innovation. We therefore have:

**Lemma 5.** There exists a decreasing function $g(M)$ such that, if $I > g(M)$, $\dot{I} < 0$, and if $I < g(M)$, $\dot{I} > 0$

Imagine an industry that begins with no submarkets. We assume that at this point there is sufficient pioneering innovation for $I$ to grow to some small positive amount. It is a minimal assumption for the industry to grow from a small number of submarkets.

**Assumption 5.** $g(0) > 0$

The path for $I$ and $M$ can be described below.

**Proposition 7.** Suppose that $I_0$ and $M_0$ are nearly zero. $M$ rises over time to some steady state level, while $I$ rises and then falls.

We analyze the system using a phase diagram. Everywhere above the $M$ axis, $M$ is rising, since $\dot{M} > 0$ if $I > 0$. $I$ is rising for $I < g(M)$, and falling for $I > g(M)$. The fact that $I$ cannot be falling and $M$ must be rising in the region where $I < g(M)$ implies that once the equilibrium path leaves that region, the equilibrium path can never reenter it. For fixed $I$, large $M$ implies the rate of decline in $I$ is maximized, so $I$ cannot remain permanently bounded above zero. The industry therefore follows a path like the one described by the arrows:
This industry begins with increases in both mature and immature submarkets. Eventually the level of pioneering research cannot sustain the level of immature submarkets, and they are maturing or dying faster than they are being created, leading to a decline in immature submarkets. In the long run all submarkets have matured, and we have a stable set of submarkets that can be thought of as the “dominant design” as in Utterback and Abernathy [1975], where the path intersects with the horizontal axis.

Since aggregate industry dynamic capabilities are proportional to $I$, they rise and then fall during the industry’s life. Dynamic capabilities per incumbent submarket, $I/N$, follow the same path as $I/M$; this is a natural measure of dynamic capabilities per firm. The rate of change of $I/M$ is proportional to

$$(\phi - \lambda - \mu) - \mu(I/M)$$
Since the first term is positive if and only if $\dot{I} > 0$, dynamic capabilities per incumbent submarket grow more slowly, and peak sooner, than aggregate dynamic capabilities. The model therefore predicts an early period of increasing dynamic capabilities per firm, and then a decline as the industry reaches maturity. An extension below allows for immature submarkets to continue forever, implying a positive steady state level of dynamic capabilities in the industry. Here the equilibrium model allows not only that dynamic capabilities matter for industry evolution, but also that the evolution drives an equilibrium level of capabilities.

4.1 Evolution of total number of submarkets

The change in the number of submarkets over time is

$$\dot{N} = \dot{M} + \dot{I} = \phi i I - \lambda I$$

The number of submarkets changes over time as new submarkets arrive (at rate $\phi i$) or die before reaching maturity (at rate $\lambda$); maturity itself simply changes a submarket from immature to mature. We first show that, from the point where immature submarkets are maximized to the point where total submarkets are maximized, growth in $N$ is slowing.

Proposition 8. Suppose $\dot{I} < 0$ and $\dot{N} > 0$ Then $\ddot{N} < 0$

This feature implies that submarkets are growing at a declining rate during the period where the number of immature submarkets is falling. On the other hand, the reverse has to be true very early in the industry’s evolution:

Proposition 9. $\lim_{N \to 0} \ddot{N} > 0$

Submarkets are rising from zero until $\dot{N} = 0$. The pattern for $N$ is $S$-shaped: first at an increasing rate, and then at a decreasing rate. Note that this pattern is not a consequence of details of the curvature of the profit function; the only assumption about how $\pi$ changes is Assumption 4; it is generated entirely by the evolution of submarkets via competition and dynamic capabilities.

The previous results pertain to the period where $\dot{N}$ is rising. Indeed that may be true throughout the dynamics. On the other hand total submarkets may decline, since $\dot{N} < 0$, if eventually $i < \lambda/\phi$. Since $i$ is decreasing in $\pi$, and $i = 0$ if $\pi = 0$, it is clear that there always exists a rate of decline in $\pi$ such that $i$ falls to the point where $\dot{N} < 0$. 

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4.2 Entry and Exit

De novo entry of firms into immature submarkets is

$$E_t^I = (i - \bar{i})I$$

There are two forces behind the evolution of entry. On the one hand, in the early part of the life cycle, $I$ is rising, which increases entry. On the other hand, as $i$ falls, the share of pioneering done by entering firms, $(i - \bar{i})/i$, falls. Since entry starts near zero, entry must initially rise to account for the existence of new firms; eventually, $\dot{I} = 0$ and therefore entry falls.

Exit occurs when a firm with a single submarket loses its leadership position. Therefore exit from firms in immature submarkets is

$$X_t^I = (1 - \phi)\omega_t I_t$$

where $\omega_t$ is the fraction of immature submarkets led by a firm with a single leadership position. In general, out of steady state, $\omega_t$ is difficult to characterize. The discontinuity in $i$, however, is guaranteed to generate a point where $X_t^I > 0$. Intuitively, exit is a reflection of accumulated past entry and hence changes continuously over time. In contrast, entry may drop discontinuously or very rapidly. In that case, the number of immature firms in the industry must drop.

**Proposition 10.** Suppose that at some date $t$, $i$ drops discontinuously to $\bar{i}$. Then $X_t^I > 0$.

When $i = \bar{i}$, de novo entry falls to zero. At the same time, there is still displacement of incumbents and therefore therefore $X_t^I > 0$. This implies a shakeout among firms operating in the immature sector, since firm numbers change over time by $-X_t^I$ during this period when entry is zero. If $\phi \bar{i} \geq \mu + \lambda$, this shakeout must occur eventually, since innovation will fall below $\bar{i}$ before $\dot{I} = 0$. On the other hand, if $\phi \bar{i} < \mu + \lambda$, de novo entry may persist forever. Beyond the point where $\dot{I} = 0$, de novo entry in immature areas $E_t^I$ is surely declining since both $i$ and $I$ are falling. When $\phi \bar{i} < \mu + \lambda$, the decline in $i$ may or may not be fast enough to generate a shakeout.

4.3 The Composition of Innovation over the Life Cycle

4.3.1 Innovation during the rise of the industry

Gort and Klepper [1982] document that the rise in firms is met with a rise
in patenting. It must be the case that innovation and firms rise early in the life cycle in our model. Our model, however, allows us to further study the composition of innovation. Total innovation in immature submarkets is \( iI \). Note that the rate of change of this variable is identical to the rate of change of immature entry; they differ by a constant. A constant fraction \( \phi \) of this innovation pioneers new submarkets. As a result, market pioneering peaks before the total number of submarkets; this is consistent with the observation in Klepper [1996a] that “major innovations tend to reach a peak during the growth in the number of producers.” In that paper, major innovations are associated with increasing versions of the product, which is a natural interpretation of the submarkets introduced in our model.\(^{30}\) In Klepper’s model, the return to process innovation changes over time as scale changes. In our model, both the return (through \( \pi \)) and aggregate cost (through the stock of incumbents with experience and a dynamic capability) of both types of innovation can change over time.

Under the interpretation that pioneering innovation corresponds to product innovation, and mature submarkets focus on process innovation, our model is also consistent with Utterback and Abernathy [1975], who stress that product innovation declines as the dominant design emerges. Since the change over time of pioneering innovation is proportional to \( i\dot{I} + i\dot{I} \), this must turn negative before \( \dot{I} = 0 \). Utterback and Abernathy [1975] also document a change from innovations that require original components, to ones that focus on adopted components and products, which fits with the notion of pioneering innovation that we use.

### 4.3.2 Persistently Innovative Industries

Adner and Levinthal [2001] stress that mature products might still be very innovative, including having many product innovations. Our model offers at least two interpretations of this fact that industries are persistently innovative. First, there is no necessity to connect product innovation exclusively to new submarkets; one could imagine new leadership positions in existing submarkets coming from either improved functionality or reduced costs.\(^{31}\) Under the assumption that product innovation is \( \phi i\dot{I} \), the model replicates

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\(^{30}\) Gort and Klepper [1982] also document a shift from major to minor innovations.

\(^{31}\) Indeed, the quality ladder model upon which the model is based can be interpreted either as a model of product or process improvements. The details of that underlying model are described in more detail in the appendix.
the rise and fall of product innovation; under the assumption that product innovation is $\phi i I + m M$, however, product innovation continues indefinitely. Both $m$ and $M$ are strictly positive in the long run. Although $m$ is declining, $M$ is rising; total mature innovation can therefore be either rising or falling. Mature innovation is

$$Mm = M(\pi(M, I) - r)$$

The long run characterization of innovation is determined by the shape of $\pi$, i.e. the impact of competition on profits. Sufficient conditions for mature innovation to be rising in the latter part of the life cycle, where $\dot{I} \leq 0$, is that $M\pi(M, I)$ increases in $M$ at least the rate $r$, i.e. competition between submarkets is not too fierce. That $M\pi(M, I)$ is increasing can be interpreted as increases in $M$ growing the market for mature submarkets sufficiently to offset the lost profits from increased competition. Under these conditions, pioneering innovation is falling in the latter part of the life cycle, while innovation in mature areas remains high.\(^{32}\)

Our model shares the demand-side characterization of innovation in Adner and Levinthal [2001], and similarly allows that innovation can persist in the long run, or decline, depending on the shape of $\pi$. One could imagine that differences in whether mature submarket innovation is product or process would be a natural way to generate different patterns of innovation ranging from the ones stressed by Utterback and Abernathy [1975] to the ones described in Adner and Levinthal [2001]. Moreover, we discuss in the appendix an extension where mature submarkets die and pioneering is perpetual, which would further allow for a channel by which product innovation does not decline in the long run, even if one thinks that product innovation is largely in immature areas.

5 Discussion: The Number of Submarkets and the Shakeout

One goal of the model is to endogenize the arrival of submarkets from birth to steady state. From Propositions 8 and 9, we know that the total number of submarkets is first growing at an increasing rate, and later at a decreasing rate. This is consistent with an S-shape for total submarkets; the S-shape has

\(^{32}\)Additionally, if mature leadership positions confer some dynamic capabilities, these may generate increasing innovation near the steady state.
been highlighted by Tong [2009]. Tong [2009] argues that an S-shape is a good assumption for the evolution of submarkets, and fits it to the experience of the tire industry. Moreover, Tong [2009] uses an exogenous S-shaped increase in submarkets to explain facts about the industry life cycle.

Following the S-shaped rise in total submarkets, total submarkets may decline; it is certain that immature submarkets decline. Declining submarket numbers near the steady state is interesting because it is related, intuitively, to the model’s ability to generate a shakeout in firm numbers. The steady state of the model mimics Klette and Kortum [2004]. In that model, firm numbers are proportional to the (exogenous) number of submarkets that exist. If submarkets are declining near the steady state, therefore, it seems natural that the model would generate a shakeout.

Proposition 10 shows that the model may deliver fast enough decline in immature submarkets to generate a shakeout in that sector. This shakeout can apply to the firm numbers as a whole, however. For instance, suppose that the ratio of immature to mature submarkets is very high. This occurs if the maturation rate is very low relative to the death rate for immature submarkets; it takes a large stock of immature submarkets to generate a few successful, mature submarkets. In that case the shakeout, led by a fall in firms operating in the immature sector, will apply to firm numbers as a whole. This line of argument naturally mirrors the notion in Klepper [1997] and Klepper and Simons [2005] that recent entrants are most susceptible to the shakeout; if that is true, then the shakeout is most likely to occur when the industry has a large number of immature submarkets relative to mature ones, and therefore a relatively large number of young firms.

A sudden “crash” in pioneering innovation guarantees the drop is fast enough, but the fall could be sufficiently fast elsewhere. From (7) it is clear that a decrease in $i$ can come from one of two sources: the demand side impact of falling profits $\pi(M,I)$ or the fall in the return to the dynamic capability. The story of the shakeout in immature firms is that rising competition eventually forces entrants without some competitive advantage out of the industry, lowering entry below exit.\(^{33}\)

The driving force in the model is changes in profits over time as com-

\(^{33}\)Note that, although the drop is to zero de novo entry, the model could allow for another stream of de novo entrants (perhaps a limited number with access to a favorable technology) such that entry was positive before and after the discontinuity. The key is that, at some point, a group of potential entrants goes from making zero profits (i.e. free entry for that group holds) to being unprofitable.
petition increases. The model does not necessarily have a prediction about aggregate (or average) profitability over time, though; even though $\pi(M, I)$ is decreasing, the composition of immature and mature submarkets is evolving. For instance, an interesting feature of the model is that, at the peak of firm numbers where the shakeout begins, total industry profits can be rising, and even average profits per submarket, despite the shakeout being caused by falling profits per submarket of a given type. This rise in profits with contraction in firm numbers might appear to be an “industry consolidation,” in the sense that fewer firms are generating more profits, but here it is not coming as a result of increased concentration at the firm level, as everything is constant returns and perfectly competitive. The profit effect comes because the composition of submarkets is changing toward mature submarkets.

Whether profits can be rising or falling depends on whether mature submarkets are more or less profitable than immature ones. The industry profit rate per submarket is

$$\frac{(M\pi + \alpha I\pi)}{(M + I)}$$

This is either increasing or decreasing in $M/I$ depending on whether $\alpha$ is smaller than or greater than one. Therefore when $\alpha < 1$, the loss in profits over time through $\dot{\pi}$ is offset if $M/I$ is rising. For $\dot{I}$ negative or positive but low, $M/I$ rises. When $\alpha < 1$, then, the shakeout can look like a consolidation, in terms of profitability, when in fact it simply coincides with the contraction of the less profitable immature sector.

6 Conclusions

Dynamic capabilities offer incumbents an innovative advantage in related areas, especially recent ones. On the other hand, the return to this capability naturally attracts entrants seeking the return for themselves. We introduce a model where both forces are present. We show that, as a result of these two forces, dynamic capabilities may impact the rate of innovation of both incumbents and entrants. Where the impact is felt depends on whether the capability generates a benefit at the margin for incumbents.

Dynamic capabilities can be also be an important driver of the industry life cycle. In the model introduced here, industries evolve as the set of submarkets changes over time. Those submarkets start out immature, but some survive to maturity. Consistent with empirical evidence, we model incumbency as generating an advantage at innovation in relatively recent areas. We
show how those capabilities evolve over the life cycle, as well as how the degree of capabilities impacts innovation rates. The model generates industry life cycle dynamics that are consistent with a variety of empirical regularities, including the shakeout of firms as the industry approaches maturity, and the evolution of innovation over the industry life cycle. The model demonstrates the central role that dynamic capabilities can have in the evolution of industry.

Our model takes, as its base, the model of innovation by incumbents contained in Klette and Kortum [2004]. We modify their model to take account of the fundamental feature of dynamic capabilities: that the advantage they confer applies to related, recent product areas. We show how such a model can be used not only to make steady state predictions of the sort highlighted in Klette and Kortum [2004], but also to study the non-stationary evolution from an industry’s birth. The model shares the desirable steady state features of Klette and Kortum [2004], while expanding it to include endogenous submarket dynamics of the sort used by Klepper and Thompson [2006] and Sutton [1998] to explain firm dynamics. The paper therefore contributes to our understanding of the role that dynamics play in the the industry life cycle, as well as the long run industry equilibrium.

An application of the model would be to use its implications to derive whether or not the dynamic capabilities in an industry are marginal or average capabilities. Doing so would provide insights into the nature of competitive advantage. The two possibilities have very different implications for the fate of incumbent firms, since marginal dynamic capabilities imply that able incumbents sow the seeds of continued leadership, whereas average capabilities allow incumbents to reap returns that attract outside innovation. In that case, the dynamic capabilities of incumbents are exactly the carrot that leads to new firms unseating incumbents and generating their own dynamic capabilities.

Appendix A: Extensions

Death of Mature Submarkets and Perpetual Market Pioneering

Our model is compatible with permanent pioneering if mature submarkets periodically die. Let mature submarkets be eliminated at rate $\delta$. This alters
the value of a mature submarket slightly:

\[ rV = \pi(M, I) - (m + \delta)V \]

The more substantive change comes about because of how it impacts the time derivative of \( M \):

\[ \dot{M} = \mu I - \delta M \]

Instead of \( M \) rising for any \( I > 0 \), now \( \dot{M} = 0 \) when \( I/M = \delta/\mu \). Below that line, \( M \) falls. The steady state, rather than having no immature submarkets, has \( M \) where both \( \dot{M} = 0 \) and \( \dot{I} = 0 \); since the latter is defined by \( g(M) \), this intersection occurs when \( M \) solves

\[ g(M)/M = \delta/\mu \]

and \( I = g(M) > 0 \). Since \( g(M) \) does not depend on \( \delta \), and \( g(M)/M \) is decreasing, the steady state number of mature submarkets is decreasing in \( \delta \).

There is perpetual market pioneering in the steady state, in order to offset the death of mature submarkets. The steady state is on the \( g(M) \) function (where it intersects \( \dot{M} = 0 \)) rather than on the \( M \) axis. Since \( I = M\delta/\mu \) and \( \dot{I} = 0 \) when \( i = (\mu + \lambda)/\phi \), we can compute the steady state mature submarkets from an analogous equation to (7):

\[ \frac{1}{(1 - \phi)c} (\alpha \pi(M, M\delta/\mu) - \mu(c - 1) - \lambda c) = (\mu + \lambda)/\phi \]

All of the earlier characterization of the shakeout near the point where de novo entry into immature submarkets crashes continues to be true. At the steady state \( i = (\mu + \lambda)/\phi \). If this is smaller than \( \tilde{i} \), it is certain that there is a shakeout; even without it, entry is declining near the steady state, since \( \dot{I} = 0 \) there, which can generate a shakeout even if \( i > \tilde{i} \).

Another implication of the submarket-death case is that it fits naturally with the idea that firms which maintain leads in frontier (immature) submarkets can persist, but ones that fall into exclusively mature submarkets face the grim prospects of having no advantage except in mature submarkets which are dying out. As a result such firms are unlikely to survive, whereas a firm with leadership positions in immature markets can continue to leverage that position into entry advantages in other immature submarkets.
More than one profiting firm per submarket

Suppose that both the leader and second-leader (i.e., the most recently displaced leader) made profits in each submarket. We then have four values to define, for leaders and followers (which we denote 1 and 2, for first and second) for each type of submarket. Denoting profits of the firms by $\pi^1$ and $\pi^2$ for leaders and followers:

\[
\begin{align*}
    rV^1 &= \pi^1(M, I) - m(V^1 - V^2) \\
    rV^2 &= \pi^2(M, I) - mV^2 \\
    rW^1 &= \alpha\pi^1(M, I) - i(1 - \phi)(W^1 - W^2) - \mu(W^1 - V^1) - \lambda W^1 + Wf(k_I(W)) - k_I(W) + \dot{W}^1 \\
    rW^2 &= \alpha\pi^2(M, I) - i(1 - \phi)W^2 - \mu(W^2 - V^2) - \lambda W^2 + Wf(k_I(W)) - k_I(W) + \dot{W}^2
\end{align*}
\]

For leader firms, arrival of an innovation in their submarket knocks them down to followers; followers are eliminated. Maturation maintains the firms rank. Here we impose, as above, that de novo entry is profitable for mature industries, although that is not necessary. Moreover we could allow the dynamic capability to differ for leaders and laggards, by making $f$ differ; here both firms maintain the capability and the value that goes with it. None of this changes the basic mechanisms of the model. The free entry conditions are

\[
\begin{align*}
    V^1(M, I) &\leq 1, \text{ with equality unless } m = 0 \\
    W^1(M, I) &\leq c, \text{ with equality unless } i < \bar{i}
\end{align*}
\]

None of the qualitative features of the model are changed; the number of equations describing the equilibrium simply rise. One could extend this analogously to 3 or more profit making firms per submarket.

Consumer Preferences and Explicit Bertrand Competition within Submarkets

In this section we show how a model of consumers preferences delivers the structure for profits we study above. Suppose that, at each instant, there is a representative consumer with utility function over consumption bundles $a$ across submarkets by

\[
\int_0^M \ln(a_j) dj + \gamma \int_0^I \ln(a_l) dl
\]

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subject to a fixed budget, normalized to one, to spend on the products:\(^{34}\)

\[ \int_0^M p_j a_j dj + \int_0^I p_l a_l dl = 1 \]

From the first order conditions for the two types of products we have that

\[ p_j a_j = \gamma p_l a_l \]

Revenue for a mature submarket is

\[ R(M, I) = 1/(M + \gamma I) \]

and \( \gamma R(M, I) \) for an immature industry.

Price, however, is determined by competition between quality levels, as in Grossman and Helpman [1991], Aghion and Howitt [1992]. In a given moment of time in submarket \( j \) there is a set of firms \( J_j \). Firm \( n \in J_j \) can produce the good at a constant marginal cost \( b_j^n \leq 1 \) per unit of quality. This allows innovations to be alternatively viewed as product innovations that raise units of quality per unit of cost, or process innovations that simply reduce cost. Firms within a submarket are ordered in a decreasing order of costs. For a given submarket \( i \), the representative consumer consumes \( a_j^n \) units of products from firm \( j \). This leads to \( d_i \) units from the submarket, where

\[ a_i = \sum_j a_i^j \]

In equilibrium consumers will all consume the lowest cost product, denoted simply \( b_j \), for each submarket. We assume that innovation reduces costs per quality unit by a factor \( \beta > 1 \). That is, if in a given submarket the lowest cost firm \( j \) has a cost \( b_j^1 \), if a new improvement is developed, it results in costs \( b_j^{i+1} = b_j^i / \beta \). The first firm to operate in a submarket has cost \( b_1^1 = 1/\beta \). For simplicity we assume that, in each submarket, if only one product has been invented, the consumers have an outside option that is provided competitively at marginal cost 1. One can interpret this as the next best alternative product that might substitute for submarket \( i \).\(^{35}\)

\(^{34}\)One interpretation of the fixed budget is that the consumer has Cobb-Douglas preferences over this industry and an outside good, and therefore has constant spending on the industry’s products.

\(^{35}\)Alternatively, the first entrant would set price equal to the unconstrained monopoly price. This would force us to have three types of firms: mature firms, immature firms, and first-movers; nothing substantive about the model or its predictions would change.
Non-lowest-cost firms price at marginal cost; to match this price, the lowest cost producer charges \( p_j^1 = 1/\beta^{j-1} \) for \( j > 1 \) and \( p_1^1 = 1 \). Profits for a mature industry are therefore

\[
R(M, I)(1 - w/p) = R(M, I)(1 - 1/\beta)
\]

Note that, if one wants to have immature industries have higher profits, despite having the industry more competitive as firms mature, one can make immature firms have a greater \( \beta \) to overcome \( \gamma < 1 \). In the language of section 2 where immature firms earned \( \alpha \) times what mature firms earn,

\[
\alpha = \gamma \frac{(1 - \beta_i)\beta_m}{\beta_i(1 - \beta_m)}
\]

Note that as long as \( \gamma < 1 \), Assumption 4 can be met for suitably chosen \( \mu \) and \( \lambda \); meanwhile \( \alpha \) can be either bigger or smaller than one.

**Appendix B: Proofs**

**Proof of Lemma 1**

*Proof.* Suppose \( V_t < 1 \). By the free entry condition it must be the case that \( m_t = 0 \). If the entry begins again in \( T \) periods, at which point the return must be 1, then the payoff is

\[
V_t \geq \int_0^T e^{-rt} \pi(M_t, I_t)dt + e^{-rT}
\]

\[
\geq \int_0^T e^{-rt} \min_{0 < t < T} \pi(M_t, I_t)dt + e^{-rT}
\]

\[
> 1
\]

where the first last inequality is by Assumption 2. Therefore the contradiction implies \( V_t = 1 \). \( \square \)

**Proof of Proposition 2**

*Proof.* For any value of \( k \), \( c\hat{f}(k) - k > cf(k) - k \). Therefore it must the maximized \( \max_k c\hat{f}(k) - k > \max_k cf(k) - k \). But then, directly from (7), \( i \) must rise to keep \( W = 1 \). \( \square \)
Proof of Lemma 2

Proof. Since

\[ \frac{d\pi}{dt} = \frac{d\pi(M, I)}{dM} \mu I + \frac{d\pi(M, I)}{dI} (i\phi - \mu - \lambda)I \]

And both derivatives of \( \pi \) are negative, it is sufficient that \( \frac{d\pi(M, I)}{dM} \mu < \frac{d\pi(M, I)}{dI} (\mu + \lambda) \).

\[ \square \]

Proof of Lemma 3

Proof. If \( I > 0 \), the result is immediate from Lemmas 7 (in the appendix) and 2; if \( I = 0 \), \( M \) and \( I \) are constant and therefore the industry remains at \( i < \bar{i} \) forever.

\[ \square \]

Proof of Lemma 4

Proof. In (8), continuity requires \( \dot{W} \) be continuous since, when there is no de novo entry, continuity of \( W \) implies continuity of \( i \). If \( \dot{W} > 0 \), it must eventually flatten out; at that point \( \dot{W} \leq 0 \) but then by (9) \( \dot{W} < 0 \), a contradiction

\[ \square \]

Proof of Proposition 4

To show this, we first show the kink in \( W \): at any such date where entry ends, \( W \) is strictly decreasing, and the the change in the slope of \( W \) is discontinuous. Denote that date by \( T \).

Lemma 6. \( \lim_{\epsilon \to 0} \dot{W}_{T-\epsilon} < 0 \) whenever \( I > 0 \)

Proof. If \( \lim_{t \to T} \dot{W} = 0 \), then \( W \) is differentiable at \( T \) and \( T \) is a local maximum of \( W \), with \( \dot{W}_T = 0 \) and \( \ddot{W}_T \leq 0 \). But then, from (9), since \( \dot{\pi}_T < 0 \), \( \dot{W}_T < 0 \), a contradiction.

Value \( W \) is continuous at \( T \), since it is an integral of future expected profits, and therefore cannot change suddenly. A kink in \( W \) requires a discontinuity in \( i \), therefore, to offset the sudden change in \( W \), and therefore proposition 4 is immediate from continuity of \( W \) and (6)

\[ \square \]

Lemma 7. Suppose the free entry condition begins binding at \( T \). Then \( i \) is continuous at \( T \) and \( \dot{\pi} \geq 0 \) for \( t \) approaching \( T \)
Proof. Note that \( \dot{W} \geq 0 \) for \( t \) near \( T \) (since \( W \) is approaching its upper bound, \( c \)), and \( i \) is rising. \( W \) is clearly continuous, and \( W \) in (6) can only be varying continuously at \( T \) if \( \lim_{t \to T} \dot{W} = 0 \) and \( i \) varies continuously at \( T \). Differentiating (8):

\[
\dot{W} = \frac{1}{r + \mu + \lambda - \phi \bar{i} - k'(W)(\phi W f'_I(k) - 1)}(\alpha \dot{\pi} + \ddot{W})
\]  

(9)

Note that, since \( W f' - 1 = 0 \) by the first order condition for the choice of \( k \), the last term \( \phi W f' - 1 < 0 \), and therefore the denominator is positive since \( k \) is increasing in \( W \). Since \( \dot{W} \geq 0 \) near \( T \) and \( \lim_{t \to T} \dot{W} = 0 \), \( \dot{W} \leq 0 \) as \( T \) is a local maximum of \( W \), and therefore \( \dot{\pi} \geq 0 \) to make this expression positive.

\[\square\]

Proof of Proposition 5

Proof. First, suppose that parameters are such that \( \phi \bar{i} > \mu + \lambda \). In this case, if there were de novo entry, i.e. \( i > \bar{i} \), new submarket generation would be \( \phi i > \phi \bar{i} > \mu + \lambda \), so the number of immature submarkets would be rising, \( \dot{I} > 0 \). As a result, \( g(M) \) must occur where de novo entry is exactly zero and \( W < c \) so that existing firms generate less than \( \phi \bar{i} \) in new submarkets. In that case, \( g(M) \) is defined by the set of points where \( W(M, I) \) reaches a point where

\[\phi f(k(W(M, I))) = \mu + \lambda\]

so that innovation by incumbents is \( i = (\mu + \lambda)/\phi \). Since \( W \) is decreasing in this region in \( M \) and \( I \), \( g(M) \) is decreasing.

On the other hand, if \( \phi \bar{i} < \mu + \lambda \), the characterization of \( g \) is more complicated. Either \( \dot{I} = 0 \) when de novo entrants are generating new submarkets, or \( I \) goes from increasing to decreasing at precisely the point where entry crashes. In the former case, define \( g_1(M) \) to be the set of points where (7) implies \( i = (\mu + \lambda)/\phi \). Since \( \pi \) is decreasing in both arguments, \( g_1 \) is decreasing. From the prior section, \( g_0 \) is the set of points where de novo entry crashes. Clearly if de novo entry crashes when \( \dot{I} > 0 \), this describes the points where \( \dot{I} \) changes signs; therefore define

\[g(M) = \min\{g_0(M), g_1(M)\}\]

Since both \( g_0 \) and \( g_1 \) are decreasing, \( g \) is decreasing. In both cases, \( I \) is decreasing if \( I > g(M) \), and increasing if \( I < g(M) \). \[\square\]
Proof of Proposition 8

Proof. Computing $\dot{N}$

$$\dot{N} = \phi i I + (\phi i - \lambda) \dot{I} \tag{10}$$

The first term is negative since $i$ is falling by Proposition 2; the second term is negative as the product of a positive $(\phi i - \lambda)$ and negative $(\dot{I})$ terms. \hfill $\Box$

Proof of Proposition 9

Proof. Note that $\dot{N} = \phi i I - \lambda I$, so, since profits and therefore $i$ is bounded, $\lim_{N \to 0} \dot{N} = 0$. For $N$ small, therefore, $\phi i I$ must be rising faster than $\lambda I$, or $N$ would become negative. This implies that $\ddot{N} > 0$. \hfill $\Box$

Proof of Proposition 10

Proof. Since entry is strictly positive for some interval before the drop in $i$, there must be a positive fraction of firms from that set who still have only one leadership position, i.e. $\omega > 0$. Therefore $X^I_t > 0$. \hfill $\Box$

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