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Dynamic risk, accounting-based valuation and firm fundamentals

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Abstract This study extends the accounting-based valuation framework of Ohlson (Contemp Acc Res 11(2):661–687, 1995) and Feltham and Ohlson (Acc Rev 74(2):165–183, 1999) to incorporate dynamic expectations about the level of systematic risk in the economy. Our model explains recent empirical findings documenting a strong negative association between changes in economy-wide risk and future stock returns. Importantly, the model also generates costs of capital that are solely a linear function of accounting variables and other firm fundamentals, including the book-to-market ratio, the earnings-to-price ratio, the forward earnings-to-price ratio, size and the dividend yield. This result provides a theoretical rationale for the inclusion of these popular variables in cost of capital (expected return) computations by the accounting and finance literatures and obviates the need to estimate costs of capital from unobservable (future) covariances. The model also generates an accounting return decomposition in the spirit of Vuolteenaho (J Finance 57(1):233–264, 2002). Empirically, we find that costs of capital generated by our model are significantly associated with future returns both in and out of sample in contrast to standard benchmark models. We further obtain significantly

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lower valuation errors in out-of-sample tests than traditional models that ignore dynamic risk expectations.

Keywords Dynamic risk · Accounting valuation · Cost of capital · Firm fundamentals

JEL Classification G12 · G14 · M41

1 Introduction

This study extends the Ohlson (1995) and Feltham and Ohlson (1999) accounting-based valuation framework to incorporate dynamic expectations about the level of risk in the economy. Our valuation model is comprehensive enough to include time varying risk, yet parsimonious enough to generate linear pricing equations. The linear pricing equation is comprised of accounting variables and a new factor that captures dynamic risk in the economy. The model yields the intuitive result that economy-wide risk and equity values are inversely related. The valuation model also generates an exact equation for equity returns and describes how returns evolve through time. These return dynamics provide a theoretical rationale for recent empirical evidence that changes in aggregate volatility are negatively associated with expected equity returns (Ang et al. 2006).

The model also provides an explicit equation for the cost of capital (expected return) expressed solely as a linear combination of accounting variables and firm fundamentals, including the book-to-market ratio, the earnings-to-price ratio, the forward earnings-to-price ratio, size, and the dividend yield.[1] Importantly, our findings provide theoretical justification for the use of popular accounting ratios and firm characteristics as determinants of future stock returns in the empirical accounting and finance literatures. In addition, we show that the equity return derived from our accounting-based valuation model can be expressed as the expected return plus cash flow news minus discount rate news, equivalent to the return decomposition of Vuolteenaho (2002). Thus we link the return decomposition literature based on book-to-mark dynamics with the accounting valuation literature based on non-arbitrage pricing. Without the inclusion of dynamic risk in the accounting-based valuation model, these results would not be possible.

Empirically, we find that costs of capital generated by our model are significantly associated with future returns both in and out of sample, unlike standard benchmark models. We also find that including an estimate of dynamic risk is important when pricing equity. More specifically, we first find that firm-level sensitivity to expected economy-wide risk is highly associated with average cross-sectional stock returns; more traditional sensitivities such as the CAPM beta or the Fama–French

[1] As we show further below, the dividend yield can be eliminated using the clean surplus relation in which case the cost of capital can be expressed solely (in addition to size) as a linear function of accounting numbers, namely, the past book-to-market ratio, the current book-to-market ratio, the earnings-price ratio, and the forward earnings-price ratio.

three-factor betas do not perform as well. Second, we show that our theoretically derived cost of capital model based solely on accounting variables and firm fundamentals provides an unbiased predictor of future average stock returns. Using the model to formulate a trading strategy generates average hedged monthly equity returns of 1.5 % and abnormal monthly returns of 1.18 % relative to the Fama–French three-factor model. Third, in comparison to benchmark valuation models, a model that includes economy-wide risk expectations consistently produces significantly lower out-of-sample pricing errors than traditional valuation models.

The remainder of the paper is organized as follows: Sect. 2 provides motivation for this study. Section 3 develops the valuation model and derives the equity return dynamics as well as the cost of equity capital (expected return) dynamics. Section 4 estimates our model-driven costs of capital and equity prices empirically and compares these estimates to those derived from benchmark models. Section 5 briefly concludes.

2 Motivation

Since the work of Ohlson (1995) and Feltham and Ohlson (1995), a large number of theoretical studies have explored how accounting data can be used to price equities.[2] This literature has largely focused on increasing the sophistication of modeling firm fundamentals, such as expected earnings, but not the dynamics of risk. There are exceptions. Feltham and Ohlson (1999) provide a theoretical foundation for more complete models that include dynamic risk and dynamic risk-free rates. In the process, they extend the standard residual income model (RIM) to include dynamic (covariance) risk.[3] Gode and Ohlson (2004) integrate time varying interest rates in the Ohlson (1995) valuation framework. However, their analysis assumes a risk neutral setting and does not incorporate dynamic risk adjustments. Motivated by Feltham and Ohlson (1999), Ang and Liu (2001) construct a generalized affine model that yields a nonlinear relation between market value and book value, which depends upon stochastic interest rates, firm level profitability, and growth in book value. Pástor and Veronesi (2003) develop a valuation model that incorporates the clean surplus relation and includes learning about accounting profits. However, these models do not provide closed-form solutions for equity prices. Instead, the price of the equity is commonly expressed as an integral (or sum) of a function that must be solved numerically. An exact closed-form solution for equity prices that incorporates dynamic risk has proven elusive but, as we shall see, is clearly desirable. Our model provides such a solution.

Empirically, the role of risk is often modeled by a backward-looking capital asset pricing model (CAPM) and other factor models based on historical estimates that do not include information about expectations of risk or future states of the economy. If historical estimates of risk are not representative of current expectations of future risk, this could be problematic when valuing stocks. For example, an equity

2 See Ohlson (2009) and the citations therein.

3 See their Corollary 3.

valuation model may suggest that a stock is undervalued using cash flow forecasts and a historical measure of risk expectations. But this could very well be the wrong conclusion and might (assuming that cash flow forecasts are reasonable) be caused by poor discount rate (risk) forecasts, such as those computed from historical betas and long term averages of excess market returns. Furthermore, there is often a disconnect between the underlying valuation model and the empirical risk measurement model. Many empirical studies use the CAPM (or the market model) to measure the firm's cost of capital and then proceed to substitute their cost of capital estimate into another valuation model, disregarding the fact that the valuation process and the return generating process are both determined simultaneously and logically should be estimated from the same model. But there is the rub: how to value the firm and estimate its cost of capital from the same model?[4] We solve this conundrum below.

Another related issue revolves around the ubiquitous use of accounting numbers to predict costs of capital without much theoretical justification. The well-known Fama and French (1993) asset pricing model is an obvious case, but there are many others. Fama and French (1993) offer no theoretical justification for using the book-to-market factor to predict expected returns. They do so because it seems to works empirically. This is hardly satisfactory from a scientific perspective. In contrast, this study offers an exact theoretical rationale for using accounting numbers to estimate costs of capital.

The papers by Nekrasov and Shroff (2009) and Penman and Zhu (2011) are directly related to ours but with substantive differences. Nekrasov and Shroff (2009) insightfully simplify the extended RIM model of Feltham and Ohlson (1999) so that it can be empirically estimated. Using historically based covariance risk measures, they compare value estimates based on their model with the CAPM and the Fama–French three-factor model and find that their model yields smaller out of sample pricing errors. The conceptual difference between our pricing model and theirs is analogous to the difference between the standard RIM and Ohlson (1995); namely, Ohlson (1995) imposes an abnormal earnings dynamic on the standard RIM, whereas we impose both an abnormal earnings dynamic and stochastic risk structure dynamic on the Feltham and Ohlson (1999) extended RIM. In contrast to Nekrasov and Shroff (2009), our primary interest is in predicting expected returns rather than equity prices, although we predict equity prices too. Also, our risk measures are forward looking and obviate the need to estimate unobservable covariances.

Penman and Zhu (2011) criticize the tendency by the literature to prematurely classify high realized returns as anomalous. Based on an accounting identity, they cogently argue that accounting variables are likely predictive of risk and, hence, of returns so that the "anomalous" results found by the literature may in fact be driven by risk. Absent from their analysis, as they acknowledge, is an equilibrium model of equity valuation so that they cannot provide an exact relation between equity returns and the accounting variables that determine returns. We provide such a relation.

[4] To the best of our knowledge, Morel (2003) is the first paper to decry this disconnect and attempt to deal with it.

3 The valuation model

Assuming that price is the discounted value of expected future dividends and that the clean surplus relation holds, Feltham and Ohlson (1999) show that the price of equity at time t (S_t) can expressed as:

$$S_t = B_t + \sum_{i=1}^{\infty} R_{f,t,t+i}^{-1} E_t[x_{t+i}^a] + \sum_{i=1}^{\infty} cov_t(m_{t,t+i}, x_{t+i}^a), \tag{1}$$

where B_t is book value at time t; E_t is the expectation operator conditional on information at time t; $R_{f,t,t+i}$ is the return on a risk-free bond from time t to $t+i$; x_t is earnings at time t; x_t^a $(= x_t - (R_{f,t-1,t} - 1)B_{t-1})$ is abnormal earnings at time t ; and $m_{t,t+i}$ is the time t stochastic discount factor for cash flows expected at time $(t+i)$. Equation (1) is the RIM valuation model extended to include dynamic stochastic discount (risk) factors as manifested in the covariance terms.

In order to derive a parsimonious linear accounting valuation model that incorporates dynamic risk, we follow Ohlson (1995) and make explicit assumptions about the dynamics of abnormal earnings and the dynamics of the stochastic discount factor. Specifically, we assume that abnormal earnings and "other value relevant information" (v_t) follow the linear autoregressive dynamic form:

$$x_{t+1}^a = \omega x_t^a + (1-\omega)x_L^a + v_t + \epsilon_{t+1} \tag{2}$$

$$v_{t+1} = \gamma v_t + u_{t+1.} \tag{3}$$

With a minor exception, these are the same dynamics as in Ohlson (1995). The original Ohlson model is set in a risk-neutral world so that the firm's cost of capital is equal to the risk-free rate. If return on equity eventually equals the firm's cost of capital, long-run abnormal earnings will converge to zero. In our risk-averse world, the cost of capital is the risk-free rate plus a risk premium so that, if the return on equity eventually converges to the cost of capital, then abnormal earnings will converge to some long-run equilibrium value above zero, which we denote x_L^a. As a consequence, we assume that next period abnormal earnings are a weighted average of this years abnormal earnings and long-run abnormal earnings. For mathematical convenience, we assume that the error term u_{t+1} is idiosyncratic and uncorrelated with the stochastic discount factor and that the error term ϵ_{t+1} is homoscedastic with variance σ_x^2.[5] Both error terms are assumed to be mean zero.

In addition, we assume a linear dynamic for the stochastic discount factor of the form:

$$m_{t,t+1} = R_f^{-1}(1 - \sigma_{m,t} e_{t+1}). \tag{4}$$

The error term e_{t+1} is assumed to have a mean of zero and a unit variance and to be (positively) correlated with ϵ_{t+1}. The term $\sigma_{m,t}$ is key and represents the level of aggregate (systematic) risk in the economy. This formulation also assumes that risk-free rates are constant. Note that this dynamic satisfies the general requirements of a

[5] The latter assumption is for simplicity. Relaxing this condition would increase the number of variables in the stock price equation by one.

discount factor, namely, that a discount factor must equal the inverse of the gross risk-free rate in expectation and take on nonnegative values only (Cochrane 2001).[6] Even though this stochastic discount factor is fairly simple, the assumed volatility structure is a close discrete-time analog of the popular SABR stochastic volatility model (Hagan et al. 2002) commonly used in option pricing. Additionally, if one assumes that shocks to the discount factor are driven by shocks to the market portfolio, this discount factor can be shown to produce standard CAPM-type models for expected returns.[7] "Appendix 1" derives this dynamic rigorously from first principles and shows its relation to CAPM.

To keep the analysis tractable, we also assume that the level of risk in the economy follows a random walk:

$$\sigma_{m,t+1} = \sigma_{m,t} + \xi_{t+1}, \tag{5}$$

where ξ_{t+1} is a mean zero random variable independent of e_{t+1}.[8]

Substituting the abnormal earnings and the stochastic discount factor dynamics into Eq. (1) yields a valuation equation that is a linear function of accounting variables, "other information," and a dynamic aggregate risk adjustment factor, as shown formally in the following proposition. Proofs for all propositions are found in "Appendix 2".

Proposition 1 *Assume that the abnormal earnings dynamics are given by* Eqs. (2) *and* (3) *and the stochastic discount factor dynamics are given by* Eq. (4) *and* (5). *The price of equity is given by:*

$$S_t = B_t + \alpha_1 x_L^a + \alpha_2 x_t^a + \alpha_3 v_t - \lambda_1 \sigma_{m,t}, \tag{6}$$

or, equivalently, by:

$$S_t = \gamma_1 x_L^a + \gamma_2 B_t + \gamma_3 x_t + \gamma_4 D_t + \gamma_5 E_t[x_{t+1}] - \lambda_1 \sigma_{m,t}, \tag{7}$$

where

$$\alpha_1 = \frac{R_f(1-\omega)}{(R_f-\omega)(R_f-1)} \geq 0, \quad \alpha_2 = \frac{\omega}{(R_f-\omega)} \geq 0, \quad \alpha_3 = \frac{R_f}{(R_f-\omega)(R_f-\gamma)} > 0,$$

$$\lambda_1 = \frac{R_f \sigma_x \rho}{(R_f-\omega)(R_f-1)} > 0, \rho = E_t(\epsilon_{t+1} e_{t+1}) > 0, \quad \gamma_1 = \frac{R_f(1-\omega)(1-\gamma)}{(R_f-\omega)(R_f-\gamma)(R_f-1)} \geq 0,$$

$$\gamma_2 = \frac{R_f(1-\omega)(1-\gamma)}{(R_f-\omega)(R_f-\gamma)} \geq 0, \quad \gamma_3 = -\frac{R_f \gamma \omega}{(R_f-\omega)(R_f-\gamma)} \leq 0,$$

$$\gamma_4 = \frac{\omega\gamma(R_f-1)}{(R_f-\omega)(R_f-\gamma)} \geq 0, \quad \text{and} \quad \gamma_5 = \alpha_3 > 0.$$

[6] Because our analysis is in discrete time, this specific form of discount factor may yield negative values. We assume that does not occur

[7] This result obtains by assuming that the market portfolio has stochastic risk. This is a straightforward extension of the basic derivations in Cochrane (2001), Chapter 9.

[8] We have also solved for a model where risk follows a mean-reverting process. The implications of our results do not change even when risk is assumed to mean-revert.

Equation (6) of Proposition 1 provides a parsimonious linear valuation equation similar to Ohlson's but with an additional economy-wide aggregate risk term (and an intercept term). Consistent with intuition, Eq. (6) shows that equity prices are positively related to firm fundamentals (e.g., abnormal earnings) but inversely related to economy-wide risk.[9] Intuitively, when uncertainty is high, such as during the 2007–2008 economic crisis, market values will be low relative to fundamentals. This effect is magnified for those firms with high α_2 coefficients, that is, firms with highly persistent earnings, because persistence compounds systematic risk shocks. Ignoring the state of uncertainty in the economy by, say, discounting using an historical CAPM or some other historical factor model, could result in considerable model error relative to the market and foster the claim that a pricing anomaly has been discovered.

Two observations regarding this valuation equation are worth highlighting. First, in a risk neutral world $\lambda_1 = 0$, resulting in the original Ohlson (1995) model (assuming $x_L^a = 0$). Second, λ_1 is increasing in the level of abnormal earnings volatility (σ_x). The latter result is consistent with the notion that increased uncertainty about firm fundamentals should reduce stock values (or increase costs of capital). However, there is a standard caveat. Even if abnormal earnings volatility is high, stock values are unaffected to the extent that shocks to abnormal earnings are uncorrelated with shocks to the stochastic discount factor ($\rho = 0$); that is, abnormal earnings volatility matters in the pricing of equities only if it is systematic.

While Proposition 1 provides the equity pricing equation, it is not immediately obvious how equity returns change over time. This issue is important if only because returns based equations are commonly used in empirical research to relate the relevance of accounting variables both to equity returns and to costs of capital (expected equity returns). Our next proposition describes the equity return dynamics and cost of capital dynamics, which are a consequence of valuation Eq. (6).

3.1 Equity returns and costs of capital

By using a stochastic discount factor approach to valuation, we need not specify the returns or expected returns dynamics exogenously. Rather, they manifest as a byproduct of the underlying assumptions that determine the valuation equation. The following proposition provides the dynamics of both returns and expected returns (costs of capital).

Proposition 2 *Let* $R_{t+1} = \frac{S_{t+1}+D_{t+1}}{S_t}$ *denote the cum dividend equity return and* $\Delta\sigma_{m,t}$ *the change in expected economy-wide (systematic) risk. Given the equity valuation* Eq. (6), *the return generating process satisfies the dynamic:*

[9] The result that λ_1 is positive presumes a positive correlation (ρ) between shocks to abnormal earnings (ϵ_{t+1}) and shocks to the stochastic discount factor (e_{t+1}). On average it is unlikely to be otherwise as long as growth in the economy and growth in firm level abnormal earnings are positively correlated. To complete the argument note that the discount factor represents the marginal rate of consumption in the economy or growth in the economy—see "Appendix 1". Therefore shocks to the discount rate factor are driven by shocks to aggregate consumption or shocks to aggregate growth, represented by e_{t+1} which, in turn, should be positively related to shocks to (abnormal) earnings ϵ_{t+1}.

$$R_{t+1} = R_f + (R_f - 1)\lambda_1 \frac{\sigma_{m,t}}{S_t} + (1 + \alpha_2)\frac{\epsilon_{t+1}}{S_t} + \alpha_3 \frac{u_{t+1}}{S_t} - \lambda_1 \frac{\Delta\sigma_{m,t}}{S_t}. \quad (8)$$

Furthermore, the cost of capital (expected return), μ_{t+1}, *is given by:*

$$\mu_{t+1} = R_f + (R_f - 1)\lambda_1 \frac{\sigma_{m,t}}{S_t} \quad (9)$$

$$= R_f\left(1 + \frac{R_f}{S_t} cov\left(\frac{x^a_{t+1}}{R_f - \omega}, m_{t,t+1}\right)\right). \quad (10)$$

Equation (8) offers considerable insight into the behavior of stock returns and their relation to costs of capital. Equation (9), the expectation of (8), shows that higher values of λ_1 increase expected equity returns (cost of capital). However, firms with the highest cost of capital or, equivalently, the largest λ_1, will experience the most dramatic movements in their stock price when there is a change in expected economy-wide risk. In particular, the last term in (8) says that firms with the highest expected returns will experience the largest downward price movements when aggregate risk increases in the economy, and vice versa when aggregate risk decreases in the economy. Thus, firms with the highest (negative) covariance with changes in aggregate risk in the economy are expected to have the highest average stock returns. This is exactly the result documented by Ang et al. (2006) in their Table 1; firms with the most *negative* loadings on changes in aggregate risk (as proxied by the VIX) have the highest future stock returns. Moreover, while other variables (such as beta in the CAPM) have not had much success at predicting stock returns, Ang et al. (2006) provide evidence that changes in aggregate risk robustly predict stock returns, adding credence to the theoretical results we have presented.

Equation (9) shows that the cost of capital (expected return) is a function of the risk-free rate, volatility of abnormal earnings, earnings persistence, and the level of risk in the economy.[10] Higher values of each of the latter variables increase the return demanded by risk-averse investors. Furthermore, simultaneous inspection of Eqs. (6) and (9) suggests that our model has the potential for cost of capital estimation in a manner that is likely to be of keen interest to accounting scholars. Specifically, the fact that the covariance term $\lambda_1\sigma_{m,t}$ is present in both the expected return Eq. (9) and in the equity price Eq. (6) provides an opportunity to substitute observable price and accounting variables for the unobservable $\lambda_1\sigma_{m,t}$ in solving for expected returns. This insight is investigated in the next section of the paper.

An open question in the accounting literature is the relation, if any, between accounting valuation models based on non-arbitrage pricing and clean surplus, such as the Ohlson (1995) model, and the accounting return decomposition models of Vuolteenaho (2002) and Callen and Segal (2004), which are based on the

[10] Note that λ_1 is the firm level driver of expected returns being a function of the persistence of abnormal earnings, the volatility of abnormal earnings, and the correlation between (shocks to) abnormal earnings and (shocks to) economy-wide systematic risk in the economy.

Table 1 Summary statistics

	R_{t+1}	Price_t	B_t	x_t	$E_t\,[x_{t+1}]$	$\frac{D_t}{S_t}$	$Size_t$	bm_t
Panel A: Firm specific summary statistics								
Mean	1.45	29.68	15.05	1.43	2.24	0.02	6.64	−0.75
Std.	12.37	26.58	14.02	5.71	2.02	0.04	1.67	0.72
Max	359.09	983.02	662.67	573.76	40.50	2.84	13.19	2.70
Min	−84.84	5.00	0.06	−281.41	−6.54	0.00	1.55	−5.54

	$R_{t+1}-1$	S_t^{-1}	B_t/S_t	x_t/S_t	$E_t\,[x_{t+1}]/S_t$	D_t/S_t
Panel B: Correlation matrix						
$R_{t+1}-1$		**0.05**	**0.08**	0.00	**0.06**	**0.01**
S_t^{-1}	**0.02**		**0.22**	**0.03**	**0.17**	**−0.06**
B_t/S_t	**0.04**	**0.27**		**0.22**	**0.56**	**0.29**
x_t/S_t	**0.03**	**0.05**	**0.45**		**0.55**	**0.18**
$E_t\,[x_{t+1}]/S_t$	**0.04**	**0.16**	**0.59**	**0.74**		**0.27**
D_t/S_t	**0.02**	**−0.21**	**0.35**	**0.40**	**0.39**	

	Mean	Std.	Max	Min
Panel C: VIX summary statistics				
VIX_t	21.43	8.53	61.41	9.82
ΔVIX_t	−0.008	4.916	39.03	−19.41

Panel A, reports descriptive statistics for 524,123 firm-months (6,778 firms) from 1980 to 2010. Price_t denotes price per share, R_{t+1} monthly cum-dividend (gross) returns, B_t book value per share, x_t earnings before extraordinary items, D_t dividends per share, $Size_t$ the log of market capitalization and bm_t the log book to market ratio. $E[x_{t+1}]$ denotes the IBES consensus forecast for one-year-ahead earnings computed as the time-weighted mean consensus analyst forecast of year t + 1 and year t + 2 earnings multiplied by common shares outstanding

Panel B, provides Pearson (the upper triangle) and Spearman (the lower triangle) correlations for the variables used in multivariate regressions. Bold numbers represent significance at the 1 % level. Non-bolded values are insignificant

Panel C, provides descriptive time-series statistics for the VIX (VXO) contracts provided by the CBOE for years 1986–2010 at the end of each month. The sample consists of 300 monthly observations. The contracts represent implied volatilities and are presented as annualized percentages. The values are obtained from the implied volatilities of contracts written on the S&P 100 (OEX) index

time-series properties of the book-to-market ratio and clean surplus. The return decomposition literature proves that returns can be decomposed into expected returns, shocks to current and future cash flows, called cash flow news, and shocks to future expected returns, called discount rate news. The following corollary of Proposition 2 shows that returns derived from our non-arbitrage based model follow a similar return decomposition.

Corollary *Consistent with the return decomposition literature, equity returns in* Eq. (8) *decompose into expected returns plus cash flow news minus discount rate news where*

$$\begin{aligned} expected\,returns &= R_f + (R_f - 1)\lambda_1 \frac{\sigma_{m,t}}{S_t}, \\ cash\,flow\,news &= (1+\alpha_2)\frac{\epsilon_{t+1}}{S_t} + \alpha_3 \frac{u_{t+1}}{S_t}, \\ discount\,rate\,news &= \lambda_1 \frac{\Delta\sigma_{m,t}}{S_t}. \end{aligned}$$

As in the return decomposition literature, a positive shock to cash flows, measured by shocks to abnormal earnings and "other information," increases equity returns, whereas a positive shock to expected returns drives down equity returns. This result cannot be demonstrated in standard accounting valuation models for which risk is constant over time.

3.2 Determinants of the cost of capital-accounting and firm fundamentals

In this section, we present one of the main findings of our paper, namely, that expected returns (costs of capital) can be expressed as a linear function of accounting variables and other firm fundamentals deflated by price. Much of the work in finance has focused on using covariances (such as beta) to measure expected returns. However, estimating these values has proven to be extremely difficult, and their success in predicting out of sample stock returns has been elusive despite extensive efforts by the literature (Daniel and Titman 1997; Subrahmanyam 2010). As opposed to focusing on unobservable covariances as the driving force of expected returns, our model allows us to substitute observable firm characteristics for unobservable covariances as in the next proposition.

Proposition 3 *The firm's costs of capital (expected returns) can be expressed as:*

$$\mu_{t+1} = 1 + \eta_1 \frac{x_L^a}{S_t} + \eta_2 \frac{B_t}{S_t} + \eta_3 \frac{x_t}{S_t} + \eta_4 \frac{E_t[x_{t+1}]}{S_t} + \eta_5 \frac{D_t}{S_t}, \tag{11}$$

or, equivalently, as:

$$\mu_{t+1} = 1 + \eta_1 \frac{x_L^a}{S_t} + \eta_2^{'} \frac{B_t}{S_t} + \eta_3^{'} \frac{x_t}{S_t} + \eta_4 \frac{E_t[x_{t+1}]}{S_t} + \eta_5 \frac{B_{t-1}}{S_t}, \tag{12}$$

where

$$\begin{aligned} \eta_1 &= \frac{R_f(1-\omega)(1-\gamma)}{(R_f-\omega)(R_f-\gamma)} \geq 0, \quad \eta_2 = \frac{R_f(R_f-1)(1-\omega)(1-\gamma)}{(R_f-\omega)(R_f-\gamma)} \geq 0, \\ \eta_3 &= -\frac{(R_f-1)R_f\gamma\omega}{(R_f-\omega)(R_f-\gamma)} \leq 0, \eta_4 = \frac{(R_f-1)R_f}{(R_f-\omega)(R_f-\gamma)} > 0, \\ \eta_5 &= \frac{(R_f-1)^2\omega\gamma}{(R_f-\omega)(R_f-\gamma)} \geq 0, \quad \eta_2^{'} = \eta_2 - \eta_5, \quad \text{and} \quad \eta_3^{'} = \eta_3 - \eta_5 \leq 0. \end{aligned}$$

This proposition represents one of the key theoretical findings in our paper and has a number of important implications for measuring costs of capital. First, Eq. (11) shows that the cost of capital (expected return) can be expressed solely as a linear function of firm fundamentals: the book-to-market ratio, the earnings-price ratio, the forward earnings-price ratio, (the inverse of) size, and the dividend yield. Alternatively, further eliminating dividends via clean surplus, Eq. (12) shows that the cost of capital can be expressed solely as a linear function of accounting variables and size where the accounting variables include the current book-to-market ratio, the past book-to-market ratio, the earnings-price ratio, and the forward earnings-price ratio. These variables have been used in empirical research as predictors of expected returns—see Subrahmanyam (2010), for example—particularly size and the book-to-market ratio, which have generated significant attention since the empirical work of Fama and French (1992). Second, there are no betas or other covariance terms on the right-hand side of these equations to estimate. Firm fundamentals alone determine costs of capital. Third, the result shows that accounting variables play a vital role in asset pricing and cost of capital measurement. Fourth, our theory not only give strong theoretical guidance for which specific firm fundamentals "should" be used to determine costs of capital but also how these firm fundamentals are to be combined as joint determinants of expected returns.

The next section tests the empirical validity of our model.

4 Empirical estimation

Our main empirical focus is on using firm fundamentals and current equity prices to predict future equity returns. We initially test whether equity returns derived from our accounting-based model are significantly associated with future realized returns in the cross-section. We then compare the equity returns forecasted by our model with the return forecasts from standard benchmarking models. Finally, we calibrate the model to real world data and test to see how well the calibrated model performs in predicting out of sample returns. In addition to returns, we also test whether the model can predict security prices by predicting out of sample stock prices relative to standard benchmarking models.

4.1 The sample

Our sample consists of a large cross-section of publicly traded firms with December fiscal year-ends from 1980 to 2010. Firm fundamentals are obtained from the annual Xpressfeed Compustat database. Analyst forecasts are obtained from the IBES summary statistics database. Stock return data are collected from CRSP. We restrict our sample to firms with positive book values, price per share greater than \$5, and at least 2 years of consecutive data. As in Nekrasov and Shroff (2009), we require that firms have 1- and 2-year-ahead analyst forecasts with a positive second year forecast, book-to-market ratios between 0.01 and 100, we also require expected earnings growth to be between 0 and 100 %.[11] After these restrictions, we are left with a total of 524,123 firm-months (6,778 firms) in the sample.

In some of our analysis, we use VIX option contract data. These data are obtained from the Chicago Board of Options Exchange (CBOE) website. The CBOE has two "VIX" contracts, one based on the S&P 100 (the OEX), which is the original VIX contract, and the new VIX, which is constructed from options written on the S&P 500. Although the new S&P 500 VIX has replaced the OEX based version, data for the new VIX are available only since 1990. Data for the old VIX contracts, renamed the VXO, are available from January 1986. We follow previous research (e.g., Ang et al. 2006) and use the old VIX contracts in order to maximize the number of observations in our sample. When we use VIX option data our sample size reduces to 441,290 firm-months (6227 firms) observations. Table 1 provides general summary statistics.

4.2 Cross-sectional stock return tests

Our initial set of tests is meant to determine how well our accounting-based cost of capital measure performs in the cross-section. The empirical regression model follows Eq. (11) and takes the regression form:[12]

$$R_{t+1} - 1 = \alpha + \frac{\eta_1}{S_t} + \eta_2 \frac{B_t}{S_t} + \eta_3 \frac{x_t}{S_t} + \eta_4 \frac{E_t[x_{t+1}]}{S_t} + \eta_5 \frac{D_t}{S_t} + \varepsilon_{t+1}. \tag{13}$$

x_t is measured by income before extraordinary items and B_t is book value. D_t is measured by dividends paid to common shareholders. Expected future earnings, $E_t[x_{t+1}]$, is measured by the time-weighted mean consensus analyst forecast of year $t + 1$ and year $t + 2$ earnings multiplied by the number of common shares outstanding as per Compustat.[13] The deflator, S_t, is last periods stock price adjusted for stock spits multiplied by the number of shares outstanding in Compustat. From Proposition 3, we expect the coefficients for the inverse of size, book-to-market ratio, expected earnings-price ratio, and dividend yield (η_1, η_2, η_4 and η_5) to be positive and the coefficient on the earnings-price ratio, η_3, to be negative.[14]

To test the model, we estimate monthly Fama–MacBeth regressions of realized next-period equity returns on firm fundamentals deflated by stock price. We initially regress returns on each explanatory variable separately and then on all of the explanatory variables simultaneously as per Eq. (13). The mean cross-sectional

[11] The restriction on earnings growth does not affect any of our results, if anything our results are stronger when we remove this restriction.

[12] We prefer to use equation (11), which includes the dividend yield, rather than Eq. (12) because of the high correlation between current and past book-to-market ratios. We refer to Eq. (11) as the accounting-based model although size and the dividend yield are not accounting variables.

[13] Particularly, we measure next period expected earning by multiplying 1 year ahead concensus IBES earnings by w_d and 2 year ahead concensus IBES earnings by $1 - w_d$ where w_d is the difference between the firms ficscal year end date and the current forecast date divided by 365.

[14] We assume that long-run abnormal earnings are cross-sectionally constant, consistent with the notion that, in the long-run, a firm will grow to the point where it resembles a cross-section of firms.

Table 2 Cross-sectional return regressions

Regression model: $R_{t+1} - 1 = \text{Intercept} + \frac{\eta_1}{S_t} + \eta_2 \frac{B_t}{S_t} + \eta_3 \frac{x_t}{S_t} + \eta_4 \frac{E_t[x_{t+1}]}{S_t} + \eta_5 \frac{D_t}{S_t}$

Intercept	0.012***	0.007**	0.014***	0.009**	0.014***	0.006
	(3.42)	(1.97)	(3.95)	(2.53)	(3.57)	(1.53)
S_t^{-1}	0.933***					0.670***
	(6.90)					(5.76)
B_t/S_t		0.012***				0.007***
		(5.68)				(3.98)
x_t/S_t			−0.002			−0.020***
			(−0.19)			(−3.11)
$E_t[x_{t+1}]/S_t$				0.061***		0.034**
				(3.33)		(2.03)
D_t/S_t					0.026	0.003
					(0.85)	(0.09)
adj-R^2	0.010	0.016	0.010	0.014	0.016	0.047

This table reports mean coefficients and t-statistics from Fama and MacBeth (1973) cross-sectional regressions of one-month-ahead excess equity returns on the variables shown. The sample consists of 524,091 firm-months from years 1980 to 2010. S_t denotes split adjusted price per share multiplied by common shares outstanding, B_t book value, x_t earnings before extraordinary items, and D_t is dividends. $E[x_{t+1}]$ denotes the IBES consensus forecast for one-year-ahead earnings computed as the time-weighted mean consensus analyst forecast of year t + 1 and year t + 2 earnings multiplied by common shares outstanding. The t-statistics are calculated from Fama–MacBeth standard errors. ** and *** denote two-tailed statistical significance at the 5 and 1 % significance levels, respectively

coefficients and Fama–MacBeth t-statistics are presented in Table 2. Moving from left to right across the table, the three most important firm fundamental variables for determining expected returns are the inverse of size, the book-to-market ratio, and the future earnings-price ratio. Importantly, focusing on the last column of the table in which all model-driven variables are included in the regression, the signs of all of the coefficients match the theoretical predictions of the model although the dividend yield is insignificant.

4.2.1 Comparison with standard covariance risk proxies

In this section, we explore how our returns model compares with benchmark CAPM and Fama–French (FF) three factor models. We follow standard estimation procedures. The firm-specific CAPM betas and FF betas are estimated using 5-year rolling windows.[15] Betas are updated every April. We estimate monthly Fama–MacBeth regressions of realized next-period equity returns on firm betas. We also include CAPM betas and FF betas in our accounting-based model to determine if these covariance variables subsume the explanatory power of firm fundamentals in explaining stock returns. Table 3 presents the results. The only beta to prove

[15] The results in this and the following tables are not sensitive to the size of the estimation window.

Table 3 Cross-sectional return regressions with covariance risk factors

Regression model: $R_{t+1} - 1 = \text{Intercept} + \frac{\eta_1}{S_t} + \eta_2 \frac{B_t}{S_t} + \eta_3 \frac{x_t}{S_t} + \eta_4 \frac{E_t[x_{t+1}]}{S_t} + \eta_5 \frac{D_t}{S_t} + \sum_i \beta_i$

Intercept	0.006	0.012***	0.011***	0.003	0.003
	(1.53)	(4.38)	(4.55)	(0.92)	(0.96)
S_t^{-1}	0.670***			0.683***	0.659***
	(5.76)			(5.90)	(6.03)
B_t / S_t	0.007***			0.007***	0.007***
	(3.98)			(4.15)	(3.95)
x_t / S_t	−0.020***			−0.017***	−0.016***
	(−3.11)			(−3.11)	(−3.14)
$E_t [x_{t+1}]/S_t$	0.034**			0.033**	0.032**
	(2.03)			(2.10)	(2.15)
D_t / S_t	0.003			0.005	0.011
	(0.09)			(0.26)	(0.60)
β		0.003		0.003	
		(1.21)		(1.30)	
β_m			0.002		0.003
			(1.05)		(1.44)
β_h			0.001		−0.000
			(0.86)		(−0.18)
β_s			0.003***		0.001
			(2.88)		(1.41)
adj-R^2	0.047	0.032	0.050	0.067	0.079

This table reports mean coefficients and t-statistics from Fama and MacBeth (1973) cross-sectional regressions of one-month-ahead excess equity returns on the variables shown. The sample consists of 524,091 firm-months from years 1980 to 2010. S_t denotes split adjusted price per share multiplied by common shares outstanding, B_t book value, x_t earnings before extraordinary items, and D_t is dividends. $E[x_{t+1}]$ denotes the IBES consensus forecast for one-year-ahead earnings computed as the time-weighted mean consensus analyst forecast of year t + 1 and year t + 2 earnings multiplied by common shares outstanding. β denotes the firm-specific beta from the CAPM. β_m, β_h, and β_s denote firm-specific betas obtained from the Fama and French (1993) three-factor model where the betas represent the firm specific slopes on the market portfolio, the high book-to-market over low book-to-market portfolio (SMB), and the portfolio consisting of small firms over large firms (HML), respectively. The t-statistics are calculated from Fama–MacBeth standard errors. ** and *** denote two-tailed statistical significance at the 5 and 1 % significance levels, respectively

significant is the size beta in the third regression. All other betas are insignificant. When we include the betas together with our accounting model-driven variables, the results are basically unchanged. Firm fundamentals carry significant associations with realized stock returns, whereas the betas do not.

4.2.2 Generating cost of capital estimates

In the previous section, we evaluated whether the variables in our theoretical model are associated with future stock returns. However finding an association does not necessarily imply that the equation actually predicts stock returns in the

Table 4 Cross-sectional cost of capital tests

	μ_{acct}	μ_{ff}	μ_{capm}	r_{t+1}
Panel A: Summary statistics				
Mean	1.38	1.05	0.82	1.27
Std.	0.64	0.60	0.39	12.30
Max	21.03	8.00	4.12	359.1
Min	−8.97	-3.45	−0.77	−84.84

	μ_{acct}		μ_{ff}		μ_{capm}	
	Coeff.	*t*-stat	Coeff.	*t*-stat	Coeff.	*t*-stat
Panel B: Cross-sectional regressions of realized returns on costs of capital						
μ	**1.181*** **	**(6.88)**	**0.193**	**(1.14)**	**0.843**	**(1.35)**
Intercept	0.395	(1.06)	1.177***	(3.07)	0.673	(1.53)
adj-R^2	0.015		0.030		0.009	

Panel A, reports summary statistics for model-determined expected return (cost of capital) forecasts and realized net equity returns. μ_{acct} denotes the accounting-based expected returns [Eq. (13)], μ_{ff} the Fama–French three-factor expected returns, and μ_{capm} the CAPM expected returns. $r_{t+1} = R_{t+1} - 1$ denotes net realized equity returns

Panel B, reports mean coefficients and *t*-statistics from Fama and MacBeth (1973) cross-sectional regressions of realized returns on each of three different costs of equity capital estimates. The sample consists of 282,810 firm-month observations from years 1990 to 2010. The *t*-statistics are calculated from Fama–MacBeth standard errors. *** denotes two-tailed statistical significance at the 1 % significance level

cross-section. Rather, if the model is a good proxy for expected returns, we should find a strong association between ex ante (expected) and ex post (average) stock returns. Following Lewellen (2011), we calibrate our model (equation 13) using 10-year (one hundred and twenty month) rolling Fama–MacBeth coefficients (with a minimum of 5 years data) along with current firm fundamentals and stock prices to predict next period's out of sample equity returns. The methodology insures that only historical data are used to form expectations about future stock returns. Returns are predicted from 1985 to 2010.

Table 4, Panel A, provides summary statistics for realized stock returns and for expected returns derived from our accounting-based model, the CAPM, and FF three-factor model. Results are again shown for 5-year rolling betas.

To test the association of each of the expected return forecasts with realized equity returns, we regress monthly realized returns on each of the cost of capital forecasts separately as follows:

$$R_{t+1} - 1 = A_1 + A_2\mu_{t,t+1} + w_{t+1}. \quad (14)$$

$\mu_{t,t+1}$ denotes period $(t + 1)$ expected return forecasts conditioned on information at time t. If $A_1 = 0$ and $A_2 = 1$, then the null that $\mu_{t,t+1}$ represents the 'true' expected equity return (cost of capital) cannot be rejected. However, verifying that $A_2 \neq 0$ is often regarded by the literature as sufficient evidence that $\mu_{t,t+1}$ is a good proxy for the "true" expected return

Table 4, Panel B, shows that expected returns based on firm fundamentals (μ_{acct}) as per Eq. (13) are strongly associated with realized returns. In particular, Fama–MacBeth regressions of Eq. (14) yield an insignificant intercept and a highly significant slope coefficient that is statistically indistinguishable from one. By contrast, cost of capital estimates using the CAPM (μ_{capm}) and the FF three factor model (μ_{ff}) have virtually no ability to predict realized equity returns. These results imply that our model, which is driven by firm fundamentals, carries considerable information about future realized returns in contrast to historical covariance estimates.

4.2.3 Portfolio returns

The cross-sectional firm-level results in the prior section indicate a statistically significant association between realized stock returns and expected returns derived from firm fundamentals. To minimize potential concerns associated with measurement error at the firm level, we next turn to a portfolio analysis. We sort firms into

Table 5 Portfolio-based time-series tests

	$E_t\ [r_{t+1}\ \vert \mu_{acct}\ ]$	$\overline{r_{t+1}}$	$E_t\ [r_{t+1}\ \vert \mu_{ff}\ ]$	$\overline{r_{t+1}}$	$E_t\ [r_{t+1}\ \vert \mu_{capm}\ ]$	$\overline{r_{t+1}}$
Panel A: Portfolio-based returns						
π_1	0.88	0.87	0.33	1.28	0.46	1.04
π_2	1.07	0.89	0.78	1.11	0.63	1.16
π_3	1.24	0.99	1.02	1.14	0.76	1.22
π_4	1.48	1.26	1.29	1.35	0.91	1.25
π_5	2.24	2.37	1.86	1.50	1.30	1.71
$\pi_5 - \pi_1$		**1.5*** **		**0.22**		**0.67**
t-stat		5.35		1.12		1.54

	μ_{acct}	μ_{ff}	μ_{capm}
Panel B: Portfolio-based alphas			
π_1	0.047	0.40	0.30
π_2	0.009	0.24	0.29
π_3	0.062	0.23	0.25
π_4	0.309	0.37	0.23
π_5	1.224	0.40	0.58
$\pi_5 - \pi_1$	**1.18*** **	**−0.001**	**0.27**
t-stat	5.73	−0.01	1.26

Panel A, reports monthly average excess returns in percentages for five equally weighted quintile portfolios, π_1 to π_5, sorted by (expected return) cost of capital measures. $r_{t+1} = R_{t+1} - 1$ denotes net realized returns.$E_t\ [r_{t+1}\ |\mu\ ]$ denotes net expected stock returns conditional on the cost of capital model, and $\overline{r_{t+1}}$ the average realized net return. μ_{acct} denotes the accounting-based expected return [Eq. (13)], μ_{ff} the Fama–French three factor expected return, and μ_{capm} the CAPM expected return

Panel B, reports portfolio-adjusted excess returns (alphas) for each quintile based on the Fama and French (1993) three-factor model. *t*-statistics are based on heteroscedasticity consistent standard errors. *** denotes two-tailed statistical significance at the 1 % significance level expected return equally weighted quintile portfolios and then determine whether an investor who allocated resources based on our expected return accounting model would have realized gains. Table 5, Panel A, shows raw portfolio returns from investing in the three proxies for expected returns. The first column lists expected next-period portfolio returns derived from our accounting-based model. The second column lists the realized next-period portfolio returns. Columns 3 through 6 show similar results based on expected return quintile portfolios derived from the FF three-factor model and the CAPM. Average realized returns are monotonically increasing for all expected return metrics. The last row in the table shows the results of a long-short strategy; that is, buying the highest expected return quintile and shorting the lowest expected return quintile. Only our firm fundamentals expected return model yields significant positive returns.

To determine whether our theoretically derived measure of expected returns is subsumed by standard risk factors, we regress each of the portfolio returns on the FF three factors. Table 5, Panel B. shows the resulting alphas; that is, the abnormal hedged returns earned by investing based on each of the expected return (cost of capital) metrics. Both of the covariance-based expected return measures generate insignificant abnormal returns. In contrast, our expected return measure generates an economically significant 1.18 % abnormal return per month. This test is important because it illustrates that our metric of (rational) expected returns is not anomalous. Instead, the excess returns are generated because of risk rather than, say, lack of attention by investors or some behavioral bias. Yet traditional asset pricing models would likely classify this excess return as anomalous. Our results are consistent with Penman and Zhu (2011), who criticize the tendency to prematurely classify high realized returns as anomalous.

4.3 Stock price estimation

The prior analysis focused on exploring whether our fundamentals-based returns equation yields a reasonable proxy for the cost of equity capital. In this section we explore whether our model does a good job of predicting equity values. First, we explore the cross-sectional explanatory power of the model. To overcome the problem that the primary variable of interest in our model, namely, economy-wide risk, is cross-sectionally constant, we estimate the sensitivity to economy-wide risk instead using a two-pass regression approach. In the first-pass, we regress next period's excess return on the VIX (and the change in the VIX) using firm-level time-series data. The regression coefficient measures the sensitivity of the firm's returns to economy-wide risk where the VIX is our empirical proxy for expected economy-wide (systematic) risk. In the second pass cross-sectional regression, we regress excess returns on the estimated "sensitivity to aggregate risk" coefficients from the first-pass regressions and on the FF factor betas over the same window as for the time-series estimation. This two-pass type estimation procedure is common to the asset pricing literature (see Core et al. 2008, for example). The details of this estimation procedure follow.

4.3.1 Estimating sensitivity to aggregate risk

Equation (8) of Proposition 2 links equity returns to both the level and changes in expected economy-wide risk. We re-arrange this equation into the form:

$$R_{t+1} - R_f = (R_f - 1)\lambda_1 \frac{\sigma_{m,t}}{S_t} - \lambda_1 \frac{\Delta\sigma_{m,t}}{S_t} + e_{r,t+1}, \tag{15}$$

where $e_{r,t+1}$ is a zero mean error term that contains the cash flow shocks. We further re-arrange Eq. (15) into the empirical regression form:

$$R_{t+1} - R_f = \lambda_0 + \lambda_1 \left[(R_f - 1)\frac{\sigma_{m,t}}{S_t} - \frac{\Delta\sigma_{m,t}}{S_t}\right] + e_{r,t+1}. \tag{16}$$

Because economy-wide risk is not observable, we use the CBOE VIX contract as a proxy. Thus, to estimate λ_1, we use the following empirical model:

$$R_{t+1} - R_f = \lambda_0 + \lambda_1 \left[(R_f - 1)\frac{VIX_t}{S_t} - \frac{\Delta VIX_t}{S_t}\right] + e_{r,t+1}. \tag{17}$$

We limit our sample to firms with at least 120 trading days of time-series data using daily CRSP stock returns, CBOE daily VIX contract data, and risk-free rates obtained from Ken French's website. To control for the well-known microstructure issues that arise from using daily data, we follow prior research (see Dimson 1979 and Bali et al. 2009, for example) and include the lagged independent variable in the regression.[16] The estimate of λ_1 is the sum of the coefficients on the contemporaneous and lagged independent variables, denoted by $\hat{\lambda}_1$. We then incorporate $\hat{\lambda}_1$ as an additional variable in the second-pass cross-sectional regression representing a firm-level metric of sensitivity to risk in the economy. A similar two-pass regression approach is followed to estimate each of the market beta and the FF three-factor betas over the same time period.

Summary statistics for annual estimates of $\hat{\lambda}_1$, firm betas, and FF three-factor betas are provided in Table 6, Panel A. The average sensitivity to changes in the VIX is 0.16, consistent with the sign predicted by our theoretical derivation. Multiplying the latter by the VIX contract value deflated by price per share yields an average value of 1.55, implying an average risk premium of around 1.55 of the risk-free rate. Given that risk-free rates from Ken French's website averaged around 4.8 % over the sample period, our model implies an average risk premium over the sample period of approximately 7.4 % and an average gross return of 12.2 %. Since the actual gross return averaged around 17 % over the sample period, our empirical model appears to underestimate average returns.

4.3.2 Does sensitivity to aggregate risk predict stock returns?

Ang et al. (2006) show that sensitivity to economy-wide risk is associated with equity returns. Nevertheless, in order to determine if sensitivity to economy-wide

[16] Letting $Z_t = (R_f - 1)\frac{VIX_t}{S_t} - \frac{\Delta VIX_t}{S_t}$, we also include $Z_{t\text{-}1}$ in the regression.

Table 6 Time-series risk estimates and association with future returns

	$\hat{\lambda}_1$	$\widehat{\lambda}\frac{VIX_t}{S_t}$	β	β_m	β_h	β_s
Panel A: Summary statistics						
Mean	0.16	1.55	0.98	1.01	0.23	0.57
Std.	0.18	2.28	0.6	0.59	1.01	0.79
Max	6.27	116.79	5.13	6.53	6.73	6.35
Min	−0.27	−8.21	−1.03	−2.63	−10.51	−3.74
Panel B: Cross-sectional return regressions						
$\widehat{\lambda_1}/S_t$		390.11***				
		(8.01)				
β				0.39*		
				(1.94)		
β_m						0.42**
						(1.98)
β_h						−0.014
						(−0.07)
β_s						0.07
						(0.33)
adj-R^2		0.032		0.036		0.077

Panel A, reports summary statistics based on time-series estimates of risk factors. The sample consists of 441,290 firm-months from April 1986 to May 2010. $\hat{\lambda}_1$ denotes the firm specific sensitivity to aggregate risk, and β the firm specific beta from the CAPM. β_m, β_h, and β_s denote firm specific betas from the Fama and French (1993) three-factor model, where the betas represent the firm specific slopes on the market portfolio, the high book-to-market over low book-to-market portfolio (SMB), and the portfolio consisting of small firms over large firms (HML), respectively

Table 6, Panel B, reports mean coefficients and *t*-statistics from Fama and MacBeth (1973) cross-sectional regressions of one-month-ahead excess equity returns on estimated risk factors. The sample consists of 441,290 firm-months from April 1986 to May 2010. The *t*-statistics are calculated from Fama–MacBeth standard errors. *, ** and *** denote two-tailed statistical significance at the 10, 5, and 1 % significance levels, respectively

risk is a reasonable risk factor for our sample, we conduct an asset pricing test and regress stock returns on this estimated risk factor (deflated by price). We further compare our "sensitivity to economy-wide risk" factor with the CAPM and with Fama–French risk factors. Specifically, we regress separately monthly excess equity returns on the sensitivity to economy-wide risk factor ($\hat{\lambda}_1$) and on the FF factor betas to determine which variables predict stock returns in the cross-section.[17]

Table 6, Panel B, presents the results. The sensitivity to economy-wide risk deflated by price is highly significant suggesting that $\hat{\lambda}_1$ proxies for priced risk. Our risk factor has a much higher level of statistical significance by comparison to the market beta. The beta coefficients for the HML and SMB factors are insignificant.

[17] The firm-level $\hat{\lambda}_1$, estimated market beta, and estimated FF factor betas are updated annually each April.

Table 7 Market-to-book cross-sectional regressions

Regression model: $\frac{S_t}{B_t} = \frac{\gamma_1}{B_t} + \gamma_2 + \gamma_3 \frac{x_t}{B_t} + \gamma_4 \frac{D_t}{B_t} + \gamma_5 \frac{E_t[x_{t+1}]}{B_t} - \hat{\lambda}_1 \frac{VIX_t}{B_t}$

${B_t}^{-1}$	181.84***					36.75***
	(24.24)					(11.81)
Intercept		2.97***				0.37***
		(65.48)				(11.30)
x_t /B_t			13.243**			−1.95***
			(56.97)			(−11.41)
$E_t\ [x_{t+1}\ ]/B_t$				18.68***		18.17***
				(63.92)		(46.81)
D_t /B_t					9.81***	0.67***
					(14.93)	(6.19)
adj-R^2	0.263	0.427	0.446	0.735	0.162	0.759

This table reports mean coefficients and *t*-statistics from Fama and MacBeth (1973) cross-sectional regressions of price-to-book ratios on the variables shown. The sample consists of 425,582 firm-month observations. S_t denotes split adjusted price per share multiplied by common shares outstanding, B_t book value, x_t earnings before extraordinary items, and D_t dividends per share. $E[x_{t+1}]$ denotes the IBES consensus forecast for one-year-ahead earnings computed as the time-weighted mean consensus analyst forecast of year t + 1 and year t + 2 earnings multiplied by common shares outstanding. $\hat{\lambda}_1$ is the firm specific sensitivity to aggregate risk. The *t*-statistics are calculated from Fama–MacBeth standard errors. *** denotes two-tailed statistical significance at the 1 % significance level

4.3.3 Predicting stock prices

We estimate the price Eq. (7) after deflating both sides by book value and substituting the VIX for economy-wide risk. This yields the following regression:

$$\frac{S_t}{B_t} = \frac{\gamma_1}{B_t} + \gamma_2 + \gamma_3 \frac{x_t}{B_t} + \gamma_4 \frac{D_t}{B_t} + \gamma_5 \frac{E_t[x_{t+1}]}{B_t} - \frac{\hat{\lambda}_1 VIX_t}{B_t} + w_{t+1}, \qquad (18)$$

where w_{t+1} is a mean-zero error term. From the theoretical derivation of Eq. (7) in Proposition 1, all regressors have are predicted to have positive signs except for earnings which is predicted to have negative sign.[18] The results of the regression are found in Table 7. All coefficients are highly significant and in the predicted direction.

To determine whether our accounting-based valuation model does a good job of estimating firm specific intrinsic values, we choose to follow previous valuation research by predicting out of sample stock values (as in Callen and Segal 2005, for example). We then compare the predicted values with the realized values and evaluate the forecast errors. More specifically, each May we use industry-specific parameters estimated from Eq. (18) with the last 10 years of data.[19] These estimates

[18] Since $\hat{\lambda}_1$ is already an estimated coefficient, we force a coefficient of one for $\frac{\hat{\lambda}_1 VIX_t}{B_t}$ when estimating Eq. (18).

[19] Industries are based on the Fama–French 48 industry classifications downloaded from Ken French's website.

Table 8 Stock price prediction pricing errors

	Full model	Ohlson model	RIM CAPM	RIM FF3
MAE	0.308	0.316***	0.451***	0.387***
MEAE	0.239	0.243**	0.392***	0.370***
AVE	0.069	0.087***	0.222***	−0.135***
Pr(AE(Full_Model)<AE(Benchmark))		0.530***	0.650***	0.660***

This table reports summary statistics for three pricing error metrics for each of five models. MAE denotes the mean absolute pricing error, MEAE the median absolute pricing error, and AVE the average pricing error. Pr(AE(Full_Model)<AE(Benchmark)) is the proportion of times that the full model produces lower absolute pricing errors than the benchmark models. The benchmark models include (i) the nested Ohlson (1995) model, (ii) the RIM CAPM for which residual income is discounted by the CAPM cost of capital as in Nekrasov and Shroff (2009), and the RIM FF3 model for which residual income is discounted by the Fama and French (1997) three-factor cost of capital as in Nekrasov and Shroff (2009)

In the first three rows, ** and *** indicate that the difference between the pricing error metric and the benchmark model is statistically significant at the 5 and 1 % significance level, respectively, using a Wilcoxon-Mann-Whitney test. In the fourth row, *** indicates that the value is significantly different from 0.5 at the 1 % significance level using a *t*-test

are then used with current firm fundamentals to produce out of sample stock price predictions. We then compare firm values from our accounting-based model with current market prices and evaluate the pricing errors relative to the pricing errors of three benchmark models. One benchmark is the Ohlson model, which is nested in our full model (with $\hat{\lambda}_1$ and γ_1 set to zero). We also compare our model to the two RIMs estimated by Nekrasov and Shroff (2009), one based on CAPM cost of capital estimates and the other on FF three-factor cost of capital estimates using 5-year rolling industry betas and 30-year rolling risk premia.[20]

Table 8, Panel A, provides summary pricing error statistics after winsorizing the top and bottom 1 % of pricing errors for each model as in Nekrasov and Shroff (2009). We also restrict the absolute pricing error to a maximum of 1 to mitigate outliers, again following Nekrasov and Shroff (2009).[21] The table provides three pricing error measures: (1) mean absolute errors (MAE) presented in the first row of the table; (2) median absolute errors (MEAE) presented in the second row; and (3) average errors (AVE) in the third row. Our model produces pricing errors that are significantly (almost always at the 1 % level) lower than each of the benchmark models. The last row shows the proportion of times that our full model produces lower pricing errors than each of the benchmark models. In all cases, our model yields lower pricing errors at a rate significantly different from 1/2. Comparing our results to the firm specific pricing errors presented by Nekrasov and Shroff (2009) in

[20] Specifically, the two RIM models used assume a functional form of $S_t = B_t + \sum_{i=1}^{T} E_t[\frac{x^a_{t+i}}{(1+r)^i}] + E_t[\frac{x^a_{t+i}(1+g)}{(1+r)^5(r-g)}]$, where B_t is book value, x^a_{t+i} is abnormal earnings at time $t + i$ and g is growth in abnormal earnings. It is assumed that $T = t + 5$ and a long-term growth rate of 0. Various other assumptions of long-term growth tended to yield higher valuation errors than the assumption of zero growth.

[21] The effects of winsorizing the data and constraining the absolute pricing error to 1 reduces the performance of our model relative to the benchmarks. If we do not apply these filters, our model does even better than what is presented in the tables relative to the benchmark models.

their Table 2 shows that the pricing errors using our model are much lower than the pricing errors of their model.

5 Conclusion

This study extends the Ohlson (1995) and Feltham and Ohlson (1999) accounting-valuation framework to include aggregate (systematic) risk in the economy. The theoretical results simultaneously provide exact solutions for the price, stock return, and cost of capital dynamics. The price equation is composed of a linear combination of five components: book value, abnormal earnings, expectations of abnormal earnings, long-term abnormal earnings, and the level of systematic risk in the economy.

Importantly, our model provides theoretical justification for the use of commonly employed firm fundamentals as determinants of the cost of capital such as size, the book-to-market ratio, the earnings-price ratio, the forward earnings-price ratio and the dividend yield. Our returns equation also conforms to the structure dictated by the accounting return decomposition literature. The model also provides a tight theoretical foundation for the findings of Ang et al. (2006), who document an economically meaningful negative association between changes in expected economy-wide uncertainty, as proxied by the VIX, and future stock returns.

Empirically, we test the model both on returns and prices. We find that the model performs well in predicting out of sample stock returns. In particular, our model-driven cost of capital predicts realized equity returns without bias and with a significant slope coefficient that is insignificantly different from one, unlike benchmark models. In regards to the pricing of stocks, we find that our model exhibits significantly smaller pricing errors by comparison to more traditional models used in valuation research that rely on historical expected return measures, such as the CAPM and the FF three-factor model.

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Appendix 1

Derivation of the discount factor

The stochastic discount factor $m_{t,t+1}$ is defined as the marginal rate substitution of aggregate consumption between two periods (see Cochrane 2001):

$$m_{t,t+1} = \frac{M_{t+1}}{M_t}, \tag{19}$$

$$M_t = \frac{\partial U(C_t)}{\partial C_t}, \tag{20}$$

where $U(\cdot)$ is a representative agent's utility function, and C_t is aggregate consumption. Assuming a standard power utility economy with a risk aversion parameter α, then

$$U(C_t) = \frac{C_t^{1-\alpha}}{1-\alpha} \tag{21}$$

$$\Rightarrow M_t = C_t^{-\alpha}. \tag{22}$$

If consumption grows at the rate g_{t+1}, then next period consumption is $C_{t+1} = C_t \exp(g_{t+1})$ so that:

$$m_{t,t+1} = \exp(-\alpha g_{t+1}) \tag{23}$$

$$\approx a - b g_{t+1}, \tag{24}$$

where a and $b > 0$ are constants. This result accords both with intuition and general linear asset pricing models. Firms whose (abnormal) earnings are positively correlated with aggregate consumption are discounted more because of risk aversion—note the negative sign in Eq. (24)—consistent with the implications of the consumption CAPM.

The expectation of $m_{t,t+1}$ must be the gross risk-free discount rate, $E_t[m_{t,t+1}] = R_f^{-1}$, which implies in turn that

$$m_{t,t+1} = E_t[m_{t,t+1}] + (m_{t,t+1} - E_t[m_{t,t+1}]) \tag{25}$$

$$= R_f^{-1}(1 - bR_f(g_{t+1} - E_t[g_{t+1}])). \tag{26}$$

Assuming that growth in aggregate consumption takes the dynamics $g_{t+1} = g + \sigma_{g,t} e_{t+1}$ and that $\sigma_{g,t+1} = \sigma_{g,t} + \xi_{t+1}$ (where e_{t+1} and ξ_{t+1} are mean zero random variables as in the text), then

$$m_{t,t+1} = R_f^{-1}(1 - bR_f \sigma_{g,t} e_{t+1}). \tag{27}$$

Calling $bR_f\sigma_{g,t} \equiv \sigma_{m,t}$ yields the same form of the dynamic discount factor used in this study:

$$m_{t,t+1} = R_f^{-1}(1 - \sigma_{m,t} e_{t+1}).$$

This discount factor readily relates to the CAPM. Assume that consumption can be proxied by the wealth in the market portfolio, then Eq. (24) can be replaced with the discount factor that generates the CAPM:

$$m_{t,t+1} = a - b r_{m,t+1}, \tag{28}$$

where $r_{m,t+1}$ is the return on the market portfolio.

Appendix 2

Proof of Proposition 1

The underlying dynamics are given by:

$$\begin{aligned}
x^a_{t+1} &= \omega x^a_t + (1-\omega)x^a_L + v_t + \epsilon_{t+1},\\
v_{t+1} &= \gamma v_t + u_{t+1},\\
m_{t,t+1} &= m_{t+1} = R_f^{-1}(1-\sigma_{m,t}e_{t+1}),\\
m_{t,t+2} &= m_{t+1}m_{t+2} = R_f^{-1}(1-\sigma_{m,t}e_{t+1})R_f^{-1}(1-\sigma_{m,t+1}e_{t+2})\\
&= R_f^{-2}(1-\sigma_{m,t}e_{t+1})(1-\sigma_{m,t+1}e_{t+2}),\\
\sigma_{m,t+1} &= \sigma_{m,t} + \xi_{t+1}.
\end{aligned}$$

All error terms are assumed to be i.i.d and are orthogonal to one another except for ϵ_{t+1} and e_{t+1}, which have a correlation coefficient of ρ. The expectation operator $E_t[\cdot]$ represents a conditional expectation given information at time t.

From Eq. (1) in the text, the price of the equity is given by:

$$\begin{aligned}
S_t &= B_t + \sum_{i=1}^{\infty} R^{-1}_{f,t,t+i}E_t[x^a_{t+i}] + \sum_{i=1}^{\infty} cov_t(m_{t,t+i}, x^a_{t+i})\\
&= B_t + \sum_{i=1}^{\infty} E_t[m_{t,t+i}x^a_{t+i}].
\end{aligned}$$

Let $z_{t+1} = x^a_{t+1}m_{t,t+1}$, then

$$\begin{aligned}
E_t[z_{t+1}] &= E_t[x^a_{t+1}m_{t+1}]\\
&= E_t[m_{t+1}]E_t[x^a_{t+1}] + E_t[(m_{t+1} - E_t[m_{t+1}])(x^a_{t+1} - E_t[x^a_{t+1}])\\
&= \frac{1}{R_f}(\omega E_t[x^a_t] + E_t[v_t] + (1-\omega)x^a_L) - \frac{1}{R_f}E_t(\sigma_{m,t}e_{t+1}\epsilon_{t+1})\\
&= \frac{1}{R_f}(\omega E_t[x^a_t] + E_t[v_t] + (1-\omega)x^a_L - \sigma_{m,t}\rho\sigma_x).
\end{aligned}$$

By the law of iterated expectations:

$$\begin{aligned}
E_t[z_{t+2}] &= E_t[E_{t+1}[z_{t+2}]]\\
&= E_t[\frac{1}{R_f^2}(1 - E_{t+1}[\sigma_{m,t}e_{t+1}])(\omega E_{t+1}[x^a_{t+1}]\\
&\quad + E_{t+1}[v_{t+1}] + (1-\omega)x^a_L - E_{t+1}[\sigma_{m,t+1}]\rho\sigma_x)]\\
&= E_t[\frac{1}{R_f}m_{t+1}(\omega x^a_{t+1} + v_{t+1} + (1-\omega)x^a_L - \sigma_{m,t+1}\rho\sigma_x)]
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{R_f} E_t[\omega z_{t+1} + m_{t+1}(v_{t+1} + (1-\omega)x_L^a - \sigma_{m,t+1}\rho\sigma_x)] \\
&= \frac{1}{R_f}\omega E_t[z_{t+1}] + \frac{1}{R_f^2}(E_t[v_{t+1}] + (1-\omega)x_L^a - \sigma_{m,t}\rho\sigma_x) \\
&= \frac{1}{R_f}\omega E_t[z_{t+1}] + \frac{1}{R_f^2}(\gamma v_t + (1-\omega)x_L^a - \sigma_{m,t}\rho\sigma_x) \\
&= \frac{1}{R_f^2}(\omega(\omega E_t[x_t^a] + E_t[v_t] + (1-\omega)x_L^a - \sigma_{m,t}\rho\sigma_x) + \gamma v_t + (1-\omega)x_L^a - \sigma_{m,t}\rho\sigma_x) \\
&= \frac{1}{R_f^2}(\omega^2 x_t^a + (\omega+\gamma)v_t + (\omega+1)(1-\omega)x_L^a - (\omega+1)\sigma_{m,t}\rho\sigma_x).
\end{aligned}
$$

Again, using the law of iterated expectations, we can write:

$$E_t[z_{t+3}] = E_t[E_{t+1}[E_{t+2}[z_{t+3}]]].$$

We solve the expectation by moving from right to left:

$$
\begin{aligned}
E_{t+2}[z_{t+3}] &= E_{t+2}[\frac{1}{R_f} m_{t+1}m_{t+2}(\omega x_{t+2}^a + v_{t+2} + (1-\omega)x_L^a - \sigma_{m,t+2}\rho\sigma_x)] \\
&= \frac{1}{R_f} E_{t+2}[m_{t+1}m_{t+2}\omega x_{t+2}^a] \\
&\quad + \frac{1}{R_f} E_{t+2}[m_{t+1}m_{t+2}(v_{t+2} + (1-\omega)x_L^a - \sigma_{m,t+2}\rho\sigma_x)].
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
E_{t+1}[E_{t+2}[z_{t+3}]] &= \frac{1}{R_f} E_{t+1}[m_{t+1}m_{t+2}\omega x_{t+2}^a] \\
&\quad + \frac{1}{R_f^2} E_{t+1}[m_{t+1}(\gamma v_{t+1} + (1-\omega)x_L^a - \sigma_{m,t+1}\rho\sigma_x)] \\
&= \frac{1}{R_f^2} E_{t+1}[m_{t+1}\omega(\omega x_{t+1}^a + v_{t+1} + (1-\omega)x_L^a - \sigma_{m,t+1}\rho\sigma_x)] \\
&\quad + \frac{1}{R_f^2} E_{t+1}[m_{t+1}(\gamma v_{t+1} + (1-\omega)x_L^a - \sigma_{m,t+1}\rho\sigma_x)].
\end{aligned}
$$

Finally,

$$
\begin{aligned}
E_t[E_{t+1}[E_{t+2}[z_{t+3}]]] &= \frac{1}{R_f^2}\omega^2 E_t[m_{t+1}x_{t+1}^a] + \frac{1}{R_f^2}\omega E_t[m_{t+1}(v_{t+1} + (1-\omega)x_L^a \\
&\quad - \sigma_{m,t+1}\rho\sigma_x)] + \frac{1}{R_f^2} E_t[m_{t+1}(\gamma v_{t+1} + (1-\omega)x_L^a - \sigma_{m,t+1}\rho\sigma_x)] \\
&= \frac{1}{R_f^2}\omega^2 E_t[m_{t+1}x_{t+1}^a] + \frac{1}{R_f^3}\omega E_t[v_{t+1} + (1-\omega)x_L^a - \sigma_{m,t+1}\rho\sigma_x] \\
&\quad + \frac{1}{R_f^3} E_t[\gamma v_{t+1} + (1-\omega)x_L^a - \sigma_{m,t+1}\rho\sigma_x] \\
&= \frac{1}{R_f^3}\omega^2(\omega E_t[x_t^a] + E_t[v_t] + (1-\omega)x_L^a - \sigma_{m,t}\rho\sigma_x)
\end{aligned}
$$

$$+\frac{1}{R_f^3}\omega(\gamma v_t+(1-\omega)x_L^a-\sigma_{m,t}\rho\sigma_x)$$
$$+\frac{1}{R_f^3}(\gamma^2 v_t+(1-\omega)x_L^a-\sigma_{m,t}\rho\sigma_x).$$

Thus,

$$\begin{aligned}E_t[z_{t+3}]&=\frac{1}{R_f^3}[(\omega^3x_t^a+\omega^2v_t+\omega^2(1-\omega)x_L^a-\omega^2\sigma_{m,t}\rho\sigma_x)\\&\quad+\omega\gamma v_t+\omega(1-\omega)x_L^a-\omega\sigma_{m,t}\rho\sigma_x\\&\quad+\gamma^2v_t+(1-\omega)x_L^a-\sigma_{m,t}\rho\sigma_x]\\&=\frac{1}{R_f^3}[\omega^3x_t^a+(\omega^2+\omega\gamma+\gamma^2)v_t+(\omega^2+\omega+1)(1-\omega)x_L^a\\&\quad-(\omega^2+\omega+1)\sigma_{m,t}\rho\sigma_x]\\&=\frac{1}{R_f^3}\left[\omega^3x_t^a+\frac{\omega^3-\gamma^3}{\omega-\gamma}v_t+\frac{\omega^3-1^3}{\omega-1}(1-\omega)x_L^a-\frac{\omega^3-1^3}{\omega-1}\sigma_{m,t}\rho\sigma_x\right].\end{aligned}$$

The pattern emerges, and we see that

$$E_t[z_{t+i}]=\frac{1}{R_f^i}\left[\omega^ix_t^a+\frac{\omega^i-\gamma^i}{\omega-\gamma}v_t+\frac{\omega^i-1}{\omega-1}(1-\omega)x_L^a-\frac{\omega^i-1}{\omega-1}\sigma_{m,t}\rho\sigma_x\right].$$

Now evaluating the summation yields:

$$\begin{aligned}\sum_{i=1}^{\infty}E_t[z_{t+i}]&=\frac{R_f(1-\omega)}{(R_f-\omega)(R_f-1)}x_L^a+\frac{\omega}{R_f-\omega}x_t^a+\frac{R_f}{(R_f-\omega)(R_f-\gamma)}v_t\\&\quad-\frac{R_f\sigma_x\rho}{(R_f-\omega)(R_f-1)}\sigma_{m,t}.\end{aligned}$$

Thus the form of the stock price is:

$$\begin{aligned}S_t&=B_t+\alpha_1x_L^a+\alpha_2x_t^a+\alpha_3v_t-\lambda_1\sigma_{m,t},\\\alpha_1&=\frac{R_f(1-\omega)}{(R_f-\omega)(R_f-1)},\\\alpha_2&=\frac{\omega}{R_f-\omega},\\\alpha_3&=\frac{R_f}{(R_f-\omega)(R_f-\gamma)},\\\lambda_1&=\frac{R_f\sigma_x\rho}{(R_f-\omega)(R_f-1)},\end{aligned}$$

which is Eq. (6) of Proposition 1.

To prove Eq. (7) of Proposition 1, substitute out "other information" as follows:

$$x_{t+1}^a = \omega x_t^a + (1-\omega)x_L^a + v_t + \epsilon_{t+1},$$
$$E_t[x_{t+1}^a] = \omega x_t^a + (1-\omega)x_L^a + v_t.$$

This implies that

$$v_t = E_t[x_{t+1}^a] - \omega x_t^a - (1-\omega)x_L^a.$$

Substituting the latter into the price equation, we can now rewrite price in terms of accounting data and expected next period abnormal earnings. Specifically,

$$\begin{aligned}
S_t &= B_t + [\alpha_1 - \alpha_3(1-\omega)]x_L^a + (\alpha_2 - \omega\alpha_3)x_t^a + \alpha_3 E_t[x_{t+1}^a] - \lambda_1\sigma_{m,t} \\
&= B_t + R_f(1-\omega)\left(\frac{1}{(R_f-\omega)(R_f-1)} - \frac{1}{(R_f-\omega)(R_f-\gamma)}\right)x_L^a \\
&\quad + \omega\left(\frac{1}{R_f-\omega} - \frac{R_f}{(R_f-\omega)(R_f-\gamma)}\right)x_t^a + \alpha_3 E_t[x_{t+1}^a] - \lambda_1\sigma_{m,t} \\
&= B_t + \left(\frac{R_f(1-\omega)(1-\gamma)}{(R_f-\omega)(R_f-\gamma)(R_f-1)}\right)x_L^a \\
&\quad - \left(\frac{\gamma\omega}{(R_f-\omega)(R_f-\gamma)}\right)x_t^a + \alpha_3 E_t[x_{t+1}^a] - \lambda_1\sigma_{m,t}.
\end{aligned}$$

Noting that

$$E_t[x_{t+1}^a] = E_t[x_{t+1} - (R_f-1)B_t],$$

and substituting, yields Eq. (7):

$$\begin{aligned}
S_t &= B_t + \gamma_1 x_L^a - (R_f-1)\alpha_3 B_t - \left(\frac{\gamma\omega}{(R_f-\omega)(R_f-\gamma)}\right)x_t^a + \alpha_3 E_t[x_{t+1}] - \lambda_1\sigma_{m,t}. \\
&= \gamma_1 x_L^a + \left(\frac{R_f(1-\omega-\gamma)+\omega\gamma}{(R_f-\omega)(R_f-\gamma)}\right)B_t - \left(\frac{\gamma\omega}{(R_f-\omega)(R_f-\gamma)}\right)x_t^a + \alpha_3 E_t[x_{t+1}] - \lambda_1\sigma_{m,t}. \\
&= \gamma_1 x_L^a + g_2 B_t - \left(\frac{\gamma\omega}{(R_f-\omega)(R_f-\gamma)}\right)(x_t - (R_f-1)B_{t-1}) + \alpha_3 E_t[x_{t+1}] - \lambda_1\sigma_{m,t}. \\
&= \gamma_1 x_L^a + g_2 B_t + g_3 x_t - (R_f-1)g_3 B_{t-1} + \alpha_3 E_t[x_{t+1}] - \lambda_1\sigma_{m,t}. \\
&= \gamma_1 x_L^a + g_2 B_t + g_3 x_t - (R_f-1)g_3(B_t - x_t + D_t) + \alpha_3 E_t[x_{t+1}] - \lambda_1\sigma_{m,t}. \\
&= \gamma_1 x_L^a + [g_2 - (R_f-1)g_3]B_t + [g_3 + (R_f-1)g_3]x_t - (R_f-1)g_3 D_t + \alpha_3 E_t[x_{t+1}] - \lambda_1\sigma_{m,t}. \\
&= \gamma_1 x_L^a + \frac{R_f(1-\omega)(1-\gamma)}{(R_f-\omega)(R_f-\gamma)}B_t - \frac{R_f\gamma\omega}{(R_f-\omega)(R_f-\gamma)}x_t + \frac{\omega\gamma(R_f-1)}{(R_f-\omega)(R_f-\gamma)}D_t + \alpha_3 E_t[x_{t+1}] - \lambda_1\sigma_{m,t}. \\
&= \gamma_1 x_L^a + \gamma_2 B_t + \gamma_3 x_t + \gamma_4 D_t + \gamma_5 E_t[x_{t+1}] - \lambda_1\sigma_{m,t}.
\end{aligned}$$

$$\gamma_1 = \frac{R_f(1-\omega)(1-\gamma)}{(R_f-\omega)(R_f-\gamma)(R_f-1)} \geq 0,$$
$$\gamma_2 = \frac{R_f(1-\omega)(1-\gamma)}{(R_f-\omega)(R_f-\gamma)} \geq 0,$$
$$\gamma_3 = -\frac{R_f\gamma\omega}{(R_f-\omega)(R_f-\gamma)} \leq 0,$$
$$\gamma_4 = \frac{\omega\gamma(R_f-1)}{(R_f-\omega)(R_f-\gamma)} \geq 0,$$
$$\gamma_5 = \frac{R_f}{(R_f-\omega)(R_f-\gamma)} > 0.$$

□

Proof of proposition 2

$$\begin{aligned} S_{t+1} &= B_{t+1} + \alpha_1 x_L^a + \alpha_2 x_{t+1}^a + \alpha_3 v_{t+1} - \lambda_1 \sigma_{m,t+1} \\ &= B_t + x_{t+1} - D_{t+1} + \alpha_1 x_L^a + \alpha_2 x_{t+1}^a + \alpha_3 v_{t+1} - \lambda_1 \sigma_{m,t+1} \\ &= B_t R_f + x_{t+1}^a + \alpha_1 x_L^a + \alpha_2 x_{t+1}^a + \alpha_3 v_{t+1} - \lambda_1 \sigma_{m,t+1} - D_{t+1} \\ &= R_f B_t + \alpha_1 x_L^a + (1+\alpha_2) x_{t+1}^a + \alpha_3 v_{t+1} - \lambda_1 \sigma_{m,t+1} - D_{t+1}. \end{aligned}$$

Therefore,

$$S_{t+1} + D_{t+1} = R_f B_t + \alpha_1 x_L^a + (1+\alpha_2) x_{t+1}^a + \alpha_3 v_{t+1} - \lambda_1 \sigma_{m,t+1}.$$

Now subtracting $R_f S_t$ from the latter equation gives:

$$\begin{aligned} S_{t+1} + D_{t+1} - R_f S_t &= \alpha_1 x_L^a + (1+\alpha_2) x_{t+1}^a + \alpha_3 v_{t+1} - \lambda_1 \sigma_{m,t+1} \\ &\quad - R_f(\alpha_1 x_L^a + \alpha_2 x_t^a + \alpha_3 v_t - \lambda_1 \sigma_{m,t}) \\ &= \alpha_1(1-R_f) x_L^a + (1+\alpha_2) x_{t+1}^a + \alpha_3 v_{t+1} - \lambda_1 \sigma_{m,t+1} \\ &\quad - R_f(\alpha_2 x_t^a + \alpha_3 v_t - \lambda_1 \sigma_{m,t}). \end{aligned}$$

and

$$\begin{aligned} x_{t+1}^a &= \omega x_t^a + (1-\omega) x_L^a + v_t + \epsilon_{t+1}, \\ (1+\alpha_2) x_{t+1}^a &= \omega(1+\alpha_2) x_t^a + (1+\alpha_2)(1-\omega) x_L^a + (1+\alpha_2) v_t + (1+\alpha_2)\epsilon_{t+1}, \\ \alpha_3 v_{t+1} &= \alpha_3 \gamma v_t + \alpha_3 u_{t+1}. \end{aligned}$$

Collecting like-terms yields:

$$\begin{aligned} S_{t+1} + D_{t+1} - R_f S_t &= \alpha_1(1-R_f) x_L^a + x_t^a((1+\alpha_2)\omega - R_f \alpha_2) \\ &\quad + [\alpha_3(\gamma - R_f) + (1+\alpha_2)] v_t - \lambda_1(1-R_f)\sigma_{m,t} \\ &\quad + (1+\alpha_2)(1-\omega) x_L^a + (1+\alpha_2)\epsilon_{t+1} + \alpha_3 u_{t+1} - \lambda_1 \xi_{t+1}. \end{aligned}$$

$$(1+\alpha_2)\omega - R_f\alpha_2 = \frac{R_f}{R_f-\omega}\omega - R_f\frac{\omega}{R_f-\omega} = 0,$$

$$\alpha_3(\gamma - R_f) + (1+\alpha_2) = \frac{-R_f}{R_f-\omega} + \frac{R_f}{R_f-\omega} = 0,$$

$$\alpha_1(1-R_f)x_L^a = -\frac{R_f(1-\omega)}{(R_f-\omega)}x_L^a = (1+\alpha_2)(1-\omega)x_L^a.$$

So,

$$S_{t+1} + D_{t+1} - R_f S_t = -\lambda_1(1-R_f)\sigma_{m,t} + (1+\alpha_2)\epsilon_{t+1} + \alpha_3 u_{t+1} - \lambda_1\xi_{t+1}.$$

Thus,

$$S_{t+1} + D_{t+1} - R_f S_t = \lambda_1(R_f-1)\sigma_{m,t} + (1+\alpha_2)\epsilon_{t+1} + \alpha_3 u_{t+1} - \lambda_1\Delta\sigma_{m,t}.$$

The gross stock return is then given by:

$$\frac{S_{t+1}+D_{t+1}}{S_t} = R_f + (R_f-1)\lambda_1\frac{\sigma_{m,t}}{S_t} + (1+\alpha_2)\frac{\epsilon_{t+1}}{S_t} + \alpha_3\frac{u_{t+1}}{S_t} - \lambda_1\frac{\Delta\sigma_{m,t}}{S_t}.$$

Taking conditional expectations provides the expected stock return:

$$E_t[\frac{S_{t+1}+D_{t+1}}{S_t}] = R_f + (R_f-1)\lambda_1\frac{\sigma_{m,t}}{S_t}.$$

□

Proof of proposition 3

From the preceding proof, we have

$$\mu_{t+1} = R_f + (R_f-1)\lambda_1\frac{\sigma_{m,t}}{S_t},$$

$$\sigma_{m,t}\lambda_1 = \frac{1}{R_f-1}(\mu_{t+1}-R_f)S_t.$$

Substitute the latter into the value equation yields:

$$S_t = B_t + \alpha_1 + \alpha_2 x_t^a + \alpha_3 v_t - \frac{1}{R_f-1}(\mu_{t+1}-R_f)S_t,$$

$$S_t(1+\frac{1}{R_f-1}(\mu_{t+1}-R_f)) = B_t + \alpha_1 + \alpha_2 x_t^a + \alpha_3 v_t,$$

$$\mu_{t+1} = (R_f-1)(\frac{B_t}{S_t} + \frac{\alpha_1}{S_t} + \alpha_2\frac{x_t^a}{S_t} + \alpha_2\frac{v_t}{S_t} - 1) + R_f,$$

$$\mu_{t+1} = R_f + (R_f - 1)(\frac{B_t}{S_t} + \frac{\alpha_1}{S_t} + \alpha_2 \frac{x_t^a}{S_t} + \alpha_2 \frac{v_t}{S_t} - 1)$$
$$= 1 + (R_f - 1)(\frac{B_t}{S_t} + \frac{\alpha_1}{S_t} + \alpha_2 \frac{x_t^a}{S_t} + \alpha_2 \frac{v_t}{S_t}).$$

To prove Eq. (11), we use (7) of Proposition 1. That is, we substitute "other information," v_t, with next period expected earnings as outlined in the proof of Proposition 1. Combining this with the results in Proposition 2 yields the (net) return dynamics:

$$R_{t+1} - 1 = (R_f - 1)\left(\gamma_1 \frac{x_L^a}{S_t} + \gamma_2 \frac{B_t}{S_t} + \gamma_3 \frac{x_t}{S_t} + \gamma_4 \frac{D_t}{S_t} + \gamma_5 \frac{E_t[x_{t+1}]}{S_t}\right) + (1 + \alpha_2)\frac{\epsilon_{t+1}}{S_t} + \alpha_3 \frac{u_{t+1}}{S_t} - \lambda_1 \frac{\Delta\sigma_{m,t}}{S_t}.$$

Taking conditional expectations of both sides yields the desired result. □

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