Channel Management for Basic Products and Ancillary Services

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We study a supply chain where a supplier, such as an original equipment manufacturer (OEM), sells a basic product and an ancillary service through a retailer. Random demands for both product and service can be influenced by costly sales efforts by the retailer. If the service attachment is sales effort independent, it is possible that a price-only contract selling the basic product at margin and charging back from the service product can coordinate the supply chain, due to the complementary effect between the basic product and its ancillary service and the inventory-riskless feature of the service product. For the general case, we develop a coordinating scheme of giving up all service margin and offering a quantity discount schedule for the basic product that can arbitrarily share the benefits of coordination. We further provide an alternative coordination scheme that can be implemented over existing wholesale prices for the product and service. This alternative scheme provides discounts to the retailer when her order quantity for the product and the attach rate for the service exceed respective thresholds. We show that in the absence of coordination, supply chain profits are higher if the retailer instead of the supplier provides the service assuming both have the same cost structure. We also show that a win-win mechanism can coordinate the system in the important case where the retailer provides the service but the supplier has a cost advantage on spare parts.

1. Introduction

There is an extensive body of supply chain literature that studies mechanisms for coordinating the channel and sharing profits from the sale of a good between retailers and suppliers. However, in increasingly competitive and commoditized markets for goods, there is little remaining profit to share. Product margins are so slim that retailers and suppliers recognize post-sales services such as extended warranties as essential to their profitability. Indeed, Berner (2004) finds that retailer profit
margins on these services can reach 50-60% – nearly 18 times higher than the typical margins on the basic products. Profits from extended warranties and other post-sales services constitute almost half of Best Buy’s operating profits and all of Circuit City’s profits. It is not surprising that these services are promoted aggressively by retailers’ sales representatives. These efforts tend to pay off: the percentage of consumers purchasing extended warranties ranges from 20% for automobiles to 75% on products such as home electronics and appliances (see Desai and Padmanabhan 2004). Even no-frills retailers like Walmart and Costco have added extended warranties and service offerings in recent years.

In addition to high profits, post-sales services help suppliers and retailers maintain a connection with their customers. This increased intimacy can lead to up-selling opportunities and higher customer loyalty. Post-sales services also extend the useful life of products, which can drive higher sales of consumables. Moreover, services are more easily differentiated than commodity products, thus providing a way for retailers and suppliers to set themselves apart from competition.

This paper introduces the notion of ancillary services into the realm of channel management research, by examining price-only contracts and coordination mechanisms in a supply chain with both basic products and ancillary services. In the model that we analyze, a single supplier sells a basic product and ancillary service to customers through a single retailer. We first investigate the possibility to coordinate the supply chain by price-only contracts. Due to the complementarity between the basic product and the ancillary service and the inventory-riskless feature of the service product, it is possible for the supplier to coordinate the supply chain by selling the basic product at margin and charge back from the ancillary service as long as the service attachment is sales effort independent. Then for the general case, we seek a mechanism to coordinate the supply chain and arbitrarily divide the benefits of coordination. We present a quantity discount schedule contract that can achieve supply chain coordination and be beneficial to both the supplier and retailer. The quantity discount schedule for services is simple and extreme as it calls for the supplier to sell the service at cost leaving all the profits from the sales of services to the retailer. The quantity discount schedule for the basic product is higher than it would be without the sales of services. The quantity discount schedule for the basic product can be designed to arbitrarily split the gains from coordination resulting in a win-win contract. Stated differently, the quantity discount schedule for the basic product embeds profit sharing for the services while providing additional profits to both parties. One further advantage of this schedule is that it forms a barrier to entry as the retailer has less of an incentive to compete by offering its own services. This is particularly true if the supplier enjoys economies of scale in the overall cost of providing the ancillary service.
In an extension of the model, we also investigate the situation where the retailer instead of the supplier provides the ancillary service. Assuming that the retailer can provide the service at the same price and at the same cost as the supplier we show that the quantity discount schedule that coordinates the supply chain is of the same form as when the supplier provides the service. The only difference is that the range of the profit sharing parameter is shifted in the favor of the retailer. We also show that total supply chain profits are higher, in an uncoordinated setting, when the retailer instead of the supplier provides the service. This is true because ownership by the retailer avoids the distortion of sales effort that occurs when each supply chain member earns a partial margin on services.

Our results further extend to the important practical situation where the retailer instead of the supplier provides the service but the supplier has a cost advantage on the procurement of spare parts and the retailer has a cost advantage in terms of labor cost and proximity to end customers. In this case the supplier can coordinate the supply chain by providing the spare parts at cost and a quantity discount schedule for the basic product that embeds the supplier’s share of profits for the parts. This scheme can be valuable when it is difficult for the supplier to compete against the retailer selling ancillary services.

The preceding results are obtained in a setting where retail prices are assumed exogenous, and the retailer controls the product order quantity and sales effort exerted for both the basic product and the ancillary service. We consider an extension of the basic model in which the retailer also controls the retail prices of the service and product. We present channel coordinating wholesale price contracts in this setting that can arbitrarily divide the benefits of coordination. One of the coordinating schedules that is dependent on the order quantity and posted retail price can be viewed as a combination of a quantity discount contract and price-discount sharing scheme.

The rest of the paper is organized as follows. Section 2 discusses the relevant literature. Section 3 describes the basic model and provides structural properties. Section 4 examines price-only contracts. Section 5 describes a quantity discount schedule that coordinates the supply chain and arbitrarily splits profits between the supplier and retailer, as well as alternative mechanisms to implement the schedule. Section 6 investigates the case where the retailer provides the service and compares supply chain profits to the case where the supplier provides the service. Section 7 considers the case when the retail prices are endogenous. Section 8 makes final remarks and indicates future research directions.
2. Literature Review

Research in managing post-sales services such as extended warranties through distribution channels is limited. Li et al. (2005) compare supply chain profits depending on whether the retailer or the supplier (not both) offers an extended warranty. Lutz and Padmanabhan (1995) and Lutz and Padmanabhan (1998) analyze the effects of third party extended warranty providers on a market where a monopolist supplier sells a bundled product and base warranty. They demonstrate the effects of a third party insurer on the supplier’s optimal base warranties and profits. Heese (2008) examine interactions in a supply chain with two suppliers competing for product sales through a common retailer who profits from selling extended warranties. Given both the manufacturer and retailer can offer the extended warranties, Hsiao et al. (2010) investigate the profitability of three strategies from the manufacturer's perspective in a distribution channel: deter, acquiesce, or foster. None of these papers examines the problem of supply chain coordination with simple price-only contracts or more sophisticated contracts.

Desai and Padmanabhan (2004) examine a model with a single supplier and a single retailer where the supplier sells a basic product and an extended warranty through the retailer. The supplier controls the wholesale price of the product and the service. The retailer controls the retail price of the product and service. There are constant marginal selling costs for the basic product and the service. Demands for both product and service are assumed to be deterministic and linear in price. They consider three cases depending on whether the service is sold through the retailer, directly to customers or through both channels. In each scenario they consider a Stackelberg game where the supplier sets the wholesale prices first and then the retail prices are determined in the second stage. They observe a “complementary goods effect” in the presence of the extended warranty. Specifically, the retailer sets a lower retail price on the basic product when it is also selling the extended warranty than it would if it were selling the basic product alone. Moreover, under some conditions on the selling costs and warranty cost, the supplier makes more profit by selling the warranty through the retailer than directly. Our work confirms many of their findings in a more general context of random, not necessarily linear, demands and sales efforts. For example, Desai and Padmanabhan (2004) only consider deterministic demand and ignore the inventory problem which is of concern in the case of stochastic demand. In the analysis of price-only contracts, we discuss a more general leverage effect, which is driven by the combination of the “complementary goods effect” and inventory-riskless feature of the service product. Moreover, we focus on the development of mechanisms to coordinate the channel, to avoid the problem of double marginalization, and
to arbitrarily split profits. These mechanisms include quantity discounts schedules that can be implemented over existing linear wholesale prices.

We design a quantity discount schedule to coordinate the supply chain with service attachments. There are many papers that consider how quantity discounts and other schemes can affect channel coordination in a supply chain concerned with selling a single product. Among these, one group of papers considers a setting in which the downstream channel member controls only his order quantity with exogenous retail price and deterministic demand. In particular, Lal and Staelin (1984) consider a price discount schedule to induce the buyer to increase his order quantity to the joint optimal order quantity. A second group of papers considers the case in which the demand rate is a deterministic but decreasing function of price. The retailer controls both the retail price and the order quantity, and the supplier controls the quantity discount. In an extension, the retailer may also control sales effort and the supplier may control product quality. These papers explore pricing mechanisms that induce channel coordination and profit sharing. Jeuland and Shugan (1983) consider a simultaneous game setting in which both channel members face fixed and variable costs without inventory or ordering costs. They show how a quantity discount schedule comprised of a fixed cost and an all-unit discount can induce channel coordination and win-win with arbitrary division of profits between the supplier and retailer. Moorthy (1987) considers the same problem, but in a Stackelberg game setting in which the supplier leads with the wholesale pricing schedule, and the retailer follows with his order quantity and retail price decision. Weng (1995) extends Jeuland and Shugan (1983) in a Stackelberg context to include ordering and holding costs as a function of order quantities, and shows that a combination of franchise fees and quantity discounts applied simultaneously can be used to ensure supply chain coordination.

We take into consideration sales effort for both the basic product and ancillary services in the channel coordination problem. Several papers consider supply chain coordination for the basic product, without services, by allowing the retailer to exert costly effort to increase demand. Cachon and Lariviere (2005) show that revenue sharing does not coordinate the supply chain when demand depends on costly retail effort. They also present a quantity discount contract, see Theorem 5 (on page 42), that falls short of coordinating the supply chain. Taylor (2002) shows that the combination of a sales rebate contract with a buy-back contract coordinates the supply chain for a multiplicative demand model. In our models we avoid the use of sales rebates because the retailer has an incentive to report sales above his order quantity if sales cannot be independently verified by the supplier. Our model, even in the absence of the ancillary service, is the first to coordinate the supply chain with costly retail effort without the use of sales rebates. Our channel
coordinating results are valid for a larger class of demand models than those treated previously in the literature. Krishnan et al. (2004) also study the combination of a sales rebate contract with buybacks. The main thrust of this work is to understand moral hazard issues that arise in supply chain coordination. In their model, they assume that sales are verifiable and allow the retailer to choose effort after observing demand or a demand signal. We assume as in Taylor (2002) and Cachon and Lariviere (2005) that effort decisions for the basic product are made before observing demand or a demand signal. We think this is a more realistic assumption as demand is typically censored and not observable. In contrast, the opportunity to attach ancillary services is limited to the number of basic units sold, so one may be tempted to apply the sales effort model in Krishnan et al. (2004) to this case. That model, however, assumes that the retailer can select a level of effort to influence the exact number of customers who will buy the service. More precisely, they assume that after observing demand they can select a multiplicative factor. We prefer to model the case where efforts by the retailer influence the probability that a customer who bought the basic product will buy the ancillary service and allow for some post-effort randomness that is outside the control of the retailer.

3. The Model
We consider a supply chain with one supplier (referred as “she”) and one retailer (referred as “he”). Both firms are risk-neutral. We assume that both the product and the service are provided by the supplier but sold through the retailer who can influence demand for both the basic product and the ancillary service by exerting sales efforts. We assume that the retail price for both the basic product and the ancillary service are exogenous. Exogenous retail prices are common in the contracting literature (see Pasternack 1985, Lariviere 1999 and Narayanan and Raman 2007) mostly to isolate contracting issues although other justifications are also discussed in these references. In Section 7 we allow the retailer to set retail prices and show that the coordinating mechanism obtained under exogenous retail prices extends to the case of endogenous retail prices.

The supplier’s goal is to design a sales mechanism for the basic product and the ancillary service to induce the retailer to make order quantity and sales effort decisions that coordinate the supply chain. The supplier also desires a sales mechanism that allows for arbitrary distributions of the gains from coordination. The sequence of events are as follows: i) the supplier presents the retailer a quantity discount schedule for the basic product and a price for the service; ii) the retailer decides on the order quantity for the product and sales effort for the product and the service; iii) demand for the product and for the service are realized and revenues are collected by the retailer. We will show, under mild assumptions, that it is indeed possible to find a mechanism that coordinates the
supply chain while allowing arbitrary distributions of the gains of coordination. In contrast to the existing literature, the mechanism we propose here does not require buy-backs or sales rebates as these instruments require the monitoring of sales by the supplier and are subject to gaming by the retailer.

The retailer faces random demand $D_\nu$ for the basic product with cumulative distribution function $F(x|\nu) = P(D_\nu \leq x)$ where $\nu$ is a parameter, such as the mean demand, that can be influenced by the retailer’s effort. As pointed out by Taylor (2002), retailers can influence demand by merchandizing, doing point-of-sale advertising, providing attractive shelf space and guiding consumer purchases with sales personnel. Price discounts can also be viewed as sales efforts to stimulate demand, while price increases can be viewed as negative effort that typically result in lower demand. Since we will be dealing explicitly with endogenous retail prices in Section 7, we can and do limit the scope to situations where efforts do not alter the retail prices. Notice that the parameter $\nu$ does not fully determine demand as demand may have random components outside the retailer’s control. This is typical of the situations pointed out by Taylor (2002). We now summarize the assumptions about $D_\nu$ that apply throughout the paper.

**Assumption 1.** $D_\nu$ is stochastically increasing and concave in $\nu$.

By stochastically increasing we mean that $\bar{F}(x|\nu) = 1 - F(x|\nu)$ is increasing in $\nu$ for each $x$. We mean increasing and decreasing in the weak sense unless we specify otherwise. For stochastic concavity, we require a common probability space such that

$$\alpha D_{\nu_1} + (1 - \alpha) D_{\nu_2} \leq D_{\alpha \nu_1 + (1 - \alpha) \nu_2}, \quad \forall \alpha \in [0, 1], \ \nu_1, \nu_2 > 0.$$ 

This assumption holds, for example, when $D_\nu = h(\nu)\theta + \epsilon$, $h(\nu)$ is concave and increasing in $\nu$ and $\theta$ is a non-negative random variable. Our setup is quite general and includes the so-called additive case and the multiplicative demand case that is the focus of Taylor (2002).

The cost of sales effort for the product, $k(\nu), \nu \geq \nu_0 \geq 0$ is assumed to be an increasing, convex and continuously differentiable function with $k(\nu_0) = 0$, e.g. $k(\nu) = 0.5B(\nu - \nu_0)^2$ with $B > 0$ defined on $\nu \geq \nu_0$. We will assume that $k(\nu)$ is public information. Later we will explain how the supplier may estimate these functions by experimenting with different wholesale prices.

If the retailer orders $Q$ units and sets the sales effort parameter at $\nu$, his sales will be $\min(Q, D_\nu)$ so the ordering and sales effort decisions are intimately related. Each unit of product sold is an opportunity to sell an ancillary service. To model the cost of attaching ancillary services to product sales we will assume the retailer makes an effort to sell the service for each of the $\min(Q, D_\nu)$
units of the basic product sold. The sales effort influences the attach rate \( a \in (0, 1) \) of the ancillary product, which is defined as the probability that a customer that bought the basic product will also buy the service. Efforts to attach services are often similar to those used to sell basic products. Spiffs (sales promotion incentive funds, see Caldieraro and Coughlan 2007), however, tend to be more common in selling services than in the sale of basic products. The cost of effort is modeled through an increasing, convex and continuously differential function \( v(a), a \geq a_0 \) with \( v(a) = 0 \) for \( a \leq a_0 \), so the cost of attaching services is \( v(a) \min(Q, D_v) \). We also assume that the cost effort function \( v(a) \) is publicly known and will later describe mechanism to estimate \( v(a) \).

If the retail price for the service is \( p_s \) and the wholesale price is \( w_s \) then \( r = p_s - w_s \) is the retailer’s margin per unit of service sold. Since the expected profit from services to the retailer is proportional to \( ra - v(a) \), he will select \( a \) to maximize \( ra - v(a) \). We will assume that \( v(a) \) increases sufficiently fast over the interval \( [a_0, 1] \) so that the equation \( v'(a) = r \) has a root \( a(r) \leq 1 \) for all \( 0 \leq r \leq \bar{r} = p_s - c_s \), where \( c_s \leq w_s \) is the service unit cost to the supplier and \( \bar{r} \) is the margin for the supply chain for each unit of service sold.

Given unit wholesale price \( w \) for the basic product, unit salvage value \( z < w \), and unit service margin \( r \), the retailer must select the order quantity \( Q \) for the basic product, and effort levels \( \nu \) and \( a \), respectively, for the basic product and the ancillary service to maximize his expected profit:

\[
\pi_R(Q, \nu, a|w, r) = -k(\nu) - wQ + (p + ra - v(a))E\min(Q, D_v) + zE(Q - D_v)^+ \\
= -k(\nu) - (w - z)Q + (p + ra - v(a) - z)E\min(Q, D_v).
\]

The expected profit for the supplier is given by

\[
\pi_S(Q, \nu, a|w, r) = (w - c)Q + (\bar{r} - r)aE\min(Q, D_v),
\]

where \( c \) is the unit cost for the product. The expected profit for a coordinated supply chain is given by \( \pi_C(Q, \nu, a) = \pi_R(Q, \nu, a|c, \bar{r}) \), and corresponds to the case where the retailer can procure at the supplier’s unit costs. The following proposition, whose proof is in the Appendix, provides sufficient conditions for concavity.

**Proposition 1.** If \( D_v \) is stochastically increasing and concave in \( \nu \), then \( \pi_R(Q, \nu, a|w, r) \) is jointly concave in \( Q \) and \( \nu \), and \( \pi_C(Q, a) = \min_{\nu} \pi_C(Q, \nu, a) \) is concave in \( Q \) for all \( a \) such that \( p + ra - v(a) \geq z \).
3.1. Retailer’s Problem, Benchmark Solution and Comparative Statics

In a decentralized setting, independent of the order quantity \(Q\), it is optimal for the retailer to select \(a = a(r)\), the solution to the first order condition \(v'(a) = r\). Due to the joint concavity, the retailer’s optimal order quantity \(Q_{R}(w,r)\) and sales effort \(\nu_{R}(w,r)\) satisfy the following first order conditions (note that these conditions are different from those proposed in Cachon and Lariviere 2005, Theorem 5):

\[
F(Q|\nu) = \frac{p(r) - w}{p(r) - \bar{z}},
\]

\[
k'(\nu) = \frac{z}{p(r) - \bar{z}} \frac{\partial E}{\partial \nu} \min(Q, D_{\nu}),
\]

where \(p(r) = p + ra(r) - v(a(r))\) is the retail price of the basic product plus the optimal expected profit for services net of the cost of effort, per unit of the basic product sold.

The benchmark solution, corresponding to a coordinated supply chain, is \(\bar{r} = r\) and \(w = c\), resulting in \(a = a(\bar{r})\), order quantity \(Q_{C} = Q_{R}(c,\bar{r})\) and effort \(\nu_{C} = \nu_{R}(c,\bar{r})\). The expected profit for the coordinated supply chain is \(\pi_{C} = \pi_{C}(Q_{C},\nu_{C},a(\bar{r}))\). The retailer’s solution when \(w_{s} = p_{s}\), or equivalently, \(r = 0\), corresponds to \(Q_{0} = Q_{R}(w,0)\) and \(\nu_{0} = \nu_{R}(w,0)\). The following Proposition confirms that larger order quantities and sales effort result from more favorable wholesale prices to the retailer.

**Proposition 2.** \(Q_{R}(w,r)\) and \(\nu_{R}(w,r)\) are decreasing in \(w \in [c, p]\) and increasing in \(r \in [0, \bar{r}]\). Moreover,

\[
Q_{0} < Q_{R}(w,r) < Q_{C}, \quad \nu_{0} < \nu_{R}(w,r) < \nu_{C} \quad \text{for all } w \in (c, p), \quad r \in (0, \bar{r}).
\]

The first inequality in Proposition 2 shows that adding an ancillary service motivates the retailer to place a larger order for the basic product and this is because the underage cost for the basic product is larger in the presence of the ancillary service. The second inequality shows that the retailer’s optimal order quantity is below the optimal order size in a coordinated supply chain with the same being true for the exerted effort. The shortfall in the order quantity and effort has to do with double marginalization.

3.2. Estimation of Retail Effort Cost Functions

The effort cost functions \(v(a)\) and \(k(\nu)\) are assumed known in the literature as in Taylor (2002), Cachon and Lariviere (2005) and Krishnan et al. (2004). In practice, these costs are not known by the supplier. Here we propose a mechanism that can be used for the supplier to estimate these costs using quadratic equations. Given any constant wholesale price scheme \((w,r)\), \(w \in (c, p), r \in (0, \bar{r})\), the
supplier can interpret the retailer’s order quantity \( Q(w, r) \) as the best-response function \( Q_R(w, r) \).

Suppose the supplier wants to fit the effort cost functions into a quadratic form, e.g., \( v(a) = 0.5A(a - a_0)^2 \) defined on \( a \geq a_0 \) and \( k(\nu) = 0.5B(\nu - \nu_0)^2 \) defined on \( \nu \geq \nu_0 \). The supplier needs to estimate the four parameters \( A, a_0, B \) and \( \nu_0 \). We can show that \( Q_R(w, r) \) is strictly decreasing in \( w \) and strictly increasing in \( r \). As a result, four different pairs of \( (w, r) \) and the corresponding order quantities \( Q(w, r) \) can be substituted into the set of equations (1)-(2) to estimate the unknown parameters. If the form is not quadratic, more pairs can be tried and a higher order curve can be fit by minimizing the mean square errors.

One may argue that the retailer may purposely distort his best response functions \( Q_R(w, r) \) to thwart the supplier’s attempt to learn about his effort cost function. While plausible, we think this is unlikely for the following reasons: i) the retailer may not know that the supplier is trying to learn his cost functions, ii) using suboptimal order quantities results in lower profits for the retailer during the experimental phase, and iii) the retailer may benefit from the coordinating scheme offered by the supplier after she estimates his cost functions.

4. Price-Only Contracts

Suppose the supplier only uses price-only contracts (see Lariviere and Porteus 2001), which are simple to implement. We investigate the optimal price-only contracts and if it is possible that they coordinate the supply chain. The supplier’s objective is to maximize profits by optimally selecting price-only contracts \( (w, r) \). For simplicity of exposure, in this section we assume that the demand of the basic hardware is sales-effort independent and we denote it by \( D \) with the subscript suppressed. Later we will discuss this simplification does not change the managerial insights we will obtain. We still maintain the assumption that the demand for the ancillary service can be influenced by retailer’s sales effort.

Given any supplier’s price-only contracts \( (w, r) \), the optimal retailer’s order quantity \( Q_R(w, r) \) satisfies the first-order-condition

\[
F(Q_R(w, r)) = 1 - \frac{w}{p(r) - \bar{z}}.
\]

The supplier faces a demand curve \( Q_R(w, r) \) as a function of \( w \) and \( r \). Her inverse demand curve of \( w \) as a function of \( Q \) and \( r \) can be written as \( w(Q, r) = (p(r) - \bar{z}) \bar{F}(Q) \). Rather than considering the decision \( (w, r) \) for the supplier, we consider the equivalent decision \( (Q, r) \). We write the supplier’s profit as a function of \( Q \) and \( r \) as

\[
\pi_S(Q, r) = (w(Q, r) - c)Q + (\bar{r} - r)a(r)E \min(Q, D).
\]
We take derivative of $\pi_S(Q,r)$ with respect to $r$ and obtain
\[
\frac{\partial \pi_S(Q,r)}{\partial r} = \frac{\partial w(Q,r)}{\partial r} Q + [(\bar{r} - r) \frac{\partial a(r)}{\partial r} - a(r)] E \min(Q,D).
\]
We know $w(Q,r) = (p(r) - z)\bar{F}(Q)$, hence
\[
\frac{\partial w(Q,r)}{\partial r} = \frac{\partial p(r)}{\partial r}\bar{F}(Q) = a(r)\bar{F}(Q),
\]
by noting that $a(r)$ is the solution to the first order condition $v'(a) = r$. If we assume the demand distribution satisfies the assumption of increasing generalized failure rate (IGFR), the optimally desired supplier’s sales $Q^*(r) = \max_{Q \geq 0} \pi_S(Q,r)$ as a function of $r$ can be characterized by the first order condition $\frac{\partial \pi_S(Q,r)}{\partial Q} = 0$ (see Lariviere and Porteus 2001). Hence,
\[
\frac{\partial \pi_S(Q^*(r),r)}{\partial r} = \frac{\partial \pi_S(Q,r)}{\partial Q} \bigg|_{Q=Q^*(r)} \frac{\partial Q^*(r)}{\partial r} + \frac{\partial \pi_S(Q,r)}{\partial Q} \bigg|_{Q=Q^*(r)} \frac{\partial a(r)}{\partial r}.
\]

In one extreme, suppose the service attach rate is sales-effort independent (i.e., $\frac{\partial a(r)}{\partial r} = 0$), in other words, it does not demand any service margin to promote sales effort and boost attach rate. Then it is optimal for the supplier to leave as little service margin as possible to the retailer, as suggested by $\frac{\partial \pi_S(Q^*(r),r)}{\partial r} < 0$ that is left with only the negative effect. This is because the service product does not involve any risk of holding inventory, hence the supplier would like to lower the wholesale price of the basic product as much as possible to alleviate the double marginalization of the basic product as long as it is possible to charge back from the service product. Note that this leverage incentive to drive down the wholesale price for basic product and jack up the wholesale price for service product is due to the following two reasons: First, there exists a complementary effect between basic product and its ancillary services, in particular, if the sales of the basic product is higher, there is more opportunity to attach the ancillaries. Second, there exists the double marginalization problem for the basic product, but not for the service product. Both reasons call for a as-low-as-possible wholesale price of the basic product which can help increase the stock level and alleviate the double marginalization. Note that this argument is still valid when the basic product sales can be influenced by the retailer’s effort. In return, the supplier charges back from the service product. If it maintains profitability for the supplier by selling to the retailer
at the marginal cost of the basic product and charging back from setting a high wholesale price for the ancillary service, as long as the service attach rate is sales-effort independent, the supplier can coordinate the chain with price-only contracts.

On the other hand, if the service attach rate depends heavily on sales effort of the retailer, the service margin left to the retailer cannot be too low. In this case, leaving enough service margin to the retailer to motivate service attachment (positive attach effect) and leveraging service margin to offer a low wholesale price for the basic product (negative leverage effect) compete in setting the optimal service margin. When the negative leverage effect dominates, e.g., it is more likely the case if the total margin on the service is more lucrative than that of the basic product, it is optimal for the supplier to sell the basic product at margin and sell the ancillary service at a moderate wholesale price, within the set of price-only contracts. It is worthwhile to note that this scheme completely removes the double marginalization problem for the basic product but the double marginalization problem for the service product remains.

5. Supply Chain Coordination through Quantity Discounts
Recall that in a coordinated supply chain it is optimal to set \( a = \bar{a} = a(\bar{r}) \). Let \( \bar{p} = p(\bar{r}) = p + \bar{r}a(\bar{r}) - v(\bar{a}) \), then the coordinated supply chain expected profit can be expressed as

\[
\pi_C(Q, \nu) = -k(\nu) - cQ + R(Q, \nu),
\]

where

\[
R(Q, \nu) = \bar{p}E \min(Q, D, \nu) + zE(Q - D, \nu)^+ = (\bar{p} - z)E \min(Q, D, \nu) + zQ.
\]

Let \( \nu(Q) \) be the smallest effort that maximizes \( \pi_C(Q, \nu) \) for any fixed \( Q \). Then \( \nu(Q) \) is the smallest solution to \( k'(\nu) = R_2(Q, \nu) \) where \( R_2 \) is the partial derivative of \( R(Q, \nu) \) with respect to \( \nu \) and

\[
\pi_C(Q) = \pi_C(Q, \nu(Q)) = -k(\nu(Q)) - cQ + R(Q, \nu(Q)).
\]

From Proposition 1, we know that \( \pi_C(Q) \) is concave. Moreover, we know that \( Q_C = Q_R(c, \bar{r}) \) maximizes \( \pi_C(Q) \).

**Theorem 1.** Setting the wholesale price of the service at \( w_s = c_s \) (or equivalently, setting \( r = \bar{r} \)) and offering a quantity discount scheduled, based on unit wholesale price

\[
w(Q) = \gamma c + (1 - \gamma) \frac{R(Q, \nu(Q)) - k(\nu(Q))}{Q}
\]

distributes the supply chain giving \( \gamma \) fraction of the chain profits to the retailer and \( 1 - \gamma \) fraction of the profits to the supplier, for \( 0 \leq \gamma \leq 1 \).
Notice that the coordinating mechanism gives all the profits of selling services to the retailer by setting \( w_s = c_s \). It has long been known that offering a product at cost and extracting a fixed payment from the retailer is a way to coordinate the supply chain for a basic product. This scheme is known as “selling the store” or “renting the store”, so it is not surprising that the idea of selling the service at cost will emerge in this context. The “fixed payment” as in a two-part tariff is embedded in the quantity discount price for the basic product. Indeed, the average wholesale price for the basic product is higher by \( (1 - \gamma)(\bar{r}a(\bar{r}) - v(a(\bar{r})))E \min(Q, D(\nu(Q)))/Q \) and this quantity is approximately equal to, but due to demand uncertainty less than, \( (1 - \gamma)(\bar{r}a(\bar{r}) - v(a(\bar{r}))) \) which is the supplier’s share \( 1 - \gamma \) times the net service profit per unit of basic product sold.

Since the coordinated solution dominates the uncoordinated solution, lower and upper bounds can be put on \( \gamma \) so that both the supplier and the retailer are better off than they would be with any constant wholesale price. This ensures a win-win solution with arbitrary distributions of profits from coordination.

Notice that we can write the quantity discount schedule as \( w(Q|\bar{r}) \) to emphasize its dependence on the gross margin \( \bar{r} \) for the service. It is easy to see that \( w(Q|\bar{r}) \) is increasing in \( \bar{r} \) so the unit wholesale price for the basic product is higher the higher the gross margin for the ancillary service. In particular, the unit wholesale price for the basic product is higher than it would be in the absence of services.

5.1. Implementation of Quantity Discount Schedule

As stated, the quantity discount schedule in Theorem 1 may be subject to legal restraints and to arbitrage opportunities in the case of multiple retailers. There are several ways of mitigating these problems. Legally, one could argue that the quantity discount schedule takes into account the cost of effort of the retailer and thereby it is justified to have different quantity discount schedules for different retailers. We refer the reader to Krishnan et al. (2004) and reference therein for a brief account of recent lawsuits related to quantity discounts that have been successfully defended. The arbitrage problem is often solved in practice by designing retailer-specific stock keeping units (SKUs) which makes it easier for the supplier to identify units diverted to other retailers.

The supplier may also wish to find mechanisms that work within existing contracts. For example, if prior to engaging in channel coordination efforts there is a contract with fixed wholesale prices for the basic product and the service then the issue is how to implement the quantity discount schedule of Theorem 1 on top of existing wholesale prices.

In this section we provide two ways to implement this. The first is in terms of a quantity discount schedule that works for order quantities that are larger than the one currently used by the
retailer. We also consider an implementation mechanism that allows the supplier to charge different wholesale prices for the basic product and the product bundled with the service, where both of these prices are associated with the total volume of the basic product ordered by the retailer.

5.1.1. Incremental Incentives Suppose that prior to engaging in channel coordination efforts the supplier uses fixed wholesale prices for both the product and service. For example, the supplier may be using wholesale prices \( w \in (c, p) \) and \( r \in (0, \bar{r}) \) that maximize his expected profits in the absence of channel coordination. The retailer will select service effort \( a \in (0, \bar{a}) \) where \( a \) is the root of \( v'(a) = r \), order quantity \( Q_R = Q_R(w, r) < Q_R(c, \bar{r}) = Q_C \) and optimal product sales effort \( \nu_R = \nu_R(w, r) < \nu_R(c, \bar{r}) = \nu_C \). The supplier can design incentives to induce the retailer to align his order quantity and sale efforts with the optimal of the whole supply chain. In designing an incremental incentive for services we find it expedient to present the retailer with an incremental margin \( r(a) \) for service effort level \( a > a(r) \). For any such \( a \) that is determined ex ante, the retailer makes expected service margin \( r(a) \) as an incentive to increase his effort from \( a(r) \) to \( \bar{a} = a(\bar{r}) \) and \( \tau(Q) \) as an incentive to increase his order from \( Q_R \) to \( Q_C \).

\textbf{Proposition 3.} The service effort incentive

\[ r(a) = \frac{\bar{r} - r(a)}{a - a(r)} \]

on the attach rate \( a > a(r) \) and the incremental quantity discount schedule

\[ \tau(Q) = \frac{(w - w(Q))Q}{Q - Q_R} \]

on orders \( Q > Q_R \) coordinate the supply chain.

Notice that the supplier does not need to offer the retailer the entire quantity discount schedules \( (4) \) and \( (5) \). It is enough to provide the retailer with \( \bar{r} = r(\bar{a}) \) as an incentive to increase his effort from \( a(r) \) to \( \bar{a} = a(\bar{r}) \) and \( \tau(Q_C) \) as an incentive to increase his order from \( Q_R \) to \( Q_C \).

5.1.2. Service Premium A different way to modify the implementation is to charge different prices for products sold with or without the service. The way Theorem 1 is stated, \( w(Q) \) should be charged on every unit of product ordered by the retailer. The retailer may not understand why he needs to pay a premium on all products. An alternative is to set \( r = \bar{r} \) and to have a base price \( w_n(Q) \) that is charged for all products and a premium \( p_{s}(Q) \) charged on the products sold with service. Then

\[ w_n(Q) = w(Q) - (1 - \gamma)[(p_s - c_s)\bar{a} - v(\bar{a})]E \min(Q, D_s(Q))/Q \]
and
\[ p_p(Q) = (1 - \gamma)[(p_s - c_s) - v(\bar{a})/\bar{a}] \]
achieve the same purpose as the quantity discount schedule \( w(Q) \). To see this notice that the retailer will still select service effort \( \bar{a} \) ex ante and his expected ordering cost is \( w_n(Q)Q + p_p(Q)\bar{a}E \min(Q, D_v(Q)) = w(Q)Q \), which would lead him to select hardware effort \( \nu(Q_C) \) and order size \( Q_C \).

5.2. Additive Demand Model and Numerical Example

We now explore in detail the additive demand model where \( D_v = \nu + \epsilon \) and \( \epsilon \) is a uniform random variable with range \([-\Delta, \Delta]\) for some \( \Delta \leq \nu_0 \). In the Appendix we show that if \( k(\nu) = 0.5B(\nu - \nu_0)^2 \) then
\[ Q_C = \nu_0 + \left[ \frac{2\phi}{\alpha(1 - \alpha)} \right] \Delta, \tag{6} \]
where \( \phi = (c - z)/(\bar{p} - z + 2\Delta B) \), and
\[ \nu_C = (1 - \alpha)\nu_0 + \alpha(Q_C + \Delta), \]
where \( \alpha = (\bar{p} - z)/(\bar{p} - z + 2\Delta B) \).

We now apply these results to the case \( c = 1000, p = 1150, z = 950, \bar{r} = 100 \). Assume that the effort cost for attaching sales is \( v(a) = 0.5A(a - a_0)^2, a \in [a_0, 1] \), where \( A = 400, a_0 = .20 \). Then \( a(\bar{r}) \) is the solution to \( 400(a - a_0) = 100 \), or equivalently \( a(\bar{r}) = .45 \) and \( v(a(\bar{r})) = 200(.25)^2 = 12.50 \) and \( \bar{p} = p + \bar{a}a(\bar{r}) - v(a(\bar{r})) = 1150 + 45 - 12.50 = 1182.50 \). Let the cost of effort be of the form \( k(\nu) = 0.5B(\nu - \nu_0)^2, \nu_0 = 500 \), then for \( \Delta = 100 \) and \( B = (\bar{p} - z)/2\Delta = 1.1625 \) we have \( \alpha = 1/2, \phi = .1075 \) so \( Q_C = 714, \nu_C = 657 \), and net expected profit \( \pi_C = $101,651 \).

This same solution and profit can be obtained by using the quantity discount schedule in Theorem 1. Namely, \( r = \bar{r} \) and
\[ w(Q) = \gamma c + (1 - \gamma) \left[ (\bar{p} - z)E \min(Q, \nu_0 + \alpha(Q + \Delta - \nu_0)) + zQ - 0.5B\alpha^2(Q + \Delta - \nu_0)^2 \right]. \]

In contrast, an uncoordinated supply chain would use a fixed wholesale price for both the product and the ancillary service. We numerically solve for the optimal wholesale price contract \((w, r)\) for the supplier resulting in \((w, r) = (1100, 40)\). The retailer then selects \( a(r) = .30 \), \( Q_R = 509 \) and \( \nu_R = 552 \), resulting in retailer’s expected profit \$27,263, supplier’s expected profit \$59,914 and total expected profit \$87,177 which is 14% lower than the coordinated solution. The retailer will be better off with the coordinated solution for any \( \gamma \geq 0.2682 \) while the supplier will be better off with any \( \gamma \leq 0.4106 \). Thus, any \( \gamma \in (0.2682, 0.4106) \) results in a win-win solution. For example, for
\[ \gamma = .30 \] the retailer’s profit would increase from $27,263 to $30,495 and the supplier’s from $59,914 to $71,155.

Consider now the mechanism that sets \( r = \bar{r} \), charges \( w = 1100 \) for the first 509 units and offers an incremental quantity discount schedule

\[
\tau(Q) = \frac{(1100 - w(Q))Q}{Q - 509}
\]

on each unit \( Q > 509 \). As shown in Proposition 3, this incremental quantity discount schedule coordinates the supply chain and is win-win for any \( \gamma \) in the interval \((0.2682, 0.4106)\).

6. Retailer-Provided Services

We now investigate what happens when the retailer introduces the ancillary service to an existing supply chain providing the basic product. We will first consider the case where the initial relationship between the supplier and the retailer is governed by a fixed wholesale price \( w \in (c, p) \). Later we will study the situation where there is initially a quantity discount schedule that coordinates the supply chain for the sale of the basic product only, with a given profit sharing parameter \( \gamma \).

We examine these scenarios in the context of the Example in Section 5.2. The benefits of channel coordination for various scenarios can be generally illustrated as in Figure 1.
6.1. Existing Wholesale Price Contract

For the constant wholesale price \( w \in (c, p) \), the order quantity and effort in the scenario without ancillary services will be solutions to the first order conditions (1) and (2) with \( r = 0 \). We will denote the solution as \( Q_R(w, 0) \) and \( \nu_R(w, 0) \) to emphasize the dependence of the procurement quantity and the effort on both \( w \) and \( p \). For the data of the Example in Section 5.2, \( w = 1100 \) and \( p = 1150 \), so \( Q_R(w, 0) = 493, \nu_R(w, 0) = 543 \) resulting in retailer’s profit $22,325 and supplier’s profit $49,300. Suppose the retailer introduces the ancillary service with unit cost \( c_s \) and retail price \( p_s \) resulting in margin \( \bar{r} = p_s - c_s \) and effort \( a(\bar{r}) \). As a consequence, his net price per unit of basic product sold increases from \( p \) to \( \bar{p} = p + \bar{r}a(\bar{r}) - v(a(\bar{r})) \). For the data of the Example in Section 5.2, \( \bar{r} = $100, \bar{p} = $1,182.50 \) and \( Q_R(w, \bar{r}) = 542, \nu_R(w, \bar{r}) = 571 \) with retailer’s profit $38,855 and supplier’s profit $54,200. Notice that the improvement in profits is respectively 74% and 10% so the supplier benefits substantially from the introduction of services by the retailer. The improvement of 10% for the supplier on selling the basic product comes completely from the increase in the retailer’s procurement quantity from 493 to 542 units.

The total supply chain profit, when the wholesale price for the basic product is \( w \) and the retailer owns the service business, orders \( Q_R(w, \bar{r}) \) and makes effort \( \nu_R(w, \bar{r}) \), is given by

\[
\pi_C(w, \bar{r}|\bar{r}) = \pi_R(Q_R(w, \bar{r}), \nu_R(w, \bar{r}), \bar{a}|c, \bar{r}).
\]

Also, let

\[
\pi_C(w, r|\bar{r}) = \pi_R(Q_R(w, r), \nu_R(w, r), \bar{a}|c, \bar{r})
\]

be the total supply chain profit when the wholesale price for the basic product is \( w \), the supplier owns the service, and the retailer gets margin \( r \in (0, \bar{r}) \). For the preceding example, we have \( \pi_C(w, \bar{r}|\bar{r}) = $93,055 \) and \( \pi_C(w, r|\bar{r}) = $87,177 \) for \( r = 40 \). The difference is not a coincidence as attested by the following result.

**Theorem 2.** For \( 0 \leq r < \bar{r} \), we have \( \pi_C(w, \bar{r}|\bar{r}) > \pi_C(w, r|\bar{r}) \).

Without coordination, the supply chain has higher efficiency when the retailer owns the service than when the supplier owns it. The reason is that when the retailer owns the service, he has a larger net profit and therefore a higher underage cost. This induces him to carry a larger quantity of the basic product and to exert more effort. In addition since he fully captures the margins on the service, there is no double marginalization problem for the ancillary service.

The supplier may propose the following quantity discount schedule to coordinate the channel and distribute the gains from coordination:

\[
w(Q|c, \bar{r}) = \gamma c + (1 - \gamma) \frac{R(Q_R(c, \bar{r})) - k(\nu(c, \bar{r}))}{Q}.
\]
Table 1  Example Revisited

<table>
<thead>
<tr>
<th>Scenario</th>
<th>(\pi_R)</th>
<th>(\pi_S)</th>
<th>(\pi_C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>without ancillary service</td>
<td>$22,325</td>
<td>$49,300</td>
<td>$71,625</td>
</tr>
<tr>
<td>service provided fully by the retailer</td>
<td>$38,855</td>
<td>$54,200</td>
<td>$93,055</td>
</tr>
<tr>
<td>service provided by the supplier ((r = $40))</td>
<td>$27,263</td>
<td>$59,914</td>
<td>$87,177</td>
</tr>
<tr>
<td>coordinated supply chain ((\gamma = 0.40))</td>
<td>$40,660</td>
<td>$60,990</td>
<td>$101,650</td>
</tr>
</tbody>
</table>

Example in Section 5.2. \(w = $1100\).

Notice that this schedule is of exactly the same form as the quantity discount schedule of Theorem 1 that applies when the supplier owns the ancillary service. What differs is the range of values of \(\gamma\) over which the quantity discount schedule is win-win. In our example, when the retailer fully owns the ancillary service the range of values that lead to a win-win solution is \(\gamma \in (0.3822, 0.4668)\) in contrast to \(\gamma \in (0.2682, 0.4106)\) which is the win-win range when the supplier owns the service and sells it through the retailer. Since \(\gamma\) represents the retailer’s share, it is clear that the retailer is in a stronger position when he owns the service. Under the proposed coordination, the choice of \(\gamma = 0.40\), for example, improves the retailer’s profits from $38,855 to $40,660 and the supplier’s profit from $54,200 to $60,990. Notice that it is compared against a solution in which the retailer owns the service and is better for both the retailer and the supplier relative to the uncoordinated solution where the supplier owns the service. Indeed, the expected profits under that case are, respectively, $27,263 and $59,914. This illustrates that sometimes the disadvantage to the supplier of not owning the service may be offset by a combination of the larger order quantity that prevails when the retailer introduces the service and the design of a win-win quantity discount schedule that coordinates the supply chain. Table 1 summarizes the profits for both parties under different scenarios.

In many practical situations we would expect one of the supply chain members to have a cost advantage in terms of providing the ancillary service. In the case of extended warranties, the supplier may enjoy a cost advantage if he either manufactures the parts or has access to them at lower prices. In this case, total supply chain profits will be higher if the supplier provides the service in conjunction with the quantity discount schedule \(w(Q)\) and \(r = \bar{r}\) as in Theorem 1. In the context of the preceding example, suppose that the unit cost of providing the service is $10 higher for the retailer and so his gross margin is \(\bar{r} = 90\) instead of \(\bar{r} = 100\). Then a coordinated supply chain with retailer-provided service will have total expected profit equal to $98,805. In the uncoordinated chain, with the retailer owning the services, he would make $36,551 while the supplier will make $53,600, so the range of win-win sharing parameters is \((.3699, .4575)\). If \(\gamma = .40\) is used the retailer gets $39,522 which is less than $40,660 he would get if the supplier owned the service.
6.2. Existing Quantity Discount Schedule for Basic Product

Suppose now there is already a quantity discount schedule \( w(Q|c, \bar{r} = 0) \) prior to retailer’s service introduction that increases the combined profit of selling per unit of product from \( p \) to \( \bar{p} = p + \bar{r}a(\bar{r}) - v(a(\bar{r})) \), where \( \bar{r} \) is the net profit of the retailer service. The retailer will then take advantage of the existing quantity discount schedule and place a larger order quantity enjoying most of the benefits of introducing the ancillary service.

To see this more formally, let \( C(Q|\bar{r}) = \max_{\nu \geq 0} \{-k(\nu) + R(Q, \nu)\} \) and \( \pi_R(\bar{r}, w(Q)) = \max_{Q \geq 0} \{-w(Q)Q + C(Q|\bar{r})\} \) denote the optimal profit of the retailer under any net revenue \( \bar{r} \) per unit of the service product and any quantity discount schedule \( w(Q) \). In particular, \( \pi_C(\bar{r}) = \pi_R(\bar{r}, c) \) is the total profit of the centralized supply chain who faces the marginal cost \( c \) per unit of product. The following result shows that with the existing quantity discount schedule unchanged, the retailer gains proportionally more than the supplier without further coordination after introducing retailer-owned services.

**Theorem 3.** For any \( \bar{r} > 0 \), \( \pi_R(\bar{r}, w(Q|c, 0)) > \gamma \pi_C(\bar{r}) > \gamma \pi_C(0) = \pi_R(0, w(Q|c, 0)), \) where \( w(Q|c, 0) = \gamma c + (1 - \gamma)C(Q|0)/Q \), \( 0 < \gamma < 1 \) is the existing coordination quantity discount schedule before the service introduction.

This result illustrates the danger of offering a quantity discount schedule that does not have an upper bound on the order quantity. It can be partially fixed by limiting orders to at most \( Q_C(c, \bar{r}) \) units. The upper bound on the order quantity is needed because without it the retailer can attach services resulting in higher order quantities at discounted prices that do not benefit the supplier who designed the quantity discount schedule without considering ancillary services. An incremental quantity discount schedule that drops the quantity discount to \( w_h = c_h \) beyond a certain order quantity is even more dangerous but can also be partially remedied by placing an upper bound \( Q \leq Q_C(c, \bar{r}) \) on the order quantity.

6.3. Comparative Advantages and Coordination to Jointly Offer Services

Consider now the scenario where the total cost of providing the ancillary service can be decomposed into two components: parts and labor. Let \( P_s \) and \( L_s \) stand for the part and labor cost respectively of the supplier’s service product, and \( P_r \) and \( L_r \) stand for the part and labor cost respectively for the retailer’s service product of the same quality as the supplier’s. We assume that the supplier has a comparative advantage in the cost of spare parts while the retailer enjoys a comparative advantage related to labor costs, i.e., \( P_s < P_r \) and \( L_s > L_r \). This usually holds in practice since the
supplier is closer to the manufacturer of spare parts and the retailer is usually closer to the end consumers making the execution of the service more cost efficient.

An optimally integrated supply chain would take advantage of the comparative advantages of each firm, with the supplier providing the parts and the retailer the labor. When the service supply chain is operated in this way, the coordination mechanism described in Theorem 1 applies, with the supplier providing the parts at cost, and the retailer providing the labor and optimizing her effort to sell services. The quantity discount schedule for the basic product contains the supplier’s profit share for the parts. The decentralized payoffs of both parties can be used to determine an incentive compatible profit sharing range in the coordinating quantity discount contract.

These results apply to situations where the retailer is already selling the service but the supplier has a cost advantage in procuring the parts. The supplier would offer the parts at cost and will apply the incremental quantity discount schedule in Proposition 3 on orders that exceed the status quo. In addition to providing the parts at cost, the supplier can help the retailer brand the service. This may be particularly valuable if the supplier has a better reputation than the retailer. This branding can help increase sales volume and may be used by the supplier in negotiating the profit sharing parameter $\gamma$.

7. Endogenous Retail Prices

In this section we will assume that demands for both the basic product and the ancillary service are price sensitive and that the retailer sets both the price $p$ of the basic product and the price $p_s$ of the ancillary service. Let $D_\nu(p)$ be the demand for the basic product under sales effort $\nu$. We will assume that $D_\nu(p)$ is strictly stochastically decreasing in $p$ and $\lim_{p \rightarrow \infty} pE[D_\nu(p)] = 0$, i.e., expected revenues decrease to zero as the price increases to infinity.

Recall that we use the attach rate $a$ to measure the service sales effort in the basic model of exogenous retail prices. For the case of endogenous retail prices, we generalize the service sales effort cost function to a function $v(a,p_s)$ of service sales effort $a$ and ancillary service retail price $p_s$. We will assume that $v(a,p_s)$ is a strictly increasing function of $p_s$ given any attach rate $a$. In other words, charging a higher retail price $p_s$ for the ancillary service results in a higher cost to maintain the same attach rate.

The retailer’s profit function is given by

$$\pi_R(Q,\nu,p,p_s,a|w,w_s) = -k(\nu) - (w - z)Q + [p - s + (p_s - w_s)a - v(a,p_s)]E\min(Q,D_\nu(p)),$$

where retail prices $p$ and $p_s$ are decision variables in addition to order quantity $Q$, the sales efforts $\nu$ and $a$. 
Let $\pi_C(Q, \nu, p, p_s, a) = \pi_R(Q, \nu, p, p_s, a | w = c, w_s = c_s)$ be the expected profit of a coordinated supply chain. We will now optimize this function with respect to the five variables under consideration. We will start with the decisions related to $a$ and $p_s$. Clearly $a$ and $p_s$ should be selected to maximize $(p_s - c_s)a - v(a, p_s)$. The first order conditions are $p_s - c_s = v_1(a, p_s)$ and $a = v_2(a, p_s)$. We will assume that the first order conditions have a unique solution $(a^*, p_s^*)$ and that this solution represents a global optimizer with respect to $a$ and $p_s$. A sufficient condition for this is that $v$ is increasingly convex in $(a, p)$ and that the mixed partial derivative $v_{12} \in (0, 1)$ for all $(a, p)$. For example, the function $v(a, p) = 0.5p(a - a_0)^2$ satisfies these properties.

The coordinated supply chain expected profit reduces to

$$
\pi_C(Q, \nu, p) = -k(\nu) - cQ + R(Q, \nu, p),
$$

where

$$
R(Q, \nu, p) = (p - z + (p_s^* - c_s)a^* - v(a^*, p_s^*))E\min(Q, D_{\nu}(p)) + zQ.
$$

Let $\nu(Q, p)$ be the smallest effort that maximizes $\pi_C(Q, p, \nu)$ for any fixed $p$ and $Q$. Then $\nu(Q, p)$ is the smallest solution to $k'(\nu) = R_2(Q, \nu, p)$ where $R_2$ is the partial derivative of $R$ with respect to $\nu$. By the same logic demonstrated in Theorem 1, we have the following coordinating nonlinear quantity discount schedule, which can be viewed as a combination of a nonlinear quantity discount schedule and a nonlinear price-discount sharing (PDS) scheme (See Bernstein and Federgruen 2005).

**Corollary 1.** For any $Q$ and $p$, the schedule $w_s = c_s$ and

$$
w(Q, p) = \gamma c + (1 - \gamma)\frac{R(Q, \nu(Q, p), p) - k(\nu(Q, p))}{Q}
$$

coordinates the supply chain giving $\gamma$ fraction of the chain profits to the retailer and $1 - \gamma$ fraction of the profits to the supplier, for $0 \leq \gamma \leq 1$.

Furthermore, suppose the maximum of the optimization problem $\max_{p, \nu} \pi_C(Q, \nu, p)$ for any fixed $Q$ exists and let us denote the maximizers by $\bar{p}(Q)$ and $\bar{\nu}(Q)$. Similar to Theorem 1, we have the following nonlinear quantity discount schedule with endogenous price $\bar{p}(Q)$.

**Corollary 2.** For any $Q$, the schedule $w_s = c_s$ and

$$
w(Q) = \gamma c + (1 - \gamma)\frac{R(Q, \bar{\nu}(Q), \bar{p}(Q)) - k(\bar{\nu}(Q))}{Q}
$$

coordinates the supply chain giving $\gamma$ fraction of the chain profits to the retailer and $1 - \gamma$ fraction of the profits to the supplier, for $0 \leq \gamma \leq 1$. 
8. Conclusion

Ancillary services represent a significant revenue opportunity. In many industries, ancillary services contribute as much or more revenue and profit margin than basic products. They also benefit revenues indirectly, by extending the useful life of the installed base and thereby ensuring steady sales of consumables for those products.

In view of the huge opportunities in the service sector, we investigated various mechanisms for a supplier selling a basic product and an ancillary service through a retailer for situations where the retailer can influence demand by exerting effort. We show how the supplier can experiment with different wholesale prices to estimate the effort cost functions for products and services. With a reliable estimate of the effort cost functions, the supplier can implement a quantity discount mechanism to coordinate the supply chain and arbitrarily share the benefits of coordination. The mechanism calls for giving up all the supplier’s profits for the service and getting the money back through the quantity discount scheme for the basic product. This contract with an appropriate sharing mechanism may be a barrier to entry for the retailer to enter the market with its own ancillary service. The mechanism can also be implemented by charging a different, higher price, for products sold with the attached service, and it can be implemented on top of existing wholesale prices. Furthermore, if the service attachment is sales effort independent, we see that it is possible to coordinate the supply chain by a totally different but simple price-only contract that calls for selling the basic product at margin but charging back through the service product.

We also investigate the case where the retailer provides the ancillary service and show that in an uncoordinated setting this leads to higher supply chain profits assuming the retailer has the same cost structure for services as the supplier. Moreover, we show that the quantity discount schedule that coordinates the supply chain and the total supply chain profits are the same regardless of who owns the service. The only difference is in the profit sharing parameter range that leads to mutual gains.

For situations where the retailer provides the service and the supplier has a cost advantage on parts, it is possible to coordinate the supply chain where the supplier provides the parts at cost and presents the retailer with a quantity discount scheme that embeds the profit for the supplier for the parts. Our results extend to endogenous retail prices for both the basic product and the service.

A direction for future research is to include competition, including several suppliers selling similar basic products and services through a common retailer who also offers ancillary services. There is a relevant mechanism to shift profits to service capacity providers that has been used successfully
by airlines in their negotiations with travel agents such as Expedia and Travelocity. In this arrangement, the airlines insist on a minimum sales volume to trigger the payment of commissions. Airlines will either get a free ride and pay no commission for sales below the trigger point or will enjoy high levels of sales. The imposition of these triggers is particularly helpful for providers with large market shares that pay small commissions as they are able to steer retail efforts in their direction at the expense of providers with small market shares that pay higher commission margins. This happens because in the absence of triggers the retailer would naturally favor the company paying higher margins. We think that some of these techniques have a potential of playing an important role in the negotiation of sales of basic products as well as ancillary services, but such techniques need to be carefully designed to be applied successfully.

Appendix. A. Proofs

Proof of Proposition 1. Lemma 1. If \( f(x, y), x, y \in \mathbb{R} \) is jointly concave in \( x \) and \( y \), non-decreasing in \( y \), and \( y(\nu), \nu \in \mathbb{R} \) is concave in \( \nu \), then \( f(x, y(\nu)) \) is jointly concave in \( x \) and \( \nu \).

Proof. Suppose \( \alpha \in [0, 1] \). For any \( x_1, x_2, \nu_1, \nu_2 \in \mathbb{R} \), we have \( \alpha y(\nu_1) + (1 - \alpha)y(\nu_2) \leq y(\alpha \nu_1 + (1 - \alpha)\nu_2) \) by the concavity of \( y(\nu) \). Further by the joint concavity and monotonicity of \( f(x, y) \),

\[
\alpha f(x_1, y(\nu_1)) + (1 - \alpha)f(x_2, y(\nu_2)) \leq f(\alpha x_1 + (1 - \alpha)x_2, \alpha y(\nu_1) + (1 - \alpha)y(\nu_2)) \leq f(\alpha x_1 + (1 - \alpha)x_2, y(\alpha \nu_1 + (1 - \alpha)\nu_2)).
\]

Lemma 2. \( E \min(Q, D_\nu) \) is jointly concave in \( Q \) and \( \nu \).

Proof. Since \( D_\nu \) is concave in \( \nu \) and \( \min(Q, D) \) is jointly concave in \( Q \) and \( D \) and increasing in \( D \), then \( \min(Q, D_\nu) \) is jointly concave in \( Q \) and \( \nu \) by a direct application of Lemma 1. More precisely,

\[
\alpha \min(Q, D_{\nu_1}) + (1 - \alpha) \min(Q, D_{\nu_2}) \leq \min(Q, D_{\alpha \nu_1 + (1 - \alpha)\nu_2}), \quad \alpha \in [0, 1].
\]

Since concavity is preserved by taking expectations it follows that \( E \min(Q, D_\nu) \) is jointly concave in \( Q \) and \( \nu \).

If \( p + ra - v(a) > s \) then \( (p + ra - v(a) - z)E \min(Q, D_\nu) \) is jointly concave in \( Q \) and \( \nu \). Subtracting the linear term \( (c - z)Q \) and the convex term \( k(\nu) \) preserves the joint concavity so \( \pi_R(Q, \nu, a|w, r) \) is jointly concave. The concavity of \( \pi_C(Q, a) \) follows from the well know projection theorem (see Sobel and Heyman 1984). □
Proof of Proposition 2. We first establish the following result.

Lemma 3. $E \min(Q, D_v)$ has increasing differences in $(Q, \nu)$. Moreover, $\pi_R(Q, \nu, a|w, r)$ has increasing differences in $(Q, \nu)$ for all $a$ such that $p + ra - v(a) > s$. Finally, $\pi_R(Q, \nu, a|w, r)$ has increasing differences in $(Q, a)$ and $(\nu, a)$ for all $a \in [a_o, a(r)]$, where $a(r)$ is the root of the equation $v(\nu) = r$.

Proof. Since $\partial E \min(Q, D_v)/\partial Q = \tilde{F}(Q|\nu)$ is increasing in $\nu$ on account of $D_v$ being stochastically increasing, it follows that $E \min(Q, D_v)$ has increasing differences in $(Q, \nu)$. Subtracting individual terms in $Q$ and $\nu$ preserves the property of increasing differences so $\pi_R(Q, \nu, a|w, r)$ also has increasing differences in $(Q, \nu)$ for any $a$ such that $p + ra - v(a) > s$. Moreover, if $r \geq v'(a)$, $\partial \pi_R(Q, \nu, a|w, r)/\partial a = (r - v'(a))E \min(Q, D_v)$ is increasing in $Q$ and $\nu$. Thus $\pi_R(Q, \nu, a|w, r)$ has increasing differences in $(Q, a)$ and $(\nu, a)$ for $a \in [a_o, a(r)]$. $\blacksquare$

We are now equipped to prove the monotonicity of $Q_R$ and $\nu_R$ in $w$ and $r$. It is easy to see that $\partial \pi_R(Q, \nu, a|w, r)/\partial Q$, $\partial \pi_R(Q, \nu, a|w, r)/\partial \nu$, and $\partial \pi_R(Q, \nu, a|w, r)/\partial a$ are decreasing in $w$ and increasing in $r$. By Lemma 3, $\pi_R(Q, \nu, a|w, r)$ has increasing differences in $(Q, \nu, a)$. Then by the comparative statics of increasing differences (see Vives 1999), the interior solutions to the concave optimization problem have the claimed monotone properties. The inequality (3) follows directly from the monotonicity of $Q_R$ and $\nu_R$ and the definitions of the boundary points $Q_0, Q_C$ and $\nu_0, \nu_C$. $\blacksquare$

Proof of Theorem 1. With $\nu = \bar{\nu}$ the retailer faces the same optimization problem as the supply chain in terms of $a$, so he selects $a = a(\bar{\nu})$. The retailer's remaining problem is

$$\pi_R(Q, \nu) = -k(\nu) - wQ + R(Q, \nu),$$

and it is optimal for him to select effort $\nu(Q)$ to maximize $-k(\nu) + R(Q, \nu)$, so his remaining problem is

$$\pi_R(Q) = -k(\nu(Q)) - wQ + R(Q, \nu(Q)).$$

Faced with a quantity discount schedule $w(Q)$ the retailer’s function becomes $\gamma \pi_C(Q)$ so he will select an order quantity that maximizes the profit of the coordinated supply chain, with $\gamma \pi_C$ going to the retailer and $(1 - \gamma) \pi_C$ to the supplier. $\blacksquare$

Proof of Proposition 3. Consider first the incentive schedule on service effort. To maximize the expected profit of services the retailer will select an ex-ante service effort level $a \geq a(r)$ to maximize

$$ra(r) + r(a)(a - a(r)) - v(a) = \bar{r}a - v(a)$$
resulting in \( a = a(\bar{r}) \). Consequently it is optimal for the retailer to select \( \nu(Q) \) for any order quantity \( Q \).

Given the rebate schedule for the basic product the retailer’s problem is given by

\[
\max_{Q \geq Q_R} \left[ R(Q, \nu(Q)) - k(\nu(Q)) - (w - \tau(Q))(Q - Q_R) - wQ_R \right].
\]

Now \((w - \tau(Q))(Q - Q_R) + wQ_R = w(Q)Q\), so the retailer’s objective reduces to

\[
\min_{Q \geq Q_R} \left[ R(Q, \nu(Q)) - k(\nu(Q)) - w(Q)Q \right],
\]

and we know that this objective is equal to \( \gamma \pi_C(Q) \), so the retailer is induced to select an order quantity to coordinate the supply chain. \( \Box \)

**Proof of Theorem 2.** The uncoordinated supply chain profits when the retailer owns the service are given by

\[
\pi_C(w, \bar{r}|\bar{r}) = (\bar{p} - z)E \min(Q_R(w, \bar{r}), D_{\nu_R(w, \bar{r})}) - (c - z)Q_R(w, \bar{r}) - k(\nu_R(w, \bar{r})).
\]

In contrast, the uncoordinated supply chain profits when the supplier owns the service are given by

\[
\pi_C(w, r|\bar{r}) = (p(r) - z)E \min(Q_R(w, r), D_{\nu_R(w, r)}) - (c - z)Q_R(w, r) - k(\nu_R(w, r)),
\]

where \( p(r) = p + \bar{r}a(r) - v(a(r)) < \bar{p} \). Notice that the net profit per unit is lower with the supplier owned service due to the double marginalization of the service which also results in less effort \( a(r) < a(\bar{r}) \). Moreover, since \( Q_R(w, r) < Q_R(w, \bar{r}) \) and \( \nu_R(w, r) < \nu_R(w, \bar{r}) \), the order quantity and the sales of the basic product are also lower and as a result \( \pi_C(w, r|\bar{r}) < \pi_C(w, \bar{r}|\bar{r}). \) \( \Box \)

**Proof of Theorem 3.** Realizing \( R(Q, \nu) = \bar{p}E \min(Q, D_{\nu}) - zE(Q - D_{\nu})^+ \) is strictly increasing in \( \bar{p} \) for any \( Q \geq 0 \) and \( \nu \geq \nu_0 \), we have \( \bar{p} = p + \bar{r}a(\bar{r}) - v(a(\bar{r})) > p \) leading to \( C(Q|\bar{r}) > C(Q|0) \) for \( \bar{r} > 0 \). Then

\[
\pi_R(\bar{r}, w(Q|c, 0)) = \max_{Q \geq 0} \{-cQ - (1 - \gamma)C(Q|0) + C(Q|\bar{r})\}
\]

\[
> \max_{Q \geq 0} \{-cQ - (1 - \gamma)C(Q|0) + C(Q|0)\} = \gamma \pi_C(0) = \pi_R(0, w(Q|c, 0)),
\]

where the last equality is due to the definition of the coordinated quantity discount schedule. \( \Box \)

**Appendix. B. Additive Model**

Suppose that \( D_{\nu} = \nu + \epsilon \) is non-negative for all \( \nu \geq \nu_0 \) where \( \epsilon \) is a mean zero random variable. Let \( F(y|\nu) = P(D_{\nu} \leq y) \). Then

\[
\frac{\partial}{\partial \nu} E \min(Q, D_{\nu}) = F(Q|\nu).
\]
This follows because $E \min(Q, D_\nu) = \int_0^Q F(y|\nu)dy$ where $F(y|\nu) = P(\nu + \epsilon \leq y) = G(y - \nu)$ so taking derivatives with respect to $\nu$ under the integral results in $\partial E \min(Q, D_\nu)/\partial \nu = \int_0^Q g(y - \nu)dy = G(Q - \nu) = F(Q|\nu)$.

For $r = \bar{r}$ and any given $Q$, an optimal effort to sell the basic product can be found by solving the equation $R_2(Q, \nu) = k'(\nu)$. This is equivalent to solving

$$(\bar{p} - z)F(Q|\nu) = k'(\nu),$$

since $R(Q, \nu) = (\bar{p} - z)E \min(Q, D_\nu) + zQ$.

If $\epsilon$ is uniform $[-\Delta, \Delta]$ with $\Delta \leq \nu_0$ and the effort cost function is $k(\nu) = 0.5B(\nu - \nu_0)^2$, then for $Q \in [\nu - \Delta, \nu + \Delta]$ the first order condition is given by

$$(\bar{p} - z)\frac{Q + \Delta - \nu}{2\Delta} = B(\nu - \nu_0)^+.$$ 

This equation has a unique solution in $\nu$ only for values of $Q \geq \nu_0 - \Delta$. Since the demand is guaranteed to be at least this large this is not a practical restriction as it will always be optimal to select $Q$ within the stated range. For values of $Q$ in the interval $[\nu_0 - \Delta, \nu_0 + \beta \Delta]$, the solution is given by

$$\nu(Q) = (1 - \alpha)\nu_0 + \alpha(Q + \Delta),$$

where $\alpha = \frac{\bar{p} - z}{\bar{p} - z + 2\Delta}$ and $\beta = (1 + \alpha)/(1 - \alpha)$, so the optimal effort is a convex combination of $\nu_0$ and $Q + \Delta$.

To maximize $R(Q, \nu(Q)) - cQ - k(\nu(Q)) = (\bar{p} - z)E \min(Q, D_{\nu(Q)}) - (c - z)Q - k(\nu(Q))$ over this interval we first investigate the derivative with respect to $Q$ for values of $Q$ in the interval $[\nu_0 - \Delta, \nu_0 + \beta \Delta]$. Since

$$E \min(Q, \nu(Q) + \epsilon) = \alpha Q + E \min((1 - \alpha)Q, \nu_0 + \alpha(\Delta - \nu_0) + \epsilon),$$

the derivative of $R(Q, \nu(Q))$ with respect to $Q$ is given by $\bar{p} - z$ times

$$\alpha + (1 - \alpha)P(\epsilon \geq (1 - \alpha)(Q - \nu_0) - \alpha \Delta) = \alpha + \frac{1 - \alpha^2}{2} - \frac{(1 - \alpha)^2}{2\Delta}(Q - \nu_0).$$

The derivatives of the other two terms are simply $-(c - z)$ and $-\alpha^2 B(Q + \Delta - \nu_0)$, so the total derivative is decreasing in $Q$ guaranteeing that the function is concave. Setting the derivative equal to zero and solving for $Q$ we obtain Equation (6).

Notice that this always gives an interior solution $Q_C \in (\nu_0 - \Delta, \nu_0 + \beta \Delta)$ since $Q_C$ is within the bounds whenever $\phi \in (0, \alpha)$ but we already know that $\phi < \alpha$, and $\phi > 0$ whenever $z < c$. 
References


