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Measuring and Understanding Brand Value in a Dynamic Model of Brand Management

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Abstract. We develop a structural model of brand management to estimate the value of a brand to a firm. In our framework, a brand's value is the expected net present value of future cash flows accruing to a firm due to its brand. Our brand value measure recognizes that a firm can change its brand equity by investing in advertising. We estimate quarterly brand values in the stacked chips category for the period 2001–2006 and explore how those values change over time. Comparing our brand value measure to its static counterpart, we find that a static measure, which ignores advertising and its ability to affect brand equity dynamics, yields brand values that are artificially high and that fluctuate too much over time. We also explore how changing the ability to build and sustain brand equity affects brand value. At our estimated parameterization, if brand equity were to depreciate more slowly, or if advertising were more effective at building brand equity, then brand value would increase. However, counterintuitively, we find that when the effectiveness of advertising is sufficiently high, increasing the rate at which brand equity depreciates can *increase* the value of a firm's brand, even as it reduces the value of the firm overall.

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1. Introduction

Brand equity is a key asset in the marketing of goods and services. Consumers use it to choose among products and services, and firms see it as a summary verdict on their marketing efforts. By its very nature, brand equity is a dynamic concept. It takes time to build brand equity and to sustain it. By the same token, once built, brand equity does not deplete right away. Consumers continue to appreciate brand equity long after it has been created (Keller 2008, Ataman et al. 2008, Erdem et al. 2008).

In this paper, we develop a structural model of brand management and use it to estimate the value of a brand from data on prices, advertising, and sales. We follow Goldfarb et al. (2009) in distinguishing between brand *equity* and brand *value*. The former refers to the extra utility consumers derive from a product because of its brand identity; the latter refers to the net present value of cash flows accruing to a firm because of its brand equity. What distinguishes the present paper from others in the literature is that we situate the problem of brand value measurement within a dynamic model of brand management. Our firms are forward looking and actively manage their brands. Specifically, they invest in advertising to sustain or enhance brand equity while accounting for brand equity depreciation, competitive reaction, and changes in market structure over time. Brand value in this framework thus captures what a brand's current equity is worth, while accounting for opportunities available to managers to change brand equity. Our structural model allows us to examine how brand value evolves in response to changes in brand equity and in firms' abilities to build and sustain brands.

Our data come from the stacked potato chip industry in the period 2001–2006. During this period, the industry was in transition. Until the fourth quarter of 2003, it was a monopoly, with Procter & Gamble's (P&G) Pringles as the only brand. Then STAX entered as a brand extension of Lay's, and it became a duopoly. The industry shows interesting dynamics in both its monopoly and duopoly periods in terms of changes in market shares, prices, and advertising.

We take advantage of these variations to estimate Pringles and STAX brand equities every quarter, using the structural methods in Goldfarb et al. (2009), and cast the resulting brand equities in a Pakes and McGuire (1994)-style quality ladder model to capture

the dynamics of brand management. In the quality ladder model, in each period, firms take stock of their existing brand equities and invest in advertising to sustain and enhance brand equity. Our brand value measure incorporates the immediate costs associated with these advertising decisions as well as the benefits they yield in the short and long run.

In 2001, at the beginning of our data set, we estimate the value of the Pringles brand to be $2.1 billion. This value slowly declines until STAX enters. It drops immediately by $137 million upon STAX's entry, even though Pringles's brand equity does not change, because increased competition limited P&G's ability to generate profits from the brand. Pringles's brand value stabilizes after STAX enters, and in the second quarter of 2006—the last period of our data—it is worth $1.6 billion. Meanwhile, STAX's brand value, which is always below Pringles's, declines steadily, and at the end of our data it is worth $692 million. (We show that a static measure that fails to account for advertising and accordingly brand equity dynamics, e.g., that of Goldfarb et al. 2009, yields brand values that are *much* higher and fluctuate *much* more over time.)

Our final set of results examines how changes in the ability to build and sustain brand equity affect brand value. To vary the ability to build and sustain brand equity, we change the brand equity depreciation rate and the effectiveness of advertising. At our estimated parameterization, as might be expected, if brand equity were to depreciate more slowly, or if advertising were more effective at building brand equity, then brand value would increase.

However, there is a counterintuitive interaction effect between the two: When the effectiveness of advertising is sufficiently high, there is an inverted-U-shaped relationship between the brand equity depreciation rate and brand value. It follows that increasing the brand equity depreciation rate can increase brand value, even as it (expectedly) reduces the value of the firm overall. So, a firm's brand becomes *more* valuable even as the brand equity that underpins it becomes harder to sustain. When the brand equity depreciation rate is low, a brand's value is relatively low because the brand equity that it possesses would be rebuilt relatively easily if it were lost. When the depreciation rate is high, a brand's value is also relatively low because the brand equity that it possesses cannot be sustained. A brand is most valuable at intermediate depreciation rates because the brand equity that it possesses can be sustained—and therefore generates high profit for the firm—and were it lost, it would be very time consuming to rebuild. In summary, when advertising is sufficiently effective, a brand is most valuable when the brand equity depreciation rate is neither too low nor too high.

2. Issues in Measuring Brand Value

In motivating our approach to measuring brand value, we emphasize the underlying goal of measuring the *financial* value of a brand to the parent firm. The emphasis on brand means that we need to separate out the contribution of the brand itself from what the product (including the brand) actually achieves in the marketplace (Keller 2008, p. 48; Fischer 2007). This calls for a comparison between a "factual" and a "counterfactual." Hence, we define brand value as the difference between the expected net present value of cash flows in a factual scenario, in which a product possesses its brand equity, and a hypothetical counterfactual scenario, in which the product is stripped of its brand equity. In both scenarios, firms retain the ability to brand build, hence in the factual scenario a firm can strive to sustain or enhance its brand equity, and in the counterfactual scenario a firm can try to rebuild its brand equity anew. Furthermore, because factual and counterfactual pricing and ad spending decisions are determined by an equilibrium, we model those decisions for each firm in every period, taking into account the prevailing brand equity levels of all firms in the industry.

Earlier approaches to measuring brand value have captured some parts of this conceptual framework, but not all. For instance, Barwise et al. (1989), Simon and Sullivan (1993), and Fischer (2007) acknowledge the need to measure the discounted present value of cash flows generated by the brand, but do not perform a full analysis of the counterfactual scenario that accounts for the impact of the brand on both consumer *and* firm behavior. Others incorporate a counterfactual scenario in which a firm is stripped of its brand and in which all firms change their behavior accordingly (Goldfarb et al. 2009, Ferjani et al. 2009), but do so in a static setting that does not account for brand building.

Fischer (2007) lists six features of an ideal measure of brand value: completeness, comparability, objectivity, future orientation, cost-effectiveness, and simplicity. While our method is not simple and it is hard to assess cost effectiveness at this point, we believe that it has important strengths in terms of the other four features.

First, a measure of brand value is regarded as *complete* only if it accounts for both the price premium and the volume premium that a firm enjoys because of its brand.[1] This encapsulates the broader point made above, i.e., that a measure must properly account for the benefits that a product (including the brand) enjoys relative to a hypothetical unbranded version of the product. Our measure captures the price and volume premiums. Furthermore, our approach suggests that the interpretation of completeness described above is itself incomplete because it fails to account for the effect of a firm's brand on brand building decisions and accordingly the evolution of the industry over time.

Second, *comparability* refers to the ability of an approach to yield brand value measures that can be compared across industries and over time. To satisfy this criterion, an approach must not give rise to unwarranted intertemporal fluctuations in a brand's value. Our approach gives rise to brand values that are relatively stable over time. Furthermore, our results show that failing to account for the effect of the brand on firms' forward-looking brand building decisions can indeed give rise to wide fluctuations in a brand value measure that are not reflective of the true value of the brand.

Third, our approach is *objective* in that we use standard data on prices, sales, and ad spending, and we provide a general framework for measuring brand value that can be adapted to suit a wide variety of industries and product categories. Finally, by definition, our approach satisfies the future-oriented criterion.

In addition to strengths that build on and improve the state of the art in the prior academic literature, we also see considerable advantages to our approach relative to using the brand values calculated by brand consulting firms. In particular, each consultancy uses its own approach, and none fully reveals its methodology.[2] As with other approaches in the academic literature, our approach has the advantage of being transparent. These brand consultancy approaches also are not complete in the sense that they do not formally consider a counterfactual scenario that explores how the absence of a brand would impact the decision making of consumers *and* firms. The brand consultancy approaches may also be less objective to the extent that the results rely on less objective data sources (surveys, focus groups, or expert panels). Therefore, relative to the brand consulting approaches, we offer a methodology that is transparent and that can therefore be more easily appraised, applied, and augmented.

A further advantage of our method is that it can be seen as valuing the brand asset with a real options approach rather than a net present value approach (Dixit and Pindyck 1994), accounting for the irreversibility of investment decisions (in brand building) and the uncertainty of the economic environment. In finance, it is now widely recognized that assets should be valued using a real options approach because failing to account for all of the possible future paths along which the economic environment might evolve can lead to incorrect valuations (see Dixit and Pindyck 1994, p. 6). In this way, we measure brands as assets in a way consistent with contemporary thinking on how assets should be valued.

3. Category Description and Data

In this section, we describe the stacked chips category and the data that we use to estimate our model.

3.1. Category

The stacked chips category originated in the late 1960s when P&G introduced Pringles. P&G's plan was to distribute Pringles over its existing distribution network, which was optimized for nonperishable products. To ensure that the chips did not spoil in transit, they were to be packed in nitrogen, which necessitated an airtight seal. This led to the now-familiar cylindrical containers and the uniformly shaped chips that could be stacked in them.[3] By the mid-1990s, Pringles had $1 billion in annual sales. In 2012 (after our data period), Pringles was sold to Kellogg's for about $2.7 billion (de la Merced 2012).

Frito-Lay, a division of PepsiCo, launched Lay's STAX in 2003. Like Pringles, STAX chips are stacked and packaged in cylindrical containers. STAX was launched with substantial marketing support, spending more on advertising than Pringles in the first quarter after entry. It immediately gained about 20% market share of the stacked chips category.

We treat stacked chips as a distinct category. We allow demand for other salty snack brands in chips, pretzels, popcorn, and cheese snacks to affect demand for stacked chips (as the outside good in a nested model) but focus on competition and strategic interaction between Pringles and STAX. We do this partly for convenience (the duopoly setting is needed for estimation), but we believe it is reasonable given the close substitutability of the two stacked chips brands.

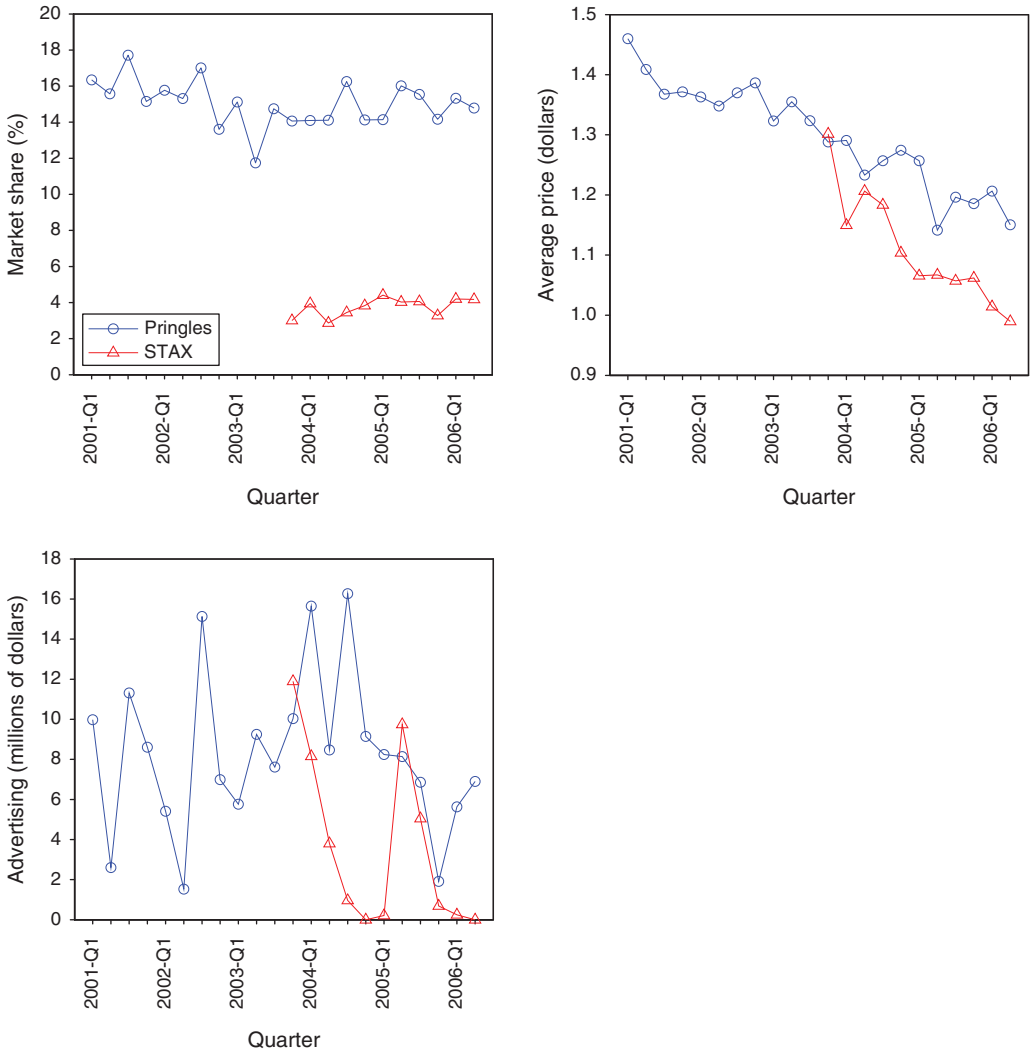
3.2. Data

Our data come from the IRI Marketing Data Set (Bronnenberg et al. 2008), which provides weekly data at the product level for 2,664 participating stores in 47 U.S. markets, between January 1, 2001, and December 31, 2006. Our quarterly estimate of brand equity is based on a static pricing game at the week level and is estimated using data at the store-week-brand level. Our estimate of brand value uses the quarter-brand as the

Table 1. Descriptive Statistics

	Mean	SD	Min.	Max.
Pringles				
Advertising ($ millions per quarter)	8.76	4.16	1.56	17.62
Average price per quarter ($)	1.28	0.11	1.12	1.52
Market share (of chips, pretzels, popcorn, and cheese snacks)	0.15	0.01	0.12	0.18
Sales ($ millions per quarter)	4.74	0.43	3.92	5.42
STAX				
Advertising ($ millions per quarter)	4.01	4.73	0.00	12.56
Average price per quarter ($)	1.08	0.09	0.98	1.25
Market share (of chips, pretzels, popcorn, and cheese snacks)	0.04	0.01	0.03	0.04
Sales ($ millions per quarter)	1.12	0.12	0.93	1.28

Figure 1. (Color online) Market Shares (in Chips, Pretzels, Popcorn, and Cheese Snacks Categories), Average Prices, and Advertising Spending



unit of observation and emphasizes quarterly advertising spending data and the quarterly estimates of brand equity derived from the static game.

This data set is well suited for this project for several reasons. First, it is very detailed: for each store and each week, the volume and average purchase price are reported. These variables are necessary for estimating brand equity (Goldfarb et al. 2009). Second, besides the entry of a new brand, we observe interesting dynamics in advertising, prices, and market shares throughout the data period, both in the monopoly phase and in the duopoly phase. The variations in market share and advertising allow us to identify the dynamic parameters in our model. Third, the IRI Marketing Data Set contains quarterly advertising spending data (primarily estimated through a media audit) for each brand in the category (provided by TNS Custom Research), spanning the period January 1, 2001–June 30, 2006.

3.3. Descriptive Statistics

Table 1 provides descriptive statistics for the 22 (respectively, 11) quarters in which Pringles (respectively, STAX) is active in our data set,[4] and Figure 1 presents plots of market shares, average price, and advertising spending. The bottom panel of Figure 1 shows substantial variability in quarterly ad spending for both brands. The top left panel of Figure 1 shows that market shares are more stable than advertising, although they also vary quite a bit over time. The top right panel of Figure 1 shows that STAX's prices are below Pringles's, with the gap increasing from almost zero in the fourth quarter of 2003 to close to 20% in the first quarter of 2006.

4. The Static Period Game

We conceptualize firms as making two types of decisions, short-run and long-run. The short-run decisions

are in prices, made weekly at each store. The long-run decisions are in advertising, and they are made quarterly at the national level.[5] We denote the week by t and the quarter by q. By "static period game" we mean the weekly competition in prices between the two firms; by "dynamic game" we mean the quarterly competition in advertising.

The static and dynamic character of these games comes from our assumption that firms behave as if brand equities are fixed in the short run, unaffected by prices, but changeable in the long run, through natural forces such as depreciation and choices such as advertising. This is as if the firm is investing quarterly and harvesting weekly.

This division between the short run and the long run, with brand equities fixed in the short run and changeable in the long run, is central to our empirical estimation strategy. We believe that it is also reasonable. We do expect brand equities to be relatively immoveable objects—if they were not, they would not be "assets." Yet at the same time, they are not fixed forever; depreciation takes its toll, and advertising can build and restore brand equity over the long run. Consistent with this framing, we model price choices as weekly and advertising choices as quarterly.

Methodologically, thinking of the period game as a static game allows us to use weekly store data on sales and prices to estimate demand-side and supply-side parameters, in particular, Pringles and STAX brand equities and firms' marginal costs. This allows us to estimate the period profit function. The estimated brand equities and the period profit function are then taken to the dynamic game described in Section 5; here, the brand equities at the beginning of a quarter serve as "start states," which depreciate during the quarter, and advertising decisions are made to achieve desirable "end states."[6]

4.1. Model

We estimate demand in a nested logit framework. As noted earlier, the structure of the stacked chips industry changed during 2001–2006, from a monopoly in the first half to a duopoly in the second half. Still, because we seek to model STAX's entry decision, we need to analyze the industry as if it were a duopoly from the beginning, comprising an incumbent firm and a potential entrant.

Our data cover 2,664 stores. We account for differences across stores by assuming that stores have different market sizes and firm-specific shocks. A firm's marginal cost varies across stores, reflecting differences in transportation costs and/or trade promotions. We incorporate this store-level heterogeneity by assuming that each store in each period draws its market size, firm-specific shocks, and firm-specific marginal costs from the same distributions. We explain below how we estimate these distributions.[7]

As explained above, we assume that firms in our model set weekly prices at the store level. While firms in reality do not set separate prices for each and every store, this assumption simplifies our model and allows us to account for the store-level heterogeneity described above. We feel that this a reasonable approach, especially given that our goal is to estimate the distributions of market sizes, firm-level shocks, and marginal costs that are common to all stores, as opposed to store-level distributions.[8]

There are two firms in our model, $n = 1, 2$, competing weekly in prices in each store, taking their brand equities as given. This is vertically differentiated price competition because one brand typically has more equity than the other. Let $\omega_n \in \{0, 1, \ldots, M\}$ represent the state of firm $n \in \{1, 2\}$ in a given quarter. States $1, \ldots, M$ describe the brand equity of a firm that is active in the product market, i.e., an incumbent firm, while state 0 identifies a firm as being inactive, i.e., a potential entrant. This formulation allows us to simultaneously capture situations in which both brands are active and situations in which only one brand is active. Thus, in the data, Pringles's state will always be one of $1, \ldots, M$, whereas STAX's state will be 0 in the weeks leading up to its entry. The industry state is $\boldsymbol{\omega} = (\omega_1, \omega_2) \in \{0, \ldots, M\}^2$.

There is a continuum of consumers in the market. Each consumer purchases at most one unit of one product in each week. The utility that consumer i shopping at store j derives from purchasing from firm n is

$$u_{ijn}(\omega_n) = B(\omega_n) - \kappa p_{jn} + \zeta_{jn} + \xi_{ij} + (1-\sigma)\varepsilon_{ijn}, \quad (1)$$

where B: $(0, 1, \ldots, M) \to \mathbb{R}$ is an increasing function that maps brand equity state ω_n into brand equity $B(\omega_n)$. We specify $B(\omega_n)$ in Section 5. (Even though we distinguish between brand equity states ω_n and brand equities $B(\omega_n)$, for ease of exposition we will sometimes refer to ω_n as "brand equity.") Furthermore, p_{jn} is firm n's price in store j, ζ_{jn} is a mean zero firm-store-specific shock (to accommodate unobserved heterogeneity across firms and stores) with standard deviation σ_ζ, and $\xi_{ij} + (1-\sigma)\varepsilon_{ijn}$ is an individual error term, where ξ_{ij} is the idiosyncratic propensity of consumer i to make a purchase in store j, and $\sigma \in [0, 1)$ determines the extent to which consumers' preferences for the firms' products are correlated. We set $B(0) = -\infty$ to ensure that potential entrants have zero demand, and hence do not compete in the product market. There is an outside alternative, product 0, which has utility

$$u_{ij0} = \xi'_{ij} + (1-\sigma)\varepsilon_{ij0}. \quad (2)$$

Assuming that the idiosyncratic preferences ε_{ij0}, ε_{ij1}, and ε_{ij2} have independent and identically distributed type-1 extreme value distributions, and that ξ_{ij} and ξ'_{ij} have distributions depending on σ such that ξ_{ij} +

$(1-\sigma)\varepsilon_{ijn}$ and $\xi'_{ij} + (1-\sigma)\varepsilon_{ij0}$ have extreme value distributions, the demand for incumbent firm n's product in store j is

$$D_{jn}(\mathbf{p}_j; \boldsymbol{\omega}, m_j, \boldsymbol{\zeta}_j) = m_j \frac{\exp((B(\omega_n) - \kappa p_{jn} + \zeta_{jn})/(1-\sigma))}{C_j + C_j^{\sigma}}, \quad (3)$$

and the demand for the outside good is

$$D_{j0}(\mathbf{p}_j; \boldsymbol{\omega}, m_j, \boldsymbol{\zeta}_j) = m_j \frac{C_j^{\sigma}}{C_j + C_j^{\sigma}}, \quad (4)$$

where $\mathbf{p}_j = (p_{j1}, p_{j2})$ is the vector of prices, κ is the price coefficient, $m_j > 0$ is the size of the market for store j (the sales of all chips, pretzels, popcorn, and cheese snacks), $\boldsymbol{\zeta}_j = (\zeta_{j1}, \zeta_{j2})$ is the vector of week-store-specific shocks to consumer utility for each brand, and $C_j = \sum_{n=1}^{2} \exp((B(\omega_n) - \kappa p_{jn} + \zeta_{jn})/(1-\sigma))$.[9] The market size m_j is assumed to have an independent normal distribution with mean μ_m and standard deviation σ_m. We assume that ζ_{jn} has a mean-zero normal distribution with standard deviation σ_ζ.

Given industry state $\boldsymbol{\omega}$, firm n chooses price $p_{jn} \geq 0$ to maximize its weekly profit from store j

$$\pi_{jn}(\boldsymbol{\omega}, c_{jn}, m_j, \boldsymbol{\zeta}_j) = \max_{p_{jn} \geq 0} D_{jn}((p_{jn}, p_{j,-n}); \boldsymbol{\omega}, m_j, \boldsymbol{\zeta}_j)(p_{jn} - c_{jn}), \quad (5)$$

where $p_{j,-n} \geq 0$ is the price charged by its rival, and $c_{jn} \geq 0$ is the marginal cost that firm n incurs when serving store j. We assume that the marginal cost is drawn from a log-normal distribution with mean μ_c and standard deviation σ_c and is independently and identically distributed across stores, firms, and weeks.

By Caplin and Nalebuff (1991), there exists a unique Nash equilibrium of the period game. We find it by solving the system of first-order conditions that arises from the firms' profit-maximization decisions via best reply iteration. Let $\pi_n^*(\boldsymbol{\omega}, \mathbf{c}_j, m_j, \boldsymbol{\zeta}_j)$ denote firm n's *quarterly* (not weekly) equilibrium profit in store j. Integrating over market size, marginal costs, and the firm-store specific shocks, and multiplying by the number of stores (2,664) and the number of weeks per quarter (13), we compute the expected equilibrium quarterly profit in industry state $\boldsymbol{\omega}$ as

$$\pi_n^*(\boldsymbol{\omega}) = 2{,}664 \times 13 \times \int_{\mathbf{c}_j, m_j, \boldsymbol{\zeta}_j} \pi_{jn}^*(\boldsymbol{\omega}, \mathbf{c}_j, m_j, \boldsymbol{\zeta}_j) \times f_c(\mathbf{c}_j) f_m(m_j) f_\zeta(\boldsymbol{\zeta}_j)\, d\mathbf{c}_j\, dm_j\, d\boldsymbol{\zeta}_j, \quad (6)$$

where $f_{\mathbf{c}}(\mathbf{c}_j)$, $f_m(m_j)$, and $f_\zeta(\boldsymbol{\zeta}_j)$ are the probability distribution functions of $\mathbf{c}_j$, m_j, and $\boldsymbol{\zeta}_j$, respectively.

4.2. Estimation

Demand Function. We follow Goldfarb et al. (2009) in estimating the parameters of the demand function. Specifically, we estimate a nested logit demand model with two nests, one for the two stacked chips brands, Pringles and STAX, and the other for the outside good, which includes nonstacked chips, pretzels, popcorn, and cheese snacks. This captures the idea that brands of stacked potato chips will compete more fiercely with each other than with other types of salty snacks.

The brand equity of each firm in each quarter is defined as the additional utility a consumer receives from consuming a branded product versus its unbranded equivalent. Operationally, this is represented as a brand-quarter fixed effect in the consumer's utility function.

Recall that the demand for good n in store j is

$$D_{jn}(\mathbf{p}_{jt}; \boldsymbol{\omega}_q, m_{jt}, \boldsymbol{\zeta}_{jt}) = m_{jt} \frac{\exp((B(\omega_{nq}) - \kappa p_{jnt} + \zeta_{jnt})/(1-\sigma))}{C_{jt} + C_{jt}^{\sigma}},$$

where ω_{nq} is firm n's brand equity state in the quarter q that includes week t, and the demand for the outside good is

$$D_{j0}(\mathbf{p}_{jt}; \boldsymbol{\omega}_q, m_{jt}, \boldsymbol{\zeta}_{jt}) = m_{jt} \frac{C_{jt}^{\sigma}}{C_{jt} + C_{jt}^{\sigma}}.$$

The market size for store j in week t, m_{jt}, is the total number of units of all chips, pretzels, popcorn, and cheese snacks sold in store j in that week. We do not observe the entire U.S. market in the IRI database because it includes a sample of U.S. drug and grocery stores in a subset of U.S. markets. So we approximate the size of the full U.S. market as follows. We first compute average weekly household spending on all chips, pretzels, popcorn, and cheese snacks in drug and grocery stores by multiplying average weekly household spending on all salty snacks in such stores (from Bronnenberg et al. 2008) by the percentage of total salty snack spending that is constituted by chips, pretzels, popcorn, and cheese snacks (computed from the IRI database). We multiply this by the total number of U.S. households to get total U.S. spending on all four of these salty snack categories in drug and grocery stores. Because market size is defined in units (not dollars), we divide this by the average price to get the total number of units of these salty snacks sold in drug and grocery stores. We then multiply this by the percentage of chips, pretzels, popcorn, and cheese snacks sales in the IRI database that are from grocery stores, rather than drug stores (97.95%), to estimate the size of the market in grocery stores. Finally, to obtain the size of the full market, which includes grocery stores, drug stores, mass stores, convenience stores, and club stores,

we scale up our estimate based on a report from P&G that implies that grocery stores contribute up 25% of Pringles's sales.[10] We divide by the number of stores in our data, yielding an estimate of $\hat{\mu}_m = 342{,}463.04$ units, implicitly scaling up the market size of each store in the data so that the stores collectively represent the entire U.S. market.[11]

Taking the difference between the log market shares of firm n in store j and the outside good in store j, we have

$$\log\left(\frac{D_{jn}(\mathbf{p}_{jt};\boldsymbol{\omega}_q,m_{jt},\boldsymbol{\zeta}_{jt})}{m_{jt}}\right)-\log\left(\frac{D_{j0}(\mathbf{p}_{jt};\boldsymbol{\omega}_q,m_{jt},\boldsymbol{\zeta}_{jt})}{m_{jt}}\right)$$
$$=B(\omega_{nq})-\kappa p_{jnt}+\sigma\log\left(\frac{D_{jn}(\mathbf{p}_{jt};\boldsymbol{\omega}_q,m_{jt},\boldsymbol{\zeta}_{jt})}{m_{jt}-D_{j0}(\mathbf{p}_{jt};\boldsymbol{\omega}_q,m_{jt},\boldsymbol{\zeta}_{jt})}\right)+\zeta_{jnt}.$$

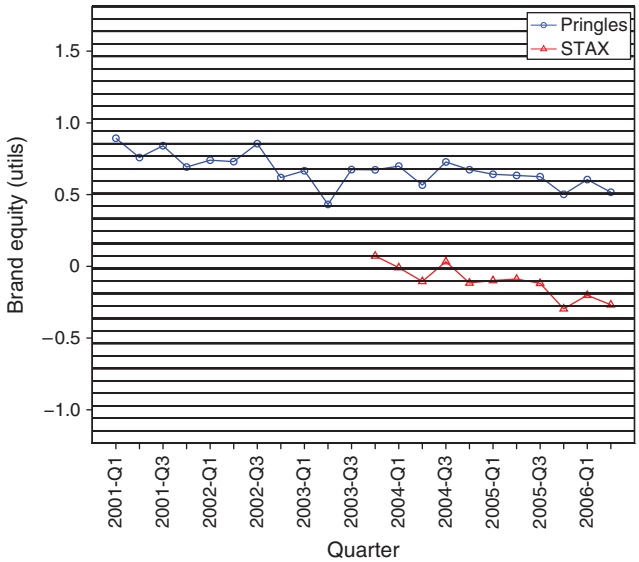
Ordinary least squares estimation of the above expression would be biased because of the endogeneity of price (it is possible that firms observe ζ_{jnt} before setting prices) and the inside share. Following Nevo (2001), we use the price of the brand in other cities, the price of the brand in other stores in the same store chain, and the price of the brand in other stores in the same chain and city as instruments. For these to be valid instruments, the stores in other cities and other stores in the chain must share a cost shock, but have distinct demand shocks. Our results are presented in Table 2 and Figure 2, the latter being a plot of the estimated brand equities of Pringles and STAX quarter by quarter, along with a discretization of the brand equity continuum that is described in Section 5.

Table 2. Demand Estimation Results

Quarter	Pringles brand equity	STAX brand equity	Quarter	Pringles brand equity	STAX brand equity
2001-Q1	0.893*** (0.007)	n/a	2004-Q1	0.698*** (0.006)	−0.009 (0.007)
2001-Q2	0.758*** (0.007)	n/a	2004-Q2	0.566*** (0.006)	−0.105*** (0.007)
2001-Q3	0.841*** (0.007)	n/a	2004-Q3	0.727*** (0.006)	0.033*** (0.007)
2001-Q4	0.692*** (0.007)	n/a	2004-Q4	0.673*** (0.006)	−0.116*** (0.007)
2002-Q1	0.740*** (0.007)	n/a	2005-Q1	0.642*** (0.006)	−0.099*** (0.007)
2002-Q2	0.730*** (0.007)	n/a	2005-Q2	0.633*** (0.006)	−0.089*** (0.007)
2002-Q3	0.855*** (0.007)	n/a	2005-Q3	0.625*** (0.006)	−0.118*** (0.007)
2002-Q4	0.618*** (0.007)	n/a	2005-Q4	0.502*** (0.006)	−0.296*** (0.007)
2003-Q1	0.666*** (0.007)	n/a	2006-Q1	0.605*** (0.006)	−0.202*** (0.007)
2003-Q2	0.432*** (0.007)	n/a	2006-Q2	0.517*** (0.006)	−0.268*** (0.006)
2003-Q3	0.674*** (0.007)	n/a	2006-Q3	0.537*** (0.006)	−0.246*** (0.007)
2003-Q4	0.673*** (0.006)	0.072*** (0.007)	2006-Q4	0.440*** (0.006)	−0.300*** (0.007)
κ (price coefficient)	−1.554*** (0.004)				
σ	0.616*** (0.003)				
σ_ζ	0.646*** (0.001)				
R^2	0.932				
Adjusted R^2	0.932				
Residual std. error	11.481	(df = 807,198)			
No. of observations	807,248				

Notes. Observations are at the store-week level. This is a nested logit demand model. The instruments are the price of the brand in other cities, the price of the brand in other stores in the same store chain, and the price of the brand in other stores in the same chain and city. Standard errors are in parentheses.
***$p < 0.01$.

Figure 2. (Color online) Brand Equity Over Time



Marginal Costs. Given the estimates of κ and σ, we can compute the marginal cost for each store-firm-week combination from the first-order condition describing the price equilibrium

$$c_{jnt} = p_{jnt} - \frac{1-\sigma}{\kappa(1 - s_{jnt}(1 + \sigma(s_{j0t}/(1 - s_{j0t}))))}, \qquad (7)$$

where s_{jnt} and s_{j0t} are the shares of firm n and the outside good, respectively, in store j in week t that are observed in the data. We can then estimate μ_c and σ_c using the first and second sample moments of c_{jnt}; this yields estimates of $\hat{\mu}_c = 0.856$ and $\hat{\sigma}_c = 0.180$.

Expected Profits. Because equilibrium prices must be computed numerically, there is no closed-form solution for $\pi_n^*(\boldsymbol{\omega})$. We therefore approximate the expected profit function (6) in each industry state through Monte Carlo simulation, integrating over the estimated distributions of $\mathbf{c}_j$ and $\boldsymbol{\zeta}_j$.

The plots of expected price, expected demand, expected market share, and expected profit are presented in Figure 3, where ω_1 and ω_2 are the brand equities of firms 1 and 2 (we use the language "firm 1 and firm 2" as opposed to "Pringles and STAX" because the firms are symmetric, and hence the results in the figure are for either Pringles or STAX). Market share, price, and, accordingly, profit increase relatively rapidly—and at an increasing rate—as brand equity increases, and they decline as rival brand equity increases.

5. A Dynamic Model of Brand Management

As we transition from short-run (weekly) to long-run (quarterly) decisions, we recognize that brand equity can depreciate, advertising can build and replenish brand equity, and because brand equity is defined relative to the outside good, it can fluctuate as the quality of the outside good changes. We also recognize that in the long run, new firms can enter.[12]

Table 3. How Advertising Spending Affects Changes in Brand Equity

	Model 1	Model 2	Model 3
Intercept	−0.0696*	−0.1088	−0.1230
	(0.0374)	(0.0499)	(0.0580)
Advertising (millions)	0.00827*	0.0111**	0.0115**
	(0.00465)	(0.00522)	(0.00536)
STAX	.	0.0589	0.0568
		(0.0500)	(0.0508)
STAX × *Advertising* (millions)	.	.	0.00141
			(0.00283)
No. of observations	32	32	32
R^2	0.095	0.137	0.144

Note. Standard errors are in parentheses.
$^*p < 0.10$; $^{**}p < 0.05$.

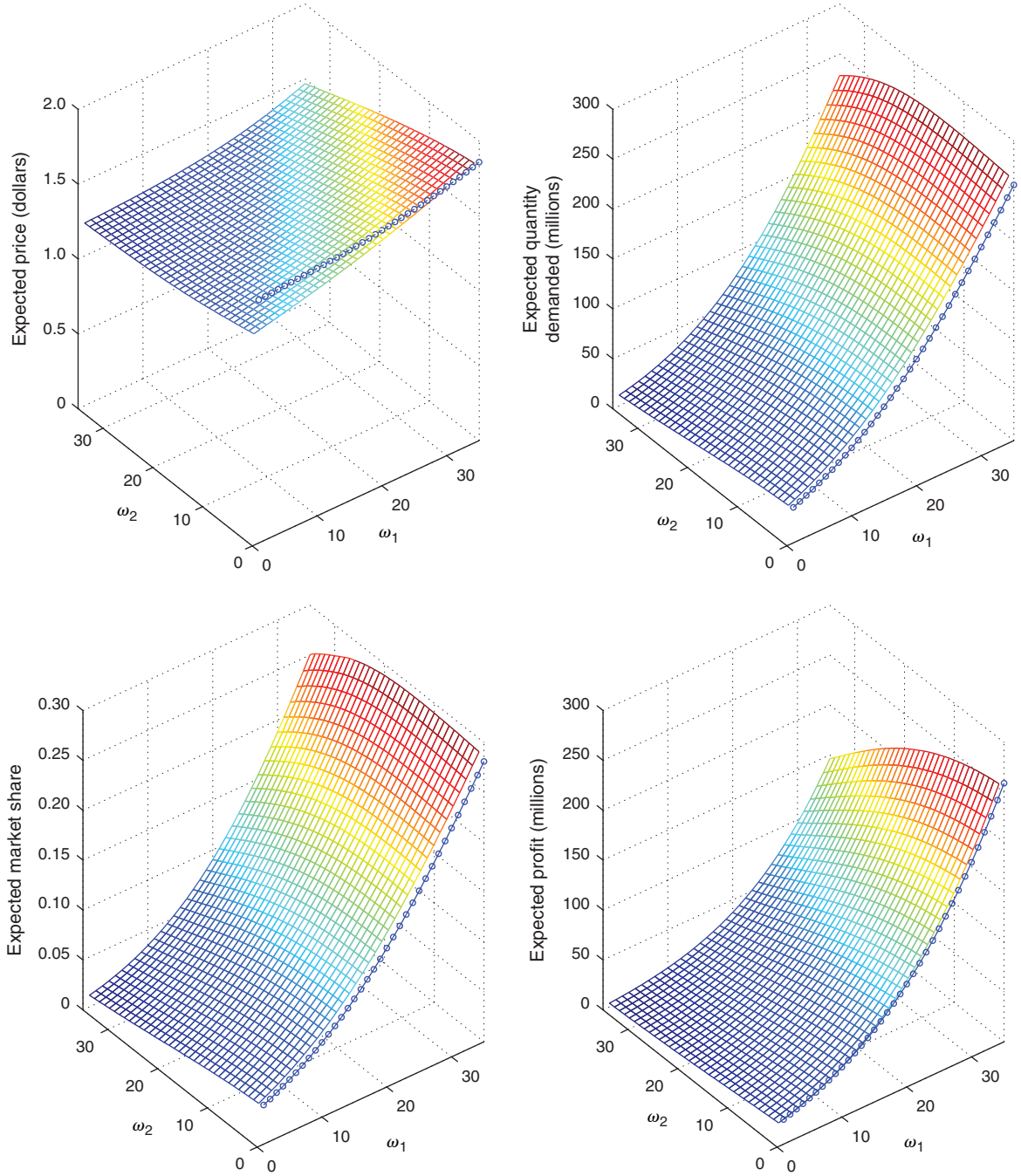
Table 3 provides suggestive evidence that advertising can have a key role in building and rebuilding brand equity. It shows a regression of quarterly brand equity changes (from quarter $q-1$ to q) on quarterly advertising expenditures. Advertising and brand equity changes are positively correlated. Our model interprets this correlation as causal.

Our dynamic model of brand management uses the quality ladder framework (Pakes and McGuire 1994, Borkovsky et al. 2012). Brand equity changes are viewed as ascensions or descensions on a brand equity "ladder." The term "quality ladder" comes from the original applications these authors had in mind. In these applications, research and development (R&D) is the investment in question, and the effect of successful R&D is to move product quality up a quality ladder. In our application, advertising replaces R&D, and brand equity replaces quality. The quality ladder framework calls for a discrete number of brand equity levels. However, the brand equities in Section 4 are estimated on a continuum. So we discretize the brand equity continuum in the following way:

$$B(\omega_n) = \begin{cases} -\infty & \text{if } \omega_n = 0, \\ w(\omega_n - 1) + l & \text{if } \omega_n > 0, \end{cases} \qquad (8)$$

where $w > 0$ is the width of an interval in the discretization, l is the lowest discrete brand equity state, and it is understood that each estimated brand equity is assigned to the discrete brand equity to which it is closest. We searched for the w and l values that would minimize w, the distance between states, while ensuring that a firm's discretized brand equity could increase and decrease by at most two units from period to period (in ways that are permitted by the model presented in Section 5.1). We allow for increases and decreases of two units to accommodate several large increases and decreases in brand equity that can be

Figure 3. Period Game Nash Equilibrium (Functions Presented Are for Firm 1; Expected Profit and Quantity Demanded Are Quarterly)



seen in Figure 2. Had we allowed brand equity to increase or decrease by at most one unit, this would have yielded a much coarser discretization. Our search yields $w = 0.08700$ and $l = -0.3196$ as the best discretization.

Using this discretization, we observe 15 discrete brand equity states in the data, $\omega_n \in \{1, \ldots, 15\}$. However, because the observed brand equities do not span the full continuum of brand equities that a firm might be able to achieve, we add additional brand equity states to the top and bottom of the state space. To determine how large the state space should be, we need to make an assumption about the lowest possible brand equity state, the state in which a firm has no brand equity. We assume that a stacked chips product with no brand equity would be only as successful in the stacked chips category as a private-label nonstacked chips product is in the nonstacked chips category (private labels are nearly irrelevant for stacked chips). Private-label nonstacked chips have a mean

share of 15.6% in the nonstacked chips category, so we add 10 states below the lowest estimated brand equity state to bring the mean inside share of a firm with no brand equity (computed as the arithmetic mean of the inside share across the rival firm's brand equity states) below 15.6%. For the sake of symmetry, we also add 10 additional states above the highest observed brand equity. Adding brand equity states to the top and bottom of the state space does not change the estimated width of an interval in our discretization ($w = 0.087$), but it does change the lowest discrete brand equity state to $l = -1.190$. In total, 35 discrete brand equity states are shown in Figure 2, where the horizontal lines represent boundaries between brand equity states, and, accordingly, the midpoints of the vertical intervals are the discrete brand equity states.

In our model, there are three forces that give rise to changes in brand equity over time. First, firms invest in advertising to increase brand equity or to stabilize it. Second, a firm's brand equity is subject to idiosyncratic depreciation, reflecting the notion that brand equity dissipates over time as the marketing activities used to build it become less salient to consumers. Finally, because brand equity is defined relative to the quality of the outside good, changes in the quality of the outside good bring about changes in the brand equity of *both* firms. This is captured by an industrywide shock that can either increase or decrease the brand equity of both firms.

5.1. Detailed Specification

Timing. We divide each quarter into two subperiods, subperiod 1 and subperiod 2. Subperiod 1 is reserved for advertising decisions and the brand equity changes resulting from advertising, firm-specific depreciation, and industrywide depreciation or appreciation, which reflect changes in the quality of the outside good. Subperiod 2 is reserved for the entry decisions of potential entrants and any changes to the industry state that result from such entry.

In subperiod 1:

1. Firms observe the prevailing industry state, $\boldsymbol{\omega}$. Each incumbent firm finds out, privately, the effectiveness of its advertising (as a random draw from a known distribution, as described below) and makes its advertising decision.

2. Advertising decisions are carried out and their uncertain outcomes are realized. A firm-specific depreciation shock is realized. An industrywide shock that causes brand equity to either increase, decrease, or remain unchanged is realized. The industry state transitions from $\boldsymbol{\omega}$ to $\boldsymbol{\omega}'$; all firms observe the new industry state.

3. Incumbent firms compete in the product market.[13]

In subperiod 2:

4. Each potential entrant draws a private, random entry (setup) cost and decides whether to enter.

5. Entry decisions are implemented, and the industry state transitions from $\boldsymbol{\omega}'$ to $\boldsymbol{\omega}''$; all firms observe the new industry state. If no entry occurs, $\boldsymbol{\omega}'' = \boldsymbol{\omega}'$.

Incumbent Firms. Suppose firm n is an incumbent firm, i.e., $\omega_n \neq 0$. Firm n's state at the end of subperiod 1 is determined by the stochastic outcome of its advertising decision, a firm-specific depreciation shock, and an industrywide shock

$$\omega_n' = \omega_n + \tau_n - \iota_n + \eta, \tag{9}$$

where $\tau_n \in \{0, 1\}$ is a random variable governed by incumbent firm n's advertising $x_n \geq 0$, $\iota_n \in \{0, 1\}$ is a firm-specific depreciation shock, and $\eta \in \{-1, 0, 1\}$ is an industrywide shock that can cause brand equity to either increase, decrease, or remain unchanged. Therefore, from quarter to quarter, a firm's brand equity can increase or decrease by up to two units.

When advertising is productive, $\tau_n = 1$, and brand equity increases by one. The advertising response function, reflecting the probability that advertising is successful, is $\alpha_n x_n/(1 + \alpha_n x_n)$, where $\alpha_n > 0$ is a measure of advertising effectiveness. In turn, $\alpha_n = e^{\gamma_n - k}$, where $k > 0$ and γ_n is a private independent draw that is made in each quarter from a gamma distribution $\Gamma(h, \theta(\omega_n))$ with shape parameter h and scale parameter $\theta(\omega_n)$. Because α_n is a strictly increasing function of γ_n, both α_n and γ_n can be regarded as measures of advertising effectiveness. For reasons explained in Section 8.2, our analysis of the relationship between advertising effectiveness and brand value focuses on γ_n. We refer to its mean ($h\theta(\omega_n)$) and its variance ($h\theta(\omega_n)^2$) as the "mean effectiveness of advertising" and the "variance of the effectiveness of advertising," respectively.

The standard assumption about the success probability in Pakes and McGuire (1994)–style quality ladder models is that α_n is a parameter. Our formulation is more flexible. First, it gives α_n a stochastic character. This allows us to rationalize the variance in advertising decisions seen in the data. Firms do not make the same advertising decisions every time they reach a particular industry state. With a gamma distribution describing advertising effectiveness, the model has the flexibility to reflect, and measure, this variability. Second, our formulation allows us to derive closed-form solutions for firms' expected advertising spending decisions (Equation (A14) in the online appendix); and the expected success probabilities stemming from those decisions (Equation (A15) in the online appendix). These closed forms allow us to incorporate random advertising shocks into the model without having to use Monte Carlo simulation when computing equilibria or estimating the model. Third, because γ_n is bounded below

by zero, the k term serves to ensure that the model retains the ability to admit "small" α_n values. Finally, because the advertising response functions are symmetric across firms (given ω_n), differences in the effectiveness of advertising across firms arise endogenously.

We denote the cumulative density function and probability density function of the gamma distribution by $G(\cdot \mid h, \theta(\omega_n))$ and $g(\cdot \mid h, \theta(\omega_n))$ and assume that

$$\theta(\omega_n) = \exp(a\omega_n^3 + b\omega_n^2 + c\omega_n + d) + 0.01, \qquad (10)$$

where $c < b^2/(3a)$ and $a < 0$. It follows that $\theta(\cdot)$ is a strictly decreasing function, and that a firm's expected advertising effectiveness is strictly decreasing in its brand equity.[14] This means that even though the firms are symmetric in their advertising response functions, they will end up with different advertising productivities (and different advertising decisions) because of different brand equities. In particular, the brand trailing in brand equity will benefit from being able to advertise more effectively (on average).

We have chosen the particular functional form in Equation (10) because it allows for a wide variety of equilibrium long-run industry structures—both symmetric and asymmetric. The canonical Pakes and McGuire (1994) model admits either symmetric long-run industry structures or *extremal* asymmetric long-run industry structures in which the laggard falls back to the lowest quality level or exits (if the model allows for exit); see Figure 4 in Borkovsky et al. (2012). We include the 0.01 term in Equation (10) to prevent number overrun errors—i.e., errors that arise when one divides by a number that is too close to zero—in our estimation algorithm. Including this term ensures that the γ_n draws are not too small. This does not affect our estimated parameterization because the minimum $\theta(\omega_n)$ value at the estimated parameterization (0.5488) is much greater than 0.01.

Our model of a firm's advertising effectiveness is motivated by the uncertainty that a firm faces over the quality of its advertising copy. The effectiveness of firm n's advertising that is realized in each quarter, α_n, reflects the quality of the advertising copy that has been produced for firm n's advertising campaign in that quarter. This quality is revealed through market testing and is observable to firm n but not to its rival. Hence, firm n decides how much to spend on advertising after having learned the quality of its own advertising, but without knowing the quality of its rival's advertising.

If $\iota_n = 1$, firm n's brand equity depreciates by one unit; this happens with probability $\delta_f(\omega_n) = \min(z(\omega_n - 1), 1)$, where $z \geq 0$. This function has been designed to capture the idea that the probability of firm-specific depreciation increases linearly in a firm's brand equity, starting from a low of zero at $\omega_n = 1$ (a firm at the bottom of the brand equity ladder does not have any equity to lose). Because firms are symmetric, differences in the firm-specific rate of depreciation across firms arise endogenously.

If $\eta \neq 0$, the industry is hit by a shock that either increases or decreases each firm's brand equity by one unit. We assume that $\eta = -1$ with probability δ_d (the industrywide brand equity depreciation rate) and $\eta = 1$ with probability δ_u (the industrywide brand equity appreciation rate), where $\delta_d, \delta_u \in [0, 1]$ and $\delta_d + \delta_u \leq 1$. We incorporate this industrywide shock because (a) it captures possible increases and decreases in the quality of the outside good and (b) our data indicate that, controlling for advertising spending, changes in brand equity are correlated across firms in each period ($\rho = 0.8501$, $p = 4.611 \times 10^{-4}$). To simplify exposition, we define the distribution of the industrywide shock as

$$\Delta(\eta) = \begin{cases} \delta_d & \text{if } \eta = -1, \\ 1 - \delta_d - \delta_u & \text{if } \eta = 0, \\ \delta_u & \text{if } \eta = 1. \end{cases}$$

The brand equity depreciation rates determine how long a given investment in brand equity can be expected to last. As with any physical asset, they are meant to reflect the depreciation of asset value due to age and obsolescence. Specifically, the industrywide brand equity depreciation rate is a stand-in for the gradual deterioration of the stacked chips category as a whole; in our data, we do observe non-stacked salty snacks taking share away from stacked chips. The firm-specific brand equity depreciation rate captures the more conventional notion of "goodwill depreciation"—the idea that all of the underpinnings of brand equity (awareness, familiarity, advertising-created associations) will fade from consumers' memories over time.

Entrants. Now suppose that firm n is a potential entrant, i.e., $\omega_n = 0$. In subperiod 2, it decides whether to enter the industry. We model entry as a transition from state $\omega_n' = 0$ to state $\omega_n'' \neq 0$. To guarantee the existence of a Markov-perfect equilibrium in pure strategies, we assume that entry costs are privately observed random variables (Doraszelski and Satterthwaite 2010). In particular, at the beginning of subperiod 2, each potential entrant draws a random entry cost ϕ_n from a log-normal distribution $F(\cdot)$ with location parameter μ^e and scale parameter σ^e. Entry costs are independently and identically distributed across firms and periods. If the entry cost is below a threshold $\tilde{\phi}_n$, then potential entrant n enters the industry; otherwise it persists as a potential entrant.[15] Upon entry, potential entrant n becomes incumbent firm n, and its state is the exogenously set initial brand equity ω^e. We do not incorporate exit because there are no instances of exit in our data.

Value and policy functions. Define $V_n(\boldsymbol{\omega}, \gamma_n)$ to be the expected net present value of incumbent firm n's future cash flows in subperiod 1 if the industry is in state $\boldsymbol{\omega} \in \{1, \ldots, M\} \times \{0, \ldots, M\}$ and it has drawn effectiveness of investment γ_n. Incumbent firm n's value function is $\boldsymbol{V}_n : \{1, \ldots, M\} \times \{0, \ldots, M\} \times (0, \infty) \to \mathbb{R}$, and its policy function $\boldsymbol{x}_n : \{1, \ldots, M\} \times \{0, \ldots, M\} \times (0, \infty) \to [0, \infty)$ specifies its advertising spending in industry state $\boldsymbol{\omega}$ given that it draws an effectiveness of advertising γ_n. Define $V_n(\boldsymbol{\omega}', \phi_n)$ to be the expected net present value of potential entrant n's future cash flows in subperiod 2 if the industry is in state $\boldsymbol{\omega}' \in \{0\} \times \{0, \ldots, M\}$ and it has drawn entry cost ϕ_n. Potential entrant n's value function is $\boldsymbol{V}_n : \{0\} \times \{0, \ldots, M\} \times (0, \infty) \to \mathbb{R}$. By integrating over its entry cost ϕ_n, we define its policy function $\boldsymbol{\xi}_n : \{0\} \times \{0, \ldots, M\} \to [0, 1]$, which specifies the probability that it enters the industry in state $\boldsymbol{\omega}'$.[16]

An abridged version of the remainder of the model is presented below. A detailed version is presented is in the online appendix.

Bellman equation and optimality conditions. We first present the problem that incumbent firm n faces in subperiod 1. Incumbent firm n's value function $\boldsymbol{V}_n : \{1, \ldots, M\} \times \{0, \ldots, M\} \times (0, \infty) \to \mathbb{R}$ is implicitly defined by the Bellman equation

$$V_n(\boldsymbol{\omega}, \gamma_n) = \max_{x_n \geqslant 0} -x_n + E\left[\pi_n(\boldsymbol{\omega}') | \boldsymbol{\omega}, x_n, x_{-n}(\boldsymbol{\omega}, \gamma_{-n}), \gamma_n\right] + \beta E\left[W_n(\boldsymbol{\omega}') | \boldsymbol{\omega}, x_n, x_{-n}(\boldsymbol{\omega}, \gamma_{-n}), \gamma_n\right], \quad (11)$$

where $\beta \in (0, 1)$ is the discount factor. The second and third terms on the right-hand side of Equation (11) are firm n's expected profit and its discounted continuation value. Solving the optimization problem on the right-hand side of equation (11), we derive the optimality condition for incumbent firm n's ad spending $x_n(\boldsymbol{\omega}, \gamma_n)$ in industry state $\boldsymbol{\omega}$ conditional on drawing an effectiveness of advertising γ_n; see equation (A6) in the online appendix.

Suppose next that firm n is a potential entrant, i.e., $\omega_n = 0$. Its value function $\boldsymbol{V}_n : \{0\} \times \{0, \ldots, M\} \times (0, \infty) \to \mathbb{R}$ is implicitly defined by the Bellman equation

$$V_n(\boldsymbol{\omega}', \phi_n) = \max\{-\phi_n + U_n(\omega^e | \boldsymbol{\omega}'), U_n(0 | \boldsymbol{\omega}')\}, \quad (12)$$

where

$$U_n(\overline{\omega}_n | \boldsymbol{\omega}') = \beta[1(\omega'_{-n} = 0)[\xi_{-n}(\boldsymbol{\omega}') V_n(\overline{\omega}_n, \omega^e) + (1 - \xi_{-n}(\boldsymbol{\omega}')) V_n(\overline{\omega}_n, 0)] + 1(\omega'_{-n} > 0) V_n(\overline{\omega}_n, \omega'_{-n})]$$

is the expected net present value of all future cash flows to potential entrant n when it is in industry state $\boldsymbol{\omega}' \in \{0\} \times \{0, \ldots, M\}$ at the beginning of subperiod 2 and it transitions to brand equity state $\overline{\omega}_n \in \{0, \omega^e\}$ during subperiod 2. Solving the optimization problem on the right-hand side of equation (12) and integrating over ϕ_n, we derive the optimality condition for potential entrant n's probability of entry $\xi_n(\boldsymbol{\omega}')$ in industry state $\boldsymbol{\omega}'$; see equation (A10) in the online appendix.

Equilibrium. We restrict attention to symmetric Markov perfect equilibria in pure strategies. Existence is guaranteed by an extension of the proof in Doraszelski and Satterthwaite (2010); see the online appendix for details. Because we solve for a symmetric equilibrium, it suffices to determine the value and policy functions of one firm, which we refer to as firm n. Solving for an equilibrium for a particular parameterization of the model amounts to finding a value function $\mathbf{V}_n(\cdot)$ and policy functions $\boldsymbol{\xi}_n(\cdot)$ and $\mathbf{x}_n(\cdot)$ that satisfy the Bellman equations and the optimality conditions for firm n.

Because of the dependence on the random advertising effectiveness γ_n (see Equation (11)), it would be both challenging and computationally burdensome to solve this system. We therefore integrate out γ_n and solve for an incumbent firm's *expected* value $V_n(\boldsymbol{\omega})$ and its *expected* advertising spending $x_n(\boldsymbol{\omega})$, instead of its value $V_n(\boldsymbol{\omega}, \gamma_n)$ and its advertising spending $x_n(\boldsymbol{\omega}, \gamma_n)$. When we integrate out γ_n, the terms representing the probability of successful advertising in the Bellman equation are replaced by the *expected* probability of successful advertising, which we denote by $\rho_n(\boldsymbol{\omega})$.

Because of the functional form of our advertising response function and the distributional assumption we make for γ_n, we are able to derive analytic closed-form expressions for expected advertising spending $x_n(\boldsymbol{\omega})$ and the expected probability of successful advertising $\rho_n(\boldsymbol{\omega})$. These closed forms completely eliminate the need for numerical integration, which would otherwise greatly increase the computational burden of equilibrium computation and model estimation.

5.2. Estimating the Dynamic Model

We use maximum likelihood estimation to estimate the dynamic model, choosing parameters to maximize the likelihood of observed advertising spending, entry, and state-to-state transitions in the data. Some parameters are not estimable, and these we fix at values that seem reasonable given the empirical setting. One such parameter is the entry cost. While we account for the entry decision in the likelihood function, we are unable to estimate the parameters of the distribution from which those entry costs are drawn because only one instance of entry is observed in the 12 quarters in which Pringles was a monopolist. We set these parameters to values that yield equilibrium entry probabilities in the region of 8% on a quarterly basis: $\mu^e = 9$ and $\sigma^e = 2$. We are also unable to identify the discount factor β, so we set $\beta = 0.99$, which corresponds to an annual interest

rate of 4.1%. We fix the parameter k in the advertising response function to $k = 10$; again, this serves to ensure that the model admits "small" values of the advertising effectiveness shock $\alpha_n = e^{\gamma_n - k}$. Finally, we set an entrant's initial brand equity state to $\omega^e = 16$, which corresponds to STAX's estimated brand equity during its first active quarter, the fourth quarter of 2003.

The key methodological innovation in our dynamic framework is that it allows for variations in advertising spending (in a given industry state $\boldsymbol{\omega}$, across time) in a way that makes both equilibrium computation and model estimation tractable. Previous papers employing the Pakes and McGuire (1994) quality ladder model assumed that a firm's investment policy function maps industry states into unique investment levels.[17] By contrast, we assume that in a given industry state, different levels of advertising spending may arise over time because there are private random shocks to the effectiveness of advertising. This allows us to rationalize the variation observed in our data—in particular, there are several industry states that are visited multiple times, and the firms make different advertising spending decisions at different times. As a result, unlike earlier empirical papers that employed the Pakes and McGuire (1994) quality ladder model (Gowrisankaran and Town 1997, Goettler and Gordon 2011), we are able to apply maximum likelihood estimation because the probability density function of advertising spending in each industry state is not degenerate. Maximum likelihood estimation is useful because it is statistically more efficient than a simulated minimum-distance estimator such as the one used in Hall and Rust (2003) and Goettler and Gordon (2011).

Incorporating private random shocks to advertising effectiveness does, however, present a major challenge. Integrating over the random shocks yields two expectations: expected advertising spending and expected advertising success probability. Our model is still tractable, however, because our modeling assumptions allow us to derive closed-form expressions for these expectations, as shown in the online appendix. An alternative approach would be to compute these expectations using numerical methods such as Monte Carlo or recursive quadrature. However, this would significantly increase computational burden because one must compute these expectations many times to compute an equilibrium.[18]

Our approach to estimating the dynamic model is described in detail in the online appendix. We provide the derivation of the values of the ad effectiveness shock γ_n that rationalize the observed advertising spending decisions. We then construct the likelihood function. Finally, we explain how we estimate the model using mathematical programming with equilibrium constraints (Su and Judd 2012).[19]

5.3. Estimated Parameters

Our estimation yields the parameter estimates in Table 4 and an equilibrium of the model that is presented in Figure 6. We calculate standard errors using a parametric bootstrap with 100 bootstrap samples.[20] The estimated parameters are sensible, though it is easier to understand their meaning in the context of the estimated equilibrium, which is discussed in Section 7.2. Both industrywide and firm-specific brand equity depreciation parameters are statistically significant. The industrywide appreciation rate is not statistically significant, but it is large enough (and the standard error is small enough) that we believe it is useful to keep appreciation in the model. We find that the scale function (10) is highly significant in the brand equity states that are spanned by the data $(11, \ldots, 25)$, which suggests that advertising is effective. Because we lack data on other states, we need to rely on our functional form assumption. The parameters of the scale function are not individually statistically significant due to correlation between the estimated values of the parameters of the polynomial function in Equation (10). Testing the scale values themselves serves to demonstrate that the combination of these parameters, which yields the scale values, is statistically significant.

6. Brand Value Estimates

As noted earlier, brand value is the expected net present value of all future cash flows that can be attributed to the brand. In other words, from the perspective of firm n in a given industry state $\boldsymbol{\omega} = (\omega_n, \omega_{-n})$, it is the difference between two firm values, $V_n(\boldsymbol{\omega})$ and $V_n(1, \omega_{-n})$, the former in the actual industry state $\boldsymbol{\omega}$ and the latter in the counterfactual industry state $(1, \omega_{-n})$, where firm n has been stripped of its brand equity. In other words[21]

$$v_n(\boldsymbol{\omega}) \equiv V_n(\boldsymbol{\omega}) - V_n(1, \omega_{-n}). \tag{13}$$

Because an equilibrium of the dynamic model includes a value function $V_n(\cdot)$, mapping industry states to firm values, the factual and counterfactual firm values needed to compute brand value are readily available once an equilibrium has been computed. Note as well that brand building and rebuilding decisions are already folded into this mapping. For instance, $V_n(1, \omega_{-n})$ already reflects the fact that firm n, being at a brand equity disadvantage in the counterfactual scenario, might try to rebuild its brand equity, and that its competitor might strive to maintain its brand equity advantage. The resulting advertising decisions affect subsequent brand equities, which in turn affect prices, advertising decisions, and market structures that arise in the short and long run.

Table 4. Estimated Parameterization of the Dynamic Game

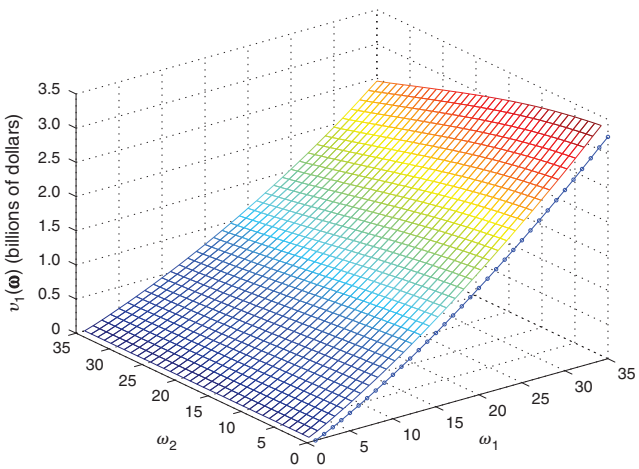
Parameter	Estimated value	Standard error
Industrywide depreciation rate, δ_d	0.4388^{***}	0.1267
Industrywide appreciation rate, δ_u	0.2571	0.1548
Firm-specific depreciation (slope) parameter, z	0.02941^{***}	4.415×10^{-3}
Shape parameter, h	9.534^{***}	1.446
Scale function values, $\theta(\omega_n) = \exp(a\omega_n^3 + b\omega_n^2 + c\omega_n + d) + 0.01$		
$\theta(1)$	1.386	233.9
$\theta(2)$	1.375	88.05
$\theta(3)$	1.364	37.20
$\theta(4)$	1.353	17.34
$\theta(5)$	1.340	8.788
$\theta(6)$	1.326	4.780
$\theta(7)$	1.312	2.761
$\theta(8)$	1.296	1.676
$\theta(9)$	1.279	1.062
$\theta(10)$	1.262^{*}	0.6977
$\theta(11)$	1.243^{***}	0.4756
$\theta(12)$	1.223^{***}	0.3383
$\theta(13)$	1.202^{***}	0.2543
$\theta(14)$	1.180^{***}	0.2049
$\theta(15)$	1.157^{***}	0.1777
$\theta(16)$	1.132^{***}	0.1641
$\theta(17)$	1.107^{***}	0.1581
$\theta(18)$	1.081^{***}	0.1564
$\theta(19)$	1.054^{***}	0.1569
$\theta(20)$	1.026^{***}	0.1587
$\theta(21)$	0.9967^{***}	0.1613
$\theta(22)$	0.9670^{***}	0.1643
$\theta(23)$	0.9365^{***}	0.1676
$\theta(24)$	0.9055^{***}	0.1715
$\theta(25)$	0.8739^{***}	0.1765
$\theta(26)$	0.8418^{***}	0.1836
$\theta(27)$	0.8094^{***}	0.1938
$\theta(28)$	0.7767^{***}	0.2072
$\theta(29)$	0.7439^{***}	0.2236
$\theta(30)$	0.7109^{***}	0.2416
$\theta(31)$	0.6780^{***}	0.2599
$\theta(32)$	0.6452^{**}	0.2771
$\theta(33)$	0.6127^{**}	0.2922
$\theta(34)$	0.5805^{*}	0.3050
$\theta(35)$	0.5488^{*}	0.3154
Parameters of the scale function, $\theta(\cdot)$		
a	-9.051×10^{-6}	2.329×10^{-4}
b	-2.666×10^{-4}	0.01465
c	-6.557×10^{-3}	0.3072
d	0.3257	2.1376

$^{*}p < 0.10$; $^{**}p < 0.05$; $^{***}p < 0.01$.

In Figure 4 we show how a firm's brand value varies as a function of its own brand equity and that of its rival. As can be seen, a brand's value increases relatively rapidly in its own brand equity and decreases relatively slowly in its rival's brand equity. These properties arise directly from the period profit function presented in Figure 3, which shows similar properties. Our estimates suggest that the maximum discounted cash flow potential of a stacked chips brand is \$3.36 billion, which arises when it has the highest possible brand equity and its rival possesses the lowest possible brand equity (industry state (35,1)). We believe the estimated values are reasonable. As a benchmark, Pringles (worldwide, including two production facilities) was sold for \$2.7 billion in 2012, which is comparable to our estimate of the value of the Pringles brand in the United States of \$1.6 billion in 2006.

Figure 5 juxtaposes the (discretized) brand equities for Pringles and STAX against the brand values those equities generate. Brand equity standard errors are

Figure 4. Brand Values (in Billions of Dollars)



small and are shown in Table 2. The figure includes 95% confidence bands, which require some explanation. While the standard errors for the brand value measures are large—for example, \$463.8 million on the \$2.08 billion estimate of Pringles's brand value in the first quarter of 2001—the measures of the changes in brand value are not. Because this figure highlights how changes in brand equity and brand value differ, we display confidence intervals that, fixing the initial level of brand value, represent the uncertainty on the change in brand value over time. When interpreting the confidence interval in other parts of this paper, our estimates can be seen as representing a very wide brand value range that shifts the narrower range shown in Figure 5.[22]

Comparing brand equity and brand value changes shows, first, that a given change in brand equity tends to result in a less than proportional change in the corresponding brand value (we explain why in Section 7.3). Second, because brand value is a function of both own brand equity and rival brand equity, a firm's brand value can change even if its brand equity does not. For example, while Pringles has the same brand equity in the third and fourth quarters of 2003, its brand value is much higher in the former (\$1.77 billion) than in the latter (\$1.63 billion). The reason is clear: in the earlier quarter Pringles was a monopolist, but in the latter quarter it faced competition from STAX. Brand value, being a cash flow–based measure, reflects the changing productivity of brand equity under different market conditions, while brand equity, being a consumer-based measure, does not. The strength of competition is a key driver of the relationship between brand equity and brand value.

Figure 5. (Color online) Brand Equities (Discretized) and Brand Values

Notes. The bars surrounding each brand value represent the 95% confidence interval on the difference between that brand value and the initial brand value (from 2001-Q1 for Pringles and 2003-Q4 for STAX).

7. Understanding the Brand Value Estimates

In this section, we will first present a useful way of thinking about brand value by identifying the two underlying factors that determine brand value. We then dig further into the results of our estimation by discussing the estimated advertising policy function and the evolution of brand equity over time, each of which helps us better understand the underlying factors. These results help frame our discussion of the benefits of using a dynamic measure of brand value relative to its static counterpart.

7.1. A Decomposition

Our model allows us to decompose brand value into two constituent parts:

1. the difference between factual and counterfactual flow profits over time (i.e., the profit premium);
2. the difference between factual and counterfactual advertising expenditures over time.

As explained above, brand value in a given industry state is the difference between firm value in the factual scenario, in which a firm possesses its brand equity, and firm value in the counterfactual scenario, in which the firm has been stripped of its brand equity. Because firm value in a given industry state $\boldsymbol{\omega}$ is the expected net present value of all future cash flows—where a firm's cash flow in a given period is its period profit minus its ad spending—we can restate firm n's brand value in industry state $\boldsymbol{\omega}$ as

$$\begin{aligned} v_n(\boldsymbol{\omega}) &= V_n(\boldsymbol{\omega}) - V_n(1, \omega_{-n}) \\ &= \sum_{q=0}^{\infty} \beta^q E[\pi_n(\boldsymbol{\omega}^{q\prime}) - x_n(\boldsymbol{\omega}^q) \mid \boldsymbol{\omega}^0 = \boldsymbol{\omega}] \\ &\quad - \sum_{q=0}^{\infty} \beta^q E[\pi_n(\boldsymbol{\omega}^{q\prime}) - x_n(\boldsymbol{\omega}^q) \mid \boldsymbol{\omega}^0 = (1, \omega_{-n})] \\ &= \sum_{q=0}^{\infty} \beta^q \big\{ \big(E[\pi_n(\boldsymbol{\omega}^{q\prime}) \mid \boldsymbol{\omega}^0 = \boldsymbol{\omega}] \\ &\qquad - E[\pi_n(\boldsymbol{\omega}^{q\prime}) \mid \boldsymbol{\omega}^0 = (1, \omega_{-n})]\big) \\ &\qquad - \big(E[x_n(\boldsymbol{\omega}^q) \mid \boldsymbol{\omega}^0 = \boldsymbol{\omega}] \\ &\qquad\quad - E[x_n(\boldsymbol{\omega}^q) \mid \boldsymbol{\omega}^0 = (1, \omega_{-n})]\big)\big\}, \end{aligned} \tag{14}$$

where the expectation is taken over industry state $\boldsymbol{\omega}^q$ that arises at the beginning of quarter q and industry state $\boldsymbol{\omega}^{q\prime}$ that arises in quarter q at the end of subperiod 1, given the initial industry state $\boldsymbol{\omega}^0$.

As can be seen from this equation, brand value is the discounted present value of the difference between factual and counterfactual expected flow profits *minus* the difference between factual and counterfactual advertising expenditures. This decomposition shows that our measure of brand value is even more comprehensive than the profit-premium measures that are commonly viewed as state of the art (see, e.g., Fischer 2007, Goldfarb et al. 2009). What this underscores is that a proper accounting of brand value must necessarily recognize that a brand has an impact not only on a firm's flow profits but *also* on its ad spending decisions. Moreover, it must account for how the different ad spending decisions made with and without the brand cause brand equity to evolve over time.

7.2. Brand Building Incentives

The decomposition presented above shows that a key determinant of brand value is the difference between factual and counterfactual advertising expenditures. To understand this difference, we must understand how brand equity affects a firm's ad spending decisions. To do that, let us examine Figure 6. Here, the top panels show the estimated policy functions for ad spending and the probability of entry, and the bottom panels show the advertising success probability and expected advertising effectiveness.[23]

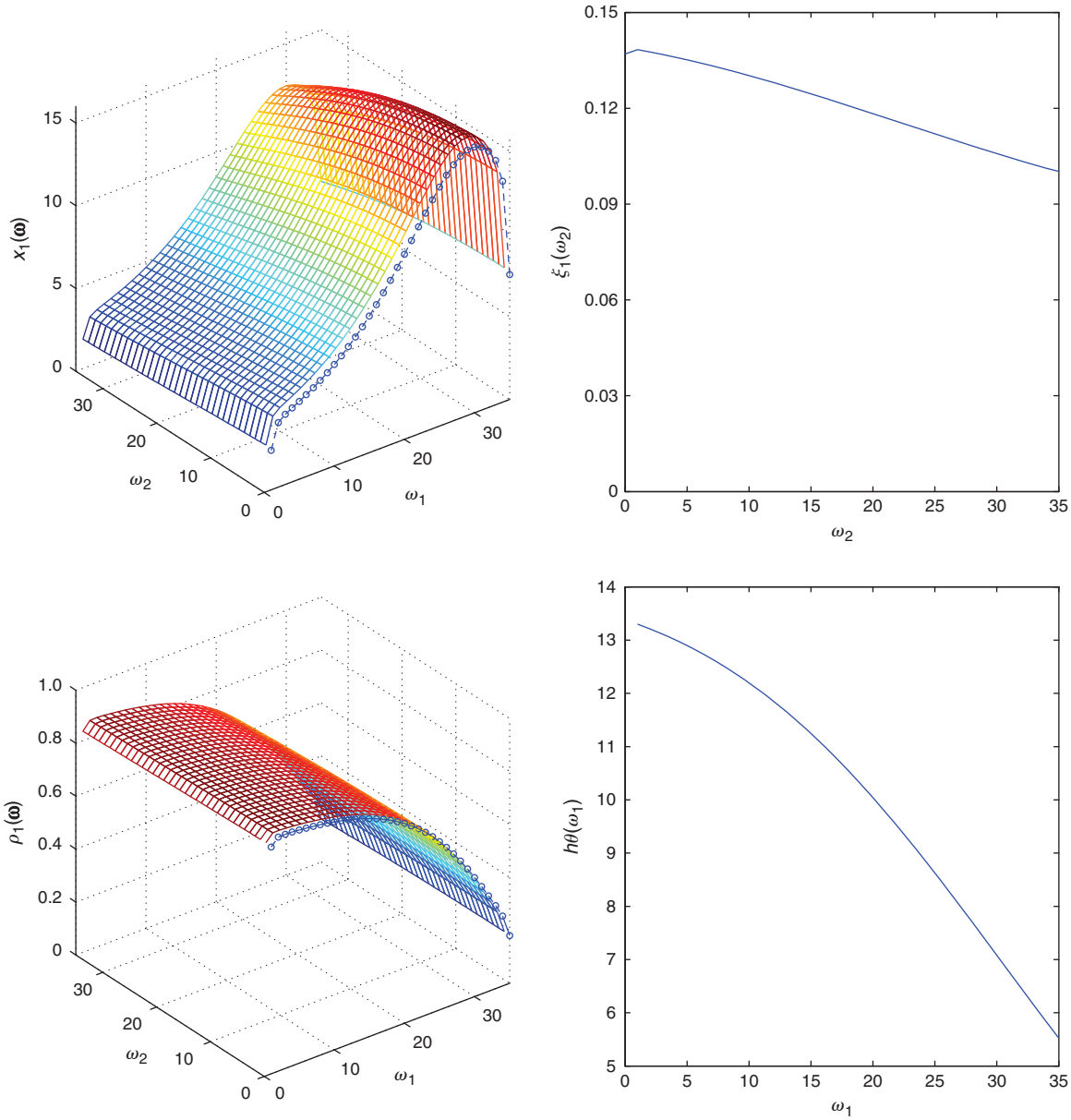
The top left panel of Figure 6 shows that there is a nonmonotonic relationship between a firm's own brand equity and its ad spending. An increase in a firm's brand equity gives rise to an increase in its ad spending, but only up to brand equity $\omega_n = 30$, after which ad spending decreases. The increase in ad spending is driven by three factors. First, as brand equity increases, a firm spends more on advertising to counteract the decreasing effectiveness of its advertising (measured by $h\theta(\omega_n)$), which is shown in the bottom right panel of Figure 6. Second, as brand equity increases, a firm spends more on advertising to counteract the increasing firm-specific brand equity depreciation rate. Finally, because the estimated period profit function is convex in a firm's own brand equity (see Figure 3), the higher a firm's brand equity, the greater the returns it earns (at least in the short run) from further increasing its brand equity and, accordingly, the greater its incentive to invest in advertising. Thus, as a firm's brand equity increases, changes in ad effectiveness, depreciation, and the profits that brand equity delivers all lead the firm to increase its ad spending.

Things change, however, for $\omega_n > 30$, because at such high levels of brand equity, a firm's ad effectiveness is very low and its firm-specific depreciation rate is very high. It becomes much more difficult to build and sustain brand equity, and this weakens advertising incentives. Thus, when a firm has high brand equity, it usually—but not always—spends more on advertising. Going forward, this insight will play a key role in helping us understand the value of the Pringles and STAX brands.

7.3. Brand Equity Over Time

The value of a brand is based on the difference between the cash flows it generates in the factual scenario, in which it possesses its brand equity, and the counterfactual scenario, in which it is stripped of its brand equity.

Figure 6. Estimated Equilibrium: Expected Ad Spending (in Millions of Dollars; Top Left), Probability of Entry (Top Right), Advertising Success Probability (Bottom Left), and Expected Ad Effectiveness (Bottom Right)

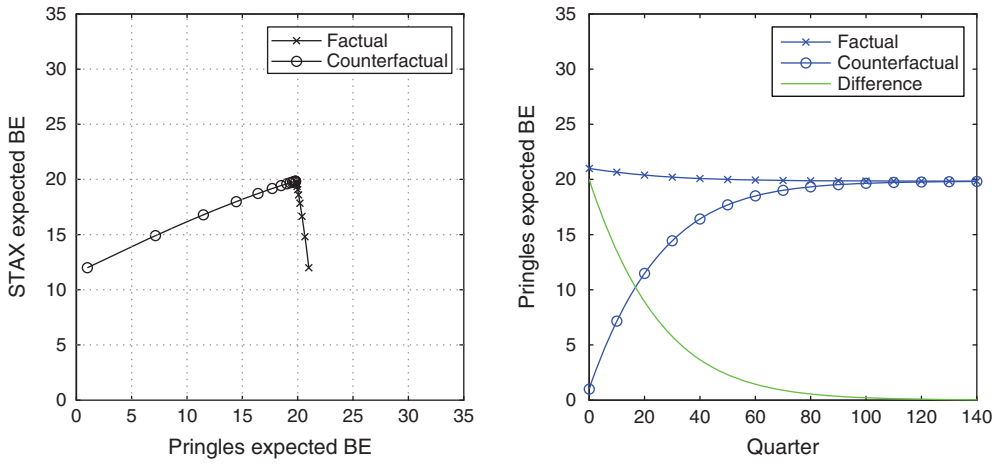


Hence, to understand brand value, we need to understand how brand equity evolves over time in each of these scenarios. Section 7.2 explored how firms behave in equilibrium. Given this understanding, we can now infer how brand equity evolves over time.

Here we focus on the industry state that arises in the last period of our data, (21,12). The left panel of Figure 7 shows how Pringles and STAX expected brand equities evolve over time in the factual scenario, which starts at (21,12), and in the Pringles counterfactual scenario, which starts at (1,12).[24] The x and o markers indicate the expected brand equities in quarters $0, 10, \ldots, 140$ for the factual and counterfactual scenarios, respectively. Pringles's brand value is determined by the difference in cash flows earned on the factual path lined with x markers and the counterfactual path lined with o markers. The analysis for STAX is qualitatively similar.

We find that in the factual scenario, by period 109, the transient distributions have converged[25] to a distribution with a modal industry state of $(20, 20)$ and the expected brand equity for both Pringles and STAX is approximately 19.8. So, while Pringles is likely to retain its brand equity advantage in the short term, STAX is likely to narrow the advantage over time and to ultimately catch up. In the Pringles counterfactual

Figure 7. (Color online) Expected Brand Equities Over Time Given Factual Initial State (21,12) and Pringles Counterfactual Initial State (1, 12)



Note. BE, Brand equity.

scenario, the industry converges to the same long-run industry structure, with convergence occurring after 125 periods.[26]

STAX ultimately catches up for multiple reasons. First, as shown in the top left panel of Figure 6, STAX never gives up—i.e., it never stops advertising—no matter how far it falls behind Pringles. Second, even though Pringles invests more in advertising than STAX when it is in the lead, STAX benefits from a higher advertising effectiveness and a lower firm-specific depreciation rate. Therefore, as seen in the bottom left panel of Figure 6, STAX is more likely to advertise successfully despite its lower advertising spending.

These results explain why changes in brand equity translate to less than proportional changes in brand value: because the long-run industry structure is symmetric, a change in a firm's brand equity affects a firm's payoffs in the short run, but not in the long run. For example, in the first quarter of 2001, a 19.4% decrease in Pringles brand equity brings about only a 10.2% decrease in its brand value; while this decline in brand equity decreases Pringles's profits in the short run, it does not change the fact that the industry will ultimately evolve to a symmetric long-run industry structure with a modal industry state of (20, 20).

This symmetric long-run industry structure is not assumed by the model, but rather what we conclude from our data. The dynamic model presented in Section 5 admits a wide variety of long-run industry structures, both symmetric (where firms are likely to have the same brand equity levels) and asymmetric (where one firm is likely to have a persistent brand equity advantage). However, all of the equilibria presented in this paper, i.e., the estimated equilibrium and all counterfactual equilibria in Section 8, give rise to symmetric long-run industry structures (though different parameterizations give rise to different symmetric industry structures). The symmetric industry structure is driven by high estimated variance of the effectiveness of advertising ($h\theta(\omega_n)^2$) at low brand equity (ω_n) states. This means that even when it is extremely difficult to build brand equity, a firm with low brand equity never completely gives up and ultimately catches up to the leader through a sequence of lucky (though anticipated) draws.[27]

Because the industry converges to a symmetric long-run industry structure, the value of a brand is derived strictly from the higher cash flows that accrue to the firm in the factual scenario—as compared to the counterfactual scenario—*before* the factual and counterfactual industry structures converge. For industry state (21,12), the factual scenario and the Pringles counterfactual scenario converge after 125 quarters; thereafter, Pringles's cash flows in the factual scenario are no higher than its cash flows in the counterfactual scenario. The value of the Pringles brand in industry state (21,12) is therefore derived from the higher cash flows that the firm earns in the factual scenario in the first 124 quarters. The longer it takes for the factual and counterfactual industry structures to converge, the greater the brand's opportunity to create value for the firm from the additional cash flows it generates.

Our model also helps us understand the drivers of brand value. Consider Pringles's brand value of \$1.6 billion in industry state (21,12). Using the profit function (Figure 3), the advertising policy function (Figure 6), and the transient distributions described above (summarized in Figure 7), we can compute the two components of the brand value decomposition presented in Section 7.1. We find that in the factual scenario, in discounted terms, Pringles earns \$1.79 billion in additional flow profits (the first component), but that it spends \$191.3 million in additional advertising expenditures (the second component). Hence,

Pringles's brand value derives from the fact that it earns *much* greater flow profits in the factual scenario, which more than overrides its greater advertising expenditures in that scenario.

7.4. The Benefits of a Dynamic Measure of Brand Value

As explained above, our measure of brand value is the dynamic counterpart of the Goldfarb et al. (2009) purely static approach. However, there is no denying that the static measure is easier to compute, so it is important to see what is being gained by taking an explicitly dynamic approach.

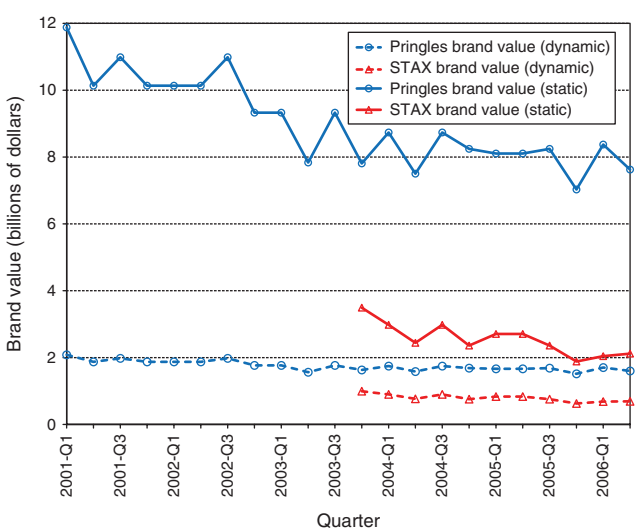
A firm's "static" brand value in industry state $\boldsymbol{\omega} = (\omega_n, \omega_{-n})$ is $\pi_n^*(\boldsymbol{\omega}) - \pi_n^*(1, \omega_{-n})$, which can be computed using the equilibrium profit function that is presented in the bottom right panel of Figure 3. To convert this to a measure of brand value for all future periods—so that it is comparable to our net present value measure—one can simply regard the brand as a perpetuity that generates cash flow $\pi_n^*(\boldsymbol{\omega}) - \pi_n^*(1, \omega_{-n})$ in all future periods and compute the present discounted value using the discount rate from the dynamic model. This is analogous to abstracting from the dynamics of brand building and assuming that the industry state does not change over time. Hence, we define

$$v_n^s(\boldsymbol{\omega}) = \frac{1}{1-\beta}(\pi_n^*(\boldsymbol{\omega}) - \pi_n^*(1, \omega_{-n})), \tag{15}$$

which we hereafter denote as "static brand value" for the sake of simplicity.

Figure 8 juxtaposes the "dynamic" brand values for Pringles and STAX—already presented in Figure 5—against the static brand values and reveals dramatic differences between the two. Specifically, the static brand values are *much* higher and fluctuate *much* more over time.

Figure 8. (Color online) Dynamic Brand Values and Static Brand Values vs. Time

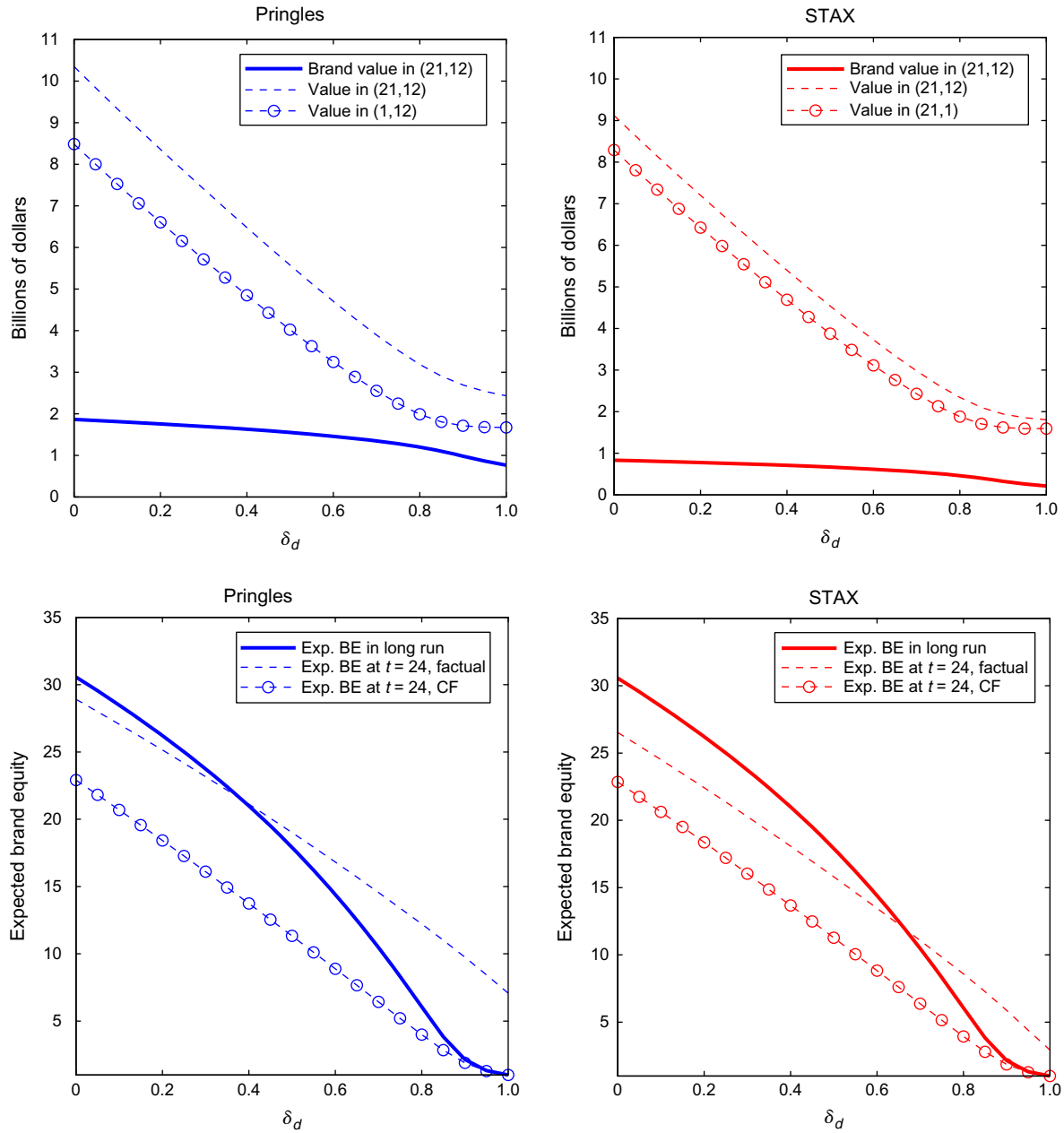


To understand why static brand values are much higher, it is useful to refer to the two components of the brand value decomposition in Section 7.1. First, and most importantly, while the static measure captures the profit premium that the brand generates in one period and assumes that it will continue to do so in perpetuity, the dynamic measure accounts for the fact that a profit premium exists only until the factual and counterfactual scenarios converge. For example, in industry state (21,12)—which prevails in the second quarter of 2006—the static and dynamic brand values for Pringles are \$7.62 billion and \$1.6 billion. Whereas the static measure implicitly assumes that the 21 units of brand equity that Pringles possesses in the factual scenario will generate additional profits of \$76.2 million per quarter indefinitely, the dynamic measure correctly recognizes that the incremental profits generated by the brand will decrease over time as the industry converges to the same symmetric long-run industry structure in both the factual and counterfactual scenarios. For example, the additional profits earned in the factual scenario decline to \$50.1 million after 16 quarters, \$28.3 million after 32 quarters, and \$14.5 million after 48 quarters.

Second, because the static measure omits advertising, it does not capture the impact of the brand on a firm's advertising expenditures. Our dynamic brand value measure accounts for the fact that a firm spends more on advertising in the factual scenario than the counterfactual scenario. For example, Pringles spends \$11.6 million on advertising in industry state (21,12) and only \$2.3 million in the corresponding counterfactual industry state (1,12).

In addition to being higher, the static brand values in Figure 8 fluctuate much more than the dynamic brand values because, as explained above, static brand values fail to account for the evolution of brand equity over time. Consider, for example, the transition from industry state (22,12) in the first quarter of 2006 to industry state (21,12) in the second quarter of 2006. This transition decreases Pringles's static brand value by \$748.6 million, from \$8.37 billion to \$7.62 billion, and its dynamic brand value by \$100.8 million, from \$1.7 billion to \$1.6 billion. The static brand value drops so starkly because this measure implicitly interprets the decline in Pringles brand equity from 22 to 21 as bringing about a persistent decrease in profits attributable to the brand of \$7.49 million per quarter. The dynamic brand value, on the other hand, decreases much less because it acknowledges that the decline in Pringles brand equity does not change the fact that the industry is slowly evolving toward its long-run industry structure; that is, the change in

Figure 9. (Color online) Industrywide Depreciation Rate Counterfactual Analysis



Notes. BE, Brand equity; CF, counterfactual.

brand equity impacts the path along which the industry is traveling, but not its destination.

We have shown that because the static measure fails to account for how the profit premium evolves over time and ignores the impact of the brand on advertising expenditures, it can produce misleading measures of brand value. This is also true of other static measures, such as a revenue premium. For Pringles and STAX, the static measure grossly overestimates brand values and also gives rise to drastic intertemporal fluctuations that are not reflective of the true values of these brands. These shortcomings limit the usefulness of such measures for accounting, legal, and brand strategy purposes.

8. How Brand Equity Depreciation and Advertising Effectiveness Affect Brand Value

Our model has three key parameters that affect the ability of firms to build and sustain brand equity: (i) the firm-specific brand equity depreciation rate, (ii) the industrywide brand equity depreciation rate, and (iii) the mean effectiveness of advertising. Here we

examine how changes in these parameters affect brand value, focusing primarily on the latter two (the results for the firm-specific depreciation rate are quite similar to those for the industrywide depreciation rate). We focus on the industry state observed in the final period of our data set, (21,12). Hence, we explore how changes in the industrywide brand equity depreciation rate and advertising effectiveness affect firm value for Pringles and STAX in industry state (21,12) relative to industry state (1,12) for Pringles and (21,1) for STAX.

8.1. Changes in the Industrywide Brand Equity Depreciation Rate

To explore the consequences of changes in the industrywide brand equity depreciation rate δ_d, we compute equilibria for $\delta_d \in \{0, 0.05, \ldots, 1\}$.[28] As we vary the industrywide depreciation rate δ_d, we assume that the ratio of the industrywide appreciation rate (δ_u) to the probability that there is no industrywide shock ($1 - \delta_d - \delta_u$) is fixed at the ratio of the estimated values,[29] i.e.,

$$\frac{\Pr(\eta = 1)}{\Pr(\eta = 0)} = \frac{\delta_u}{1 - \delta_d - \delta_u} = \frac{0.2571}{0.3040} = 0.8458.$$

All other parameters are held fixed at the estimated values.

Firm vs. Brand Value. Our first observation is that a given change in the industrywide brand equity depreciation rate has a much greater impact on firm value than on brand value. As shown in the top panels of Figure 9, firm value in both the factual and counterfactual scenarios declines rapidly as the industrywide depreciation rate increases, but brand value declines much more slowly. For instance, increasing δ_d from 0 to 1 decreases Pringles's firm value by \$7.9 billion in the factual scenario and by \$6.8 billion in the counterfactual scenario. This rapid decline in firm value occurs because, as shown in the bottom left panel of Figure 9, an increase in the industrywide depreciation rate decreases a firm's expected brand equity both in the short run (after 24 quarters) and the long run. Therefore, at a higher industrywide depreciation rate, a firm traverses a less profitable path toward the long-run industry structure *and* earns less profit once the industry reaches its long-run industry structure. By contrast, brand value is much less sensitive to changes in the industrywide depreciation rate. Increasing δ_d from 0 to 1 decreases the Pringles brand value by only \$1.1 billion—from \$1.9 billion to \$763 million. Results for STAX are similar. Intuitively, an increase in the industrywide depreciation rate has a relatively small impact on brand value because it has a similarly detrimental impact on firm value in both the factual and counterfactual scenarios.

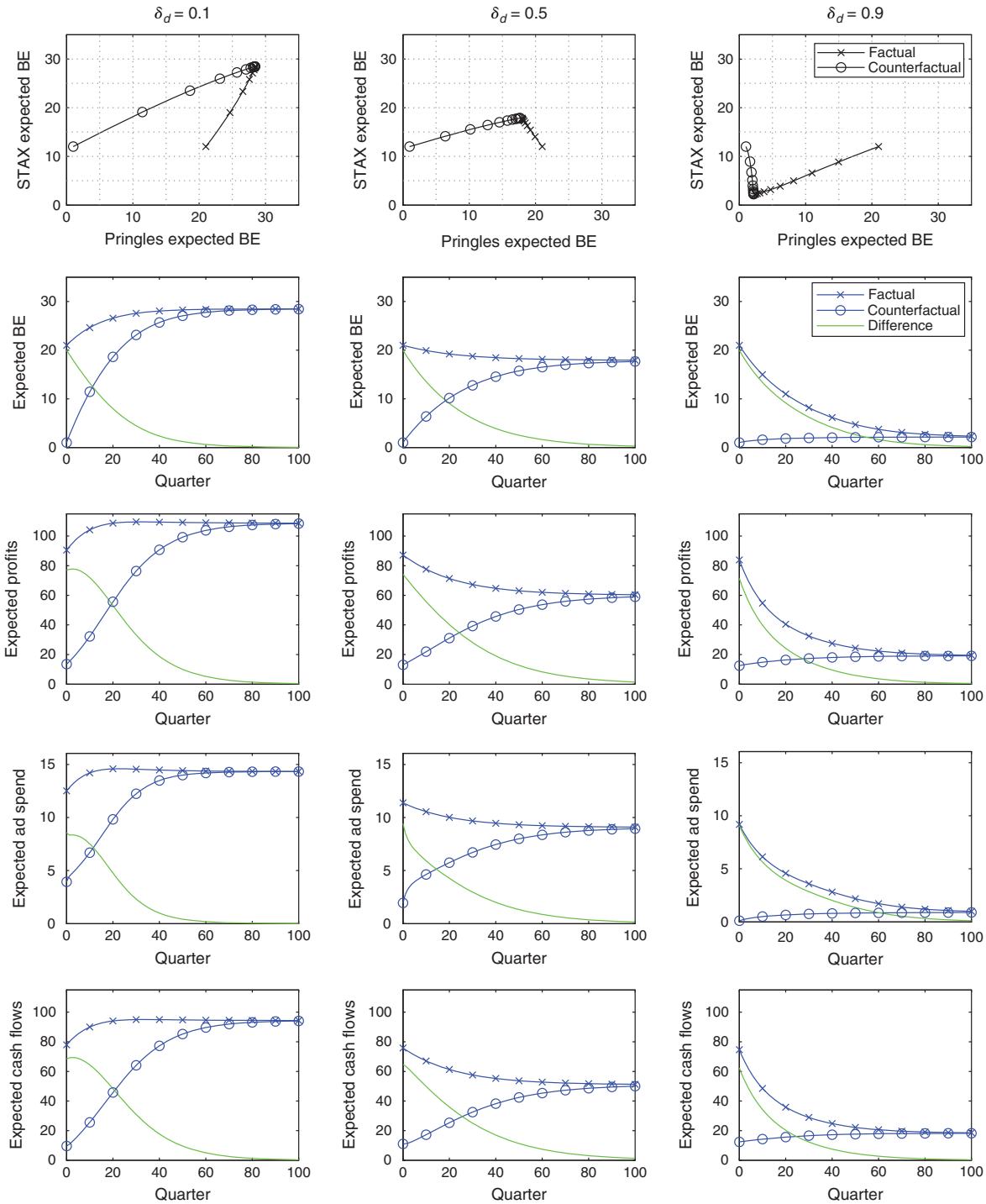
While the brand equity that a firm possesses is valuable regardless of circumstances, the source of a brand's value does depend on the circumstances. This is shown in the bottom panels of Figure 9. When $\delta_d = 0$, the 21 units of brand equity that Pringles possesses in the factual scenario are highly valuable because they give Pringles a big head start in its journey toward its expected long-run brand equity of 30.6. On the other hand, when $\delta_d = 1$, despite being unable to sustain any brand equity whatsoever, the 21 units of brand equity that Pringles possesses in the factual scenario are still valuable because Pringles is able to harvest them and in doing so earn significant profits even as they erode. In the counterfactual scenario for $\delta_d = 1$, Pringles earns relatively little profit because it starts with no brand equity and it fails to build any. The sources of STAX's brand value are analogous, though smaller because its initial brand equity is lower.

Explaining the Decrease in Brand Value. Figure 10 helps us understand why brand value is decreasing in the industrywide depreciation rate by examining the impact of the industrywide depreciation rate on (i) the time required for factual and counterfactual industry structures to converge and (ii) the two components of the brand value decomposition. We restrict attention to Pringles, but the results for STAX are qualitatively similar.

We explained above that the value of a brand is derived strictly from the higher cash flows that the firm earns in the factual scenario—relative to the counterfactual scenario—*before* the factual and counterfactual industry structures converge. We first explore whether an increase in the industrywide depreciation rate decreases brand value because of its impact on the time required for the factual and counterfactual scenarios to converge. The first two rows of Figure 10 present the evolution of the expected industry state and Pringles's expected brand equity in the factual and counterfactual scenarios for three levels of the depreciation rate. While the time required for convergence does vary across depreciation rates,[30] the differences have a negligible impact on brand value because at each depreciation rate, more than 99% of Pringles's brand value is generated in the first 100 quarters. Having ruled out this possible explanation, we turn to the two components of the brand value decomposition.

The third row of Figure 10 shows how Pringles's profits differ across the factual and counterfactual scenarios—the first factor in the brand value decomposition. An increase in the industrywide depreciation rate decreases the difference between factual and counterfactual profits in the sufficiently early quarters. (In later quarters, this relationship does not necessarily hold because of the difference across depreciation rates in the time required for factual and counterfactual industry structures to converge. However, the overall brand value results are driven by the earlier periods

Figure 10. (Color online) Industrywide Depreciation Rate Counterfactual Analysis (Pringles)



Note. BE, Brand equity.

because the difference between factual and counterfactual expected brand equity is higher and the flow profits are less discounted.)

The smaller difference (between factual and counterfactual profits) for higher depreciation rates stems from the fact that an increase in the industrywide depreciation rate leads to a decline in Pringles's long-run expected brand equity; see the bottom left panel of Figure 9 and the top row of Figure 10. For $\delta_d = 0.1, 0.5$, and 0.9, the expected long-run brand equities are 28.5, 17.9, and 2.2, respectively. The difference between factual and counterfactual flow profits is greater when

firms are moving toward a higher long-run expected brand equity because a firm's period profit function is highly convex in its own brand equity (see Figure 3).[31]

Consider the factual and counterfactual paths presented in the top left panel of Figure 10 for $\delta_d = 0.1$. The factual scenario yields much greater flow profits because the industry travels along the steep portion of the period profit function, while in the counterfactual scenario, the industry starts on the flat portion and reaches the steep portion only much later. Alternatively, for $\delta_d = 0.9$, in both the factual and counterfactual scenarios, the industry travels along the relatively flat portion of the period profit function. Intuitively, because a brand delivers greater and greater period profits—at an increasing rate—as its brand equity increases, a firm that is able to build and sustain a high level of brand equity benefits greatly from being able to start with *relatively* high brand equity instead of having to build all of its brand equity from scratch.

The fourth row of Figure 10 shows how advertising expenditures differ across the factual and counterfactual scenarios—the second factor in the decomposition. It shows that the impact of an increase in the industrywide depreciation rate on ad spending is qualitatively similar to its impact on flow profits, but smaller in magnitude because—irrespective of the industrywide depreciation rate—ad spending is less sensitive to changes in brand equity than flow profits.

The fifth row of Figure 10 encapsulates all of the data in the above rows by presenting factual and counterfactual cash flows over time as well as the difference between them. Pringles's brand value is the present discounted value of the differences in expected cash flows presented in the fifth row (but for all future quarters, not only the first 100), as in the decomposition.

Overall, an increase in the industrywide depreciation rate gives rise to three effects on brand value: (i) it has a negligible effect on the time required for the factual and counterfactual industry structures to converge, which on its own would have a negligible impact on brand value; (ii) it decreases the difference between factual and counterfactual flow profits, which on its own would *decrease* brand value; and (iii) it decreases the difference between factual and counterfactual advertising expenditures, which on its own would *increase* brand value. As the fifth row of Figure 10 shows, ultimately brand value is decreasing in the industrywide depreciation rate because effect (ii) overwhelms effect (iii).

Intuitively, the brand equity that a firm possesses is most valuable when the industrywide depreciation rate is low because (i) a firm is able to sustain and augment its existing brand equity and thus earn high profits, and (ii) were it lost, the firm would have to travel a much less profitable path until it ultimately built a high level of brand equity.

In Sections 8.2 and 8.3, we further explore how model parameters that affect the ability of firms to build and sustain brand equity affect brand value, focusing on results and the underlying intuition. Thorough analyses of these counterfactuals are provided in the online appendix.

8.2. Changes in the Effectiveness of Advertising

We explore the effects of changes in the effectiveness of advertising by varying $h\theta(\omega_n)$, the mean of γ_n, while holding fixed its variance, $h(\theta(\omega_n))^2$.[32] To do that, let the estimated values of parameters d and h, reported in Table 4, be denoted by $\hat{d}$ and $\hat{h}$, and let $\theta(\omega_n)$ evaluated at the estimated values of a, b, c, and d be denoted by $\hat{\theta}(\omega_n)$. To vary the mean effectiveness of advertising by the multiplicative factor λ, we set $d = \hat{d} + \ln(1/\lambda)$ and $h = \hat{h}\lambda^2$. This yields a mean effectiveness of advertising of $\hat{h}\hat{\theta}(\omega_n)\lambda$ while leaving the variance unchanged at $\hat{h}\hat{\theta}(\omega_n)^2$.[33]

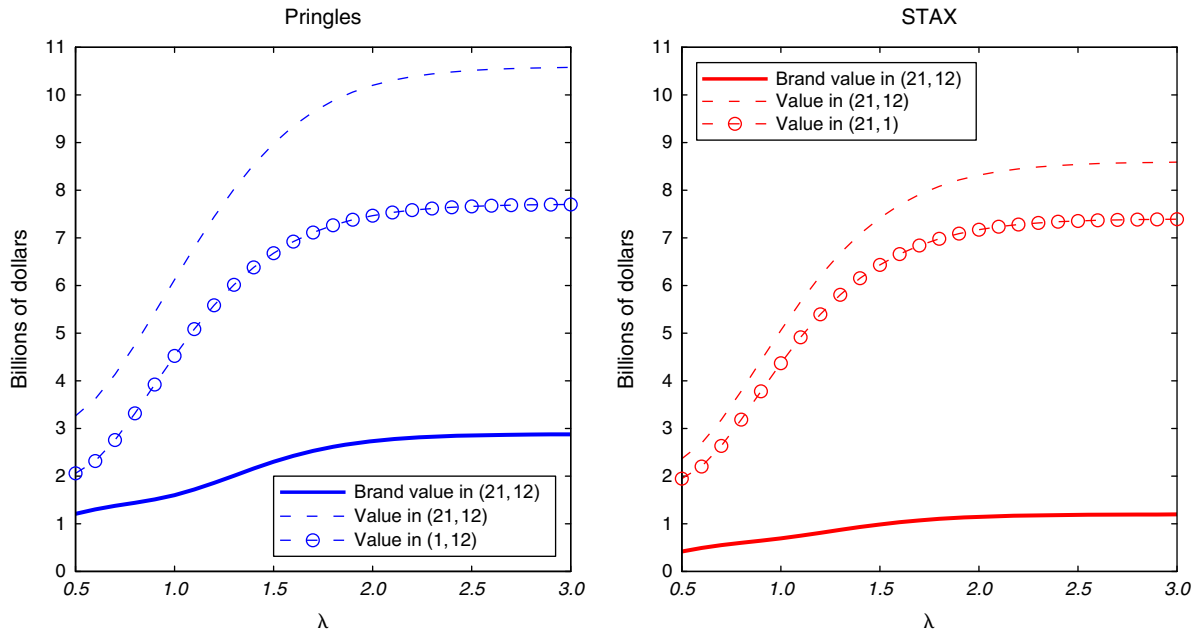
We compute equilibria for $\lambda \in \{0.5, 0.6, \ldots, 3\}$. Figure 11 shows that an increase in the mean effectiveness of advertising increases brand value. The intuition underlying this result is similar to the intuition provided in Section 8.1. The main point is that as it becomes easier to build brand equity, firms build more brand equity in the long run, and this gives rise to a greater difference between factual and counterfactual flow profits.

8.3. Nonmonotonicities in the Effect of Industrywide Brand Equity Depreciation Rate on Brand Value

In the two counterfactuals presented above, we varied either the industrywide brand equity depreciation rate or the mean effectiveness of advertising, while holding the other constant. We now conduct a richer analysis by bringing firm-specific brand equity depreciation into the mix and by varying two parameters simultaneously.[34] In doing so, we show that there are important interaction effects between these parameters which lead to a nonmonotonic relationship between the industrywide depreciation rate and brand value.

The top panels of Figure 12 show how changes in both the industrywide depreciation rate and the firm-specific depreciation parameter z affect brand value. The bottom panels of Figure 12 show how changes in both the industrywide depreciation rate and the mean effectiveness of advertising affect brand value. The monotonically decreasing relationship between the industrywide depreciation rate and brand value presented in Figure 9 can be found at the $z = 0.0294$ cross-sections of the figures in the top panels and the $\lambda = 1$ cross-sections of the figures in the bottom panels. The top panels of Figure 12 show that if we decrease

Figure 11. (Color online) Mean Effectiveness of Advertising Counterfactual Analysis



the firm-specific depreciation parameter z from the estimated value to $z = 0.018$ or lower, the relationship between the industrywide depreciation rate and brand value becomes nonmonotonic. The bottom panels of Figure 12 show that if we increase the mean effectiveness of advertising by at least 30% ($\lambda \geq 1.3$), the relationship between the industrywide depreciation rate and brand value becomes nonmonotonic.

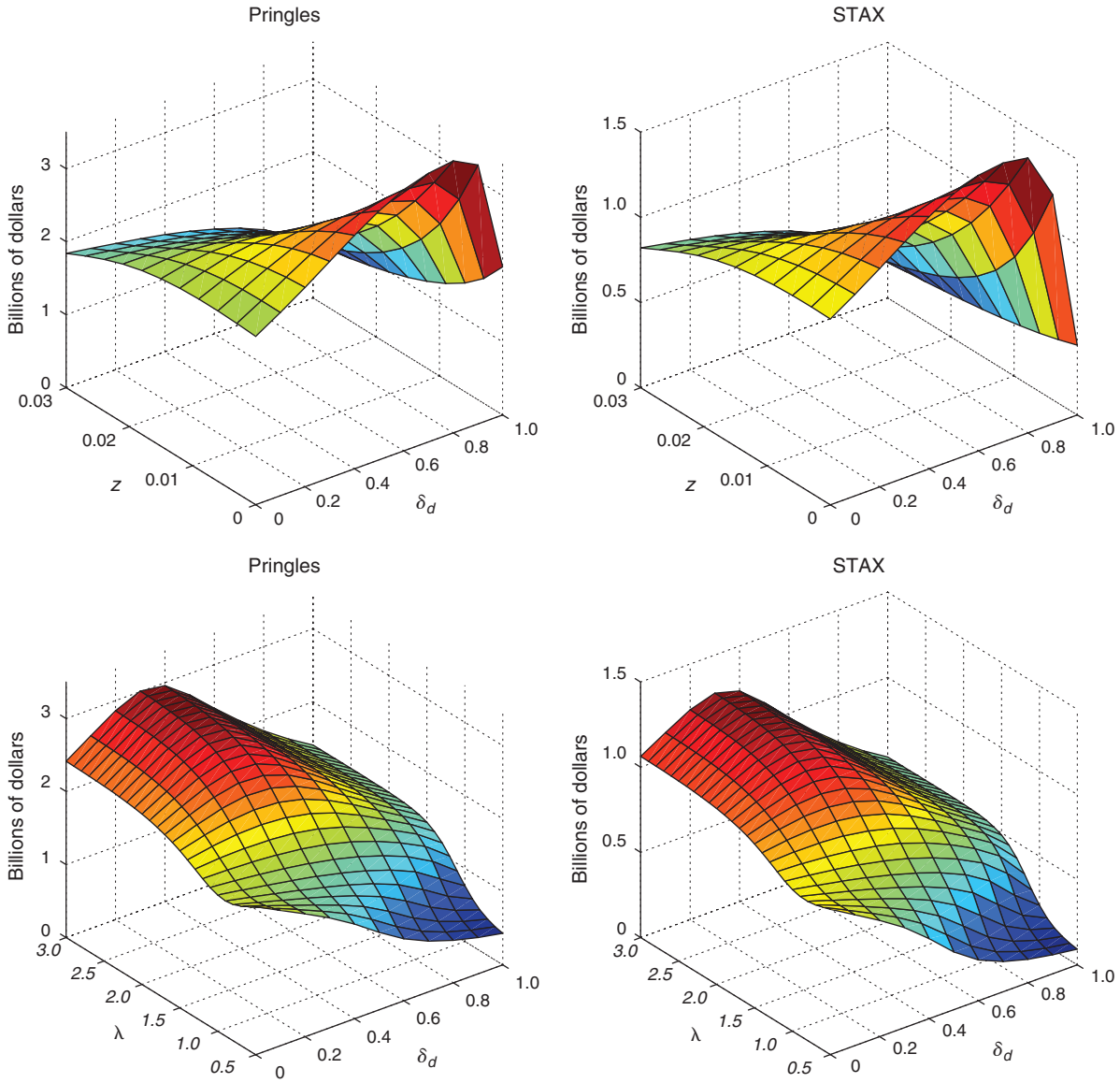
Decreasing Firm-Specific Depreciation. To understand the nonmonotonicities in the top panels of Figure 12, we fix the firm-specific depreciation parameter to $z = 0$ and present firm values and brand values for different industrywide depreciation rates in Figure 13. When there is no firm-specific depreciation, to build and sustain brand equity a firm need only counteract industrywide depreciation. Figure 13 shows that while an increase in the industrywide depreciation rate decreases firm value, it can *increase* brand value. The first observation is not surprising; we do expect firms to suffer when it becomes harder to sustain brand equity. However, the second perhaps is. It leads to the insight that brand equity can become *more* valuable even as it becomes harder to sustain.

In Section 8.1, we learned that at the estimated parameterization, brand value is maximized at $\delta_d = 0$. However, when there is neither firm-specific depreciation ($z = 0$) nor industrywide depreciation ($\delta_d = 0$), firms can build brand equity relatively easily. It follows that a firm in the counterfactual scenario rebuilds brand equity quickly. Hence, the factual and counterfactual scenarios converge quickly and, accordingly, the brand has little value. When the industrywide depreciation rate is very high (e.g., $\delta_d = 0.95$), brand value is low because a firm is unable to sustain its brand equity, and it spends heavily on advertising to slow its decline, which erodes cash flows. However, at intermediate levels of industrywide depreciation, brand value is high because (i) a firm is able to sustain and enhance its brand equity and therefore generate high cash flows (relative to the counterfactual scenario), and (ii) if brand equity were lost, it would be very time-consuming to rebuild.

Increasing the Mean Effectiveness of Advertising. The source of the nonmonotonicities in the bottom panels of Figure 12 (for $\lambda \geq 1.3$) is similar because an increase in the mean effectiveness of advertising and a decrease in the firm-specific depreciation parameter have similar impacts; both make it easier for firms to build and sustain brand equity. In the online appendix, we fix $\lambda = 2$ and present firm values and brand values as in Figure 13. Here we simply emphasize the key insights. First, when ad effectiveness is sufficiently high—just as when firm-specific depreciation is sufficiently low—while an increase in the industrywide depreciation rate decreases firm value, it can *increase* brand value. Second, brand value is highest at moderate industrywide depreciation rates for the very same reasons described above—i.e., the firm is able to build and sustain a high level of brand equity and, should it be lost, the rebuilding process would be very time-consuming and much less profitable.

Summary of Counterfactual Analyses. The counterfactual analyses conducted in Section 8 have generated four important insights. First, in all of the counterfactual analyses, a change in brand building ability has a bigger impact on firm value than on brand value. Because

Figure 12. Brand Value in (21,12) vs. (i) Firm-Specific Depreciation Parameter z and Industrywide Depreciation Rate δ_d (Top Panels), and (ii) Multiplier for the Mean Effectiveness of Advertising λ and Industrywide Depreciation Rate δ_d (Bottom Panels)



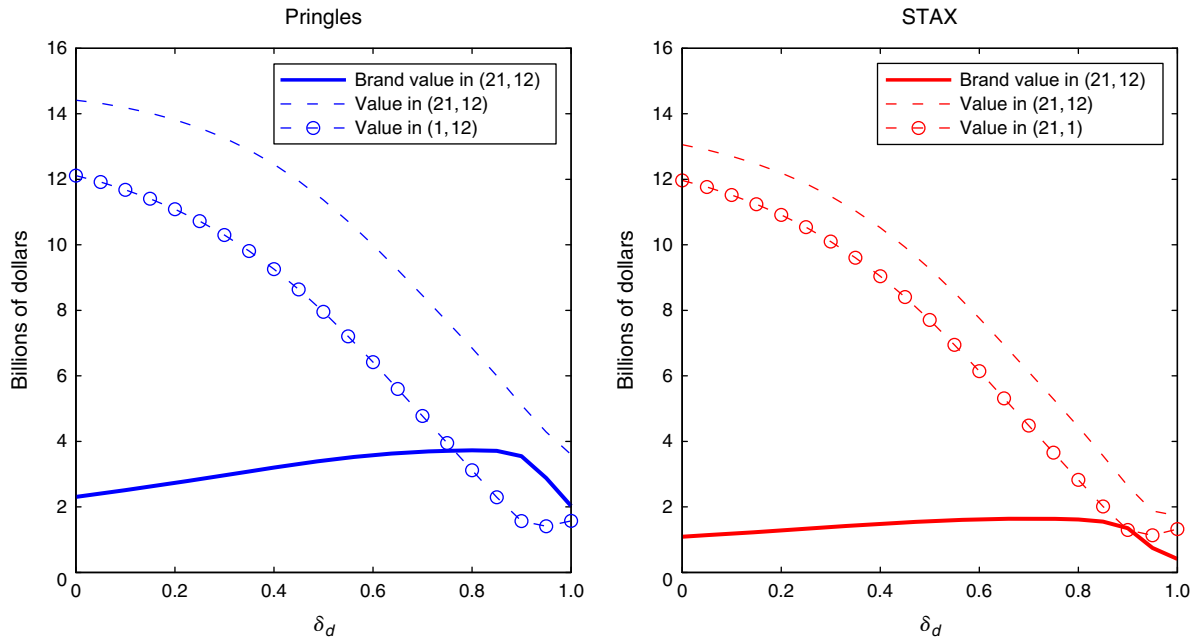
a given change in brand building ability has a similar impact (positive or negative) on firm value in both the factual and the counterfactual scenarios, brand value—the difference between the two—is affected much less.

Second, in all of our counterfactual analyses, brand value is maximized under the same set of circumstances: (i) a firm is able to build and sustain a relatively high level of brand equity and thus earn high profits, and (ii) were the firm to lose its brand equity, the process of rebuilding it would be very time consuming and much less profitable.

Third, at the estimated parameterization, a change in an industry fundamental causes brand value to vary monotonically. Therefore, the circumstances under which brand value is maximized are at the extremes—either a very low industrywide depreciation rate or a very high mean effectiveness of advertising. These extremal values make it possible for a firm to achieve condition (i) because both low industrywide depreciation and high ad effectiveness make it easier to sustain and enhance brand equity.

Fourth, while the relationship between the industrywide depreciation rate and brand value is monotonically decreasing at the estimated parameterization, this is not necessarily so at other parameterizations. In particular, if a change in industry fundamentals makes it easier for a firm to sustain and enhance brand equity—e.g., a reduction in firm-specific depreciation or an

Figure 13. (Color online) Industrywide Depreciation Rate Counterfactual Analysis; No Firm-Specific Depreciation ($z = 0$)



increase in ad effectiveness—then this can give rise to an inverted-U relationship between the industrywide depreciation rate and brand value. It follows that brand value is highest at an intermediate industrywide depreciation rate.

9. Conclusion

We develop a structural model of brand management to estimate the value of a brand to a firm. Our brand value measure is forward looking and accounts for competition within the category, brand equity depreciation, and opportunities to build and sustain brand equity via advertising. In our framework, a brand's value is the expected net present value of future cash flows accruing to a firm due to the brand. Pinning down the specific contribution of the brand calls for a comparison between a factual scenario, in which a product possesses its brand equity, and a hypothetical counterfactual scenario, in which the product is stripped of its brand equity. Our structural model allows us to account for the different decisions that consumers and firms make across these scenarios. For example, in the counterfactual scenario, a firm is stripped of its brand equity, and this affects consumers' purchasing decisions and, in turn, the pricing and advertising decisions of the firm and its rival over time.

Using data from the U.S. stacked chips category for the period 2001–2006, we provide quarterly estimates of brand equity and brand value for the Pringles and STAX brands. We find that a given change in brand equity gives rise to a less than proportional change in brand value. We also compare our brand value measure to an analogous static measure that ignores advertising and, accordingly, its ability to shape brand equity dynamics. The static measure yields brand values that are grossly overestimated and that fluctuate too much over time.

Our structural model allows us to explore how changes in a firm's ability to build and sustain brand equity affect the value of its brand. Perhaps most notably, we find that when industry conditions make it easier to brand build, an increase in the industrywide brand equity depreciation rate can *increase* brand value, even as it (expectedly) reduces firm value. So, a brand becomes *more* valuable even as the brand equity on which it is constituted becomes harder to sustain.

We believe our framework breaks new ground by offering a way to measure, and understand, brand value that is integrated into a dynamic model of brand management. Furthermore, we believe that our framework could be adapted to industries whose details might differ from those of the stacked chips category, the subject of this study, and augmented to answer other important questions on brand valuation and brand management. First, while we study the national-level brand management decisions observed in the stacked chips category, in some product categories, geographic differences in consumers' preferences may induce firms to manage their brands more locally (Bronnenberg et al. 2007). Second, while the assumption that firms build brand equity via investments in advertising applies well to the stacked chips category in the period 2001–2006, in other product categories, especially in more recent times, firms are building brand equity in a variety of different ways, of which advertising is only one component. Third, while we have restricted attention to a

product category in which each firm operates a single brand, one could extend our methods to product categories in which firms maximize profits over multiple brands.

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Endnotes

[1] Rosen (1974), Holbrook (1992), and Swait et al. (1993) estimate brand equity using a price premium approach. Kamakura and Russell (1993) and Srinivasan et al. (2005) estimate brand equity using a volume premium approach. Srinivasan and Park (1997) estimates brand equity using both approaches. Ailawadi et al. (2003) measure brand value using a revenue premium approach, which accounts for both the price and volume premiums.

[2] Interbrand, WPP/Millward Brown, and Prophet use discounted cash flow techniques based on the economic profits attributable to the brand. Brand Finance uses a discounted cash flow approach that is more transparent because it is based on royalties that a firm would have to pay were it stripped of its brand. BAV and CoreBrand use measures that are largely survey based. Because the various methods are valuable intellectual property, the details of how the values are derived are not public. See Salinas (2009) for an excellent overview.

[3] Frederic J. Baur was so proud of having invented the Pringles container that he asked that his ashes be buried in one. When he died in 2008, his children honored his wish (Caplan 2008).

[4] Here and elsewhere, all dollar amounts are deflated to year 2000 dollars using the Consumer Price Index.

[5] This assumption is based on institutional evidence that advertising budgets were confirmed quarterly in the period 2001–2006 (interview with a former P&G vice president with detailed knowledge of the Pringles business during that time period who wishes to remain anonymous, August 2012).

[6] In previous applications of the Ericson and Pakes (1995) framework, states have typically been observed in the data. For example, in the Goettler and Gordon (2011) study of R&D competition between Intel and AMD, the authors observed processor speed (their *quality* state variable). In our study, brand equities at the beginning of a quarter serve as the state variables for the dynamic game, but we do not observe them in the data; therefore, we need to estimate them prior to estimating the dynamic model.

[7] Alternatively, we could have estimated these distributions at the store level. The benefit of our approach is that it allows us to incorporate store-level heterogeneity without increasing the computational burden substantially—we simply approximate the expected profit function by integrating out over the aforementioned distributions using Monte Carlo simulation. Had we estimated those distributions at the store level, we would have had to conduct this same exercise *for each store*. This approach would significantly increase the computational burden of both the period profit approximation and the calculation of standard errors of the dynamic model. In fact, because we calculate standard errors of the dynamic model using a parametric bootstrap with 100 bootstrap samples, and for each bootstrap sample we approximate a separate period profit function, this approach to computing standard errors would be computationally infeasible.

[8] We are abstracting away from the role of the retailer. Effectively, we are assuming that the retail sector is perfectly competitive, which is likely reasonable for grocery retailing. Our assumption has the virtue of reducing the computational burden of estimating the model and running counterfactuals. Alternatively, we could have assumed a monopolistic retailer, as in Sudhir (2001) and Goldfarb et al. (2009). However, because brand equity states are derived exclusively from retail prices and market shares, changing the industry structure at the retail level would not affect the brand equity states that we estimate in the first stage. What would change is the period profit function. In principle, our framework can be applied using whichever assumption about industry structure at the retail level is most appropriate for the application at hand.

[9] To simplify exposition, we suppress the dependence of C_j on $\mathbf{p}_j$, $\boldsymbol{\omega}$, and $\boldsymbol{\zeta}_j$.

[10] See http://www.pg.com/en_US/downloads/investors/investor_presentation.pdf.

[11] We need not estimate σ_m because the market size m_j enters the demand function (3) multiplicatively and is independent of all other model parameters. It follows that μ_m is multiplicatively separable from the rest of the profit function (6). Therefore, in approximating the profit function, we can simply replace m_j with its expectation μ_m.

[12] These dynamic implications build on prior work that emphasizes optimal advertising policies for forward-looking firms, starting with the seminal Nerlove and Arrow (1962) paper and more recently in work that has used the Nerlove and Arrow (1962) framework to estimate how sales and market shares respond to advertising (Horsky 1977, Chintagunta and Vilcassim 1992, Dubé et al. 2005, Dubé and Manchanda 2005, Sriram and Kalwani 2007), the relationship between advertising and goodwill (Doganoglu and Klapper 2006), and advertising and brand perceptions (Clark et al. 2009).

[13] Because product market competition occurs at the weekly level, in Step 3, firms compete in the product market 13 consecutive times (i.e., for one quarter), with the prevailing brand equities held fixed before moving on to subperiod 2. Product market competition takes place after advertising decisions have been implemented because advertising—unlike R&D—affects both current and future payoffs.

[14] While the literature offers no clear guidance on the relationship between brand strength and advertising effectiveness (e.g., Tellis 1988, Aaker and Biel 1993), Bagwell (2007, p. 1739) notes that the preponderance of evidence supports diminishing returns to advertising.

[15] This decision rule can be represented either with the cutoff entry cost $\tilde{\phi}_n$ or with the probability $\xi_n \in [0,1]$ that firm n enters the industry in state $\boldsymbol{\omega}$, for there is a one-to-one mapping between the two via $\xi_n = \int 1(\phi_n \le \tilde{\phi}_n) dF(\phi_n) = F(\tilde{\phi}_n)$, where $1(\cdot)$ is the indicator function.

[16] While we define value and policy functions (as well as brand value in definition (13)) as if $n = 1$, the analogous functions for firm 2 ($n = 2$) are defined similarly.

[17] See Gowrisankaran and Town (1997), Laincz (2005), Doraszelski and Markovich (2007), Borkovsky et al. (2010), Goettler and Gordon (2011), Narajabad and Watson (2011), Borkovsky et al. (2012), Goettler and Gordon (2014), and Borkovsky (2016).

[18] For example, to compute an equilibrium at the estimated parameterization, these expectations were computed 1,397,656 times using our closed-form expressions.

[19] Schmidt-Dengler (2006), Igami (2015), and Takahashi (2015) also estimate dynamic games using a full-solution method.

[20] To accurately calculate standard errors for the dynamic model, we need to account for the estimation error in our brand equity estimates. To do so, we sample brand equities from the estimated distributions (see Table 2) 100 times. For each sample of brand equities, we create a new discretization as in Equation (8), compute the expected profit function (as in the bottom right panel of Figure 3), and reestimate our dynamic model. The most burdensome part of this process is computing the expected profit function, as this entails re-solving the static pricing game for *each* industry state many times (once for each pair of $\mathbf{c}$ and $\boldsymbol{\zeta}$ that are drawn). Furthermore, this must be done separately for each of the 100 samples of brand equities because, as explained above, each gives rise to a different discretization. To make this process more efficient, we do not resample the price coefficient κ, the nested parameter σ, or the standard deviation σ_ζ of the mean-zero firm-store-specific shock ζ_{jn} because the standard errors for these parameters are small relative to their estimated values. This approach allows us to use the same Monte Carlo samples and corresponding equilibrium price calculations as in the initial expected profit function calculation via an importance sampler.

[21] Because firms are symmetric, the value of firm 1's brand in industry state $\boldsymbol{\omega}$ is equal to the value of firm 2's brand when the equities are reversed, i.e., $v_1(\omega_1, \omega_2) = v_2(\omega_2, \omega_1)$.

[22] The intuition for the difference between the standard error of the brand value and the change in brand value is that the brand value measures are comparisons of the equilibrium value function in two quite different industry states (factual and counterfactual), whereas the change in brand value compares nearby industry states, as the counterfactuals get differenced out. The comparison of nearby states is more precise than the comparison of distant states, and so the standard errors are smaller.

[23] We include the entry policy function for completeness but do not discuss it in detail. As noted in Section 5.2, the entry cost parameters are not estimated because entry occurs only once in the data.

[24] Firm n's expected brand equity in quarter q is $E_q(\omega_n) = \sum_{\omega\in\{1,\ldots,M\}^2} \omega_n \times \mu_q(\boldsymbol{\omega} \mid \boldsymbol{\omega}^0)$, where $n = 1$ denotes Pringles, $n = 2$ denotes STAX, and the transient distribution over the set of industry states for quarters $q = 1, \ldots, 140$, given initial industry state $\boldsymbol{\omega}^0$, is denoted by $\mu_q(\boldsymbol{\omega} \mid \boldsymbol{\omega}^0)$.

[25] We say that the industry has converged to its long-run industry structure when the sup norm of the difference between successive transient distributions is less than 10^{-5}.

[26] Irrespective of the initial industry state, the industry always converges to the same long-run industry structure.

[27] We do not run counterfactuals on the variance of the effectiveness of advertising because we find such counterfactuals hard to motivate. Such counterfactuals do give rise to asymmetric long-run industry structures at certain values for the variance of the effectiveness of advertising.

[28] We compute counterfactual equilibria using a Gauss–Jacobi algorithm that is similar to the Pakes and McGuire (1994) algorithm in that it incorporates iterated best reply and value function iteration, but it does so using the closed-form expressions (Equations (A14) and (A15) in the online appendix) and the expected value function (Equation (A18) in the online appendix). As detailed in the online appendix, we have searched for multiple equilibria using a simple continuation method and have not found any instances of multiplicity.

[29] By computing equilibria for all (δ_d, δ_u) pairs that satisfy the restriction $\delta_d + \delta_u \leq 1$, we have verified that our results are robust to this assumption about how δ_u varies as we vary δ_d.

[30] At $\delta_d = 0.1, 0.5$, and 0.9, it takes 115, 128, and 195 quarters, respectively, for the factual and Pringles counterfactual industry structures to converge.

[31] The convexity of the period profit function should *not* be regarded as simply an artifact of the exponential nature of the nested logit demand system in Section 4.2. Rather, the convexity reflects the fact that an increase in a firm's brand equity gives rise to both a higher equilibrium price *and*—despite the higher price—a higher quantity demanded. Accordingly, this convexity arises in even much simpler quality ladder models.

[32] As discussed in Section 5.1, α_n is an increasing function of γ_n, so it too captures the effectiveness of advertising. We focus our analysis on γ_n simply because the mean of α_n is undefined if $\theta(\omega_n) \geq 0.5$, which happens in some industry states for almost all of the counterfactual equilibria that we compute.

[33] The mean $h\theta(\omega_n)$ and variance $h\theta(\omega_n)^2$ of firm n's effectiveness of advertising are functions of its brand equity state ω_n. In varying λ, we are varying the mean effectiveness of advertising *for each brand equity state* by a multiplicative factor of λ while holding fixed the variance of the effectiveness of advertising *for each brand equity state*. However, for any given λ value, the mean and variance vary across brand equity states.

[34] In particular, we compute equilibria for $(\delta_d, \lambda) \in \{0, 0.1, \ldots, 1\} \times \{0.5, 0.6, \ldots, 3\}$ and $(\delta_d, z) \in \{0, 0.1, \ldots, 1\} \times \{0, 0.003, \ldots, 0.03\}$. We do not compute equilibria for $z > 0.03$ because at $z = 0.03$, the firm-specific depreciation rate in the highest brand equity state is 1, i.e., $\delta_f(35) = 1$.

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