Marketing Science

Publication details, including instructions for authors and subscription information:
http://pubsonline.informs.org

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MARKETING SCIENCE
Vol. 33, No. 6, November–December 2014, pp. 849–870
ISSN 0732-2399 (print) | ISSN 1526-548X (online)

http://dx.doi.org/10.1287/mksc.2014.0869

Organizational Structure and Gray Markets

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Conventional wisdom suggests that when firms face a negative externality like gray marketing (i.e., the selling of branded goods outside of the manufacturer's authorized channels), an effective strategy to reduce the negative impact is to centralize decision making. Nevertheless, in industries with significant gray marketing, we observe many firms with decentralized decision making. Our study assesses whether decentralized decision making can be optimal when a manufacturer faces gray market distribution. We consider a market where a focal firm competes with an existing competitor that produces a differentiated product and a gray marketer that sources an identical product from a lower-priced foreign market. We find that decentralization is optimal under quantity-based competition, provided the gray market is relatively uncompetitive and the level of competitive intensity between the focal firm and the competitor is high. Decentralization leads a firm to make aggressive production decisions, which leads to lower prices, yet it also leads to higher market share for the firm compared to centralization. When the level of competitive intensity between a firm and its competitor is high, the gain in market share more than offsets the loss due to lower prices. As a result, the focal firm is better off decentralizing its operations independent of (a) whether the competitor operates in the foreign market, and (b) the competitor's organizational structure. This finding contradicts the belief that centralized decision making is always optimal when authorized manufacturers attempt to limit the negative impact of gray markets. The findings also provide insight to understand why firms might employ decentralized decision making in industries where gray markets are active.

Keywords: gray markets; diversion; foreign market entry; decision rights
History: Received: February 4, 2013; accepted: June 2, 2014; Preyas Desai served as the editor-in-chief and John Zhang served as associate editor for this article. Published online in *Articles in Advance* September 8, 2014.

1. Introduction

Gray marketing is the selling of genuine branded goods by third parties ("gray marketers") that operate outside of manufacturer-authorized channels. When manufacturers sell their branded goods at different prices in different markets or channels, gray marketers often import these goods from a lower-priced market and resell them in a higher-priced market such as the United States *to compete with* the manufacturer's authorized channel.[1] Gray marketing affects a wide range of categories: common examples include clothing, books, electronics, automobiles, and luxury goods. Although the importance of gray marketing varies by category, its impact is large. A 2009 analysis by Deloitte LLP estimates lost U.S. sales of up to $63 billion (4.5% of sales) per year in the consumer products sector (Wolf 2009), and a 2008 study by KPMG estimates lost sales of $58 billion per year (8% of global sales) in the information technology (IT) sector (KPMG 2008).

From a legal perspective, there is little manufacturers can do to stop gray marketing. High court rulings in the United States and Europe have upheld the legality of gray market sales as being in the best interest of consumers. For example, in a recent case, the U.S. Supreme Court ruled six-three in favor of a Thai citizen (Kirstaeng) who had legally purchased several hundred thousand dollars' worth of textbooks published by John Wiley & Sons at bookstores in Thailand, and then resold them in the United States in competition with authorized distributors (*Kirstaeng v. John Wiley & Sons* 2013). Given firms' inability to impede gray marketing through legal means, manufacturers have turned to other strategies to mitigate the gray market's impact. A popular strategy has been for manufacturers to centralize authority over their subsidiaries.[2] This

[1] Although we frame this paper as gray market diversion from a foreign to a domestic market, the model is not restricted to geographically separate markets. It also applies to a situation where a branded manufacturer attempts to price discriminate across different markets or channels and leakage across the markets or channels takes place.

[2] Other popular strategies include restricting after sales service (such as warranties) to products bought through authorized distribution

strategy is motivated by the conventional wisdom that centralized decision making is an effective strategy to manage a negative externality like gray markets (Varian 1992).

Assmus and Wiese (1995), in a study assessing gray market deterrents, observe that a multinational can implement centralized control over its subsidiaries through a variety of measures including: having the head office make all volume and retail pricing decisions, mandating a company-wide, uniform set of policies, and indirectly controlling retail prices by setting higher transfer prices to subsidiaries in geographic regions that are sources for gray market goods. Moreover, empirical studies find evidence of firms implementing centralized decision making to reduce gray market volumes. For example, Myers (1999), Myers and Griffith (1999), Michael (1998), and Doyle (1997) confirm (a) the effectiveness of centralized decision making as a way to reduce gray market volume, and (b) that gray market volume can be minimized either by restricting the autonomy of employees in subsidiaries or by implementing the "utmost cooperation" between the domestic and international division. Finally, anecdotal evidence also supports the use of centralization to minimize gray market distribution. For example, Yeung and Mok (2013) find that the implementation of centralized control over a foreign provider of automobiles was an effective deterrent to the parallel importation of automobiles.

Our study challenges this conventional wisdom. Specifically, our objective is to assess whether *decentralizing* authority can be an optimal strategy when firms face gray markets.[3] Our research question is motivated by the observation that firms often implement decentralized decision making in industries where gray markets are active.[4] For example, we infer that gray markets have a significant impact on John Wiley & Sons' profitability, given Wiley's attempts to stop the gray market through legal means. Nevertheless, Wiley "often assigns to its wholly owned foreign subsidiary (Wiley Asia) the rights to publish, print, and sell foreign editions of Wiley's English language textbooks abroad" (*Kirstaeng v. John Wiley & Sons* 2013, p. 1). Thus, despite the gray market threat, Wiley has implemented decentralized control by allocating decision rights to its foreign subsidiary.

To address the research question, we employ a model of differentiated competition. The model consists of two firms, each of which sells a differentiated product, that compete in both a domestic market and a lower-priced foreign market. If a firm enters the foreign market, we assume that the gray market diverts foreign product back to the domestic market to capitalize on the gap in retail prices between the two markets. In this setting, our analysis finds that both firms can be more profitable by decentralizing decision making. This counterintuitive result hinges on two observations that relate to markets where gray markets are common.

The first observation is that the nature of competition in categories where gray markets thrive varies substantially. For example, in some markets, firms compete fiercely on price. In these markets, it is relatively easy to adjust the volume of product available if price decreases lead to a significant increase in demand. Categories that fit this description include most digital products (software and music) and even pharmaceuticals where the cost of underage is much higher than the cost of overage (Arrow et al. 1951).[5] In other categories, competition is constrained by the amount of stock that is produced. In these markets, products typically have long production lead times or capacity constraints that make it difficult to adjust production quantities after forecasts have been confirmed. Categories that fit this description include fashion clothing, shoes, and even books.[6]

The second observation concerns an inference regarding the price differential between lower and higher priced markets and what this differential implies about the market power of the gray market institution. Specifically, gray marketing occurs consistently in many categories and the price differential between markets persists, despite significant volumes of gray market product being diverted into the higher priced market.[7]

channels, and differentiating products across markets such that the product produced for foreign consumers is different from the product authorized for sale in the domestic market. These strategies have their own shortcomings, however. For example, in the case of after sales services, large gray marketers now provide their own warranties for products when authorized distributors refuse to provide them, diminishing the effectiveness of after sales service as a gray market deterrent. In the case of differentiating products, there are both explicit costs (e.g., higher manufacturing and advertising costs) and implicit costs (e.g., increasingly homogenous global consumer preferences) that can make differentiation unattractive.

[3] Firms might decentralize authority for reasons unrelated to gray marketing (e.g., to make better use of the superior local information of regional managers). We exclude these tensions from our model to ensure that our results are driven by the gray market factor. We acknowledge this potential limitation in §6.

[4] Evidence in Robinson and Stocken (2013) suggests that multinationals have become increasingly decentralized over time. Moreover, this shift is quite strong in industries characterized by pervasive gray markets. For example, 77% of subsidiaries in the wholesale trade industry operate with decentralized authority.

[5] The prevalence of gray markets in these categories is discussed in McDougall (2007) and Kanavos and Costa-Font (2005).

[6] Gray market activity in these categories is discussed in *Kirstaeng v. John Wiley & Sons, Inc.* (2013) and NERA (1999).

[7] Admittedly, there are categories where gray marketing has visibly narrowed the gap between the low and high priced country (for example, tobacco and pharmaceutical products). Discussion of how the gray market has affected the price of cigarettes can be found in Luk et al. (2007) and Gilbert (2011).

We argue that this differential persists if and only if the gray market has a degree of market power. In our robustness tests, we illustrate that, as the gray market becomes perfectly competitive (i.e., the number of gray marketers approaches infinity), the potential volume of product that might be transferred from the low-price jurisdiction to the high-price jurisdiction is so high that it leads both firms to implement uniform pricing across markets. In turn, uniform pricing precludes arbitrage between the markets. It follows that, when we observe an industry with both active gray markets and a disparity in retail prices across markets, at least some degree of market power lies within the gray market. Said differently, the player(s) in the gray market ship a volume of product to the higher-priced jurisdiction that does not completely eliminate the price differential across markets.

To incorporate our first observation regarding the heterogeneous nature of competition across categories, we model an economy where the underlying consumer preferences are constant but where the nature of competition varies. First, we examine a market based on Cournot competition to reflect markets where competition is constrained by the quantity of product that is available (see Kreps and Scheinkman 1983). Second, we examine a market based on Bertrand competition to reflect markets where competition is unfettered by either capacity or quantity considerations.

To incorporate our second observation, we vary the degree of market power that exists within the gray market. To reflect a situation where the gray market is relatively powerful, we first model an economy with a single gray marketer. To modulate the market power of the gray market institution, we then increase the number of gray marketers.

Our main finding is that when a manufacturer competes in a market characterized by Cournot competition and the gray market institution is relatively concentrated, decentralization can be more profitable than centralization. This result is obtained when the level of competitive intensity between firms is high (e.g., the products of the competing manufacturers are good substitutes for each other). Most interesting about this finding is that decentralization can be a focal firm's optimal organizational structure choice independent of whether only one firm or both firms are active in the foreign market. This finding also arises independent of the competing firm's decision-making structure.

A second important finding of our analysis is that a manufacturer does not always optimize its profitability by minimizing gray market volume. This result is important, as the prevailing assumption in the literature regarding the benefits of centralized decision making is that a firm is better off when gray market volume is minimized.[8] Our analysis demonstrates that this prevailing wisdom is incorrect. There are cases where (a) decentralized decision making is optimal yet gray market volumes are lower under centralization, and (b) centralized decision making is optimal yet gray market volumes are lower under decentralization.[9] Our analysis underlines the importance of focusing on the *profit* implications of whether or not decision making should be centralized, as using the simple heuristic of minimizing gray market volume can reduce a firm's profits.

Our analysis also demonstrates that centralized decision making is strictly preferred in markets characterized by price-based competition. Here, centralization is dominant because, in contrast to quantity-based competition where the decisions of firms are strategic substitutes, the decisions of the players under price-based competition are strategic complements. The main objective of firms is to use a "strategic decision" such as organizational structure to reduce competition. As decentralization leads to more aggressive pricing, it also exacerbates competition such that any increase in market share is not sufficient to counter the lost margin due to lower pricing.

We also investigate how our findings are affected by the degree to which the gray market institution is competitive. We find that the optimality of decentralization in a Cournot market continues to persist, provided there are comparatively few gray marketers. When there are many gray marketers however, any price differential between the foreign and domestic markets leads to a flood of gray market product in the domestic market. Here, the benefit of having more aggressive country managers is completely outweighed by the negative impact of flooding the domestic market. In this situation, the simple heuristic of "minimizing gray market volume" is optimal and this outcome is best achieved through centralized decision making.

The paper proceeds as follows. We review the literature in §2. Following that, we present our model in §3

[8] For example, Assmus and Wiese (1995, p. 34) suggest that the multinational's objective should be to "proactively prevent or at least restrict gray market activities before they occur"; Myers and Griffith (1999, p. 6) state that the effective management of distribution channels relies on a company's ability "to restrict unauthorized 'leakage' from its supply chain operations"; Myers (1999, p. 111) posits that "managerial decision makers responsible for multiple markets and understanding the underlying cost structures of all export ventures ... will be more likely to dedicate the necessary efforts to combat or prevent gray market distribution"; and Gallini and Hollis (1999) highlight that a firm has to prevent or limit gray marketing.

[9] Decentralization leads to a more aggressive subsidiary in the lower priced market but it also results in a more aggressive subsidiary in the higher priced market. For this reason, gray market volumes under decentralization can be higher or lower than those observed under centralization.

and our results in §4. Finally, we assess two model extensions in §5 and conclude in §6.

2. Literature Review

A standard explanation for multichannel marketing is the proliferation of both customer segments and channels (Kotler and Keller 2009). In fact, the driving force behind most new channels is the opportunity to serve new customers profitably (such as consumers in a foreign country). Of course, there are costs to adding channels beyond the cost of simply managing and dealing with another customer. As noted by Coughlan et al. (2006), these costs include "conflict" that may occur when these channels compete for the same customers. This is precisely the situation when gray market firms divert product from a lower priced foreign market back to the domestic market.

Academics have analyzed the topic of gray markets to better understand their overall effect on industry. Antia et al. (2006), Assmus and Wiese (1995), Weigand (1991), Cespedes et al. (1988), Li and Robles (2007), and Cavusgil and Sikora (1988) take the position that gray markets are a problem for manufacturers for reasons that include losing control of distribution, a decreased ability to price discriminate, the erosion of brand equity, and the stifling of multinationals' incentives to invest in research and development. Although firms often suffer as a result of gray marketing, Maskus and Chen (2004) and Autrey and Bova (2012) show that global surplus is often increased by gray market activity. As noted in the introduction, this may explain why courts are reluctant to rule against firms that facilitate the sale of gray market goods. In any event, diversion creates a complexity that firms need to manage.[10]

Not surprisingly, significant research is dedicated to analyzing the alternatives that manufacturers have to limit the impact and magnitude of gray markets. There are also scores of articles in the business press that highlight the negative impact of gray markets and provide guidance on how gray marketing can be minimized.[11] As noted in the literature cited above, a popular strategy to reduce the negative effect of gray markets is for a manufacturer to centralize its decision making. After all, the presence of gray market goods invariably requires a source and this source typically does not act in the best interest of the manufacturer. In many (if not all) cases, the source of gray market goods is a foreign subsidiary (or a third party) that has the rights to market the product in another territory. Our objective is to challenge the conventional wisdom that centralization is universally effective to reduce the negative impact of gray markets.

A key contribution of our analysis is to examine this question in an environment where two manufacturers compete with each other. An important limitation of many prior gray market studies is that they do not consider the impact of competition on optimal strategies. Specifically, the standard approach is to examine the challenge of a monopolist that loses the ability to price discriminate as a result of gray markets. By including an existing competitor in the market, we can assess both the direct and the indirect effects of the gray market on a focal manufacturer's profits. The direct effect of the gray market, assessed by our model and most extant models, is the cannibalization of demand for the focal firm's product in the domestic market. The indirect effect of the gray market, highlighted in our analysis, is its impact on the strategic behavior of the existing competitor. In the next section, we present the model.

3. Model Setup

We consider a setting where two risk-neutral firms compete in a domestic market with differentiated products. Each firm may also sell its product in a foreign country (where competitive prices are lower) through a wholly owned foreign subsidiary.[12] If either firm enters the foreign market, the gray market might divert product from the foreign market to the domestic market.[13] Although sales to the gray marketer generate incremental profit in the foreign market, the negative effect of diversion needs to be accounted for.

For simplicity, we assume that the product sold in the foreign market by each firm is identical to the product in the domestic market. We also assume that (a) the product's marginal cost is constant and zero (without loss of generality), (b) profits in the domestic and foreign market are of equal value, and (c) the cost of shipping gray marketed product back to the

[10] Interestingly, a few studies identify situations in which gray markets do not generate a negative externality for manufacturers. Bucklin (1993) suggests that price erosion in the home market is frequently offset by an increase in unit sales, and Ahmadi and Yang (2000) suggest that gray markets might extend the firm's global reach and improve global profits. Raff and Schmitt (2007) demonstrate that letting retailers trade unsold inventories to the gray market increases retailer orders given demand uncertainty and can lead to higher manufacturer profit. Chen (2009) shows conditions under which gray markets may help a firm segment its home market via the service level selected by authorized retailers.

[11] For a typical example, see Dove and Hamilton (2008).

[12] For simplicity, we examine a model with a domestic market with higher price levels and one foreign market with lower price levels. However, the model applies to the diversion of product from any number of lower priced markets back to a higher priced domestic market.

[13] We assume that domestic consumers are not capable of purchasing goods in the foreign market. Relative to consumers, the gray marketer has specialized knowledge about where to find appropriate goods and has expertise in transhipping. Gray markets exist because the per-unit transaction costs of domestic consumers to acquire foreign goods are significantly higher than those of the gray marketer.

domestic market is negligible. The model thus focuses on a situation where gray market diversion is as high as possible in order to understand how it can be managed effectively.

In addition to the foreign market entry decision, a firm that decides to enter also makes a decision about organizational structure. The model considers two organizational options, decentralized and centralized control. Decentralized control means that a firm gives the domestic and foreign subsidiaries profit responsibility and control over production (or pricing). With decentralization, the objective of each subsidiary is to optimize local profits.[14] Centralized control means that decisions are made at the head office and thus production quantities (or prices) are set in each market in order to maximize the global profits of the firm. Thus, in the first stage of the game, each firm can choose no entry (N), decentralized entry (D), or centralized entry (C).

For the two competitive contexts we model (Cournot and Bertrand), we consider two situations. The first situation assumes only one domestic firm has the ability to enter the foreign market. In this case, the firm without the ability to enter is limited to choosing no entry. In the second situation, both firms have the ability to enter the foreign market and all three organizational options are available to both firms.

To explore when decentralization and centralization strategies emerge in the presence of gray markets, we present three models. Our first is a model of quantity-based competition with a monopolist gray market firm. Firms 1 and 2 (the Stackelberg leaders) choose quantity first, anticipating the subsequent quantity choices by the gray marketer (the Stackelberg follower).[15] A number of papers use this structure to analyze gray markets in a context of Cournot-based competition (Li and Robles 2007, Chen 2009, Autrey and Bova 2012). Kreps and Scheinkman (1983) show that a market where capacities are set before prices are chosen can be accurately represented using a model of Cournot (quantity-based) competition. Importantly for our model, whereas the analysis of Kreps and Scheinkman is based on competition between homogeneous goods, it is straightforward to apply the Kreps and Scheinkman insights to markets where products are differentiated (Martin 2001).[16]

In §5, we present two alternative models. The first alternative considers the question of organizational structure in a market characterized by Bertrand price competition. In a second alternative model, we examine the optimal organizational structure when there are competing gray marketers.

3.1. Cournot Timeline

After the organizational structure decisions, firms 1 and 2 (the Stackelberg leaders) choose quantities in each market to maximize profits. Note that, as firm 1 and firm 2 are the originators of all products in both markets, it is intuitive that they set quantity first. Next, the gray marketer (the Stackelberg follower) assesses the profitability of acquiring a quantity of product in the foreign market and reselling that same quantity in the domestic market at the market-clearing price. The gray marketer chooses the quantity that maximizes its profit. When the gray market is active, the volume sold in the domestic market is higher and this leads to a lower equilibrium domestic price. Note that the gray market will not function if the price in the foreign market is sufficiently high. Alternatively, if the equilibrium price in the foreign market is sufficiently low, domestic firms will not enter the foreign market.[17]

Consistent with standard Cournot models, product prices in the domestic and foreign markets (respectively) are determined such that the market clears (Mas-Colell et al. 1995). The game structure assumes that (a) foreign consumers and the gray marketer pay the same price for product purchased in the foreign market, and (b) the gray marketer and the respective domestic firm receive the same price for product sold in the domestic market, as their respective products are perfect substitutes. The timing of the decisions in the game is shown in Figure 1.

3.2. Cournot Demand Structure

The demand structure we use is based on a quadratic direct utility function subject to an income constraint for a market with two products as in Singh and Vives (1984)

$$U = \alpha_1\hat{q}_1 - \frac{\beta_1}{2}\hat{q}_1^2 + \alpha_2\hat{q}_2 - \frac{\beta_2}{2}\hat{q}_2^2 - \gamma\hat{q}_1\hat{q}_2 + \mathrm{N} \qquad (1)$$

subject to

$$I > p_1\hat{q}_1 + p_2\hat{q}_2 + \mathrm{N}. \qquad (2)$$

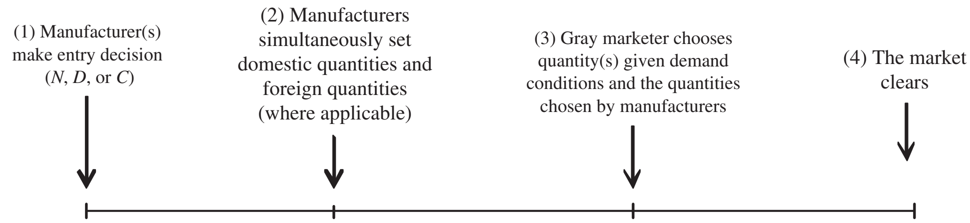
[14] Note that our model setup differs from that of McGuire and Staelin (1983) and similar setups in Arya et al. (2008). The McGuire and Staelin model assesses the optimal distribution choices of two competing manufacturers. Each manufacturer can either distribute through an independent exclusive retailer or it can "vertically integrate" the retailer and distribute directly to end consumers.

[15] The Cournot results are robust to modeling the gray marketer as price-taking follower with a cost disadvantage. We do not present this model but the details are available from the authors on request.

[16] A limitation of Kreps and Scheinkman (1983) is raised by Davidson and Deneckere (1986) who show that the findings depend on the type of rationing used to determine best responses in the pricing subgame. Despite this limitation, Davidson and Deneckere do recognize the relevance of the Kreps and Scheinkman model in markets where firms make their capacity decisions well in advance of pricing decisions for technological reasons. Moreover, the markets in which gray markets thrive are more often than not characterized by rationing that leads to the Kreps and Scheinkman results (Tirole 1990, pp. 212–214).

[17] When the price in the foreign market is sufficiently low, gray market goods collapse profits in the domestic market and the only beneficiary of foreign entry is the gray marketer.

Figure 1 **Cournot Timeline**



In Equations (1) and (2), $\hat{q}_i$ $(i = 1, 2)$ are the quantities of Firm i's product available in the market, p_i $(i = 1, 2)$ are the clearing prices for each product, N is the numeraire commodity, and I represents the income constraint. As in Singh and Vives (1984), the following functions are obtained when utility is maximized and positive quantities of the numeraire commodity are consumed

$$p_i = \alpha_i - \beta_i \hat{q}_i - \gamma \hat{q}_j, \quad i, j = 1, 2, \ i \neq j. \tag{3}$$

To focus on the case of symmetric firms, we set $\alpha_1 = \alpha_2 = \alpha$ and $\beta_1 = \beta_2 = \beta$. We normalize α, the intercept in the domestic market to 1 without loss of generality. The intercept in the foreign market is set to $F < 1$ so that foreign market prices are lower than domestic prices. To further simplify the analysis, we normalize β, the coefficient on the firm's own quantity, to 1. These normalizations mean that γ, the coefficient on the competitor's quantity, is not a fully flexible measure of substitutability and its interpretation is different than were α and β unrestricted. However, the normalizations allow us to define a feasible range for γ between $(0, 1)$.[18] We define γ as the degree of competitive intensity between the products of firms 1 and 2. As γ approaches 0, the products are perfectly differentiated and competitive intensity is low. As γ approaches 1, the products become perfect substitutes and competitive intensity is high.

We now present the pricing functions in each market, recognizing that $\hat{q}_i$ in the domestic market consists of the quantity produced domestically by firm i, q_i, and the perfect substitute diverted by the gray market, q_{Gi}, from firm i's foreign market

$$p_i = 1 - (q_i + q_{Gi}) - \gamma(q_j + q_{Gj}), \quad i, j = 1, 2, \ i \neq j. \tag{4}$$

In the foreign market, $\hat{q}_i$ is simply q_{Fi}, the quantity sold at price p_{Fi} by firm i to consumers in the foreign market. Because the intercept in the foreign market is F, we write the pricing functions in the foreign market as follows:

$$p_{Fi} = F - q_{Fi} - \gamma q_{Fj}, \quad i, j = 1, 2, \ i \neq j. \tag{5}$$

The gray marketer's demand for each firm's product, q_{Gi}, is determined endogenously to maximize profit given local demand that incorporates the quantity decisions made by firms 1 and 2. The gray marketer diverts product from the foreign market and sells it in the domestic market implying that its profit per unit is $p_i - p_{Fi}$, the price differential between the domestic and foreign markets. Because the gray marketer sells its product in the domestic market, its sales satisfy domestic consumer demand but not foreign consumer demand; accordingly, the gray market quantity, q_{Gi}, is incremental to the local demand of foreign consumers, q_{Fi}.[19] The total production quantity of firm i's foreign subsidiary is as follows:

$$Q_i = q_{Fi} + q_{Gi}, \quad i = 1, 2. \tag{6}$$

An important benefit of our demand structure is that it involves no "rescaling": γ is both the coefficient on the cross product of q_1 and q_2 in the quadratic direct utility function and the measure of competitive intensity in Equations (4) and (5). When the parameters from a direct utility function are rescaled in the demand function, difficulty can arise in the interpretation of comparative statics (Staelin 2008). Since this formulation involves no rescaling, results for different values of γ can be compared directly.

3.2.1. Decentralized Decision Making. The objective functions for each firm and the gray marketer are as follows $(i = 1, 2)$:

$$\pi_i = p_i q_i \quad \text{and} \quad \pi_{Fi} = p_{Fi} Q_i, \tag{7}$$

$$\pi_G = (p_1 - p_{F1}) q_{G1} + (p_2 - p_{F2}) q_{G2}, \tag{8}$$

subject to (4), (5), and (6).

[18] These normalizations are common in models of differentiated Cournot competition (Arya and Mittendorf 2008; Arya et al. 2008, 2010; Zanchettin 2006 and Qiu 1997). To examine whether our results depend on the normalizations made across the domestic and foreign markets, we conducted robustness checks for the one-firm entry case. The checks show that the findings hold if we allow the foreign demand slope to vary with $\beta \in (0, \infty)$. Moreover, the model insights are qualitatively identical even when the firms have asymmetric demand intercepts in the domestic and foreign settings.

[19] We assume a single gray marketer in the initial model, but we relax this assumption in §5.2.

Using backward induction, we solve the gray marketer's problem first and this leads to reaction functions for q_{G1} and q_{G2} as a function of the quantities produced by the manufacturers in the first stage. These are then substituted into the objective functions of firms 1 and 2. These objective functions are optimized with respect to q_1, q_{F1}, q_2, and q_{F2}, respectively, to create a system of four equations in four unknowns.

When only one firm can enter the foreign market, we assume without loss of generality that firm 1 is the player with the capability to enter the foreign market. In this case, $q_{F2}=0$, $q_{G2}=0$, and since firm 2 has no foreign sales, p_{F2} does not exist. Here, the gray marketer has a single reaction function for q_G (we omit the firm number in the subscript to distinguish the one-entry and two-entry cases). This q_G is then substituted into the objective functions for firm 1's domestic subsidiary, firm 1's foreign subsidiary, and firm 2's domestic subsidiary. These functions are optimized with respect to q_1, q_2, and q_F, respectively, to create a system of three equations in three unknowns.

3.2.2. Centralized Decision Making. The objective functions for each firm and the gray marketer are as follows ($i=1,2$):

$$\begin{aligned} \Pi_i &= p_i q_i + p_{Fi} Q_i, \\ \pi_G &= (p_1 - p_{F1}) q_{G1} + (p_2 - p_{F2}) q_{G2}, \end{aligned} \tag{9}$$

subject to (4), (5), and (6). Note that the constraints (i.e., the demand functions) are unaffected by the organizational structure of the firms.

As before, the gray marketer's problem is solved first and this generates reaction functions for q_{G1} and q_{G2} as a function of the quantities produced by the manufacturers in the first stage. These are then substituted into the objective functions of firms 1 and 2. In contrast to the decentralized case, each firm has a single objective function that it optimizes with respect to q_i and q_{Fi} simultaneously. This generates a system of four equations in four unknowns.

When only one firm can enter the foreign market, we again set $q_{F2}=0$, $q_{G2}=0$, and since firm 2 has no foreign sales, p_{F2} does not exist. The gray marketer again has a single reaction function for q_G, which is then substituted into each firm's objective function. This generates a system of three equations in three unknowns.

3.2.3. Asymmetric Decision Making. Here, we assume that firm 1 operates with a centralized structure and firm 2 operates with a decentralized structure and that both firms enter the foreign market. Firm 1's objective function is given in Equation (9), firm 2's objective functions are given in Equation (7), and the gray marketer's objective function is given in Equation (8). As noted above, the constraints are unaffected. Thus, these objective functions are also optimized subject to (4), (5), and (6).

To solve the asymmetric game, the gray marketer's reaction functions for q_{G1} and q_{G2} as a function of the quantities produced by the manufacturers are substituted into the respective objective functions of firms 1 and 2. These functions are then optimized with respect to q_1, q_{F1}, q_2, and q_{F2}, respectively, to create a system of four equations in four unknowns.

4. Model Analysis

In this section, we present the analysis for markets characterized by Cournot competition. The cases of Bertrand competition and a competitive gray market are considered in §5. To analyze the Cournot case, we first derive each firm's optimal profits under the alternative organizational structures. For brevity, we do not present the first-order conditions or quantities, but provide them in the appendix. For each structure (decentralized and centralized), we first analyze the simple setting when only firm 1 can enter the foreign market and then the case when both firms can enter. To determine the optimal structure, we compare the profits of each alternative and we then discuss the implications.

For all settings, we restrict the quantities to be nonnegative. Equilibrium domestic quantities are always nonnegative, but the gray market quantity is only nonnegative when F is sufficiently low. We define $m_i(\gamma)$ and $n_i(\gamma)$ as the upper bounds of F for which $q_{Gi} \geq 0$ with a decentralized and centralized structure, respectively. We also require $q_{Fi} \geq 0$; F must be sufficiently large such that the optimal quantity consumed by foreign consumers is nonnegative.[20]

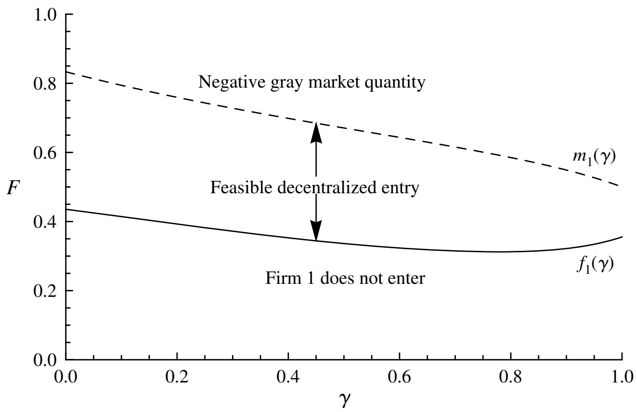
It should be noted that the equilibrium profit for the firms when neither enters the foreign market is $\Pi = 1/(\gamma+2)^2$. Therefore, a decision to enter the foreign market by one of the firms (given that the competitor does not have a foreign subsidiary) must yield an increase over this level of profits.

4.1. Decentralized Structure

4.1.1. Only Firm 1 Enters the Foreign Market (Decentralized). In the decentralized case when only firm 1 enters the foreign market, the optimal profits

[20] This constraint never binds with centralized entry, and only binds for a small range of parameter values when both firms choose decentralized entry (in particular, when γ is very close to 1 and F is very low). We define $d(\gamma)$ as the lower bound of F such that $F > d(\gamma)$ ensures $q_{Fi} \geq 0$ (shown in Figure A.3 in the appendix).

Figure 2 **The Feasible Range for Gray Marketing in the Decentralized Case Where Only One Firm Enters the Foreign Market**



for the three players (firm 1's profit is the sum of its domestic profit and foreign profit) are

$$\Pi_1 = ((7(4-2\gamma-\gamma^2)+2F(4-3\gamma^2))^2+3(4-2\gamma-\gamma^2 +2F(8-5\gamma^2))^2)\cdot(32(13-8\gamma^2)^2)^{-1}, \tag{10}$$

$$\pi_2 = \frac{(26-19\gamma+2F\gamma)^2(2-\gamma^2)}{32(13-8\gamma^2)^2}, \tag{11}$$

$$\pi_G = \frac{(5(4-2\gamma-\gamma^2)-2F(12-7\gamma^2))^2}{64(13-8\gamma^2)^2}. \tag{12}$$

We define $f_1(\gamma)$ as the lower bound of F for which firm 1 enters the foreign market with a decentralized structure. A small F has two effects on firm 1's entry decision. First, there is lower profit potential from entering the foreign market. Second, the gray market's cost base is correspondingly lower and the gray market cannibalizes more of firm 1's domestic sales. For F sufficiently low, firm 1 does not enter the market because the loss of domestic sales to the gray market more than offsets the profit potential of entering the foreign market. The expression for $f_1(\gamma)$ is provided in the appendix. Figure 2 shows the parameter region where firm 1 will enter the market (above $f_1(\gamma)$, the solid line). Figure 2 also illustrates the infeasible region above the cutoff $m_1(\gamma)$, where the gray market quantity is zero. In this region, the market price in the foreign market is higher than in the domestic market. Said differently, this is theoretically a region where the gray market would flow in the opposite direction: from the domestic market to the foreign market. Because the focus of our analysis is to understand the challenge of a domestic firm entering a low-priced foreign market where a gray marketer diverts product back to the domestic market, we restrict attention to the feasible region between $f_1(\gamma)$ and $m_1(\gamma)$.

4.1.2. Both Firms Enter the Foreign Market (Decentralized). In the decentralized case when both firms enter the foreign market, the optimal profits for the three players are

$$\Pi_1 = \Pi_2 = \frac{(2F+3\gamma+7)^2+3(4F+2F\gamma+1)^2}{2(13+13\gamma+3\gamma^2)^2}$$
$$\text{and} \quad \pi_G = \frac{(5+3\gamma-6F-4F\gamma)^2}{2(1+\gamma)(13+13\gamma+3\gamma^2)^2}. \tag{13}$$

4.2. Centralized Structure

4.2.1. Only Firm 1 Enters the Foreign Market (Centralized). Solving the centralized case when only firm 1 enters the foreign market, the optimal profits for the three players are

$$\Pi_1 = (3(4-2\gamma-\gamma^2)^2+3F(4-3\gamma^2)(4-2\gamma-\gamma^2) +F^2(48-60\gamma^2+19\gamma^4))\cdot(2(12-7\gamma^2)^2)^{-1},$$
$$\pi_2 = \frac{(6-4\gamma+F\gamma)^2(2-\gamma^2)}{2(12-7\gamma^2)^2}, \tag{14}$$
$$\pi_G = \frac{(4-2\gamma-\gamma^2-2F(2-\gamma^2))^2}{4(12-7\gamma^2)^2}.$$

We define $g_1(\gamma)$ as the lower bound of F for which firm 1 enters the foreign market with a centralized structure. Similar to the decentralized case, when F is sufficiently low, firm 1 does not enter the market because the loss of domestic sales to the gray market more than offsets the profit potential of entering the foreign market.[21] The expression for $g_1(\gamma)$ is provided in the appendix. The feasible zone for gray marketing in the case of a centralized structure is shown in Figure 3. Figure 3 also illustrates the infeasible region above the cutoff $n_1(\gamma)$, where the gray market quantity violates the nonnegativity constraint under a centralized organizational structure. As in the decentralized case, the market price in the foreign market is higher than in the domestic market in this region.

4.2.2. Both Firms Enter the Foreign Market (Centralized). Solving the centralized case when both firms establish foreign subsidiaries yields the following optimal profits for the three players:

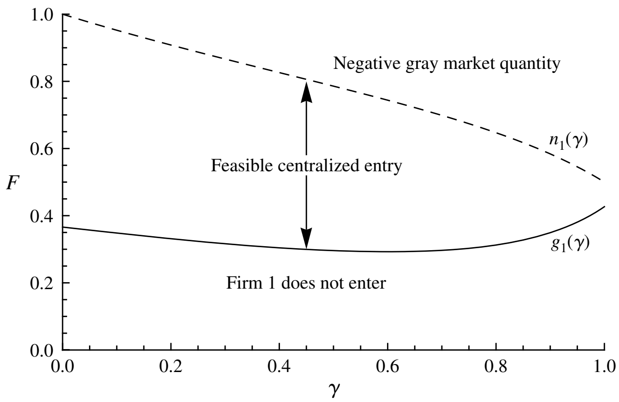
$$\Pi_1 = \Pi_2 = \frac{2}{3}\frac{1+F+F^2}{(2+\gamma)^2} \quad \text{and}$$
$$\pi_G = \frac{2}{9}\frac{(1-F)^2}{(2+\gamma)^2(1+\gamma)}. \tag{15}$$

4.3. Asymmetric Structure

For the asymmetric case when firm 1 is centralized and firm 2 is decentralized, we derive the optimal quantity and profit expressions for each firm in the appendix.

[21] A firm also earns profit on gray market volume but the price (or the absolute margin) on gray market volume is significantly less than the price earned in the domestic market.

Figure 3 The Feasible Range for Gray Marketing in the Centralized Case Where Only One Firm Enters the Foreign Market



4.4. Equilibrium Analysis

We denote the strategy set for both firms in the first stage of the game as $s_i = \{N = \text{no entry}, D = \text{entry with decentralized control}, C = \text{entry with centralized control}\}$, $i = 1, 2$. A decision to enter the foreign market by one of the firms (given that the competitor does not have foreign operations) must yield an increase versus the profits earned by the firm when neither firm enters the foreign market (denoted $\{N, N\}$). Accordingly, we take the $\{N, N\}$ equilibrium as the benchmark case and analyze when the firms have a profitable incentive to deviate from this outcome.

For ease of exposition and without loss of generality, we frame the discussion of best responses in terms of the focal firm, firm 1. We then derive and discuss the implications of the resulting equilibria.

4.4.1. Equilibrium When Only One Firm Can Enter the Foreign Market. In this scenario, firm 2 is assumed to not have the ability to enter the foreign market. If firm 1 chooses decentralized foreign entry, the domestic subsidiary chooses q_1 to maximize π_1, the foreign subsidiary chooses q_{F1} to maximize π_{F1}, and firm 2 chooses only q_2 to maximize π_2. With decentralized entry, firm 1's profit expression is provided in (10). If firm 1 chooses centralized foreign entry, the domestic head office chooses q_1 and q_{F1} to maximize the sum of π_1 and π_{F1}, and firm 2 chooses q_2 to maximize π_2 (it does not enter the foreign market). With centralized entry, firm 1's profit expression is provided in (14). If firm 1 chooses not to enter it earns $1/(\gamma+2)^2$ and entry only occurs if firm 1 increases its profits from this level, i.e., when $F > f_1(\gamma)$ if firm 1 chooses a decentralized structure or $F > g_1(\gamma)$ when its structure is centralized.

We define the cutoff value B_N as the boundary separating the regions where firm 1's decentralized profit is higher than its centralized profit when firm 2 does not enter the foreign market. The results when only firm 1 enters the foreign market are summarized in Proposition 1. To clarify the exposition, it is useful to define $\gamma_N^* = \frac{1}{2}\sqrt{2}\sqrt{3-\sqrt{3}} \approx 0.79623$. Moreover, the boundary B_N is comprised of two functions defined over the interval $[\gamma_N^*, 1)$: the definitions of $\bar{F}_N$ and $\underline{F}_N$ are provided in the appendix.

Proposition 1. *When only firm 1 has the ability to enter the foreign market, and given that firm 1 prefers entry, firm 1's optimal decision is decentralization when $\gamma > \gamma_N^*$, $F \in (\underline{F}_N, \bar{F}_N)$. Otherwise, firm 1's optimal decision is centralization.*

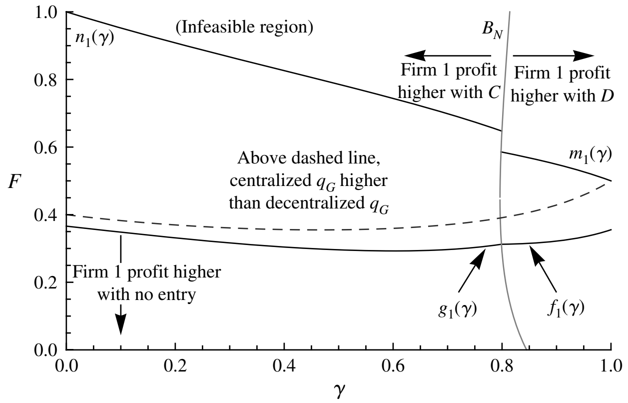
In other words, when the level of competitive intensity between the firms is high and firm 1 is motivated to enter the foreign market, the optimal decision is to enter with a decentralized organization. This is surprising because under decentralization, neither the domestic nor the foreign subsidiary account for the profit implications of their choices on the subsidiary that operates in the other country. Indeed, the failure to incorporate demand interdependencies leads both subsidiaries to choose higher quantities than in a centralized setting. In particular, firm 1's domestic arm does not account for its quantity choice having a negative impact on gray market demand under decentralization (and this indirectly affects the foreign subsidiary's profitability). Moreover, the aggressive stance of the two subsidiaries under decentralization reduces the magnitude of domestic industry profits.

The counterpoint to the detrimental effect that firm 1's decentralization has on domestic industry profits is the beneficial effect that firm 1's decentralization has on firm 2's production decision. Because decentralization leads to aggressive production decisions by both of firm 1's subsidiaries, decentralization weakens firm 2: the higher the level of competitive intensity between the firms, the more pronounced the effect. The increased level of competition in the domestic market causes firm 2 to choose a lower production quantity: under Cournot competition, the decisions of competing firms are strategic substitutes (Tirole 1990). The reduced quantity chosen by firm 2 can lead to higher global profit for firm 1. In other words, foreign entry with a decentralized structure is associated with a tension between two effects:

1. A reduction in the industry profits available to firms in the domestic market.
2. An increase in firm 1's market share (i.e., capturing a larger portion of these available profits).

When γ is sufficiently high (i.e., the level of competitive intensity is sufficiently high), the increase in firm 1's share of total domestic profits under decentralization more than offsets the reduced size of the domestic profits and leads to higher profits for firm 1—both domestically and globally.

Figure 4 **Comparison of Gray Market Quantity Under Decentralization versus Centralization**



Finally, the dominance of decentralization when product differentiation is low is not explained by higher gray market volume under decentralization. As depicted in Figure 4, with decentralization, gray market quantities can be higher or lower than the quantities observed under centralized control. The dashed line separates the region where gray market volume is higher with centralization compared to decentralization. Above the dashed line, centralization leads to higher gray market volume than decentralization; below the dashed line, the opposite is true.

This observation highlights an important aspect of managing gray markets in markets characterized by Cournot competition. For firms, a key benefit of foreign borders (not to mention the geographic distance that often separates countries) is the opportunity it provides to charge different groups of customers different prices. This is critical in the conditions we examine: the "willingness to pay" for two groups of customers is significantly different. Cursory analysis would suggest that the most straightforward approach to retain profits in these conditions is to *minimize* or *eliminate* leakage between the groups of customers. In fact, the legal actions and literature we cite in the introduction reflect this perspective on gray markets. Our analysis shows that unless a firm can prohibit gray marketing through legal action (which is difficult outside of categories like pharmaceuticals), minimizing gray market volume is not necessarily the correct criterion. Sometimes the optimal strategy for a firm is to allow a *larger* gray market volume. This is precisely the case in Figure 4 above the dashed line to the left of B_N and below the dashed line to the right of B_N.

Taken together, the key factor in determining a firm's optimal organizational structure is the competitiveness of the gray market firm relative to the domestic competitor. Specifically, we have the following:

1. When a firm has a significant degree of market power in the domestic market (γ is low), the most important criterion is to manage the gray market such that the externality is internalized. This is achieved with a centralized structure.

2. In contrast, when a firm does not enjoy market power (γ is high), the most important criterion is to use the gray market as a weapon to weaken the domestic competitor. This is achieved with a decentralized structure.

We now consider a situation where both domestic firms can enter the foreign market. Similar to this section, we assume that diverted product from the foreign subsidiary is a perfect substitute for the domestic version of that firm's product.

4.4.2. Equilibrium When Both Firms Can Enter the Foreign Market. When firm 2 also chooses whether to enter the foreign market and the organizational structure to adopt in the event of entry, the analysis and resulting intuition parallel the decision of firm 1 in §4.4.1 (where only firm 1 can enter the foreign market). If firm 2 chooses to enter the foreign market and it adopts a decentralized structure, firm 1's optimal response in terms of entry and structure is shown in Figure A.1 in the appendix. If firm 2 chooses to enter the foreign market and it adopts a centralized structure, firm 1's optimal response is shown in Figure A.2 in the appendix.

To clarify the analysis, we define B_{S_2} as the boundary in (γ, F) space that delineates the optimal organizational structure for firm 1 as a function of s_2, firm 2's chosen strategy for the foreign market, where $s_2 \in \{N, D, \text{or } C\}$. Not surprisingly, the mathematical definition of B_{S_2} depends on whether firm 2 centralizes, decentralizes, or does not enter the market. For the three cases, we provide the mathematical definitions of B_{S_2} in the appendix (B_N is defined in §4.4.1. The definitions of B_D and B_C are provided in the proof of Lemma 1). The boundary definitions are indeed different but in all three cases (a) B_{S_2} is a border that divides the parameter space into low and high levels of γ, and (b) B_{S_2} is a threshold to the left of which firm 1's optimal organizational structure is centralized, and to the right of which firm 1's optimal structure is decentralized (given that the feasibility constraints are satisfied). Similar to the analysis in §4.4.1, for each of the three cases, we define a critical value $\gamma^*_{S_2}$ and two functions defined over the interval $[\gamma^*_{S_2}, 1)$: $\bar{F}_{S_2}$ and $\underline{F}_{S_2}$ that form the basis for the boundary (these are defined in the appendix). The lemma is written for the general case of B_{S_2}, and similar to the single firm entry game, the best response is not related to the level of gray market volume.

Lemma 1. *Given firm* 1 *prefers entry and a strategy* $s_2 \in \{N, D, \text{or } C\}$ *by firm* 2, *firm* 1*'s optimal strategy is decentralization when* $\gamma > \gamma^*_{S_2}$, $F \in (\underline{F}_{S_2}, \bar{F}_{S_2})$. *Otherwise firm* 1*'s optimal strategy is centralization.*

Although the best responses and resulting intuition parallel the single firm entry game, there is an important difference between the one firm and two firm entry contexts. In particular, firm 2 faces a symmetric situation to firm 1. The objective of the analysis is to find a strategy set for the two firms in the first stage of the game where neither firm has an incentive to deviate. As explained earlier, firm i's strategy is denoted as $s_i \in \{N, D, \text{ or } C\}$. We define equilibrium as a strategy pair $\{s_1, s_2\}$, where both strategies are best responses to the strategy of the competitor. In certain regions of the parameter space, multiple equilibria are possible. For simplicity, when there are multiple equilibria in a given parameter region, we highlight the Pareto optimal equilibrium for that region (details of the other equilibria are provided in the appendix). This is pertinent given the starting point of our analysis (i.e., neither firm is operational in the foreign market). For example, we assume that firms will not enter the foreign market if $\{N, N\}$ is one of several equilibria, because $\{N, N\}$ is Pareto superior to the other equilibria when it is a fixed point in the first stage.

As in the one-firm entry case, we find that if both firms prefer to enter the foreign market, then both firms choose centralized entry only if competition is sufficiently weak.[22] Conversely, when (γ, F) lies to the *right* of B_C then both firms choose *decentralized* entry and this is the unique equilibrium strategy. To clarify the exposition, it is useful to define $\gamma_C^* = \frac{1}{3}\sqrt{3}\sqrt{6-\sqrt{6}\sqrt{7-\sqrt{13}}}$. The boundary B_C is comprised of two functions, $\bar{F}_C$ and $\underline{F}_C$, which we define in the appendix.

We formalize the resulting equilibrium of the game in which both firms can enter the foreign market in Proposition 2.

Proposition 2. *When both firms can enter the foreign market and entry is preferred, $\{D, D\}$ is the unique, pure strategy equilibrium when $\gamma > \gamma_C^*$, $F \in (\underline{F}_C, \bar{F}_C)$. Otherwise, $\{C, C\}$ is either a unique pure strategy equilibrium or a Pareto-dominant pure strategy equilibrium.*

When $\gamma > \gamma_C^*$ and $F \in (\underline{F}_C, \bar{F}_C)$ and firms choose to enter the foreign market, decentralized entry by both firms (i.e., $\{D, D\}$) is the unique pure strategy equilibrium. Note that the firms would realize higher profits if they could both commit to entering with centralized structures. However, in this region, the best response to centralized entry by the competitor is decentralized entry and the best response to decentralized entry is also decentralized entry. Thus, this region is characterized by prisoners' dilemma type payoffs. Both firms are worse off because the option of a decentralized structure is available. The primary implication of Proposition 2 is that the equilibrium organizational structure of decentralization does not depend on only one firm having the capability to enter the foreign market (the key finding of §4.4.1). Rather, our findings arise independently of whether one or two firms enter the foreign market. Naturally, in this model, the choice of organizational structure is irrelevant when there is no gray market: the production decisions of both centralized and decentralized managers are identical when demand in each country is independent. However, when there are gray markets, it is interesting to find that decentralization can dominate centralization for a focal firm's foreign expansion independent of the competitor's organizational structure or foreign expansion plans. In summary, the analysis shows that the incentive to "internalize" the gray market externality is mitigated by the preexisting level of competitive intensity in the domestic market.

5. Model Extensions

In this section, we examine two additional settings to provide a comprehensive perspective on the impact of organizational structure on performance when gray markets are active. The first setting is a model of Bertrand price competition. We show that in this setting, decentralization is never optimal. In the second setting, we return to quantity competition and assume that multiple gray marketers can divert products from the foreign market to the domestic market. We illustrate that the tenor of our results hold provided that the gray market is not overly competitive.

5.1. Bertrand Competition with a Price-Taking Gray Marketer

As noted earlier, gray markets are also prominent in categories where production lead times are short and quantities can be adjusted easily. Accordingly, we analyze a model where the markets in both the domestic and foreign markets are characterized by price competition. Even in markets characterized by price competition, the gray marketer chooses the quantity of product to be diverted from the low-priced market to the high-priced market. As in the Cournot model, firms 1 and 2 set prices first, anticipating the subsequent quantity choice by the gray marketer. However, in this setting, the gray marketer is "price-taking" and chooses its quantity as a function of the prices in the two markets.[23]

[22] When competition is moderate, there are multiple equilibria: $\{C, C\}$, $\{D, D\}$, and a mixed strategy equilibrium. We assume that $\{C, C\}$ is the chosen equilibrium in this region because it Pareto dominates both $\{D, D\}$ and the mixed strategy equilibrium.

[23] In contrast to the main model, where the gray marketer is a follower making the same type of decision as the firms (production quantities), it is difficult to represent the gray marketer as a sequential price setter. The reason is that the manufacturers' prices are not best responses to the gray marketer's price when the gray marketer

As in §§3 and 4, each firm first chooses its organizational structure, which determines the decision maker for the pricing decisions (i.e., the local subsidiary or the head office), followed by price competition with their rival, which in turn is followed by gray market diversion and local demand realization. The model is solved by backward induction.

In this setting, the gray marketer does not face a downward-sloping demand curve in the domestic market but instead faces a supply cost disadvantage that is positively related to gray market volume. This reflects the idea that as the volume the gray marketer wishes to acquire increases, it is increasingly difficult to find and source stock in the foreign market. The gray marketer sells a profit-maximizing quantity in the domestic market at the prevailing domestic price. The objective function for the gray marketer is as follows:

$$\pi_G = \left(p_1 - p_{F1} - \tfrac{1}{2}q_{G1}\right)q_{G1} + \left(p_2 - p_{F2} - \tfrac{1}{2}q_{G2}\right)q_{G2}. \quad (16)$$

The gray marketer optimizes Equation (16) with respect to q_{G1} and q_{G2}, which yields $q_{Gi} = p_i - p_{Fi}$, $i = 1, 2$. Similar to the Cournot model, gray market demand, q_{Gi}, is incremental to local demand from foreign customers, q_{Fi}.

Anticipating this level of gray market demand, firms 1 and 2 engage in Bertrand price competition in both markets. To obtain *foreign* demand curves that correspond to the Cournot model, we invert Equation (5) and solve for q_{Fi}

$$q_{Fi} = \frac{(F - p_{Fi}) - \gamma(F - p_{Fj})}{1-\gamma^2}, \quad i, j = 1, 2,\ i \neq j. \quad (17)$$

This step holds market demand constant across the Cournot and Bertrand settings; thus, the only difference is how the firms compete with each other. Next, each firm's residual *domestic* demand equals total domestic demand less the quantity diverted by the gray marketer, q_{Gi}. The resulting residual demand function is identical to the demand function obtained by simply inverting Equation (4) and solving for q_i, which yields

$$q_i = \frac{(1 - p_i) - \gamma(1 - p_j)}{1-\gamma^2} - q_{Gi}, \quad i, j = 1, 2,\ i \neq j. \quad (18)$$

The objective functions of firms 1 and 2 are identical to the objective functions of §§3.2.1 through 3.2.3; however, both firms optimize with respect to prices p_i and p_{Fi} instead of quantities. Finally, the optimal prices (and the resulting profits) are used to determine the organizational structures chosen by the firms in stage 1. Similar to §4, we present only the profit results for the decentralized and centralized cases; see the appendix for the respective optimal prices and gray market quantities. For brevity, we present the analysis in which only firm 1 has the ability to enter the foreign market (i.e., $q_{F2} = 0$, $q_{G2} = 0$); the results in the two-firm entry case have identical intuition to the one-firm entry case.

The optimal firm profits for the three players in the *decentralized* case are (firm 1's profit is the sum of its domestic profit and foreign profit)

$$\Pi_1^{\text{DBer}} = \frac{(1-\gamma)(2-\gamma^2)(4+F+\gamma(2+F))^2}{9(1+\gamma)(5-3\gamma^2)^2} + \frac{(2-\gamma-\gamma^2+F(8-5\gamma^2))^2}{18(5-3\gamma^2)^2}, \quad (19)$$

$$\pi_2^{\text{DBer}} = \frac{(1-\gamma)(15+\gamma(4+F)-\gamma^2(7-F))^2}{36(1+\gamma)(5-3\gamma^2)^2},$$
$$\text{and} \quad \pi_G^{\text{DBer}} = \frac{(2-\gamma-\gamma^2-F(2-\gamma^2))^2}{8(5-3\gamma^2)^2}. \quad (20)$$

The optimal profits for the three players in the *centralized* case are

$$\Pi_1^{\text{CBer}} = ((1-\gamma)(2+F+\gamma(1+F))(12+6\gamma-\gamma^2(4-F) - \gamma^3(2-F)))\cdot(36(1+\gamma)(2-\gamma^2)^2)^{-1} + \frac{F}{24}\left(2+5F-\frac{2(F+\gamma)}{2-\gamma^2}\right), \quad (21)$$

$$\pi_2^{\text{CBer}} = \frac{(1-\gamma)(6+\gamma(2+F)-\gamma^2(2-F))^2}{36(1+\gamma)(2-\gamma^2)^2},$$
$$\text{and} \quad \pi_G^{\text{CBer}} = \frac{(2+\gamma)^2(2(1-\gamma)-F(2-\gamma))^2}{288(2-\gamma^2)^2}. \quad (22)$$

Given these equilibrium profit levels, firm 1 always prefers centralized decision making to decentralized decision making, when its rival cannot enter the foreign market. As we note above, we omit the details of the two-firm entry case because the intuition is identical: firm 1's best response is to choose centralization over decentralization independent of whether firm 2 chooses centralized entry, decentralized entry, or no entry.

Proposition 3. *Under Bertrand competition, centralized decision making is always preferred to decentralized decision making.*

The above result aligns with the conventional wisdom that centralization is an effective strategy to internalize the negative externality of gray markets. It is important to note that these findings do not arise because we model the gray marketer as a price taker that chooses quantity. If we model the gray marketer

chooses price last. Accordingly, the gray market firm is modeled as a price-taking quantity setter. This approach follows dominant firm models as in Carlton and Perloff (2000), pp. 109–119.

as a price taker that chooses quantity (instead of a Stackelberg follower) in the Cournot setting, firm 1 still chooses to *decentralize* as the level of competitive intensity increases.[24]

The findings arise because, when the market is characterized by Bertrand competition, the pricing decisions of firms are strategic complements. As a result, a competitor's best response to an aggressive decision is more aggression, not retreat. Decentralizing reduces total industry profit but does not increase firm 1's share of the industry profit. Thus, decentralization is not optimal in a category characterized by Bertrand competition.

The key takeaway from this analysis is that firms in a market characterized by Bertrand competition are better off impeding the gray market by reducing its importance; this is achieved through centralized decision making. This leads to two predictions about markets characterized by price-based competition where gray marketing takes place. First, firms are more likely to impose centralized control on the behavior of their foreign subsidiaries. Second, the heuristic of choosing the organizational structure that minimizes gray market volume is more effective than in markets characterized by quantity-based competition.

5.2. Competition in the Gray Market

In the final setting, we assess the optimal organizational structure when the gray market is more competitive. As noted earlier, we examine this setting in a context where firms are Cournot competitors. The objective functions for centralized and decentralized firms are unchanged from the main model, except that we modify the Cournot domestic demand and the foreign total quantity demanded to reflect additional gray market firms as follows:

$$p_i = 1 - \left(q_i + \sum_{n=1}^{N} q_{Gin}\right) - \gamma\left(q_j + \sum_{n=1}^{N} q_{Gjn}\right), \quad i, j = 1, 2,\ i \neq j, \tag{23}$$

$$Q_1 = q_{F1} + \sum_{n=1}^{N} q_{G1n} \quad \text{and} \quad Q_2 = q_{F2} + \sum_{n=1}^{N} q_{G2n}. \tag{24}$$

We use the same solution technique as in the main model (see §§3 and 4). Each of the N gray marketers chooses a quantity of each firm's product, q_{Gin}, to maximize profit. Their combined demand is again incremental to that of local customers, q_{Fi}. To generate the reaction functions to the quantity choices of firms 1 and 2, respectively, each gray marketer maximizes $(n = 1, \ldots, N)$

$$\pi_{Gn} = (p_1 - p_{F1})q_{G1n} + (p_2 - p_{F2})q_{G2n}. \tag{25}$$

[24] The model results for a Cournot setting where the gray marketer is a price-taking follower are available from the authors on request.

The resulting functions are provided in the appendix. To solve the model, we compute the equilibrium firm quantities and profits as a function of N and γ. This allows us to define a generalized boundary $B_C(N, \gamma)$, to the right of which there is a pure strategy equilibrium where each firm enters the foreign market with a decentralized structure (i.e., $\{D, D\}$). To clarify the exposition, it is useful to define

$$\gamma_C^*(N) = \sqrt{\frac{2+4N-\sqrt{2}\sqrt{(1+2N)(2+4N+N^2-\sqrt{N^2(4+8N+N^2)})}}{1+2N}}.$$

Moreover, for each discreet value of $N \in [1, \infty)$ the boundary $B_C(N, \gamma)$ is comprised of two functions defined over the interval $[\gamma_C^*(N), \infty)$: $\bar{F}_{\text{Cmult}}$ and $\underline{F}_{\text{Cmult}}$ are defined in the appendix.

Proposition 4. *Provided firms prefer entry and $N \leq 4$, $\{D, D\}$ is a unique, pure strategy equilibrium when $\gamma \in (\gamma_C^*(N), 1)$ and $F \in (\underline{F}_{\text{Cmult}}, \bar{F}_{\text{Cmult}})$. Otherwise, $\{C, C\}$ is either a unique, pure strategy equilibrium or a Pareto-dominant, pure strategy equilibrium. If firms prefer entry and $N \geq 5$, $\{C, C\}$ is either a unique, pure strategy equilibrium or a Pareto-dominant, pure strategy equilibrium for all $\gamma \in (0, 1)$.*

Proposition 4 illustrates that our initial results are affected by the degree of competition in the gray market. If there is too much competition in the gray market (i.e., more than four gray market firms), decentralization is not an equilibrium. In other words, the gray market must not be too competitive for decentralization to dominate as a best response. In an untabulated analysis, we also model a perfectly competitive gray market by taking the limit as $N \to \infty$. As $N \to \infty$, in both the decentralized and centralized settings, each firm's equilibrium prices are the same in both markets (i.e., $p_i = p_{Fi}$). A firm competing in markets with fully competitive gray markets is obviously constrained because the gray market has an incentive to divert additional volume anytime there is a price differential between the domestic and foreign markets. With a decentralized structure, the foreign subsidiary always has an incentive to increase volume if there is a price differential. Why? Because the foreign subsidiary (which is motivated by its own interest) reaps the entire benefit of increased gray market volume. Of course, this increased volume represents a direct reduction in the profit earned by the domestic subsidiary. In a sense, a decentralized firm cannot prevent the foreign subsidiary from increasing production and this drives the domestic market price downward. Ultimately, for a decentralized firm, an equilibrium is only possible when (a) there is no price differential between the markets, and (b) the foreign subsidiary has no incentive to increase production. The only situation that satisfies these conditions is one in which the price is zero in both markets (and this is obtained when both subsidiaries produce quantities that lead to prices of zero).[25]

With centralization, the head office has a different basis for making decisions. The head office accounts for both the increased profit for the foreign subsidiary and lost profit for the domestic arm in its choice of foreign production. In a nutshell, the head office has less incentive to increase the volume of production in the foreign market. Ultimately, the centralized structure generates volumes that lead to uniform but strictly positive prices across markets, leading to the dominance of $\{C, C\}$ when the gray market is perfectly competitive. Interestingly, this finding suggests that the amount of gray market volume observed, ex post, is not necessarily increasing in the magnitude of the threat posed by the gray market, ex ante. Specifically, when the gray market is most destructive (i.e., a setting where the gray market is perfectly competitive), the optimal strategy for both firms independent of organizational structure is to choose quantities that lead to uniform pricing across subsidiaries (i.e., charging equal retail prices across international jurisdictions). Uniform pricing eliminates the arbitrage opportunity and reduces gray market volume to zero. However, the lack of gray market volume observed ex post cannot be interpreted as the gray market having a minimal impact on firm decisions, ex ante. Rather, it is the seriousness of the ex ante gray market threat that forces firms to set quantities that lead to uniform pricing across markets—a costly decision for most firms because it eliminates the ability to price discriminate across borders.

These findings also extend the results of §4. When the gray market institution has an incentive to limit gray market volume and competitive intensity is high, the competitor remains a firm's most important competitive threat; in these conditions, decentralized decision making is optimal. As the gray market becomes more competitive however, the gray market becomes the firm's biggest challenge independent of the competitive intensity between the two firms. Accordingly, it follows that in this setting, centralization is optimal for all γ.

6. Conclusion

6.1. Limitations

As with all analytical models, these results are subject to limitations. For example, we abstract away from the individual players that may be active in gray market distribution. In particular, resellers and distributors may also participate in gray market activity. To assess the impact of including additional intermediaries on the results, we reanalyze the model when one firm enters the foreign market, but assume that an intermediary can impose an additional cost of $c \in (0, 1)$ per unit of product on the gray marketer. The additional per-unit cost could represent the cost charged by a purchasing agent in the foreign market to obtain a given volume, or it could be a per-unit markup charged by the gray marketer's distributor in the domestic market. In untabulated analysis, when the gray market existence constraints are met, we find that there exists a boundary to the right of which decentralization is the optimal organizational structure for all c. Thus, the tenor of our results appear to be robust to the inclusion of additional intermediaries in the distribution of gray market products.

Additionally, the organizational structure decision represents a method for coping with gray marketing over the long term but firms do address the gray market in the short term with other measures. These short-term measures include conducting investigations, refusing to honor warranties for gray market sales, and penalizing sellers for breaching contract terms. To keep the analysis tractable, these short-term measures are not reflected in our analysis. Moreover, by making the foreign and domestic products perfect substitutes of one another, we do not consider any differences in substitution patterns or own price effects across the two economies. Also, there are a number of reasons that a firm might choose to decentralize authority that are unrelated to gray market activity, including motives related to incentive alignment and superior local information possessed by regional managers. We do not explicitly include these motivations in our model. Finally, our single-period model does not reflect the intertemporal trade-offs that firms contemplating global expansion consider in their planning. For example, our model is static and does not reflect the potential growth that may occur in a foreign market or the value that gaining an early foothold in such a market may deliver (despite the immediate problem it creates for the firm in terms of gray market goods). We leave analysis arising from these limitations to future research.

6.2. Summary

Because the global firm is the ultimate originator of all gray market goods, centralization seems to be the obvious solution to the gray market problem. It allows the firm to internalize the negative externality of the gray market. Our analysis demonstrates a limitation to this logic. This limitation has important implications in markets where gray markets are active.

[25] The finding of equilibrium prices equal to zero needs to be interpreted in the context of the normalizations that we employ. Essentially, prices are driven down in a decentralized structure until the markup over marginal cost is zero.

In markets where lead times are long and quantities are determined months before products are brought to market, foreign entry accompanied by decentralized management is advantageous when the gray market is concentrated and the level of competitive intensity between firms is high. The finding holds in situations where either one or both firms enter the foreign market. The advantage of decentralized management arises because decision makers only account for local performance, which makes them more aggressive. When the intensity of competition between firms is high, this can be advantageous as the aggressive posture of a firm's subsidiaries inflicts damage on the competitor.

In a sense, the firm has to manage the tension between maximizing the global industry profit pie and maximizing its share of global industry profits. Maximizing the global industry profit pie is achieved with centralized decision making because the firm internalizes the externality created by gray markets. Maximizing a firm's share of the global profit pie occurs with decentralized control because of more aggressive decisions by locally motivated decision makers. When the level of competitive intensity between products is high, the primary need of each firm is to capture as much of the market as possible. This is achieved with decentralized control. When the level of competitive intensity between the products is low, the primary need of each firm is to limit the damage caused by the gray market. This is achieved with centralized control.

Our analysis makes a different prediction for markets where quantities can be adjusted quickly (i.e., markets where the nature of competition is price based). The fundamental difference between price-based competition and quantity-based competition is that the decisions of competitors are strategic complements as opposed to strategic substitutes. As a result, gray market goods are damaging not only because they lead to lost volume but also because they drive prices downward. In these conditions, even when the level of competitive intensity in the market is high, the primary threat to either firm is the gray market. As a result, centralization is optimal in markets characterized by Bertrand competition. Finally, when there are few barriers to becoming a gray marketer (there are many gray market players), centralized control of foreign subsidiaries should also dominate.

Three practical implications follow from our analysis. First, in industries where stock is produced well before actual sales to consumers, a firm that plans to expand internationally needs to recognize the strategic advantage of decentralization over and above factors such as local knowledge, the need to motivate local management, and regulatory restrictions. Second, in industries where stock is produced well in advance of actual sales, managers should not necessarily be instructed to minimize gray market volume. Sometimes profits are optimized when gray market volume is higher. Finally, it is difficult to assess the impact of gray markets solely by looking at the quantity of gray market goods being sold. The model demonstrates that gray market volume may be significant even in markets where the primary threat for each firm is the direct competitor and not the gray marketer. Conversely, gray markets may not exist at all when a firm manages each subsidiary to "equalize" price across markets. In practice, it is impossible to ascertain the seriousness of the gray market threat by simply observing gray market volume.

Acknowledgments
The authors would like to thank many colleagues and the participants at the 2010 Marketing Science Conference in Cologne and the University of Illinois at Urbana–Champaign brown bag series for their helpful comments and suggestions.

Appendix

Sketch of Solution for Decentralized Entry (Only Firm 1 Enters Foreign Market)

The gray marketer's problem is to optimize $\pi_G = (p_1 - p_F)q_G$ with respect to the choice of q_G. This implies that the gray marketer's optimization is

$$\frac{\partial \pi_G}{\partial q_G} = \frac{\partial}{\partial q_G}(q_G - Fq_G - q_1 q_G + q_F q_G - \gamma q_2 q_G - q_G^2) = 0$$
$$\Rightarrow \quad q_G = \frac{1}{2}(q_F - q_1 - \gamma q_2 + 1 - F). \tag{26}$$

Substituting in q_G and differentiating the objective functions for firm 1 (the domestic and foreign profit functions) and firm 2, we obtain the following first-order conditions:

$$\begin{aligned}
\frac{\partial \pi_1}{\partial q_1} &= \frac{1}{2}F - q_1 - \frac{1}{2}q_F - \frac{1}{2}\gamma q_2 + \frac{1}{2} = 0,\\
\frac{\partial \pi_2}{\partial q_2} &= -\frac{1}{2}\gamma + \frac{1}{2}F\gamma - 2q_2 - \frac{1}{2}\gamma q_1 - \frac{1}{2}\gamma q_F + \gamma^2 q_2 + 1 = 0,\\
\frac{\partial \pi_F}{\partial q_F} &= 2F + \frac{1}{2}q_1 - 3q_F + \frac{1}{2}\gamma q_2 - \frac{1}{2} = 0.
\end{aligned} \tag{27}$$

Solving these expressions for q_1, q_2, and q_F, leads to the following unique solution:

$$\begin{aligned}
q_1 &= \frac{1}{52 - 32\gamma^2}(F(8 - 6\gamma^2) + 7(4 - 2\gamma - \gamma^2)),\\
q_2 &= \frac{1}{52 - 32\gamma^2}(26 - 19\gamma + 2F\gamma),\\
q_F &= \frac{1}{52 - 32\gamma^2}(2F(18 - 11\gamma^2) - (4 - 2\gamma - \gamma^2)),\\
q_G &= \frac{1}{52 - 32\gamma^2}\left(5\left(2 - \gamma - \frac{1}{2}\gamma^2\right) - F(12 - 7\gamma^2)\right).
\end{aligned} \tag{28}$$

For firm 1 to enter the market, $\Pi_1 > 1/(\gamma+2)^2$ is a necessary condition. This implies that $F > f_1(\gamma)$ is a necessary condition where

$$\begin{aligned} f_1(\gamma) &= ((-208+\gamma(-104+\gamma(2+\gamma)(124+\gamma(1-3\gamma)(13+3\gamma))) \\ &\quad + \sqrt{(2+\gamma)^2(13-8\gamma^2)^2(224-336\gamma^2+48\gamma^3+120\gamma^4-24\gamma^5-3\gamma^6)})) \\ &\quad \cdot (2(2+\gamma)^2(52-66\gamma^2+21\gamma^4))^{-1}. \end{aligned} \tag{29}$$

In addition, for the gray market to exist, $p_1 > p_F$ is a necessary condition. This implies that $F < (5(4-2\gamma-\gamma^2))/(2(12-7\gamma^2)) = m_1(\gamma)$.

Sketch of Solution for Decentralized Entry (Both Firms Enter the Foreign Market)

The gray marketer's problem is to optimize $\pi_G = (p_1 - p_{F1})q_{G1} + (p_2 - p_{F2})q_{G2}$ with respect to the choice of q_{G1} and q_{G2}. This leads to the following first-order conditions for the gray marketer:

$$\frac{\partial \pi_G}{\partial q_{G1}} = -F - 2q_{G1} + q_{F1} - q_1 - 2\gamma q_{G2} + \gamma q_{F2} - \gamma q_2 + 1 = 0, \tag{30}$$

$$\frac{\partial \pi_G}{\partial q_{G2}} = -F - 2q_{G2} + q_{F2} - q_2 - 2\gamma q_{G1} + \gamma q_{F1} - \gamma q_1 + 1 = 0. \tag{31}$$

This leads to the following reaction functions for the gray marketer:

$$\begin{aligned} q_{G1} &= \frac{1}{2(\gamma+1)}(q_{F1} - F - q_1 + \gamma q_{F1} - \gamma q_1 + 1), \\ q_{G2} &= \frac{1}{2(\gamma+1)}(q_{F2} - F - q_2 + \gamma q_{F2} - \gamma q_2 + 1). \end{aligned} \tag{32}$$

Substituting and differentiating the objective functions for firms 1 and 2 (the domestic and foreign profit functions), we obtain the following first-order conditions:

$$\frac{\partial \pi_1}{\partial q_1} = \frac{1}{2}(1 + F - 2q_1 - q_{F1} - \gamma q_2 - \gamma q_{F2}) = 0, \tag{33}$$

$$\frac{\partial \pi_2}{\partial q_2} = \frac{1}{2}(1 + F - 2q_2 - q_{F2} - \gamma q_1 - \gamma q_{F1}) = 0, \tag{34}$$

$$\frac{\partial \pi_{F1}}{\partial q_{F1}} = \frac{1}{2}\left(3F + q_1 - 6q_{F1} - 3\gamma q_{F2} - \frac{1-F}{1+\gamma}\right) = 0, \tag{35}$$

$$\frac{\partial \pi_{F2}}{\partial q_{F2}} = \frac{1}{2}\left(3F + q_2 - 6q_{F2} - 3\gamma q_{F1} - \frac{1-F}{1+\gamma}\right) = 0. \tag{36}$$

Solving these expressions for q_1, q_2, q_{F1}, and q_{F2} yields the following unique solution:

$$\begin{aligned} q_1 = q_2 &= \frac{7+3\gamma+2F}{13+13\gamma+3\gamma^2} \quad \text{and} \\ q_{F1} = q_{F2} &= \frac{F(9+11\gamma+3\gamma^2)-1}{(1+\gamma)(13+13\gamma+3\gamma^2)}, \end{aligned} \tag{37}$$

$$q_{G1} = q_{G2} = \frac{5+3\gamma-6F-4F\gamma}{2(1+\gamma)(13+13\gamma+3\gamma^2)}. \tag{38}$$

Sketch of Solution for Centralized Entry (Only Firm 1 Enters Foreign Market)

The gray marketer's problem is unaffected by the organizational structure of the competing firms so the reactions function for the gray marketer in the centralized conditions are identical to Equation (26). Substituting and differentiating the objective functions for firms 1 and 2, we obtain the following first-order conditions:

$$\begin{aligned} \frac{\partial \Pi_1}{\partial q_1} &= -q_1 - \frac{1}{2}\gamma q_2 + \frac{1}{2} = 0, \\ \frac{\partial \pi_2}{\partial q_2} &= -\frac{1}{2}\gamma + \frac{1}{2}F\gamma - 2q_2 - \frac{1}{2}\gamma q_1 - \frac{1}{2}\gamma q_F + \gamma^2 q_2 + 1 = 0, \\ \frac{\partial \Pi_1}{\partial q_F} &= 2F - 3q_F + \frac{1}{2}\gamma q_2 - \frac{1}{2} = 0. \end{aligned} \tag{39}$$

Solving these expressions for q_1, q_2, and q_F leads to the following unique solution:

$$\begin{aligned} q_1 &= \frac{1}{2(12-7\gamma^2)}(12 - 6\gamma - 3\gamma^2 - F\gamma^2), \\ q_2 &= \frac{1}{12-7\gamma^2}(6 - 4\gamma + F\gamma), \\ q_F &= \frac{1}{2(12-7\gamma^2)}(16F - 9F\gamma^2 - (4 - 2\gamma - \gamma^2)), \\ q_G &= \frac{1}{2(12-7\gamma^2)}(4 - 2\gamma - \gamma^2 - 4F + 2F\gamma^2). \end{aligned} \tag{40}$$

For firm 1 to enter the market, $\Pi_1 > 1/(\gamma+2)^2$ is a necessary condition. This implies that $F > g_1(\gamma)$ is a necessary condition, where

$$\begin{aligned} g_1(\gamma) &= ((-192 + 3\gamma(-32 + \gamma(2+\gamma)(40 + 4\gamma - 12\gamma^2 - 3\gamma^3)) \\ &\quad + \sqrt{(2+\gamma)^2(12-7\gamma^2)^2(192-288\gamma^2+48\gamma^3+104\gamma^4-24\gamma^5-3\gamma^6)})) \\ &\quad \cdot (2(2+\gamma)^2(48 - 60\gamma^2 + 19\gamma^4))^{-1}. \end{aligned} \tag{41}$$

In addition, for the gray market to exist, $p_1 > p_F$ is a necessary condition. This implies that $F < (4-2\gamma-\gamma^2)/(4-2\gamma^2) = n_1(\gamma)$.

Sketch of Solution for Centralized Entry (Both Firms Enter the Foreign Market)

The gray marketer's problem is unaffected by the organizational structure of the competing firms so the reactions function for the gray marketer in the centralized conditions are identical to Equation (32). Substituting and differentiating the objective functions for firms 1 and 2 (the domestic and foreign profit functions), we obtain the following first-order conditions:

$$\frac{\partial \Pi_1}{\partial q_1} = -q_1 - \frac{1}{2}\gamma q_2 + \frac{1}{2} = 0, \tag{42}$$

$$\frac{\partial \Pi_2}{\partial q_2} = -q_2 - \frac{1}{2}\gamma q_1 + \frac{1}{2} = 0, \tag{43}$$

$$\begin{aligned} \frac{\partial \Pi_1}{\partial q_{F1}} &= \frac{1}{2(\gamma+1)}(4F + 3F\gamma - 6q_{F1} - 6\gamma q_{F1} \\ &\qquad - 3\gamma q_{F2} - 3\gamma^2 q_{F2} - 1) = 0, \end{aligned} \tag{44}$$

$$\frac{\partial \Pi_2}{\partial q_{F2}} = \frac{1}{2(\gamma+1)}(4F+3F\gamma-6q_{F2}-3\gamma q_{F1} - 6\gamma q_{F2}-3\gamma^2 q_{F1}-1)=0. \quad (45)$$

Solving these expressions for q_1, q_2, q_{F1}, and q_{F2} leads to the following unique solution:

$$q_1=q_2=\frac{1}{\gamma+2} \quad\text{and}\quad q_{F1}=q_{F2}=\frac{4F+3F\gamma-1}{3(2+3\gamma+\gamma^2)} \quad\text{and}\quad q_{G1}=q_{G2}=\frac{1-F}{3(2+3\gamma+\gamma^2)}. \quad (46)$$

Sketch of Solution for Asymmetric Entry (Both Firms Enter the Foreign Market)

As before, the gray marketer's problem is unaffected by the organizational structure of the competing firms so the reactions function for the gray marketer in the centralized conditions are identical to Equation (32). Substituting and differentiating the objective functions for firms 1 and 2 (the domestic and foreign profit functions), we obtain the following first-order conditions:

$$\frac{\partial \Pi_1}{\partial q_1}=-q_1-\frac{1}{2}\gamma q_2+\frac{1}{2}=0, \quad (47)$$

$$\frac{\partial \pi_2}{\partial q_2}=\frac{1}{2}F-\frac{1}{2}q_{F2}-q_2-\frac{1}{2}\gamma q_{F1}-\frac{1}{2}\gamma q_1+\frac{1}{2}=0, \quad (48)$$

$$\frac{\partial \Pi_1}{\partial q_{F1}}=\frac{4F+3F\gamma-6q_{F1}-6\gamma q_{F1}-3\gamma q_{F2}-3\gamma^2 q_{F2}-1}{2(\gamma+1)}=0, \quad (49)$$

$$\frac{\partial \pi_{F2}}{\partial q_{F2}}=\frac{4F+3F\gamma-6q_{F2}+q_2-3\gamma q_{F1}-6\gamma q_{F2}+\gamma q_2-3\gamma^2 q_{F1}-1}{2(\gamma+1)}=0. \quad (50)$$

Solving these expressions for q_1, q_2, q_{F1}, and q_{F2} leads to the following unique solution:

$$q_1=\frac{3\gamma^3-4F\gamma-6\gamma^2-14\gamma+2F\gamma^2+26}{3\gamma^4-26\gamma^2+52}, \quad q_2=\frac{(2-\gamma)(14+4F-3\gamma^2)}{3\gamma^4-26\gamma^2+52}, \quad (51)$$

$$q_{F1}=\frac{104F+6\gamma+24F\gamma+3\gamma^2-66F\gamma^2-6F\gamma^3+9F\gamma^4-26}{3(\gamma+1)(3\gamma^4-26\gamma^2+52)}, \quad (52)$$

$$q_{F2}=\frac{108F+20\gamma+28F\gamma-3\gamma^3-66F\gamma^2-6F\gamma^3+9F\gamma^4-12}{3(\gamma+1)(3\gamma^4-26\gamma^2+52)}, \quad (53)$$

$$q_{G1}=\frac{52-52F-30\gamma+36F\gamma-15\gamma^2+18F\gamma^2+9\gamma^3-12F\gamma^3}{6(\gamma+1)(3\gamma^4-26\gamma^2+52)}, \quad (54)$$

$$q_{G2}=\frac{30-36F-11\gamma+8F\gamma-9\gamma^2+12F\gamma^2+3\gamma^3-3F\gamma^3}{3(\gamma+1)(3\gamma^4-26\gamma^2+52)}. \quad (55)$$

These quantities lead to the following profit expressions. To highlight the different structures associated with each firm, we replace the firms identifiers, 1 and 2, with the subscripts *cent* and *decent*.

$$\begin{aligned}\Pi_{\text{cent}}=&((2{,}704+1{,}248\gamma^3+36\gamma^6)(1+F+F^2)-3{,}224\gamma-104\gamma^2\\&-255\gamma^4-126\gamma^5-4{,}160F\gamma+832F\gamma^2-480F\gamma^4-72F\gamma^5\\&-2{,}912F^2\gamma-416F^2\gamma^2-180F^2\gamma^4-144F^2\gamma^5)\\&\cdot(6(3\gamma^4-26\gamma^2+52)^2)^{-1},\end{aligned}$$

$$\Pi_{\text{decent}}=\frac{(2-\gamma)^2(208(1+F+F^2)-12\gamma^2(7+4F+8F^2)+3\gamma^4(3+4F^2))}{2(3\gamma^4-26\gamma^2+52)^2},$$

$$\begin{aligned}\pi_G=&(6{,}304-5{,}824\gamma-2{,}400\gamma^2+3{,}396\gamma^3-207\gamma^4-495\gamma^5\\&+108\gamma^6-14{,}048F+12{,}272F\gamma+6{,}360F\gamma^2-7{,}776F\gamma^3\\&+204F\gamma^4+1{,}224F\gamma^5-252F\gamma^6+7{,}888F^2-6{,}448F^2\gamma\\&-4{,}176F^2\gamma^2+4{,}416F^2\gamma^3+84F^2\gamma^4-756F^2\gamma^5+144F^2\gamma^6)\\&\cdot(36(1+\gamma)(3\gamma^4-26\gamma^2+52)^2)^{-1}.\end{aligned}$$

Proof of Proposition 1

When only firm 1 is capable of entering the foreign market, firm 1's profit from decentralized entry is given in (10), from centralized entry is given in (14), and nonentry profit is $1/(\gamma+2)^2$. We previously showed that $\Pi_1^D > 1/(\gamma+2)^2$ if and only if $F > f_1(\gamma)$ and $\Pi_1^C > 1/(\gamma+2)^2$ if and only if $F > g_1(\gamma)$ (see derivations at (29) and (41)). When both $F < f_1(\gamma)$ and $F < g_1(\gamma)$, firm 1's best option is nonentry. Therefore it is only in firm 1's interest to enter the foreign market when $F > f_1(\gamma)$ or $F > g_1(\gamma)$.

Next, we derive the boundary B_N by calculating $\Pi_1^D - \Pi_1^C$, which is quadratic in F. Solving for F^* yields an expression in γ with the following discriminant:

$$DS_N=\sqrt{(-2\gamma^4+6\gamma^2-3)(13-8\gamma^2)^2(12-7\gamma^2)^2(4-2\gamma-\gamma^2)^2}. \quad (56)$$

The discriminant $DS_N=0$ when $\gamma=\gamma_N^*=\frac{1}{2}\sqrt{2}\sqrt{3-\sqrt{3}}$. At γ_N^*, $F^*=(3/218)(60+\sqrt{3}-2\sqrt{1{,}611-807\sqrt{3}})$. When $\gamma<\gamma_N^*$, F^* has complex roots. The upper and lower roots of F^* are as follows:

$$\begin{aligned}\bar{F}_N=&(-624+312\gamma+2{,}304\gamma^2-1{,}074\gamma^3-2{,}501\gamma^4+982\gamma^5\\&+1{,}031\gamma^6-270\gamma^7-135\gamma^8+DS_N)\cdot(2(1+\gamma)(1-\gamma)\\&\cdot(624-1{,}260\gamma^2+843\gamma^4-187\gamma^6))^{-1}, \quad (57)\end{aligned}$$

$$\begin{aligned}\underline{F}_N=&(-624+312\gamma+2{,}304\gamma^2-1{,}074\gamma^3-2{,}501\gamma^4+982\gamma^5\\&+1{,}031\gamma^6-270\gamma^7-135\gamma^8-DS_N)\cdot(2(1+\gamma)(1-\gamma)\\&\cdot(624-1{,}260\gamma^2+843\gamma^4-187\gamma^6))^{-1}. \quad (58)\end{aligned}$$

When $\gamma>\gamma_N^*$, the boundary B_N is defined by $\bar{F}_N$ for $F > (3/218)(60+\sqrt{3}-2\sqrt{1{,}611-807\sqrt{3}})$ and $\underline{F}_N$ for $F<(3/218)\cdot(60+\sqrt{3}-2\sqrt{1{,}611-807\sqrt{3}})$.

When the market characteristics (γ, F) lie to the right of B_N and $F > f_1(\gamma)$ then firm 1's profit is higher under decentralized entry than centralized entry or no entry. Conversely, when the market characteristics (γ, F) lie to the left of B_N and $F > g_1(\gamma)$, firm 1 is better off under centralized entry. This completes the proof of Proposition 1.

Proof of Lemma 1

When both $F < f_1(\gamma \mid s_2)$ and $F < g_1(\gamma \mid s_2)$, firm 1's best option is nonentry. Therefore, we restrict our proof to region in which $F > f_1(\gamma \mid s_2)$ or $F > g_1(\gamma \mid s_2)$. We consider three cases: (a) firm 2 chooses nonentry, (b) firm 2 chooses decentralized entry, and (c) firm 2 chooses centralized entry. For each case, we derive the respective boundary as in Proposition 1 by calculating the cutoff F where $\Pi_1^D - \Pi_1^C$ (given firm 2's choice) changes sign. Boundary B_N is given in Equations (57) and (58). We next determine the remaining boundaries B_D and B_C.

First, we derive the boundary B_D by calculating $\Pi_1^D - \Pi_1^C$ when firm 2 has entered the foreign market with *decentralized* control. Solving this expression for F yields an expression in γ with the following discriminant:

$$DS_D = (3(-169+338\gamma^2+338\gamma^4-234\gamma^6+27\gamma^8) \cdot (52-26\gamma^2+3\gamma^4)^2(13+13\gamma+3\gamma^2)^2)^{1/2}. \quad (59)$$

The discriminant $DS_D = 0$ when $\gamma = \gamma_D^* = \frac{1}{3}[\frac{1}{2}(39-7\sqrt{39}+\sqrt{78(8-\sqrt{39})})]^{1/2}$. At γ_D^*, $F^* = \frac{1}{4}\sqrt{13}-\frac{1}{4}$. When $\gamma < \gamma_D^*$, F^* has complex roots. The upper and lower roots of F^* are as follows:

$$\bar{F}_D = ((-8{,}788-4{,}056\gamma+43{,}940\gamma^2+32{,}448\gamma^3-22{,}984\gamma^4 -22{,}542\gamma^5+1{,}911\gamma^6+5{,}148\gamma^7+612\gamma^8-378\gamma^9-81\gamma^{10} +DS_D))\cdot(2(1-\gamma)(8{,}788+16{,}900\gamma+1{,}352\gamma^2-10{,}816\gamma^3 -3{,}549\gamma^4+1{,}989\gamma^5+819\gamma^6-117\gamma^7-54\gamma^8))^{-1}, \quad (60)$$

$$\underline{F}_D = ((-8{,}788-4{,}056\gamma+43{,}940\gamma^2+32{,}448\gamma^3-22{,}984\gamma^4 -22{,}542\gamma^5+1{,}911\gamma^6+5{,}148\gamma^7+612\gamma^8-378\gamma^9-81\gamma^{10} -DS_D))\cdot(2(1-\gamma)(8{,}788+16{,}900\gamma+1{,}352\gamma^2-10{,}816\gamma^3 -3{,}549\gamma^4+1{,}989\gamma^5+819\gamma^6-117\gamma^7-54\gamma^8))^{-1}. \quad (61)$$

When $\gamma > \gamma_D^*$, the boundary B_D is defined by $\bar{F}_D$ for $F > \frac{1}{4}\sqrt{13}-\frac{1}{4}$ and $\underline{F}_D$ for $F < \frac{1}{4}\sqrt{13}-\frac{1}{4}$. As depicted in Figure A.1, when the market characteristics (γ, F) lie to the right of B_D and $F > f_1(\gamma \mid D)$ then firm 1's profit is higher under decentralization than centralization or no entry. Conversely, when the market characteristics (γ, F) lie to the left of B_D and $F > g_1(\gamma \mid D)$, firm 1 is better off under centralized entry.

We next derive the boundary B_C by calculating $\Pi_1^D - \Pi_1^C$ when firm 2 has entered the foreign market with *centralized* control. Solving this expression for F yields an expression in γ with the following discriminant:

$$DS_C = \sqrt{3(-16+16\gamma^2+44\gamma^4-24\gamma^6+3\gamma^8)(52-26\gamma^2+3\gamma^4)^2}. \quad (62)$$

Figure A.1 F **Constraints for Firm 1 Given Decentralized Entry by Firm 2: No Decentralized Entry for** $F < f_1(\gamma \mid D)$**; No Centralized Entry for** $F < g_1(\gamma \mid D)$

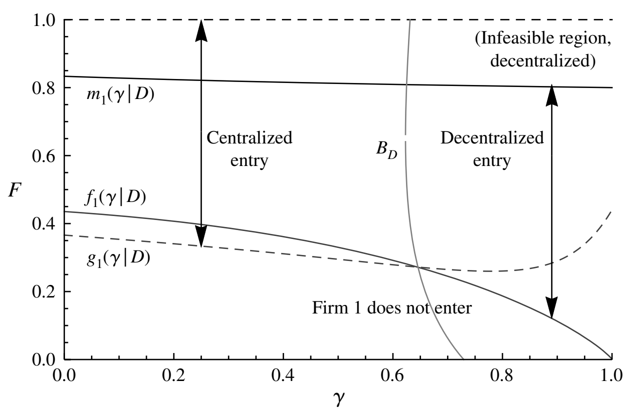
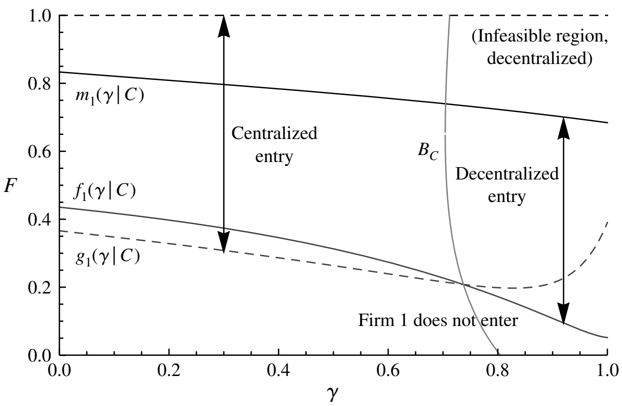


Figure A.2 F **Constraints for Firm 1 Given Centralized Entry by Firm 2: No Decentralized Entry for** $F < f_1(\gamma \mid C)$**; No Centralized Entry for** $F < g_1(\gamma \mid C)$



The discriminant $DS_C = 0$ when $\gamma = \gamma_C^* = \frac{1}{3}\sqrt{3}[6-\sqrt{6}\sqrt{7-\sqrt{13}}]^{1/2}$. At γ_C^*, $F^* = \frac{1}{4}\sqrt{13}-\frac{1}{4}$. When $\gamma < \gamma_C^*$, F^* has complex roots. The upper and lower roots of F^* are as follows:

$$\bar{F}_C = \frac{(208-880\gamma^2+544\gamma^4-120\gamma^6+9\gamma^8-DS_C)}{8(-52+76\gamma^2-28\gamma^4+3\gamma^6)}, \quad (63)$$

$$\underline{F}_C = \frac{(208-880\gamma^2+544\gamma^4-120\gamma^6+9\gamma^8+DS_C)}{8(-52+76\gamma^2-28\gamma^4+3\gamma^6)}. \quad (64)$$

When $\gamma > \gamma_C^*$, the boundary B_C is defined by $\bar{F}_C$ for $F > \frac{1}{4}\sqrt{13}-\frac{1}{4}$ and $\underline{F}_C$ for $F < \frac{1}{4}\sqrt{13}-\frac{1}{4}$. As depicted in Figure A.2, when the market characteristics (γ, F) lie to the right of B_C and $F > f_1(\gamma \mid C)$ then firm 1's profit is higher under decentralization than centralization or no entry. Conversely, when the market characteristics (γ, F) lie to the left of B_C and $F > g_1(\gamma \mid C)$, firm 1 is better off under centralized entry.

To summarize,

$$B_{S_2} = \begin{cases} B_N & \text{if firm 2 chooses nonentry;} \\ B_D & \text{if firm 2 chooses decentralized entry;} \\ B_C & \text{if firm 2 chooses centralized entry.} \end{cases}$$

When the market characteristics (γ, F) lie to the left of B_{S_2}, then firm 1's profit is higher under centralized entry than under decentralized entry. Conversely, when the market characteristics (γ, F) lie to the right of B_{S_2}, then firm 1's profit is higher under decentralized entry than under centralized entry. Provided entry is preferred to nonentry, firm 1's best choice is determined by comparing the market characteristics versus the relevant boundary. This proves Lemma 1.

Sketch of Proof of Proposition 2

This proof proceeds as follows. First, we establish the details of firm 1's best response function. Second, we solve for all the equilibrium regions and for any region with multiple equilibria, we determine the Pareto optimal equilibrium.

1. When both firms can enter the foreign market, firm 2 has three possible choices: no entry, decentralized entry, and centralized entry. We modify our notation to denote firm 2's choice as follows: "|N," "|D," and "|C." When firm 2 chooses no entry, firm 1's best responses are given in Proposition 1.

The decentralized and centralized entry responses are solved in a similar manner, resulting in the following expressions:

$$\begin{aligned} g_1(\gamma \mid D) = {} & ((-1{,}827{,}904+3{,}233{,}984\gamma+1{,}379{,}040\gamma^2 \\ & -4{,}937{,}504\gamma^3+786{,}864\gamma^4+2{,}517{,}008\gamma^5 \\ & -903{,}448\gamma^6-480{,}792\gamma^7+226{,}932\gamma^8+32{,}004\gamma^9 \\ & -19{,}134\gamma^{10}-702\gamma^{11}+513\gamma^{12}) \\ & +4(3(676-754\gamma^2+247\gamma^4-24\gamma^6)^2(456{,}976 \\ & -1{,}476{,}384\gamma+1{,}388{,}504\gamma^2+275{,}808\gamma^3 \\ & -1{,}120{,}132\gamma^4+393{,}432\gamma^5+204{,}724\gamma^6-118{,}560 \\ & \cdot\gamma^7-23{,}490\gamma^8+23{,}166\gamma^9-3{,}627\gamma^{10}))^{1/2}) \\ & \cdot(2(1{,}827{,}904-1{,}968{,}512\gamma-2{,}547{,}168\gamma^2 \\ & +3{,}266{,}432\gamma^3+940{,}992\gamma^4-1{,}881{,}152\gamma^5 \\ & +53{,}560\gamma^6+439{,}296\gamma^7-72{,}132\gamma^8-36{,}864\gamma^9 \\ & +8{,}694\gamma^{10}+27\gamma^{12}))^{-1}, \qquad (65) \end{aligned}$$

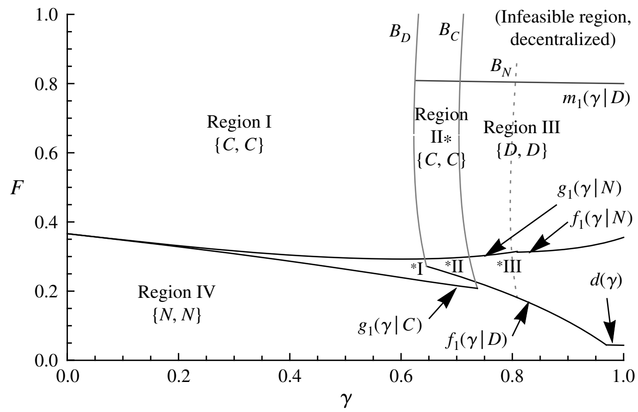
$$\begin{aligned} g_1(\gamma \mid C) = {} & (2((-144+72\gamma+192\gamma^2-66\gamma^3-73\gamma^4+15\gamma^5+6\gamma^6) \\ & +(3(12-7\gamma^2)^2(144-144\gamma-168\gamma^2+132\gamma^3+89\gamma^4 \\ & -30\gamma^5-21\gamma^6))^{1/2}))\cdot(576-696\gamma^2-24\gamma^3+202\gamma^4 \\ & +12\gamma^5+3\gamma^6)^{-1}, \qquad (66) \end{aligned}$$

$$\begin{aligned} f_1(\gamma \mid D) = {} & ((-7{,}436\gamma+54{,}418\gamma^2+15{,}574\gamma^3-24{,}219\gamma^4 \\ & -8{,}640\gamma^5+2{,}323\gamma^6+1{,}248\gamma^7+171\gamma^8) \\ & +4(-8{,}788+((169+169\gamma-65\gamma^2-104\gamma^3-24\gamma^4)^2 \\ & \cdot(9{,}464-10{,}816\gamma-8{,}606\gamma^2+8{,}684\gamma^3+4{,}013\gamma^4 \\ & -1{,}638\gamma^5-1{,}092\gamma^6))^{1/2}))\cdot(2(35{,}152+32{,}448\gamma \\ & -35{,}490\gamma^2-40{,}612\gamma^3+3{,}003\gamma^4+12{,}470\gamma^5 \\ & +3{,}301\gamma^6+78\gamma^7+9\gamma^8))^{-1}, \qquad (67) \end{aligned}$$

$$\begin{aligned} f_1(\gamma \mid C) = {} & ((-59{,}904+92{,}352\gamma+47{,}104\gamma^2-132{,}384\gamma^3 \\ & +16{,}864\gamma^4+64{,}592\gamma^5-23{,}144\gamma^6-12{,}504\gamma^7 \\ & +6{,}376\gamma^8+1{,}044\gamma^9-696\gamma^{10}-54\gamma^{11}+36\gamma^{12}) \\ & +(3(1{,}248-624\gamma-1{,}352\gamma^2+676\gamma^3+436\gamma^4 \\ & -218\gamma^5-42\gamma^6+21\gamma^7)^2(2{,}688-5{,}184\gamma+224\gamma^2 \\ & +4{,}320\gamma^3-1{,}336\gamma^4-1{,}232\gamma^5+398\gamma^6+192\gamma^7 \\ & -67\gamma^8))^{1/2})\cdot(119{,}808-119{,}808\gamma-170{,}528\gamma^2 \\ & +195{,}072\gamma^3+71{,}536\gamma^4-112{,}192\gamma^5-3{,}512\gamma^6 \\ & +26{,}880\gamma^7-3{,}068\gamma^8-2{,}352\gamma^9+414\gamma^{10} \\ & +9\gamma^{12})^{-1}. \qquad (68) \end{aligned}$$

We illustrate firm 1's best response functions for decentralized and centralized entry in Figures A.1 and A.2, respectively. Figures A.1 and A.2, respectively, also show the infeasible regions above the decentralized cutoff, $m_1(\gamma \mid D)$ and $m_1(\gamma \mid C)$, where the $q_{G1} < 0$; however, the corresponding cutoff with centralized entry never binds, i.e., $n_1(\gamma \mid D)$ and $n_1(\gamma \mid C) \geq 1$.

Figure A.3 **Equilibrium Regions When Both Firms Can Enter Foreign Market**



Note. Multiple equilibria denoted by "*".

2. The equilibrium regions are determined as follows. Figure A.3 depicts these regions; subregions i, ii, and iii are included in region IV. Areas with multiple equilibria are denoted by "*".

For convenience, we define $\gamma_{B_{S_2}} \equiv B_{S_2}^{-1}(\gamma)$, for $s_2 \in \{N, D, \text{or } C\}$. We begin by noting that for any given $F \in (0,1)$, $\gamma_{B_D} < \gamma_{B_C} < \gamma_{B_N}$ (in plain terms, B_D is strictly to the left of B_C and B_C is strictly to the left of B_N). Proposition 2 pertains to the case in which both firms prefer entry.

(a) Region I: unique $\{C, C\}$ equilibrium. Region I is bounded by B_D on the right because irrespective of whether firm 2 chooses decentralized or centralized entry, to the left of B_D firm 1's best response is centralized entry or no entry. When will entry be preferred to the left of B_D (i.e., what is region I's lower boundary)? Since $\gamma_{B_D} < \gamma_{B_N}$, for all $F > g_1(\gamma \mid N)$ entry will be uniquely preferred, and $\{C, C\}$ is the unique Nash equilibrium. However, when F lies below $g_1(\gamma \mid N)$, and above $g_1(\gamma \mid C)$, there are three equilibria: $\{C, C\}$, $\{N, N\}$, and a mixed strategy. In this parameter region, $\Pi_1^{NN} > \Pi_1^{CC}$ so the Pareto dominant equilibrium is $\{N, N\}$, and thus entry is not preferred by the firms. Last, for all $F < g_1(\gamma \mid C)$, $\{N, N\}$ is the unique Nash equilibrium. Therefore, region I is bounded below by $g_1(\gamma \mid N)$.

(b) Region II: Pareto-dominant $\{C, C\}$ equilibrium. Region II is bounded by B_D on the left and B_C on the right. When the market characteristics (γ, F) lie between B_D and B_C, there are three equilibria. First, suppose entry is preferred. If firm 2 chooses D, then because the market characteristics are to the right of B_D, firm 1 will choose D as well, yielding a $\{D, D\}$ equilibrium (provided $F < m_1(\gamma \mid D)$). If firm 2 chooses C, then because the market characteristics are to the left of B_C, firm 1 will choose C yielding a $\{C, C\}$ equilibrium. As before, a mixed equilibrium also exists. In this region, Π_1^{CC} is the most profitable of the three equilibria, so it Pareto dominates.[26] When will entry be preferred when the market characteristics (γ, F) lie between B_D and B_C? Since $\gamma_{B_C} < \gamma_{B_N}$, for all $F > g_1(\gamma \mid N)$, entry is preferred irrespective of Firm 2's strategy. However, when F lies below $g_1(\gamma \mid N)$ and above

[26] This equilibrium refinement also creates the most conservative possible boundary for our counterintuitive equilibrium, $\{D, D\}$.

$g_1(\gamma \mid C)$, there are multiple equilibria (including several mixed strategies), which now include $\{N, N\}$. Of all of these equilibria, Π_1^{NN} is the most profitable, so the Pareto dominant equilibrium is $\{N, N\}$ for $F < g_1(\gamma \mid N)$, and thus entry is not preferred by the firms. Last, for all $F < g_1(\gamma \mid C)$, the unique Nash equilibrium is $\{N, N\}$. Therefore, region II is bounded below by $g_1(\gamma \mid N)$.

(c) Region III: unique $\{D, D\}$ equilibrium. Region III is bounded by B_C on the left because irrespective of whether firm 2 chooses decentralized or centralized entry, to the right of B_C firm 1's best response is decentralized entry or no entry (provided $F < m_1(\gamma \mid D)$). When will entry be preferred to the right of B_C? Since $f_1(\gamma \mid D) < \min[f_1(\gamma \mid N), g_1(\gamma \mid N)]$, then for all F above both $f_1(\gamma \mid N)$ and $g_1(\gamma \mid N)$, entry will be uniquely preferred, and $\{D, D\}$ is the unique Nash equilibrium. However, when F lies below $\min[f_1(\gamma \mid N), g_1(\gamma \mid N)]$ and above $\max[f_1(\gamma \mid D), d(\gamma)]$, there are three equilibria: $\{D, D\}$, $\{N, N\}$, and a mixed strategy. Of all of these equilibria, Π_1^{NN} is the most profitable, so the Pareto dominant equilibrium is $\{N, N\}$ for $F < \min[f_1(\gamma \mid N), g_1(\gamma \mid N)]$, and thus entry is not preferred by the firms. Last, for $F < \max[f_1(\gamma \mid D), d(\gamma)]$, the unique Nash equilibrium is $\{N, N\}$. Therefore, region III is bounded below by $\min[f_1(\gamma \mid N), g_1(\gamma \mid N)]$.

(d) Region IV: Unique or Pareto-dominant $\{N, N\}$ equilibrium. This region is comprised of all of the parameter ranges excluded from regions I, II, and III because $\{N, N\}$ is either Pareto optimal or the unique equilibrium. In this region, entry is not preferred by firms.

This completes the proof of Proposition 2.

Bertrand Competition: Equilibrium Prices and Gray Market Quantities

Decentralized optimal prices and gray market quantity in the Bertrand setting

$$p_1^{\text{DBer}} = \frac{(1-\gamma)(4+F+\gamma(2+F))}{3(5-3\gamma^2)}$$
$$p_F^{\text{DBer}} = \frac{(2-\gamma-\gamma^2+F(8-5\gamma^2))}{6(5-3\gamma^2)},$$
$$p_2^{\text{DBer}} = \frac{(1-\gamma)(15+\gamma(4+F)-\gamma^2(7-F))}{6(5-3\gamma^2)},$$
$$q_G^{\text{DBer}} = \frac{2-\gamma-\gamma^2-F(2-\gamma^2)}{2(5-3\gamma^2)}.$$

Centralized optimal prices and gray market quantity in the Bertrand setting

$$p_1^{\text{CBer}} = \frac{(1-\gamma)(2+F+\gamma(1+F))}{3(2-\gamma^2)}$$
$$p_F^{\text{CBer}} = \frac{1}{12}\left(2+5F-\frac{2(F+\gamma)}{2-\gamma^2}\right),$$
$$p_2^{\text{CBer}} = \frac{(1-\gamma)(6+\gamma(2+F)-\gamma^2(2-F))}{6(2-\gamma^2)},$$
$$q_G^{\text{CBer}} = \frac{(2+\gamma)(2(1-\gamma)-F(2-\gamma))}{12(2-\gamma^2)}.$$

Proof of Proposition 3

We subtract the decentralized profit, Π_1^{DBer}, derived in (19), from the centralized profit, Π_1^{CBer}, derived in (21), yielding the difference

$$\Pi_1^{\text{CBer}} - \Pi_1^{\text{DBer}} = \frac{1}{72(10-11\gamma^2+3\gamma^4)^2}(4(2-\gamma-\gamma^2)^2(7-4\gamma^2) + F^2(112-276\gamma^2+237\gamma^4-82\gamma^6+9\gamma^8) + 8F(1-\gamma)(2+\gamma)(11-5\gamma^2(3-\gamma^2))).$$

By assumption, $F > 0$, $\gamma > 0$, and $\gamma < 1$. Thus,

$$72(10-11\gamma^2+3\gamma^4)^2 > 0$$
$$4(2-\gamma-\gamma^2)^2(7-4\gamma^2) > 0$$
$$F^2(112-276\gamma^2+237\gamma^4-82\gamma^6+9\gamma^8) > 0$$
$$8F(1-\gamma)(2+\gamma)(11-5\gamma^2(3-\gamma^2)) > 0.$$

Therefore, $\Pi_1^{\text{CBer}} > \Pi_1^{\text{DBer}}$. This proves Proposition 3.

Solution for the *n*th Gray Marketer's Reaction Functions

The nth gray marketer's problem is to optimize Equation (25) subject to (23) and (5), with respect to the choice of q_{G1n} and q_{G2n}. This leads to the following first-order conditions for each of the N gray marketers:

$$\frac{\partial \pi_{Gn}}{\partial q_{G1n}} = 1 - q_1 - 2q_{G1n} - \sum_{m\neq n} q_{G1m} - \gamma\left(q_2 + 2q_{G2n} + \sum_{m\neq n} q_{G2m}\right) - F + q_{F1} + \gamma q_{F2} = 0, \tag{69}$$

$$\frac{\partial \pi_{Gn}}{\partial q_{G2n}} = 1 - q_2 - 2q_{G2n} - \sum_{m\neq n} q_{G2m} - \gamma\left(q_1 + 2q_{G1n} + \sum_{m\neq n} q_{G1m}\right) - F + q_{F2} + \gamma q_{F1} = 0. \tag{70}$$

Solving simultaneously and applying symmetry yields the following best response functions for each gray marketer:

$$\begin{aligned} q_{G1n} &= \frac{1}{(1+N)(1+\gamma)}(1-q_1-q_1\gamma-F+q_{F1}+\gamma q_{F1}),\\ q_{G2n} &= \frac{1}{(1+N)(1+\gamma)}(1-q_2-q_2\gamma-F+q_{F2}+\gamma q_{F2}). \end{aligned} \tag{71}$$

Sketch of Proof of Proposition 4

The case of $N = 1$ is shown in Proposition 3. To extend the proof to $N > 1$ (i.e., multiple gray marketers), we first compute the equilibrium quantities and profits as a function of N using the optimal gray market quantities in (71) and following the same process as in §4.4.2.

Next we derive the generalized boundary $B_C(N, \gamma)$, firm 1's boundary between optimally choosing decentralized versus centralized entry, given that firm 2 chooses *centralized* entry with N gray marketers. We denote firm 1's profits in this situation as Π_1^{Dmult} for choosing decentralized entry and Π_1^{Cmult} for choosing centralized entry. We derive $B_C(N, \gamma)$ by solving $\Pi_1^{\text{Dmult}} - \Pi_1^{\text{Cmult}}$ for F, which yields an expression with the following discriminant:

$$DS_{\text{Cmult}} = ((1+2N)((4-\gamma^2)^2(1+2N)+2N^2(2-\gamma^2))^2 \cdot (\gamma^4(4-\gamma^2)^2(1+2N)-4N^2(2-\gamma^2)^2))^{1/2}. \tag{72}$$

The discriminant $DS_{\text{Cmult}} = 0$ when

$$\gamma = \gamma_C^*(N) = \sqrt{2 - \frac{\sqrt{2(1+2N)(2+4N+N^2-\sqrt{N^2(4+8N+N^2)})}}{1+2N}}.$$

We note that $\gamma_C^*(N)$ is increasing in N for $\forall N \in [1, \infty)$ and that $\gamma_C^*(N) < 1$ for $\forall N < \frac{3}{4}(3+\sqrt{13}) \simeq 4.95$. Given that the number of gray market competitors is an integer, the maximum number of gray market competitors for which $\gamma_C^*(N) \in (0, 1)$ is 4.

We define $F_C^*(N) \equiv F^*|_{\gamma=\gamma_C^*(N)}$. F^* has complex roots when $\gamma < \gamma_C^*(N)$. The upper and lower roots of F^* are as follows:

$$\begin{aligned}\bar{F}_{\text{Cmult}} = {} & ((-\gamma^2(4-\gamma^2)^3(1+4N)+4N^4(2-\gamma^2)^2+4N^2(4-\gamma^2)^2 \\ & \cdot(2N(1-\gamma^2)+1-5\gamma^2+\gamma^4))-DS_{\text{Cmult}})\cdot(-4N(1+N) \\ & (N^2(2-\gamma^2)^2+(4-\gamma^2)^2(1-\gamma^2)(1+2N)))^{-1}, \qquad (73)\end{aligned}$$

$$\begin{aligned}\underline{F}_{\text{Cmult}} = {} & ((-\gamma^2(4-\gamma^2)^3(1+4N)+4N^4(2-\gamma^2)^2+4N^2(4-\gamma^2)^2 \\ & (2N(1-\gamma^2)+1-5\gamma^2+\gamma^4))+DS_{\text{Cmult}})\cdot(-4N(1+N) \\ & (N^2(2-\gamma^2)^2+(4-\gamma^2)^2(1-\gamma^2)(1+2N)))^{-1}. \qquad (74)\end{aligned}$$

When $\gamma > \gamma_C^*(N)$, the boundary $B_C(N, \gamma)$ is defined by $\bar{F}_{\text{Cmult}}$ for $F > F_C^*(N)$ and $\underline{F}_{\text{Cmult}}$ for $F < F_C^*(N)$. When the market characteristics (γ, F, N) lie to the right of $B_C(N, \gamma)$ and $N \leq 4$, firm 1's profit is higher under decentralization than centralization. Conversely, when the market characteristics (γ, F, N) lie to the left of $B_C(N, \gamma)$, firm 1 is better off under centralized entry.

Next, we confirm that to the right of $B_C(N, \gamma)$ firm 2's best response to firm 1 decentralization is also decentralization. Using the same procedure as for $B_C(N, \gamma)$, we derive the discriminant DS_{Dmult}, critical point $\gamma_D^*(N)$ (a lengthy expression available on request from the authors), and generalized boundary $B_D(N, \gamma)$. We also note that $\gamma_D^*(N) < 1$ for all N. Comparing boundaries reveals that $B_D(N, \gamma)$ lies strictly to the left of $B_C(N, \gamma)$ (i.e., $B_D^{-1}(N, \gamma) < B_C^{-1}(N, \gamma)$). Thus, for $N \leq 4$, to the right of $B_C(N, \gamma)$ decentralization is the optimal response to decentralization, and $\{D, D\}$ is a unique, pure strategy equilibrium provided both firms enter the foreign market.[27]

Finally, we verify that there exists a region to the right of $B_C(N, \gamma)$ where both firms prefer entry to nonentry, and the gray market is nonnegative. For $N \leq 4$ and to the right of $B_C(N, \gamma)$, the upper bound of the feasible region is strictly greater than the lower bound where firms prefer entry, i.e., $m_1(\gamma \mid D) > f_1(\gamma \mid D)$. Thus, there exist feasible market characteristics (γ, F, N) that lie to the right of $B_C(N, \gamma)$ and in this region $\{D, D\}$ is a unique, pure strategy equilibrium.

Thus, provided there are 4 or less gray marketers, there exist feasible market characteristics (γ, F) that lie to the right of $B_C(N, \gamma)$ where $\{D, D\}$ is a unique, pure strategy equilibrium. It follows that, when there are 4 or less gray marketers, there are feasible market characteristics (γ, F) that lie to the left of $B_C(N, \gamma)$ where $\{C, C\}$ is either a unique, pure strategy equilibrium or a Pareto-dominant, pure strategy equilibrium. Further, if $N \geq 5$, then $\gamma_C^*(N) > 1$, and $\{C, C\}$ is either a unique, pure strategy equilibrium or a Pareto dominant, pure strategy equilibrium for all feasible $\gamma \in (0, 1)$. This proves Proposition 4.

[27] Note that $\gamma_D^*(N) < 1\ \forall N$ suggests that there is always a region to the right of $B_D(N, \gamma)$. Thus, provided that firms prefer entry and the gray market is nonnegative, there always exists a $\{D, D\}$ equilibrium $\forall N$, although it is Pareto inferior to a $\{C, C\}$ equilibrium for $N \geq 5$.

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