Rewarding Sequential Innovators: Prizes, Patents and Buyouts*

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Abstract

This paper presents a model of cumulative innovation where firms are heterogeneous in their research ability. We study the optimal reward policy when the quality of the ideas and their subsequent development effort are private information. Monopoly power is a scarce resource to be allocated across innovators that arrive at various times. The optimal assignment of property rights must counterbalance the incentives of current and future innovators. The resulting mechanism resembles a menu of patents that have infinite duration and fixed scope, where the latter increases in the value of the idea. This optimal patent menu can be implemented with a simple *buyout scheme*: The innovator commits at the outset to a price ceiling at which he will sell his rights to a future inventor. By paying a larger fee initially, a higher price ceiling is obtained. Any subsequent innovator must pay this price and purchase its own buyout fee contract. We relate this mechanism to the proposed compulsory licensing schemes.

Keywords: Sequential Innovation, Patents, Mechanism Design, Compulsory Licensing.

JEL: D43, D82, L51, O31.

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1 Introduction

A central feature of innovative activity is that research is cumulative. Scotchmer (2005) provides an extensive list of sectors for which this feature is particularly important; in biotechnology research, each generation of a genetically modified crop is used as a base of future enhancements; computer programs are based on the features introduced by prior generations of software and faster and more reliable computer hardware builds on previous less-advanced products. Many other products have been configured by a succession of improvements of diverse origin.

A variety of methods are available to reward innovators. Two of the most commonly discussed (in Wright (1983), for instance) are patents and research prizes. The latter is simply a transfer to the innovator for development of a particular invention. The former consists of the granting of some sort of market power to the innovator, perhaps in exchange for a fee. It is well understood that, when information is complete and absent other imperfections, it is optimal to choose a prize as the reward, since it does not result in any of the distortions that may accompany market power. When the principal charged with rewarding innovators does not have complete information about the benefits of an invention, however, it has been shown, for instance in Scotchmer (1999) and Cornelli and Schankerman (1999), that it may be optimal to grant a patent, since the value of the reward is then tied to the innovation's value through its potential profits in the market.

In this paper we argue that the cumulative nature of innovation is relevant to the way in which research is rewarded. If research is rewarded through the granting of particular property rights, as for instance in a patent, the cumulative structure leads to the natural question of what to do when an improvement arises. Keeping the promise of property rights to the first innovator limit what can be offered to the second innovation.

We study the trade-off between patents and prizes in a model where new ideas arrive continually, and there is moral hazard. Existing models have studied either environments with asymmetric information or cumulative inventions. Here we incorporate both. The model of cumulative innovation we employ is similar to the one in O'Donoghue, et al. (1998),[1] but also is in the spirit of

[1] A key difference is that we assume the patent office's information about the quality of innovations is incomplete,

cumulative innovation models of Scotchmer and Green (1990) and Green and Scotchmer (1995). New innovations build on the knowledge embodied in previous innovations.

We allow an authority to reward innovators with a variety of instruments, in the spirit of Wright (1983), but force them to operate with limited information about the potentially patentable innovations. Using market power as a reward is useful, as in the single innovation models, when value is unknown. It is here, though, that a scarcity arises. Promising future market power to the current innovator limits what can be offered to future innovators in the same market. Lines of what constitutes a "sufficient" improvement to warrant a new patent must be drawn. We use an extreme model where, if only one innovator is ever to arrive, the optimal patent policy implements the efficient level of research. We then show that in the same model, but with multiple innovators arriving in sequence with cumulative innovations, it is impossible to achieve the efficient level of research. This reinforces the idea that the cumulative nature of innovation is very relevant to policy. Aligning the innovator's incentives with society's requires allocating to each innovator the rights to the entire future. Since the future must be shared, the planner must trade off incentives for today's innovator versus saving property rights for future innovators.

Our optimal policy prescribes a threshold rule for the minimum size of the future superseding innovation, which is similar to the notion of "breadth" used in O'Donoghue, et al. (1998). Although the contract between the innovator and the authority could include patents of any length or breadth, the optimal reward can be interpreted as a patent with no statutory expiration date, but rather provides a constant amount of protection against future improvements forever. Current patent policy has a clear sense in which the level of protection declines suddenly, at the end of the patent's statutory life. The optimal policy here suggests that patents should end only because something better arrives, and not because of some imposition of a statutory time limit for the protection. Under plausible conditions the optimal patent policy involves different types of protection for different innovations; bigger improvements get greater protection.

We show that the optimal mechanism can be instituted through a system of mandatory buyout

and we allow for any combination of breadth and length of patents, while they consider only infinite breadth and infinite length patents. On the other hand, they consider heterogeneity on the part of the consumers as well as a form of bargaining between innovators; we have neither.

fees. In order to market an innovation, the innovator must pay a prearranged buyout amount to the owner of the prior state-of-the-art innovation. Then the new innovators may choose from a menu of buyout fees to be paid to him by an innovation that wishes to supplant him, with a greater buyout fee requiring a greater up-front payment, made to the planner. The optimal policy generates information about the quality of innovations, which may reduce the burden on the courts.[2] Moreover, we show that the menu of contracts offered by the patent office in order to implement the optimal reward is not time or history dependent, which is particularly appealing in terms of realistically using such a method.[3]

The idea of mandatory licensing dates back at least as far as the 1800's, when such a rule passed the House of Lords (Machlup and Penrose (1950)). Such a rule is not in effect today in the United States, but as Lerner (2002) documents, it is contemplated in the patent statute of most developed countries although its use is restricted to exceptional circumstances. The difference in our case is that in the optimal policy derived here, innovators, as part of the granting of a patent, must commit to a price at which they will relinquish their rights. This commitment mitigates any bargaining power a patent holder might exert on future innovators. Tandon (1982) uses similar buyout fees in a complete information model to mitigate monopoly costs. This price acts as the breadth of the patent: the bigger the fee, the greater the patent's implied breadth, since future innovators will need a substantial improvement to find optimal to pay the larger buyout fee.

Our point about prizes is similar to that in Scotchmer (1999) and others: prizes are useful if information is complete enough. Whereas, in Scotchmer (1999), the reason for prizes is to avoid monopoly distortion, here prizes would be attractive because of the scarcity of monopoly rights to assign. Other authors have studied different rationales for prizes. Notably, Wright (1983) argues that prizes may mitigate problems associated with patent races. In our formulation, with ideas private to a single innovator, this argument for prizes is not present; we have only that prizes today, if substituted for monopoly power, give the planner more freedom to use monopoly power

[2] See Llobet (2003) for a study on how the legal environment affects cumulative research.

[3] Interestingly, while the current US statute does not explicitly allow for multiple breadths, it has been claimed by papers such as Allison and Lemley (1998) that, in fact, the patent courts do provide additional protection for products which represent large improvements.

in the future. Shavell and van Ypersele (2001) suggest offering an optional reward, so that some patents would be replaced with rewards.

Kremer (1998) takes a different approach to elicit the private information held by the innovator. He considers the case where potential competitors have information which is correlated with the value of the invention. In this case, an auction among competitors to obtain monopoly rights on the invention generates information useful to a prize-granting authority. We study, on the other hand, an abstract approach to the classic patent/prize design problem, where the planner has access to transfers and contingent exclusion rights – the ability to offer an innovator a period of rights to exclude others from producing the innovation. Our approach does not rely on information from competitors; in the context of sequential innovation, today's competitors are likely to be future innovators, and for this reason, the dynamic implications of his mechanism (including collusion) might be less clear.

The next section introduces the model and sets the stage by studying a special case where there will only ever be one innovator. When costs are unknown but value of the project is observable, the reward for an innovator is a cash prize. When value is unobservable, patents are employed as a reward. In the model below, the efficient level can be implemented in either case. These results mirror the findings of several papers in the patent literature and are similar to the spirit of mechanisms used in the regulation literature. The main points come in the third section. There, the general model of many innovations is introduced and the optimal policy is contrasted with the one for the single innovator. The planner must trade-off the incentives for current against future innovators. In Section 4 we show that the optimal policy can be decentralized by means of a buyout scheme. Section 5 concludes.

2 One Innovation

2.1 Innovations, Markets, and Rewards

There is a single non-durable good differentiated by quality q and an infinite horizon of discrete time periods. The future is discounted according to a discount factor β. The marginal cost of production of the good is normalized to always be zero. The only costs are in improving the product's quality, and we will refer to the cost of research as simply the innovator's "cost." A product of quality zero is freely available and sold competitively, therefore at a price of zero. There is a single, infinitely lived consumer with time-additively separable preferences and per-period utility $q - p$, where p is the price of the good. Because it abstracts from static inefficiency, this market structure is particularly appropriate to isolate the dynamic implications of market power.

Each innovation begins with an "idea", denoted by z. If it is not implemented no cost is incurred. If it is implemented, it must be researched to be made into a viable product. That product can be freely imitated unless the innovator is given some specific property rights (a patent). Let q be the outstanding quality of the leading firm over which the innovator seeks to improve. The cost of the improvement is a function not only of z but also of the size Δ of the improvement (in the quality space) over the state of the art, so that the purchase of the improved product provides utility $q + \Delta - p$. The cost of an improvement Δ given an idea z is $c(\Delta, z)$. It is assumed that[4]

Assumption 1 *$c_1 > 0$, $c_2 < 0, c_{11} > 0$ and $c_{12} < 0$.*

Costs are increasing and convex in the size of the invention. The higher is z, the *lower* are marginal costs c_1. Therefore the social planner prefers that firms which draw high z spend more on research. The higher is the firm's z, the lower are its costs.[5]

One can interpret a higher z as a better quality idea, or emanating from a more productive research firm, in the sense that a higher z makes it easier to improve on the product. We will sometimes refer to higher z as "better," although it actually only makes higher quality less expensive to implement. The distribution of z is continuous and described by the density function $\phi(z)$ with the associated cumulative distribution $\Phi(z)$. The support of $\Phi(z)$ is an interval $[0, 1]$.

Since Δ is an improvement to quality, if the quality of the product before improvement is q, after the innovation it is $q' = q + \Delta$. Given this fact and the form of preferences, the size of the

[4] Numerical subscripts of c refer to partial derivatives.

[5] Since everyone is assumed risk neutral, one can think of innovations and costs to be expressed in terms of expected values.

innovation Δ is also the amount of social surplus it generates each period. For instance, if the product is sold competitively at marginal cost 0, then the homogeneous consumer enjoys $q + \Delta$ units of surplus; if the product is sold at a higher price, profits rise one-for-one with lost consumer surplus.

In the first best allocation that a fully informed social planner would choose, $\Delta^*(z)$, the optimal innovation size as a function of the innovator's cost type z, maximizes the present value of the increase in total surplus net of the cost of the innovation. That is, it solves

$$\Delta^*(z) = \arg\max_{\Delta \geq 0} \frac{\Delta}{1-\beta} - c(\Delta, z).$$

As a result, in the first best,

$$\frac{1}{1-\beta} = c_1\left(\Delta^*, z\right). \tag{1}$$

Our assumptions imply that, whenever $\Delta^*(z) > 0$ it is increasing in z, so that better ideas (in the sense that higher quality is cheaper to implement) lead to larger innovations.[6]

Finally, we restrict ourselves to what we call *contingent exclusion rights*; the planner can reward the innovator with the right to exclude temporarily the sales of future improvements that might arise. The planner can also charge a fee $F(z)$. Notice that this family of contracts includes not only patents but also other mechanisms considered in the literature such as prizes (which can be understood as a negative fee and no monopoly power).

When a regulator faces a situation of unknown demand (as in this and many patent examples), there is an incentive to monitor via consumers. We follow the typical approach and rule out such avenues. As described in Lewis and Sappington (1988), monitoring demand can be difficult when the product in question is bundled with other goods or services, as is the case with many innovations that are only a small part of a product. If the firm providing the innovation also chooses the levels of the other components of the final product, monitoring transactions of the final product

[6] In general, not all ideas will generate an increase in welfare that justifies the cost. There will be a threshold value $\underline{z}^*$ so that only $z > \underline{z}^*$ will be optimally developed into innovations. This minimum size is defined by

$$\frac{\Delta^*(\underline{z}^*)}{1-\beta} - c(\Delta^*\left(\underline{z}^*\right), \underline{z}^*) = 0.$$

is manipulable.

If the regulator uses a prize, however, such fears might be assuaged, since the product would not be sold by the innovator. Still, using reports from consumers might be difficult, since many innovations are inputs into many different products. Suppose there are a continuum of goods indexed j that incorporate the innovation and a single consumer for each good. Let the value of product j to its consumer be $a(j)q$, where q is the quality of the innovation as described by the ladder. The total value of the innovation is then $q \int a(j)dj$. To calculate this via reports from consumers the planner would need a large number of reports for the various goods.[7] It is easy to see how this could be prohibitively expensive, even at a small cost per report. Moreover, it would be perfectly possibly to let $a(j)$ evolve stochastically over time, subject to the restriction that $a(j)q$ is always rising. Each innovation would bring the need for a huge number of *additional* reports to be useful.

To simplify the exposition, we will consider only the case in which monopoly power for the leading innovation does not preclude earlier innovations from being sold. We show later that this is without loss of generality. Hence, a firm with improvement Δ sells it for price $p = \Delta$ during the patent term and $p = 0$ once it is superseded.

2.2 Rewarding an Innovation

In order to set the stage for the model with cumulative innovations, it is useful to start by addressing how to reward an innovator if it is the only idea that will ever arise. What can the planner do to encourage innovation in this case? First, suppose that the planner can observe q, and hence Δ. Then the first best can simply be achieved by offering a reward $F(z) = -c(\Delta^*(z), z)$ for an innovator of type z who undertakes an innovation of size $\Delta^*(z)$, and offering nothing if an innovator of type z chooses anything else. Rewarding innovators does not require granting monopoly power.[8]

[7] If the innovator is slightly risk averse, basing the prize on a sample of reports of $a(j)q$ would not be effective if $a(j)$ varies greatly across products.

[8] Notice that, if z were private information, $\Delta^*(z)$ can still be implemented in a manner identical to the mechanism in Baron and Myerson (1982).

In reality, the true quality of an innovation is hard to ascertain, and therefore a prize system may be difficult to implement effectively. As a result, Scotchmer (1999) shows that it may be useful to offer patents, since the value of a patent is tied to the value of the innovation. There the planner offers to allow a longer patent in exchange for payment of a larger fee. Only a monopolist with a relatively big innovation will find it worthwhile to pay the fee, since the flow value of the monopoly power is increasing in the size of the improvement.

Suppose, then, that Δ is unobservable. A prize structure like the one previously defined will not be effective, since everyone will claim to have made a large innovation, and there will be no way to verify if the research had actually been undertaken. However, since market power only affects the distribution of the surplus, the efficient outcome can be attained, but only through the granting of monopoly rights. In fact, if the duration of the patent T is infinite and $F = 0$, it is easy to verify that the innovator chooses $\Delta^*(z)$, since the profits of the firm are exactly equal to the social surplus, and so profit maximization and the definition of $\Delta^*(z)$ coincide. However, doing so requires giving away all future market power for this product.

3 Multiple Innovations

3.1 Allocating Market Leadership

Suppose that in each period, a new innovator arises. There are three important forces that make this problem of rewarding sequential innovators interesting. First, we assume that the innovation size Δ is unobservable (moral hazard), so it is not possible to simply reward all innovators with a negative fee that implements the first best of the previous section. For simplicity we assume that z is observable; we drop that assumption in Section 4. Second, given that market power will be part of the reward, a crucial feature is that the total discounted length of market power to be allocated across all innovators is finite. At any point in time, there are $\frac{1}{1-\beta}$ units of "discounted time" to be allocated: the sum of all future time periods t, discounted by β^t. Allocating monopoly power to one innovator may curtail the planner's ability to award monopoly power in the future. Finally, the heterogeneity of innovations is important. When evaluating how much length of market leadership to give to the current innovator, the planner trades off promised market leadership to today's leader against the value of offering that reward to a future innovator who may have a higher z, and therefore a cheaper way to increase quality. As shown below, there are some fundamental differences between the problem with and without heterogeneity.

From the innovator's perspective, all that matters is how long monopoly power will last for the innovation. The innovation can make profits Δ in any period that it is the highest quality product allowed to be sold, and zero otherwise. Denote the *expected discounted duration* of the monopoly power granted under the patent by d. A unit of d gives the innovator the right to make Δ profits for one immediate period, $1+\beta$ units give it the right for the current period and the next period, and so on. Consequently, the innovator's expected revenue from sales of the good are $d{\cdot}\Delta$. The innovator solves

$$\Delta(d, z) = \arg\max_{\Delta}\ d{\cdot}\Delta - c(\Delta, z) - F(z). \tag{2}$$

The corresponding first order condition is

$$d = c_1(\Delta, z)$$

where Δ denotes the optimal choice made by the firm. Due to the assumptions on the cost function, it is immediate that the function $\Delta(z, d)$ is increasing in both its arguments. The more duration is granted, the more innovation will be undertaken, since it will be marketable for longer. Of course, the cost is that duration precludes future innovations that might be worthwhile, since all the duration that the planner can allocate is $\frac{1}{1-\beta}$. The duration can involve for example T periods of monopoly power, so that $d = \frac{1-\beta^T}{1-\beta}$ or more generally, rules that depend on all the ideas obtained by past as well as future innovators.

Since we are assuming that z is observable in this section, the fee is simply a transfer that has no effect on the optimal policy we construct. Therefore we will focus on the allocation of duration. It is clear from the form of profits in (2) that, if the innovator were allowed to exclude prior innovations, so that he faced competition $\Delta + x$ units of quality inferior to his innovation instead of Δ units inferior, this would not change the incentives to innovate; instead the innovator would face the problem of

$$\max_{\Delta} \; d{\cdot}(\Delta + x) - c(\Delta, z) - F(z).$$

If, in the revised problem, $F(z)$ is increased by $d{\cdot}x$, the problems are identical, and so none of the results in this section rely on the assumption that the innovator faces competition from all prior innovations.

The previous expression together with equation (1) implies that underinvestment will in general occur; the innovator only internalizes the welfare effects of the improvement during his incumbency. This externality resembles the mechanism emphasized in the literature on Moral Hazard in teams (see for example, Holmstrom (1982)), where each agent only internalizes his reward from the effort exerted. As in our case, the optimal level of effort cannot be achieved because it would imply promising all the surplus to each agent.

We study a planner who seeks to maximize the present value of total surplus from innovations, using *contingent exclusion rights.* At any given time, the planner has made *duration* promises to prior innovators, in the form of a time allocation where other further innovations will be excluded. Although the planner would like to always implement the new innovations, he is constrained by the duration promised to prior innovators. Denote by D the cumulative units of duration promised to prior innovators entering the period. We will argue below that D is a sufficient statistic for the planner's problem of choosing duration d at any history. When an idea z arrives, the planner must then decide how to allocate duration between the new idea and the old products. Exclusion rights are contingent in the sense that the planner might use z in deciding how to allocate these rights between current and prior innovators. The planner must, then, decide how much duration $d_c(z)$ to allocate to the current innovation that arrives at time t and how much duration $d_p(z)$ to allocate to previous innovators.

The planner faces the following constraints. Both $d_p(z)$ and $d_c(z)$ cannot be negative for any z. The sum of $d_p(z)$ and $d_c(z)$ cannot exceed $\frac{1}{1-\beta}$, the total expected discounted duration. Without loss of generality we take $d_p(z) + d_c(z) \geq 1$; since there is no monopoly distortion from assigning duration, assigning (at least) all duration until the next idea arrives (in one unit of time) is always optimal, since it generates more innovation and therefore higher welfare.

Finally, the planner must keep his promises of duration made before. Promise keeping (PK) requires that

$$D = \int d_p(z)\phi(z)dz.$$

The current innovation will generate a social benefit Δ any time that the innovator that created it or any subsequent improvement is produced, since by the nature of cumulative innovation each future improvement incorporates all the previous innovations. If there were no promised duration to prior innovators, then, the benefits would be attained for $\frac{1}{1-\beta}$ discounted periods; but given the choice of $d_p(z)$, the current innovation will be excluded during $d_p(z)$ units of time. Therefore, the contribution to social welfare of innovator z is

$$R(d_p(z), d_c(z), z) = \left(\frac{1}{1-\beta} - d_p(z)\right)\Delta(d_c(z), z) - c(\Delta(d_c(z), z))$$

where the function $\Delta(d, z)$ is as defined in equation (2).

We write the problem recursively. Let D be the state variable, and $V(D)$ represent the expected present value of all future innovations given that D units of duration have already been allocated. Suppose the planner grants $d_p(z) + d_c(z)$ units of duration in a given period. One unit of this duration accrues during that period, leaving a balance – starting next period – of $\tilde{D}(z) = \frac{1}{\beta}(d_p(z) + d_c(z) - 1)$.

Take the "static" return at any given period to be the entire expected discounted social return from the innovation that occurs at time t. Then

$$V(D) = \max_{d_p(z), d_c(z)} \int \left(R(d_p(z), d_c(z), z) + \beta V(\tilde{D}(z))\right)\phi(z)dz \tag{3}$$

$$\text{s.t.} \quad \begin{array}{c} D = \int d_p(z)\phi(z)dz \qquad \text{(PK)} \\ 1 \leq d_p(z) + d_c(z) \leq \frac{1}{1-\beta} \\ \tilde{D}(z) = \frac{1}{\beta}(d_p(z) + d_c(z) - 1) \\ d_p(z) \geq 0, d_c(z) \geq 0. \end{array} \tag{4}$$

The next proposition shows under general conditions that the function V is concave and therefore, the first order conditions of this problem are sufficient.[9]

[9] We thank one of the referees for providing this general result. The proof is contained in the appendix.

Proposition 1 *The function $V(D)$ is weakly concave.*

For the remainder of the paper, we maintain a monotonicity assumption on the shape of Δ:

Assumption 2 (sorting) $\Delta_{12}(d,z) > 0$.

Under the sorting assumption, the impact of duration is greater for higher z, i.e. ideas where quality is less expensive to implement. In other words, there is a complementarity between a good idea and the reward that an innovation receives. This goes beyond the monotonicity assumption $c_{12} < 0$; not only would a fully informed planner want to spend more resources on higher z, but a constrained planner with only duration at his disposal wants to allocate more of *that* scarce resource to ideas with higher z. An example of a cost function satisfying this condition is the following:

Example 1 *The cost function $c(\Delta, z) = \frac{\Delta^{\alpha}}{z} + k$ with $\alpha > 1$ satisfies assumptions 1 and 2.*

By analyzing the first order conditions, it can be shown that the optimal $d_c(z)$ is increasing in z. This result stems from the fact that $\Delta_{12}(d,z) > 0$ so that the marginal social return of allocating duration to innovators with higher z is greater.

Proposition 2 *The optimal policy function $d_p(z)$ is decreasing and $d_c(z)$ is increasing in z and strictly increasing in the range where $d_p(z) = 0$ and $d_c(z) > 0$.*

Given that the optimal $d_p(z)$ is decreasing in z, this proposition implies that the optimal duration policy is contingent on the quality of new innovations. [10] So a fixed patent length is not optimal. Moreover, expected patent length should increase with the quality of an innovation. Notice that the existence of renewal fees in the current patent system has some implications that

[10] Absent assumption 2 examples of non-motonocity can be constructed. More details are available from the authors.

are qualitatively similar with the latter property, as less profitable innovations may opt for not renewing.

In order to further characterize the optimal contract, it is useful to think about possible structures that $d_p(z)$ and $d_c(z)$ might take. When both $d_p(z)$ and $d_c(z)$ are strictly positive, rights to future rents are given to more than one party. In this case, the current innovator is given a *promise* of a patent that starts at some future date. Or equivalently, letting $d(z) = d_p(z) + d_c(z)$ the allocation of a patent with duration $d(z)$ could be randomized between the new and previous innovator, with probabilities $d_c(z)/d(z)$ and $d_p(z)/d(z)$, respectively.[11]

An opposite situation -which we provide sufficient conditions for below- is the case where $d_p(z) = 0$ whenever $d_c(z) > 0$, so that granting of rights to today's innovator terminates rights to previous innovators. Since $d_c(z)$ is increasing, this implies that there is some threshold report of z today that terminates the previous innovators' rights. This is a sort of patent breadth for the incumbent rights holder; only if z is sufficiently large (which translates to sufficiently high improvement Δ) will the new innovator be allowed to produce.

Definition 1 *A patent system is* exclusive *if, for all D, $d_c(z) > 0$ implies $d_p(z) = 0$ almost everywhere in z.*

We first analyze the implications of an optimal patent policy when it is exclusive and then discuss conditions that rule out the alternative case.

3.2 Exclusive patent systems

Assume that the optimal policy is exclusive. Since d_c is increasing, the choice of $d_c(z)$ must be a threshold rule, in the sense that the set of z where $d_c(z)$ is zero must fall below some threshold, denoted $\underline{z}$. For those values where $d_c(z) = 0$, concavity of V immediately implies that for all $z \leq \underline{z}$, $d_p(z)$ is constant. Moreover, because contingent on not having implemented a new innovation the problem is the same every period, concavity also means that the planner will choose the same

[11] Under this last interpretation, the innovator chooses Δ prior to the result of the randomization.

duration across periods, and therefore $d_p(z) = 1 + \beta D$.

Proposition 3 *If the optimal patent is exclusive, then the set* $\{z : d_c(z) = 0\}$ *is an interval* $[0, \underline{z}(D))$. *Moreover,* $d_p(z) = 1 + \beta D$ *in this range and* $\underline{z}(D)$ *is increasing in* D.

This proposition can be interpreted as an optimal rule that takes the form of a constant breadth patent. Either something greater than $\underline{z}(D)$ comes along, in which case the innovator will be granted duration $d_c(z)$, or the current innovator gets nothing, and the protection for the previous innovator remains D from tomorrow on. Conditional on not being replaced ($d_p(z) > 0$), the protection afforded is just a constant threshold $\underline{z}$. Using the promise-keeping constraint, the duration can be easily expressed as

$$1 + \beta D = \frac{1}{1 - \beta \Phi(\underline{z})} \tag{5}$$

which means that $\underline{z}$ is increasing in D. When replaced, the duration of the new innovator $d_c(z)$ takes the form of a new $\underline{z}$. Since $d_c(z)$ is increasing in z, it must be the case that this new threshold is larger the larger is the innovator's z.[12]

There is some evidence that courts follow something like this rule. The most common way to invalidate a patent is to show the courts that it is not a very "big" improvement. In such cases, the patent may be invalidated (i.e. zero breadth), or it may be that it is quite easy for other products to be sold. Allison and Lemley (1998) study a sample of 299 patents litigated in 239 cases. These represent all the suits in the period 1989-1996 started by competitors in order to invalidate existing patents. They find that the most argued reason to limit the original innovator's property right is the obviousness of the patented invention, used in 42% of the cases. In this model, small improvements – or $z < \underline{z}(D)$ – get no protection, while larger inventions, according to proposition 2 obtain an increasing $d_c(z)$. This additional protection, is, of course, costly. The proof of this proposition, together with the first order condition, implies that the derivative $V' < 0$, so promises of breadth by the patent office decrease future prospects.[13]

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[12] Note that there is no way to affect $\underline{z}$ through Δ, so there is no incentive to increase Δ to increase breadth.

[13] Note that a direct argument can be used to show that V is decreasing. Since $R(d_p, d_c, z)$ is decreasing in d_p and increasing in d_c, lowering D (through any reductions in $d_p(z)$ satisfying the promise keeping constaint) while increasing $d_c(z)$ to maintain $\tilde{D}(z)$ must increase the planners payoff.

Notice that our basic environment is very similar to the model of O'Donoghue, et al. (1998). They assume that patents are the only possible reward structure, and they describe a patent in terms of its length (how long it lasts) and its breadth (in terms of how much improvement Δ is required for a new innovation to be allowed). We have a similar notion of a patent, arising as an optimal response with moral hazard. Without moral hazard, the optimal response would be to reward with prizes. In their language, our patents have infinite statutory length and finite breadth $\underline{z}(D)$.[14]

Also note that under our regularity condition, proposition 2 implies that $d_c(z)$ is strictly increasing when positive. The optimal policy thus grants more breadth of protection to better innovations, since higher z leads to greater quality improvement Δ. This rule may seem quite complicated to implement, as it requires a patent authority to determine z. In section 4, we consider a simple decentralization scheme. The decentralization applies even when z is not observed.

3.3 When is the optimal patent system exclusive?

In this section we provide a regularity condition under which the patent system is exclusive.

Two reasons operate for the patent system to be exclusive. The first is related to the redistribution of duration across states in the same period. The second, is due to the social value of the improvement that the innovator does not internalize.

We develop these two effects by considering the possibility that the system is not exclusive. We take two values of z where both d_p and d_c are strictly positive. We then ask when the system can be improved by the following variation. We increase the reward to current innovators for the higher value of z, and lower it for the lower value of z. We make the opposite changes to the reward for past innovators, so that the sum of current and previous allocations remains constant for each z, and promise keeping is still satisfied. This variation keeps everything constant from the next period onward, so we can look at the impact of such a change purely in terms of the planner's current return. We use the argument to develop a regularity condition such that the change will always

[14] Breadth in O'Donoghue, et al. (1998) is described in terms of Δ and ours is in terms of z. Since $d_2(z)$ is non-decreasing in z implying that Δ and z are one-to-one in our model, the depictions are identical.

increase surplus, and therefore the planner should always give the duration to the higher value of z, for whom the marginal impact of duration, in terms of Δ, is the greatest. As a result, the patent system must be exclusive. We formally provide the result as proposition 4.

Start with the possibility that for more than one value of z the existing innovator is not dismissed after a new implemented idea arrives. In particular, consider two values $z_2 > z_1$ where $d_p(z_2), d_c(z_2), d_p(z_1)$,and $d_c(z_1)$ are all strictly positive. Take the following alternative plan. Decrease $d_p(z_2)$ and increase $d_p(z_1)$ by small magnitudes keeping $d_p(z_2)\phi(z_2) + d_p(z_1)\phi(z_1)$ constant. This variation keeps the expected value of d_p unchanged so it preserves the promise-keeping constraint. At the same time, change $d_c(z_1)$ and $d_c(z_2)$ in the opposite direction in order to keep the sums $d(z_1)$ and $d(z_2)$ constant.

To examine the effects of this variation, we examine total surplus for a given new innovation z, given by:

$$\left(\frac{1}{1-\beta} - d_p\right)\Delta(d_c, z) - c(\Delta(d_c, z)) + \beta V(d_p + d_c)$$

where for notational convenience we have suppressed the dependence of d_p and d_c on z. Since the variation considered keeps $d_p + d_c$ constant, we may ignore the last term. For fixed z, the remaining terms can be separated as follows:

$$\left(\frac{1}{1-\beta} - d_p - d_c\right)\Delta(d_c(z), z) + [d_c\Delta(d_c, z) - c(\Delta(d_c, z), z)]$$

where the term in square brackets are the profits of the firm. It is easy to see that this term is strictly convex in d_c provided $\Delta_1(d_c, z) > 0$ (for the same reason that profits are convex in prices). As $d = d_p + d_c$ are kept constant, using the envelope condition for Δ, the only effects of the variation that need to be considered are the following:

$$G(d_c, z) \equiv \left(\frac{1}{1-\beta} - d\right)\Delta_1(d_c, z) + \Delta(d_c, z). \tag{6}$$

If this expression increases with z the variation suggested gives a higher value. The second term can be interpreted as the *static* effect of the reallocation of durations across different states in the same period. To see it, suppose that after the current innovator no more ideas are ever implemented. This would imply that it is optimal that $d(z) = \frac{1}{1-\beta}$, and the private incentives of the current innovator and social returns are perfectly aligned. Using the previous assumptions we obtain that the reallocation leads to higher value. The intuition is simple; the variation assigns more duration to innovators for which the impact in terms of the size of the improvement is larger. This effect comes from assumption 2; bigger z makes the return to duration larger, and therefore larger z leads to larger Δ.

The first term corresponds to a *dynamic* effect, the social return that the innovator does not internalize. In this case two possibly countervailing effects occur. Under the assumption that d_p and d_c are strictly positive for z_2 and z_1, it can be shown from combining the two first order conditions from (3) that $d(z_1) < d(z_2)$. Together with our assumptions, this implies that the direct effect of increasing s is positive. However, an indirect effect operates through the change in d_c. Since $d_c(z_1)$ is greater than $d_c(z_2)$ this effect will be positive (negative) if function $G(d_c, z)$ is convex (concave) in d_c.

Summing up, we obtain that unless Δ is sufficiently concave in d_c to offset the convexity of profits π (static effect) and the direct effect of z, the total effect will be positive and thus the original plan will not be optimal. This argument implies that as far as $G(d_c, z)$ is convex in d_c, $d_p(z)$ and $d_c(z)$ cannot be both positive in any interval, so almost surely $d_c > 0$ implies $d_p = 0$.

Assumption 3 *$\left(\frac{1}{1-\beta} - d\right) \Delta_1\left(\tilde{d}_c, \tilde{z}\right) + \Delta\left(\tilde{d}_c, \tilde{z}\right) > \left(\frac{1}{1-\beta} - d\right) \Delta_1(d_c, z) + \Delta(d_c, z)$ whenever $\tilde{d}_c > d_c$ and $\tilde{z} > z$ for all $d \geq d_c$.*

A sufficient condition (very far from necessary, from our previous discussion) is that $\Delta_{11} > 0$. It should be noted that while this condition guarantees the convexity of $G(d_c, z)$ in d_c it does not imply that the surplus function $\left(\frac{1}{1-\beta} - d_p\right) \Delta(d_c, z) - c(\Delta(d_c, z))$ is convex in d_c for fixed d_p. So the extreme policy described above is not the result of assuming convexity of the planner's return; it comes from the combination of the fact that bigger innovations are obtained when higher z arrive, and that the marginal return to assigning duration to the current innovator does not fall too fast.

Proposition 4 *Under the regularity condition (assumption 3), the patent system is exclusive.*

It should be noted that an extreme failure of the convexity assumption can generate innovations on hold, in the sense that old innovations are not displaced when a new idea comes along. For example, if $c(0, z) = 0$ so that there is no fixed cost of innovation and the Inada condition $\Delta_1(0, z) = \infty$ holds for all z, allocating duration only to the largest ideas will not be optimal. As a result, the planner will provide positive duration to all ideas in order to equate their marginal contribution to social welfare. Any outstanding duration in a given period implies that this period's innovator will be kept on hold, or that duration will only be allocated to the current innovator randomly.[15]

4 Adverse Selection and Decentralization through Buyouts

In the previous section the quality of the idea z was assumed to be publicly observable. Now suppose that it is private information of each innovator. A standard approach would be for the planner to use a revelation mechanism where innovators report z and receive an expected discounted duration of $d_c(z)$ by paying a fee $F(z; D)$, where as before, D is the sum of all durations promised to previous innovators.

We propose an alternative decentralization for exclusive patent systems that takes the form of mandatory buyouts. At any point in time a new innovator must pay a buyout b to the current leader in order for a new innovator to be allowed to displace him. The innovator, in addition to paying the buyout b, must also choose a buyout amount b' that the *next* innovator who wants to produce will have to pay. In exchange for setting the buyout amount b', the innovator must transfer, at the time of the innovation, a payment τ to the planner. Since τ and b' will be associated with a given z (and may vary across z), we write $\tau(z)$ and $b'(z)$. Naturally, *buying* greater protection (in the form of a higher required buyout b') requires a greater up-front payment.

We construct the buyouts that decentralize the allocation of the previous section, in the sense that it gives the same durations $d_c(z)$. We start with the standard approach using a revelation mechanism. An innovator with idea z will report $\widehat{z}$, the solution to

$$\max_{\widehat{z}} \; d_c(\widehat{z})\,\Delta\left(d_c(\widehat{z}), z\right) - c\left(\Delta\left(d_c(\widehat{z}), z\right), z\right) - F(\widehat{z}; D)\,.$$

[15] Notice that the possibility of such an example benefits from the assumption that innovations improve the quality frontier for future improvements immediately, even if they are not marketed immediatly. If innovations did not improve the state-of-the-art until they were marketed, the potential for innovations on hold would be reduced.

Since $d_c(z)$ is monotone according to proposition 2, the planner can implement truth-telling by setting

$$F(z;D) = d_c(z)\Delta(d_c(z),z) - c(\Delta(d_c(z),z),z) + \int_{\underline{z}(D)}^{z} c_2(\Delta(d_c(x),x),x)dx. \tag{7}$$

This is just the usual formulation from the mechanism design literature, where payments make the lowest type implemented ($\underline{z}(D)$) earn zero profits. Every other type has the appropriate incentive to report z truthfully due to the rents from private information they obtain from $F(z;D)$. Since this implements the solution of the previous section where z is observed, it must be the best the planner can achieve when z is private information.

If the patent policy is exclusive, we can go further than the description of (7) and reinterpret the payments as a combination of payments to a planner and mandatory buyout fees. To do so, note that we can rewrite (7) in the additive form[16]

$$\begin{aligned} F(z;D) &= d_c(z)\Delta(d_c(z),z) - c(\Delta(d_c(z),z),z) + \int_0^z c_2(\Delta(d_c(x),x),x)dx \\ &\quad - \int_0^{\underline{z}(D)} c_2(\Delta(d_c(x),x),x)dx \\ &\equiv \sigma(z) + \gamma(D), \end{aligned}$$

so that the payment F can be broken into a part that depends on the current innovator's type z (the first three terms) and a part that depends on the previous innovators' level of protection D (the last term), denoted by $\sigma(z)$ and $\gamma(D)$ respectively. Note that, since the system is exclusive, D is simply the protection afforded the previous innovator.

Now consider the following buyout system. A new innovator, of type z, in order to be allowed to produce, needs to first pay the incumbent a buyout $b = \gamma(D)$, as described above. In order to extend his protection to future periods he must pay a fee $\tau(z)$ to the planner that entitles him to a buyout $b'(z) = \gamma(d_c(z))$ paid by the future innovator that displaces him. Because the arrival of a future innovator that can displace him is uncertain, the expected present value of that payment to the current innovator is

$$\frac{1 - \Phi\left(\underline{z}(d_c(z))\right)}{1 - \beta\Phi\left(\underline{z}(d_c(z))\right)}\beta b'(z).$$

[16] We once again thank the referee for replacing our complicated argument with this much simpler one.

Now define

$$\tau(z) \equiv \sigma(z) + \frac{1-\Phi(\underline{z}(d_c(z))}{1-\beta\Phi(\underline{z}(d_c(z))}\beta b'(z).$$

Since each innovator pays exactly

$$F(z;D) = \tau(z) + \gamma(D) - \frac{1-\Phi(\underline{z}(d_c(z))}{1-\beta\Phi(\underline{z}(d_c(z))}\beta b'(z),$$

this mechanism implements the optimum of the previous section. The first term is paid to the planner, the second is paid as a buyout fee, and the last is later returned to the innovator as his future buyout fee.

Notice that

$$b = -\int_0^{\underline{z}(D)} c_2(\Delta(d_c(x), x), x)dx > 0,$$

and since according to (5) $\underline{z}(D)$ increasing in D, the previous expression implies that b is also increasing in D. Since larger innovations command larger duration, the buyout is larger for innovations that displace bigger improvements.

The buyout fee system is particularly simple. The planner posts a list of fees and buyouts $\{\tau(z), b'(z)\}$. At any point in time, the current innovator can choose to become the market leader if it is willing to buy out the incumbent by paying b and obtain a future buyout $b'(z)$ at a price $\tau(z)$ which is independent to the duration granted to previous innovators. Moreover, once these payments are made, nothing about the previous innovators must be carried forward; all that matters in the future is the duration $d_c(z)$ granted to the new market leader, as operationalized through the current buyout amount, and, in each period, the type of the future innovator contemplating entry.

Clearly this system still requires the planner to possess the knowledge about the structure of the problem typical in mechanism design problems; here, that amounts to knowing the cost of development $c(\Delta, z)$ and the distribution $\Phi(z)$. Nonetheless, the typical (complete information) optimal patent design approach requires even more knowledge: the knowledge of the actual cost function for the particular innovator, as well as the ability to observe whatever characteristic defines breadth. In practice, though, the definition of breadth in the patent statute tends to be vague. Given that vast resources are spent in patent litigation, it seems that enforcement is not at all trivial.

An advantage of the proposal offered above is that the mechanism itself generates information about the quality of the innovations. If two products are on the same ladder, the report of the type z is sufficient to determine infringement and future protection, instead of a costly court system to determine what qualities have arisen. The buyout system is a particularly simple decentralization, where each innovator simply decides if he is willing to pay the current buyout amount in order to "infringe."

Another advantage of the buyout scheme implemented here is that it precludes any negotiation between the incumbent and the innovator that might replace him. Such a negotiation results in a hold-up problem identified among others by Green and Scotchmer (1995). The reason for this inefficiency is that the negotiation reduces the surplus that the new innovator can obtain, decreasing the incentives to enter. For this reason, papers such as O'Donoghue et al. (1998) obtain that a wide patent with a shorter duration might be optimal, since it reduces the period of time for which the hold-up occurs. The present paper suggests that optimal pre-specified licensing payments avoid the need for a short and wide patent.

5 Conclusion

The patent statute involves a single, somewhat vague definition of breadth for all innovations, and leaves most of the job of deciding property rights to the courts. Here, breadth is, as in models such as O'Donoghue, et al. (1998), defined in terms of quality. There are two ways to view this. The first is that, on average, the greater the quality difference, the greater the physical or intellectual difference between products; these differences are the typical notion of breadth. In the legal literature there is also a sense that quality differences matter for what is considered infringing or non-infringing; Merges and Nelson (1990) say that "The more significant the technological advance represented by the allegedly infringing device, the less willing the courts should be to find it an equivalent to the patentee's device." In constructing rewards for innovators, however, it is possible to generate information on the improvement achieved through the self-selection of patent protection. In fact, we show here how patent breadth can be implemented through a system of compulsory licensing, replacing much of the burden now placed on the courts.

This is especially important if one wishes to offer different patent breadth to different inventions. It is not surprising that in light of the heterogeneity of inventions, the optimal reward policy may reward different innovations with different breadth.

In this paper, the optimal policy is characterized, and it is found that in many cases such differentiation is optimal. This is an important practical point: it may be both optimal as well as feasible to offer multiple patent breadths. In addition, the optimal policy has patents of infinite statutory duration. The sort of patent policy described here would involve a more complicated set of patents offered, but a less complicated system to determine infringement, since the choice of patent would determine who had a right to produce.

A feature of the optimal policy is that patents have infinite statutory duration. They expire, effectively, only when a sufficiently good idea comes along. This results because it is always better to transfer profits to the leader in a state of the world when some amount of time has passed, but nothing good enough to supplant it has come along, rather than by giving extra breadth precluding a useful innovation. Whereas the patentee cares only about the probability of being the leader, the patentor cares about the size of the innovation when the new leader comes along.

This is important because it differs from the lessons of the one innovation case, which has been studied in slightly different environments. The cumulative research formulation used here suggests, as in other papers such as O'Donoghue et al. (1998), that breadth is a central part of the definition of a patent when further innovations will arrive. It may be more difficult to achieve efficient research outcomes in the cumulative case. The optimal policy might require differentiation between innovations through breadth in the cumulative case. This complements Hopenhayn and Mitchell (2001), where different breadths might be assigned to patents with different degrees of fertility.

The results also suggest an intuition regarding the question of the optimal length of patent protection. While this is a classic subject dating back to Nordhaus (1969) and Arrow (1969), among others, the formulation used here provides a new way to look at the role of statutory length when patents may become obsolete before the end of the statutory life of the patent. Long lived patents are beneficial in the sense that they shift the patent's enforcement to relatively low value projects, rather than precluding higher value projects for a smaller length of time.

Here, an infringing improvement is never able to be produced by way of some licensing agreement. This assumption is made to highlight the role of patents in dissuading future innovators. For the patent problem to be interesting, of course, licensing must be imperfect, lest the Coase theorem lead to an efficient outcome. It is possible to imagine that a compulsory licensing scheme such as the one suggested here might facilitate transactions of patents, since the protection they provide would be more clearly delineated than under the current patent law, where the outcome is left entirely to the court's discretion. It is clear that any policy which encourages licensing would have that as an extra benefit.

The idea that patents might help solve the hidden information problem faced in designing rewards for innovators is not new. In fact, John Stuart Mill argued in favor of patents on this basis, stating (from Machlup and Penrose (1950))

> ....an exclusive privilege, of temporary duration is preferable; because it leaves nothing to anyone's discretion; because the reward conferred by it depends upon the invention's being found useful, and the greater the usefulness, the greater the reward....

When the rewarder's information is limited, prizes may not be useful because the rewarder cannot determine how much to reward. Monopoly rights, on the other hand, have a reward related to the value of the innovation. The mechanism proposed here leaves nothing to the discretion of the patentor; the patentee makes incentive-compatible choice of the appropriate protection. It may be, however, that the duration should not have a fixed length, but rather only end when a sufficiently good report arrives at the patent office.

One key simplification of the model is that demand for the product is perfectly inelastic, so that there are no static welfare costs of monopoly. The existing literature shows that the effect of adding this feature to the model depends on particulars of how the monopoly distortions are modeled; some papers (Gilbert and Shapiro (1990)) suggest that such a force encourages long patents, while others

(Klemperer (1990)) suggest that it might go either way. An elastic demand allows the usage of additional instruments like output subsidies.

Important questions remain. An important issue is that of strategic behavior by inventors. Here each innovator in the sequence is different. This may overlook the fact that patentees routinely are thinking about future innovations that they will patent themselves when making research and patenting decisions. Another central question is the role of licensing. Incorporating some form of imperfect licensing would add an important element of the role of patents. All of these issues can be addressed within the structure introduced here, taking account of both the cumulative nature of research as well as the asymmetry of information that makes the rewarding of innovation a difficult task of government.

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A Proofs

Throughout the appendix we remap, without loss of generality, ϕ to the uniform distribution

Before establishing the concavity of V (proposition 1) we need to state an additional result.

Lemma 5 *If $f(x,z)$ is continuous in its two arguments and weakly increasing in x, the function*

$$F(\overline{x}) \equiv \max_{a \le x(z) \le b} \int_0^1 f(x(z), z)\, dz$$
$$s.t. \int_0^1 x(z)\, dz = \overline{x}$$

is continuous, non-decreasing, and weakly concave.

Proof. Continuity is immediate from the Theorem of the Maximum, and weakly increasing is direct from f being weakly increasing. Take any two $\bar{x}_1 < \bar{x}_2$ with optimal actions $x_1(z)$ and $x_2(z)$. We have ■

Claim 6 *There exists a set A such that*

$$\begin{aligned}
\int_A x_1(z)dz &= \lambda\bar{x}_1 \\
\int_A x_2(z)dz &= \lambda\bar{x}_2 \\
\int_A f(x_1(z), z)dz &= \lambda F(\bar{x}_1) \\
\int_A f(x_2(z), z)dz &= \lambda F(\bar{x}_2)
\end{aligned}$$

Proof. Define four measures

$$\begin{aligned}
\mu_1(E) &= \int_E x_1(z) \\
\mu_2(E) &= \int_E x_2(z) \\
\mu_3(E) &= \int_E f(x_1(z), z) \\
\mu_4(E) &= \int_E f(x_2(z), z)
\end{aligned}$$

and the mapping from measurable subsets of [0,1] into R^4

$$\mu(E) = \{\mu_1(E), \mu_3(E), \mu_3(E), \mu_4(E)\}$$

Note that the range of μ includes $\{0,0,0,0\}$ and $\{\bar{x}_1,\bar{x}_2,F(\bar{x}_1),F(\bar{x}_2)\}$, moreover, by Theorem 5.5 in Rudin (1991), the range of μ is convex, so there must be some set A with $\mu(A) = \{\lambda\bar{x}_1,\lambda\bar{x}_2,\lambda F(\bar{x}_1),\lambda F(\bar{x}_2)\}$

Let $\bar{x} = \lambda\bar{x}_1 + (1-\lambda)\bar{x}_2$ and define

$$\hat{x}(z) = \begin{matrix} x_1(z) \text{ if } z \in A \\ x_2(z) \text{ if } z \in A^c \end{matrix}$$

where A is as described in the claim. Then $\int \hat{x}(z)dz = \lambda\bar{x}_1 + (1-\lambda)\bar{x}_2$ ($\hat{x}(z)$ is feasible at $\bar{x}$), so $F(\bar{x}) \geq \int f(\hat{x}(z),z)dz$. But $\int f(\hat{x}(z),z)dz = \lambda F(\bar{x}_1) + (1-\lambda)F(\bar{x}_2)$, so F is concave. ■

Proof of Proposition 1. The problem in (3) can be reformulated as

$$\max_{0\leq d_p(x)\leq\frac{1}{1-\beta}} \int_0^1 h\left(d_p\left(z\right),z\right)dz$$
$$\text{s.t.} \int_0^1 d_p\left(z\right)dz \geq D$$

where z is re-normalized to be uniform $[0,1]$ and

$$h\left(d_p,z\right) \equiv \max_{d_c\geq 0}\left(\frac{1}{1-\beta}-d_p\right)\Delta\left(d_c,z\right) - c\left(\Delta\left(d_c,z\right),z\right) + \beta V\left(d_p+d_c\right)$$
$$\text{s.t. } 1 \leq d_p + d_c \leq \frac{1}{1-\beta}.$$

Note that the constraint $\int_0^1 d_p\left(z\right)dz \geq D$ will always be met with equality as h is strictly decreasing in d_p.

For $d_p(z) = -x(z)$, $D = -\bar{x}$, and $h(-d_p,z) = f(x,z)$, this problem satisfies the conditions of lemma 5 and the result follows. ■

In order to prove proposition 2 we first need the following two intermediate results:

Lemma 7 *Suppose $s \in [0,1]$. Let $u(s,x)$ be a function with increasing differences, where $x \in [0,1]$. Then, the solution to*

$$\max_{\{x(s)\}} \quad \int u(s,x(s))ds$$
$$s.t. \quad \int x(s)d(s) = M,$$

$x^*(s)$, is increasing in s.

Proof. For any $\lambda > 0$, the function

$$u(s, x(s)) - \lambda x(s)$$

is supermodular in s, x. By the Kuhn-Tucker theorem, there exists $\lambda > 0$ such that $x(s)$ maximizes

$$\int [u(s, x(s)) - \lambda x(s)]\, ds.$$

This must also maximize pointwise $u(s, x(s)) - \lambda x(s)$ for each s and by supermodularity the solution $x(s)$ is increasing. ■

Definition 2 *A patent system is* exclusive *at D if $d_c(z) > 0$ implies $d_p(z) = 0$ almost everywhere in z.*

Lemma 8 *Suppose that, at some point $D_0 \in (0, \frac{1}{1-\beta})$, the patent system is not exclusive. Then V is differentiable at D_0*

Proof. Let $d_p(z)$ and $d_c(z)$ be the optimal choices for D_0. For D in a neighborhood of D_0, define $\hat{d}_p(z, D) \equiv \alpha(D) d_p(z)$. Consider a measurable set A where $d_p(z)$ and $d_c(z)$ are both strictly positive; this set has positive measure since the patent system is not exclusive. Set $\alpha(D)$ to satisfy

$$\int_A \alpha(D) d_p(z) dz = D - D_0 + \int_A d_p(z) dz.$$

Notice that α is differentiable and that the duration $D - D_0$ is assigned proportionally in the interval. On this interval A, define $\hat{d}_c(z, D)$ so that $\hat{d}_c(z, D) + \hat{d}_p(z, D) = d_c(z) + d_p(z) \equiv \tilde{D}(z)$. For other points z let $\alpha(D) = 1$.

Let

$$W(D) = \int \left(R(\hat{d}_p(z, D), \hat{d}_c(z, D), z) + \beta V(\tilde{D}(z)) \right) dz,$$

where

$$R(\hat{d}_p(z, D), \hat{d}_c(z, D), z) = \left(\frac{1}{1-\beta} - \tilde{D}(z) \right) \Delta(\hat{d}_c(z, D), z) + \hat{d}_c(z, D) \Delta(\hat{d}_c(z, D), z) - c(\Delta \left(\hat{d}_c(z, D), z), z \right).$$

Since R is differentiable in D, $W(D)$ is also differentiable. Of course it is a feasible strategy for all D so it must be that $W(D) \leq V(D)$, and $W(D_0) = V(D_0)$. A direct application of the result of Benveniste and Scheinkman implies that V is differentiable at D_0. ■

Proof of Proposition 2. In order to prove the result, suppose towards contradiction that, for some D, there is an interval $[a, b]$ of z where this does not hold.

By lemma 8, if the patent system is not exclusive at D then $V(D)$ is differentiable. Without loss of generality, consider three alternative cases: 1) both are decreasing; 2) both are increasing; 3) d_p is increasing and d_c decreasing. It is useful to rewrite the reward function as

$$R(d_p(z), d_c(z), z) = \left(\frac{1}{1-\beta} - d_p(z) - d_c(z)\right)\Delta(d_c(z), z) + \pi(d_c(z), z)$$

where $\pi(d_c(z), z)$ is the discounted profits of the innovator granted $d_c(z)$ units of duration.

Case 1. If both are decreasing, let

$$u(z, d_c) = \left(\frac{1}{1-\beta} - d_p(z) - d_c\right)\Delta(z, d_c) + \pi(d_c, z) + \beta V\left(\frac{1}{\beta}(d_p(z) + d_c - 1)\right).$$

Because $\pi_1(d_c, z) = \Delta(z, d_c)$, we have that

$$u_2(z, d_c) = (\frac{1}{1-\beta} - d_p(z) - d_c)\Delta_2(z, d_c) + V'\left(\frac{1}{\beta}(d_p(z) + d_c - 1)\right).$$

Since d_p is decreasing, $\frac{1}{1-\beta} - d_p(z) - d_c$ is increasing in z; we have assumed that Δ_2 is increasing in z, and since $V'$and d_p are decreasing, the last term is increasing in z. Therefore u has increasing differences and lemma 7 applies, contradicting d_c decreasing

Case 2. If both are increasing, let

$$u(z, d_p) = \left(\frac{1}{1-\beta} - d_p - d_c(z)\right)\Delta(z, d_c(z)) + \pi(d_c(z), z) + \beta V\left(\frac{1}{\beta}(d_p + d_c(z) - 1)\right)$$

and we have

$$u_2(z, d_p) = -\Delta(z, d_c(z)) + V'\left(\frac{1}{\beta}(d_p + d_c(z) - 1)\right).$$

Note that this is decreasing in z, and since the assumptions of lemma 7 apply to $(z, -d_p)$, d_p must be decreasing, a contradiction.

Case 3. Without loss of generality suppose d_c is strictly decreasing in z, so it is invertible. Let $d_p(d_c)$ denote $d_p(z)$ for d_c in the image of $[a, b]$. then

$$\begin{aligned} u(z, d_c) &= \left(\frac{1}{1-\beta} - d_p(d_c) - d_c\right) + \pi(d_c, z) + \beta V\left(\frac{1}{\beta}(d_p(d_c) + d_c - 1)\right) \\ &= [(T_1 - d_p(d_c))\Delta(z, d_c)] + [\pi(d_c, z) + (T_2 - d_c)\Delta(z, d_c)] + \beta V\left(\frac{1}{\beta}(d_p(d_c) + d_c - 1)\right) \end{aligned}$$

where $T_1 > d_p,\ T_2 > d_c$ and $T_1 + T_2 = \frac{1}{1-\beta}$. Consider the first term in square brackets. Taking its derivative with respect to z gives

$$(T_1 - d_p(d_c))\Delta_1(z, d_c)$$

which is increasing in d_c because d_p is decreasing and $\Delta_{12} > 0$. The derivative of the second term in square brackets with respect to d_c is (using envelope condition) $(T_2 - d_c)\,\Delta_2\,(z, d_c)$, which is positive since $\Delta_{12} > 0$. This implies that $u(\cdot)$ has increasing differences in (d_c, z) as it is the sum of functions with increasing differences, and therefore lemma 7 implies and d_c must be increasing, a contradiction.

The strict monotonicity of $d_c\,(z)$ when $d_p\,(z) = 0$ and $d_c\,(z) > 0$ follows by applying the Implicit Function Theorem to first order conditions together with assumption (A2) and concavity of V.

If, on the other hand, the patent system is exclusive at D, then we establish the result in three steps.

(1) It must be that there exists $\tilde{z}$ such that $d_p(z) = 0$ for almost all $z > \tilde{z}$ and $d_c(z) = 0$ for almost all $z < \tilde{z}$. If not, take $z < \tilde{z} < z'$ with $d_p(z') > 0$ and $d_c(z) > 0$. By setting $\hat{d}_c(z') = d_c(z)$ and $\hat{d}_c(z) = d_c(z')$,the planner gets strictly greater current reward (due to the sorting condition) and identical reward from tomorrow on.

(2) For $z < \tilde{z}$, it must be an optimum to make $d_p(z)$ constant due to concavity of the value function.

(3) For $z' > z > \tilde{z}$, suppose that $d_c(z)$ is not strictly increasing, i.e. $d_c(z') \leq d_c(z)$. Then by setting $\hat{d}_c(z') = d_c(z)$ and $\hat{d}_c(z) = d_c(z')$,the planner gets strictly greater current reward (due to the sorting condition) and identical reward from tomorrow on. ■

Proof of Proposition 3. Immediate from the arguments in the text. ■

Proof of Proposition 4. Note that by lemma 8, if the system were not exclusive for some D, then $V(D)$ is differentiable. For that case, define the change of variables $d(z) = d_p(z) + d_c(z)$, and the associated problem

$$\begin{aligned} V(D) \quad = \quad & \max_{d(z), d_c(z)} \int \left(\left(\frac{1}{1-\beta} - d(z) + d_c(z) \right) \Delta(d_c(z), z) - c\left(\Delta(d_c(z), z), z\right) + \beta V\left(\frac{1}{\beta}(d(z) - 1) \right) \right) dz \\ & \quad D = \textstyle\int (d(z) - d_c(z)) dz \quad \text{(PK)} \\ & s.t. \quad 1 \leq d(z) \leq \tfrac{1}{1-\beta} \\ & \quad\quad d(z) \geq d_c(z) \geq 0 \end{aligned}$$

Consider the set Z of points z where $d(z) > d_c(z) > 0$, i.e. both $d_p(z)$ and $d_c(z)$ are strictly positive. The first order condition for the choice for $d\,(z)$ in this set is

$$-\Delta(d_c(z), z) + V'\left(\frac{1}{\beta}(d(z) - 1)\right) - \mu = 0, \tag{8}$$

where $\mu \leq 0$ is the Lagrange multiplier on the promise-keeping constraint (PK). It is immediate that V concave and $d_c(z)$ increasing imply that $d(z)$ is decreasing on Z. The first order condition for the choice of $d_c(z)$ in this range is (after replacing $c_1(\Delta, z) = d_c(z)$ from the innovator's first order condition)

$$\left(\left(\frac{1}{1-\beta} - d(z) \right) \Delta_1(d_c(z), z) + \Delta(d_c(z), z) \right) + \mu = 0. \tag{9}$$

By the regularity condition (3) the left hand side of (9) is strictly increasing in z for the range $d(z) > d_c(z) > 0$, and hence the equality cannot hold for multiple z. ∎