Profit-Increasing Asymmetric Entry

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Abstract
Despite conventional wisdom that firm profits decrease with competitive entry, the empirical literature finds a number of situations where the entry of an equivalent-quality competitor into a market led to higher profits for the incumbents. Our paper uses a standard linear Hotelling (1929) market to study how location choices affect the possibility that profits increase for the incumbents with competitive entry by a new rival. We show that profits of all incumbents can increase after competitive entry even if the new competitor is located such that it competes directly with only one of the incumbents. This asymmetric entry demonstrates two separate mechanisms that can lead to profit increases: first, profits can increase because a rival’s pricing incentives have changed, causing the rival firm to increase its price. Second, entry can change a firm’s own pricing incentives, which in turn cause the competitor to increase its price anticipating a higher-priced equilibrium. Either of these effects on their own can be sufficient for profits to increase. Of course, increased prices do not imply increased profits. Rather, profits only increase in scenarios where the higher prices offset the lost sales. As such, we note that profits for a monopolist are always higher than profits for firms facing any level of competition. Further, we show that average industry profits must eventually go to zero. Thus, the relationship between profits and the number of firms in the market for industries with profit-increasing entry follows a down-up-down shape. We discuss the implications this has on interpreting results from the empirical entry literature.

Key words: market entry, spatial models, direct and indirect competitors, marketing battlefield, dimensions of competition.
1. Introduction

The literature in economics, industrial organization and marketing relies on the assumption that increased levels of competition lead to lower average prices, reduced profits, and higher levels of total welfare. In fact, competition policy is designed to reduce market concentration and ensure healthy levels of competition within categories. Similarly, conventional wisdom states that in order to protect above-normal levels of profit, incumbent firms should erect “barriers” that make it difficult for new firms to enter the market.¹

Empirical observation, however, paints a more-complicated picture about the relationship between entry and profits. For example, it has been observed that the opening of a new Starbucks may lead to higher profits for existing coffee shops that serve a market.² Profits have also been observed to increase after the entry of new competitors in the energy, soap and fashion industries. One can often explain profit increases after competitive entry by complementarities or by increased levels of marketing within a category (for example, Lipitor’s entry in the statin market led to significantly higher levels of category marketing spending).³ However, in many observed cases of entry leading to increased profits for incumbents, there are no complementarities and the new product is clearly a substitute for the incumbent products.

In this paper, we analyze the potential for profit-increasing entry using a model of horizontal differentiation. The new firm enters at a location where it only competes with one of the incumbent firms. We show that profits can increase for both incumbents. The basic intuition for why profits can increase with entry is that competitive entry can lead to increased, rather than decreased, prices, and that the increase in prices sometimes offset the losses from having fewer customers post-entry.

Several papers show that prices can increase with entry (Hauser and Shugan 1983, Perloff et. al. 1996, Thomadsen 2007 and Chen and Riordan 2008), but price increases are not equivalent to profit increases. In fact, in the above papers, firms that raise their prices all experience declining profits. This obtains because the papers consider models where the market moves from monopoly to duopoly. In fact, firms are always better off as a monopolist than as a firm facing competition. This result can be stated as a theorem: Under any standard choice model, a monopolist has weakly

¹ Bain (1956) notes that incumbents can act together to create barriers to entry and thereby maintain their profitability in the face of potential entry.
² See Clark (2007).
³ For further details, see Lipitor (A) by Reinhard Angelmar, INSEAD published case 2006.
lower profits after the entry of a second firm in the absence of a complementarity or market-size externality. The essence of the theorem is that a monopolist makes more sales at any price than a duopolist would make at the same price. Further, a monopolist must make more profits by charging the monopoly price than it would obtain as a monopolist charging the price that would be set under a duopoly by revealed preference.4

The same reasoning, however, does not extend to entry in a market where there are two or more incumbents. In such an environment, each firm’s optimal price is affected by their competitors’ prices as well as their own demand conditions. Entry by a new competitor in a range of locations can increase the incumbents’ profits through two mechanisms. First, the presence of a new entrant can change the incentives of an incumbent, causing that firm to charge a higher price, which allows the other incumbent to raise prices without losing many customers. We call this the direct effect. Second, the presence of a competitor can act as a commitment device by a focal firm to be soft on pricing, which then can result in the competitor raising price enough to offset the customers that the focal firm loses to the new entrant. We call this the indirect effect. We are able to illustrate and separate these two effects by looking at asymmetric entry.

Thus, we find that profits of an incumbent firm can increase with entry, but never from the entry of a second firm in the market. Combined with the fact that profits eventually decrease with entry in any model where there is a maximum number of potential customers, we find that profits can exhibit a down-up-down relationship with respect to the number of firms in the market.

Our results have important and surprising implications for empirical researchers and managers. For example, we show that profits can increase for a firm even if it is the only firm that competes directly with the new entrant. Thus, a manager deciding on how to respond to a new entrant may prefer to accommodate entry and raise prices versus taking action to prevent entry. In general, incumbents often work to obstruct entry by competitors, expending costly resources lobbying politicians, retailers or other groups to make entry difficult for new rivals. Our results identify conditions under which firms should not only avoid the costs of spending these resources, but where they would be better off encouraging the entrant.

Further, the finding that profits can go up, and that the relationship between profits and the number of firms in the market can have a down-up-down shape is important for academic researchers studying the impact of competition. For example, the down-up-down shape of profits

4 We use duopoly in this sentence, but the logic applies to any number of firms.
changes the interpretation of Bresnahan and Reiss (1991)-style tests of competitiveness. It also has implications for the reasonableness of functional forms used in empirical entry games, where the functional forms of existing models preclude the identification of profit-increasing entry, even in those industries where such phenomena occur.

1.1 Related Research

To our knowledge, there are only two papers that identify profit-increasing entry using standard product differentiation models. The first, Chen and Riordan (2007), looks at the Spokes model, which is an address-based model of global competition. In this model, each firm competes symmetrically with every other firm. Thus, profits increase in Chen and Riordan from a combination of the direct and indirect effects we mentioned earlier. However, Chen and Riordan cannot isolate the two effects, so we cannot assess whether entry acting as a commitment device or entry acting to soften the pricing incentives of the incumbents drives the increase in profits. Similarly, Ishibashi and Matsushima (2009) shows that entry by a low-quality firm (a vertically differentiated entrant) that competes symmetrically with the incumbents can increase the profits of high-quality incumbents. The mechanisms here are the same as in Chen and Riordan.

In contrast to the Spokes model and the model of Ishibashi and Matsushima, real markets are characterized by asymmetric competition: new entrants often compete head-on with some firms and indirectly with others. In these scenarios, only the direct or only the indirect effects may apply to each firm. We demonstrate that either of the two effects can be sufficient for profits to increase.

Further, when the market parameters are such that profit increasing entry occurs in the Spokes model, the relationship between the number of entrants and profits is positive for all entry after the first profit-increasing entrant, independent of how many additional firms enter the market (up to the maximum allowed in the model). This finding of a U-shaped relationship between profits

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5 There are some papers which demonstrate profit-increasing entry through alternative mechanisms. Most of these papers show that profits can increase when entry creates a positive externality on incumbent firms or brings new customers to the market that were not attainable before the presence of the new entrant, even at very low prices. For example, the literature on geographic agglomeration shows that firms may choose to locate near competitors due to the fact that consumers prefer shopping where there is a cluster of stores (e.g. Dudey 1990 and Gauri, Sudhir and Talukdar 2007). Zhu, Singh and Dukes (2011) show that some supermarkets located near a Walmart or a Target can experience increased traffic in their grocery items. Alternatively, Thomadsen (2012) demonstrates that profits for all firms can increase when existing incumbent firms offer new products because the desire to avoid intra-firm cannibalization enforces a commitment by incumbents to be softer competitors.
and the number of firms is inconsistent with real markets. In real markets, the addition of new entrants eventually drives down both industry and average firm profitability. The positive relationship between the number of competitors and firm profitability obtains because the Spokes model does not capture the notion of “market crowding” which seems essential for a model that examines the relationship between profitability and market entry. Our model captures this notion, and our analysis shows that the market-crowding effect eventually dominates and erodes profits. As a result, the pattern of profits with respect to the number of firms in the market follows a down-up-down pattern in a scenario where profit-increasing entry occurs. As we note in Section 3, this shape has important implications for the modelling approaches used by empiricists who study entry.

Our paper is also related to Pazgal, Soberman and Thomadsen (2013), who identify conditions in which a shrinking market can lead to increased profits for incumbent firms. The key insight that paper provides is that “distant captive consumers” exacerbate competition between existing competitors: their departure relaxes competition and ultimately, enhances profitability. In contrast, we consider conditions where competition for captive consumers increases due to the entry of a new competitor. While there are some similarities between these scenarios, there are also key differences. First, in order for profit-increasing entry to occur, there must be consumers who do not buy in the category prior to entry. We believe an explanation for the limited attention that profit-increasing entry has received from academics is that the key papers that examine product differentiation assume full market coverage before entry. Second, while entry by a competitor makes it difficult for an incumbent to capture customers that switch to the entrant, these consumers are nevertheless “contestable” if the incumbent firm cuts price. Taken together, these constraints make it more difficult to find conditions where profit-increasing competitor entry occurs compared to conditions where profit-increasing customer exit occurs. Moreover, we identify conditions that lead to profit increasing entry even when new entrants choose their locations endogenously.

1.2 Empirical Relevance

Our paper demonstrates two mechanisms under which profits can increase from competitive entry. These the results are not merely theoretical; rather, they are observed in several industries.
As a first example, consider what happened to an actual online entertainment and gaming site, which we will refer to as GENERAL ENTERTAINMENT CO. This website is the oldest – and one of the largest – websites for its form of entertainment, and it offers members a wide range of programs and shows, from kid-friendly to very-violent entertainment. The site garners well over half a million unique visitors from the US, Europe and Asia each month, with clients ranging in ages from 6 to 40 years old. For about $5 per month these visitors become members and enjoy unlimited high definition access (and download ability) to all offerings on the site. Its main competitor, which we call FAMILY CO, offers mostly family oriented material and charges a similar monthly fee. Over the years GENERAL ENTERTAINMENT CO experimented with different monthly rates ranging from 40% lower to 30% higher; none of these generated higher profits.

At the beginning of 2011, a new competitor, VIOLENT CO, entered the same market offering a selection of more violent shows, games and programing content. Overall, the themes of VIOLENT CO’S offering were more mature, suggesting a target audience that was older on average. In this market, it is apparent that VIOLENT CO competes directly with GENERAL ENTERTAINMENT CO but has little overlap with FAMILY CO. This market structure will be the basis for the model we propose in Section 2.

After VIOLENT CO’s entry, GENERAL ENTERTAINMENT CO was uncertain about its best reaction. The owner of GENERAL ENTERTAINMENT CO decided to experiment by increasing its monthly price by almost 20%. This resulted in a profit increase of between 5%-6% despite a net loss of 15% of its customer base. Interestingly, FAMILY CO also increased its monthly fees by about 10% despite not facing any new direct competition. While prices exhibited small fluctuations over the coming year, prices in the market stabilized at these higher levels. To the surprise of the owner of GENERAL ENTERTAINMENT CO, increased competition had made his business more profitable.

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6 The management of this website has shared their details with us, but asked us not to reveal their identity within entertainment and gaming. We found that other managers – especially those at large firms – who experienced profit-increases in the face of competitive entry also did not want the fact that profits increased to be made public – partially out of a fear that this may weaken the firm’s negotiating power.
The gaming example is not an isolated incident. Pauwels and Srinivasan (2004) find that Dove, Lever 2000 and Dial soap experienced increased profits from store-brand entry.7 The pricing and quantity patterns behind these increased profits are consistent with the theory advanced in this paper: all of the firms increased their prices, and some of the firms (Ivory and to some extent Dial) had sharp declines in sales, while others (Lever 2000 and Dove, who likely compete less directly with the new entrant) experienced increased sales. It is possible that the store-brand product may have been perceived to be lower-quality; however, recent work on private labels speaks to their “quality equivalence” implying that horizontal differentiation may be as important as vertical differentiation in today’s retail context for private label (Soberman and Parker 2006). With this as a backdrop, our results imply that profits for Dial soap could increase after the store-brand entry, even if the store brand was positioned such that it competed primarily with Dial.

Pauwels and Srinivasan also show that it is possible that profits can increase with entry for some but not all of the firms. In the toothbrush category, Colgate raised its prices and experienced higher profits with the entry of store-brand toothbrushes. Reach also increased its prices, although the information provided in the paper is not sufficient for us to judge whether its profits increased or not. Oral-B, however, decreased their prices slightly, although they experienced higher revenues and higher sales.

We also observe that in many markets a necessary (but not sufficient) condition for our effect – prices rising with competition – occurs. For example, we analyze a set of airport car-rental data from a week in March 2004. Singh and Zhu (2008) use a dataset that is similar except for the dates of the rental car.8 We consider the daily rental rates for cars at airports with 3 or fewer car rental companies, and regress the rental rate on the number of firms in the market, 9 a series of dummy variables indicating the brand of the rental company, an indicator for whether the rental rate is a weekday or weekend rate, the number of passengers per rental firm and the total air traffic. Our results both without and with the “airport traffic” and “traffic per rental company” control variables are reported in Table 1. We find that prices are statistically higher by about 7% when the

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7 Pauwels and Srinivasan (2004) does not directly measure profits, but it provides price and quantity information. We assume that higher revenues and lower unit sales (meaning lower total costs), or higher prices and higher quantity together lead to higher profits.
8 We thank Vishal Singh for sharing this data.
9 Our number of firm variables are (1) the presence of 2 or more rental companies at the airport and (2) the presence of exactly 3 rental companies at the airport. The coefficient on the presence of 2 or more rental companies at the airport will capture the difference in price of going from a monopolist to a duopoly, while the coefficient on the presence of exactly 3 rental companies will be the difference in price of going from a duopoly to a triopoly.
airport has 3 rental companies compared to 2. Conversely, prices are relatively similar under monopoly or duopoly. This is consistent with our model where it is the third entry that leads to increased profits.

Table 1: Car Rental Price Regression results

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Presence of two of more rental companies</td>
<td>0.93</td>
<td>-0.01</td>
<td>-0.35</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td>(1.55)</td>
<td>(0.03)</td>
<td>(1.52)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>Presence of three rental companies</td>
<td>3.11</td>
<td>0.07</td>
<td>2.50</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>(1.40)</td>
<td>(0.03)</td>
<td>(1.16)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>Include Rental Car Fixed Effects</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Include Weekend Dummy</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Include Airport Traffic &amp; Traffic per Rental Co.</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Standard errors in parentheses. Numbers in **Bold** are significant at the 95% confidence level.

The car rental example is not the only case where we see prices increase with entry. Geroski (1989) finds that entry increases profit margins earned by incumbents in 6% of the industries studied. Similarly, Perloff et. al. (1996), Ward et. al (2002), Yamawaki (2002), Simon (2005), Goolsbee and Syverson (2008) and McCann and Vroom (2010) show that entry can lead to price increases in the pharmaceutical, consumer packaged goods, luxury car, magazine, airline and hotel industries, respectively.

The rest of the paper is organized as follows. In Section 2, we present a base model with a linear market. With this model, we identify conditions that are sufficient for a new entrant to lead to higher profits for the incumbents. In Section 3, we examine more-broadly how profits are related to the number of firms in the market. We find that industries where profits increase with competitive entry exhibit a "down-up-down" relationship between profits and the number of firms in the market. We discuss the implications of this finding on empirical entry research. Section 4 concludes the paper.

2. Model

The model we use to analyze the impact of an entrant on the profitability of incumbents is a finite linear market of length \( L \) (Hotelling 1929). To simplify, we assume that consumers are uniformly
located along the market with a constant density of 1. Firms are located at various locations along
the linear market and Consumer $i$’s utility from buying and consuming the product from firm $j$ can
be represented as

$$U_{ij} = V - p_j - d_{ij},$$  \hspace{1cm} (1)

where $V$ is the monetary benefit a consumer realizes from a product that perfectly suits her taste,
$p_j$ is the price charged by Firm $j$ and $d_{ij}$ is the distance between consumer $i$ and Firm $j$.\textsuperscript{10} A consumer can choose not to buy in which case they consume an outside good and earn a normalized utility of zero. We assume that all firms have a constant marginal cost, so firm profits are given by $(p_j - c)q_j$ where $q_j$ is the quantity sold.\textsuperscript{11} Without loss of generality, we set the marginal cost to zero, implying that $p$ is the absolute mark-up over marginal cost.

In what follows, we demonstrate the existence of conditions under which entry leads to profit increases for incumbent firms that are internally located in our Hotelling market. Specifically, the analysis considers the change in incumbent profits between two scenarios. In the pre-entry scenario, the two incumbents are alone in the market and compete with each other. By compete, we mean that a) the marginal consumer between the two firms is indifferent between them, and b) the marginal consumer between the two firms strictly prefers buying to consuming the outside good. Post-entry, the incumbents retain the same locations as in the pre-entry scenario, but a third firm is added to the market at a different location. The three firms then set prices and compete for consumers. We examine conditions under which profits for the incumbents increase after the entry of the third firm.

The incumbents are assumed to be located at interior market locations (not near the endpoints). We do not speculate why these incumbent firms have the positions they do, although we note that such an outcome could be consistent with technological limits on producing certain sets of attributes or with having “market needs” evolve away from the initial positions of the pioneers over time. To simplify the exposition, we give names to the two incumbents. The first is Firm $I$, which is the incumbent for which the distance to an end of the market is shortest. The distance from incumbent $I$ to the nearest end of the market is $A$. The second incumbent is Firm $C$ (centrally located) and the distance to an end of the market for Firm $C$ is longer than for Firm $I$. We denote the distance between the two incumbents as $D$.

\textsuperscript{10} The seemingly more general expression $U = v - aP - td$ is equivalent under the normalization $V = v/t$ and $p = tP/a$.

\textsuperscript{11} Our results can hold for a model with increasing marginal (convex) costs.
We assume the entrant, $E$, selects a location adjacent to incumbent $C$ but not between the incumbents. To illustrate the forces that underlie profit-increasing entry, we consider a context where $E$’s location is exogenous. We then demonstrate the robustness of the finding by having $E$ choose its location endogenously. The distance between incumbent $C$ and entrant $E$ is denoted by $S$. The distance between Firm $E$ and the far end of the market is denoted by $B$. Figure 1 illustrates the market.\(^{12}\)

![Figure 1: The Linear Market](image)

Before presenting the analysis, we highlight the mechanism through which Firm $E$’s entry can increase the profits of Firm $C$ and Firm $I$. Suppose that $E$’s entry is at a location that induces Firm $C$ to increase its price. Firm $I$ will be strictly better off because its nearest competitor sets a higher price. Firm $I$ can raise its price and increase the quantity sold, leading to higher profits. The key question is: can Firm $C$ be better off when this occurs? The answer is yes. Firm $I$’s best response to $C$’s higher price is to raise its price, which clearly helps offset the loss of profits from $C$’s lost volume to the new entrant. Whether Firm $C$ is better off depends on the relative impact of the price increase and the lost demand (to Firm $E$). We show conditions under which the price increase has a larger impact and Firm $C$’s profits do increase.

Note that the mechanisms leading to profit increases for Firms $I$ and $C$ are different. Firm $I$ benefits from the *direct effect* of Firm $E$’s entry changing the price-sensitivity of Firm $C$’s demand. Firm $E$’s presence reduces Firm $C$’s ability to attract consumers (near $E$) with a low price and this causes Firm $C$ to charge a higher price. However, this also leads to an *indirect effect* of Firm $E$’s entry on Firm $C$’s profits. In particular, once Firm $E$ enters, Firm $I$ increases its price because it anticipates that Firm $C$ will charge a higher price. Firm $I$’s reaction, in turn, confers a benefit on Firm $C$. It is this mechanism that we call the *indirect effect*. We show that Firm $C$’s profits can increase due to the indirect effect, even though there is no direct impact of $E$’s entry on

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\(^{12}\) We assume Firm $I$ is to the left of Firm $C$ without loss of generality.
Firm I’s demand. These effects provide intuition for the mechanism behind how incumbents’ profits can increase when a new competitor enters the market.

Our objective is to describe the conditions when both incumbents gain due to competitive entry and we start by presenting an example. We then move to a general theorem that identifies precise conditions for the phenomenon.

Suppose that Firms $I$ and $C$ are located such that $A = \frac{4}{15}, D = \frac{1}{6}, V = \frac{1}{2}$ and the line is of length $L = \frac{179}{150}$. Before entry, the firms both choose a price of $\frac{7}{30}$ and earn profits of $\frac{49}{600}$. Now suppose that Firm $E$ enters a distance $S = \frac{51}{100}$ away from Firm $C$. Then the following prices form an equilibrium: $p_E = \frac{1}{4}, p_C = \frac{6}{25}$ and $p_I = \frac{211}{900}$. These prices lead to firm profits of $\pi_E = \frac{1}{8}, \pi_C = \frac{613}{7500}$ and $\pi_I = \frac{44521}{540000}$, respectively. Thus, profits for Firms $C$ and $I$ (after Firm $E$’s entry) both increase, i.e. $\pi_I$ and $\pi_C > \frac{49}{600}$. This example demonstrates that equilibrium prices and profits can increase after entry. Note, too, that the new entrant earns monopoly profits in our example. Because monopoly profits are the maximum that can be earned (independent of whether competitors are in the market or not), $E$’s location is consistent with endogenous entry given $C$ and $I$’s locations. In what follows, we generalize this result and derive precise conditions that lead to both incumbents being better off after Firm $E$ enters.

As a first step, we characterize the equilibria that support this phenomenon. In spatial models, when the monetary benefit associated with a consumer’s preferred product, $V$, is sufficiently low, the demand curves of competing firms may have kinks in them (Salop 1979 and Perloff 1996). The kinks occur due to the fact that when prices are low (relative to $V$), the firm’s marginal consumer substitutes between the firm’s product and its rival’s product. In such a case, it is difficult to attract consumers with a price cut because the more distant a consumer is from the firm in question, the closer that same consumer is to the competitor. In contrast, when prices are high, a firm’s marginal consumer substitutes between the firm’s product and the outside good. Note that the attractiveness of the outside good is independent of how far the consumer is from the

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13 The calculations match those found in the proof of Theorem 1.
firm, so here, demand is more responsive to price. The solid lines in Figure 2 presents a figure of this demand. The flatter demand curve at higher prices reflects demand that is more responsive to a change in price.

Of course, demand curves do not always have kinks. When a firm’s competitors are close enough such that all of its customers prefer the rival’s product over the outside good, the firm’s demand curve will not have a kink. These are typically the conditions analyzed with spatial models i.e. $V$ is sufficiently large that the market is completely covered (Osborne and Pitchik 1987).

When $V$ is such that the two incumbents price at a kink-point, infra-marginal consumers realize positive surplus but the marginal consumer between the firms is indifferent between buying from Firm $I$, buying from Firm $C$ and not buying at all. When this occurs, the utility of consumers as a function of their location is as shown in Figure 3.\footnote{The properties of these kinks are described in-depth by Salop (1979) and Perloff et. al. (2006).}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Demand Curve with a Kink-Point}
\end{figure}
The optimal price for a firm can occur along any part of its demand curve, including at a kink point. If the optimal price occurs at a kink-point in a firm’s demand curve then first-order conditions do not identify the price. Also, each linear segment in the demand curve corresponds to a different first-order condition. The optimal price only occurs along a particular segment of the demand curve if the first-order condition for that line-segment is satisfied within the range of prices corresponding to that segment. Because marginal revenues are monotonically decreasing, it must be that the first-order conditions are satisfied on exactly one of the line segments or that the optimal price occurs at a kink-point.\(^{15}\)

To present the conditions that can cause both incumbents' profits to increase, we first present a lemma that outlines what an equilibrium with increased profits must look like. We then discuss the details of firm locations that lead to profit-increasing entry.

The lemma explains that in order for the profits of both Firm \(I\) and \(C\) to increase post-entry, Firm \(C\) must price at a kink-point after the entry of Firm \(E\).

**Lemma:** Any locations for Firms \(I\), \(C\), and \(E\) that lead to (1) a pure strategy pricing equilibria for all firms before and after entry, and (2) higher profits for both Firms \(I\) and \(C\) after Firm \(E\)’s entry

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\(^{15}\) With linear travel costs, there is a discontinuity in the demand curve at the price that makes consumers at the rival firm’s location indifferent between the two products because at this price the firm “steals” all of the rival’s demand. This discontinuity interferes with the monotonicity of the marginal revenue and can prevent the existence of pure-strategy pricing equilibria. We confirm that such undercutting does not occur in the conditions identified by our theorem.
than each firm obtains before the entry, must entail Firm C setting prices such that its first-order conditions are satisfied along the linear portion of its demand curve where the marginal consumer to the left of Firm C substitutes between Firms C and I before Firm E’s entry, and Firm C prices at a kink-point in its demand curve after Firm E’s entry.

The proof is provided in the Appendix. The basic intuition behind the lemma is that if prior to entry, Firm C prices along a linear portion of its demand curve where the marginal consumers substitute between Firm C and the outside good, then Firm C must be charging the monopoly price and earning monopoly profits, i.e. Firms I and C are so far apart, that they serve different markets. We mentioned in the introduction that profits weakly decrease with competitive entry in such a scenario. If Firm C prices at a kink-point in demand before entry and entry causes Firm C to raise its price, then Firm C’s profits must decrease since it had the option to increase its price before the entry of Firm E and sell to all consumers who attain a positive utility from that higher price but chose not to price in that way. Similarly, if Firm E’s entry causes Firm C to decrease its price then Firm I’s profits obviously decrease.

This implies that any situation where the entry of Firm E results in higher profits for Firms I and C must be associated with Firm C setting a price (before entry) that corresponds to the linear part of Firm C’s demand curve where the marginal consumers to the left of Firm C substitute between Firms C and I. In the Appendix, we show that the profits of both Firms I and C can only increase after entry when Firm C prices at a kink-point post-entry. Such a situation is presented in Figure 4 below. Before entry, Firm C’s demand curve is given by the solid lines, and Firm C prices at the dot on the solid line. After entry, Firm C’s demand curve shifts out to the dotted line, and Firm C increases its price to the dot at the kink-point in the demand curve.

The lemma above provides necessary, but not sufficient, conditions for Firms C and I’s profits to increase. The theorem below provides sufficient conditions Firm E’s entry to increase C and I’s profits.
Figure 4: Demand Curve for Firm C Before and After Entry

**Theorem 1:** If the incumbents I and C are in a market of finite length where

a) The distance between I and the edge of the market, \( A \geq V \left( \frac{43}{47} - \frac{2}{67} \sqrt{10} \right) \),

b) The distance between the incumbents, \( D \in \left( \frac{6}{7+5\sqrt{10}} V, \frac{5}{7} V \right) \) and

c) The length of the market, \( L \geq A - V \left( \frac{97.2}{46.9} - \frac{2}{67} \sqrt{10} \right) \).

and E enters at a distance \( S \in \left( \text{Max} \left[ \frac{16}{49} V - \frac{47}{49} D + \frac{4}{49} \sqrt{18V^2 - 30VD - 12D^2}, \frac{598}{295} V - \frac{16}{85} D, D \right], \frac{11}{10} V - \frac{1}{5} D \) \) from incumbent C, then there exists an equilibrium such that profits increase for both incumbents.

Intuitively, Firms I and C need to be sufficiently close so that a pure-strategy price equilibrium exists along the steep part of the demand curve in the pre-entry (2-firm) scenario. In addition, the incumbents cannot be too close or they will undercut each other’s prices. The range on S ensures that Firm E is (a) sufficiently distant from Firm C such that Firm C prices at a kink-point in its demand curve after Firm E enters the market and (b) sufficiently close to Firm C such that Firm E’s realized market touches the realized market for Firm C.

**2.1 Endogenous Locations**

In the proceeding analysis, we assume that the locations of all firms are chosen exogenously. This may accurately reflect industries where limits on technology or other factors impact where the
products are positioned. Alternatively, it is possible that firms may be able to set their locations optimally.

We treat the locations of $I$ and $C$ as being exogenous to the profit-increasing scenario we consider. However, we note that the above results can justify the optimality of the location at which the new firm enters. This is due to the fact that after $E$’s entry, Firms $C$ and $E$ both price at a kink-point. From $C$’s perspective, what matters is not $E$’s price, but the location of the marginal consumer. Thus, the above results are consistent with $E$ choosing a location where it charges the monopoly price and enjoys demand of $V$ and earns monopoly profits. As noted in the introduction, monopoly profits are the highest possible profits for the firm so there is no incentive for $E$ to charge a different price. This location choice is not generally unique.

There are also cases where the entrant would locate in a way that leads to profit increasing entry where $E$ does not charge the monopoly price. However, solving all of these cases leads to a series of complex conditions. For purposes of exposition, we present sufficient conditions that are consistent with endogenous location choices for $E$, as a function of $I$ and $C$’s initial locations, where $E$ charges the monopoly price.

**Theorem 2:** If the incumbents $I$ and $C$ are in a market of finite length where

a) The distance between the incumbents, $D \in \left(\frac{6}{7+5\sqrt{10}} V, \frac{6}{4} V\right)$.

b) The distance between $I$ and the edge of the market, $A \geq \frac{3}{5} V - \frac{D}{5}$ and

c) The length of the market is

$$L \in \left(A + \max \left[\frac{76V + 2D + 4\sqrt{18V^2 - 30VD - 12D^2}}{49}, \frac{134V + 67D}{85}, \frac{4V}{7} + D\right], A + \frac{8V + 4D}{5}\right)$$

then $E$ enters at a distance $S = L - A - D - \frac{V}{2}$ from incumbent $C$, and profits increase for both incumbents.

3. Broader Relationship of Incumbent Profits to Entry by New Firms

The purpose of this section is to understand the pattern that exists between incumbent profits and the number of firms in the market, where firms choose their locations endogenously.
As noted earlier, a monopoly’s profits weakly exceed the profits a firm earns in the same market when more than one firm is present. In this section, we show that once a threshold number of firms are present in a spatial market, profits of incumbents decrease with the number of competitors. Thus, in conditions where entry does lead to higher profits for incumbents, the relationship between profits and the number of firms in the market has a down-up-down pattern (or potentially a pattern with several ups and downs) rather than the monotonic down pattern, which is the basis for most thinking in economics and industrial organization regarding the effect of competition. We then discuss the implications of this result on the empirical product-entry literature.

We start our analysis by first noting that profits trend towards zero when there are enough firms present in the market. Formally, we note that in a market with total size $M$, if consumer $i$ purchases product $j$ she obtains utility $U_{ij} = V - p_j - f(d_{ij})$, where $f$ represents a positive-valued non-decreasing function of distance, and $U_{i0} = 0$ if the consumer consumes the outside good, then the average profits for each firm when there are $N$ firms in the market are bounded by $\frac{VM}{N} \rightarrow 0$ as $N \rightarrow \infty$. The proof of this statement is straightforward: The average demand for each firm is bounded by $\frac{M}{N}$ and prices must be strictly below $V$ (or else consumers choose not to buy).

The fact that profits approach zero after enough entry, along with the fact that profits are highest when only one firm is present in the market, as shown in the introduction, suggests that profits will either steadily decrease, or else decrease, then increase, and finally decrease again as more firms enter. An example of this relationship is presented in Figure 5. Note that prices can increase even as the market moves from a monopoly to a duopoly, but that firm profits must be lower in the duopoly market than in the monopoly market. Similar to the findings of Section 2, higher equilibrium prices when the third firm enters is a necessary condition for entry to be profit increasing. However, we recover the conventional economic relationship between competition and prices once there are a sufficient number of firms in the market.

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16 As noted earlier, a monopolist always earns higher profit than a duopolist in the absence of complementarities so the relationship between incumbent profit and the number of firms must always be downward sloping at $n=1$.

17 Figure 5 is the result of a game from a circular model where there are 6 potential locations and the firms choose locations sequentially in a subgame-perfect manner. The precise derivation is not important for the points we are making in the paper, but appear in the appendix for the convenience of the reader.
The observation that profits may not decline monotonically in the number of competitors is not purely of theoretical interest. For example, Bresnahan and Reiss (1991) study the impact of competition on profits. They consider entry by only a small number of firms, and find that entry by the first 3 firms reduces profits, but entry by the 4th or 5th firm has almost no effect. They conclude that markets with 3 firms must have achieved a “competitive state.” However, our results suggest that the monotonic decline in profits with the number of firms in an industry can reverse before the industry reaches a level close to full competition. Thus, the findings of Bresnahan and Reiss may be unrelated to the notion that 3 firms are sufficient for an industry to achieve a “competitive state.”

It is also interesting that a number of recent structural estimations of entry games rely on a reduced-form relationship between profits and the number of competitors. For example, Berry (1992) studies the factors that affect profitability and adopts the generic-like assumption that profits decrease in the number of competitors. Our analysis shows such an assumption in an empirical analysis may be less innocent than it sounds. The assumption in Berry (1992) is not an

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18 In Section 4 of their paper, Bresnahan and Reiss find that when they look at price data, they find that prices are lower when there are more than 5 firms in the market, consistent with our model but inconsistent with the conclusion that full competition is attained after 3-5 firms enter the market.
isolated example; the analysis of structural games is often based on a reduced-form profit function with the number of competitors as an explanatory variable. Seim (2006) assumes that profits are a linear combination of several factors and a linearly-decreasing function of the number of competitors, where the coefficient on the number of competitors is a function of the distance between the firm and each competitor. Singh and Zhu (2008), Ciliberto and Tamer (2009) and Datta and Sudhir (2012) also model profits as declining linearly in the number of competitors in their respective entry games. Similarly, Cleeren et. al. (2010) study inter-format competition and assume that profits decline with competitive entry, although they do not assume linearity.

Because the results of empirical analysis rely on the legitimacy of the reduced form at the heart of the model, it is important to recognize that profits can increase with a new entrant, especially when analyzing entry in markets with a small number of firms. While the above papers make important contributions, it is worth noting that the papers assume away the possibility that profits may increase, which can lead to distortions in the empirical estimates for industries where profits may increase with entry. Even in industries where profits monotonically decrease with entry, the forces that push towards profit-increasing entry may be present, but not in sufficient strength to offset the loss of demand. In such cases, the relationship between profits and the number of firms in the market might look something like Figure 5, only instead of having a slight increase in profits the relationship may have a slight decrease in profits. Thus, assuming something akin to a linear or even logarithmic relationship between profits and the number of firms would be qualitatively inappropriate.

Further, counter factual simulations using the estimated results of the models listed above (and papers with similar assumptions) will not show profits increasing with entry, even under conditions where profits do increase because the possibility of an increase is ruled out by the functional form. Thus, the fact that the vast majority of empirical studies provide no evidence of profit-increasing entry is unsurprising; the result is guaranteed by the functional form of the models and is not a result of the data. Thus, the perception among academics might be that profit-increasing entry is rarer than it actually is.

4. Conclusion
This paper studies how the locations of incumbents and entrants affect the possibility that profits for incumbent firms increase after entry. We provide conditions on locations for entry that can lead
to higher profits for all firms, even when only one firm competes directly with a new entrant. By studying asymmetric entry, we isolate two separate effects – a direct and an indirect effect – that can lead to profit-increasing entry. We focus our analysis on an example where profits increase with the entry of a third firm into the market; however, profit-increasing entry can theoretically occur for the entry of a third or greater firm.19 Further, we note that markets where profit-increasing entry occurs are characterized by a down-up-down relationship between profits and the number of firms in the market. We also discuss the implications of this finding on empirical research on entry.

Our results have important implications for managers and empirical researchers. Managers often expend resources trying to prevent entry by competitors – perhaps lobbying governments to put restrictions on entry or lobbying retailers to keep competing manufacturers off store shelves. Our results suggest that in some cases, firms should avoid expending these resources, and instead accommodate entry and raise prices. Managers need to know that their profits can increase even if their firm is the only one that competes directly with the new entrant or if the new entrant is located closer in “product space” than the incumbent’s competitor. We also provide insight for empirical researchers seeking to measure the relationship between profits and the number of firms in a market. Notwithstanding endogeneity concerns that often arise in measuring these types of relationships, our findings suggest that researchers need to reflect on the functional form that relates the number of competitors to profits. In particular, our findings raise concern about the plausibility of the linear (or log-linear) functional form and suggest that researchers need to consider functional forms that allow for the possibility of non-monotonic (and non-U-shaped) relationships between the number of firms and profits. Importantly, this functional form issue is not merely an issue for reduced form estimation. It is also an issue that affects studies of entry – and entry games – that while “structural” in spirit use reduced-form profit functions as building

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19 An example can be constructed for any $N \geq 3$ using a linear market of long enough length where consumers have linear travel costs by placing the incumbent $(N - 1)$ firms an equal distance apart, and allowing the entrant, the $N$th firm, to locate slightly less than $V/2$ units away from the edge of the realized market. Note that $V$ must be at least $21/16$ times as large as the distance between each of the incumbent outlets, and $V$ cannot be too large or no pure-strategy pricing equilibrium will exist. The equilibrium before entry is one where all firms price on linear part of their demand function according to their first-order conditions, while after entry the entrant prices as a monopoly (which is also a kink-point), the firm closest to it prices at a kink point and the other $(N-2)$ firms price according to their standard first-order conditions. As a specific example of profit-increasing entry by a 4th firm, let $V = 3$, the distance between Firms 1 and 2 (and between Firms 2 and 3) = 1, and the distance between Firm 3 and the new entrant, Firm 4 = 3.1. There is an equilibrium where the price for Firm 4 after entry is the monopoly price of 1.5, and Firm 3 prices at a kink point. In this equilibrium, profits for Firm 1 increase by 0.23%, profits for firm 2 increase by 1.61%, and profits for Firm 3 increase by 0.17%, where the firms are ordered from left to right on the line.
blocks. The results of any model-based analysis rely on the legitimacy of the reduced form. Our analysis shows that profits can increase when new competitors enter markets with a small number of firms. This is the precise situation considered in many of these studies.
References


Appendix

Proof of Lemma
Before entry, \( C \) can (1) price along a linear segment of the demand curve where all of the marginal consumers are considering the outside good, (2) price at a kink-point, or (3) price along a linear segment of the demand curve with some of the marginal consumers substituting between \( C \)'s product and \( I \)'s product. If \( C \) is pricing on the segment corresponding to case (1), the firm charges the monopoly price and earns monopoly profits, in which case profits can only decrease. If before entry the firm is pricing at a kink-point, then either entry causes \( C \)'s prices to increase, in which case \( C \)'s profits decrease because before entry \( C \) had the option to increase its price and to sell to all consumers who would earn a positive utility at this higher price; Revealed preference demonstrates that the profits must be lower. Alternatively, entry could cause \( C \)'s prices to decrease, in which case firm \( I \) will be worse off, since it makes lower profits for any price it could set relative to profits at the same price before entry. Also when \( C \)'s prices do not change, \( C \) must sell weakly less and earn weakly lower profits.

Therefore, any equilibrium where \( C \) and \( I \)'s profits both increase must involve case 3: \( C \) charging a price before entry corresponding to a linear segment of the demand curve where some of the marginal consumers substitute between \( C \) and \( I \). The corresponding first order condition for \( C \) before entry is \( q_c + p_c \frac{\partial q_C}{\partial p_c} = 0 \), where \( \frac{\partial q_C}{\partial p_c} = -1.5 \). Thus, \( p_{c2} = \frac{2}{3} q_{c2} \), where the 2 in the subscript denotes the total number of firms in the market. Profits are then \( \pi_{c2} = \frac{2}{3} q_{c2}^2 \).

When there are 3 firms in the market, either the third firm is located far enough apart that \( E \)'s realized market does not overlap with \( C \)'s market, \( C \) prices at a kink-point such that the marginal consumer for \( C \) in the direction of \( E \) gets zero utility, or \( \frac{\partial q_C}{\partial p_c} = -1 \). (Note that it is impossible that the new equilibrium will involve \( C \) and \( I \) serving separated market areas, or even having the marginal consumer between \( C \) and \( I \) obtaining zero surplus. Both of these outcomes would require firm \( C \) raising its price. However, the first-order condition for \( C \) can be written as \( p_c = - \frac{q_c}{\frac{\partial q_C}{\partial p_c}} \). If \( C \) and \( I \) serve different markets but \( C \) is not at a kink-point where the marginal
consumer between $E$ and $C$ gets zero utility, $\frac{\partial q_c}{\partial p_c} = -1.5$. However, at any price, $q_c$ is lower, so the first-order condition would be satisfied at a lower price than $p_{c2}$, which is a contradiction.)

We now rule out that profits will increase if either $C$ and $E$’s markets do not touch, or if $\frac{\partial q_c}{\partial p_c} = -1$. In the first case, there is no overlap in the market areas, so $C$ and $I$’s profits remain unchanged. Suppose instead that $C$ prices according to its first-order conditions, where $\frac{\partial q_c}{\partial p_c} = -1$.

The first-order conditions must satisfy $p_{c3} = q_{c3}$ and $\pi_{c3} = q_{c3}^2$. Note that $\pi_3 > \pi_2$ iff $q_{c3}^2 > \frac{2}{3} q_{c2}^2$, or $q_3 > \sqrt{\frac{2}{3}} q_2$. However, it must be that $q_{c2} - q_{c3} \geq \frac{17}{12} (p_{c3} - p_{c2})$. The term represents the maximum change in sales from the change in price $(p_{c3} - p_{c2})$, and can be parsed as follows: on the side of the outlet where the new entrant enters, the quantity sold cannot be more than the firm would have sold if it had an uncontested region on that side; The rate of change in quantity on that side if it were uncontested is $\frac{\partial q_c}{\partial p_c} = -1$. On the side where the other incumbent firm is located, $\frac{\partial q_c}{\partial p_c} = -\frac{1}{2}$, but the best response of the other firm is to increase its price by $\frac{1}{6}$ times the change in its opponent’s price, so the firm regains $\frac{1}{6} \cdot \frac{1}{2}$ consumers. In sum, the total effect is $\frac{\partial q_c}{\partial p_c} \leq -\frac{17}{12}$, with the even steeper decline occurring when the new firm steals even more customers from the incumbent outlet. This means that $q_{c3} \leq q_{c2} - \frac{17}{12} (p_{c3} - p_{c2}) \leq q_{c2} - \frac{17}{12} (q_{c3} - \frac{2}{3} q_{c2}) \leq q_{c2} - \frac{17}{12} \left( \sqrt{\frac{2}{3}} - \frac{2}{3} \right) q_{c2} < \sqrt{\frac{2}{3}} q_2$, which contradicts $q_3 \geq \sqrt{\frac{2}{3}} q_2$. □

**Proof of Theorem 1**

When only $C$ and $I$ are present in the market, the demand for $I$ is $V - p_I + \frac{p_{cI} - p_I + D}{2}$, and the demand for $C$ is $x_{CI} = D - \frac{p_{cI} - p_I + D}{2} + x_{CS} = V - p_C$. This implies that profits for each firm are $\pi_I = p_I \left( V - p_I + \frac{p_{cI} - p_I + D}{2} \right)$ and $\pi_C = p_C \left( V - p_C + \frac{p_I - p_C + D}{2} \right)$. Simultaneous optimization of
these expressions and solving leads to \( p_I = p_C = \frac{2}{5}(V + D/2) \) and profits of \( \pi_I = \pi_C = \frac{3}{50}(2V + D)^2 \). These calculations assume that the realized market for \( I \) and \( C \) touch, i.e. \( V - p_I + V - p_2 \geq D \), which becomes \( \frac{7}{6}D \leq V \) after substitution and rearranging. We also ensure that neither firm will undercut the other firm’s market and steal all of their customers. A firm undercutting their competitor must charge a price of \( p_{uc} = \frac{2}{5}(V + D) - D = \frac{2}{5}V - \frac{4}{5}D \). The undercutting firm’s profits are then \( 2p_{uc}(V - p_{uc}) \). The deviation will not be profitable when \( \frac{3}{50}(2V + D)^2 \geq 2p_{uc}(V - p_{uc}) \) which occurs when \( V \leq \frac{1}{3}(7 + 5\sqrt{10}) \frac{D}{2} \).

After entry we need an equilibrium where the profit of the incumbent firms increases. It is easy to verify that the first order conditions (Best responses) for prices are given by:

\[
p_I = \frac{1}{3}V + \frac{D}{6} + \frac{1}{6}p_C \quad p_C = \frac{1}{4}p_I + \frac{1}{4}p_E + \frac{1}{8}S + \frac{1}{8}D \quad p_E = \frac{1}{3}V + \frac{1}{6}S + \frac{1}{6}p_C \tag{A14}
\]

However, we know from Theorem 5 that \( C \) cannot price according to the FOC condition. Hence we look for an equilibrium where Firm \( I \) prices according to its FOC while \( C \) and \( E \) price at a kink point. Let \( p_C \) be the price charged by Firm \( C \) in the new equilibrium. Then \( p_E = 2V - S - p_C \) and \( p_I = \frac{2V + D}{6} + p_C \).

We need to ensure that there is no separation between the set of customers who consume \( C \) and those who consume \( E \). This occurs when \( p_E \geq \frac{1}{2}V \) (the monopoly price). The reason for this constraint is that if price is lower than \( \frac{1}{2}V \) and the firms price at a kink point then \( E \) can always increase its price to the monopoly price, sell to anyone who obtains a positive utility, and earn monopoly profits. The condition that \( p_E \geq \frac{1}{2}V \) is equivalent to the condition that \( p_E = 2V - S - p_C \geq \frac{1}{2}V \). \( \tag{A15} \)

Furthermore \( p_E = 2V - S - p_C \) must be below the price suggested by the FOC condition (A14). We know this because if the first-order conditions hold at a lower price than \( 2V - S - p_C \) then it must be that profits are increasing as prices decrease due to the optimization arguments that support pricing at first-order conditions. On the other hand, if the price that satisfies condition (A14) is greater than \( 2V - S - p_C \) then equation (A14) cannot describe the optimal price because equation (A14) assumes that the realized markets for each product touch, while these markets do not touch.
when prices are above $2V - S - p_C$. Combining condition (A15) with $2V - S - p_C \leq \frac{1}{3}V + \frac{1}{6}S + \frac{1}{6}p_C$ leads to the constraint:

$$\frac{10}{7}V - p_C \leq S \leq \frac{3}{2}V - p_C.$$  \hspace{1cm} (A16)

Following the same logic as explained from $E$’s pricing, in order for firm $C$ to choose a kink-point $p_c$ as its price, this price must be lower than the price dictated by the FOC in (A14). Substituting $p_I = \frac{1}{4}V + \frac{1}{6}D + \frac{1}{6}p_C$ and $p_E = 2V - S - p_C$ into $p_C = \frac{1}{4}p_I + \frac{1}{4}p_E + \frac{1}{4}S + \frac{1}{4}D$ yields

$$p_C \leq \frac{14}{39} \left( V + \frac{D}{2} \right).$$  \hspace{1cm} (A17)

Further, $C$’s profits, $\frac{1}{12} p_C (14V + 7D - 17p_c)$, must be higher than its profits before entry. This leads to

$$\frac{2}{5} \left( V + \frac{D}{2} \right) \leq p_C \leq \frac{36}{85} \left( V + \frac{D}{2} \right).$$  \hspace{1cm} (A18)

(Note that the lower bound condition guarantees that the price charged by $C$ is higher than the price before entry). Condition (A17) is not binding as condition (A18) is more restrictive. Clearly Firm $I$ increases its profits and Firm $E$ makes positive profits in such an equilibrium. We need to check that the markets between $I$ and $C$ touch: $2V - p_I - p_C \geq D$ or

$$p_C \leq \frac{10}{7}V - D.$$  \hspace{1cm} (A19)

Combining conditions (A16), (A18) and (A19) yields: $\max \left[ \frac{598}{595}V - \frac{18}{85}D, D \right] \geq S \leq \frac{11}{10}V - \frac{1}{5}D$.

We also must check that in equilibrium no firm will want to undercut its competitors:

**E will not undercut C:** $p_{C}^{\text{dev}} = p_C - S \leq 2p_C - \frac{10}{7}V \leq \frac{72}{85}V + \frac{36}{85}D - \frac{10}{7}V \leq$

$$\frac{72}{85}V + \frac{36}{85}\frac{6}{7}V - \frac{10}{7}V = -\frac{26}{119}V < 0$$

Where the first inequality is due to (A16) the second due to (A18) and the third is due to the range for $D$.

**C will not undercut E:** $p_{C}^{\text{dev}} = 2V - S - p_C - S$

$$\leq 2V - p_C - \frac{20}{7}V + 2p_C \leq \frac{-6}{7}V + \frac{36}{85}V + \frac{18}{85}D \leq \frac{-6}{7}V + \frac{36}{85}V + \frac{18}{85}\frac{6}{7}V = -\frac{30}{119}V < 0.$$
Where the first inequality is due to (A16) the second due to (A18) and the third is due to the range for D.

C will not undercut I: \[ p_C^{\text{dev}} = \frac{2V + D}{6} + \frac{p_C}{6} - D \]. The condition for lower profits at the new lower price is

\[ p_c > \frac{14}{31}V - \frac{3}{31}D - \frac{4}{31}\sqrt{-11V^2 + \frac{67}{2}VD + 49D^2} \equiv LB \quad \text{and} \quad p_c < \frac{14}{31}V - \frac{3}{31}D + \frac{4}{31}\sqrt{-11V^2 + \frac{67}{2}VD + 49D^2} \equiv UB. \]  

(A20)

It is easy to check that \[ \frac{2}{5}(V + \frac{D}{2}) \geq LB \] and \[ UB \geq \frac{36}{85}(V + \frac{D}{2}) \] so constraint (A20) is not binding and is subsumed by (A5).

I will not undercut C: \[ p_I^{\text{dev}} = p_C - D \]. The condition for lower profits at the new lower price are

\[ p_c < \frac{22}{49}V + \frac{47}{49}D - \frac{4\sqrt{6}}{49}(3V + D)(V - 2D) \quad \text{or} \quad p_c > \frac{22}{49}V + \frac{47}{49}D + \frac{4\sqrt{6}}{49}(3V + D)(V - 2D). \]  

(A21)

Clearly when \( D \geq \frac{1}{2}V \) the conditions hold. However when \( D \leq \frac{1}{2}V \) the condition may bind.

In summary, the following is the complete characterization of the parameter space where the incumbents earn higher profits in the context of a pure strategy equilibrium: \( D \in \left( \frac{6}{7+3\sqrt{10}}V, \frac{6}{7}V \right) \)

and \( S \in \left( \max\left[ \frac{49}{49}V - \frac{47}{49}V + \frac{\sqrt{18V^2 - 30VD - 12D^2}}{49}, \frac{98}{85}V - \frac{18}{5}D, \frac{11}{10}V \right], \frac{1}{5}D \right) \). A graphic representation of the parameter space is given below:
Finally, we need to calculate the Minimum Length of the Line given $V$.

The maximum size of $S$ is $\frac{11}{10}V - \frac{1}{5}D$ and the allowable range for $D$ is $D \in \left(\frac{6}{7+5\sqrt{10}}, \frac{6}{7}\right)$. However, we need to specify the minimum size of $A$ and the minimum size for $B$ given $D$ and $S$ such that assumption of substitution with the outside good for extreme consumer at either end of the served market is justified. If the most distant consumer to the left of $I$ (pre-entry) obtains zero surplus then

$$V - \frac{2}{5}\left(V + \frac{D}{2}\right) - A = 0 \Rightarrow A > \frac{3}{5}V - \frac{1}{5}D.$$ 

Note that because $D > \frac{6}{7+5\sqrt{10}}V$, a sufficient condition for this to hold is that $A \geq V\left(\frac{44}{67} - \frac{2}{67}\sqrt{10}\right)$. Similarly, the calculations in the above proof depend on the most-distant consumer from $E$ obtaining a zero surplus when $E$ charges its monopoly price, i.e. $p_E = \frac{V}{2}$. This implies that $B > \frac{V}{2}$. We now identify the minimum length of the line $L > A + D + S + B$ such that all conditions are satisfied. One possibility is that $D = \frac{6}{7+5\sqrt{10}}V$. This implies that $A + D + S + B = \frac{11}{5}V + V\left(\frac{6}{67}\sqrt{10} - \frac{42}{335}\right)$. The second possibility is that $D = \frac{6}{7}V$. This implies that $A + D + S + B = \frac{19}{7}V$. It is straightforward to show that

$$\frac{19}{7}V > \frac{11}{5}V + V\left(\frac{6}{67}\sqrt{10} - \frac{42}{335}\right).$$

Therefore, if $L > \frac{19}{7}V$ and the conditions of Theorem 1 are satisfied, then the entry of Firm $E$ will lead to increased profits for both Firms $I$ and $C$. However, because $A$ is unlimited, we must ensure that $L \geq \frac{19}{7}V + A - \left(V\left(\frac{43}{67} - \frac{2}{67}\sqrt{10}\right) = A - V\left(\frac{672}{469} - \frac{2}{67}\sqrt{10}\right)\right)$. 

Proof of Theorem 2

The conditions from Theorem 1 all continue to hold. We now add the condition that $p_E = \frac{V}{2}$ and that $B$ from Figure 1 is equal to $\frac{V}{2}$ (this is sufficient but not necessary – $E$ could optimally enter in a location that encroaches $C$’s market area while still earning monopoly profits, even if there is an opportunity to locate further to the right). The length of the market is then $A + D + S + B$. $A$ and
D are set before the entry, $B = \frac{V}{2}$, and the rest of the length is the endogenously chosen $S$. The theorem uses a shorthand for the conditions, but a full set of conditions is that when

\[
\frac{6V}{7 + 5\sqrt{10}} < D \leq \frac{1700\sqrt{3918 - 42122}}{226081} V
\]

then the supported length of the market is

\[
L \in \left( A + \frac{76V + 2D + 4\sqrt{18V^2 - 30VD - 12D^2}}{49}, A + \frac{8V + 4D}{5} \right).
\]

When

\[
\frac{1700\sqrt{3918 - 42122}}{226081} V < D \leq \frac{598}{721} V
\]

then the supported length of the market is

\[
L \in \left( A + \frac{134V + 67D}{85}, A + \frac{8V + 4D}{5} \right).
\]

Finally, when $\frac{598}{721} V < D \leq \frac{6}{7} V$ the supported length of the market is

\[
L \in \left( A + \frac{4V}{7} + D, A + \frac{8V + 4D}{5} \right).
\]

**Equilibrium Analysis Behind Figure 5**

To demonstrate the possibility of the down-up-down pattern, consider a situation where firms are sequentially allowed to enter into a circular market. Consumers have utility as given by Equation 1. Suppose that the monetary benefit of consuming an ideal product is $V = \frac{1}{2}$ and that the market has 6 equally-spaced discrete locations zoned for entry. This means a new firm can enter in any zone that is unoccupied (by another firm). We assume that there are 6 potential entrants and in each period only one new firm enters the market. Thus, in a period, the entrant chooses its location, and then firms compete by choosing prices. The game proceeds for 6 periods, and the objective of each entrant is to maximize its sum of profits over all periods in which it operates.

**Theorem:** In a circular market with the rules for entry and optimization as described above, there exists a Subgame Perfect Equilibrium that involves the first firm entering the market and earning a monopoly profit of $1/8$ in period 1. In period 2, the second firm locates at a distance $1/3$ away from the first firm, and both firms earn profits of $8/75$. The third firm locates at a distance $1/3$ away from each of the two incumbents, and all 3 firms earn profits of $1/9$. The fourth, fifth and sixth firms locate at the remaining available locations. In such a case, the first three firms earn
profits of either $1/16$ or $49/576$ in period 4, profits of either $100/3249$ or $169/3249$ in period 5, and profits of $1/36$ in period 6.$^{20}$

Proof: We begin by noting that across all of the numbers of firms that can be present in the market, there are only 12 total potential layouts for these firms. We first summarize the profits in each of these scenarios; we then examine the different paths along which the game can proceed.

**Monopoly:** A monopolist will price at $1/4$ and earn a profit of $1/8$.

**Duopoly:** There are 3 scenarios for what can happen if there are two firms in the market – these firms can locate $1/6$, $1/3$ or $1/2$ of the distance in the circle apart.

**Case 2A:** The firms are located $1/6$ of the circle apart. In this case, because $D \in \left( \frac{3}{7 + 5\sqrt{10}}, \frac{3}{7}V \right)$, we can use the first-order conditions $p_j = \frac{V}{3} + \frac{D}{3} + \frac{p_{-j}}{6}$, which give profits of $\frac{6}{25}(V + D)^2 = \frac{49}{600} \approx 0.0817$, where the decimal value is provided only to aid the reader when we consider what choices the firms will make.

**Case 2B:** The firms are located $1/3$ of the circle apart. Again, $D \in \left( \frac{3}{7 + 5\sqrt{10}}, \frac{3}{7}V \right)$, so profits are $\frac{6}{25}(V + D)^2 = \frac{8}{75} \approx 0.1067$.

**Case 2C:** The firms are located $1/2$ of the circle apart. In this case, both firms are *de facto* monopolists, and earn profits of $1/8 = 0.125$.

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$^{20}$ There are many ways that the firms could choose their locations when multiple locations provide equal profits. The equilibrium above assumes that the firms enter their locations randomly – i.e., an entrant chooses one of $k$ locations with probability $1/k$ when the $k$ locations offer equal and maximum profit for the entrant. That said, the equilibrium is not unique, and there are strategies that lead to different equilibria.
**Three firms**: There are 3 scenarios for what can happen: each of the 3 firms can occupy adjacent spots, two of the firms can occupy adjacent spots with a gap between these two firms and the third firm, or all 3 firms can be spaced equally far apart.

**Case 3A**: All 3 firms are located adjacently. The locations look like the following picture

In this case, the market will not be covered. Profits for each of the outer firms will be

\[ p_o = \left( V - p_o + D + \frac{p_i - p_o}{2} \right), \]

where \( p_o \) is the price of the outer firms, and \( p_i \) is the price of the inner firms. Profits for the inner firm, given the symmetry of the outer firms, will be \( p_i(2D + p_o - p_i) \).

Solving the first-order conditions, and calculating the subsequent demand yields profits of

\[ \pi_o = \frac{75}{968} = 0.0775 \quad \text{and} \quad \pi_i = \frac{169}{4356} = 0.0388. \]

**Case 3B**: Two firms are located adjacent. The third firm is located away.
In this case, the market will be covered, but firms A and C will price at kink-points on their demand curve (i.e., \( p_A = \frac{1}{2} - p_C \)). Firm B will price according to its standard first-order conditions:

\[
p_B = \frac{1}{8} + \frac{p_A + p_C}{4} = \frac{1}{8} + \left( \frac{1}{2} - p_C \right) + \frac{p_C}{2} = \frac{1}{4}.
\]

We also know that B will sell \( \frac{1}{4} \) units, so \( \pi_B = \frac{1}{16} = 0.0625 \). It must also be that A and C do not wish to increase their prices above the kink-point. If the firms increased their prices any higher, the profit functions and first-order conditions would be:

\[
\pi_A = p_A \left( \frac{1}{2} - p_A + \frac{1}{12} + \frac{p_B - p_A}{2} \right) \rightarrow p_A = \frac{17}{72}.
\]

\[
\pi_C = p_C \left( \frac{1}{2} - p_C + \frac{1}{6} + \frac{p_B - p_C}{2} \right) \rightarrow p_C = \frac{19}{72}.
\]

Note that at these prices, \( p_A = \frac{1}{2} - p_C \). Thus, they must be the equilibrium prices. (Firms wouldn’t lower their prices further – any further decline would yield even less demand than in the profit functions above.) Equilibrium profits are then \( \pi_A = \frac{289}{3456} \approx 0.0836; \pi_B = \frac{1}{16} = 0.0625; \pi_C = \frac{361}{3456} \approx 0.1045 \).

**Case 3C:** All three firms are evenly spaced at a distance of 1/3 to each other. Profits are then \( \pi = 0.1111 \), as shown in the example from Section 4.

**4 Firms:** There are 3 scenarios for what can happen if there are four firms in the market – the two empty spots can be 1/6, 1/3 or 1/2 of the distance in the circle apart.

**Case 4A:** The four incumbent firms operate in adjacent spots. In such a case, there are two inside firms that have competitors on both sides of them, and two outside firms that have a competitor on one side and an empty spot on the other side. The equilibrium entails the two outside firms pricing at a kink-point, and the inside firms pricing along a linear portion of their demand curve. Profits for the inside firm are \( \pi_i = p_i \left( \frac{1}{6} + \frac{p_i' + p_o - p_i}{2} \right) \), where \( p_i' \) represents the price of the other “inside” firm. There are a continuum of equilibria, but a unique equilibrium that has the two outside
firms and the two inside firm pricing symmetrically. This symmetric equilibrium prices are \( p_o = \frac{1}{4} \) and \( p_i = \frac{7}{36} \), and profits are \( \pi_o = \frac{11}{144} \approx 0.0764; \pi_i = \frac{49}{1296} \approx 0.0378 \).

There are also asymmetric equilibria. To find these, we introduce the following notation for the firms at the different locations:

We then have the following conditions:

(W1) \( p_{oA} = \frac{1}{2} - p_{oB} \)

Ensures that there is a kink-point equilibrium.

(W2) \( p_{IA} = \frac{1}{12} + \frac{p_{IB} + p_{oA}}{4} \)

First-order condition for \( I_A \).

(W3) \( p_{IB} = \frac{1}{12} + \frac{p_{IA} + p_{oB}}{4} \)

First-order condition for \( I_B \).

(W4) \( p_{oA} \geq \frac{7}{36} + \frac{p_{IA}}{6} \)

Price must be above the price that would be optimal for the linear part of the demand curve above the kink, where the marginal customer on one side of the firm is indifferent between the firm’s product and the outside good.

(W5) \( p_{oA} \geq \frac{7}{36} + \frac{p_{IB}}{6} \)

Equivalent to W4.

(W6) \( p_{oA} \leq \frac{1}{6} + \frac{p_{IA} + p_{oB}}{4} \)

Price must be below the price that would be optimal for the linear part of the demand curve below the kink, such that
marginal customers on both sides of the firm earn positive utility.

\[ p_{Oe} \leq \frac{1}{6} + \frac{p_{Ie} + p_{Oe}}{4} \]

Equivalent to W6.

Substituting W3 into W2 yields

\[ p_{Ie} = \frac{1}{12} + \frac{1}{12} \cdot \frac{4}{4} \cdot \frac{p_{Oe}}{4} + \frac{p_{Oe}}{4} \cdot \frac{5}{48} + \frac{p_{Ie}}{16} + \frac{1}{16} \cdot \frac{p_{Oe}}{4} \]

which becomes

\[ p_{Ie} = \frac{13}{90} + \frac{p_{Oe}}{5} \]

Plugging this and W1 into W4 and W6 yields

\[ p_{Oe} \in \left[ \frac{59}{261}, \frac{59}{216} \right] \]

Because the same limits apply to O_B, the actual lower limit must be

\[ \frac{1}{2} - \frac{59}{216} = \frac{49}{216}, \]

so

\[ p_{Oe} \in \left[ \frac{49}{216}, \frac{59}{216} \right] \]

Note that the symmetric solution is in this interval. This yields a price range of

\[ p_{Ie} \in \left[ \frac{41}{216}, \frac{43}{216} \right] \]

which in turn implies profit ranges of

\[ \pi_{O} \in \left[ \frac{3481}{46659}, \frac{3577}{46659} \right] \approx [0.0746, 0.0767], \]

and

\[ \pi_{I} \in \left[ \frac{1681}{46659}, \frac{1847}{46659} \right] \approx [0.0360, 0.0396] \]

Case 4B: The two empty spots can be located 1/3 of the circle apart. The configuration is as follows:
Then all firms price according to traditional first-order conditions. It is easy to confirm the following prices and profits:

\[ p_L = \frac{1}{4}, \pi_L = \frac{1}{16} = 0.0625; \quad p_M = \frac{5}{24}, \pi_M = \frac{25}{576} = 0.0434; \quad p_N = \frac{7}{24}, \pi_N = \frac{49}{576} = 0.0851. \]