# Incentives and the Structure of Teams*

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**Abstract**

This paper studies the relationship between moral hazard and the matching structure of teams. We show that team incentive problems may, on their own, generate monotone matching predictions in the absence of complementarities or anti-complementarities in production technology. We also derive sufficient conditions on the primitives of the model leading to the optimality of positive and negative matching of team members.

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# 1 Introduction

Team production and team performance-based compensation schemes have been adopted by many firms over the last 30 years (as documented by Hamilton et al. (2003)). Given that employees usually differ in their skills and productivity, it is important to understand what kind of teams would maximize firms' profit. Namely, should the high skilled workers be teamed with high skilled or low skilled partners? This question is the main focus of our paper.

It is well known from the matching literature that the matching patterns crucially depend on the properties of the payoff function: in the frictionless world with transferable utility supermodularity leads to positive matching, while submodularity induces negative matching (e.g., Becker (1973)). Many models assume that the technology has some complementarity structure which the payoff function inherits (e.g. Kremer (1993)). By contrast, we study an environment where the technology is neither supermodular nor submodular, and show that predictions about matching structure can arise purely because of the team incentive problem.

Our results are derived in a standard model of moral hazard, in which a profit maximizing planner (firm owner) decides how to organize agents (workers) into teams. The team output depends on the efforts exerted by the team members. Workers differ in their productivity (or the cost of exerting effort). The planner observes the types of the workers, but not the actual exerted effort. The production technology is chosen in such a way that, under complete information, the planner is indifferent between the possible matching structures. In order to make the results as plain as possible, we focus on a very stark model, in which the planner has to form two teams of two out of four workers, given that two of the workers have high productivity and the other two are less productive.

Then we develop three sets of results concerning the optimal matching structure when efforts are not observable. First, we show that if types are ordered by the level of input they provide, the ordering of rewards among those types determines the optimal matching structure. In particular, if higher input types are rewarded more in the event of a success, then negative matching is optimal. If higher input types are rewarded less, then the teams should be matched positively.

The intuition is that the team structure generates payments that can be broken into two categories. On the one hand, the agent may be paid for a success for which his effort was responsible. On the other hand, he may be also paid as a result of the effort of his teammate. We call this second kind of payment an “accidental” payment. To minimize the costs associated with accidental payments, the firm owner should team the high input worker with the one whose rewards are low, so that frequently paid accidental compensations are small in magnitude. Correspondingly, if the high input worker is promised a relatively low reward, positive matching is optimal; if, on the contrary, high input workers are paid high rewards, it is better to form mixed teams.

Then we go on to develop results about the impact of the shape of the underlying technology on the optimal matching structure. As is standard in the literature, incentive compatibility stipulates a particular relationship between the effort workers give and the reward given to the worker in the event of a success. This relationship is determined only by the properties of exogenous production technology and the incentive compatibility constraints. We show that, when input levels are interior, positive matching is associated with cases where the reward required to implement a particular effort level is a convex function of effort, while negative matching becomes optimal if the reward function is sufficiently concave.

This result is related to the first one, about the relationship between rewards and efforts. Consider two agents, one with lower cost. It is natural (and, indeed, optimal), for the agent with lower cost to be made to give more effort. But since that agent has lower cost, a particular level of effort can be achieved with a smaller reward. The question is how much more effort should be chosen for the low cost agent, and how much higher a compensation level that will necessitate. When the required reward is a very convex function of effort, it is prohibitively expensive to make effort substantially higher for the low cost agent, and therefore the main difference between the agents is not effort levels, but the required cost to provide a given effort level. As a result, the higher input agent (the low cost type) is paid less in the event of a success, and positive matching is optimal. By contrast, when the reward required to implement a given level of effort is a concave function, the low cost agent can be asked to give a great deal higher effort, so much so that the low cost agent gets a greater reward in the event of a success. Our result about input levels and rewards implies that this case is exactly the one where negative matching is optimal.

Our final set of results includes corner solutions on effort, for two particular functional forms. We show how matching can go from positive to negative as parameters move into the region where lower corners become relevant. When the low skill type becomes sufficiently inefficient, it is optimal to eventually not to elicit effort from the low skill types, and instead spread out the high skill types across projects to mitigate team incentive problems. By contrast, we show in a different example that matching can go from negative to positive as upper corners begin to bind.

Although our results are developed in a principal-agent model with risk neutrality and limited liability, we extend the model to show that the intuition we develop applies in related settings. We discuss how the results relate to a case with either a looser limited liability constraint or a case where agents are risk averse and there is no limited liability. We show the sense in which the force we describe is part of those situations, although we show that such an environment also introduces a variety of other effects that make it analytically intractable. We present numerical results to show that similar results to the limited liability case can be obtained.

Our results are derived in the environment in which the firm owner organizes the workers in teams with the objective of maximizing profit. In our second extension we consider an alternative model in which the workers form partnerships and agree to split the team's output between themselves. This formulation is also not tractable analytically, even for particular functional forms. Thus we perform a series of numerical exercises, and find that, as in our benchmark model, the properties of the underlying technology may similarly lead to either positive or negative matching patterns in partnership formation.

## 1.1 Related Literature

Our work is closely related to two strands of literature. First, moral hazard has been extensively discussed in the literature as a natural feature of team production. A large number of studies have analyzed the implications of the free-riding incentives as well as the properties of the optimal compensation to the team members (e.g., Holmstrom (1982), McAfee and McMillan (1991)). To our knowledge, however, our paper is the first attempt to understand whether moral hazard in teams may have certain implications for the optimal team structure in the presence of worker heterogeneity.

Second, there has been a lot of work done, dating back to Becker (1973) focusing on matching patterns among heterogeneous agents. Recently, a number of studies analyzed how matching predictions might be affected by various economic frictions. For instance, one line of research pioneered by Shimer and Smith (2000) focuses on the role of search frictions and argues that in search models positive assortative matching may fail even if the joint production function is supermodular. Another friction that is likely to weaken the effects of technological complementarities is described by Kaya (2008), who illustrates that if the types of the matching partners are not observable, some of the low-type agents cannot be deterred from mimicking the high-type agents and, therefore, positive assortative matching cannot be sustained in the equilibrium. Our paper studies a different friction (moral hazard) and obtains quite different results: in contrast to the findings discussed above, we show that, depending on the parameters of the model, moral hazard can either reduce or decrease the degree of technological complementarity embodied in production technology.

The effects of moral hazard on matching entrepreneurs to projects have been investigated by Thiele, Wambah (1998) and Newman (2007). They study a problem of assigning heterogeneous in wealth, risk averse entrepreneurs to the projects with different amount of risk. In the absence of frictions, wealthier (and hence less risk averse) entrepreneurs would undertake riskier projects. However, if the entrepreneur's effort is unobservable, the opposite matching pattern may arise if the utility is linear in effort. This is because richer agents have lower marginal utility of income, and should be offered higher compensation to induce a particular effort level. While our paper also emphasizes the role of moral hazard, our question, as well as modeling environment, is very different from theirs, and the mechanism outlined in these papers does not play any role in generating our results. The matching in their papers is inter-firm, whereas we are concerned with matching within a given production process .

Finally, Prat (2002) poses a question related to ours, of how to organize workers in teams, but he studies the effects of a different friction. The paper introduces learning into a team production model and shows that, in spite of this modification, the matching patterns are still determined by the super- or sub- modularities in the production technology. In contrast, in our model all the matching predictions arise solely due to moral hazard friction and are not driven by the properties of the underlying technology.

# 2 Model

Consider a technology owned by a risk neutral principal that requires two agents to be operable. Output from the technology is stochastic and depends on the input from each agent $\theta_i$. Denote that input by $x_i$. The output is (by normalization) one with probability $g(x_1) + g(x_2)$, where $g(x)$ is increasing and satisfies $0 \leq g(x) < 1/2$. Otherwise output is zero. We will term output of one a "success."

Our choice of functional form for the success probability is driven by two considerations. First, we want the choice to be such that, in the absence of information frictions, matching is irrelevant. That is satisfied because of the additive separability in the inputs $x_1$ and $x_2$, as we will show formally below. Second, we want the function to be such that there is always some chance of failure, and therefore the incentive problem will be unavoidable when inputs are unobserved. This motivates the admissible range for $g(x)$.

Each agent is risk neutral and has cost of effort $c(x, \theta)$, where $\theta$ is the agents type. We assume that $c(x, \theta)$ is increasing in $x$. The agent has limited liability, in the sense that state by state, the wages paid cannot be lower than zero. We discuss relaxing the limited liability constraint below. Without loss of generality, we can restrict attention to contracts that pay zero for failures and $w_\theta \geq 0$ for successes.

In order to address the question of whether to choose homogeneous or heterogeneous teams, we will suppose that the principal operates two teams and is faced with four agents, two each of types $\theta = l$ and $\theta = h$, respectively. The principal then has to decide whether to match like types or different types.

# 3 Benchmark: Complete Information

A planner who could observe the inputs $x$ could simply choose $x^*_\theta$ for each agent to maximize

$$g(x^*_\theta) - c(x^*_\theta, \theta)$$

and pay

$$w_\theta = c(x^*_\theta, \theta)/(g(x^*_\theta) + g(x^*_{\theta^-}))$$

where $\theta^-$ is the type of the agent's teammate. Four agents, then ,would generate expected output of $2g(x^*_l) + 2g(x^*_h)$ and be paid expected wages of $2c(x^*_l, l) + 2c(x^*_h, h)$ regardless of the matching structure.

# 4 Incomplete Information

In the rest of the paper we focus on the case where only success or failure is observable, but inputs are not. It is straightforward to add another input provided by workers, for which their input is observable and additively separable, and they are paid their marginal product. Since that payment is independent of the matching structure, it does not impact our results. One can think of our wage payment in that setting as a bonus for better than normal results, which come about as a result of the unobserved effort, and $x$ as being the worker's input into generating this extraordinary outcome.

The principal chooses a team structure, either matching like types (positive matching) or different types (negative matching). He chooses efforts $x_\theta$ and wages $w_\theta$ in a way that satisfies the incentive constraint

$$x_\theta \in \arg\max_x g(x)w_\theta - c(x, \theta)$$

An important feature of our additive structure for the underlying technology is that this incentive constraint is valid regardless of the team structure. Of course, the agent's compensation depends on the type of his partner – the agent also collects $g(x_{\theta^-})w_\theta$ – but that is not relevant to the choice of $x$, and hence is left out of the incentive constraint. In other words, the agent's partner plays no role in the provision of incentives. Below we discuss alternative formulations where this is not the case, and argue that the intuition we develop for this stark model will naturally carry over.

The principal collects all output, net of wage payments. Formally, the principal can choose either to match positively, in which case he solves

$$\begin{aligned} \max_{x_h, x_l, w_h, w_l} \; & 2g(x_h)(1-2w_h) + 2g(x_2)(1-2w_l) \\ s.t. \quad & x_\theta \in \arg\max_x g(x)w_\theta - c(x,\theta), \theta \in \theta_h, \theta_l \end{aligned} \tag{1}$$

or negatively, in which case he solves

$$\begin{aligned} \max_{x_h, x_l, w_h, w_l} \; & 2(g(x_h) + g(x_l))(1 - w_h - w_l) \\ s.t. \quad & x_\theta \in \arg\max_x g(x)w_\theta - c(x,\theta), \theta \in \theta_h, \theta_l \end{aligned} \tag{2}$$

## 4.1 Payment vs. Inputs

We first establish the following basic result relating matching patterns with the correlation between effort and reward specified by the optimal contract.

**Claim 1** *Negative (positive) matching can be the optimal choice of the planner only if for such matching structure high-input types receive high (low) compensation in the event of success.*

**Proof.** Suppose to contradiction that the planner chooses to sort workers positively and offers higher compensation to the types exerting more effort. We will show that the planner's profit would increase if, instead, the workers were rematched negatively.

For brevity, denote $g(x_l)$ and $g(x_h)$ by $g_l$ and $g_h$. The total surplus of the firm owner matching his teams positively is equal to

$$\Pi^+ = 2g_l(1-2w_l) + 2g_h(1-2w_h).$$

As was noted above, the same contract will still be incentive compatible if the teams are rematched negatively (since IC constraint (3) is affected only by the worker's own type). By rematching the workers negatively, and offering the same contract, the firm owner would obtain profit

$$\Pi^{-} = 2(g_l + g_h)(1 - w_l - w_h).$$

It is easy to verify that

$$\Pi^{-} - \Pi^{+} = 2(g_l - g_h)(w_l - w_h),$$

which is positive if high-input workers receive high rewards. Hence, the planner would increase his expected profit by rematching the workers.

The opposite result is based on a symmetric argument. ■

This result relates an endogenous (but possibly empirically observable) variable, relative pay across workers of different types, to the matching structure. The intuition is straightforward. Agents make "earned" income $g(x_\theta)w_\theta$ and "accidental" income $g(x_{\theta^-})w_\theta$. To keep accidental income at a minimum, agents with high inputs (i.e. high $x$, and therefore high $g(x)$) should be matched with agents with low wages. This insures that the relatively likely accidental payments are kept as small as possible. Note that the result does not require any special assumptions about the shape of $g(x)$ and $c(x, \theta)$.

The result shows that matching may be irrelevant only if, for any matching structure, either both types provide the same input or both types receive the same wages.[1] In general, differences in cost will generate different optimal inputs, and different inputs will require different payments, given the differences in costs. In other words, typically there will be non-trivial matching predictions.

Notice that the statement of Claim 1 is conditional on the properties of the variables that are endogenously determined within our model, and it cannot be immediately stated how the efforts and rewards should be related to each other in the optimal incentive compatible contract. The reason is that the workers with the higher cost of effort should, on one hand, obtain a higher compensation for every unit of effort they provide, but, on the other hand, would also be requested to exert lower effort level. These two effects act in the opposite directions and

[1] Notice that the matching structure becomes irrelevant if, for instance, effort is bounded from above and all the workers are asked to exert the maximum possible effort. The same would be true if the effort choice is discrete - work or do not work - and the optimal contract induces everyone to work.

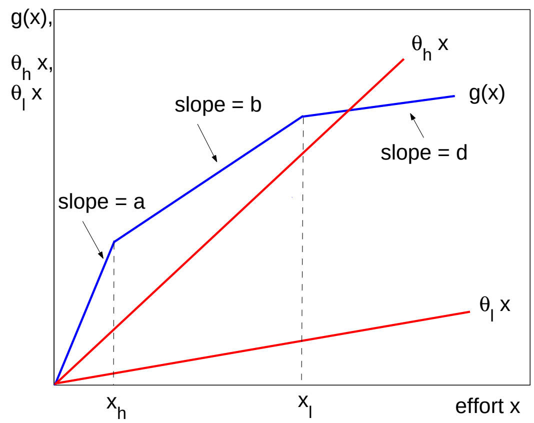


Figure 1: An example with piecewise linear $g(x)$.

make the relationship between efforts and rewards ambiguous. In the following section we develop an example illustrating that, depending on the properties of $g(x)$, either of the two effects may dominate, implying that moral hazard may potentially generate either positive or negative matching.

## 4.2 The shape of $g$ and matching: an example

To illustrate how shape of $g(x)$ may be related to the optimal matching structure, consider the following example. Suppose that $c(x, \theta) = \theta x$ with $\theta \in \{\theta_l, \theta_h\}$ and $\theta_l < \theta_h$. Let $g(x)$ be piecewise linear, with slope $a$ for low $x$, slope $b < a/2$ for intermediate values of $x$ and slope $d < b/2$ for large $x$. Suppose also that $a/2 > \theta_h > b$ and $b/2 > \theta_l > d$. An example of such $g(x)$, $\theta_l x$ and $\theta_h x$ is illustrated on Figure 1. Notice that it is always optimal to set $x_h$ at the lower kink point and $x_l$ at the higher as it is illustrated on Figure 1. The corresponding payments should be $w_h = \theta_h/a$ and $w_l = \theta_l/b$.[2]

To see why this shape of $g(x)$ may endogenously lead to either matching structure, consider adjusting $x_h$, while holding fixed $x_l$, $g(x_l)$, and $b$. The left plot of Figure 2 gives an example of function $g(x)$, for which negative matching is optimal: when $x_h$ is sufficiently low and, correspondingly, $a$ is sufficiently high,

[2]Note that $a/2 > \theta_h > b$ and $b/2 > \theta_l > d$ imply that the planner's expected payoff is positive for either matching structure. This guarantees that the firm owner would not shut any of the teams down. It is still possible, however, that shutting one of the workers down would generate higher payoffs. Additional conditions may be imposed on the parameters to eliminate such possibility.

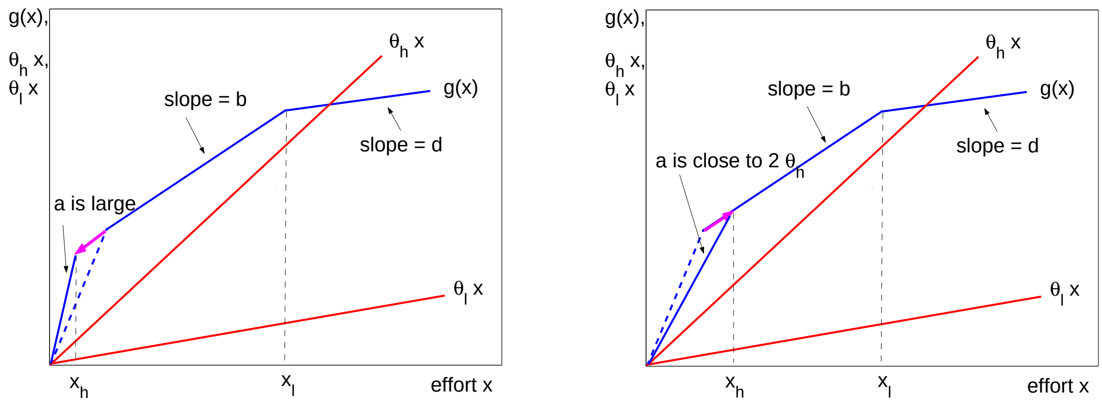


Figure 2: Adjusting $a$ and $x_h$ to achieve negative matching (on the left) and positive matching (on the right)

the agent who contributes less receives smaller payment, since $w_h = \theta_h/a$ becomes smaller than $w_l = \theta_l/b$. If, on the other hand, $x_h$ is sufficiently high so that $a$ is arbitrary close to $2\theta_h$ (as illustrated on the right plot), the payment to the high-cost agent becomes bigger than the payment to the lower-cost agent, and positive matching becomes optimal.

## 4.3 Interior solutions

Suppose that $g(x)$ is concave and three times continuously differentiable, and, as in the previous section, the cost function for type $\theta$ is $c(x, \theta) = \theta x$.[3] In this case the incentive constraint for any interior effort level simplifies to

$$g'(x_\theta)w_\theta = \theta. \tag{3}$$

We can rewrite this as

$$w_\theta = \theta/g'(x),$$

or, letting $r(x) = 1/g'(x)$,

$$w_\theta = \theta r(x).$$

We interpret $r(x)$ as the shape of the reward as a function of effort.

[3] If $c(x, \theta)$ is linear in $\theta$, i.e. $c(x, \theta) = \theta\tilde{c}(x)$, it is without loss of generality to assume that $c(x, \theta) = \theta x$, because it is always possible to redefine the variables in such a way that either $g(x)$ or $\tilde{c}(x)$ is linear in $x$.

Note that for $\theta_h > \theta_l$, it is easy to show that any optimal contract must have $x_h < x_l$; otherwise the agents' inputs should be reversed.[4] To see how the shape of $r(x)$ is related to the matching structure, suppose there is negative matching. We can use the first order condition to write the principal's payoff just as a function of the inputs:

$$(g(x_l) + g(x_h))(1 - \theta_l r(x_l) - \theta_h r(x_h)) \tag{4}$$

The first order conditions are

$$\begin{aligned} g'(x_l)(1 - \theta_l r(x_l) - \theta_h r(x_h)) &= \theta_l r'(x_l)(g(x_l) + g(x_h)) \\ g'(x_h)(1 - \theta_l r(x_l) - \theta_h r(x_h)) &= \theta_h r'(x_h)(g(x_l) + g(x_h)) \end{aligned}$$

So we have that

$$\frac{g'(x_l)}{g'(x_h)} = \frac{\theta_l r'(x_l)}{\theta_h r'(x_h)}$$

or

$$\frac{\theta_h r(x_h)}{\theta_l r(x_l)} = \frac{r'(x_l)}{r'(x_h)}.$$

Since this is a negative match, the left hand side must be less than one, as it is the ratio of the pay of the low input type to the high input type. However, the right hand side can't be less than one if $r$ is convex and $x_l > x_h$. So convex $r(x)$ implies that we could not have it be optimal to match negatively. Put in terms of $g(x)$, we have the following result:

**Claim 2** *Suppose that $-\frac{g''(x)}{(g'(x))^2}$ is (weakly) increasing and that in the mixed teams the solution is interior. Then it is optimal to sort the teams positively.*

A variety of commonly used examples meet this sufficient condition, including $g(x) = .5(1 - e^{-x})$, $g(x) = A - B/x$ , $g(x) = x - Ax^2$, and $g(x) = A + \ln x$.

Another way to think about the condition is in terms of coefficients of absolute risk aversion and absolute prudence, taking $g(x)$ to be analogous to the utility function. Than the sufficient condition is that coefficient of absolute prudence is larger than twice the coefficient of absolute risk aversion. Similar conditions arise

[4] If $x_h > x_l$ then by reversing inputs and adjusting the rewards to satisfy the agents' incentive constraints, the planner will keep the probability of success at the same level but will be able to reduce the total compensation paid in case of successful output.

in other matching settings, such as Newman (2007) and Theile and Wambach (1999).

We can also prove a partial converse, relating concave $r(x)$ to negative matching.

**Claim 3** *Suppose that* $-\frac{g(x)g''(x)}{(g'(x))^2}$ *is (weakly) decreasing and for the positively matched team the optimal effort level is interior. Then the conditions of Claim 1 hold, and thus it is optimal to sort the teams negatively.*

**Proof.** For a positively matched team, the FOC from (4) is

$$g'(x)\left(1-\frac{2\theta}{g'(x)}\right)+g(x)\cdot\frac{2\theta g''(x)}{(g'(x))^2}=0. \tag{5}$$

It can be simplified to

$$\frac{g'(x)}{\theta}=2\left(1-\frac{g(x)g''(x)}{(g'(x))^2}\right). \tag{6}$$

or

$$\frac{1}{w_\theta}=2\left(1-\frac{g(x_\theta)g''(x_\theta)}{(g'(x_\theta))^2}\right)$$

If $-\frac{g(x)g''(x)}{(g'(x))^2}$ is decreasing, then the right hand side is decreasing in $x$; therefore Then if, for instance, higher $\theta$ leads to higher $x_\theta$ (making the RHS smaller), it must be the case that higher $\theta$ makes $w_\theta$ larger. But then higher inputs are associated with higher bonuses, and negative matching is optimal, a contradiction to the asserted positive matching. ■

In this case, we need that $r(x)$ is sufficiently concave in order to get negative matching. However, note that this does not mean that $g(x)$ must be terribly unusual; for instance, $g(x)=x^\alpha$ meets the requirement for all $\alpha$ strictly between zero and one. In the utility function language, the coefficient of absolute prudence, plus the ratio of $g'(x)$ to $g(x)$, must be smaller than twice the coefficient of absolute risk aversion.

Intuitively, the convexity/concavity of the reward function is important for matching predictions because it determines whether the differences in compensations are mostly driven by the differences in marginal costs or by the differences in effort levels. As we emphasized earlier, the relationship between the agent's cost of effort and his compensation is generally ambiguous: an agent with the lower cost would be asked to exert more effort, but is compensated less per unit of effort. The question, then, is how much greater a level of effort should be chosen for the low cost agent, and how much more cost that higher effort will necessitate. When the required reward is a very convex function of effort, it is prohibitively expensive to make effort substantially higher for the low cost agent, and therefore the main difference between the agents is not effort levels, but the required cost to provide a given effort level. As a result, the higher input agent (the low cost type) is paid less in the event of a success, and positive matching is optimal. By contrast, when the reward required to implement a given level of effort is a concave function, the low cost agent can be asked to give a great deal higher effort, overcoming the lower cost and implying that the low cost agent gets a greater reward in the event of a success. The first set of results implies that this case is exactly the one where negative matching is optimal.

It is also instructive to point out that Claims 2 and 3 establish sufficient conditions for super- and sub- modularity of the planner's profit function (under the assumption that solution is interior). This means that the mechanism described in our model illustrates how complementarities or anti-complementaries in the payoff function can endogenously arise due to the presence of moral hazard friction.

Although we focused on teams of two, these results for interior solutions can be extended directly larger teams. In other words, when solutions are interior, the matching structure can be described by the shape of the $g(x)$ function using exactly the conditions described in that section.

For that case, then, team size can only impact the matching structure through the impact it has on choosing corners. As teams grow, there will be a tendency to move toward corner solutions, in which some of the team members provide no unobservable effort, since there will be greater accidental payment for any level of bonus promised to the agent, coming from the agents relatively small role in the larger and larger team. As a result, we may see switches from positive to negative matching as it is discussed in the following section.

### 4.4 The impact of the corner solutions on the matching structure

All the results in the previous Section are formulated for interior solutions only. For some parameter values, however, the planner would choose to assign the extreme effort levels to the agents. Here we discuss how the matching structure may change if the corners arise.

Suppose that effort levels can be chosen from the interval $[\underline{x}, \overline{x}]$. Without loss of generality, we can normalize $\underline{x} = 0$. The presence of the upper bound $\overline{x}$ is due to the assumption $g(x) < 1/2$, which guarantees that the probability of success of a team is below 1. Naturally, if $\lim_{x\to\underline{x}} g'(x)$ is finite and $\lim_{x\to\overline{x}} g'(x) > 0$ then, independently of the matching structure, both types of agents would be asked to exert zero unobservable effort[5] if $\theta_l$ is sufficiently large, and maximum effort if $\theta_h$ is sufficiently low. In either case the model would obviously have no matching predictions.

If, however, only one of the marginal costs is extreme, only one type of agents would be assigned an extreme effort. Naturally, the appearance of such corners may reverse the matching structure chosen by the principal. In what follows we argue that particular corners tend to favor particular matching structures, namely that setting the effort of one of the agents to minimum creates incentives for the planner to form mixed teams, while setting the effort of one of the agents to maximum may induce the planner to rematch the mixed teams positively.

Intuitively, if the effort of the high-cost agent is very expensive, he would not be asked to exert any effort for any matching pattern, and only the low-cost types would be working. In this case it is optimal to assign the low-cost agents to separate teams in order to mitigate the moral hazard problem. Thus the corner solutions where the high-cost types exert minimal effort may reverse the optimal matching structure from positive to negative. Alternatively, if the marginal cost of effort for one of the types is very low, the agent would be asked to exert maximal effort. Any further decrease in the marginal cost would not lead to higher exerted

[5]Note that corner solutions here need not be interpreted as agents not working. It is simply that those agents don't contribute unobserved effort and receive bonus payments. Agents with $x = 0$ could still be providing observable effort essential to the operation of the technology, and be compensated in accordance with their cost of effort. Here all those efforts and payments are fixed at zero, but none of the results require that those payments do not exist.

effort and, hence, would unambiguously lower the payment to this agent in case of success. Thus, if the parameters are such that in the interior solution higher effort is compensated with higher reward (i.e. negative matching is the optimal outcome), a significant decline in the marginal cost for one of the agents would increase his effort to the maximum $\overline{x}$ and drive his compensation down below his partner's. Consequently, switching the matching structure from negative to positive would reduce the total value of 'accidental' payoffs. We formally illustrate these intuitive arguments for particular forms of $g(x)$, exponential and power, for which, respectively, positive and negative matching is optimal in the interior solution.

### 4.4.1 The effect of the corner $x = 0$: exponential case

Suppose that $g(x) = (1 - e^{-x})/2$. We know from the prior section that any interior case will always have the feature that matching is positive. We then only have to consider the outcome for this interior, positive matched case, as compared with the possible corner solutions for both matching structures.

For the interior case, the planner's surplus from a team of type $\theta$ is found by solving

$$\max_x 2g(x)(1 - 2\theta/g'(x))$$

The first order condition is[6]

$$e^{-x} = 4\theta e^x$$

which implies $e^{-x} = 2\sqrt{\theta}$ and the maximized payoff is

$$1 - 4\sqrt{\theta} + 4\theta. \tag{7}$$

For comparison, consider a team with one team member giving zero effort. It is easy to show that the zero effort agent will always be the higher cost type, if the team is heterogenous. Therefore the planner must only choose an effort for the (weakly) lower type $\theta$, in order to solve

$$\max_x g(x)(1 - \theta/g'(x))$$

[6]It is also shown in the Appendix that whenever positive matching is optimal, the second order condition for the positively matched teams is satisfied.

In this case the first order condition is

$$e^{-x} = 2\theta e^{x}$$

so we have that $e^{-x} = \sqrt{2\theta}$ for the agent that works $x > 0$, and a maximized payoff of

$$1/2 - \sqrt{2\theta} + \theta \tag{8}$$

For a team with two like types, then, the decision to choose $x = 0$ for one of them occurs at $\theta^* = \frac{3}{2} - \sqrt{2}$; for small $\theta$, both work, but for high $\theta$ only one works.

It is immediate that if $\theta^* < \theta_l < \theta_h$, so that any team would have one worker providing zero effort, it is optimal that the planner chooses negative matching, so that a low quality type provides effort in both teams. If, by contrast, $\theta_l < \theta_h < \theta^*$, positive matching has both agents on both teams working an interior amount. The alternative is negative matching with only the $\theta_l$ types working. The payoff in the positive case is

$$1 - 4\sqrt{\theta_l} + 4\theta_l + 1 - 4\sqrt{\theta_h} + 4\theta_h$$

whereas the negative case has payoff

$$2(1/2 - \sqrt{2\theta_l} + \theta_l).$$

It is easily verified that the positive matching payoff is larger for all $\theta_l$ and $\theta_h$ below $\theta^*$.

The final case is the one where $l < \theta^* < \theta_h$. In this case, negative matching remains with the same payoff of

$$2(1/2 - \sqrt{2\theta_l} + \theta_l)$$

but positive matching gives a payoff of

$$1 - 4\sqrt{\theta_l} + 4\theta_l + 1/2 - \sqrt{2\theta_h} + \theta_h$$

since the team of two $\theta_h$ types has only one providing positive $x$. For fixed $\theta_l$, as $\theta_h$ rises, the payoff to the positive matching case falls until the team is no longer productive at all, and the payoff to the negative matching outcome remains constant. For large $h$ the negative case always dominates, and for $\theta_h$ near $\theta^*$ the positive case dominates. Therefore, for every $\theta_l$, there is a cutoff $\theta_h^*(\theta_l)$ such that negative matching is chose in $\theta_h$ is greater than $\theta_h^*(\theta_l)$. It is easy to verify that $\theta_h^*(\theta_l)$ is decreasing in $\theta_l$, with $\theta_h^*(\theta^*) = \theta^*$.

Intuitively, for fixed $\theta_l$, negative matching is associated with a sufficiently high cost for $\theta_h$-type workers. When both types are sufficiently good, all agents give effort and the form of $g(x)$ guarantees interior solutions. However, as the higher cost agent gets worse, the payoff from that team declines. Eventually the team of $\theta_h$ type moves to a corner solution where only one agent provides effort. Eventually the payoff from that team doesn't even justify its existence, and the principal does better by switching to positive matching.

#### 4.4.2 The effect of the corner $x = \overline{x}$ : power case

Suppose that $g(x) = x^\alpha$, with $\alpha \in (0, 1/2)$ and feasible effort $x \in [0, \overline{x}]$.[7] Then $-\frac{g(x)g''(x)}{(g'(x))^2} = \alpha(1-\alpha)$, which, by Claim 3, implies that the optimal matching structure is negative. From the first order conditions, the optimal effort levels satisfy

$$\begin{aligned} \alpha x_l^{\alpha-1} &= \frac{\theta_l}{\alpha}\left[1 + \left(\frac{\theta_l}{\theta_h}\right)^{\frac{\alpha}{1-2\alpha}}\right] \\ \alpha x_h^{\alpha-1} &= \alpha x_l^{\alpha-1}\left(\frac{\theta_l}{\theta_h}\right)^{\frac{\alpha-1}{1-2\alpha}} = \frac{\theta_h}{\alpha}\left[1 + \left(\frac{\theta_h}{\theta_l}\right)^{\frac{\alpha}{1-2\alpha}}\right] \end{aligned} \tag{9}$$

In the Appendix it is verified that the solution to (9) also satisfies the second order condition as long as $\alpha < 1/2$. It also follows from (9) that the rewards paid to the agents (found as $r_i = \theta_i/\alpha x_i^{1-\alpha}$) do not exceed 1/2, so the planner's profit is positive. Obviously, $\theta_l < \theta_h$ implies that $x_l > x_h$.

For a given $\theta_h$, find $\underline{\theta_l}(\theta_h)$ from

$$\alpha \overline{x}^{\alpha-1} = \frac{\underline{\theta_l}(\theta_h)}{\alpha}\left[1 + \left(\frac{\underline{\theta_l}(\theta_h)}{\theta_h}\right)^{\frac{\alpha}{1-2\alpha}}\right]$$

[7] The restriction $x \le \overline{x}$ is imposed to guarantee that $g(x_l) + g(x_h) \le 1$.

As long as $\theta_h > 2\alpha^2\overline{x}^{\alpha-1}$, such $\underline{\theta_l}(\theta_h)$ is well defined and for a team $(\theta_h, \underline{\theta_l}(\theta_h))$ the effort assignment is $x_h < x_l = \overline{x}$ and (9) is satisfied. If $\theta_h$ is kept fixed and $\theta_l$ falls below $\underline{\theta_l}(\theta_h)$ then it is optimal to set $x_l = \overline{x}$ and $x_h = \min\{\overline{x}, \widehat{x_h}\}$, where $\widehat{x_h}$ solves

$$\alpha\widehat{x_h}^{\alpha-1}\left(1 - \frac{\theta_l}{\alpha}\overline{x}^{1-\alpha}\right) - \frac{\theta_h}{\alpha} = \frac{\theta_h}{\alpha}(1-\alpha)\overline{x}^{\alpha}\widehat{x_h}^{-\alpha} \tag{10}$$

The above equation is the planner's first order condition with respect to $x_h$ when $x_l$ is fixed at $\overline{x}$. In the Appendix we show that the solution $\widehat{x_h}$ to (25) exists, is unique, is strictly positive and that the second order condition is satisfied. It is also straightforward to see that as $\theta_l$ declines, $\widehat{x_h}$ rises and the reward paid to the high cost agent $r_h = \theta_h/\alpha\widehat{x_h}^{1-\alpha}$ increases (it converges to a positive value as $\theta_l \to 0$). Hence, there exists $\underline{\theta_l^*}(\theta_h) < \underline{\theta_l}(\theta_h)$ such that $r_l = \theta_l/\alpha\overline{x}^{1-\alpha} < r_h = \theta_h/\alpha\widehat{x_h}^{1-\alpha}$ for all $\theta_l < \underline{\theta_l^*}(\theta_h)$. Since in this case the agent exerting higher effort receives lower reward, Claim 1 implies that the planner would obtain higher profit if he switches from negative to positive matching.[8] Note that, since Claim 1 provides necessary, but not sufficient, conditions, it may be optimal to switch to the the positive matching structure even for some $\theta_l \in (\underline{\theta_l^*}(\theta_h), \underline{\theta_l}(\theta_h))$. In this case the gains from the switch would be driven by effort readjustment, not by reducing the accidental payments at given effort levels.

Finally, if $\theta_h \leq 2\alpha^2\overline{x}^{\alpha-1}$, then both agents in the mixed teams would be asked to exert maximal effort $\overline{x}$. If the agents were rematched positively, the effort assignment for type $\theta$ would be given by $\min\left\{\overline{x}, \left(\frac{\alpha^2}{2\theta}\right)^{\frac{1}{1-\alpha}}\right\}$, which for $\theta_l < \theta_h \leq 2\alpha^2\overline{x}^{\alpha-1}$ also simplifies to $\overline{x}$. Hence, for these parameter values matching structure is irrelevant since both types exert $x = \overline{x}$ in any match.

We can summarize the above arguments as follows. If $\theta_h > \alpha^2\overline{x}^{\alpha-1}$ then there exists such $\theta_l^*(\theta_h)$ that negative matching is optimal for all $\theta_l \geq \theta_l^*(\theta_h)$, but it becomes optimal to rematch the teams positively if $\theta_h$ falls below $\theta_l^*(\theta_h)$. If $\theta_h$ is sufficiently low ($\theta_h \leq \alpha^2\overline{x}^{\alpha-1}$), both types exert maximum effort in either match, and the matching structure becomes irrelevant.

[8]Since $\widehat{x_h}$ solving (25) is decreasing in $\theta_l$, it is possible that for $\theta_l < \underline{\theta_l^*}(\theta_h)$ the solution to (25) exceeds $\overline{x}$. This would be true if $\underline{\theta_l^*}(\theta_h) < \alpha\overline{x}^{\alpha-1} - \theta_h$. In this case, both agents are asked to exerts maximum effort, and there is no gain for the planner from switching to positive matching while keeping the efforts at the same levels. However, note that the planner's profit might still increase if after rematching the agents positively he asks some of them to exert less effort.

# 5 Extensions

## 5.1 Relaxing Limited Liability

Our results were all generated in the context of a model where moral hazard frictions arise due to limited liability. Since agents are risk neutral, limited liability is essential to this friction. It is, however, instructive to point out that our results are not driven by the restriction of non-negativity of rewards. Instead, we could require that $w_i \geq \underline{w}_i$, where $\underline{w}_i$s are negative (and potentially different across different types), and our result would still apply as long as this constraint is binding for all the workers in all the matching structures, i.e. if $\underline{w}_i$s is not too small. When $\underline{w}$ becomes sufficiently low, the workers' individual rationality constraints would become binding in all the matches for all the types, effort allocation would coincide with the first best, and matching would become irrelevant.[9]

Perhaps a better way to understand whether limited liability is the major driving force of our results would be to consider a model in which the workers are risk averse, so that we can dispense with limited liability.[10] Under this modification, the decision problem of the firm owner becomes

$$\begin{aligned} \max_{x_l, x_h, r_l, r_h} \; & [g(x_i) + g(x_j)] \left(1 - w_l^1 - w_h^1\right) - [1 - (g(x_i) + g(x_j))] \left(w_l^0 + w_h^0\right) \\ \text{s.t.} \quad & x_i = \arg\max_x \left\{ [g(x_i) + g(x_j)]\, u(w_i^1) + [1 - g(x_i) - g(x_j)]\, u(w_i^0) - c(x_i, \theta_i) \right\}, \quad i = l, h \\ & [g(x_i) + g(x_j)]\, u(w_i^1) + [1 - g(x_i) - g(x_j)]\, u(w_i^0) - c(x_i, \theta_i) = \underline{u}, \quad i = l, h. \end{aligned} \tag{11}$$

To get intuition for how this problem relates to the one we have studied so far, consider an agent whose partner is described by $p$, the partner's contribution to the probability of success. In the event of success utility for the agent is $u(r_i^1) = u_1$, failure is $u(r_i^0) = u_0$. The participation constraint can be written as

$$(p + g(x))u_1 + (1 - p - g(x))u_0 - c(x) = u$$

[9] The cases with intermediate values of $\underline{w_i s}$, for which individual rationality constraint binds occasionally (for some types in some matches) are hard to characterize in general.

[10] For concreteness, we formally study the case where compensation provided by the planner is the only source of the workers' consumption; nothing changes if the worker has outside income.

or

$$(p + g(x))\sigma + u_0 - c(x) = u$$

where $\sigma = u_1 - u_0$. The IC constraint is

$$g'(x)\sigma = c'(x)$$

So, for fixed $x$, $\sigma$ is fixed from the IC constraint. As $p$ increases in the participation constraint, for fixed $\sigma$, $u_0$ and $u_1$ must decline, so the wage payments decrease in $p$ for any $x$. On the other hand, the high state is achieved more often, so we need to know what the net impact is on the planner's payout, as $p$ changes.

Let $u_0(p)$ be the value of $u_0$ as a function of $p$, which according to the prior argument has $u_0'(p) = -\sigma$, and let $\rho(u)$ be the resource cost to the planner of delivering $u$ units of utility. The planner pays

$$(p + g(x))\rho(\sigma + u_0(p)) + (1 - p - g(x))\rho(u_0(p))$$

Payments change in $p$ at the rate

$$\rho(\sigma + u_0(p)) - \rho(u_0(p)) - \sigma((p + g(x))\rho'(\sigma + u_0(p)) + (1 - p - g(x))\rho'(u_0(p)))$$

The limited liability case with no risk aversion has the feature that $u_0'(p) = 0$, as you always pay zero in the event of a failure, so payments always increase when you get a better partner, for a fixed . This leads to the matching result described in the case with no risk aversion. Clearly this force is still in the model with risk aversion, but so is another force, coming from the fact that $u_0(p)$ is declining. This force works in the opposite direction, so generally the optimal matching structure could go in either direction in the case with risk aversion, even when we can derive a matching structure for the case of limited liability and risk neutrality.

We numerically solved an example with linear $c(x, \theta)$, piecewise-linear $g(x)$ (introduced in section 4.2) and logarithmic utility. We found that the the model with risk aversion and binding individual rationality constraints generates predictions that are qualitatively similar to the ones obtained in our benchmark model: negative matching is optimal when $x_h$ is low, and positive matching is

The properties of an example with log utility and no limited liability

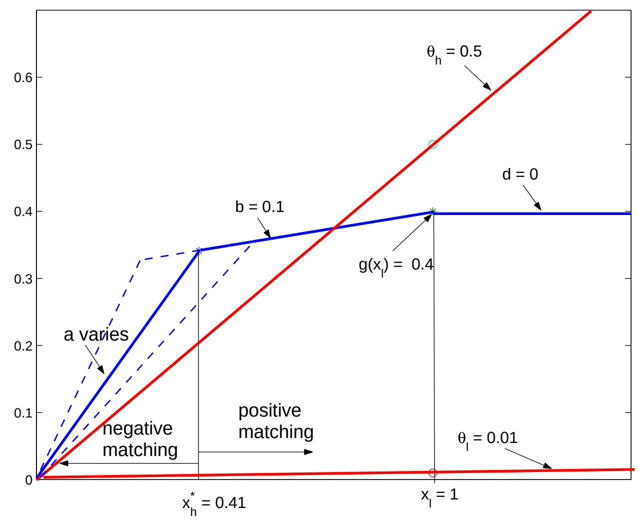


Figure 3: Matching predictions in the model with piecewise-linear $g(x)$ and logarithmic utility, $\underline{u} = \ln(0.05)$.

optimal when $x_h$ is high. Figure 3 illustrates an example for a particular set of parameters and reports the cutoff level $x_h^*$ at which the planner is indifferent between the two matching structures. It also turns out that this cutoff level is greater than the one that would be obtained in the model with risk neutrality and binding limited liability (where $x_h^* = 0.06$)[11], suggesting that, at least for some specifications, relaxing limited liability and introducing risk aversion may favor negative matching.

## 5.2 Matching in partnerships

In our model the principal's profit is always positive; in other words, $w_l + w_h < 1$. Since the first best efforts satisfy $g'(x_i) = \theta_i$, while in our model $g'(x_i)w_i = \theta_i$, it seems natural that the total surplus might increase if the agents bypass the firm owner and obtain $w_l + w_h = 1$ by forming a partnership. In this section we explore such a case, and show numerically that both negative and positive matching may arise as the equilibrium outcome if such partnerships are formed. The structure of matching is similar to the model of moral hazard in teams that we study.

[11]Such $x_h^*$ is computed to equalize the rewards of the two types, $\theta_h/a = \theta_l/b$.

Suppose that in a partnership of two agents, the first agent receives share $s_1$ of total output, and the second agent gets the rest. The incentive compatible efforts are

$$\begin{aligned} g'(x_1)s_1 &= \theta_1 \\ g'(x_2)(1-s_1) &= \theta_2. \end{aligned} \tag{12}$$

Correspondingly, we can determine the pair of utilities derived by the agents for a given sharing rule:

$$\begin{aligned} W(s_1) &= (g(x_1)+g(x_2))\left(1-\frac{\theta_1}{g'(x_1)}\right)-\theta_2 x_2 \\ v(s_1) &= (g(x_1)+g(x_2))\frac{\theta_1}{g'(x_1)}-\theta_1 x_1. \end{aligned} \tag{13}$$

By varying the sharing rule $s_1$ we obtain the frontier $W(v)$ of values delivered by the contract. This frontier is not linear[12], implying that this model can be classified as the model with non-transferable utility studied in the matching literature. Legros and Newman (2007) provide sufficient conditions for positive (negative) equilibrium matching in this environment.

Denote by $W_{ij}(v)$ the frontier for the pair of agents with marginal costs $\theta_i$ and $\theta_j$ respectively, and by $v_{ij}^{-1}(W)$ the inverse of this frontier. Suppose that $\theta_H > \theta_L$. Legros and Newman (2007) show that the equilibrium allocation is characterized by *positive* assortative matching if $v_u(v) = v_{LL}^{-1}(W_{HL}(v)) \geq v_{LH}^{-1}(W_{HH}(v)) = v_d(v)$, and negative matching for the reverse inequality.

This insight allows us to run a large number of numerical exercises for $g(x) = (1-e^{-x})/2$ and for $g(x) = x^\alpha$, $\alpha \in (0,1)$. Figures 4 and 5 illustrate two examples of the solutions for these functional forms. Remarkably, for all the parameter values that we tried, we found positive matching when $g(x)$ is exponential and GDD condition holds if $g(x)$ is a power function, implying positive matching in the former case and negative in the latter. Recall that in our benchmark model exactly the same matching predictions are obtained for these functional forms, suggesting that our intuition could perhaps be applicable to a broader class of models.

[12]Not that $W(v)$ is not monotone in $s_1$, so we would focus on the intermediate values of $s_1$, for which $W(v)$ is decreasing.

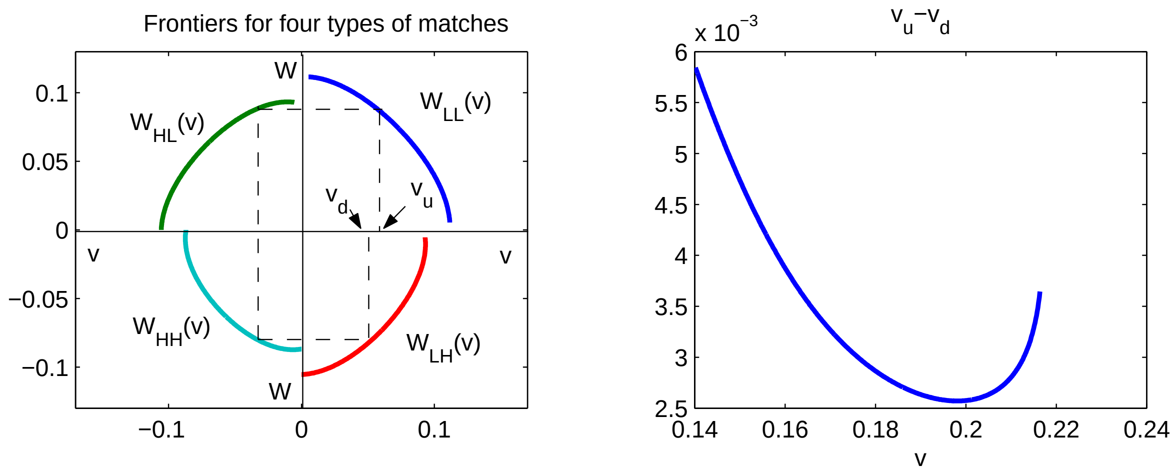


Figure 4: Frontiers and $v_u(v) - v_d(v)$ for $g(x) = (1 - e^{-x})/2$, $\theta_l = 0.1$, $\theta_h = 0.11$. Since $v_u(v) - v_d(v) > 0$ for all $v$, GID condition holds and positive matching obtains in equilibrium.

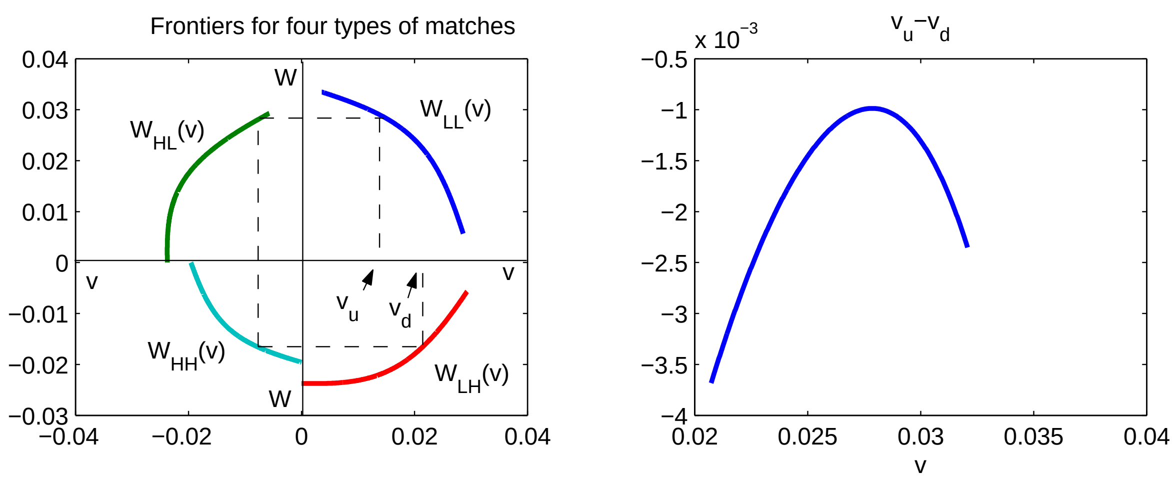


Figure 5: Frontiers and $v_u(v) - v_d(v)$ for $g(x) = x^{0.55}$, $\theta_l = 3$, $\theta_h = 4$. Since $v_u(v) - v_d(v) < 0$ for all $v$, GDD condition holds and negative matching obtains in equilibrium.

# 6 Conclusion

We develop a model of team production where, in the absence of information frictions, there is no reason for any matching structure to prevail. Once we add a team moral hazard problem, there is typically a non-trivial matching decision for the principal. We formulate the solution to this decision in three ways. First we show that the decision is directly linked to the relationship between inputs across agents of different types, and the bonus they received. Second, we show that the structure is related to the shape of the relationship between an individual agents input and the team probability of success, in a way that is familiar from other matching structures that do not include a team component. Finally, for special cases, we show that the matching structure can depend on the way in which corner solutions arise. Corner solutions can switch the matching structure from positive to negative matching. The force seems to survive moving either to a world with risk aversion by agents, or to a model of partnerships.

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# 7 Appendix

## 7.1 Second order conditions for positively matched teams

If both workers in the team have marginal cost $\theta$, then the planner's profit is given by

$$\Pi(x) = 2g(x)\left(1 - \frac{2\theta}{g'(x)}\right). \tag{14}$$

Correspondingly,

$$\Pi'(x) = 2g'(x)\left(1 - \frac{2\theta}{g'(x)}\right) + 4\theta g(x)\frac{g''(x)}{(g'(x))^2}, \tag{15}$$

and

$$\Pi''(x) = 2g''(x) - 2\theta\left(g(x)\frac{-g''(x)}{(g'(x))^2}\right)'_x. \tag{16}$$

Therefore, the second order condition is satisfied if $g(x)\frac{-g''(x)}{(g'(x))^2}$ is increasing (whenever the optimal matching structure is positive), but may be violated otherwise.

## 7.2 Second order conditions for negatively matched teams

The planner's profit in the team consisting of two different workers, with marginal costs $\theta_l$ and $\theta_h$, is given by

$$\Pi(x_l, x_h) = (g(x_l) + g(x_h)) \left(1 - \frac{\theta_l}{g'(x_l)} - \frac{\theta_h}{g'(x_h)}\right). \tag{17}$$

Thus

$$\frac{\partial \Pi(x_l, x_h)}{\partial x_l} = g'(x_l) \left(1 - \frac{\theta_l}{g'(x_l)} - \frac{\theta_h}{g'(x_h)}\right) + \theta_l \frac{g''(x_l)}{g'(x_l)^2} \left(g(x_l) + g(x_h)\right), \tag{18}$$

$$\frac{\partial^2 \Pi(x_l, x_h)}{\partial x_l^2} = g''(x_l) \left(1 - \frac{\theta_h}{g'(x_h)}\right) - \theta_l \left(\frac{-g''(x_l)}{g'(x_l)^2} \left(g(x_l) + g(x_h)\right)\right)'_{x_l} \tag{19}$$

Note that, as for the positively matched teams, $\frac{\partial^2 \Pi(x_l, x_h)}{\partial x_l^2} < 0$ if positive matching is optimal but the second order condition may be violated if $g(x)\frac{-g''(x)}{(g'(x))^2}$ is decreasing (i.e., if the negative matching structure is optimal).
Since the problem is symmetric, the same is true for $\frac{\partial^2 \Pi(x_l, x_h)}{\partial x_h^2}$.

The cross-derivative is

$$\frac{\partial^2 \Pi(x_l, x_h)}{\partial x_l \partial x_h} = \theta_h g'(x_l) \frac{g''(x_h)}{(g'(x_h))^2} + \theta_l g'(x_h) \frac{g''(x_l)}{(g'(x_l))^2}, \tag{20}$$

which, by the first order conditions, can be simplified to

$$\frac{\partial^2 \Pi(x_l, x_h)}{\partial x_l \partial x_h} = 2\theta_h g'(x_l) \frac{g''(x_h)}{(g'(x_h))^2} = 2\theta_l g'(x_h) \frac{g''(x_l)}{(g'(x_l))^2}. \tag{21}$$

Unfortunately, we cannot say anything conclusive regarding the sign of

$$\frac{\partial^2 \Pi(x_l, x_h)}{\partial x_l^2} \frac{\partial^2 \Pi(x_l, x_h)}{\partial x_h^2} - \left(\frac{\partial^2 \Pi(x_l, x_h)}{\partial x_l \partial x_h}\right)^2,$$

even if the positive matching is optimal. However, we can verify that the second order conditions hold for the particular functional forms of $g(x)$ considered below.

### 7.3 Second order conditions for the interior solution for power function

Plugging $g(x) = x^\alpha$ into (19) and (21), we obtain that

$$\frac{\partial^2 \Pi(x_l, x_h)}{\partial x_l^2} = \alpha(\alpha - 1)x_l^{\alpha-2} < 0, \tag{22}$$

and

$$\frac{\partial^2 \Pi(x_l, x_h)}{\partial x_l \partial x_h} = 2(\alpha - 1)\theta_l x_l^\alpha x_h^{\alpha-1} = 2(\alpha - 1)\theta_h x_h^\alpha x_l^{\alpha-1}. \tag{23}$$

Correspondingly,

$$\begin{aligned}
&\frac{\partial^2 \Pi(x_l, x_h)}{\partial x_l^2}\frac{\partial^2 \Pi(x_l, x_h)}{\partial x_h^2} - \left(\frac{\partial^2 \Pi(x_l, x_h)}{\partial x_l \partial x_h}\right)^2 = \\
&\qquad = \alpha^2(1-\alpha)^2 \frac{x_l^{\alpha-1}x_h^{\alpha-1}}{x_l x_h} - 4\theta_l\theta_h(\alpha-1)^2\frac{1}{x_l x_h} = \\
&\qquad = \frac{(1-\alpha)^2}{x_l x_h}\theta_l\theta_h\left[\frac{1}{\alpha^2}\left(1 + \left(\frac{\theta_l}{\theta_h}\right)^{\frac{\alpha}{1-2\alpha}}\right)\cdot\left(1 + \left(\frac{\theta_h}{\theta_l}\right)^{\frac{\alpha}{1-2\alpha}}\right) - 4\right],
\end{aligned} \tag{24}$$

which is definitely positive if $\alpha \in (0, 1/2]$.

### 7.4 Some properties of corner solution for power function

Consider the case when $\theta_h > 2\alpha^2\overline{x}^{\alpha-1}$ and $\theta_l$ is sufficiently small so that the planner chooses the corner solution $x_l = \overline{x}$. As mentioned in the main text of the paper, the optimal effort of the high cost agent in this case is $x_h = \min\{\overline{x}, x\}$, where $x$ solves

$$\alpha x^{\alpha-1}\left(1 - \frac{\theta_l}{\alpha}\overline{x}^{-\alpha}\right) - \frac{\theta_h}{\alpha} = \frac{\theta_h}{\alpha}(1-\alpha)\overline{x}^\alpha x^{1-\alpha} \tag{25}$$

Denote the left hand side and the right hand side of the above equation by $L(x)$ and $R(x)$ respectively. Obviously, both $L(x)$ and $R(x)$ are decreasing. Observe that $\alpha < 1/2$ implies that $\lim_{x\to 0} L(x)/R(x) = \infty$ and $\lim_{x\to\infty} L(x)/R(x) = 0$. Next, it can be verified that $\min_x L(x) - R(x) < 0$, implying that the above equation has at least one solution. To see that this solution is unique, it suffices to notice that $L'(x) > R'(x)$ implies that $L(x) < R(x)$. Hence, $L(x)$ and $R(x)$ have a unique intersection at which $L'(x) < R'(x)$, implying that there exists unique $x_h \in (0, +\infty)$ which maximizes the planner's expected payoff.