Market Outcomes and Dynamic Patent Buyouts[1]

Alberto Galasso
University of Toronto and NBER

Matthew Mitchell
University of Toronto

Gabor Virag
University of Toronto

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Abstract

Patents are a useful but imperfect reward for innovation. In sectors like pharmaceuticals, where monopoly distortions seem particularly severe, there is growing international political pressure to identify new reward mechanisms which complement the patent system and reduce prices. Innovation prizes and other non-patent rewards are becoming more prevalent in government's innovation policy, and are also widely implemented by private philanthropists. In this paper we describe situations in which a patent buyout is effective, using information from market outcomes as a guide to the payment amount. We allow for the fact that sales may be manipulable by the innovator in search of the buyout payment, and show that in a wide variety of cases the optimal policy still involves some form of patent buyout. The buyout uses two key pieces of information: market outcomes observed during the patent's life, and the competitive outcome after the patent is bought out. We show that such dynamic market information can be effective at determining both marginal and total willingness to pay of consumers in many important cases, and therefore can generate the right innovation incentives.

1 Introduction

Innovation is the main engine of economic growth, and the consensus among economists, beginning with Arrow (1962), is that the positive externalities from R&D imply under-investment relative to the socially optimal level. For example, a recent study by Bloom et al. (2013) estimates that the gross social rate of return to R&D substantially exceeds the private return, with the socially optimal R&D level being more than twice as high as the currently observed R&D expenditure. A central policy question, therefore, is how one can best devise mechanisms that encourage innovation.

The patent system is one of the main instruments governments use to increase R&D incentives. Recently, increased attention has been paid to alternative reward mechanisms, which complement the patent system and can preserve innovation incentives, especially for breakthrough technologies generating large social welfare. McKinsey estimates that the total funds available from innovation prizes have more than tripled over the last decade to surpass $375 million with a large number of philanthropists entering the business of rewarding innovators (McKinsey, 2009). For example, Qualcomm and Nokia have offered multi-million dollar prizes for the development of affordable devices that can recognize and measure personal health information. Similarly, the Gates Foundation has offered an innovation award to immunize children in the poorest parts of the world, and the X PRIZE Foundation offered a $10 million Ansari Prize for a private space vehicle to launch a reusable manned spacecraft into space twice within two weeks (Murray et al, 2012). At the same time, government interest in innovation prizes has also increased substantially. In the United States, President Obama's Strategy for American Innovation strongly encouraged the use of innovation prizes, and the America Competes Reauthorization Act of 2011 provided all federal agencies with the power to offer innovation prizes (Williams, 2012).

Despite this growing trend, there is relatively little theoretical work on the design of innovation prizes. This paper contributes to the recent literature focusing on designing prizes that infer demand from various market signals, and use that information to design a reward at least partially based on a cash prize. We study the problem of a philanthropic or government agency interested in rewarding a breakthrough innovation with limited information on the research cost and the social welfare generated by the new technology. Following the mechanism design literature, we refer to such agency as the "social planner". We show that, in a wide variety of environments, social welfare can be improved by prizes in the form of a buyout of patent rights over time. The buyout system replaces some of the rents that are obtained through monopoly rights with a prize.

If the planner cannot discern the quality of innovations, pure prizes are difficult to implement because the value of the prize cannot be tied to the surplus generated by the innovation, as the demand is unknown by the planner. Worse still, it might be the case that market signals can be manipulated by an innovator, was the innovator to know that a prize was tied to market outcomes. Even if the planner could obtain precise information about the number of units sold at a given price (for instance, by observing units sold under perfect competition), this is insufficient to construct the inframarginal values of consumers, which is essential to estimating the full value of an innovation. Our approach addresses the need to estimate the surplus of inframarginal consumers. In order to accomplish this, we stress the dynamic approach to innovation rewards, since one point on the demand curve will generally be insufficient for reconstructing demand. In contrast with the previous literature, we assume that the planner can learn over time about market conditions by observing price and quantity realizations that arise from the choice of the innovator and the underlying demand function. As information about the market demand is revealed, the reward mechanism that maximizes social welfare may change according to the revealed information. Eventually, the planner can resort to allowing perfect competition, which generates additional information about the demand for the innovation.

In all but the least-manipulable environments we study, the optimal policy begins with market power for the innovator, and gradually moves toward competitive pricing as information is generated by the experience of the innovation. We show that even in the most manipulable environments, where the true price that gave rise to the observed sales can be completely obscured by the innovator, the optimal mechanism involves some reward through a contingent prize near the end of the period in which the innovator is rewarded. The optimal mechanism is a hybrid between a patent and a prize in the sense that it rewards innovators through prices above marginal cost initially, but then moves toward a reward that is focused on a cash prize and prices closer to, or reaching, marginal cost.

Our results are directly relevant to the rising number of philanthropists who have entered the business of rewarding innovators. Recent proposals have considered linking prize rewards to specific market outcomes. For example, the Center for Global Development advised that philanthropists willing to sponsor the development of a malaria vaccine pay the innovator 14 dollar for each of the first 200 million treatments sold at 1 dollar to the recipients (Glennerster, et al. 2006). Our results suggest one approach to this philanthropy: use resources to buyout patents that have a track record of success.[1]

The results might also be of interest to policy makers. From the policy maker's perspective, our findings provide guidance to government agencies looking for tools that complement patents and can spur innovation while minimizing product market distortions. We show that an effective mechanism is a patent buyout scheme whose reward depends on the observed market outcomes. The computation of the reward resembles structural estimation studies which typically estimate the primitives of a model from local price variation and exploit these estimates for out-of-sample welfare analysis. In other words, the buyout is facilitated by information from data in much the same way that the impact of a merger on consumer surplus is assessed through estimation of an econometric model. An important feature of the mechanism is that it does not require any change in the functioning of the current patent system. Policy makers have simply the option to complement patents with buyout schemes that depend on market outcomes. From this perspective, we believe that this policy tool is well suited for selected high-value technologies for which the welfare impact of free access is expected to exceed the cost of the public funds associated with the buyout.

Our model can also provide insights for a regulator, antitrust or otherwise, who faces firms with monopoly granted through IP. For example, the Australian government offers copayments for selected drugs to mitigate monopoly distortions. Similarly, the FDA is involved in the administration of ex post rights for pharmaceuticals through the orange book program and the rights granted therein. More generally, Hovenkamp (2004) describes the sense in which

[1] Similar ideas have appeared in AgResult, an initiative launched by the governments of Australia, Canada, Italy, the United Kingdom, the United States, the Bill and Melinda Gates Foundation and the World Bank to mitigate R&D underinvestment in tropical agriculture. A key feature of the initiative is to focus on incentive schemes that link payments to demonstrated results.

antitrust policy might respond to growing IP protection.

We develop a model with discrete time and infinite horizon where the planner commits to a reward structure that depends on the history of prices and quantities realizations observed over time. The planner's problem in designing an appropriate prize is observing total benefit of the innovation. As in Kremer (1998), this requires information about the quality of the innovation; Weyl and Tirole (2012) point out that this problem is magnified by the need to discern the willingness to pay of non-marginal consumers. Our mechanism attacks both issues. Our first result is that, in the absence of demand manipulation, the first best can be approached arbitrarily closely in a large set of demand functions that includes those typically used in the industrial organization literature.

Discussing a number of extensions of the baseline model, we argue that the assumptions required to reach the first best are those typically imposed in structural industrial organization studies which identify the primitives of a model from local price variation and exploit the estimated parameters to conduct out-of-sample welfare analysis (Figure 1 case A). In our context, the planner can request the innovator to generate price variation that will be used to identify the underlying demand curve of the technology and to compute a patent buyout transfer that compensates the innovator for the surplus generated. By keeping the price variation concentrated around the marginal cost of production, the planner can limit the loss of surplus associated with learning to a minimum (Figure 1 case B).

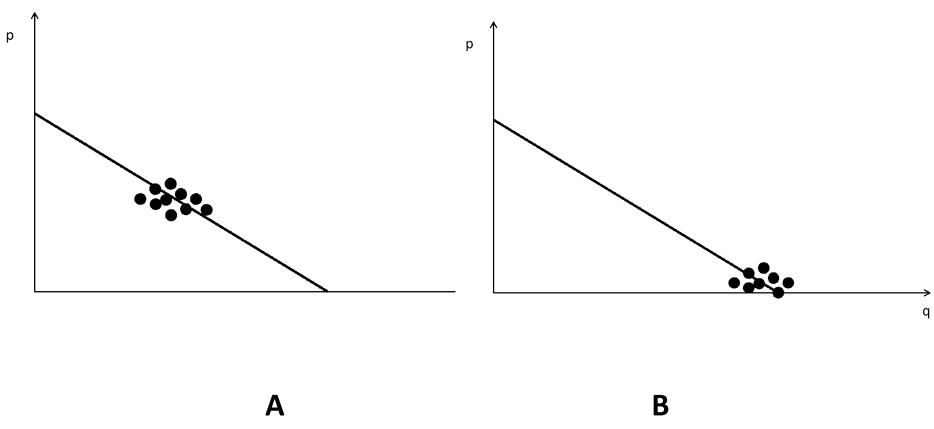

Figure 1: Market Outcomes and Demand Identification

We then investigate the case in which the innovator can manipulate demand. In some cases, such as pharmaceuticals, quantity may be relatively well measured, but prices may be more opaque and companies have an incentive to manipulate their prices in order to obtain higher reimbursements through public funding.[2] In keeping with the pharmaceutical price manipulation example, we assume that quantity is observable, while price may not be. We show that it is crucial to distinguish between the case in which demand manipulation is possible after the buyout takes place, and the case in which post-buyout demand is non-manipulable. We show that pre-buyout manipulation, even if costless, may be ignored as long as manipulation after buyout is not possible. This is because the planner can generate price variation after the buyout to learn the demand and to punish the innovator in the case of manipulation. This implies that market outcomes are relevant even after the buyout, because they are useful to detect and avoid manipulations.

The case in which the planner cannot generate price variation after the buyout is more complicated. We consider the case in which after the buyout the patent is sold in a competitive market, and neither the planner nor the innovator can manipulate this outcome. We show that in this case, as long as pre-buyout manipulation is costly, the planner can construct a buyout scheme that generates the same R&D incentives as a patent and increases total welfare. Intuitively, the planner can induce the innovator to reveal the true monopoly profits by requiring a stream of pre-buyout outcomes that are too costly to manipulate.

Finally, we characterize the optimal mechanism when price manipulation is costless for the innovator. We show that even in this case, the optimal mechanism differs substantially from a patent. It is optimal for the planner to induce the innovator to produce quantities that are above the monopoly level, and larger output for innovations generating lower surplus.

The paper is organized as follows. Section 2 reviews the related literature. Section 3 presents the baseline model. Section 4 examines the optimal policy in the absence of demand manipulation. Section 5 introduces costly demand manipulation. Section 6 studies the optimal mechanism in the presence of costless demand manipulation. Section 7 summarizes and

[2] For example, in March 2001 the State of Wisconsin reached a $4.2 million settlement agreement with Merck, Schering and Warrick Pharmaceuticals in litigation charging the companies with defrauding the Wisconsin Medicaid Program. Wisconsin alleged that the pharmaceutical manufacturers manipulated wholesale prices information, knowing that Medicaid would rely on these prices to determine Medicaid reimbursement.

concludes. All the proofs are in the appendix.

2 Related Literature

This paper is connected to various strands of the literature on the economics of innovation. In an influential paper, Kremer (1998) suggests a buyout mechanism, which is linked to an auction to incentivize research and maximize welfare. The role of the auction is to reveal information to the planner about the private value generated by the innovation. Innovation incentives are maximized because the planner would pay for the patent the private value times a fixed markup that compensates for the difference between social and private surplus. Consumer welfare is also maximized because the innovation would be placed in the public domain once acquired by the planner. An important assumption underlying the buyout scheme suggested by Kremer is that the competitors of the innovator know the value (and the cost) of the innovation and are willing to take part in the auction. In our model, we depart from this assumption and assume that only the innovator knows how valuable an innovation is at the aggregate level. Therefore, the planner needs to design a mechanism that aggregates the information contained in consumers' individual valuations.

Wright (1983) and Shavell and Van Ypersele (2001) provide a comparison of prizes and patents as mechanisms to incentivize innovation in a static framework. Scotchmer (1999) studies the optimal mechanism to reward innovation when the planner offers a menu of patents that differ in length and application fee. She shows that if market outcomes are not observed, then in the presence of asymmetric information on the cost and benefit of research, patent renewal mechanisms are optimal in the sense that every incentive compatible and individually rational direct revelation mechanism can be implemented with a renewal mechanism. Cornelli and Schankerman (1999) characterize the optimal innovation mechanism in a model with moral hazard and adverse selection where innovators have unobservable productivity parameters. As in Scotchmer (1999), the planner offers the innovator a menu of patents that differ in length and application fee. They show that the optimal patent scheme is typically differentiated and can be implemented through menu of patent renewals.[3]

[3] Gans and King (2007) extend the innovative environment to include timing as an important choice. They demonstrate that a finitely lived, but broad, patent can be socially desirable.

Hopenhayn and Mitchell (2001) and Hopenhayn, Llobet and Mitchell (2006) study the optimal patent design when innovation is cumulative and each discovery is a building block to future innovations. Hopenhayn and Mitchell (2001) consider the case in which the quality of the idea is private information and there are two generations of the technology. They show that to maximize innovation incentives, patents must vary in breadth, i.e. the policy maker needs to vary the set of products that at any given time may be prevented by the patent holder. Hopenhayn, Llobet and Mitchell (2006) study a dynamic framework with multiple cumulative innovations and private information about the quality of ideas and R&D investments. They show that in such an environment the optimal mechanism is a patent buyout scheme where the innovator commits to a price ceiling at which he sells his rights to a future inventor.

Acemoglu and Akcigit (2012) develop a dynamic framework with cumulative innovation and show that full patent protection is not optimal, whereas state-dependent property rights are preferable. Akcigit and Liu (2014) study the optimal mechanism to reward research investments when multiple firms compete with private information about the value of innovation.

We are aware of only few studies which consider observable market outcomes. The first one is Weyl and Tirole (2012) that studies the optimal reward structure in the presence of multidimensional heterogeneity and non-manipulable market outcomes. In a static framework, they show that the optimal policy requires some market power but not full monopoly profits. Such a policy is similar to Mitchell and Moro (2006), who study a planner who trades off deadweight loss against over-transferring to a group that "loses" from elimination of the distortion generating deadweight loss. Our setup differs from Weyl and Tirole (2012) because we introduce dynamics and allow the innovator to manipulate market outcomes.

The second paper is by Chari, Golosov and Tsyvinski (2012) who compare prizes and patents when the planner can observe market signals over time. They develop a dynamic framework where both the innovator and his product market competitors (but not the planner) know the value of the innovation that is represented by a unidimensional parameter. Their main finding is that patents are necessary if the innovator can manipulate market signals. Our model departs from their setting in a number of dimensions. First, we assume that only the innovator knows the value of his technology and we do not require the presence of informed competitors. Second, we allow for multidimensional heterogeneity in innovation quality. Because in a multidimensional setting observing one market outcome is not enough to learn the entire demand curve, in our model the planner faces a nontrivial learning problem even under full (i.e. non-manipulated) observation of the market signals. This implies that, differently from Chari, Golosov and Tsyvinski (2012), in our model the planner finds it optimal to use information acquired over time in a truly dynamic way. Third, we do not restrict the planner to use either patents or prizes and we consider a large set of reward structures that depend on the quantity and prices practiced by the innovator. In particular, we allow for patents of different "strength" in which the price charged by the innovator differs from both the monopoly and the competitive prices.

Finally, Brynjolfsson and Zhang (2007) in a policy paper propose a "statistical couponing" mechanism to assess the value of digital goods (e.g. software or music files). Their idea is to identify consumer willingness to pay exploiting coupons for a small (but representative) sample of consumers. The planner infers total market valuation from the behavior of this sample of consumers, and rewards appropriately the innovator while keeping price at marginal cost level. Our results show that a similar outcome can be reached in the absence of coupons, by exploiting dynamic market information. Our analysis also stresses that innovators have strong incentives to manipulate the coupon market to obtain larger rewards.

3 The Model

Time is discrete and the horizon is infinite. Each innovation is characterized by variables c and θ. These two parameters determine the cost and the value of a particular innovation. First, the ex-ante cost of creating the innovation is $c \in C \subset \mathbb{R}_+$. Second, the demand function $q = D(p, \theta)$ depends on the demand parameter θ. We assume that $\theta \in \Theta$, a compact subset of $\mathbb{R}^N$, and that D is continuous in θ. Demand and cost parameters, θ and c are private information for the innovator and are distributed according to a smooth probability density function $\psi(\theta, c)$ that is common knowledge among the planner and the innovator. We make the regularity assumption that D is twice continuously differentiable in p. To ensure the concavity of the static profit function in p, we assume that $D_p(p, \theta) < 0$ and $D_p(p, \theta) + pD_{pp}(p, \theta) < 0$ for each $p \geq 0$. Let $\overline{p} > 0$ be the minimum price at which $D(\overline{p}, \theta) = 0$. The marginal cost of production is normalized to zero.[4]

We assume that a planner observes perfectly the quantities in each period but the innovator can manipulate the price observed by the planner. Specifically, if the real price charged is p_t, the innovator can make the planner observe $\widehat{p}_t$ by sustaining a cost $\phi\left(\widehat{p}_t, p_t, \theta\right)$ with $\phi\left(p_t, p_t, \theta\right) = 0$. Most of our analysis will focus on two polar cases: (i) no manipulation where

$$\phi = \infty \text{ if } \widehat{p}_t \neq p_t$$

and (ii) costless manipulation where

$$\phi = 0 \text{ for all } \widehat{p}_t.$$

Section 5 provides examples and a micro-foundation for the manipulation cost $\phi\left(\widehat{p}_t, p_t, \theta\right)$. For instance, a positive manipulation cost arises if the innovator can offer secret discounts which cost more to the innovator than they are worth for the consumers.

Let us indicate with $h_t \in H_t$ the public history at time t, that can be defined recursively as $h_t = (h_{t-1}, r_t)$ where $r_t = (q_t, \widehat{p}_t)$ is the information revealed in period t and $h_0 = \varnothing$. Thus $H_t \in \mathbb{R}_+^{2t}$, the set of public histories at time t is the Cartesian product (t times) of the set of observable price-quantity pairs.

The planner maximizes expected total discounted surplus, with discount factor δ between periods. Following the literature, we assume that the planner has full commitment power to any announced policy.[5] The planner designs a reward schedule that in each period transfers to the innovator a sum, $g_t(h_t)$, that depends on the history $h_t \in H_t$. The planner has also the option to set up a non-manipulable irreversible competitive market in period $T + 1$. The switching time may depend on the history, and can be infinite (i.e. switching to competition may never occur). Our preferred interpretation of this reward schedule is that the innovator owns a patent up to period T and that at period $T + 1$ patent rights are revoked and the innovation is placed in the open domain. When $g_t(h_t) > 0$ and T is finite, the policy implies

[4] The parameter θ, by capturing the global shape of the demand function can be interpreted as a proxy for the perceived quality of the innovation. Our model abstracts from consumption externalities.

[5] This is usually justified by reputational concerns of the planner. We further discuss the case of limited commitment at the end of Section 4.2.

that the planner is paying the innovator to remove patent protection at time T, i.e. the planner buys the patent out at time T.

A strategy of the innovator is a sequence of pair of prices $(\widehat{p}_t, p_t)$ for each period t that satisfies the constraint that prices are set to zero after switching has occurred. Let $\alpha \in A$ denote any such strategy and A denote the set of all possible strategies.[6] The function $T(\alpha)$ captures the time period in which the planner's policy calls for a switch to the competitive and non-manipulable regime. This time is deterministic from the perspective of the innovator since it depends only on his strategy. In Appendix 2 we provide a recursive definition of $T(\alpha)$ given the planner's policy function.

Then the innovator's maximization problem upon pursuing the innovation is

$$\max_{\alpha \in A} \sum_{t=1}^{T(\alpha)} \delta^{t-1} \left(p_t D\left(p_t, \theta\right) + g_t(h_t) - \phi\left(\widehat{p}_t, p_t, \theta\right)\right) + \sum_{t=T(\alpha)+1}^{\infty} \delta^{t-1} g_t(h_t). \tag{1}$$

To simplify the notation, we leave the relationship between the switching time and the strategy of the innovator implicit and indicate $T(\alpha)$ as T in the remainder of the paper.[7]

Let us indicate the optimal revealed and actual price for period t with $\widehat{p}_t^*(\theta)$ and $p_t^*(\theta)$ and with $h_t^*(\theta)$ the public history revealed by this optimal equilibrium play. Investment in innovation takes place if the net present value of the profits of the innovator (1) exceeds c. Let us indicate with $\Theta^*(c)$ the set of types for which this condition is satisfied.

The social surplus (net of manipulation costs) in the product market if the planner chooses functions $\{g_t\}_{t=1,2,3,\ldots}$ and T is equal to:

$$W(\theta) = \sum_{t=1}^{T} \delta^{t-1} \left[S(p_t^*(\theta), \theta) - \phi\left(\widehat{p}_t^*(\theta), p_t^*(\theta), \theta\right)\right] + \sum_{t=T+1}^{\infty} \delta^{t-1} S(0, \theta)$$

with

$$S(p_t, \theta) = p_t D\left(p_t, \theta\right) + \int_{p_t}^{\overline{p}} D\left(z, \theta\right) dz.$$

The social planner chooses functions g_t and T (taking the optimal strategy of the innovator

[6]Note that the innovator's strategy is formed upon observing the planner's switching policy, and that the planner's switching policy affects the set of strategies for the innovator.

[7]We assume that the innovator canno manipulate the market after time T. To prevent manipulation, the planner can implement a whistle-blowing system or exclude the innovator from producing after period T and only allow the competitive fringe to manufacture the good.

as given) to maximize the expected total social welfare created by the innovation:

$$\max_{g_t,T} \int_c \int_{\theta\in\Theta^*(c)} [W(\theta) - c]\, \psi(\theta, c) d\theta dc.$$

The *first best* can now be defined formally: in the first best it holds that $p_t = 0$ for all $t \geq 1$, the innovator does not distort the observed price ($\widehat{p}_t = p_t$), and the innovation is developed if and only if

$$c \leq \sum_{t=1}^{\infty} \delta^{t-1} S(0, \theta).$$

The first best can be easily implemented by the planner if θ is known. To do so, the planner transfers the entire surplus to the innovator if he observes the competitive quantity and punishes the innovator if a different quantity is observed (i.e. $g_t = S(0, \theta)$ if $q_t = D(0, \theta)$ and $g_t = -\infty$ if $q_t \neq D(0, \theta)$). In this case one can interpret the punishment as part of the contract that potential prize winners agree to.

The functions g_t and T allow the planner to implement a number of different reward mechanisms. We provide some examples below.

Patents

When $g_t(h_t) = 0$ and $T = \varsigma$ the planner offers a ς-period patent that generates innovation incentives through product market profits. The setting also accommodates the payment of renewal fees. For example we can introduce a fee, f, to be paid at time $T_1 < \varsigma$, with expiration of the patent in the absence of payment:

$$\begin{aligned} g_t(h_t) &= \begin{cases} -f & \text{if } \ t = T_1 \ \text{ and } \ \widehat{p}_{T_1} > 0 \\ 0 & \text{else} \end{cases} \\ T &= \begin{cases} \varsigma & \text{if } \widehat{p}_{T_1} > 0 \\ T_1 & \text{else} \end{cases}. \end{aligned}$$

Simple Buyout

The following specification

$$\begin{aligned} g_t(h_t) &= \begin{cases} 0 & \text{if } \ t < \varsigma \\ K & \text{if } \ t = \varsigma \end{cases} \\ T &= \varsigma \end{aligned}$$

captures a simple buyout scheme in which the planner commits to buy the patent after ς periods at a pre-specified amount K. The setting also allows to implement more complex buyout mechanisms where transfer price K and acquisition time T may depend on observed market outcomes.

4 Optimal Mechanism in the Absence of Demand Manipulation

In this section we characterize the optimal mechanism when the government can dictate prices and the innovator cannot manipulate demand, i.e. $\phi = \infty$ if $\widehat{p}_t \neq p_t$. In this case the planner can essentially dictate prices by requiring that the prize winner follows the prescribed price path. Existing prizes, like the malaria prize offered by the Center for Global Development (described below), include a pricing requirement.

To develop the intuition, let us consider a simple setting where the demand is linear $q_t = \theta_1 - \theta_2 p_t$. In this simple environment the planner can identify the intercept of the demand by inducing a price equal to zero in the first period so that $q_1 = \theta_1$. In the second period he can induce $p_2 = \varepsilon > 0$ and identify θ_2 by inverting $q_2 = q_1 - \theta_2\varepsilon$. This means that it takes only two periods for the planner to learn the demand function and the surplus generated by the innovation. Notice that the planner can set ε arbitrarily close to zero and minimize the deadweight loss generated by above marginal cost pricing. If the entire surplus generated by the innovation is transferred to the innovator, innovation incentives are set at the first best level.[8] This generates two benefits: lower deadweight loss, and the prize can reward inframarginal values that should motivate innovation, but are not included in simple monopoly pricing.

The above example suggests that transfers that depend on market outcomes can be powerful mechanisms to incentivize innovation. The planner finds it optimal to use market information in a truly dynamic way that allows him to approximate the complete information (first best) solution. In particular, by conditioning rewards on quantities and prices, the planner can obtain the information required to trace-out the demand curve. Once the demand is known, the surplus generated by the innovation is transferred to the inventor to maximize his innovation

[8] A transfer that approximates the first best is $g_1(h_1) = 0$ for all h_1; $g_2(h_2) = S(\varepsilon) + S(0)/\delta$ if $h_2 = \{q_1, 0, q_2, \varepsilon\}$ and $g_2(h_2) = -\infty$ otherwise; $g_t(h_t) = S(0)$ for $t > 2$ if $h_t = \{q_1, 0, q_2, \varepsilon, q_3, 0, .., q_t, 0\}$ and $g_t(h_t) = -\infty$ otherwise.

incentives. In the linear case, the demand can be learned by observing only two data points: the quantity sold at marginal cost and the quantity sold at any strictly positive price. Exploiting this feature of the demand, the planner will learn the demand by inducing the innovator to sell at an arbitrarily small price. This makes the deadweight loss negligible and allows the planner to approximate the first best solution.

Relative to static multidimensional screening mechanisms as the one characterized by Weyl and Tirole (2012), it is natural that the dynamic model can do more, since it offers more instruments. The result obtained in the simple linear setting shows that this gain can be substantial. Our next result shows that this logic expands to a broad set of demand functions. We start with the definition of an analytic demand function.

Definition 1 *(Judd, 1998) A demand function $D(\cdot,\theta)$ is analytic on X if and only if for every $p \in X$ there is an r, and a sequence c_k such that whenever $|z-p| < r$:*

$$D(z,\theta) = \sum_{k=0}^{\infty} c_k (z-p)^k.$$

We generalize the result obtained for linear demands to analytic functions.

Proposition 1 *If $D(\cdot,\theta)$ is analytic on $[0,\overline{p}] \subset \mathbb{R}$, then the first best can be approached arbitrarily closely.*

Our proof builds on Aghion et al (1991) who show in the context of an uninformed decision maker that when a payoff function is analytic the approximate derivative at a single point can be used to estimate the global behavior of the function. We show that the demand function can be approximated by collecting price and quantity observations over a small neighborhood around a single price. These observations are used to approximate the derivatives of $D(\cdot,\theta)$ around that price and to learn about the global behavior of $D(\cdot,\theta)$.

By choosing a smaller and smaller neighborhood around $p = 0$, the planner minimizes the welfare losses associated with learning and increases the accuracy of the estimates of the derivatives of $D(\cdot,\theta)$. In the proof we show that exploiting a step-wise analytic continuation technique the planner can approach arbitrarily closely the first best even if $D(\cdot,\theta)$ can be expanded in a power series locally but not globally.

Proposition 1 substantially generalizes the result for linear demands. Polynomials, exponentials, logarithms, power functions and a number of other demand functions that are typically used in applied theory are analytic functions. Fox and Ghandi (2011) show how analyticity of the market demand is a property of various well know demand models used for structural estimation as the linear random coefficients model, the almost ideal demand system of Deaton and Muellbauer (1980) and the mixed logit of Berry, Levinsohn and Pakes (1995).

It is important to notice that Proposition 1 does not require the planner to know the specific functional form of the demand, she only needs to know that it is an analytic function. For instance, if the planner knows that the demand is one of N functional forms (or a linear combination of the N forms), experimentation means not only learning the shape of a given functional form, but experimenting across functional forms. Given that N can be arbitrarily large, this specification allows us to consider an arbitrarily rich set of possible demand functions.

One may also wonder about the role of the planner in an environment in which demand is known but marginal costs are not. This alternative problem is likely to be simpler for the planner, because it involves using variation in market outcomes to identify the marginal value of the cost function rather than to identify both the marginal and inframarginal willingness to pay as in our baseline model. As long as the planner can set-up a non-manipulable competitive market with a fringe producing at the same (unknown) marginal cost as the innovator, then our approach remains valid. The planner can simply observe the competitive price and back-out the marginal cost of production.

4.1 Discussion

The prior result can be generalized in many ways and in some (but not all) cases still allow the planner to implement the first best.[9]

4.1.1 Non-Stationary Demand

Demand Shifts. Following Battaglini (2005) we assume that the demand has two states high (H) and low (L) with $D_H(p,\theta) \geq D_L(p,\theta)$ for each p and that transition between states follows a Markov process. We show that also in this setting, if demand functions are analytic

[9] In Appendix 2 we describe these extensions in more detail.

the planner can maximize innovation incentives by approximating the first best outcome. To understand the intuition for the proof consider the case of linear demand. The planner can identify the intercepts of the two demand functions by dictating a price equal to zero and maintaining it until two different quantities are observed. Then he will set $p = \varepsilon$ until two different quantities are observed. With two observations along each demand line the planner learns the demand and welfare functions. By setting ε arbitrarily close to zero, the dead weight loss generated by above marginal cost pricing is minimized and the first best is approached arbitrarily closely. An interesting feature of this result is that the optimal incentive scheme is non-stationary and has unbounded memory even if the demand shifts follow a Markov process and the relevant economic environment has a memory of only one period.

Demand Growth. As we show it in Appendix 2, Proposition 1 may not generalize if demand grows over time. In such setting, when the demand does not grow too quickly, the planner will be able to approximate each level of demand and approach the first best. Nevertheless, the planner may not have enough time to learn the various demand levels when growth is fast and the first best may not be approached.

4.1.2 Public Funds and Menu Costs

Social Costs of Public Funds and Imperfect Capital Markets. When the innovator does not have access to a frictionless capital market, he prefers to be paid as soon as possible. However, a simple buyout mechanism delays much of the payment until after the buyout occurs, which can pose a long delay because learning the entire demand curve takes time. This is not a problem if the planner can raise funds at no cost, that is, if taxation does not have any deadweight losses. In this case, the planner can easily reimburse the innovator even before the buyout. Intuitively, the function g_t allows the planner to act as a capital market removing any friction the innovator may face in raising funds. In other words, the planner can spread the reward to the innovator over time adjusting the amount paid depending on the market outcomes observed in the past.

The assumption that there is no loss incurred by society when raising revenue (to buy the patent out) is a typical assumption in the economics of innovation literature (Chari et al., 2012; Weyl and Tirole, 2012). The assumption is justifiable in our setting where buyouts may be conducted by philanthropic foundations and may not be associated with distortionary taxation.

In our third extension we show that Proposition 1 is robust to dropping this assumption. We extend our model considering the case in which the government finances transfer T at a cost $(1+\kappa)T$ where $\kappa \geq 0$ denotes the cost of public funds due to the deadweight loss associated with taxation (as in Laffont and Tirole, 1993). The planner faces a trade-off between two types of welfare distortions: the cost of raising money through public taxation, κ, and the surplus losses due to market power. We show that in this case the first best involves a positive price, and that market power does not prevent the planner to approximate the welfare maximizing outcome arbitrarily closely. To see the intuition of this result, consider a mechanism design approach in which the innovator reports to the planner a type, $\widetilde{\theta}$, and the planner indicates a path of market outcomes. If the type is reported truthfully, the first best will be approximated by the planner by choosing market outcomes arbitrarily close to the welfare maximizing outcome for that type. Truthful revelation will occur because the planner can exploit market outcomes to learn the analytic demand and punish the innovator if the reported type is not consistent with the identified demand.

The above analysis also implies that even if there is a cost of raising public funds, the planner can still bring the payments of the innovator forward in time, if it is socially beneficial that the innovator is getting reimbursed as soon as possible. In particular, the planner can still implement the welfare maximum and pay out the innovator arbitrarily early.

Menu Costs. In our model the only cost of price variations is their impact on consumer's welfare.[10] In the presence of menu costs the planner's problem becomes substantially more complex. In fact, introducing a cost of changing the price leads to a trade-off between the marginal welfare benefit from observing an additional data point along the demand curve and the cost of changing the price.

4.1.3 Additional Identification Challenges

*Asymmetric Production Costs***.** Our baseline model assumes that the marginal cost of production for the innovator is the same as the one of the competitive fringe producing after buyout. In the Appendix, we show that Proposition 1 extends to the case in which the innovator has

[10] This is the typical assumption in the innovation literature, and can be justified by the mixed empirical evidence on the significance of menu costs (Nakamura and Steinsson, 2008) especially after the diffusion of the internet and modern information and communication technologies (Brynjolfsson and Smith, 2000).

a cost advantage. Interestingly, the optimal policy no longer involves a buyout. Instead, even when the planner has full knowledge of the demand curve, the innovator is selected to produce and sell the product at his own (lower) marginal cost. We also show that in the case in which the competitive fringe has a cost advantage over the innovator, Proposition 1 goes through if it is assumed that: (i) either the production technology of the fringe is available to the innovator through licensing or contract manufacturing; (ii) or it is possible for the planner to levy a tax after the buyout to learn the demand after the buyout. Under case (i) the planner requires the innovator to pick one firm in the competitive fringe as exclusive licensee of the innovation, and induces price variation around the marginal cost of the manufacturer. This is a natural assumption in cases where the innovator lacks complementary assets required for efficient large-scale production. Under case (ii) the planner places the innovation in the competitive market immediately, and generates price variation through taxation. If both assumption (i) and (ii) are violated, Proposition 1 does not hold and the first-best cannot be achieved. In this case, learning the demand curve can only occur by experimenting at prices that are above the post-buyout production costs.

Demand is Observed with Error. We consider the case in which the demand is observed with error and assume that $q_t = D(p_t, \theta) + \varepsilon_t$ where ε_t is a mean zero i.i.d. noise over the support $[-\bar{\varepsilon}, \bar{\varepsilon}]$. Even in this case, analyticity of the demand function is sufficient to approach the first best arbitrarily closely. In the linear demand case, the planner can use the following two step scheme. In the first stage the planner induces the firm to charge $p = 0$ and obtains a sample of N quantities for this price. Then he sets a price equal to ε and obtains another sample of N quantities. The weak law of large numbers guarantees that, for N large enough, the sample averages are unbiased estimates of $D(0, \theta)$ and $D(\varepsilon, \theta)$ and therefore the demand parameters can be learned by the planner. While the law of large numbers guarantees that the estimate is unbiased, the variance of the estimate depends on the price variation and is smaller when the variation is larger. This is not an issue in our setting because we assumed that the planner and the innovator are both risk neutral. Even in the case of risk aversion, the variance can be made arbitrarily small by letting the sample size, N, be very large. For a given sample size, one can employ econometric techniques to estimate the residual uncertainty about the demand curve and the induced total surplus when pricing at marginal cost. Back of the envelope calculations show that even for a relatively low sampling error, the variance in the estimate of higher order derivatives of the demand function can be substantial when the sample size is not large enough. For a discussion of additional challenges faced in structural demand estimation see Chintagunta and Nair (2011).

Small Sample. Proposition 1 assumes that the planner can collect an arbitrarily large sample of market outcomes. The use of large samples which assure asymptotic convergence is the norm in modern applied econometrics. Nonetheless, one way to study robustness of our results is to assume instead that only a small finite sample of N market outcomes can be observed. Because our interpolation technique relies on large sample convergence, the small sample implies that the demand is approximated with an error and thus the first-best is no longer achievable. A more interesting observation concerns the optimal prices set by the planner in this case. For small samples, the interpolation literature suggests that the demand will be learned more precisely when data points are not all draw from a small interval (Mastroianni and Milovanovic, 2008). In our context this implies that the planner faces a trade-off. If he only charges prices close to the marginal cost, then the deadweight loss from experimentation is low, but the demand is not learned precisely. If the planner sometimes charges higher prices, learning is accelerated at the cost of increasing current deadweight losses for consumers. While a formal analysis is beyond the scope of our work (we do not report this extension in the Appendix), it is important to notice that with small samples a clear trade-off exists between generating demand information to incentivize innovation and mandating low prices to reduce the deadweight loss. However, the more general lesson that learning the demand is useful for incentivizing innovations, and that such learning can occur by dynamically experimenting taking into account consumer welfare remains valid with small samples.

In general, Proposition 1 does not hold in settings where structural demand identification is not feasible. When the demand is not analytic, local price variation cannot be exploited to identify to estimate the global behavior of the demand function. In the same way, when demand grows very fast it may be unfeasible to collect the price-quantity observations necessary to identify demand. These issues are typical in structural modeling, where it is assumed that the structural parameters identified through local data variation can be used to perform counterfactuals or policy simulations (Reiss and Wolak, 2007). From this perspective, Proposition 1 does not require extra assumptions to those typically imposed in structural industrial organization studies. Without demand manipulation (studied in Section 5), then, in many contexts market signals can be used to construct a prize that improves outcomes by lowering deadweight loss from monopoly while at the same time rewarding inframarginal value from the innovation.

When the first best is possible to obtain, the planner may need to generate a large number of observations by experimenting at different price points near the marginal cost. The rate of convergence to full learning, and thus to the first best surplus, depends on the demand parameters. It remains for future research to study the speed of this convergence and to identify additional properties of the estimator.[11]

4.2 Implementation

Proposition 1 suggests that variation in prices and quantities may provide useful information for a planner who aims to maximize welfare by providing innovation incentives and minimizing distortions in the product market. For a large class of demand functions, we have shown that a policy maker can learn the surplus generated by the innovation and minimize market distortions by generating a price variation that is close to the marginal cost of production. This allows the planner to implement an outcome arbitrarily close to the first best.

The most intuitive way to generate this price variation is by awarding the innovator a patent that confers him the exclusive right to sell the product and to commit to a *patent buyout* scheme whose reward depends on the observed market outcomes. In other words, the planner can dictate to the patentee a price path and commit to buyout the patent if the innovator follows the path with a reward that depends on the quantities sold. The computation of the reward resembles structural estimation studies which typically estimate the primitives of a model from local price variation and exploit these estimates for out-of-sample welfare analysis. An implication of our result is that policy-makers may affect innovation incentives by designing reward systems that exploit these techniques.

[11] However, in the proof of Proposition 3 we show that even if experimentation away from the marginal cost is desirable, the chosen price is always strictly below the monopoly price.

In the context of the malaria vaccine, the Center for Global Development proposes to reward the innovator with a prize if 200 million treatments are sold at 1 dollar to the recipients. The suggested prize is 2.8 billion (14 dollars per treatment). A possible concern with such scheme is that vaccine development may inefficiently not take place if such reward is too small compared to the social welfare generated by the vaccine. An implication of Proposition 1 is that this prize scheme can be improved by requesting the successful innovator to sell the 200 million treatments at different prices, even if the overall price level remains close to the 1 dollar benchmark. This is because the market outcomes generated by such price variation will allow the sponsor to obtain an estimate of the product market-surplus generated by the new vaccine. Such estimate will provide useful guidance in determining the reward and avoiding under-payment (or overpayment) for the innovation.

But buyouts are not the only way to implement the first best. An alternative approach is to start from a perfectly competitive market in which the product is sold at marginal cost. The price variation can then be generated by the planner, perhaps interpreted as a government, taxing the firms and shifting their marginal costs of production. The information generated in this way will be the same as the one generated by the buyout scheme and can be exploited by the planner to implement the first best. An implication of this alternative implementation method is that market power is not essential to solve the asymmetric information problem between the policy maker and the innovator. In other words, for a large class of demand functions the socially optimal innovation level can be reached through minor perturbations of a competitive market.

The mechanism proposed in this section assumes that the planner is able to commit to truthfully revealing the observed price-quantity pair, and all the market participants agree with the revelation. Suppose instead that the government can freely manipulate the observed price, and thus can decide how much the innovator is paid. This case is similar to the case where the innovator can costlessly manipulate the price signal in the sense that the price variable becomes non-contractible, since it is not verifiable in front of the court.[12] The analysis of price

[12] This approach is the one followed by the incomplete contracts literature, see Tirole (2003) for a discussion. A caveat that is pointed out in the literature is that if both the agent and the principal can observe a variable, then it is possible to enforce a contract by requiring both agents to report the value of that variable. In case of disagreement both the principal and the agent can be punished, which allows truthtelling to be an equilibrium

manipulation by the innovator in Section 5 is therefore also applicable to the case where the planner has limited commitment.

5 Demand Manipulation

The analysis in Section 4 focused on the case of no demand manipulation. In this Section we consider the case in which the innovator can manipulate the market outcomes.

5.1 Buyouts and Price Variation

In the general model described in Section 3, the innovator can affect the market outcomes and manipulate market signals received by the planner up to period T but not after T. Our model also assumes a constant competitive market outcome from $T + 1$. A natural interpretation of this assumption is that the patent is acquired by the planner at T, so in the following we will refer to T as the buyout time.[13]

For a moment, let us depart from that model and assume that the planner (but not the innovator) can affect market outcomes after T. In this setting the first best can be approximated as in the case in which manipulation is not possible. This is the case both if manipulation is costly and if it is costless. To see this, consider the case in which the demand is linear. Then the planner can acquire the patent in the first period, sell the innovation at $p_1 = \varepsilon$ and $p_2 = 0$ and reward the innovator in the second period. In other words, the planner can appropriate the patent, generate the market outcomes required to learn the surplus generated by the innovation and then compensate the innovator. Alternatively, the planner can induce the innovator to generate the market outcomes necessary to learn the surplus and use additional post-buyout market outcomes to detect demand manipulations. For example, the patentee can be required to sell at $p_1 = \varepsilon$ and $p_2 = 0$ in the first two periods. The planner can then acquire the patent and practice $p_3 = \varepsilon$ and $p_4 = 0$ in the third and fourth periods. If the outcomes generated by the innovator coincide with those generated by the planner, the innovator will

outcome. If that caveat is accepted, then limited commitment on the part of the planner does not prevent the planner from implementing the first best.

[13] Noticed that in the previous Section we ignored $T(h_T)$ and focused on $g(h_t)$. This is because, in the absence of price manipulation the planner can generate a competitive outcome using only $g(h_t)$ by punishing the innovator if $p_t \neq 0$.

be rewarded with a transfer that approximates the surplus generated. If there are differences between market outcomes generated by the innovator and those generated by the planner, the innovator receives no transfer.

The basic insight is that pre-buyout manipulation, even if costless, can be avoided as long as manipulation after buyout is not possible and the planner can generate price variation after buyout to identify the demand and detect manipulation. Therefore, for manipulation to distort away from the first best, it has to be the case that either (i) manipulation by the innovator is feasible both before and after the buyout or (ii) the ability of the planner to generate price variation after the buyout is limited. In the next Section we study case (ii) from above.

5.2 Post-Buyout Competitive Outcome

We now consider the case in which after the buyout time T the innovation is sold in a competitive market and that neither the innovator nor the planner can affect (manipulate) this outcome. The quantity of product sold can be perfectly observed by the planner but the price and hence the revenue can be distorted by the innovator, as described in Section 3. This may arise, for example, when the innovator awards secret discounts to his consumers.

To provide a micro-foundation of the manipulation cost ϕ, we assume that the innovator can convince the planner that he is selling at $\widehat{p} > p$ by sustaining a cost equal to $\phi(\widehat{p}, p, \theta) = \widehat{\phi}((\widehat{p} - p)\, D(p, \theta))$ with $\widehat{\phi}$ being twice differentiable, and $\widehat{\phi} > 0$, $\widehat{\phi}'' \geq 0$. Intuitively, the planner observes sales equal to $\widehat{p}D(p, \theta)$ whereas the true revenue is equal to $pD(p, \theta)$ and $(\widehat{p} - p)\, D(p, \theta)$ are fake revenues undermined by secret price discounts. A simple justification of a positive manipulation cost is that the secret discounts offered are wasteful, that is they cost more to the innovator to offer than they are worth for the consumers. Alternatively, there may be a difference between the cost of external and internal financing. As argued by Aghion and Tirole (1994), for innovative firms this difference arises naturally because of the informational asymmetries involving new products and technologies. In this case, to convince the planner that the revenue is equal to $\widehat{p}D(p, \theta)$ the innovator will have to borrow $(\widehat{p} - p)\, D(p, \theta)$ sustaining a cost of $\widehat{\phi}\left((\widehat{p} - p)\, D(p, \theta)\right)$.[14] A simple functional specification for the manipulation cost is

[14] Another microfundation of the cost $\widehat{\phi}$ is that with some probability the planner will detect the manipulation and the innovator will pay a fine that depends on the fake proceeds.

$i\,(\widehat{p} - p)\,D(p, \theta)$, if $i > 0$ there is a positive cost of manipulating sales.

Proposition 2 *A patent of length T is Pareto dominated by a patent buyout scheme that depends on market outcomes.*

The proposition shows that for any patent of length T the planner (philanthropist) can design a buyout scheme that improves welfare. The planner commits to buy out the patent at a price that depends on the market outcomes observed during the first $\widehat{T} < T$ periods. The buyout time $\widehat{T}$ is chosen to allow the planner to learn about the value of the innovation and to remove the incentives of the innovator to manipulate sales. At this optimal time the marginal cost of manipulating sales for $\widehat{T}$ periods is equal to the marginal benefit of obtaining extra buyout reward.

In the linear case the optimal buyout time $\widehat{T}$ is pinned down by the formula

$$\frac{\delta^{\widehat{T}} - \delta^{T}}{1 - \delta^{\widehat{T}}} = i \tag{2}$$

that indicates how patent buyout takes place sooner as i gets larger. This result is reminiscent of Chari et al. (2012) who consider patents and prizes, and show that shorter patents are more likely to be optimal when manipulation costs are higher, but longer patents need to be used when manipulation costs are lower. In the next Section, we show that even with costless manipulation of the price signals (when an infinitely lived patent is implied by (2)), one can do better by considering mechanisms that are different from both prizes and patents.[15]

With additional assumptions on the relationship between surplus and monopoly profits, innovation incentives can be increased even more. Take for example the setting of Weyl and Tirole (2012) with $D(p, \theta) = \sigma Q(\frac{p}{m})$ where $\theta = (\sigma, m)$, m is the monopoly price, σ is the quantity sold at marginal cost price and $Q()$ is a function known to the planner. In their setting there is proportionality between monopoly profits $m\sigma Q(1)$ and surplus at zero price $m\sigma S(0)$. By inducing truthful revelation of monopoly profits, the buyout allows the planner to back out the surplus and to transfer the entire surplus to the innovator from period $\widehat{T} + 1$.

[15] It is important to note that there are two important differences between the setup of Chari et al (2012) and ours. First, we allow heterogeneous innovation costs. Second, Chari et al (2012) rule out positive transfers by allowing the innovator to produce a fake (and useless) "innovation".

The innovator will obtain the monopoly profits before the buyout and the entire consumer surplus for the post-buyout period. In this way consumers enjoy greater surplus than the case of a T-period patent and the innovator has greater innovation incentives. In particular, the outcome resembles the first best after the buyout, because there is marginal cost pricing and all the surplus is transferred to the innovator.[16]

One may speculate that when $\phi(\widehat{p}, p, \theta) = 0$ patents cannot be improved upon. This is not the case, as the next proposition shows; even with fully manipulable prices, the planner can improve on patents.[17]

Proposition 3 *When $\phi(\widehat{p}, p, \theta) = 0$ for all $\widehat{p}$, p and θ, there is a per unit subsidy level τ that Pareto dominates patents that last forever.*

Proposition 3 shows that even when price manipulation is costless, the planner can improve upon patents by exploiting the observed quantities. In the proof we show that a small quantity subsidy increases product market surplus by reducing the market price and increasing firm's profits. We also show that for τ small enough, such positive welfare effect dominates any loss generated by entry of inefficient innovators induced by the subsidy.

Overall, Propositions 2 and 3 show that for a broad class of demand functions patents are not the optimal mechanism to incentivize innovation when the planner can observe market outcomes, even when the innovator may substantially manipulate sales. In the next Section, in a simplified environment, we characterize the optimal mechanism.

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[16] In the linear specification, if we interpret $i > 0$ as the difference between the cost of external and internal financing the planner can reduce manipulation incentives even more by combining the buyout of the patent with the requirement to purchase a bond. Specifically, the planner can request the innovator to purchase a bond that costs $\gamma\widehat{p}D(p,\theta)$, pays no interest and expires after T^B periods. If $pD(p,\theta)$ is the only revenue available to the innovator, he will have to borrow $\gamma\widehat{p}D(p,\theta) - pD(p,\theta)$ for T^B periods at a cost of

$$i\left(\gamma\widehat{p}D(p,\theta) - pD(p,\theta)\right)\frac{1-\delta^{T^B}}{1-\delta}.$$

This extra manipulation cost generated by the bond allows to accelerate the buyout time and therefore increases consumer welfare.

[17] It is also possible to show that patents of finite lengths can also be improved upon by a simple per unit subsidy mechanism but this result is somewhat more tangential to what we discuss below.

6 Optimal Mechanism with Costless Manipulation

In this Section we study the optimal incentive system in which the quantity produced is observable by the planner, but the innovator can manipulate the price costlessly, so the price will not be contracted on. This assumption captures a situation where the innovator can offer secret price discounts to buyers at no cost (other than lowering revenues). As in the previous Section, we assume that after the buyout the innovation is sold in a competitive market and that neither the innovator nor the planner can affect (manipulate) this outcome.

We will study the problem with a mechanism design approach in which the innovator reports to the planner a type, $\widehat{\theta}$, and the planner requires that in period t the innovator produces a specific quantity, $q_t(\widehat{\theta})$, and receives a payment $\tau_t(\widehat{\theta})$. To simplify the analysis we focus on the linear demand case

$$D(p) = \theta_1 - \theta_2 p.$$

While this demand function allows us to simplify substantially the exposition, our key results hold with a more general demand of the form $D(p) = \theta_0 - \sum_{i=1}^{K} \theta_i p^{i-i}$ where K is known and the $\theta_i \geq 0$ are unknown.[18]

First, we show that there is no loss of generality in assuming that the planner knows the intercept θ_1.[19] More precisely, we can approximate the welfare of an auxiliary problem where the planner knows θ_1 from the outset arbitrarily closely. This is an upper bound because the planner cannot do better than in the hypothetical case where he observed θ_1 at the beginning.

Lemma 1 *The planner can approximate the welfare that can be induced under full information about θ_1 arbitrarily close.*

This result is quite intuitive: the planner can perfectly learn the demand intercept when the market becomes perfectly competitive and punish the innovator if θ_1 was not reported truthfully. Exploiting this Lemma, we focus on the linear demand case with known intercept

[18] In particular, all the steps in the proof of Proposition 4 would hold as the argument readily generalizes to any case where first-order conditions are sufficient for optimality, for which it is sufficient if D is concave in p.

[19] We do not need the innovator to report his cost, c, because in our setting, as in Scotchmer (1999), the innovator's compensation cannot depend on the true c since he cannot be punished for lying about c.

(normalized to 1) and unknown slope that for simplicity we rewrite as $\theta_2 = 1/2\theta$. The demand is therefore

$$q = 1 - \frac{p}{2\theta}$$

and larger θ are associated with steeper demand curves and larger consumer surplus. Notice that the monopoly quantity is independent of θ and it is equal to $q^M = 1/2$.

6.1 Static Mechanisms

We first study a static setting where the profits are realized only for one period after the innovator reports his type. Let $p(\widehat{\theta}, \theta) = 2\theta(1 - q(\widehat{\theta}))$ be the price at which the innovator can sell quantity $q(\widehat{\theta})$ if the actual demand is characterized by θ. The profits from reporting $\widehat{\theta}$ when the type is θ (gross of innovation costs) are:

$$\begin{aligned} U(\widehat{\theta}, \theta) &= \tau(\widehat{\theta}) + p(\widehat{\theta}, \theta)q(\widehat{\theta}) \\ &= \tau(\widehat{\theta}) + 2\theta(1 - q(\widehat{\theta}))q(\widehat{\theta}). \end{aligned}$$

Letting $V(\theta) = U(\theta, \theta) - c$ denote the rent under truth-telling, the envelope theorem implies that

$$V'(\theta) = \frac{\partial}{\partial\theta} U(\widehat{\theta}, \theta) \mid_{\widehat{\theta}=\theta} =$$

$$= q(\theta)\frac{\partial}{\partial\theta} p(\widehat{\theta}, \theta) \mid_{\widehat{\theta}=\theta} = 2q(\theta)(1 - q(\theta)). \tag{3}$$

The above condition (3) is a first order condition. The following result states a necessary and sufficient condition for implementability:

Lemma 2 *A schedule $q(\theta)$ can be implemented if and only if q is weakly decreasing in θ.*

Lemma 2 shows that the optimal mechanism requires the quantity sold to be decreasing in θ. Therefore, as the surplus created by the innovation increases, the quantity produced is reduced. The intuition for this result is the following. The planner exploits market power to induce truthful revelation and screen consumers' willingness to pay. When θ is large consumers are willing to pay high prices for the product and the innovator is likely to prefer market power to lump-sum transfers. Conversely, when θ is low consumers are price sensitive and market power would not be attractive to the innovator.

We are ready to formulate the planner's problem. First, note that the total surplus when q is implemented for an innovator with type θ is $W(q,\theta) = \int_0^q 2\theta(1-x)dx = \theta(2q-q^2)$. Let $\widehat{c}(\theta)$ be the highest cost innovator who enters (endogenously determined by the mechanism by $V(\theta)=0$). Then the objective function can be written as

$$\Pi = \int_{\underline{\theta}}^{\overline{\theta}} \int_0^{\widehat{c}(\theta)} \psi(c,\theta)(W(q(\theta),\theta)-c)dcd\theta.$$

The planner's problem is

$$\begin{aligned} & \max_{q(\theta)} \Pi \\ \text{s.t.} \quad \widehat{c}'(\theta) &= V'(\theta) = 2q(\theta)(1-q(\theta)), \text{ and } q'(\theta) \leq 0 \ \forall \theta \in [\underline{\theta}, \overline{\theta}]. \end{aligned}$$

The main challenges are twofold: first, the monotonicity constraint on q; second, the fact that the state variable $\widehat{c}(\theta)$ has free initial *and* end conditions, a combination that is uncommon for standard dynamic optimization problems. To obtain a solution to this problem, let us assume uniform independent distributions for c and θ on $[0,1]$ and $[\underline{\theta},1]$ for some $\underline{\theta} > 0$. Then the problem is equivalent to

$$\begin{aligned} & \max_{q(\theta)} \int_{\underline{\theta}}^1 [\theta(2q(\theta)-q^2(\theta))\widehat{c}(\theta) - \frac{\widehat{c}^2(\theta)}{2}]d\theta \\ \text{s.t.} \quad \widehat{c}'(\theta) &= 2q(\theta)(1-q(\theta)), \text{ and } q'(\theta) \leq 0 \ \forall \theta \in [\underline{\theta}, 1]. \end{aligned}$$

Optimal static mechanism

In the next proposition we characterize the optimal quantity schedule in the presence of costless price manipulation.

Proposition 4 *In the optimal static mechanism, there exists $\overline{\theta} \in (\underline{\theta}, 1)$ such that it holds that q is strictly decreasing on interval $[\underline{\theta}, \overline{\theta}]$ and then constant on $[\overline{\theta}, 1]$. Moreover, $q(\theta) \geq 2/3 > 1/2 = q^{monopoly}$ for all θ and $q(\underline{\theta}) = 1 = q^{first\ best}$.*

To gain intuition for this result, our starting point is Lemma 2, which implies that the quantity schedule needs to be weakly decreasing to be incentive compatible. In the proof of the Proposition, we show that in the relaxed problem where the monotonicity constraint on q is ignored, the optimal solution is such that $q(\underline{\theta}) = q(1) = 1$.[20] Given this, it is not surprising

[20] The reason is that otherwise function q could be increased uniformly by the same amount ε, and the value of $\widehat{c}(\underline{\theta})$ adjusted so that an increase in welfare is induced without violating any incentive constraints.

that when one reintroduces the monotonicity constraint on q it is still true that $q(\underline{\theta}) = 1$. It is also not surprising that now $q(1) < 1$, because $q(1) = 1$ and the monotonicity constraint would imply that $q(\theta) = 1$ for all θ, that is all possibility for screening would be given up.

Proposition 4 shows that the optimal mechanism differs substantially from a patent system even if the innovator can manipulate price signals costlessly. The optimal quantity schedule has three important characteristics. First, the quantity produced varies across types. This is a fundamental difference with the patent system that implements only the monopoly quantity that in our setting is constant across types. Second, the quantity produced by each type is above the monopoly quantity. Thus, despite costless price manipulation, information on the quantity produced allows the planner to reward the innovation generating less distortions than a traditional patent system. Finally, the optimal quantity is strictly decreasing in θ for low values of θ and constant for high surplus innovations as depicted in Figure 2.

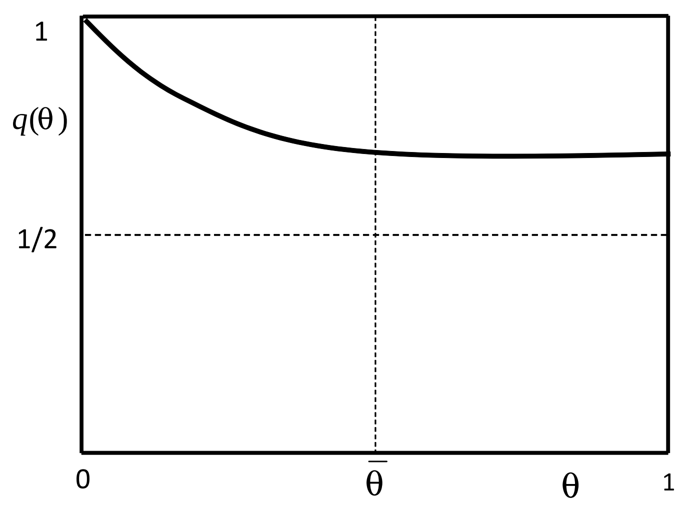

Figure 2: Optimal quantity schedule with costless price manipulation

The intuition behind this result is that the planner's welfare maximization involves a trade-off between a 'consumer welfare' effect and a 'screening' effect. When quantities decrease with θ, the planner can use market power to screen consumers' willingness to pay. Nevertheless, maximization of consumer surplus implies that larger quantities should be offered for innovation with larger θ since the impact on welfare of an increase in q is greater the greater is θ. For low values of θ, the 'screening effect' dominates and the planner exploits market power to screen willingness to pay. This is intuitive since for low θ it is crucial for the planner to avoid excess entry of low value innovators. As θ increases, the innovations have larger impact on consumer surplus and the planner has lower incentives to distort the market for screening purposes. For θ large enough, the 'consumer welfare' effect dominates and the planner implements a quantity schedule that is constant in θ. The idea that market power can be exploited to screen willingness to pay is similar to the logic in Weyl and Tirole (2012).[21]

6.2 Optimal Dynamic Mechanism and Discussion

Having characterized the optimal quantity schedule in the static setting, we now consider the dynamic problem where the planner can choose a path $(q_t(\theta), \tau_t(\theta))$ for every $t \geq 0$.[22] Our main result shows that repeating the same quantity over time for all types θ is optimal.

Proposition 5 *It is optimal for the planner to set a policy where $q_t(\theta)$ is constant in time for any θ, that is to adopt the optimal static mechanism.*

A constant mechanism (over time) is optimal because of the desirable features of quantity (and price) smoothing over time. This is due to the fact that the total surplus is concave in the quantity (and price), so inducing a temporal variation in quantities (as patents do) introduces extra distortion in the product market without improving innovation incentives. This finding resembles the result of Gilbert and Shapiro (1990).[23]

Proposition 5 confirms that the optimal mechanism *differs from a patent system* even if manipulation (of prices) is costless. Welfare is maximized with the innovator selling a quantity that is above the monopolistic quantity until the buyout occurs, unlike the (optimal) patent system described by Scotchmer (1995). This result is related to our earlier finding (Proposition 3), which shows that a small quantity subsidy always improves welfare.

[21] They restrict their attention to Cobb-Douglas reward policies (in our setting this restriction would generate a constant level of q across types). They show that q decreases with the variance of the type distribution. In our setting, we show that even with a fixed type distribution, the planner may use different quantities to screen for different types.

[22] Since no new information is revealed to the agent (the innovator), it is without loss of generality to concentrate on mechanisms where the agent reports his type only at the outset.

[23] They conclude that the optimal patent policy calls for infinitely lived patents when patent breadth is increasingly costly in terms of deadweight loss. In our setting, lowering the quantity produced can be thought of as an increase in patent breadth because a lower quantity reduces consumer surplus and increases the profits of the innovator.

Notice the apparent tension between Proposition 5 and Lemma 1. Proposition 5 requires the planner to implement a constant quantity over time whereas Lemma 1 requires the planner to move to the competitive outcome for at least one period in order to learn the intercept of the demand function. This tension identifies a key trade-off. On one hand, the planner would like to smooth market outcomes over time to increase welfare. On the other hand, the planner would like to generate variation of market outcomes to learn the underlying demand parameters. In the linear context, this tension leads to a mechanism that resembles a buyout where the patent is bought out after a long time (as long as possible) has elapsed.[24]

Proposition 5 also highlights the fact that learning from market signals over time may be substituted by an initial screening process where the innovator self reports his type. The literature on dynamic mechanism design cautions us that this result (no learning is optimal until the buyout) is only true because our agent (the innovator) has strictly superior information over the planner, and this advantage is maintained over time.[25] However, in a large number of applications this may be a realistic assumption. In such applications, the optimal mechanism does not utilize learning on the part of planner, rather it relies on a single report of the innovator at the outset. Such a policy can be implemented by offering a menu of R&D subsidies and per unit quantity subsidies.

It is beyond the scope of our work to characterize the optimal mechanism in a general framework of dynamic market signals, but a few characteristics of our proposed mechanism appear to be robust. First, prices need not be set at the extremes of monopoly pricing (i.e. full patent protection) or fully competitive pricing. Second, buyout itself can be viewed in terms of its ability to generate information, an important aspect that has been overlooked by the previous literature. Relatedly, observed demand information *after* the patent buyout can be used to incentivize innovation.

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[24] In reality, there may be legal or political reasons why the buyout cannot be delayed indefinitely. For example, it may happen that the product becomes obsolete, and in this case the planner may not be able to commit to a buyout that may not seem to promote consumer welfare ex-post.

[25] Baron and Besanko (1984) show that if adverse selection parameters are perfectly correlated over time, then under full commitment the optimal policy is the repetition of the optimal static contract. Our result is not a direct consequence of theirs as we allow the agent (the innovator) to manipulate price signals costlessly.

7 Conclusions

In this paper we have examined the problem of a social planner aiming to maximize consumer welfare and innovation incentives while observing prices and quantities practiced by the innovator over time. We have shown that information about market outcomes may allow the planner to generate more welfare than a traditional patent system through patent buyouts.

There are a number of historical experiences in which governments bought patents out. The most famous example of patent buyout took place in July 1839 when the French government purchased the patent for the Daguerreotype photography process. The inventor, Luis Jacques Daguerre, was not able to find buyers for the process, but obtained the support of a politician that convinced the government to acquire the patent and put the rights in the public domain. Within a short period of time the process spread around the country to become the technology standard in photography (Kremer, 2001). In recent academic and policy debates pharmaceutical patent buyouts have been suggested as a strategy to improve health in low income countries. For example, Banerjee et al. (2010) propose that a Health Impact Fund compensate drug manufacturers that sell in low income countries at marginal cost. They suggest that the compensation to a given manufacturer would depend on use of the drug and evidence of realized health benefits.[26]

Our paper provides two main insights into the design and application of such buyout schemes. First, the planner may find it beneficial to collect market data before the buyout and use them to estimate the surplus generated by the innovation. In practice, surplus may be estimated through structural econometric models that allow policy makers to estimate the primitives of consumer preferences and to generate out of sample predictions (Cho and Rust, 2008). Such estimates can provide useful guidance in the determination of the buyout compensation for the innovator. Second, the planner should consider the welfare cost associated with collecting price-quantity observations. As long as local variation in market outcomes can be exploited to learn about the global properties of the demand, prices close to marginal costs minimize the loss in consumer surplus.

[26] A similar policy proposal is described in Guell and Fischbaum (1995).

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Appendix 1: Proofs

Proof of Proposition 1

First, note that if $r \geq \overline{p}$ then $D(\cdot,\theta)$ can be expanded globally on $[0,\overline{p}]$ and we can construct a global estimate of the demand function given approximate knowledge of the function $D(\cdot,\theta)$ around the point $(0, D(0,\theta))$. The global estimate is obtained with the following polynomial:

$$\sum_{i=0}^{k} \alpha_i p^i,$$

where α_i is an appropriate estimate of the i'th derivative of D with respect to p at $p = 0$ divided by $i!$ to use Taylor's formula. Notice that the coefficients of the polynomial can be estimated by charging $k+1$ distinct prices close to 0.

The basis for this is that as k gets large, the approximation of the derivatives improves and thus our estimate of D approaches the true value of D arbitrarily close.[27] To formalize this, suppose that we have taken a sample of $k+1$ observations such that the price was always below some $\widehat{x} > 0$. The error term (in absolute value) for the estimate of the i'th derivative can be bounded by $\max\limits_{\theta\in\Theta, i, x\in[0,\widehat{x}]} \mid D^{(i+1)}(x,\theta)x^{i+1}/(i+1)! \mid \leq K\widehat{x}^{i+1}$, which can be made arbitrarily small (in absolute value) if $\widehat{x}$ is small (Mastroianni and Milovanovic, 2008). Here we used the fact that there exists a $K > 0$ such that $\max\limits_{x\in[0,\widehat{x}]} \mid D^{(i+1)}(x,\theta)/(i+1)! \mid < K$ for all $i = 1, 2, ...$ and $\theta \in \Theta$. To establish that this is indeed true, note that by D being analytic there exists $\widetilde{K}(\theta)$ such that $\max\limits_{x\in[0,\widehat{x}]} \mid D^{(i+1)}(x,\theta)/(i+1)! \mid < \widetilde{K}(\theta)$ for all $i = 1, 2, ...$ Moreover, Weierstrass's theorem implies that there exists K such $\widetilde{K}(\theta) \leq K$ for $\theta \in \Theta$, because Θ is compact and $\widetilde{K}$ is a continuous function of θ because all the derivatives of D are continuous in θ by assumption.[28]

If $r < \overline{p}$ then $D(\cdot,\theta)$ can only be expanded locally and approximation by polynomial is valid only in intervals around p^* of size less than r. To estimate the demand in this case we apply an analytic continuation technique as in Aghion et al (1991). Let us define $l = \overline{p}/n$ and

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[27] If the derivatives at 0 can be estimated with a known error ε, then the total error at $p > 0$ is less than $\varepsilon(p + p^2 + ...) = \frac{\varepsilon}{1-p}$ if $p < 1$. If the choking price cannot be bounded away from 1 (from above), then this procedure does not suffice, and local expansion is needed similarly to what is suggested below for the case where $r < \overline{p}$.

[28] Note, that directly we only assumed that D itself is continuous in θ. However, if D is analytic, then continuity of D in θ implies that all the c_k coefficients are continuous in θ, which implies that all derivatives of D are also continuous in θ.

take n large enough such that $l < r$. We can approximate $D(\cdot,\theta)$ in the interval $[p^*, p^* + l]$ by setting $k = n^2$ and calculating

$$\sum_{i=0}^{n^2} \alpha_i (p - p^*)^i,$$

and approximate the first $(n^2+1) - n$ derivatives of $D(\cdot,\theta)$ by the first $(n^2+1) - n$ derivatives of the polynomial. Next, let $\langle \beta_i | 0 \leq i \leq n^2 - n \rangle$ be the values of these derivatives at $x^* + l$. We can now approximate $D(\cdot,\theta)$ in the interval $[p^* + l, p^* + 2l]$ by

$$\sum_{i=0}^{n^2-n} \beta_i (p - p^* - l)^i$$

and approximate the first $(n^2+1) - 2n$ derivatives of $D(\cdot,\theta)$ by by the first $(n^2+1) - 2n$ derivatives of the polynomial. Proceeding this way one reaches $\overline{p}$ after at most n steps and similarly proceeding leftward one can estimate $D(\cdot,\theta)$ up to zero. Also in this case by choosing p^* arbitrarily small and n arbitrarily large the demand is approximated arbitrarily closely at a very low welfare cost.

Proof of Proposition 2

Consider the following mechanism. The innovator is awarded a patent for $\widehat{T} \leq T$ periods as long as the same prices and quantities $(\widehat{p},\ D(p,\theta))$ are observed by the planner for the entire patent duration $\widehat{T}$. After $\widehat{T}$ periods the patent is acquired by the planner that will pay the innovator $\widehat{p}D(p,\theta)$ per period for the remaining $T - \widehat{T}$ periods and the innovation is sold at marginal cost. With $\widehat{p} \geq p$ the payoff of the innovator is

$$\frac{1-\delta^{\widehat{T}}}{1-\delta}\left[pD(p,\theta) - \widehat{\phi}\left((\widehat{p}-p)D(p,\theta)\right)\right] + \frac{\delta^{\widehat{T}} - \delta^T}{1-\delta}\widehat{p}D(p,\theta).$$

Now consider setting $\widehat{T}$ such that

$$\delta^{\widehat{T}} - \delta^T = (1-\delta^{\widehat{T}})\widehat{\phi}'(0)$$

so that the marginal benefit of manipulation when $\widehat{p} = p$ is exactly equal to the marginal cost.[29] Setting $\widehat{p} = p$ is then optimal for the innovator because the first order condition

[29]If $\widehat{T}$ is not an integer set it equal to the smallest integer for which $\delta^{\widehat{T}} - \delta^T < (1-\delta^{\widehat{T}})\phi'(0)$.

holds by construction and the objective function is concave in $\widehat{p}$. This removes the innovator's incentive to manipulate. Maximizing the payoff with respect to p (with $\widehat{p} = p$) gives:

$$\begin{aligned} & (1-\delta^{\widehat{T}})\left[pD'(p,\theta)+D'(p,\theta)-\widehat{\phi}'(0)\,\widehat{p}D'+\widehat{\phi}'(0)\,(pD'(p,\theta)+D'(p,\theta))\right]+(\delta^{\widehat{T}}-\delta^{T})\widehat{p}D'(p,\theta) \\ = \; & (1-\delta^{T})(pD'(p,\theta)+D'(p,\theta)) \end{aligned}$$

so the innovator will truthfully report the monopolistic profits. The profits of the innovator will be the same as with a patent of length T but consumers will be better off.

Proof of Proposition 3

When a per unit subsidy is awarded there are two main changes in total welfare. First, the set of types who enter becomes larger as the profit of the innovator increases. Second, for a fixed type who enters even without a subsidy, the total surplus on the market changes as prices go down due to the subsidy. Both effects increase welfare when τ is small as we show below.

The following argument shows that there is a small enough per unit subsidy $\tau > 0$ such that for *any* specific value of θ social welfare is larger than in the absence of any subsidies $(\tau = 0)$. To save notation, we do not explicitly indicate that the optimal price is a function of θ, and not only of τ.

The profits for the patentee in the presence of a quantity subsidy are equal to $(p + \tau)D(p, \theta)$ where τ is the per unit subsidy. The first order and second order conditions are:

$$\begin{aligned} (p+\tau)D'(p,\theta)+D(p,\theta) &= 0 \\ 2D'(p,\theta)+(p+\tau)D''(p,\theta) &\leq 0. \end{aligned}$$

Let us indicate with $p(\tau)$ the optimal price charged by the monopolist. Now we exploit the FOC and the implicit function theorem to obtain

$$\frac{dp(\tau)}{d\tau} = -\frac{D'(p,\theta)}{2D'(p,\theta)+(p+\tau)D''(p,\theta)} < 0$$

because $D'(p, \theta) < 0$ and the second order condition is satisfied. Profits of the firm when optimally charging price $p(\tau)$ can be written as $\pi(\tau) = R(\tau, p(\tau)) = (p(\tau) + \tau)D(p(\tau), \theta)$. The envelope theorem implies that

$$\pi'(\tau) = \frac{dR}{d\tau} = D(p(\tau), \theta) > 0,$$

so innovation incentives become larger as τ increases. Next, for a given θ the product market surplus S (net of subsidies) is equal to

$$S(\tau) = p(\tau)D(p(\tau), \theta) + \int_{p(\tau)}^{\infty} D(z, \theta)dz$$

and thus

$$\begin{aligned} S'(\tau) &= D(p(\tau), \theta)\frac{dp(\tau)}{d\tau} + p(\tau)D'(p(\tau), \theta)\frac{dp(\tau)}{d\tau} - D(p(\tau), \theta)\frac{dp(\tau)}{d\tau} \\ &= p(\tau)D'(p(\tau), \theta)\frac{dp(\tau)}{d\tau} > 0. \end{aligned}$$

Total welfare can be written as $W(\tau) = \int_{\underline{c}}^{\pi(\tau)} (S(\tau) - x)\psi(\theta, x)dx$. Thus for τ close to zero we obtain

$$W'(\tau) = \int_{\underline{c}}^{\pi(\tau)} S'(\tau)\psi(\theta, x)dx + \pi'(\tau)(S(\tau) - \pi(\tau))\psi(\theta, \pi(\tau)) > 0,$$

because

$$S(\tau) - \pi(\tau) = p(\tau)D(p(\tau), \theta) + \int_{p(\tau)}^{\infty} D(z, \theta)dz - (p + \tau)D(p, \theta) = \int_{p(\tau)}^{\infty} D(z, \theta)dz - \tau D(p, \theta) > 0$$

for τ close to zero. Take any $\tau > 0$ such that $\int_{p(\tau,\theta)}^{\infty} D(z, \theta)dz - \tau D(p(\tau, \theta), \theta) > 0$ for all θ.[30] By the above, any such subsidy level τ increases total welfare for all θ. In other words the same level τ is applicable to all θ.

Proof of Lemma 1

Take the hypothetical problem where the planner observes θ_1 so the innovator needs to report only θ_2. As we show it in the next Section, the optimal mechanism prescribes a quantity $q_t(\theta_2) = q^*(\theta_2)$ that is constant in time (t). Now, take our original problem where the planner does not observe θ_1 at the outset, and suppose that the planner provides a buyout at time T

[30] When τ goes to zero the difference $\int_{p(\tau,\theta)}^{\infty} D(z, \theta)dz - \tau D(p(\tau, \theta), \theta)$ is strictly positive for every θ. Therefore, as long as $D(p(0, \theta), \theta)$ is bounded below by a positive uniform bound for all θ, then there is a τ that works uniformly for all θ. If such a uniform bound is not available, then the proof goes through with a few straightforward modifications.

and sets the quantities produced before time T equal to the $q^*(\theta_2)$.[31] After the buyout, when the market becomes perfectly competitive the intercept will be observed by the planner. At that stage the innovator can be punished if the quantity sold at marginal cost, θ_1, differs from the report of the innovator $\widehat{\theta}_1$. By making the punishment large enough the innovator has no incentive to misreport. Moreover, letting T become arbitrarily large the welfare induced by this mechanism approximates the welfare under full information about θ_1.

Proof of Lemma 2

First, let us write up the incentive conditions $U(\widehat{\theta},\theta) \leq U(\theta,\theta)$ and $U(\theta,\widehat{\theta}) \leq U(\widehat{\theta},\widehat{\theta})$. Adding these constraints up and substituting $p(\widehat{\theta},\theta) = 2\theta(1-q(\widehat{\theta}))$ we obtain

$$2\theta\left[(1-q(\widehat{\theta}))q(\widehat{\theta}) - (1-q(\theta))q(\theta)\right] \leq 2\widehat{\theta}\left[(1-q(\widehat{\theta}))q(\widehat{\theta}) - (1-q(\theta))q(\theta)\right]$$

Because quantities are higher than the monopoly quantities (1/2) then q has to be decreasing in θ. On the other hand, if q is decreasing in θ, then by choosing an appropriate transfer schedule τ the quantity schedule can be implemented.

Proof of Proposition 4

Part 1: Solution of the relaxed problem

To develop intuition for the optimal static mechanism as characterized in Proposition 4, we simplify the problem by looking at the optimal control problem ignoring the $q'(\theta) \leq 0$ constraint first. To obtain a solution continuos in θ, we follow Hellwig (2009) and specify the following Hamiltonian:

$$H = \lambda(\theta)2q(\theta)(1-q(\theta)) + [\theta(2q(\theta) - q^2(\theta))\widehat{c}(\theta) - \frac{\widehat{c}^2(\theta)}{2}].$$

The state variable $\widehat{c}$ has neither an initial nor an end condition, which makes it different from other optimal control problems. The first order condition for the control variable is

$$0 = \frac{\partial H}{\partial q} = \lambda(\theta)2(1-2q(\theta)) + 2\theta(1-q(\theta))\widehat{c}(\theta),\ \forall\theta. \tag{4}$$

The other co-state equation is

$$-\lambda'(\theta) = \frac{\partial H}{\partial \widehat{c}} = \theta(2q(\theta) - q^2(\theta)) - \widehat{c}(\theta). \tag{5}$$

[31] By standard arguments, there is a payment schedule τ that makes this quantity schedule incentive compatible.

Moreover, Hellwig (2009) shows that in this class of problems:

$$\lambda(\underline{\theta}) = \lambda(1) = 0. \tag{6}$$

The above conditions lead to the following result.

Lemma 3 $q(\underline{\theta}) = q(1) = 1.$

Proof. From (4) we obtain that

$$q(\theta) = \frac{\lambda(\theta) + \widehat{c}(\theta)\theta}{2\lambda(\theta) + \widehat{c}(\theta)\theta} \tag{7}$$

that is equal to 1 when $\theta = 1$ and when $\theta = \underline{\theta}$. ■

This result shows that in the relaxed problem there is efficient production both for the innovations that create the largest surplus and for those that create the smallest surplus. One may conjecture that the solution of the relaxed problem is then a prize and all innovations are produced without market distortions. The next proposition shows that this is not the case, and that the optimal quantity schedule is non-monotonic.

Lemma 4 *There exists a* $\overline{\theta}$ *such that* $q(\overline{\theta}) < 1$ *and* $q'(\overline{\theta}) = 0$. *Moreover* $q' \leq 0$ *for* $\theta \in [\underline{\theta}, \overline{\theta})$ *and* $q' > 0$ *for* $\theta \in (\overline{\theta}, 1]$.

Proof. Differentiating (4) with respect to θ and dividing through by 2 yields

$$\lambda'(\theta)(1 - 2q(\theta)) - 2q'(\theta)\lambda(\theta) + (1 - q(\theta))\widehat{c}(\theta) - \theta q'(\theta)\widehat{c}(\theta) + \theta(1 - q(\theta))\widehat{c}'(\theta) = 0.$$

Substituting in from (5) and also using the formula for $\widehat{c}'$ yields

$$\left(\widehat{c}(\theta) - \theta(2q - q^2)\right)(1 - 2q(\theta)) + (1 - q(\theta))\widehat{c}(\theta) + \theta(1 - q(\theta))2q(1 - q) = q'(2\lambda + \theta\widehat{c}),$$

so the sign of q' is equal to the sign of

$$\left(\widehat{c}(\theta) - \theta(2q - q^2)\right)(1 - 2q(\theta)) + (1 - q(\theta))\widehat{c}(\theta) + \theta(1 - q(\theta))2q(1 - q)$$

$$= \widehat{c}(2 - 3q) + \theta q[2(1 - q)^2 - (2 - q)(1 - 2q)]$$

$$= \widehat{c}(2 - 3q) + \theta q^2. \tag{8}$$

From (7) it follows that for all $\theta \geq \underline{\theta}$ it holds that $q(\theta) \leq 1$, therefore $q'(\underline{\theta}) \leq 0$ holds because $q(\underline{\theta}) = 1$ by the previous Lemma. Because $q(\underline{\theta}) = q(1) = 1$, it means that there exists a $\overline{\theta} \in (\underline{\theta}, 1)$ such that $q'(\overline{\theta}) = 0$ and $q''(\overline{\theta}) > 0$. Now assume that there exists some $\widetilde{\theta} > \overline{\theta}$ for which $q'(\widetilde{\theta}) < 0$. This means that there exists a $\theta' \in \left(\overline{\theta}, \widetilde{\theta}\right)$ such that $q'(\theta') = 0$ and $q''(\theta') < 0$. Notice that $q'(\theta') = 0$ implies that $A'\left(\theta'\right) = q\left(6q^2 - 9q + 4\right)$ that is strictly positive for any value of $q > 0$. This implies that if $q'(\theta') = 0$ then $q''(\theta') > 0$ that contradicts the existence of $\widetilde{\theta}$ and implies that $q' > 0$ for each $\theta > \overline{\theta}$. ■

The intuition for this result is related to the fact that ignoring the monotonicity constraint on q is essentially equivalent to ignoring the global optimality conditions of the innovator (agent), just taking the first order conditions of his problem into account. Therefore, the relaxed problem still includes some aspects of the incentive constraints of the innovator to report truthfully. The result indicates that a non-constant quantity schedule can be used to screen the different types of the innovators and make sure that (first-order) innovation incentives reflect the underlying demand conditions. This feature will play a substantial role in the solution of the original problem.

Part 2: The optimal static mechanism

We now reintroduce the monotonicity constraint $q'(\theta) \leq 0$. We first show that there is efficient production for the lowest innovation type ($q(\underline{\theta}) = 1$), since for such a type there is no incentive to misreport in general. Suppose that $q(\underline{\theta}) = q^* < 1$. Then take a small deviation where for all $\theta \in [\underline{\theta}, \underline{\theta} + \varepsilon]$ the quantity is set at $\widetilde{q}(\theta) = 1$, and for other values of θ we maintain the original candidate optimum. We show that this increases welfare, and still satisfies all the constraints. First, it is obvious that the monotonicity constraint is still satisfied. Second, we keep $\widehat{c}(\theta)$ unchanged for all θ outside the interval. This means that for all $\theta \in [\underline{\theta}, \underline{\theta} + \varepsilon]$ it holds that the modified entry function $\widetilde{c}(\theta) = \widehat{c}(\underline{\theta} + \varepsilon)$ because $\widetilde{c}'(\theta) = 0$ for all $\theta \in [\underline{\theta}, \underline{\theta} + \varepsilon]$ as $\widetilde{q}(\theta) = 1$ for such values of θ. The original value of the entry cost is such that for all $\theta \in [\underline{\theta}, \underline{\theta} + \varepsilon]$ it holds that the $\widehat{c}(\theta) = \widehat{c}(\underline{\theta} + \varepsilon) - \int_{\theta}^{\underline{\theta}+\varepsilon} 2q(x)(1 - q(x))dx$. But then $\widetilde{c}(\theta) - \widehat{c}(\theta) = \int_{\theta}^{\underline{\theta}+\varepsilon} 2q(x)(1 - q(x))dx$ which goes to zero when ε goes to zero. Therefore, the component of the change in welfare that results from changing the entry unction for types in $[\underline{\theta}, \underline{\theta} + \varepsilon]$ is second order in ε. The gain in welfare that comes from the fact that quantities are increased is first order in ε. Therefore, for a small enough ε this change is welfare improving. This concludes the proof that $q(\underline{\theta}) = 1$.

We know from above that $q(\underline{\theta}) = 1$ and that the entire solution must be constrained, since the relaxed problem has an optimal solution that violates the monotonicity constraint. Therefore, there exist $\theta', \overline{\theta}$ such that $1 \geq \theta' > \overline{\theta} > \underline{\theta}$ and the solution involves $q(\theta) = q^*$ for all $\theta \in [\overline{\theta}, \theta']$, and $q(\theta)$ is strictly decreasing on $[\underline{\theta}, \overline{\theta}]$.[32] We provide a proof by contradiction. Suppose that there exist exist $\theta'' < 1$ and $\theta''' > \theta''$ such that q is strictly decreasing on $[\theta'', \theta''']$, while $q(\theta) = q^*$ for all $\theta \in [\overline{\theta}, \theta'']$. We derive a contradiction for such a point θ'' to conclude our proof. To derive this contradiction we study an auxiliary problem. Take the solution for interval $[\underline{\theta}, \overline{\theta}]$ as given, and let us maximize the objective function $\int_{\overline{\theta}}^{1} [\theta(2q(\theta) - q^2(\theta))\widehat{c}(\theta) - \frac{\widehat{c}^2(\theta)}{2}] d\theta$ taking $q(\overline{\theta}), \widehat{c}(\overline{\theta})$ as given, and placing the further condition that

$$q(\theta) \leq q(\overline{\theta}) \text{ for all } \theta \geq \overline{\theta}. \tag{9}$$

We show that the solution of this problem is a constant path on interval $[\overline{\theta}, 1]$, and thus the required θ'', θ''' cannot exist. The Hamiltonian is unchanged as the extra constraint (9) is incorporated as a standard Kuhn-Tucker condition:

$$H = \lambda(\theta)2q(\theta)(1 - q(\theta)) + [\theta(2q(\theta) - q^2(\theta))\widehat{c}(\theta) - \frac{\widehat{c}^2(\theta)}{2}].$$

The binding monotonicity constraint on $[\overline{\theta}, \theta'']$ means that $\frac{\partial H}{\partial q} \mid_{q=q^*} \geq 0 \; \forall \theta \in [\overline{\theta}, \theta'']$, and in particular

$$\frac{\partial H}{\partial q} \mid_{q=q^*, \theta=\overline{\theta}} \geq 0. \tag{10}$$

The fact that the monotonicity constraint ceases to bind at θ'' means that

$$\frac{\partial H}{\partial q} \mid_{q=q^*, \theta=\theta''} = 0. \tag{11}$$

Using that $q(\theta) = q^*$ for all $\theta \in [\overline{\theta}, \theta'']$ we obtain that

$$\frac{\partial H}{\partial q} \mid_{q=q^*} = 2\lambda(\theta)(1 - 2q^*) + 2\theta\widehat{c}(\theta)(1 - q^*),$$

and thus

$$\frac{\partial^2 H}{\partial q \partial \theta} \mid_{q=q^*} = 2\lambda'(\theta)(1 - 2q^*) + 2\left(\theta\widehat{c}(\theta)\right)'(1 - q^*).$$

[32] In other words, $\overline{\theta}$ is the lowest type where the monotonicity constraint binds in the solution of the original problem.

We know that

$$\lambda'(\theta) = \widehat{c}(\theta) - \theta(2q^* - (q^*)^2),$$

and

$$\widehat{c}'(\theta) = 2q^*(1 - q^*).$$

Therefore,

$$\frac{\partial^2 H}{\partial q \partial \theta} \mid_{q=q^*} = 2(1-2q^*)\lambda'(\theta) + 2(1-q^*)[\theta\widehat{c}'(\theta) + \widehat{c}(\theta)] =$$

$$= 2(1-2q^*)\left(\widehat{c}(\theta) - \theta(2q^* - (q^*)^2)\right) + 2(1-q^*)\widehat{c}(\theta) +$$

$$+2(1-q^*)\theta 2q^*(1-q^*) =$$

$$= 2(\widehat{c}(2-3q^*) + \theta\,(q^*)^2). \tag{12}$$

Because the monotonicity constraint starts binding at $\theta = \overline{\theta}$, we can conclude two observations at that point. First, ignoring the monotonicity constraint there locally is valid, second in the relaxed problem $q'(\overline{\theta}) = 0$ holds[33]. Then the same argument as above (see (8)) implies that $\widehat{c}(\overline{\theta}))(2-3q^*) + \overline{\theta}\,(q^*)^2 = 0$. Therefore, $\frac{\partial^2 H}{\partial q \partial \theta} \mid_{q=q^*, \theta=\overline{\theta}} = 0$ must hold by (12). Also, $\frac{\partial}{\partial \theta}\left(\frac{\partial^2 H}{\partial q \partial \theta} \mid_{q=q^*}\right) = 2(\widehat{c}'(2-3q^*) + (q^*)^2) = 2(2q^*(1-q^*)(2-3q^*) + (q^*)^2) = 2q^*(2(1-q^*)(2-3q^*) + q^*) > 0$ for all relevant values of q^*. Therefore, together with $\frac{\partial^2 H}{\partial q \partial \theta} \mid_{q=q^*, \theta=\overline{\theta}} = 0$ we obtain that for all $\theta \in (\overline{\theta}, \theta'']$

$$\frac{\partial^2 H}{\partial q \partial \theta} \mid_{q=q^*} > 0. \tag{13}$$

But comparing (10), (11), and (13) yields a contradiction, which concludes our proof of the shape of q. Finally, $\widehat{c}(\overline{\theta}))(2-3q^*) + \overline{\theta}\,(q^*)^2 = 0$ implies that $q^* > 2/3$, which provides the last result.

Proof of Proposition 5

Take any (potentially non-constant) path q_t, τ_t. The proof establishes that the same entry function $\widehat{c}$ can be induced by an appropriate policy that is constant over time. Moreover, total welfare is higher under this policy as the sum of consumer and producer surplus is larger than under the original non constant policy. First, it is clear that a one-time up-front transfer is

[33] This is an instance of the smooth pasting condition at point $\overline{\theta}$ where the function switches from being strictly decreasing to being flat.

without loss of generality as the innovator only cares about the present value of the transfers. The utility from reporting $\widehat{\theta}$ when the type is θ is

$$U(\widehat{\theta},\theta)=\tau(\widehat{\theta})+\sum_{t=0}^{\infty}\delta^{t}p_{t}(\widehat{\theta},\theta)q_{t}(\widehat{\theta}).$$

By construction, $p_t(\widehat{\theta},\theta)=2\theta(1-q_t(\widehat{\theta}))$, and thus $U(\widehat{\theta},\theta)=\tau(\widehat{\theta})+2\theta\sum_{t=0}^{\infty}\delta^t q_t(\widehat{\theta})(1-q_t(\widehat{\theta}))$. Letting $V(\theta)$ denote the rent (under truth-telling), the envelope theorem implies that

$$\begin{aligned} V'(\theta) &= \frac{\partial}{\partial\theta}U(\widehat{\theta},\theta)\mid_{\widehat{\theta}=\theta}= \\ &= 2\sum_{t=0}^{\infty}\delta^{t}q_{t}(\widehat{\theta})(1-q_{t}(\widehat{\theta})). \end{aligned} \tag{14}$$

A similar argument as in Lemma 2 implies that incentive compatibility requires that $\sum_{t=0}^{\infty}\delta^t q_t(\theta)$ is decreasing in θ. Take a constant quantity scheme that satisfies $\sum_{t=0}^{\infty}\delta^t q^*(\widehat{\theta})(1-q^*(\widehat{\theta}))=\sum_{t=0}^{\infty}\delta^t q_t(\widehat{\theta})(1-q_t(\widehat{\theta}))$. This will then guarantee that the payoffs of the innovator, and thus the entry function is preserved.[34] It is then sufficient to prove that for any θ the realized total surplus is larger than the one under the original policy. That is, it is sufficient to show that for all θ it holds that $\sum_{t=0}^{\infty}\delta^t\theta q^*(\theta)(2-q^*(\theta))\widehat{c}(\theta)\geq\sum_{t=0}^{\infty}\delta^t\theta q_t(\theta)(2-q_t(\theta))\widehat{c}(\theta)$ or

$$\sum_{t=0}^{\infty}\delta^{t}q^{*}(\theta)(2-q^{*}(\theta))\geq\sum_{t=0}^{\infty}\delta^{t}q_{t}(\theta)(2-q_{t}(\theta))$$

if $\sum_{t=0}^{\infty}\delta^t q^*(\theta)(1-q^*(\theta))=\sum_{t=0}^{\infty}\delta^t q_t(\theta)(1-q_t(\theta))$. Using Jensen's inequality this follows if we show that $x(2-x)$ is a concave transformation of $x(1-x)$ restricting x to be on $[0.5,1]$. Letting $y=x(2-x)$ and $z=x(1-x)$ it holds that $y=z+x$. So, it is sufficient to show that y is concave in z for which it is sufficient that x is concave in z. But this holds because z is a concave and decreasing function of x.

[34] The incentive conditions are not affected either, see (14).

Appendix 2: Additional Results

Formalization of Switching Time

The primitive of the planner's buyout policy is a function $\tau_t : H_t \to \{0,1\}$ indicating whether the switch to a competitive market has occurred at or before time t given the history h_t. Let us define as $H_j(h_t)$ the set of histories at time $j > t$ following a history h_t. To interpret $\tau_t(h_t)$ as an irreversible switch to a competitive market we require that $\tau_t(h_t) = 1 \Rightarrow \tau_j(H_j(h_t)) = 1$ for each $j > t$.

We start by defining the set of admissible histories in each period $t \geq 1$. The set of admissible histories in period 1 consists of all positive price-quantity pairs if $\tau_0 = 0$ but the price is restricted to be equal to zero if $\tau_0 = 1$. Formally:

$$H_1 = \{x \in \mathbb{R}^2_+ : x = (q, \widehat{p}), q \in \mathbb{R}_+, \widehat{p} = 0 \text{ if } \tau_0 = 1\}.$$

An inductive step defines the set of admissible histories H_t for all $t \geq 2$

$$H_t = \{x \in \mathbb{R}^{2t}_+ : x = (y, q, \widehat{p}),\ y \in H_{t-1}, q \in \mathbb{R}_+, \widehat{p} = 0 \text{ if } \tau_{t-1}(y) = 1\}.$$

We are ready to define the switching time T taking the planner's policy and the innovator's strategy as given. Given any strategy of the innovator $\alpha \in A$, let α_t denote the truncation of α up to period t. We indicate with $h_t(\alpha_t)$ the admissible public history generated by α_t. Taking the policy of the planner $\tau = (\tau_0, \tau_1, \tau_2, ...)$ as given, the switching time $T(\alpha)$ is defined as follows: $\tau_k(h_k(\alpha_k)) = 0$ for all $k \leq T-1$ and $\tau_T(h_T(\alpha_T)) = 1$.[35]

Markov Shifts

We extend our setting and assume that the demand has two states. Let us indicate with $D_L(p,\theta)$ the quantity consumed in the low demand state and with $D_H(p,\theta)$ the quantity consumed in the high demand state. For simplicity, we assume that $D_H(p,\theta) \geq D_L(p,\theta)$ for each p and that the inequality is strict if $D_H(p,\theta) > 0$.[36] We follow Battaglini (2005) and denote with $\Pr(D_L \mid D_k) \in (0,1)$ the probability that state L is reached if the demand is in

[35] Note, that function τ_T is defined only on histories such that switching has not occurred by period $T-1$, but this is satisfied by assumption here.

[36] Proposition A1 holds as long as $D_H(p,\theta) \neq D_L(p,\theta)$ for $p \in (0,\varepsilon)$ with ε arbitrarily close to zero.

state k. At date zero the prior on the demand states are (μ_H, μ_L). In this extended setting the problem for the inventor is to choose

$$\max_{p_t} \tau_t(r(p_t), h_{t-1}) + \delta E\left[V\left(D\left|h_t, \theta, D_t\right.\right)\right]$$

where $V\left(D\left|h_t, \theta, D_t\right.\right)$ is the value function of an innovator type θ after public history h_t at the demand state D_t. Investment in innovation takes place if $\mu_H V\left(D\left|h_0, \theta, D_H\right.\right) + \mu_L V\left(D\left|h_0, \theta, D_L\right.\right) \geq c$ and the total social welfare created by the innovation is

$$\int_c \int_{\theta \in \Theta^*(c)} \left[\sum_{t=0}^{\infty} \sum_{i \in \{L,H\}} \delta^t S(D_t(p_t^*)) \Pr(D_t = D_i) - c\right] \psi(\theta, c) d\theta dc.$$

Also in this setting the planner can maximize innovation incentives by approximating the first best outcome.

Proposition A1 *If D_L and D_H are analytic the first best can be approached arbitrarily closely.*

Proof. As in the proof of Proposition 1 we approximate the demand functions by polynomials that are estimated by charging $n^2 + 1$ distinct prices close to $p^* = 0$. For the estimation we now need two different quantities for each of these prices. The smaller quantity observed at a price is used for the estimation of D_L and the larger one to estimate D_H. Once the two demand functions have been approximated around $p^* = 0$, analyticity can be exploited to learn their global behavior by following the procedure in the proof of Proposition 1. By choosing and experimentation interval arbitrarily close to $p^* = 0$ and n arbitrarily large the demands are approximated arbitrarily closely at an arbitrarily low welfare cost. ■

Demand Growth

A natural assumption with new technologies is that demand grows over time. Suppose, for example that for τ periods the demand is $D_L(p, \theta)$ and it becomes $D_H(p, \theta)$ from period $\tau + 1$ with $D_H(p, \theta) > D_L(p, \theta)$. If the functions are polynomials:

$$D_\alpha(p, \theta) = \sum_{i=0}^{I} c_i^\alpha(\theta) p^i$$

with $\alpha \in \{L, H\}$ then, under the restriction that only one price-quantity observation can be obtained in each period, the amount of time required to identify the low state demand is increasing in the complexity of the demand.

This simple specification suggests that when the demand does not grow too quickly, the first best can be implemented since the planner can learn the parameters of the demand fast enough. In particular, when $\tau \geq I + 1$ the first best can be approached arbitrarily closely: it takes $I+1$ distinct price-quantity observations to identify all the coefficients of the polynomial. By requiring the innovator to charge in each period a distinct p_t arbitrarily close to zero the welfare cost of learning is minimized.

Nevertheless, the planner may not have enough time to learn the demand when growth is fast. Take for example the case in which the demand is linear and the planner can observe only one price-quantity combination for the low demand regime. In this case the planner cannot approach the first best and will have to reward the innovator for the surplus generated in the low demand state by granting a one period patent or by using the Weyl and Tirole (2012) mechanism for one period.

It is important to notice that when $\tau < I + 1$ it is not optimal to give a τ period patent and then learn costlessly the demand $D_H(p, \theta)$. This is because a patent that lasts τ periods generates a loss in consumers' surplus in each period. The planner can improve the overall welfare by granting a patent that lasts only for one period and observe the quantities and prices practiced by the innovator. For periods 2 to τ the planner can transfer an amount equal to the observed first period profits to the innovator under the requirement that the product is sold at marginal cost. In this case the innovation incentives are the same as with a τ periods patent but the loss in consumer surplus is substantially lower.

Demand identification may be problematic also when the demand starts at a high level and then suddenly drops or disappears. This may occur when a follow-on superior technology is developed. Also in this case, the planner may not be able to reach the first best if the high demand state does not last for a period of time long enough to identify the demand curve.[37]

This discussion suggests that it is crucial for the planner to collect market outcomes in a timely manner. Nonetheless, the restriction that only one price-quantity can be observed in each period can be relaxed if the planner can generate variation geographically. When

[37] In this case the planner may actually use intellectual property protection to prevent the new innovator to sell the innovation until the surplus generated by the previous innovator is estimated. Nonetheless this delay would affect negatively consumers surplus. A more careful examination of how market outcomes may help designing patent protection in the presence of cumulative innovation is left to future research.

sudden demand shifts are expected, the planner may prefer to collect market outcomes through geographic (cross-markets) price variation rather than intertemporal (within market) price variation.

Asymmetric Production Costs

The following result extends Proposition 1 to the case in which the innovator has a cost advantage.

Proposition A2 *Suppose that the competitive fringe has a higher marginal cost of production than the innovator. Then the first-best is still attainable under no manipulation.*

Proof. Take the same mechanism as in the baseline case with the difference that after the "buyout" the innovator does the production and not the fringe, and the price is set at the marginal cost of the innovator. The maximization problem of the innovator does not change as his post-buyout profit is still zero, and his pre-buyout profits are unchanged by construction. Therefore, the first-best is still approached arbitrarily well. ■

The next proposition examines the case in which the competitive fringe has a cost advantage.

Proposition A3 *Suppose that the competitive fringe has a lower marginal cost of production than the innovator. Then the first-best is still attainable under no manipulation if*

i) the production technology of the fringe is available to the innovator through licensing or contract manufacturing,

or

ii) it is possible for the planner to levy a tax after the buyout to vary prices after the buyout.

Proof. If the production technology of the fringe is available to the innovator, then the pre-buyout experimentation can be done at prices close the production cost of the fringe. Then the same buyout mechanism works as in the baseline case to attain the first-best level of welfare. If it is possible for the planner to levy a tax after the buyout, then a buyout can be done at the very beginning. Upon the buyout, the planner can generate price variation by changing the per unit tax levied. Then the same argument as in the baseline case would yield that the seller can learn the entire demand curve using local price experimentation. The innovator's compensation is set such that the present value of transfer is equal to the present value of the total surplus when the first-best is repeated every period. ■

Social Cost of Public Funds

Following Laffont and Tirole (1993) and Galasso and Tombak (2014) we assume that the government finances transfer G at a cost $(1+\kappa)G$ where $\kappa \geq 0$ represents the cost of public funds due to the deadweight loss associated with taxation. We start by characterizing the first best in the case in which the planner knows the demand parameter θ. Consider a constant price p and a transfer of G per period.[38] The per-period product market surplus net of the deadweight loss associated with taxation is equal to

$$S(p,\theta) = pD(p,\theta) + \int_p^{\infty} D(z,\theta)\,dz - \kappa G.$$

The innovator invests if and only if

$$\frac{pD(p,\theta)+G}{1-\delta} \geq c.$$

Let us indicate with $\widehat{c}$ the marginal innovator (whose profits are zero) and let $\widehat{C} = \widehat{c}(1-\delta)$. If the planner aims to induce entry of innovator $\widehat{c}$ the problem becomes

$$\max_{T,p\geq 0} \frac{1}{1-\delta}\left(pD(p,\theta) + \int_p^{\infty} D(z,\theta)\,dz - \kappa G\right)$$
$$\text{such that} \quad pD(p,\theta) + G - \widehat{C} = 0$$

The corresponding Lagrangian is

$$L = \frac{1}{1-\delta}\left(pD(p,\theta) - \int_{\overline{p}}^{p} D(z,\theta)\,dz - \kappa G\right) + \lambda\left(\widehat{C} - pD(p,\theta) - G\right),$$

and the first order conditions are[39]

$$\begin{aligned} \frac{1}{1-\delta}\left(D + pD' - D\right) - \lambda\left(D + pD'\right) &= 0 \\ -\frac{\kappa}{1-\delta} - \lambda &= 0. \end{aligned}$$

[38] We discuss the optimality of a constant price path in Section 6.2.

[39] By using an equality version of the first order condition with respect to p, we assumed that $p>0$ in the optimum. The positivity of p follows if $D_p(0) > -\infty$ because then increasing the price slightly above zero yields a second order loss for the consumers but a first order gain in terms of need for pulic funds.

These conditions yield that the optimal price p^* satisfies

$$\frac{\varepsilon(p^*)}{1+\varepsilon(p^*)} = -\kappa \tag{15}$$

where $\varepsilon(p) = pD'/D$ is the (negative) elasticity of the demand. To induce an innovator with cost $\widehat{c}$ to enter, the planner faces a trade-off between two types of welfare distortions: the cost of raising money through public taxation and the surplus losses due to market power. In a simple linear setting where $p = 1-q$ the condition implies an optimal price $p^* = \kappa/(2\kappa+1)$ that ranges from 0 (in the case of no cost of public funding) to the monopoly level (when $\kappa = \infty$).

Let us denote with $p^*(\kappa, \theta)$ the price that satisfies condition (15) and with $G(\theta, \kappa, \widehat{c}) = \widehat{c} - p^* D(p^*, \theta)$ the transfer that induces entry of innovators with $c \leq \widehat{c}$. The maximization problem of the planner is then equivalent to finding the optimal entry level $\widehat{c}$:

$$\max_{\widehat{c}} \int_0^{\widehat{c}} \frac{1}{1-\delta} \left(p^*(\kappa,\theta) D\left(p^*(\kappa,\theta),\theta\right) + \int_{p^*(\kappa,\theta)}^{\infty} D\left(z,\theta\right) dz - \kappa G(\theta,\kappa,\widehat{c}) - c \right) \psi(\theta, c) dc.$$

Let us denote with $\widehat{c}^*(\kappa, \theta)$ the solution to this maximization problem. From the values of $\widehat{c}^*(\kappa, \theta)$ and $p^*(\kappa, \theta)$ we obtain the optimal transfer $G^*(\theta, \kappa)$.[40]

The above results show that with known demand, the first best is reached with prices equal to $p^*(\kappa, \theta)$ quantity $D\left(p^*(\kappa, \theta), \theta\right)$ and transfer $G^*(\theta, \kappa)$. We now show that the first best can be approximated arbitrarily closely even if the demand, θ, is unknown. We attack the problem with a mechanism design approach in which the innovator reports to the planner a type, $\widehat{\theta}$, and the planner requires that in period t the innovator produces a specific market outcome $q_t(\widehat{\theta}, h_t)$, $p_t(\widehat{\theta}, h_t)$ and receives a payment $\tau_t(\widehat{\theta}, h_t)$.

Consider the following mechanism. The innovator reports $\widehat{\theta}$ and the planner requires N quantity observations arbitrarily close to $D\left(p^*(\kappa, \widehat{\theta}), \widehat{\theta}\right)$. Set N large enough so that the analytic demand can be identified. Each time a quantity is produced, the planner assesses whether the prices are consistent with the revealed demand function, i.e. the market outcome lies on the demand curve $D\left(., \widehat{\theta}\right)$. If the quantity produced is not the one requested by the planner or the price is not consistent with the demand, then the innovator is punished and τ_t is set to $-\infty$ ever after. If the quantity is the one requested by the planner, then the transfer is $G^*(\theta, \kappa)$. After the first N observations the demand has been identified, and the reward is

[40] In the linear case $\widehat{c}^* = (2\kappa + \kappa^2 + 1)/(8\kappa + 8\kappa^2 + 2)$ and $T^*(\theta, \kappa) = \left(3\kappa^2 + 4\kappa + 1\right)/2\left(2\kappa + 1\right)^2$.

equal to $G^*(\theta,\kappa)$ per-period ever after conditional on observing $D\left(p^*(\kappa,\widehat{\theta}),\widehat{\theta}\right)$. It is easy to see that the innovator has no incentive to report his type untruthfully and the first best is approximated arbitrarily closely.

Demand is Observed with Error

Our setting assumes that the planner can perfectly observe the demand. We can relax this assumption and consider the case in which the demand is observed with error. To analyze such a setting, we assume that:

$$q_t = D(p_t,\theta) + \varepsilon_t, \tag{16}$$

where ε_t is a mean zero i.i.d. noise over the support $[-\overline{\varepsilon},\overline{\varepsilon}]$. In the next proposition we show that even in this case the planner can estimate the surplus generated by the innovation and transfer it to the innovator.

Proposition A4 *If D is analytic, the first best can be approached arbitrarily closely.*

Proof. As in the proof of Proposition 1 we approximate the demand function by a polynomial estimated by charging n^2+1 distinct prices close to $p^*=0$. For the estimation we now need N different quantities for each of these prices. Once N quantities are observed at a price p, $N^{-1}\sum_{i=1}^{N} q(p)$ is used for the estimation of D. Because of the weak law of large numbers, the sample average converges in probability to $D(p,\theta)$. Once the demand function has been approximated around $p^*=0$, its analyticity can be exploited to learn its global behavior exploiting the procedure illustrated in the proof of Proposition 1. By choosing and experimentation interval arbitrarily close to $p^*=0$ and N, n arbitrarily large, the demand is approximated arbitrarily closely at an arbitrary low welfare cost. ■